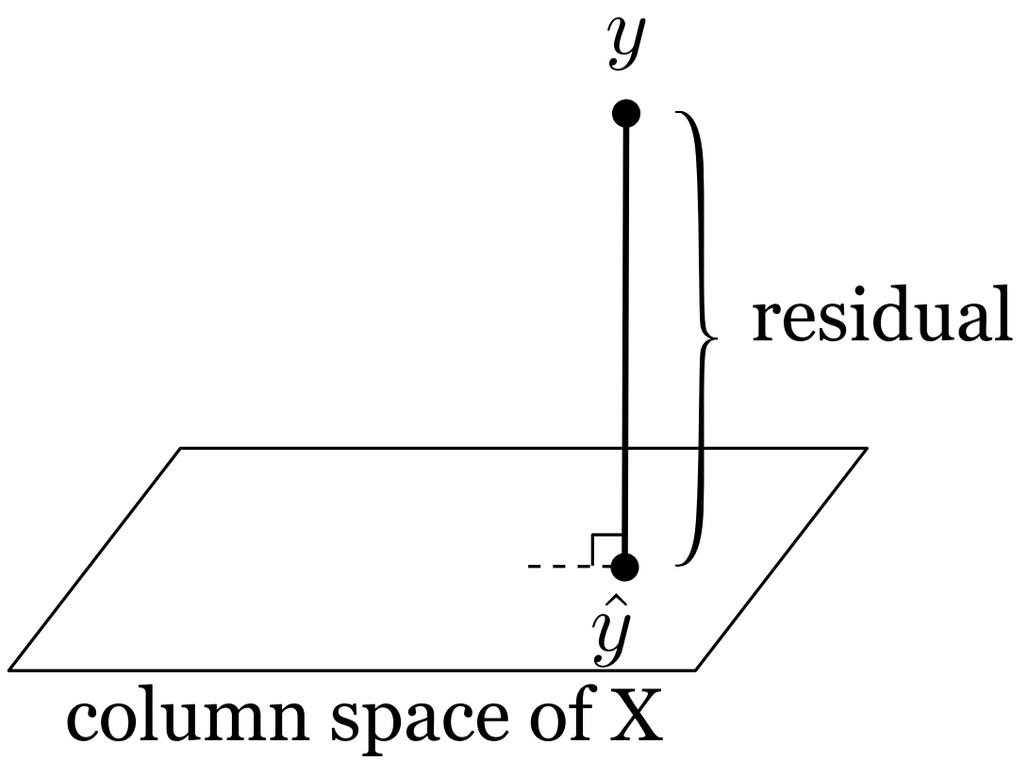
CS109/Stat121/AC209/E-109
Data Science
Bias and Regression

Hanspeter Pfister, Joe Blitzstein, and Verena Kaynig



This Week

- HW1 due tonight at11:59 pm (Eastern Time)
- HW2 posted soon

Census: Everybody's moving into their parents' basements

By Brad Plumer June 20, 2012 Follow @bradplumer

Ever since the financial crisis hit, Americans have found it harder and harder to live on their own. According to a new report (pdf) from the Census Bureau, the number of "shared households" increased by a whopping 2.25 million between 2007 and 2010:

Daniel Sherrett, 28, prepares dinner with his mother as part of his deal to live at home. Parents and children are sharing homes for longer than expected. (Michael Temchine/The Washington Post)

> In spring 2007, there were 19.7 million shared households. By spring 2010, the number of shared households had increased by 11.4 percent, while all households increased by only 1.3 percent.

This number does not include co-habitating or married couples. Rather, it's specifically a measure of the growing fraction of Americans who are either living with roommates or shacking up with relatives. And the bulk of the increase came from kids who are living at home with their parents: "Between 2007 and 2010, the number of adult children who resided in their parents'

Most Read Business

1 Here is everything we know about whether gentrification pushes poor people out 

2 Honey isn't as healthy as we think 

3 These are the hardest places for minimum wage workers to live 

4 Why Americans dress so casually 

5 Is your adviser truly protecting your retirement? 

The Most Popular All Over

The Atlantic
The Rise of Victimhood Culture 

Magazine

1657 COMMENTS

It's Official: The Boomerang Kids Won't Leave

By ADAM DAVIDSON JUNE 20, 2014

Census Data from the Current Population Survey (CPS)

"It is important to note that the CPS counts students living in dormitories as living in their parents' home."

– Census Bureau, http://www.census.gov/prod/2013pubs/p20-570.pdf

Some Forms of Bias

- selection bias
- publication bias (file drawer problem)
- non-response bias
- length bias

Where did the data come from? What don't we have?

What 'is ppl who don't answe are diff from ppl who did?

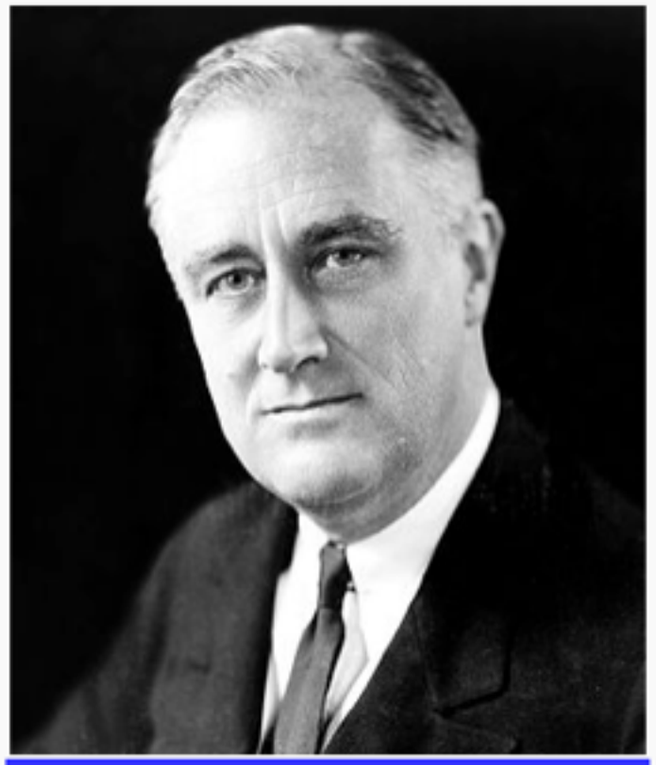
What can actually be replicated? Controversy over p-values. Only article w/ p-value < 0.05.

1936 Presidential Election, Landon vs. FDR

1932 ← **November 3, 1936** → 1940

531 electoral votes of the Electoral College

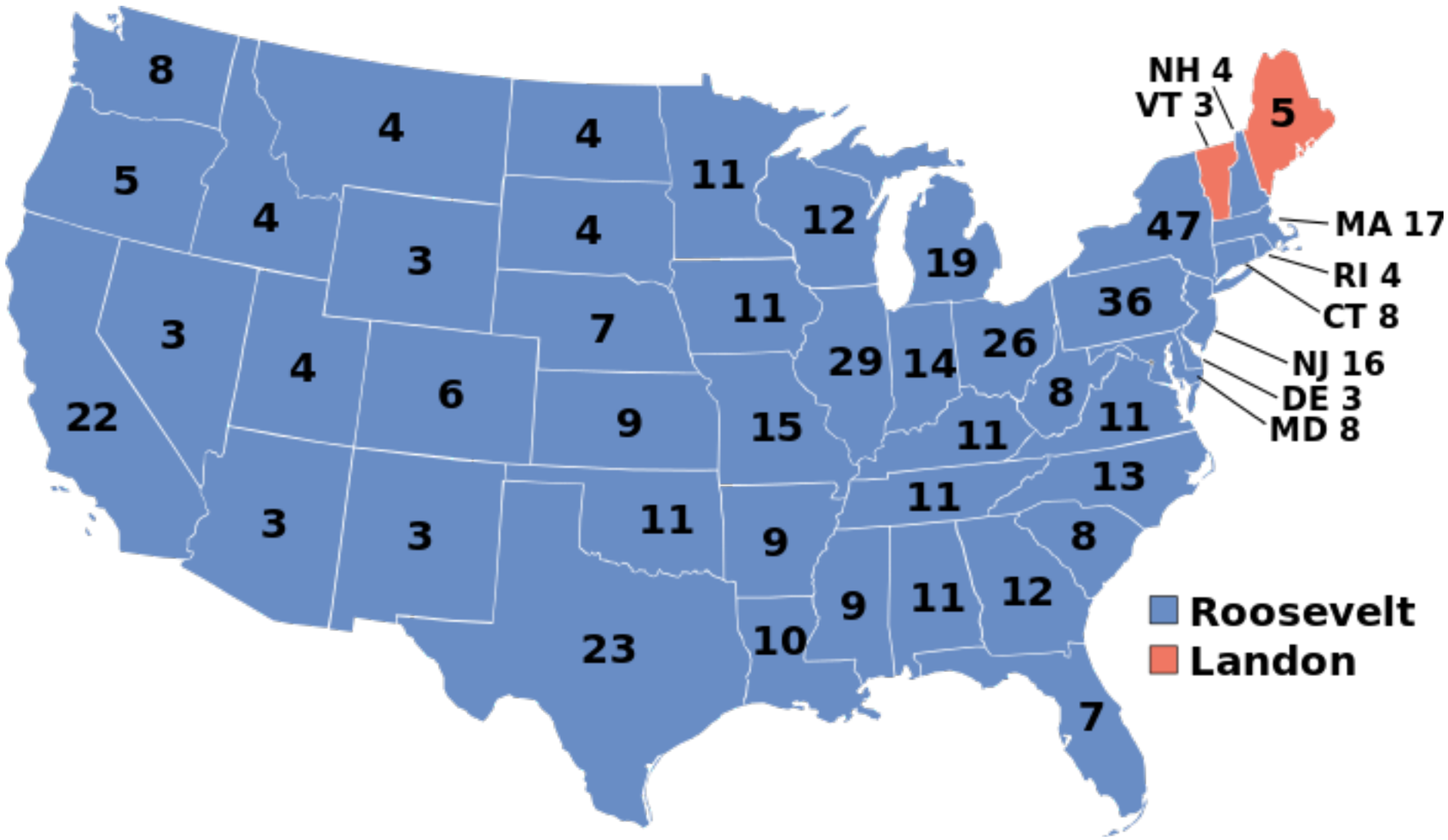
266 electoral votes needed to win



Nominee	**Franklin D. Roosevelt**	Alf Landon
Party	Democratic	Republican
Home state	New York	Kansas
Running mate	**John N. Garner**	Frank Knox

1936 Presidential Election, Landon vs. FDR

Literary Digest predicted Landon
would win with 370 electoral votes,
based on sample size of 2.4 million.



source: https://en.wikipedia.org/wiki/United_States_presidential_election,_1936

1936 Presidential Election, Landon vs. FDR

Literary Digest got responses from 2.3 million out of 10 million people surveyed.

Selection bias.

To collect their sample, they used 3 readily available lists:

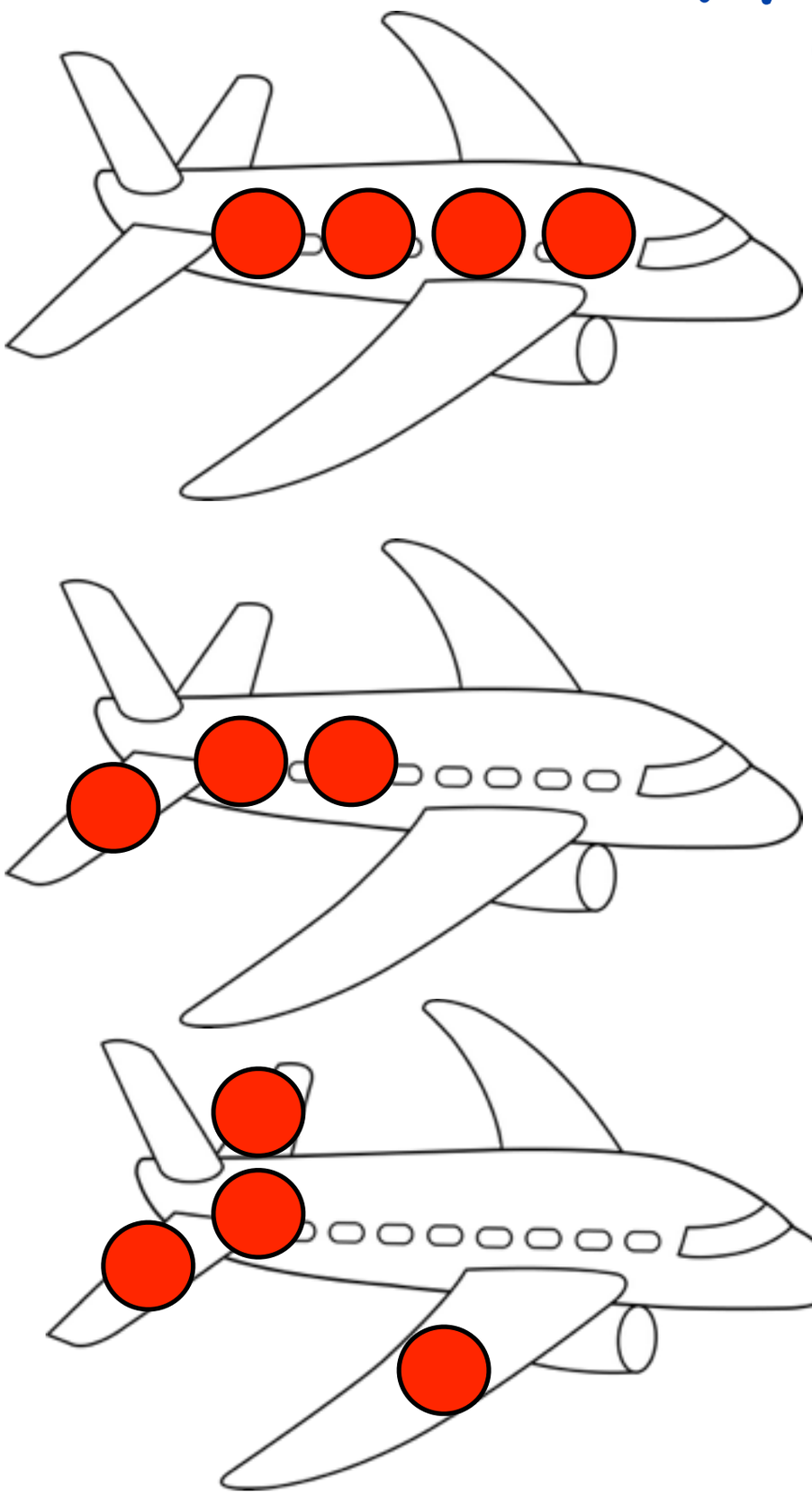
- readers of their magazine
- car registration list
- phone directory

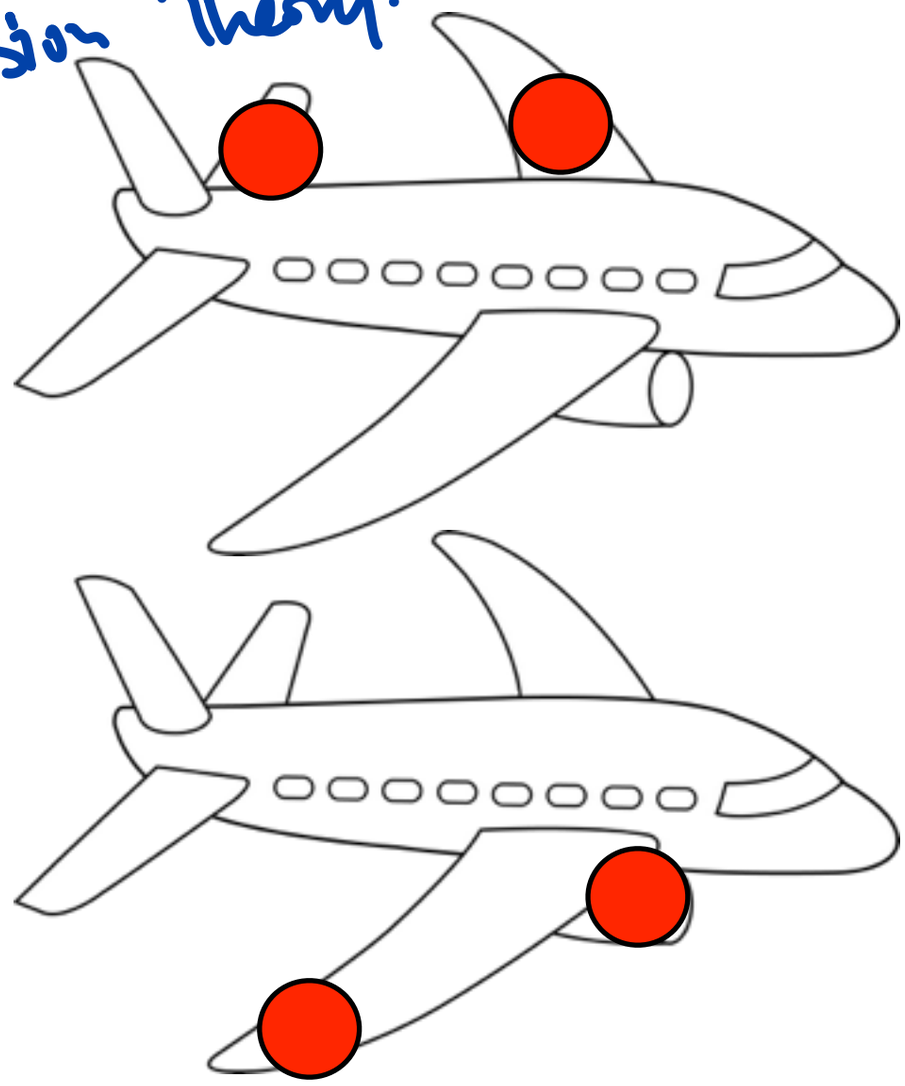
Not representative of all Americans.

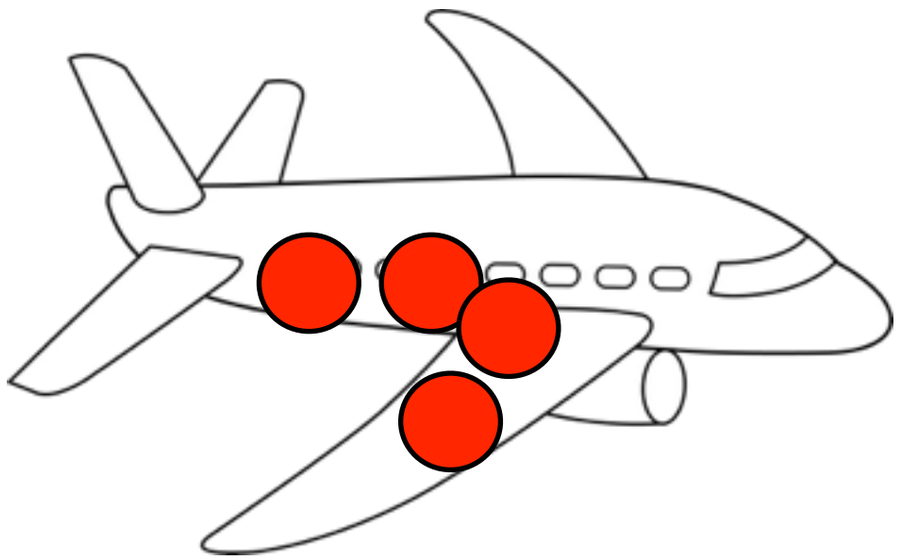
- Most ppl didn't own cars
- Not everyone had a car.

Wald and the Bullet Holes

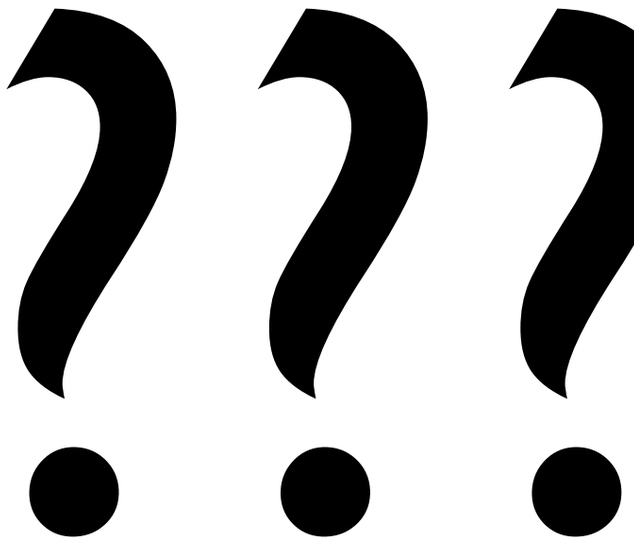
↳ Abraham Wald
↳ Decision Theory.







What about the *unobserved* planes? Missing data!



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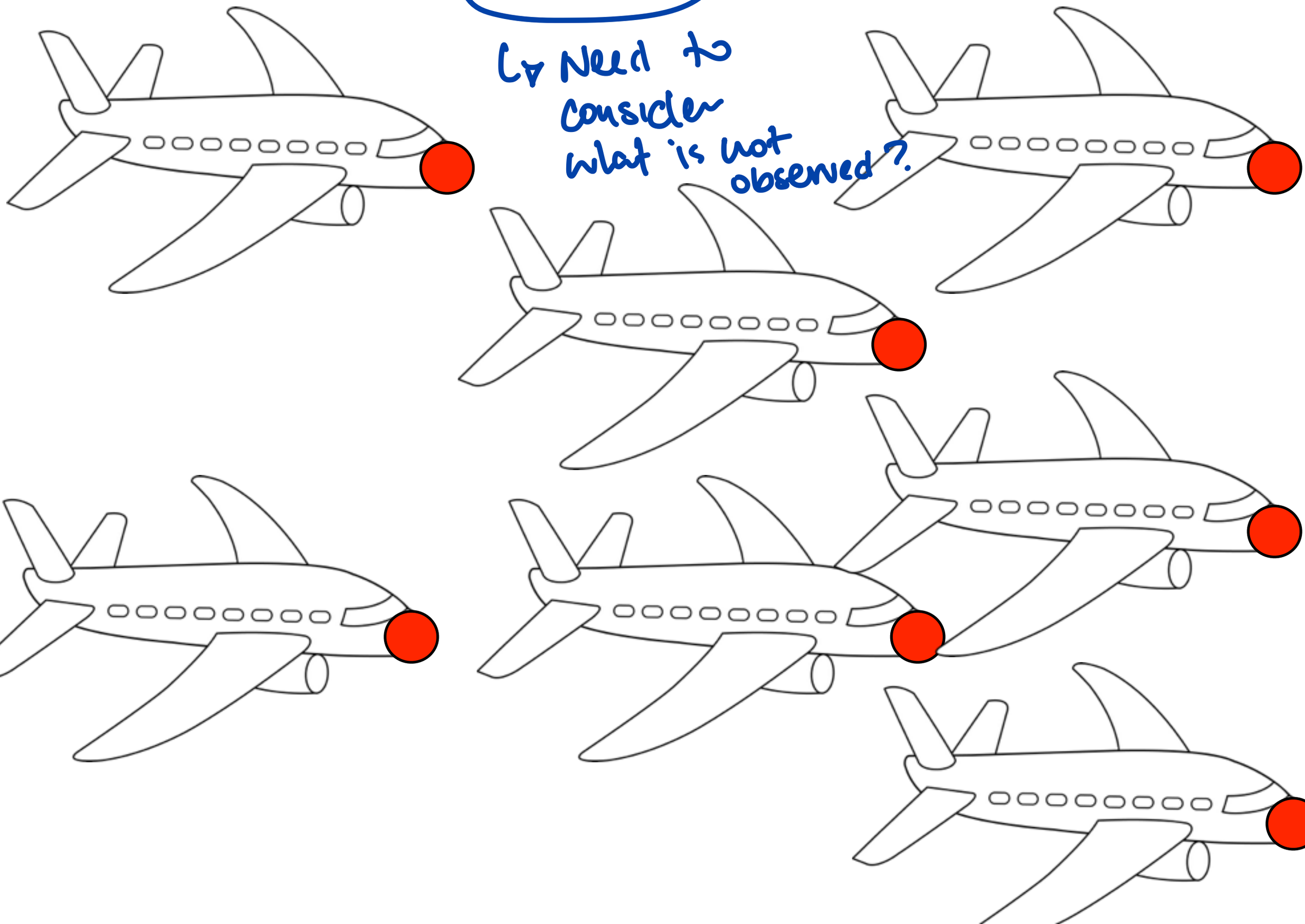


Longevity Study from Lombard (1835)

Profession	Average Longevity
chocolate maker	73.6
professors	66.6
clocksmiths	55.3
locksmiths	47.2
students	20.2

Need to think about this deep to avoid inaccurate conclusion.

Sources: Lombard (1835), Wainer (1999), Stigler (2002)

Class Size Paradox

Why do so many schools boast small average class size but then so many students end up in huge classes?

Use to start

Simple example: each student takes one course; suppose there is one course with 100 students, fifty courses with 2 students.

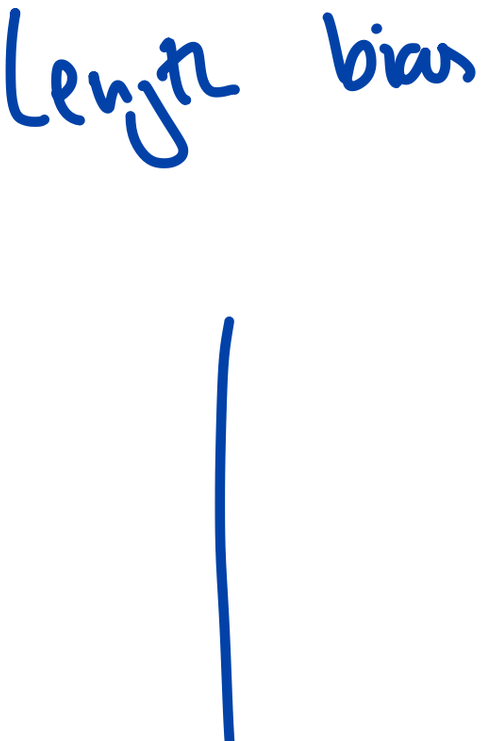
Dean calculates: (100+50*2)/51 = 3.92

Students calculate: (100*100+100*2)/200 = 51

Very diff experience based on vantage point.

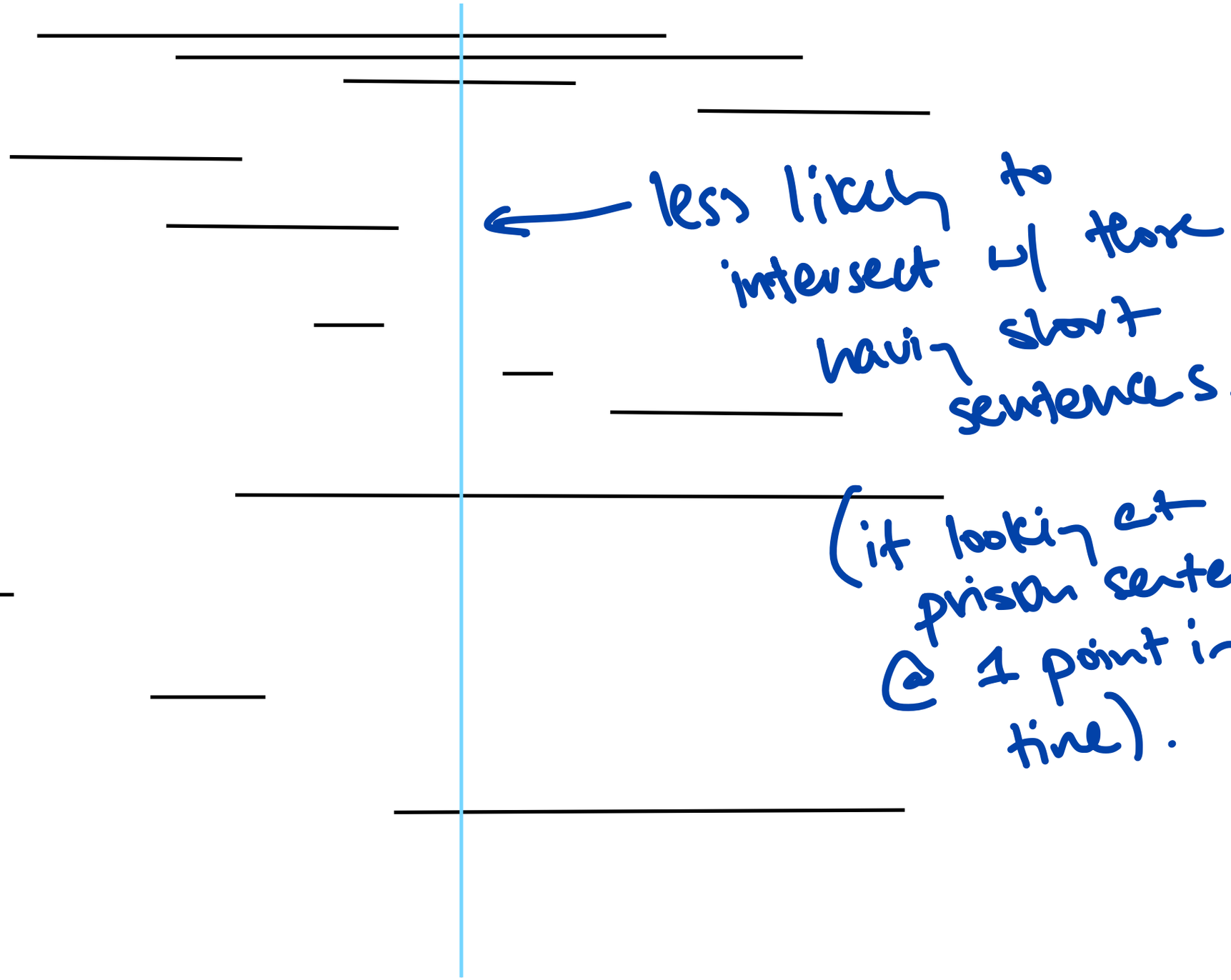
“About 10 percent of the 1.6 million inmates in America’s prisons are serving life sentences; another 11 percent are serving over 20 years.”

source: http://www.nytimes.com/2012/02/26/health/dealing-with-dementia-among-aging-criminals.html?pagewanted=all



Length-Biasing Paradox

How would you measure the average prison sentence?



(Stat definition)

Bias of an Estimator

The *bias* of an estimator is how far off it is on average:

$$\text{bias}(\hat{\theta}) = E(\hat{\theta}) - \theta$$

hat is estimator.

truth.

So why not just subtract off the bias?

(1) Don't know the bias → don't know θ

(2) Bias-variance tradeoff

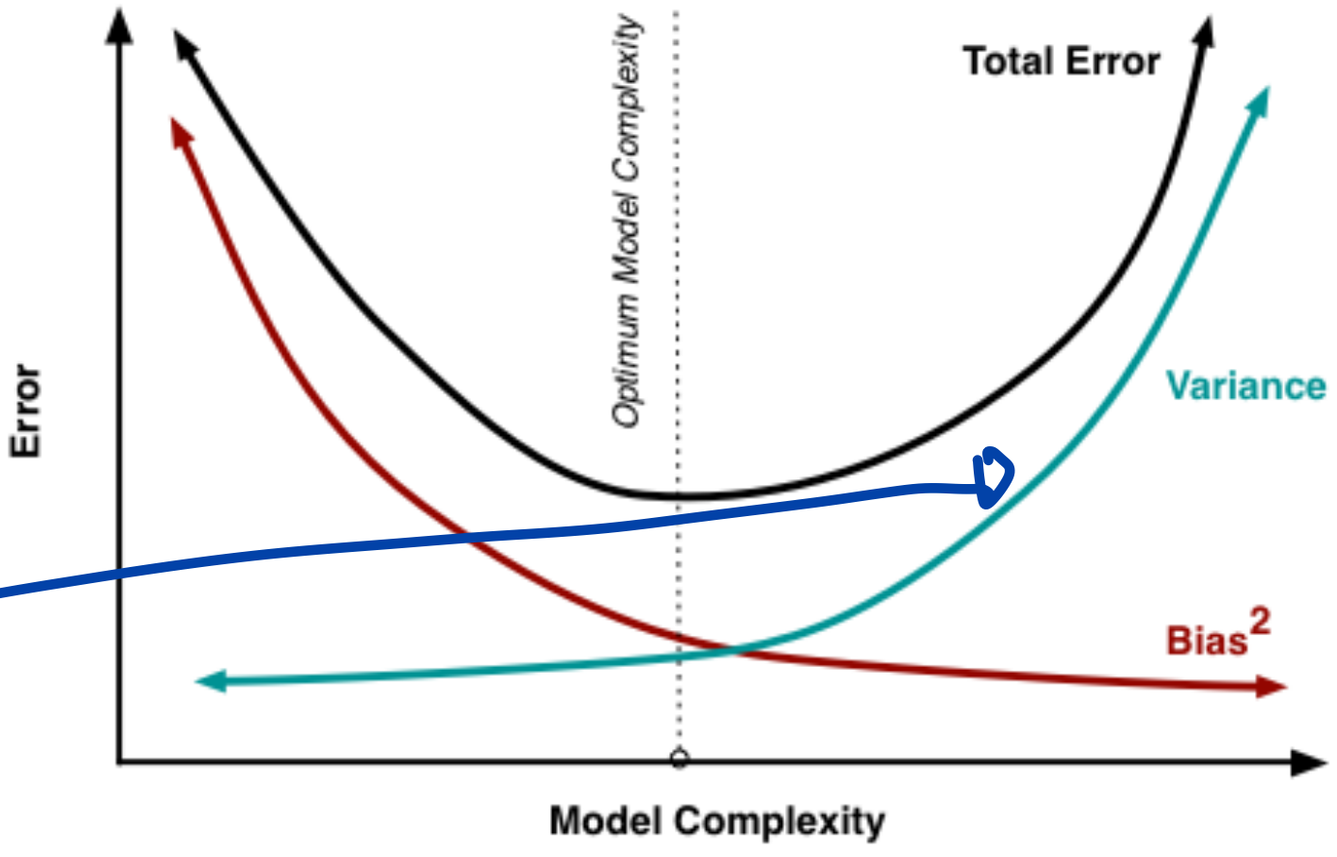
Bias-Variance Tradeoff

one form: $\mathrm{MSE}(\hat{\theta}) = \mathrm{Var}(\hat{\theta}) + \mathrm{bias}^2(\hat{\theta})$

(Most important in stats / ML).

often a little bit of bias can make it possible to have much lower MSE

mean squared estimator.



b/c estimating too many things @ once.

Goal: not min bias, but min MSE

Trade bias for variance.

http://scott.fortmann-roe.com/docs/BiasVariance.html

Unbiased Estimation: Poisson Example

$$X \sim \text{Pois}(\lambda)$$

rate

most widely used for count data.

Goal: estimate $e^{-2\lambda}$

$(-1)^X$ is the best (and only) unbiased estimator of $e^{-2\lambda}$

sensible?

↳ don't make sense,
but is unbiased. Don't use.

Fisher Weighting

How should we combine independent, *unbiased* estimators for a parameter into one estimator?

$$\hat{\theta} = \sum_{i=1}^{k} w_i \hat{\theta}_i$$

The *weights* should sum to one, but how should they be chosen?

$$w_i \propto \frac{1}{\mathrm{Var}(\hat{\theta}_i)}$$

(Inversely proportional to variance; why not SD?)

Nate Silver Weighting Method

Lcombine polls, demographics, historical data.

- Exponential decay based on recency of poll
- Sample size of poll (bigger better)
- Pollster rating

rate has been fine-tuned overtime

http://fivethirtyeight.com/features/how-the-fivethirtyeight-senate-forecast-model-works/

L Silver also uses an iterative approach.

New thing from Silver → how well did pollsters do in the past?
→ hard to do them for now, but not for past.

Multiple Testing, Bonferroni

How should we handle p-values when testing multiple hypotheses?

Issue in DS (fishing expeditions)

For example, what if we are looking at diet (with 10 kinds of food) and disease (with 10 diseases)?

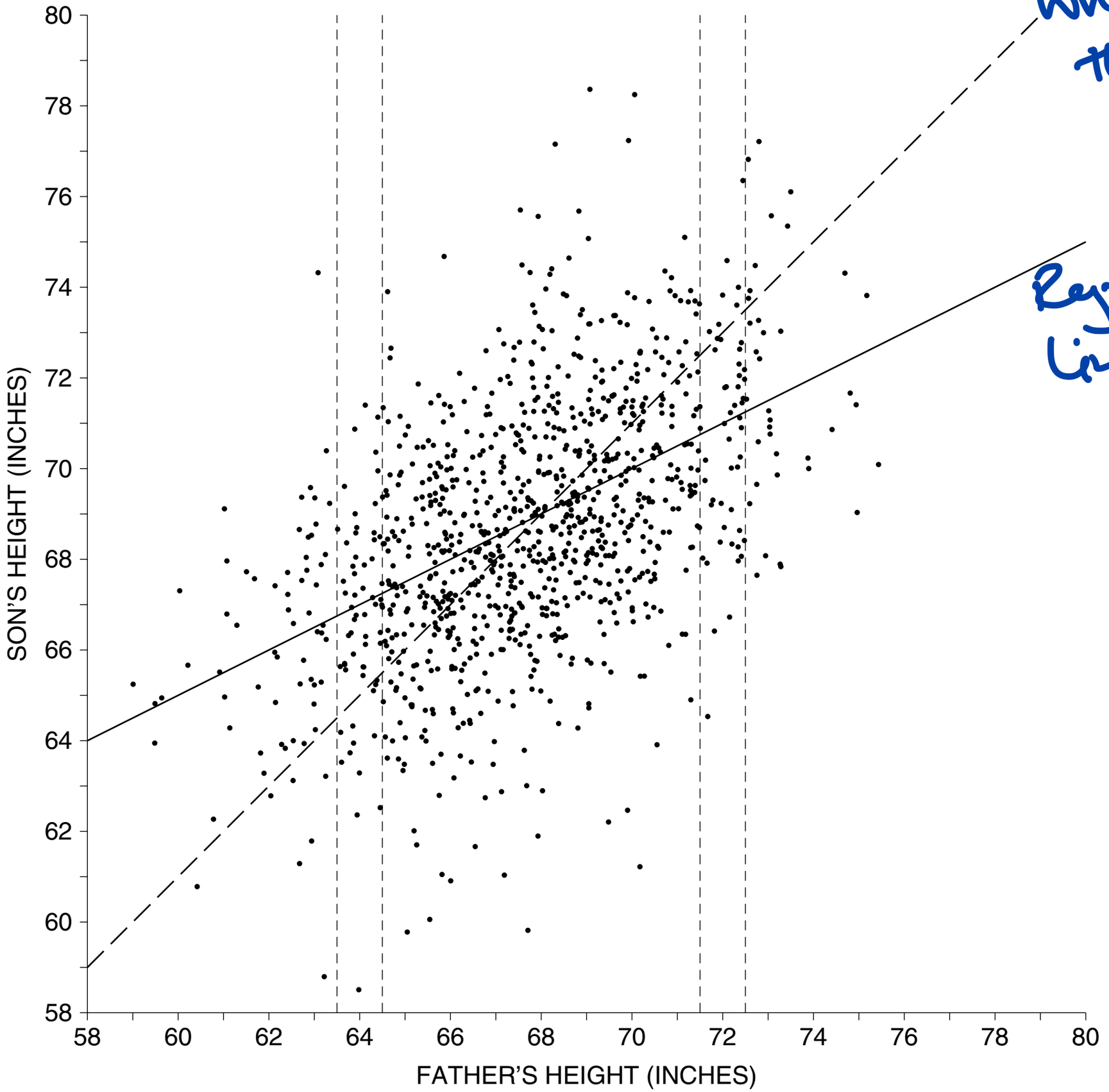
100 diff potential relationships; likely to find some low p-value.

A simple, conservative approach is Bonferroni: divide significance level by number of hypotheses being tested.

$$FWER = Pr\left\{\bigcup_{I_0}\left(p_i \leq \frac{\alpha}{m}\right)\right\} \leq \sum_{I_o}\left\{Pr\left(p_i \leq \frac{\alpha}{m}\right)\right\} \leq m_0\frac{\alpha}{m} \leq m\frac{\alpha}{m} = \alpha$$

CD family wise Error rate

https://en.wikipedia.org/wiki/Bonferroni_correction



plot from Freedman, data from Pearson-Lee

Regression Toward the Mean (RTTM)

Examples are everywhere...

Test scores
Sports
Inherited characteristics, e.g., heights
Traffic accidents at various sites

This is where π term came from.

Daniel Kahneman Quote on RTTM

I had the most satisfying Eureka experience of my career while attempting to teach flight instructors that praise is more effective than punishment for promoting skill-learning....

[A flight instructor objected:] "On many occasions I have praised flight cadets for clean execution of some aerobatic maneuver, and in general when they try it again, they do worse. On the other hand, I have often screamed at cadets for bad execution, and in general they do better the next time. So please don't tell us that reinforcement works and punishment does not..."

This was a joyous moment, in which I understood an important truth about the world: because we tend to reward others when they do well and punish them when they do badly, and because there is regression to the mean, it is part of the human condition that we are statistically punished for rewarding others and rewarded for punishing them.

Regression Paradox

mean = 0
Std = 1

RTTM

y: child's height (standardized)
x: parent's height (standardized)

Regression line: predict y = rx;
think of this as a weighted average of
the parent's height and the mean

Now, what about predicting the parent's height from
the child's height? Use x = y/r? Correct math, wrong statistically

Regression line is x = ry, the r stays the same!

"

y = xr

OLS = criteria
linear = model.

Linear Model

using x to predict y.

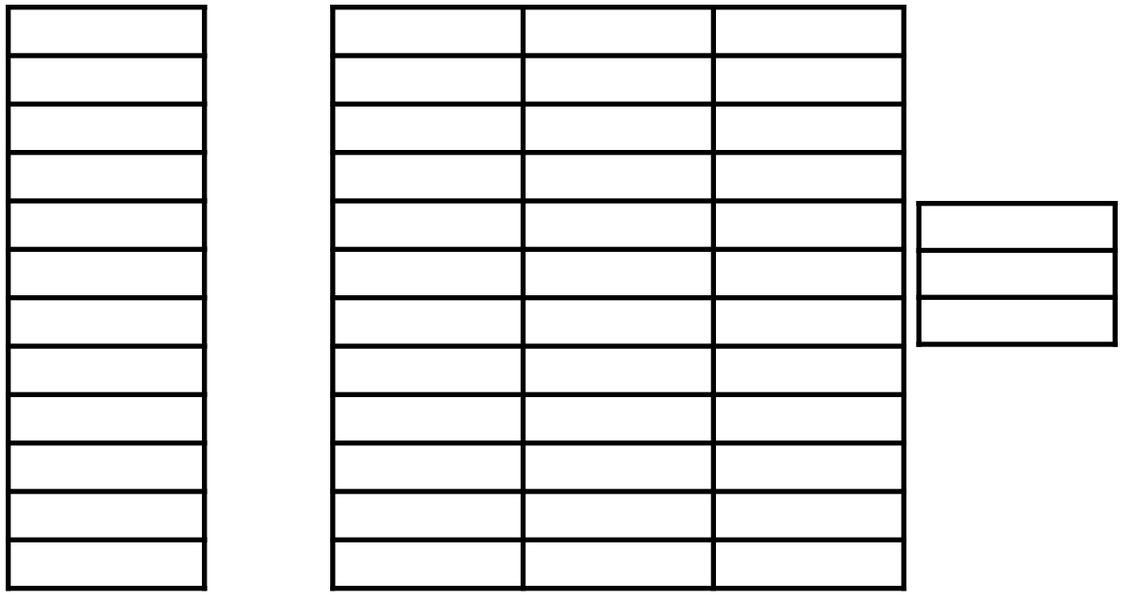
often called "OLS" (ordinary least squares), but that puts the focus on the procedure rather than the model.

$$\underbrace{y}_{n\times 1} = \underbrace{X}_{n\times k}\underbrace{\beta}_{k\times 1} + \underbrace{\epsilon}_{n\times 1}$$

same as X

error

matrix notation



What's linear about it?

$$\underbrace{y}_{n\times 1} = \underbrace{X}_{n\times k}\underbrace{\beta}_{k\times 1} + \underbrace{\epsilon}_{n\times 1}$$

Linear refers to the fact that we're taking linear combinations of the predictors.

Still linear if, e.g., use both x and its square and its cube as predictors.

↳ x^2 => still linear.

is linear comb from β

Sample Quantities vs. Population Quantities

sample version (think of x and y as data vectors)

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

population version (think of x and y as r.v.s)

$$y = \beta_0 + \beta_1 x + \epsilon$$

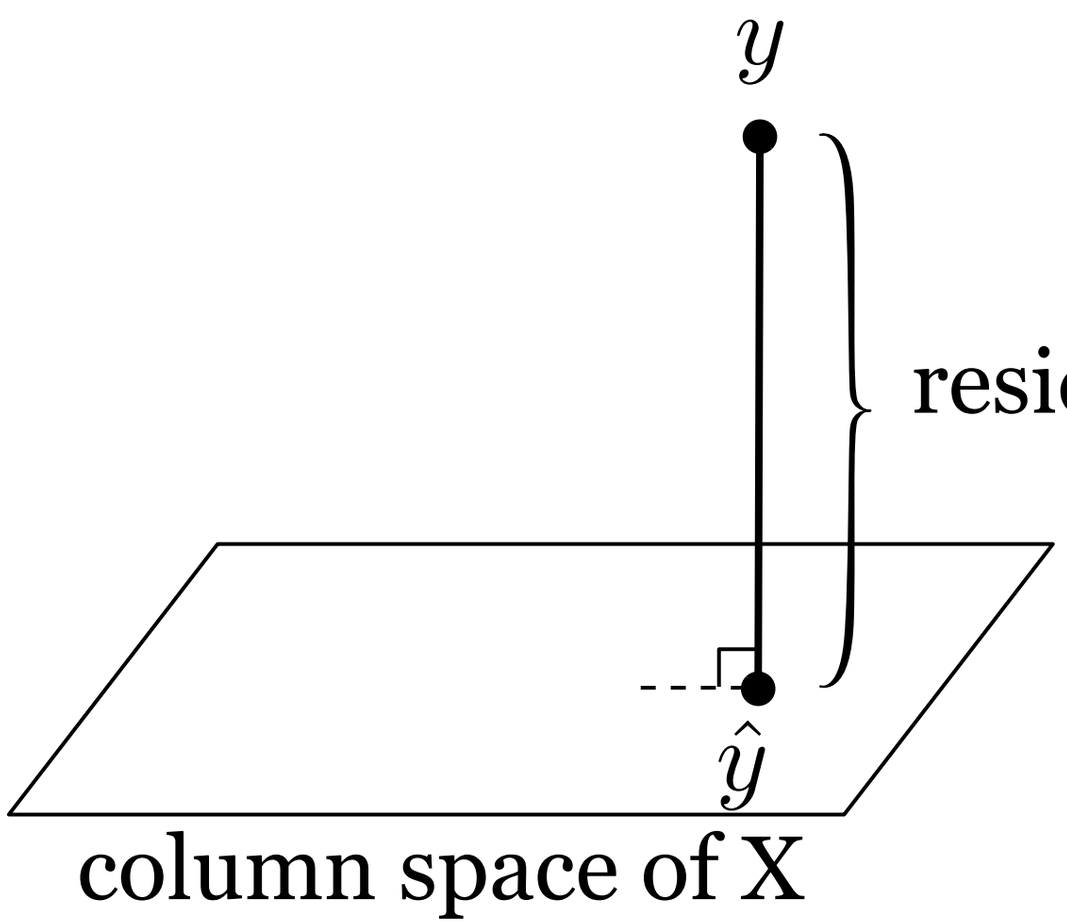
$$E(y) = \beta_0 + \beta_1 E(x)$$

$$\text{cov}(y, x) = \beta_1 \text{cov}(x, x)$$

Is analog of variance of two variable.

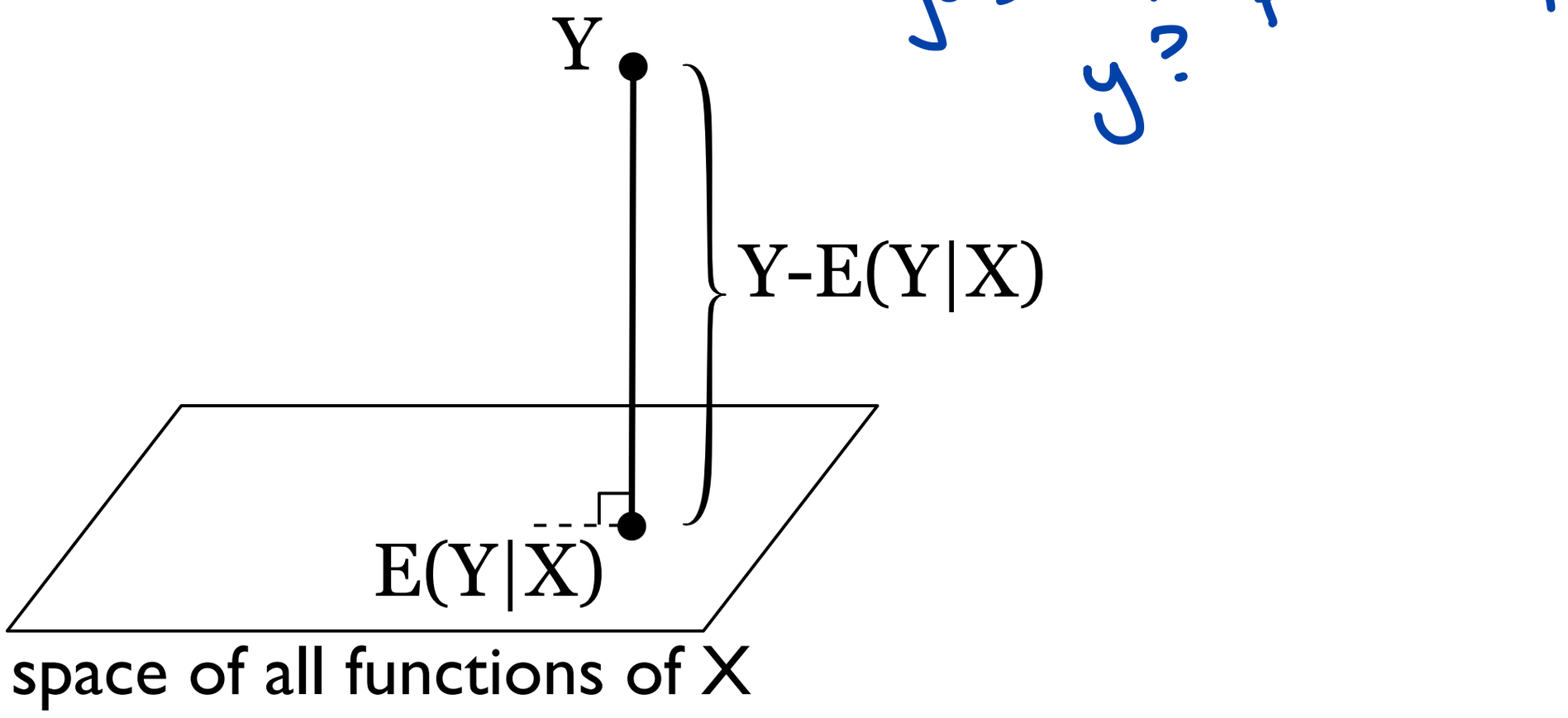
visualize regression as a *projection*

looking for a point in space that is closest to another point



or as a *conditional expectation*

what function of x can I use best 'job of predicting y?



Gauss-Markov Theorem

Famous in regression

Theoritical evidence for looking at OLS

Consider a linear model

$$y = X\beta + \epsilon$$

where y is n by 1, $\mathbf{X}$ is an n by k matrix of covariates, β is a k by 1 vector of parameters, and the errors ϵ_j are uncorrelated with equal variance, $\epsilon_j \sim [0, \sigma^2]$. The errors do not need to be assumed to be Normally distributed.

Then it follows that...

estimate of β

$$\hat{\beta} \equiv (X'X)^{-1}X'y$$

is **BLUE** (the **B**est **L**inear **U**nbiased **E**stimator).

For Normal errors, this is also the MLE.

Residuals

$$y = X\hat{\beta} + e$$

mirrors

$$y = X\beta + \epsilon$$

The residual vector e is *orthogonal* to all the columns of X.

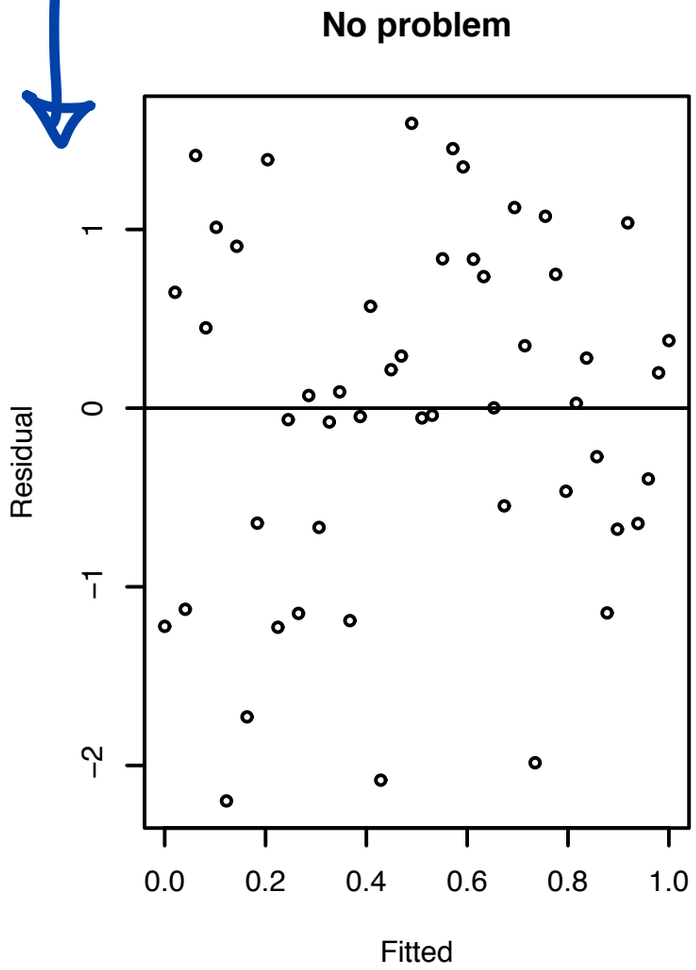
Never will know the ϵ

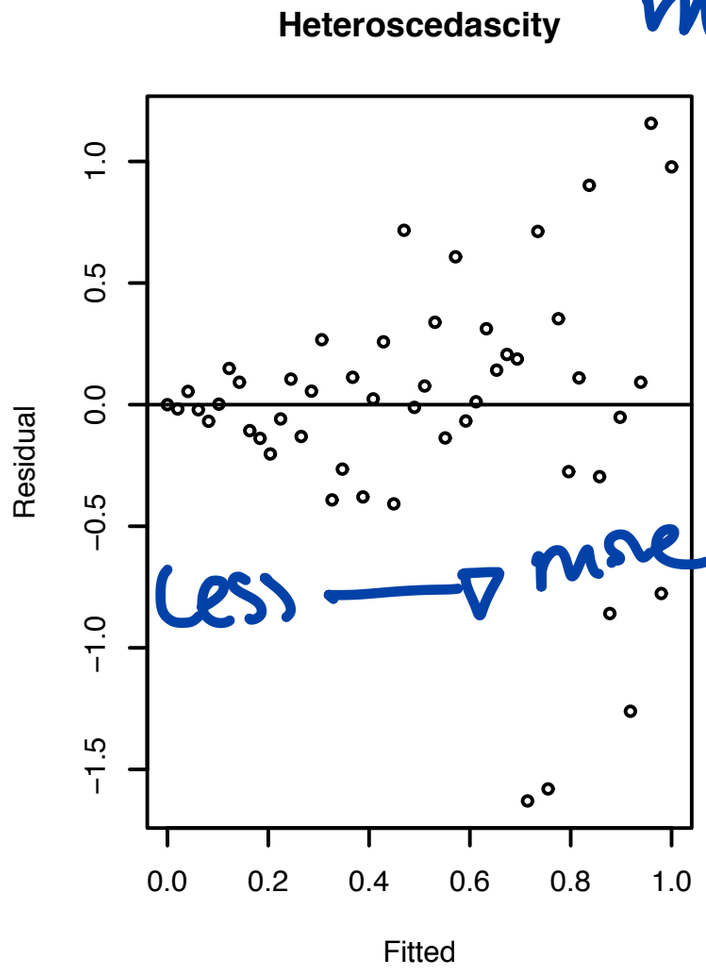
OLS estimate β

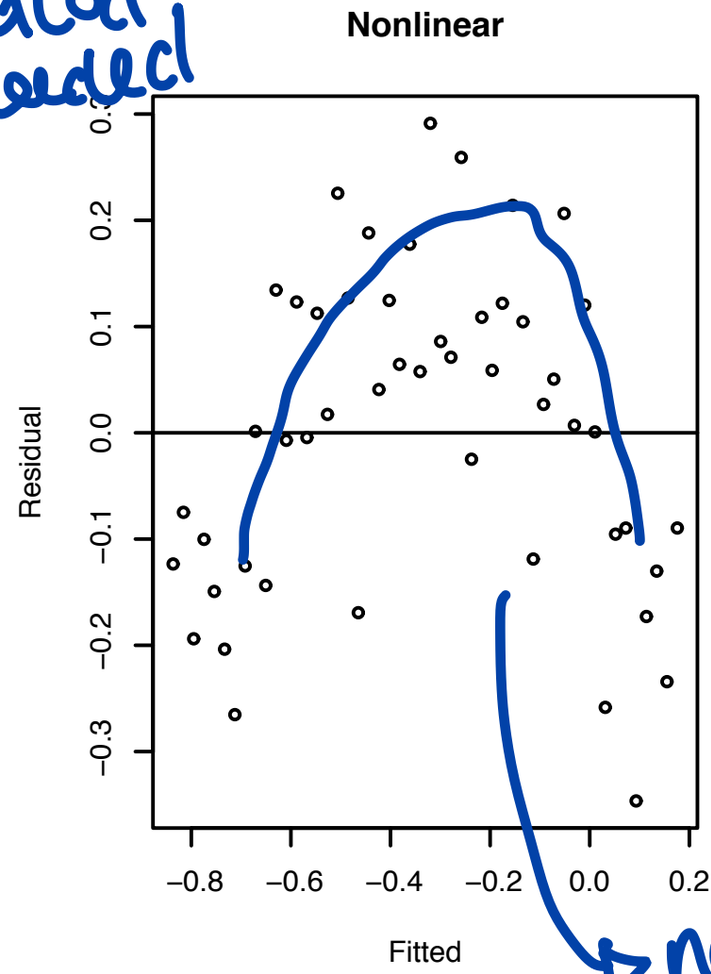
actual value of y vs. pred

Residual Plots

Always plot the residuals! (Plot residuals vs. fitted values, and residuals vs. each predictor variable)







Faraway, http://cran.r-project.org/doc/contrib/Faraway-PRA.pdf

"Explained" Variance

decomposing variance.

$$\text{var}(y) = \text{var}(X\hat{\beta}) + \text{var}(e)$$

$$R^2 = \frac{\text{var}(X\hat{\beta})}{\text{var}(y)} = \frac{\sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2}$$

R^2 measures goodness of fit, but
it does *not* validate the model.
Adding more predictors can only increase R^2.

▷ not great on new data.

▷ overused; how useful is the model for prediction?

Penalties are crude fixes.