A Novel Approach to Teaching Phased Array Antenna Systems

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***Abstract*— We describe a simple, yet elegant method to provide a full sensory experience that describes the operation of a phased array antenna system. Our method is based on using an audio source that feeds an array of speakers appropriately spaced apart such that when a listener walks around the array, they experience the null points as well as the primary lobe and minor lobes. It is extremely profound for the student to experience a relatively loud tone, and then when moving just a few inches perceives the complete absence of the tone. The apparatus is economical and relatively easy to implement and test as compared to an RF antenna phased array. The ideas presented are applicable to other courses in electrical and mechanical engineering that cover signal processing and acoustics. Student assessment has shown that the proposed method greatly enhances understanding of phased array systems.**

***Keywords:* phased arrays; antenna arrays; antenna education**

I. Introduction

Phased array antenna systems provide a practical method of achieving gain and directivity, particularly at the lower frequencies where ordinary directional antennas would not be practical [1]. Phased arrays are part of the larger field of adaptive signal and array processing and have a wide variety of applications in other areas such as beamforming, underwater acoustics, and biomedical ultrasound imaging. This topic is introduced in most undergraduate signal processing courses. The general theory of array processing is described in references [2] and [3] and the theory specific to antennas is found in references [1], [4], and [5]. Acoustical phased arrays and beamforming are describe by Beranek [6]. Ferren [7] implements a phased array using speakers.

While it is important to cover phased arrays in an undergraduate antennas course, it is however, a challenge to build and measure the pattern for practical RF systems. It requires at least an antenna range that is free of obstructions, which may not be readily available or economical for urban environments. Furthermore, there may be regulatory issues associated with RF antennas (i.e. interference to other radio services, etc.). Hence instructors resort to computer simulations to illustrate this concept. While simulations provide design flexibility as well as accurate and precise characterizations of array patterns, there is strong evidence that they may not necessarily give the student the necessary insight and understanding of phased arrays and other beam forming concepts.

This paper proposes the use of an acoustic system as an alternative to using an RF antenna array to explain a phased array system. The core of our concept is to use a configuration of *n*-speakers separated by some wavelength or fraction thereof and then measure the intensity pattern as a function of angle. Instead of an RF signal radiating from a set of antennas, an audio signal radiates from a set of speakers. Note we achieve a physically realizable array at audio frequencies since one wavelength at a 1 kHz audio frequency is roughly 0.3 meters. Note also that the effective spacing (relative to fraction of wavelength) between radiators is easily changed by simply varying the audio frequency. Concepts learned are readily extrapolated to an antenna system. The method can also be used to illustrate beam forming and other adaptive filtering applications. Especially noteworthy is that as the student walks around the array and records the sound intensities, they undergo a full sensory experience of the null points, minor lobes and maximum response at the center of the array. Even more profound is that when they change position by inches, they sound level goes from relatively loud to complete silence.

II. Beamforming Equation Development

Our array is shown below in Figures 1 and 2. Figure 1 shows a phased array of eight transmit antennas, or in this case, speakers, and Figure 2 shows a magnified view of the first 3 elements. Note in this particular illustration, the antennas are separated by an acoustical distance of Δ and the angle of departure, θ is approximately 45 degrees. The speakers are fed in parallel by one single tone generator as shown in Figure 3. It should be noted that while the acoustical distance between speakers is Δ, because the speed of sound is

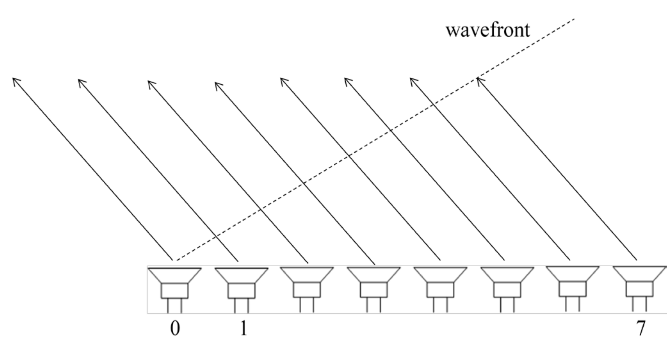

Fig. 1. An array of 8 speakers, separated by acoustical distance Δ, and radiating a tone at departure angle of θ degrees from the normal.

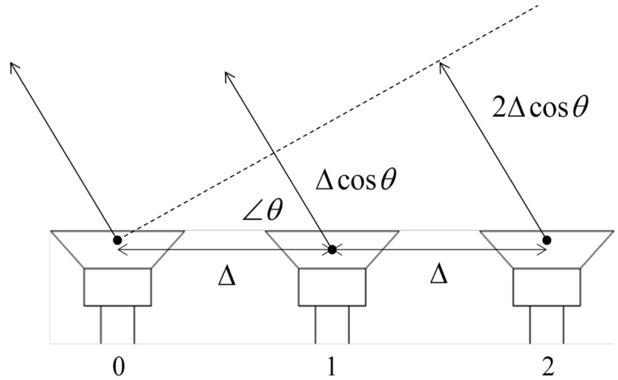

Fig. 2. Expanded view of speaker array from Figure 1, showing relative path length differences to the user of 0, $\Delta\cos\theta$, $2\Delta\cos\theta$,... for speakers #s 0, 1 and 2.

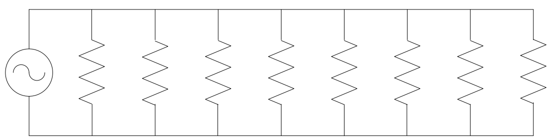

Fig. 3. Equivalent electrical circuit whereby the speakers are represented by resistors and fed by a single tone source.

orders of magnitude less than the speed of electrical current means the electrical separation between the speakers is essentially zero.

If an individual were standing "broadside" to the array (i.e. at angle of arrival of $\theta = 90^0$), and the transmitted signal was a single tone sinusoid, each speaker component would arrive at that point "in phase," because all propagation paths to the individual from all radiators are of equal lengths. If the individual shifts to the left as depicted in Figure 2, $\theta < 90^0$, and the signals no longer arrive in phase. In fact, we see from Figure 2 that the propagation paths vary in length by integer multiples of $\Delta\cos\theta$, and while the composite signal is still the arithmetic sum of sinusoids, those sinusoids are no longer in phase. Students learn that the composite signals strength at the user's position can then be represented as a sum of signal vectors (versus scalars).

Assuming the signal arriving at the user's position from antenna #0 has unity amplitude and zero phase, we can express that signal as

$$x(t) = \cos 2\pi f_0 t\,. \tag{1a}$$

The signal arriving at the second antenna (antenna #1) would be

$$x(t) = \cos(2\pi f_0 t - \phi), \tag{1b}$$

the phase delay of ϕ due to the added distance of $\Delta\cos\theta$. We readily see that at the 8[th] antenna, the received signal would be

$$x(t) = \cos(2\pi f_0 t - 7\phi). \tag{1c}$$

In terms of phases, we can express the vector sum of the antenna outputs as

$$S = 1 + e^{-j\phi} + e^{-j2\phi} + \ldots \qquad + e^{-j7\phi}. \tag{2}$$

Instead of expressing the received signals in terms of phase, let's express the phase delay of ϕ in terms of the relative traveling distances. It can be shown that $\phi = 2\pi\Delta\cos\theta$. giving

$$S = 1 + e^{-j2\pi\Delta\cos\theta} + e^{-j2\pi\times 2\Delta\cos\theta} + \ldots.. + e^{-j2\pi\times 7\Delta\cos\theta} \tag{3}$$

Before we look at the more general case where we can point the antennas in various directions, let's take the simplest case first where the antenna is pointed directly toward the source. In this case, the angle $\theta = 90^0$ or $\pi/2$ radians, and therefore, $\cos\theta = 0$. Thus, regardless of antenna spacing or wavelengths, the exponential terms in Eq. 3 will be 0, and thus $S = 1 + e^{-j0} \ldots. = 8$. This is something we would intuitively expect, that is eight antennas should give us 8X the received power as compared to 1 antenna. Let's now consider what happens when the antenna is not pointed directly toward the source. Do we still get more received power with 8 antennas as compared with only a single antenna, and could there be some angles where everything gets canceled out such that $|S| \cong 0$? Let's do some additional analysis on Eq. (3) to determine an analytical expression for $|S|$ versus arrival angle θ. We re-write Eq. (3) more compactly as

$$S = \sum_{n=0}^{7} e^{-j2\pi n\Delta\cos\theta} = \sum_{n=0}^{7} \left(e^{-j2\pi\Delta\cos\theta}\right)^n \tag{4}$$

Using the identity of $\sum_{n=0}^{N} a^n = \dfrac{1-a^{N+1}}{1-a}$ Eq. (4) is expressed as

$$S = e^{-j7\pi\Delta\cos\theta}\,\frac{\sin\left(8\pi\Delta\cos\theta\right)}{\sin\left(\pi\Delta\cos\theta\right)} \Rightarrow |S| = \frac{\sin\left(8\pi\Delta\cos\theta\right)}{\sin\left(\pi\Delta\cos\theta\right)} \tag{5}$$

As a check to verify the validity of our derivation, let's see if in the case of $\theta = \pi/2$ Eq. (5) gives us the same answer as Eq. (3). With $\lim_{x\to 0} \dfrac{\sin Nx}{\sin x} = N$ and substituting $\theta = \pi/2$ into Eq. (5) we get $|S| = 8$. Note that Δ is some fraction and/or multiple of a wavelength. In other words, $\Delta = k\lambda$ where typically the array spacing is λ, $\lambda/4$ or $\lambda/2$ giving us $\Delta = 1$, $1/4$ or $1/2$.

III. Experimental Procedure and Results

We now consider a polar plot for the theoretical and measured pattern of an array. Our experimental setup consists of an array of eight speakers in parallel that are driven by a single tone audio source. They are spaced approximately $\lambda/2$ which was about one foot and then fed by a single tone signal at 600 Hz. The distance between the center of the speaker array and the sound meter is 30 feet. The speaker array is shown in Fig. 4. Precise sound intensity measurements were made using a sound meter, and rotating around the array at a constant 30 foot distance which was maintained by a piece of string attached between the array center and the sound meter. Hearing the peaks and nulls however, provided a much more qualitative and thus useful measurement. The azimuth angle was based on calculating the $\sin^{-1}(d/30)$, where d is the normal distance from the sound meter (i.e. measured point) to the axis of the speaker array. The intensity pattern as a function of angle is shown in Fig. 5. Note we had to subtract out the ambient noise. Fig. 5 plots both the theoretical (solid line) and measured beam pattern. The arrows pointing outward indicate the *measured* peak locations, whereas the arrows pointing inward indicate the *measured* nulls. Considering the lack of precision in our instrumentation, the measured and theoretical peak and null locations were remarkably close. *Thus we achieved the main objective of the experiment which was show a beamforming pattern for a phased array.*

Fig. 4. Speaker array test setup. The yellow line is the speaker array axis.

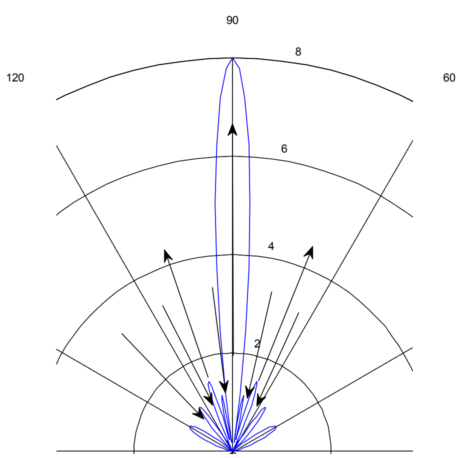

Fig. 5. Acoustic power as a function of azimuthangle, showing *theoretical* pattern (solid) and *measured* peaks (arrows out) and nulls (arrows in). Speaker array was driven by a 600Hz source and measurements were taken at a 30 foot radial distance.

To see how the pattern is affected by array element spacing, the effective spacing could be altered by simply varying the frequency. For example, by doubling the frequency, the effective spacing is changed from $\lambda/2$ to λ. This is not so easily done with RF applications since multiple changes in wavelength require drastic changes in the antenna's physical dimensions. This is particularly the case when dealing with HF or VHF systems. Testing was not without its challenges. These include the fact that in most environments the ambient sound level was relatively high, and that the speed of sound varies with air temperature, and pressure. While we could not obtain precise measurements that would accurately replicate the theoretical pattern, there was no question that as the listener rotated around the array, they would experience the peaks and nulls in the radiated sound intensity and thus better understand the pattern produced by a phased array.

IV. Qualitative Educational Assessment

For many students, the abstract nature of beam forming is difficult to understand. Preliminary assessment in our Electronic Navigation course (where students learn about phased array "localizer" antenna systems) suggests to us that those students who have experienced this acoustic beam forming experiment have better intuition, and have a more solid understanding of the concept of vector signal addition that is so fundamental to understanding phased arrays.

Based on the success thus far, this experiment will be integrated into the laboratory portion of the antennas course at the U.S. Coast Guard Academy where additional assessment will be done. In fact, the increased understanding of phased array concepts being able to function as localizers have impacted the content of the senior capstone design projects.

V. Conclusions and Future Work

By using this simple apparatus, our students were able to obtain a full sensory experience of the beam forming concept. It was especially gratifying to see their reactions when the perceived sound volume went from extremely loud to almost complete silence with only inches of movement, thereby transforming abstract equations into a readily understandable sensory experience. The measurement process and results gave the students a practical appreciation of how phased array systems can greatly increase the effective radiated power (ERP) of a signal.

As a complement to a system of speakers, audio frequencies and the ear, a beam former/phased array system could be implemented using an array of quarter wave ground plane antennas driven by an RF source with the resulting RF intensity measured by a portable spectrum analyzer. In fact, such a system is currently being tested and yielding results similar to that shown Fig. 5. However, from a pedagogical perspective, the sensory results of this acoustical system is considerably more effective at conveying beam forming concepts than a comparable RF system.

Future work would include: (a) adding a band pass filter to the speaker system to minimize background noise, (b) reducing the number of speakers from eight to four in order to simplify the radiation pattern without compromising the learning objectives, and (c) adding variable delay elements to the system and allow for steering of the beam pattern.

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