AI1103 Assignment-1
I.Rajasekhar Reddy – CS20BTECH11020

March 15, 2021

QUESTION:
A manufacturer has three machine operators A, B and C. The first operator A produces 1% defective items, where as the other two operators B and C produce 5% and 7% defective items respectively. A is on the job for 50% of the time, B is on the job for 30% of the time and C is on the job for 20% of the time. A defective item is produced, what is the probability that it was produced by A?

ANSWER:
P(A) : be the probability that item is produced by operator A.
P(B) : be the probability that item is produced by operator B.
P(C) : be the probability that item is produced by operator C.
D be the event that item produced is defective.
So from question it is clear that
P(A) $= \frac{50}{100} = \frac{1}{2}$, P(B) $= \frac{30}{100} = \frac{3}{10}$, P(C) $= \frac{20}{100} = \frac{1}{5}$. And————(1)

P$(\frac{D}{A})$: is the probability of defective item produced by A.
P$(\frac{D}{B})$: is the probability of defective item produced by B.
P$(\frac{D}{C})$: is the probability of defective item produced by C.
From the question it is clear that
P$(\frac{D}{A}) = \frac{1}{100}$, P$(\frac{D}{B}) = \frac{5}{100}$, P$(\frac{D}{C}) = \frac{7}{100}$ ————(2)

P(D) = P(A)P$(\frac{D}{A})$ + P(B)P$(\frac{D}{B})$ + P(C)P$(\frac{D}{C})$ As the defective item can be produced from either operator A or B or C.
P(D) $= (\frac{1}{2})(\frac{1}{100}) + (\frac{3}{10})(\frac{5}{100}) + (\frac{1}{5})(\frac{7}{100}) = \frac{34}{1000}$

We need to find probability that item was produced by operator A given that item is defective P$(\frac{A}{D})$
From conditional probability,
P$(\frac{A}{D}) = \frac{P(AD)}{P(D)} = \frac{\frac{P(AD)}{P(A)}}{\frac{P(D)}{P(A)}} = \frac{P(\frac{D}{A})}{\frac{P(D)}{P(A)}} = \frac{P(\frac{D}{A})P(A)}{P(D)}$

On substituting values from (1) and (2)
P$(\frac{A}{D}) = \frac{(0.01)(0.5)}{(0.034)} = \frac{5}{34}$

Required probability is 0.147058