Irodov Problem 1.13

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1.13 Point A moves uniformly with velocity v so that the vector $\vec{v}$ is continually 'aimed' at point B which in its turn moves rectilinearly and uniformly with velocity $u < v$. At the initial moment of time $\vec{v} \perp \vec{u}$ and the points are separated by a distance l. How soon will the points converge?

I want to discuss the solution of the above problem from 'Problems in General Physics' by I.E. Irodov [1]. Let T be the time taken for points A and B to converge. So, we need to find an expression for T in terms of the quantities u, v, and l.

1 School kid's solution

Let r be the distance between the points A and B, and θ be the angle between the velocities $\vec{v}$ and $\vec{u}$. We consider the relative velocity of A with respect to B along AB and orthogonal to AB to obtain the following relation.

$$\dot{r} = -v + u\cos\theta \tag{1}$$

$$r\dot{\theta} = -u\sin\theta \tag{2}$$

First, we will eliminate $\dot{\theta}$ by differentiating equation 1 with respect to time and then substituting the expression for $\dot{\theta}$ from equation 2.

$$\ddot{r} = -u\dot{\theta}\sin\theta$$
$$\implies r\ddot{r} = u^2\sin^2\theta$$

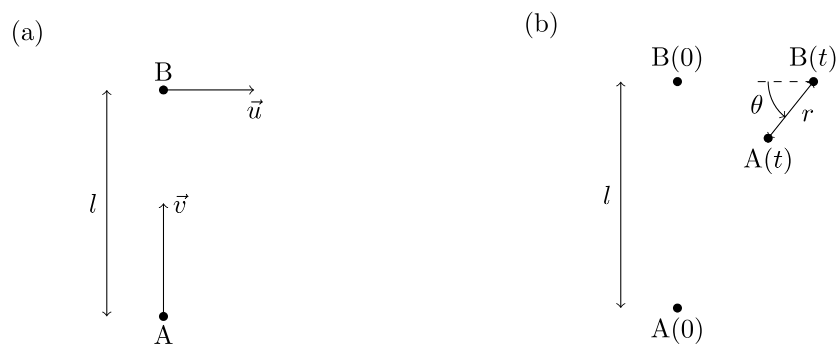

Figure 1: (a) Velocities of points A and B separated by a distance of l at the initial moment of time $t = 0$ (b) Velocities of points A and B separated by a distance of r at time $t \in (0, T)$.

Now, we will eliminate θ from the above expression by using the trigonometric identity $\cos^2\theta+\sin^2\theta = 1$, and equation 1.

$$\begin{aligned}
r\ddot{r} &= u^2\sin^2\theta \\
&= u^2(1-\cos^2\theta) \\
&= u^2\{1-\left[\frac{\dot{r}+v}{u}\right]^2\}
\end{aligned}$$

Simplifying the above expression we obtain the following second order differential equation.

$$\begin{aligned}
& r\ddot{r}+\dot{r}^2+2\dot{r}v+v^2-u^2=0 \\
\implies & \frac{d}{dt}(r\dot{r}+2rv)+v^2-u^2=0
\end{aligned}$$

Now, integrating with respect to time from $t=0$ to T we get the following expression.

$$\begin{aligned}
& \int_{t=0}^{t=T}\frac{d}{dt}(r\dot{r}+2rv)dt+\int_{t=0}^{t=T}(v^2-u^2)dt=0 \\
\implies & r(T)\dot{r}(T)+2r(T)v-r(0)\dot{r}(0)-2r(0)v+(v^2-u^2)T=0
\end{aligned}$$

We will use the fact that

$$\begin{aligned}
r(0) &= l \\
\dot{r}(0) &= -v+u\cos\pi/2 && \text{(from equation 1)} \\
&= -v \\
r(T) &= 0 && \text{(since A and B converge at time T)}
\end{aligned}$$

to get the final expression for T.

$$T=\frac{lv}{v^2-u^2} \tag{3}$$

2 Numerical solution

Interestingly, the above analytical solution does give us information about the trajectory of A. Given numerical values of u, v, and l, the trajectory of point A relative to B can be found by considering the ordinary differential equations 1 and 2 to formulate an initial value problem as $r(0)=l$ and $\theta(0)=\pi/2$. On applying the forward Euler's method we arrive at the following scheme to find the trajectory of A.

$$r_{n+1}=r_n+\Delta t(u\cos\theta-v) \tag{4}$$

$$\theta_{n+1}=\theta_n-\Delta t u\sin\theta \tag{5}$$

where Δt is the time step considered for the discretisation of time, that is,

$$\begin{aligned}
r_n &\approx r(n\Delta t) \\
\theta_n &\approx \theta(n\Delta t)
\end{aligned}$$

Given below is a python function to simulate the motion of A relative to B.

```python
import numpy as np
import matplotlib.pyplot as plt

def numSol(u, v, l, dt, tolFrac = 1e-4, NMax = 100_000):
    # Initial Condition
    r = l
    th = np.pi/2

    rList = []
    thList = []

    tol = tolFrac * l
    # Forward Euler steps till r > tol
    stepCount = 0
    while r > tol and stepCount < NMax:
        stepCount += 1
        rList.append(r)
        thList.append(th)

        rNext = r + dt * (u*np.cos(th) - v)
        thNext = th - dt * u*np.sin(th) / r

        r,th = rNext,thNext

    # T_answer = stepCount * dt
    return stepCount, np.array(rList), np.array(thList)
```

The consistency of the analytical result from equation 3 with the simulated relative position of A with respect to B for $u, v, l = 3, 5, 10$ can be seen in figure 2. We observe this consistency for a wide range of problem parameters u, v and l.

3 Physics major's solution

To be written please wait

Discussion

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References

[1] I. E. Irodov, *Problems in General Physics*, Mir Publishers, 1988.

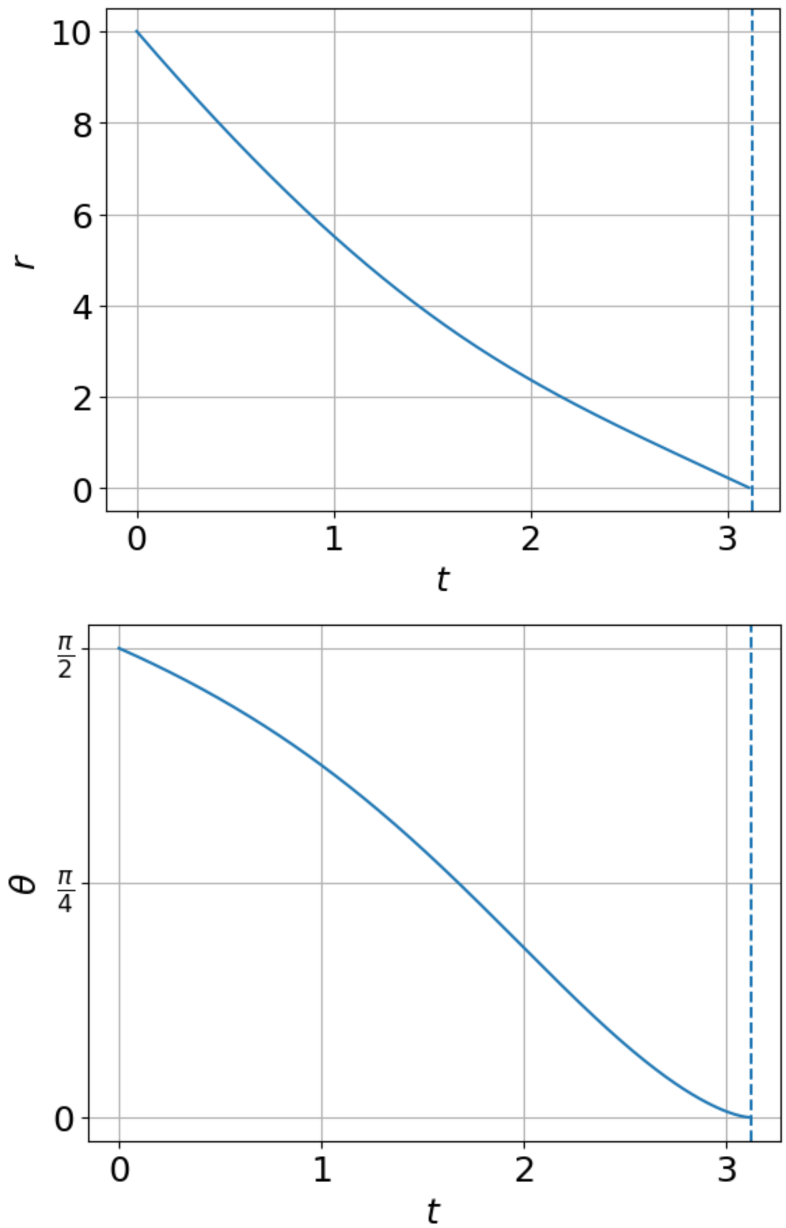

Figure 2: Simulation of r and θ for $u, v, l = 3, 5, 10$, where r is the distance AB and θ the angle between the velocities of A and B. The dashed line indicates the time of contact T, according to the analytical solution given by equation 3, $T = lv/(v^2 - u^2)$.