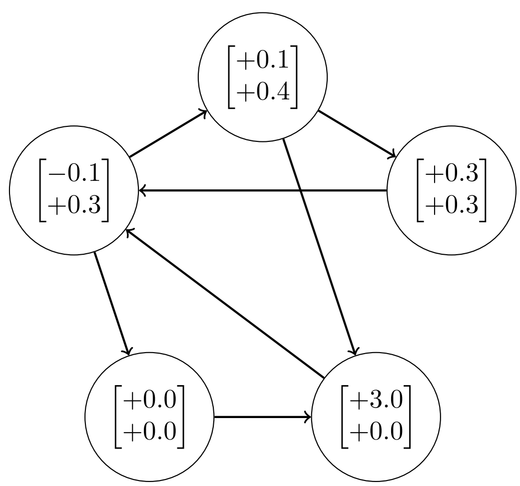
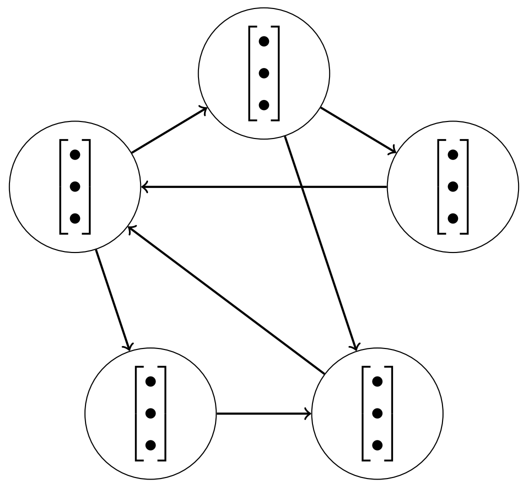
- Set of nodes $|V| = n$
- Set of edges $E \subseteq V \times V$
- Set neighbours of node v is $N(v) = \{u : (u, v) \in E\}$
- In-degree is $|N(v)|$
- Adjacency matrix A

$$a_{i,j} = \begin{cases} 1 & \text{if } (i,j) \in E \\ 0 & \text{otherwise} \end{cases}$$

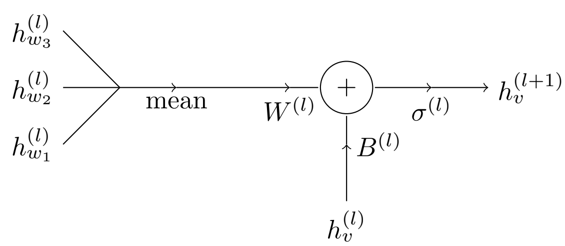
- Mesh consists of Vertices, Edges and Faces
- Graph Convolution Network (GCN)
  is a special case of Graph Neural Network (GNN)



- Layer $l = 0$
- $n_l = 2$
- $h_v^{(l)} \in \mathbb{R}^{n_l}$

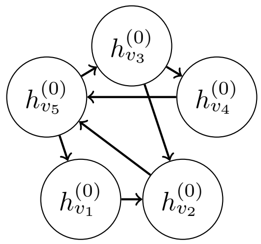


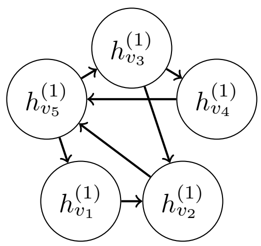
- Layer $l = 1$
- $n_l = 3$
- $h_v^{(l)} \in \mathbb{R}^{n_l}$
- Same graph
- Transformed node vectors

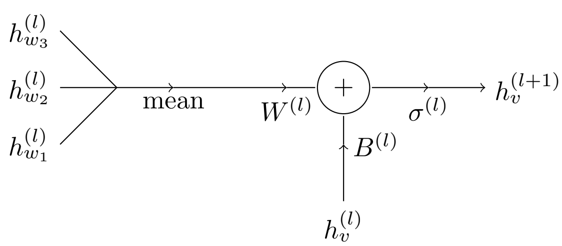


Layer l, node v, $N(v) = \{w_1, w_2, w_3\}$

- $h_v^{(l)} \in \mathbb{R}^{n_l}$
- Each layer the node vector is transformed
- Layer l has activation function $\sigma^{(l)} : \mathbb{R} \to \mathbb{R}$
- Trainable parameters $W^{(l)}, B^{(l)} \in \mathbb{R}^{n_{(l+1)} \times n_l}$
- Forward pass $h_v^{(l+1)} = \sigma^{(l)}(W^{(l)} \sum_{w \in N(v)} \frac{h_w^{(l)}}{|N(v)|} + B^{(l)} h_v^{(l)})$ for $l = 0, 1, 2 \ldots$







$\rightarrow$

Layer 0

Layer 1

We can rewrite the forward-pass relation in the following manner

$$h_v^{(l+1)} = \sigma^{(l)}(W^{(l)} \sum_{w \in N(v)} \frac{h_w^{(l)}}{|N(v)|} + B^{(l)} h_v^l)$$
$$\implies H^{(l+1)} = \sigma^{(l)}(D^{-1} A H^{(l)} W^{(l)\intercal} + H^{(l)} B^{(l)\intercal})$$

Where,

$$H^{(l)} = \begin{bmatrix} h_1^{(l)\intercal} \\ \vdots \\ h_n^{(l)\intercal} \end{bmatrix} D = \begin{bmatrix} d_1 & & \\ & \ddots & \\ & & d_n \end{bmatrix}$$
$$d_v = |N(v)|$$

This is more suitable for implementation purposes as matrix vector operations are highly optimised