Indian Institute of Technology Jodhpur

Probability, Statistics and Random Processes- MA221

Semester II (2016 - 2017)

Assignment I

1. In a city, suppose that there is a 60% chance that it will rain today, 50% chance that it will rain tomorrow, and 30% chance that it does not rain either day. Find the probabilities that

    (a) it will rain today or tomorrow.

    (b) it will rain today and tomorrow.

    (c) it will rain today but not tomorrow.

    (d) it either will rain today or tomorrow, but not both.

2. Suppose that two balls are to be randomly drawn, one after another, from a bag containing 2 black and 4 white balls. Calculate the probabilities of the events (sampling without replacement) -

    (a) The first ball is white.

    (b) The second ball is white.

    (c) The first ball is white given that the second ball is white.

3. You get a stick of length 1 unit and break it randomly into three pieces. What is the probability that you can make a triangle using the three pieces?

4. Suppose we pick a random number N from the set $\{1, 2, \cdots, 10\}$. Let A be the event that $N < 7$ and B be the event that N is even. Are A and B independent?

5. Two basketball players play a game in which they alternatively shoot a basketball at a hoop. The first one to make a basket wins the game. On each shot, player 1 has probability of success p_1, while player 2 has probability of success p_2. The shots are assumed to be independent.

    (a) Find the probability that player 1 wins the game.

    (b) What relation between p_1 and p_2 will make the game fair?

6. Let $C_1, C_2, \cdots, C_n$ be a partition of sample space S, and A and B be any two events. It is known that A and B are conditionally independent given C_i for all $i \in \{1, 2, \cdots, n\}$. Also, B is independent of all C_i's. Show that A and B are independent.

7. In a city, it's rainy one third of the days. Given that it is rainy, there will be heavy traffic with probability 1/2, and given that it is not rainy, there will be heavy traffic with probability 1/4. If it's rainy and there is heavy traffic, you arrive late for work with probability 1/2. The probability is 1/8 if it is not rainy and there is no heavy traffic.

In other two situations, rainy and no traffic, not rainy and traffic, the probability of being late is 1/4. You pick a random day.

(a) What is the probability that it's not raining and there is heavy traffic and you are late?

(b) What is the probability that you are late?

(c) What is the probability that it rained today, given that you arrived late at work?

8. A chord is drawn random in a circle of radius R. The probability that its length exceeds that of a side of an inscribed equilateral triangle can be calculated in three different ways -

(a) By using its distance d from the center.

(b) By using the angle θ it subtends at center.

(c) By using the location of its mid point.

Are these probabilities same? If not, is it a paradoxical situation?

9. Find the probability that the sum of two randomly chosen positive numbers (both ≤ 1) will not exceed 1 and that their product will be $\leq 2/9$.

10. You choose a point (A, B) randomly in the unit square $\{(x, y) : x, y \in [0, 1]\}$. What is the probability that the equation $AX^2 + X + B = 0$ has real roots.

11. A random variable X may assume 4 values with probabilities $(1+3x)/4, (1-x)/4, (1+2x)/4$ and $(1-4x)/4$. Find the condition on x so that these values represent the probability distribution of X.

12. A circular board with radius 1 is sectioned in n concentric discs with radii $\frac{1}{n}, \frac{2}{n}, \cdots, 1$. A dart is thrown randomly inside the circle. If it hits the ring between the circles of radii $\frac{i}{n}$ and $\frac{i+1}{n}$, for $i = 0, 1, \cdots, n-1$, $(n-i)$ monetary units are won. Let X be the random variable that denotes the amount of money won. Find the probability mass function of X.