Assignment - Mathematics

1. In a n/w, data packets are sent through a shared channel. Suppose there are 10 devices attempting to send data.
In any given time slot, each device has a 20% chance of attempting to send a packet, independently of other devices. A collision occurs if two or more devices attempt to send a packet in the same time slot. What is the prob that in a given time slot there will be no collision?

→ What is prob that 1 specific device sends (0.2) and other 9 do not send (0.8)?

(n) no. of devices = 10

(p) Prob of sending = 20% = 0.2

(1-p) Prob. of not sending = 1-0.2 = 0.8

(k) One device send - 1 Success

$$\boxed{P(k) = \binom{n}{k} \cdot p^k \cdot (1-p)^{n-k}}$$ formula.

Binomial prob Mass Function (PMF) used for discrete random variables when counting exact no. of success in independent trails.

$$\binom{10}{1} \times (0.2)^1 \times (0.8)^9$$

$$= 10 \times 0.2 \times (0.8)^9$$

$$P = 10 \times 0.2 \times 0.1342$$

$$\boxed{P = 0.269}$$

P(No collision) ≈ 0.269

2. A System uses 8-char Passwords. Each char can be any lowercase letter (a-z), uppercase letter (A-Z) or digit (0-9)

Prob of randomly generated 8 char password contains at least one digit, char repetition allowed?

\- - - - - - - - a-z A-Z 0-9 Prob. at least 1 digit? Char repeat.

classic prob + complement Rule

↳

Total chrs : 26+26+10 = 62 chrs

$P(\text{at least one digit}) = 1 - P(\text{no digit})$

Total password $= 62^8$

Password no digit $= 26+26 \Rightarrow 52^8$

$P = 1 - \frac{52^8}{62^8} \approx 0.755$ //

P(at least one digit) = 0.755 | 75.5% chance

3. A software project has 2 modules, A & B.

The prob of A has bug is 0.15

The prob of B has bug is 0.10

both prob of bug is 0.03

a) what is prob the A has bug, given B has bug

b) What prob that B has a bug, A doesn't

↳ $P(A) = 0.15$

$P(B) = 0.10$

$(A \cap B) \Rightarrow P(A \& B) = 0.03$

Conditional prob

(a)

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

~~$p(A \cap B) = P(B) \cdot P(A|B)$~~

$$A(A|B) = \frac{0.03}{0.10} = 0.3$$

the prob of module A having bug given b has a bug is 0.3

(b) Conditional prob

$P(\text{not } A) = 1 - P(A)$

$1 - 0.15$

$= 0.85$

$P(B \ \& \ \text{not } A) = P(B) - P(A \& B)$

$= 0.10 - 0.03$

$= 0.07$

$$P(B|\text{not } A) = \frac{P(B \ \& \ \text{not } A)}{P(A')} = \frac{0.07}{0.85} \approx 0.0824$$

4.

A robot uses a sensor to detect an obstacle. The sensor has a 95% chance of correctly detect an obstacle when one is present (true the rate).
It also has 2% chance of indicating an obstacle when there is no one (false the rate). In the robot's env, obstacles are present 10% of the time.

What prob that obstacle is truly present, given that the sensor indicates an obstacle?

↳ $P(\text{obstacle}) = 10\% = 0.10$ — Bayes' thm

$P(\text{No obstacle}) = 90\% = 0.90$

$P(\text{sensor indicates obstacle} \mid \text{obstacle present}) =$ True the $\Rightarrow 95\% = 0.95$

$P(\text{Sensor indicates obstacle} \mid \text{No obst}) =$ False the $\Rightarrow 2\% = 0.02$

$P(\text{obstacle} \mid \text{sensor says obstacle}) = ?$

$$\boxed{P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}}$$

A → obstacle present
B → sensor says obstacle

$$P(\text{obstacle} \mid \text{sensor the}) = \frac{P(\text{sensor the} \mid \text{obst}) \cdot P(\text{obs})}{P(\text{sensor the})}$$

i) obstacle & sensor correctly detect ii) no obstacle & sensor gives false the.

$$P(\text{sensor the}) = (0.95 \times 0.10) + (0.02 \times 0.90)$$
$$= 0.113$$

→ $$\frac{0.95 \times 0.10}{0.113} = \frac{0.095}{0.113} \approx 0.807$$

5. A spam filter uses a keyword 'money' to identify spam.

- 10% of all emails are spam
- If a email is spam, the word 'money' appears in it 80% of time.
- If an email is not spam, the word 'money' appears in it 5% of the time.

If an email contains the word 'money'; what is the prob that is spam?

↳ P(spam) = 10% = 0.10 ✓

Baye's thm.

P(not spam) = 90% = 0.90

P(word money & spam) = 80% = 0.80 ✓

P(word money & not spam) = 5% = 0.05

P(spam & money) = ?

$$\boxed{P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}}$$

A - email is spam

B - word money.

P(B) = P(money word) = ?

$= (0.80 \times 0.10) + (0.05 \times 0.90)$

$= 0.125$

$$P(A|B) = \frac{0.80 \times 0.10}{0.125} = \frac{0.08}{0.125} = 0.64 //$$

6. A rare disease affects 1 in 10,000 people. A diagnostic test for the disease has a 99% true +ve,
0.5% false +ve
If a randomly selected person tests +ve for the disease, what is the probability have the disease?

↳

P(disease) = 1 in 10,000 = 0.0001 Bayes Thm
P(No disease) = 1 − 0.0001 = 0.9999
P(positive test & Disease): True positive = 0.99
P(positive test & No disease) = False positive = 0.005
P(disease & positive test) = ?

$$\boxed{P(A/B) = \frac{P(B/A) \cdot P(A)}{P(B)}}$$

A → disease
B → positive test

(i) True positive - disease person +ve (ii) false positive healthy test person +ve

$P(B) = (0.99 \times 0.0001) + (0.0005 \times 0.9999)$

$= 0.0050985$

→

$$\frac{0.99 \times 0.0001}{0.0050985} \approx 0.01942$$

$\approx 1.94\%$

Scanned with OKEN Scanner

7. A batch of 500 MC is produced. The prob that a single MC is defective is 0.02. Let X be the number of defective MC in a random sample of 20 MC from this batch.

Assume the sample is taken with replacement, or the batch is large enough that we can model this as independent trials.

a) What is the prob that exactly 3 MC in the sample are defective?

b) What is the prob that at least 2 MC in the sample are defective.

↳ $P(X=3)$ & $P(X \geq 2) = ?$

$$\boxed{P[X=k) = \binom{n}{k} \cdot p^k \cdot (1-p)^{n-k}}$$ binomial

i) $P(X=3) = \binom{20}{3} \times (0.02)^3 \times (0.98)^{17}$

$$= \left(\frac{20 \times 19 \times 18}{3 \times 2 \times 1}\right) \times (0.00008) \times (0.6984)$$

$= 1140 \times 0.00008 \times 0.6983$

$$\boxed{P(X=3) = 0.01478}$$

ii) $P(X \geq 2) = 1 - [P(X=0) - P(X=1)]$ + Complement

$P(X=0) = \binom{20}{0} \times (0.02)^0 \times (0.98)^{20}$

$= 1 \times 1 \times 0.6676$

$$\boxed{P(X=0) = 0.6676}$$

$P(X=1) = \binom{20}{1} \times (0.02)^1 \times (0.98)^{19}$

$= 20 \times 0.02 \times 0.6813$

$$\boxed{P(X=1) = 0.2724}$$

∴ $1 - 0.6676$ $\boxed{P(X \geq 2) = 0.0595}$

from scipy.stats import binom

n=20

p=0.02

prob-exact-3 = binom.pmf(3,n,p)

prob-0 = binom.pmf(0,n,p)

prob-1 = binom.pmf(1,n,p)

prob-at-least-2 :

1 - (prob-0 + prob-1)