Indian Institute of Technology Jodhpur

Probability, Statistics and Random Processes- MA221

Semester II (2016 - 2017)

Assignment IV

1. A driver is eagerly eyeing a precious parking space some distance down the street. There are five cars in front of the driver, each of which having a probability 0.2 of taking the space. What is the probability that the car immediately ahead will enter the parking space?

2. In oil exploration, the probability of an oil strike in the North Sea is 1 in 500 drillings. What is the probability of having exactly 3 oil-producing wells in 1000 explorations?

3. At least one half of an airplane's engines are required to function in order for it to operate. If each engine independently functions with probability p, for what value of p is a 4-engine plane more likely to operate than a 2-engine plane?

4. The time required to repair a machine is exponentially distributed with mean time 1 hour. What is the probability that a repair time exceeds 2 hours? What is the conditional probability that a repair takes at least 5 hours, given that its duration exceeds 2 hours?

5. Let X be a binomially distributed random variable with parameters n and p. Find the value of p that maximizes $P(X = k)$ for $k = 0, 1, \cdots, n$. Also, show that

$$E\left(\frac{1}{X+1}\right) = \frac{1-(1-p)^{n+1}}{(n+1)p}.$$

6. Let X be uniformly distributed random variable on the interval $(0, 1)$. Show that $Y = -\beta \ln(1 - X)$ has an exponential distribution with mean $\beta > 0$.

7. The time (in minutes) required to obtain a response in a human exposed to tear gas A has gamma distribution with paramaters $\alpha = 2$ and $\beta = 1/2$. The distribution for a second tear gas B is also gamma but has paramaters $\alpha = 1$ and $\beta = 1/4$.

   (a) Calculate the mean time required to get a response in a human exposed to each tear gas formula.

   (b) Calculate the variance for both the distributions.

   (c) Which tear gas is more likely to cause a human response in less than 1 minute?

8. Suppose that the marks on an examination are distributed normally with mean 76 and standard deviation 15. Of the best students 15% obtained A grade and of the worst students 10% lost the course and obtained F.

9. The distribution of resistance for resistors of a certain type is known to be normal, with 10% of all resistors having a resistance exceeding 10.256 ohms and 5% having a resistance smaller than 9.671 ohms. What are the mean value and standard deviation of the resistance distribution?

10. Suppose a diameter at breast height (in.) of trees of a certain type is normally distributed with mean 8.8 and standard deviation 2.8.

    (a) What is the probability that the diameter of a randomly selected tree will be at least 10 in.?

    (b) What is the probability that the diameter of a randomly selected tree will be between 5 and 10 in.?

    (c) What value of c is such that the interval $(8.8 - c, 8.8 + c)$ includes 98% of all diameter values?