

HW2: TRAJECTORY GENERATION FOR NONLINEAR SYSTEMS

Exercise 1

This exercise is part of a series of activities leading to the implementation of an interactive MIMO control law for the **Flying-Chardonnay**, an automatic drink delivery device. This exercise exploits the MATLAB model implemented previously, and follows the following configuration its parameters (in S.I. units):

$$\overline{m_d = 1 \quad m_c = 1 \quad l = 1 \quad l_d = 1 \quad J = 1 \quad C_D = 0.01 \quad g = 10}$$

1. (10pts) Find a trajectory $(\mathbf{x}(t), \mathbf{u}(t))$ that **shifts the drone horizontally in 4m** while keeping the level of water reasonably high. Additionally, please follow the following design constraints:

- (a) Avionics (sensors/actuators) sample frequency: 50Hz;
- (b) Maneuver maximum time: 2s.

To measure the water level and animate your trajectory, please use the MATLAB files [Exercise Support] MATLAB: Flying-Chardonnay Animation posted on the LMS. You can also use the [Lecture Example] MATLAB: MPC for Automatic Rocket Landing entry to look for inspiration.

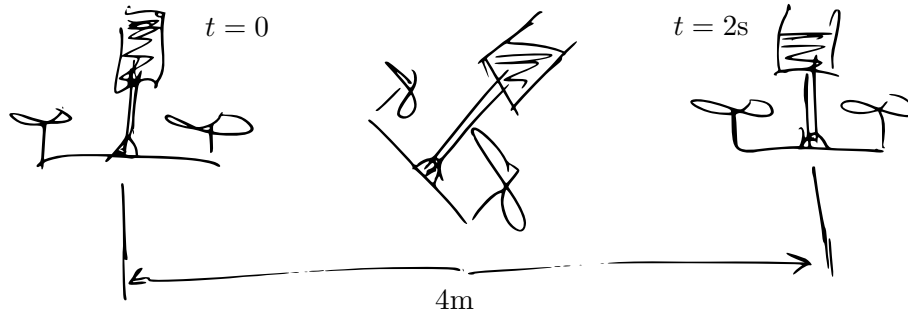


Figure 1: An trajectory example.

Exercise 2

Consider the nonlinear discrete-time system represented by the state equations:

$$\begin{aligned} \mathbf{x}_{k+1} &= \begin{pmatrix} \cos u_k & -\sin u_k \\ \sin u_k & \cos u_k \end{pmatrix} \mathbf{x}_k \\ \mathbf{y}_k &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x}_k \end{aligned}$$

where $\mathbf{x}_k \in \mathbb{R}^2$, $\mathbf{y}_k \in \mathbb{R}^2$, and $u_k \in \mathbb{R}$. Assume the system is initially at $\mathbf{x}_0 = (1, 0)$, and that its desired final position at $k = N$ is $\mathbf{x}_N = (0, 1)$. The final objective of this exercise will be to find **by hand** the optimal trajectory $\{(\mathbf{x}_0, u_0), (\mathbf{x}_1, u_1), \dots, (\mathbf{x}_N, u_N)\}$ that minimizes the following cost function:

$$J = \sum_{k=0}^N \alpha |\mathbf{y}_k|^2 + \beta u_k^2 \quad (1)$$

where $\alpha > 0$ and $\beta > 0$.

1. **(7pts)** Find the optimal trajectory for a prediction horizon $N = 2$.
2. **(3pts)** Find the optimal trajectory for a prediction horizon $N = 2024$.