# Indian Institute of Technology Jodhpur

Probability, Statistics and Random Processes- MA221

Semester II (2016 - 2017)

Assignment III

1. The time, in minutes, required for a student to travel from home to a morning class is uniformly distributed between 20 and 25. If the student leaves home promptly at 7:38 a.m., what is the probability that the student will not be late for class at 8:00 a.m.?

2. A pair of dice is rolled n times.

   (a) Find the probability that "seven" will not be shown at all.

   (b) Find the probability of obtaining double six at least once.

   (b) Find the number of throws required to assure a 50% success of obtaining double six at least once.

3. n points are randomly placed in an interval $(0, T)$. What is the probability that these points are in interval $(t_1, t_2)$?

4. A homeowner has just installed 20 light bulbs in a new home. Suppose that each has a probability 0.2 of functioning more than three months. What is the probability that at least five of these function more than three months? What is the average number of bulbs the homeowner has to replace in three months?

5. Let $X$ be a Poisson random variable with parameter $\lambda$.

   (a) Find $E(\cos(\theta X))$.

   (b) If $P(X = 2) = 9P(X = 4) + 90P(X = 6)$, then find the variance of $X$.

6. Suppose that a random variable $X$ is distributed (arbitrarily) over the interval $a < X < b$. Show that

   (a) $E(X)$ is bounded by the same limits.

   (b) $Var(X) \leq (b - a)^2/4$.

7. Let random variable $X$ be uniformly distributed over interval $0 \leq X \leq 2$. Determine a lower bound for $P(|X - 1| \leq 0.75)$ using the Chebyshev inequality and compare it with the exact value of this probability.

8. A target is made of three concentric circles of radii $3^{-1/2}, 1$ and $3^{1/2}$ feet. Shots within the inner circle count 4 points, within the next ring 3 points, and within the third ring 2 points. Shots outside of the target count 0. Let R be the random variable representing distance of the hit from the center. Suppose that the pdf of R is

$$f_X(x) = \begin{cases} \frac{2}{\pi(1+r^2)} & r > 0 \\ 0 & \text{otherwise} \end{cases}$$

Compute the mean score of each shot.

9. Let $X$ be a random variable with pdf $f(x) = \frac{e^{-x}x^3}{6}, 0 < x < \infty$. Find the moment generating function of $X$.

10. Let $X$ be a random variable with characteristic function given by $\phi_X(t) = e^{2(e^{it}-1)}$. Determine $E(2X^2 - 5X + 1)$.