AI1103 - Assignment 5

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Download all latex codes from

https://github.com/rajasekhar156/Assignment−5/blob/main/main.tex

QUESTION

Let X_1 and X_2 be a random sample of size two from a distribution with probability density function

$$f_\theta(x) = \theta\left(\frac{1}{\sqrt{2\pi}}\right)e^{-\dfrac{1}{2}x^2} + (1-\theta)\left(\frac{1}{2}\right)e^{-|x|},$$

$-\infty < x < \infty$,
where $\theta \in \left\{0, \dfrac{1}{2}, 1\right\}$. If the observed values of X_1 and X_2 are 0 and 2, respectively, then the maximum likelihood estimate of θ is

1) 0
2) $\frac{1}{2}$
3) 1
4) not unique

ANSWER

Given X_1 =0, X_2 =2, n=2 and

$$f_\theta(x) = \theta\left(\frac{1}{\sqrt{2\pi}}\right)e^{-\dfrac{1}{2}x^2} + (1-\theta)\left(\frac{1}{2}\right)e^{-|x|} \quad (0.0.1)$$

Then log of likelihood function is given by

$$l(\theta) = \sum_{i=1}^{i=n} \log f_\theta(x_i) \quad (0.0.2)$$

$$= \log f_\theta(x_1) + \log f_\theta(x_2) \quad (0.0.3)$$

$$= \log\left(\theta\left(\frac{1}{\sqrt{2\pi}}\right)e^{-\dfrac{1}{2}0^2} + (1-\theta)\left(\frac{1}{2}\right)e^{-|0|}\right) + \log\left(\theta\left(\frac{1}{\sqrt{2\pi}}\right)e^{-\dfrac{1}{2}2^2} + (1-\theta)\left(\frac{1}{2}\right)e^{-|2|}\right) \quad (0.0.4)$$

$$= \log\left(\theta\left(\frac{1}{\sqrt{2\pi}}\right) + (1-\theta)\left(\frac{1}{2}\right)\right) + \log\left(\theta\left(\frac{1}{\sqrt{2\pi}}\right)e^{-2} + (1-\theta)\left(\frac{1}{2}\right)e^{-2}\right) \quad (0.0.5)$$

$$= 2\log\left(\theta\left(\frac{1}{\sqrt{2\pi}}\right) + (1-\theta)\left(\frac{1}{2}\right)\right) - 2 \quad (0.0.6)$$

Since likelihood $L(\theta) = e^{l(\theta)}$.

Likelihood function $L(\theta)$ at $\theta = 0, \frac{1}{2}, 1$ is given by

1) At $\theta = 0$ $\quad L(\theta = 0) = \frac{1}{4}e^{-2} = 0.0338$
2) At $\theta = 1$ $\quad L(\theta = 1) = \frac{1}{2\pi}e^{-2} = 0.0215$
3) At $\theta = \frac{1}{2}$ $\quad L(\theta = \frac{1}{2}) = \left(\frac{1}{2\sqrt{2\pi}} + \frac{1}{4}\right)^2 e^{-2} = 0.0273$

Hence the maximum likelihood estimate of θ is at $\theta = 0$