HW5: Controllable and Unobservable Spaces

Exercise 1

This exercise is part of a series of activities leading to the implementation of an interactive MIMO control law for the `Flying-Chardonnay`, an automatic drink delivery device. This exercise exploits the MATLAB model implemented previously, and follows the following configuration its parameters (in S.I. units):

$m_d = 1$	$m_c = 1$	$l = 1$	$l_d = 1$	$J = 1$	$C_D = 0.01$	$g = 10$

Using the hovering (i.e., steady drone in rest) linear state-space model approximation computed in a previous assignment,

1. **(1pts)** Is this system stable? Justify.
2. **(2pts)** Is the system controllable? Is it stabilizable? Justify.
3. **(2pts)** Is this system under-actuated, fully-actuated or over-actuated? Justify.

Exercise 2

Consider the following linear system:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u \tag{1}$$

1. **(3pts)** Is $(0, 1)$ on its controllable subspace?
2. **(2pts)** What is the smallest achievable distance $\boldsymbol{x}(t)$ can have with respect to $(0, 1)$ by arbitrarily actuating over $u(t)$ and starting from the origin (i.e., $\boldsymbol{x}(0) = \mathbf{0}$)?

Exercise 3 (MIMO Exam 2021)

Figure 1 illustrates the final rendezvous approach maneuver of a vehicle with the ISS under intense solar flare. The final approach dynamics is linearized with respect to the ISS trajectory and yields

$$\frac{d}{dt} \begin{pmatrix} \Delta r \\ \Delta\theta \end{pmatrix} = \begin{bmatrix} 10 & -2 \\ 2 & 5 \end{bmatrix} \begin{pmatrix} \Delta r \\ \Delta\theta \end{pmatrix} + \begin{bmatrix} 0 & 3 & 2 \\ 1 & 5 & 1 \end{bmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} \tag{2}$$

where $\Delta r(t) = r(t) - R_{ISS}$ and $\Delta\theta(t) = \theta(t) - \theta_{ISS}(t)$ are the relative position of the vehicles in polar coordinates. A successful docking requires $\Delta r \to 0$ and $\Delta\theta \to 0$.

1. **(1pts)** Is the docking operation fully controllable or partially controllable?
2. **(1pts)** Is the docking operation overactuated, fully-actuated or underactuated?

After a system malfunction due to the solar flare, two actuator channels got jammed to $u_1(t) = 0$ and $u_2(t) = 1$. Fortunately, $u_3(t)$ is still operational. Under this new scenario, and considering the system is currently at $(\Delta r, \Delta\theta) = (-1, -1)$, answer the following:

1. **(3pts)** Plot on the same figure the new configuration of the controllable subspace, and the trajectory (on polar coordinates) of the partially jammed system with $u_3(t) = 0$ for $0 \le t \le 1$.

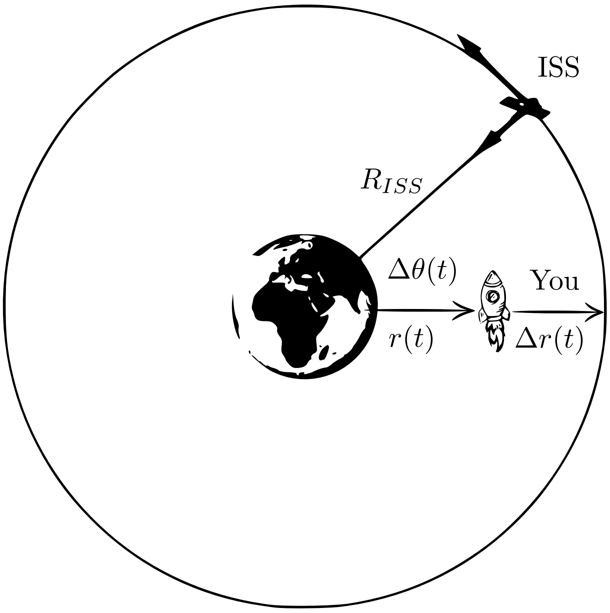

Figure 1: Final rendezvous approach.

2. **(3pts)** Can docking still be achieved with partially jammed controls by steering $u_3(t)$? Justify.

3. **(2pts)** Assuming the life support system holds for more 30 units of time, can our heroes make it safely to the ISS? Justify.