AI1103 - Assignment 5

I.Rajasekhar Reddy – CS20BTECH11020

Download all latex codes from

https://github.com/rajasekhar156/Assignment−5/blob/main/main.tex

QUESTION

Let X_1 and X_2 be a random sample of size two from a distribution with probability density function

$f_\theta(x) = \theta \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} + (1-\theta)\frac{1}{2}e^{-|x|}$,

$-\infty < x < \infty$,

where $\theta \in \{0, \frac{1}{2}, 1\}$. If the observed values of X_1 and X_2 are 0 and 2, respectively, then the maximum likelihood estimate of θ is

1) 0
2) $\frac{1}{2}$
3) 1
4) not unique

ANSWER

Given X_1 =0, X_2 =2, n=2 and

$$f_\theta(x) = \theta \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} + (1-\theta)\frac{1}{2}e^{-|x|} \tag{0.0.1}$$

Then log of likelihood function is given by

$$l(\theta) = \sum_{i=1}^{i=n} \log f_\theta(x_i) \tag{0.0.2}$$

$$= \log f_\theta(x_1) + \log f_\theta(x_2) \tag{0.0.3}$$

$$= \log(\theta \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}0^2} + (1-\theta)\frac{1}{2}e^{-|0|}) + \log(\theta \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}2^2} + (1-\theta)\frac{1}{2}e^{-|2|}) \tag{0.0.4}$$

$$= \log(\theta \frac{1}{\sqrt{2\pi}} + (1-\theta)\frac{1}{2}) + \log(\theta \frac{1}{\sqrt{2\pi}} e^{-2} + (1-\theta)\frac{1}{2}e^{-2}) \tag{0.0.5}$$

$$= 2\log(\theta \frac{1}{\sqrt{2\pi}} + (1-\theta)\frac{1}{2}) - 2 \tag{0.0.6}$$

Since likelihood $L(\theta) = e^{l(\theta)}$.

Likelihood function $L(\theta)$ at $\theta = 0, \frac{1}{2}, 1$ is given by

1) At $\theta = 0$ $\quad L(\theta = 0) = \frac{1}{4}e^{-2} = 0.0338$
2) At $\theta = 1$ $\quad L(\theta = 1) = \frac{1}{2\pi}e^{-2} = 0.0215$
3) At $\theta = \frac{1}{2}$ $\quad L(\theta = \frac{1}{2}) = (\frac{1}{2\sqrt{2\pi}} + \frac{1}{4})^2 e^{-2} = 0.0273$

Hence the maximum likelihood estimate of θ is at $\theta = 0$