

Anti-windup design & limit cycle migration

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Consider the following plant:

$$G(s) = \frac{1}{1 + 0.2s} \frac{1}{s(s + 0.1)} \quad (1)$$

with control input u and main output $y = G(s)u$. Both y and its derivative \dot{y} are assumed available for feedback.

1. Design a PID controller with the following structure:

$$u(t) = K_i \int_0^t (r(\tau) - y(\tau)) d\tau - K_p y(t) - K_d \dot{y}(t) \quad (2)$$

so that the three dominant poles of the closed-loop system are:

$$[-1 \quad -(1 \pm j)/\sqrt{2}]$$

Verify that the non-placed dynamic that corresponds to the actuator remains stable and significantly faster than the placed ones.

2. We now assume that **the control signal is limited**:

$$|u(t)| \leq 0.1$$

Implement your controller and the above saturation in a simulation diagram and check its effect for constant step references $r(t) = r_0$ with growing magnitudes r_0 starting from 0.3 up to 1.

- (a) Check that the nominal behavior is not affected when $r_0 = 0.3$
 - (b) What happens with $r_0 = 1$?
 - (c) Determine by simulations, a critical value for which oscillations appear.
 - (d) Conclude on the existence of a limit-cycle. Characterize its pulsation, magnitude and nature.
3. Confirm the above observations with the describing function method based on the first harmonic approximation.

- (a) Remove the saturation block from the simulation diagram to construct the interconnection $M(s)$ "seen" by the saturation (with the negative feedback convention). Obtain a state-space representation of $M(s)$ with the `linmod` function.
- (b) Visualize the nyquist plot of $M(s)$ and characterize possible intersections with the critical locus of the saturation.
- (c) Deduce from the above intersection the possible existence of a limit-cycle. Evaluate its pulsation and magnitude. Justify the use of the approximated describing function for the saturation (that coincides with that of the relay function)
- (d) Compute the ratio:

$$R = \frac{M(3j\omega_c)}{M(j\omega_c)}$$

and conclude on the validity of the approximation here. Compare the results obtained in (c) with the simulation-based values obtained in question 2.

4. We would like now to extend the operating domain of our system so that it behaves safely for $r_0 = 1$. To do so, we propose to implement a standard anti-windup strategy that consists of limiting the integral action when the saturation is active. The control law is modified as follows:

$$u(t) = K_i \int_0^t (r(\tau) - y(\tau) - K_{aw}\epsilon(\tau)) d\tau - K_p y(t) - K_d \dot{y}(t) \quad (3)$$

where the anti-windup signal $\epsilon(\tau)$ characterizes the activity of the saturation on $[0, t]$. In nominal conditions, when the saturation is not active, $\epsilon = 0$ and the anti-windup action is switched off.

$$\epsilon(\tau) = u(\tau) - \text{sat}(u(\tau)) \quad (4)$$

- (a) Implement this modified controller in your linear analysis oriented diagram and extract new $M(s)$ for different values of K_{aw} in the interval $[0, 1]$.
- (b) Visualize on the same figure several nyquist plots and show that there exists a critical value K_{aw}^* above which no limit cycle will appear.
- (c) Select and implement an anti-windup gain $K_{aw} > K_{aw}^*$ in your nonlinear simulation diagram and conclude.