HW7: Transmission Zeros, Zero-Pole Cancellation and Sigma Plots

Exercise 1 (MIMO Exam 2021)
Figure 1 illustrates the top view of a skyscraper in downtown Montreal.

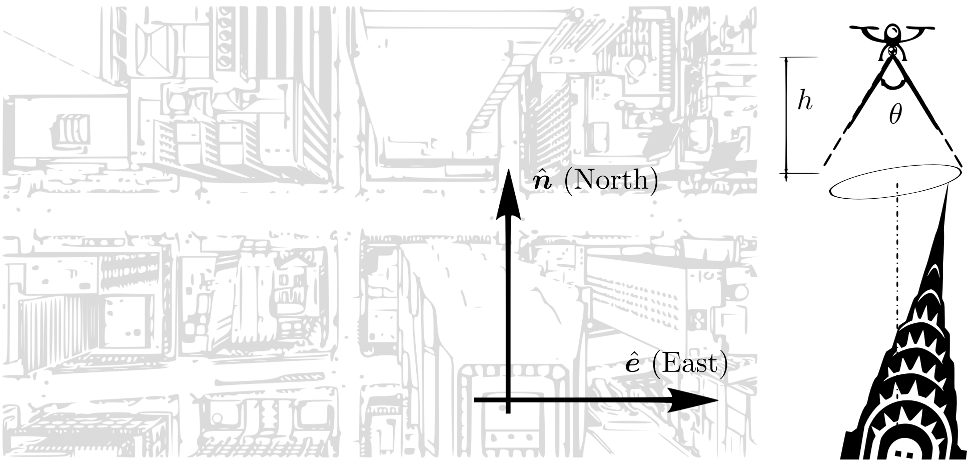

Figure 1: A Building in Downtown Montreal.

The structural department in your company identified the frequency response of the building with respect to the wind and obtained $\boldsymbol{p} = G(s)\boldsymbol{w}$, where

$$G(s) = \begin{pmatrix} \frac{-0.25}{s^2+0.2s+0.25} & \frac{0.25}{s^2+0.2s+0.25} \\ \frac{-1}{s+1} & \frac{-1}{s+1} \end{pmatrix} \tag{1}$$

and $\boldsymbol{p}(s) = (p_n(s), p_e(s))$ and $\boldsymbol{w}(s) = (w_n(s), w_e(s))$ denote, respectively, the displacement of the apex of the tower, and wind velocity, both described in north-east coordinates according to the picture above. All values are given in S.I. units.

1. **(5pts)** Which wind frequency yields the highest building vibration amplitude?

2. **(5pts)** Which **wind** direction $\hat{\boldsymbol{u}} \in \mathbb{C}^2$ corresponds to highest vibration amplitude?

3. **(5pts)** And what is the resulting **building** vibration direction $\hat{\boldsymbol{y}} \in \mathbb{C}^2$?

4. **(5pts)** A drone is flying right above the building to record the apex displacement in a video (see Fig. 1). Assuming the camera has an effective angle of view of $\theta = 60^o$ and a circular frame shape (fisheye), what is the minimum value of h necessary to record the full building apex trajectory response to the following wind input:

$$\begin{cases} w_n(t) = 2\cos(10\pi t) \\ w_d(t) = 3\sin(10\pi t) \end{cases} \tag{2}$$