Anti-windup design & limit cycle migration

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Consider the following plant:

$$G(s) = \frac{1}{1 + 0.2s} \; \frac{1}{s(s + 0.1)} \tag{1}$$

with control input u and main output $y = G(s)u$. Both y and its derivative $\dot{y}$ are assumed available for feedback.

1. Design a PID controller with the following structure:

$$u(t) = K_i \int_0^t (r(\tau) - y(\tau))\, d\tau - K_p y(t) - K_d \dot{y}(t) \tag{2}$$

   so that the three dominant poles of the closed-loop system are:

$$[-1 \quad -(1 \pm j)/\sqrt{2}]$$

   Verify that the non-placed dynamic that corresponds to the actuator remains stable and significantly faster than the placed ones.

2. We now assume that **the control signal is limited**:

$$|u(t)| \leq 0.1$$

   Implement your controller and the above saturation in a simulation diagram and check its effect for constant step references $r(t) = r_0$ with growing magnitudes r_0 starting from 0.3 up to 1.

   (a) Check that the nominal behavior is not affected when $r_0 = 0.3$
   (b) What happens with $r_0 = 1$?
   (c) Determine by simulations, a critical value for which oscillations appear.
   (d) Conclude on the existence of a limit-cycle. Characterize its pulsation, magnitude and nature.

3. Confirm the above observations with the describing function method based on the first harmonic approximation.

(a) Remove the saturation block from the simulation diagram to construct the interconnection $M(s)$ "seen" by the saturation (with the negative feedback convention). Obtain a state-space representation of $M(s)$ with the linmod function.

(b) Visualize the nyquist plot of $M(s)$ and characterize possible intersections with the critical locus of the saturation.

(c) Deduce from the above intersection the possible existence of a limit-cycle. Evaluate its pulsation and magnitude. Justify the use of the approximated describing function for the saturation (that coincides with that of the relay function)

(d) Compute the ratio:

$$R = \frac{M(3j\omega_c)}{M(j\omega_c)}$$

and conclude on the validity of the approximation here. Compare the results obtained in (c) with the simulation-based values obtained in question 2.

4. We would like now to extend the operating domain of our system so that it behaves safely for $r_0 = 1$. To do so, we propose to implement a standard anti-windup strategy that consists of limiting the integral action when the saturation is active. The control law is modified as follows:

$$u(t) = K_i \int_0^t (r(\tau) - y(\tau) - K_{aw}\epsilon(\tau))\, d\tau - K_p y(t) - K_d \dot{y}(t) \qquad (3)$$

where the anti-windup signal $\epsilon(\tau)$ characterizes the activity of the saturation on $[0\,,\ t]$. In nominal conditions, when the saturation is not active, $\epsilon = 0$ and the anti-windup action is switched off.

$$\epsilon(\tau) = u(\tau) - sat(u(\tau)) \qquad (4)$$

(a) Implement this modified controller in your linear analysis oriented diagram and extract new $M(s)$ for different values of K_{aw} in the interval [0 1].

(b) Visualize on the same figure several nyquist plots and show that there exists a critical value K_{aw}^* above which no limit cycle will appear.

(c) Select and implement an anti-windup gain $K_{aw} > K_{aw}^*$ in your nonlinear simulation diagram and conclude.