Trend and Seasonality

Luana Lima

03/04/2021

Setting R code chunk options

First R code chunk is used for setting the options for all R code chunks. The choice echo=TRUE means both code and output will appear on report, include = FALSE neither code nor output is printed.

Loading packages and initializing

Second R code chunk is for loading packages. By setting message = FALSE, the code will appear but not the output.

```
library(lubridate)
library(ggplot2)
library(forecast)
library(Kendall)
library(tseries)
```

Importing data

Let's continue working with our inflow data for reservoirs in Brazil.

```
#Importing time series data from text file#
raw_inflow_data <- read.table(file="../Data/inflowtimeseries.txt",header=FALSE,skip=0)

#Trim the table to include only columns you need
nhydro <- ncol(raw_inflow_data)-2
nobs <- nrow(raw_inflow_data)

#If your file does not have header like this one you can add column names after
#creating the data frame
colnames(raw_inflow_data)=c("Month","Year", "HP1", "HP2","HP3","HP4", "HP5",
                            "HP6","HP7", "HP8","HP9","HP10", "HP11","HP12",
                            "HP13", "HP14","HP15")

#Checking data
head(raw_inflow_data)
```

```
##   Month Year  HP1  HP2  HP3  HP4  HP5  HP6 HP7  HP8 HP9 HP10 HP11 HP12 HP13
## 1   Jan 1931 4782 4076 2518 2450 2649 1462 450  968 246 2636  452 4870  452
## 2   Feb 1931 7323 7681 4188  150 2401  758 554  219  74 4158  457 4550  796
## 3   Mar 1931 8266 5921 3253 2389 3261  707 615  333 123 3847  631 6537  804
## 4   Apr 1931 6247 4600 2449 1253 2006  469 474  297 113 3291  510 7298  644
## 5   May 1931 3642 2789 1651 2374 2454 3167 378 3295 938 1956  276 4942  421
## 6   Jun 1931 2425 2062 1270 2672 2433 3236 301 2547 951 1371  201 2478  305
##    HP14  HP15
## 1 17342 31270
## 2 21530 43827
## 3 33299 49884
## 4 34674 43962
## 5 15184 35156
## 6  8611 25764
```

```
str(raw_inflow_data)
```

```
## 'data.frame':    972 obs. of  17 variables:
##  $ Month: chr  "Jan" "Feb" "Mar" "Apr" ...
##  $ Year : int  1931 1931 1931 1931 1931 1931 1931 1931 1931 1931 ...
##  $ HP1  : int  4782 7323 8266 6247 3642 2425 2158 1854 1839 1896 ...
##  $ HP2  : int  4076 7681 5921 4600 2789 2062 1644 1301 1439 1340 ...
##  $ HP3  : int  2518 4188 3253 2449 1651 1270 1204 1152 1297 1259 ...
##  $ HP4  : int  2450 150 2389 1253 2374 2672 1238 605 1016 674 ...
##  $ HP5  : int  2649 2401 3261 2006 2454 2433 1798 1160 1584 1563 ...
##  $ HP6  : int  1462 758 707 469 3167 3236 1957 844 1937 1484 ...
##  $ HP7  : int  450 554 615 474 378 301 256 244 222 355 ...
##  $ HP8  : int  968 219 333 297 3295 2547 2585 1173 3596 1140 ...
##  $ HP9  : int  246 74 123 113 938 951 883 404 378 211 ...
##  $ HP10 : int  2636 4158 3847 3291 1956 1371 1186 1049 1162 1507 ...
##  $ HP11 : int  452 457 631 510 276 201 213 196 161 208 ...
##  $ HP12 : int  4870 4550 6537 7298 4942 2478 1905 1647 1453 1358 ...
##  $ HP13 : int  452 796 804 644 421 305 261 246 250 328 ...
##  $ HP14 : int  17342 21530 33299 34674 15184 8611 5939 4259 3282 3305 ...
##  $ HP15 : int  31270 43827 49884 43962 35156 25764 18109 13320 8225 8900 ...
```

Creating the date object

Here we use the function my() from package lubridate.

```
#using package lubridate
my_date <- paste(raw_inflow_data[,1],raw_inflow_data[,2],sep="-")
my_date <- my(my_date)  #function my from package lubridate
head(my_date)
```

```
## [1] "1931-01-01" "1931-02-01" "1931-03-01" "1931-04-01" "1931-05-01"
## [6] "1931-06-01"
```

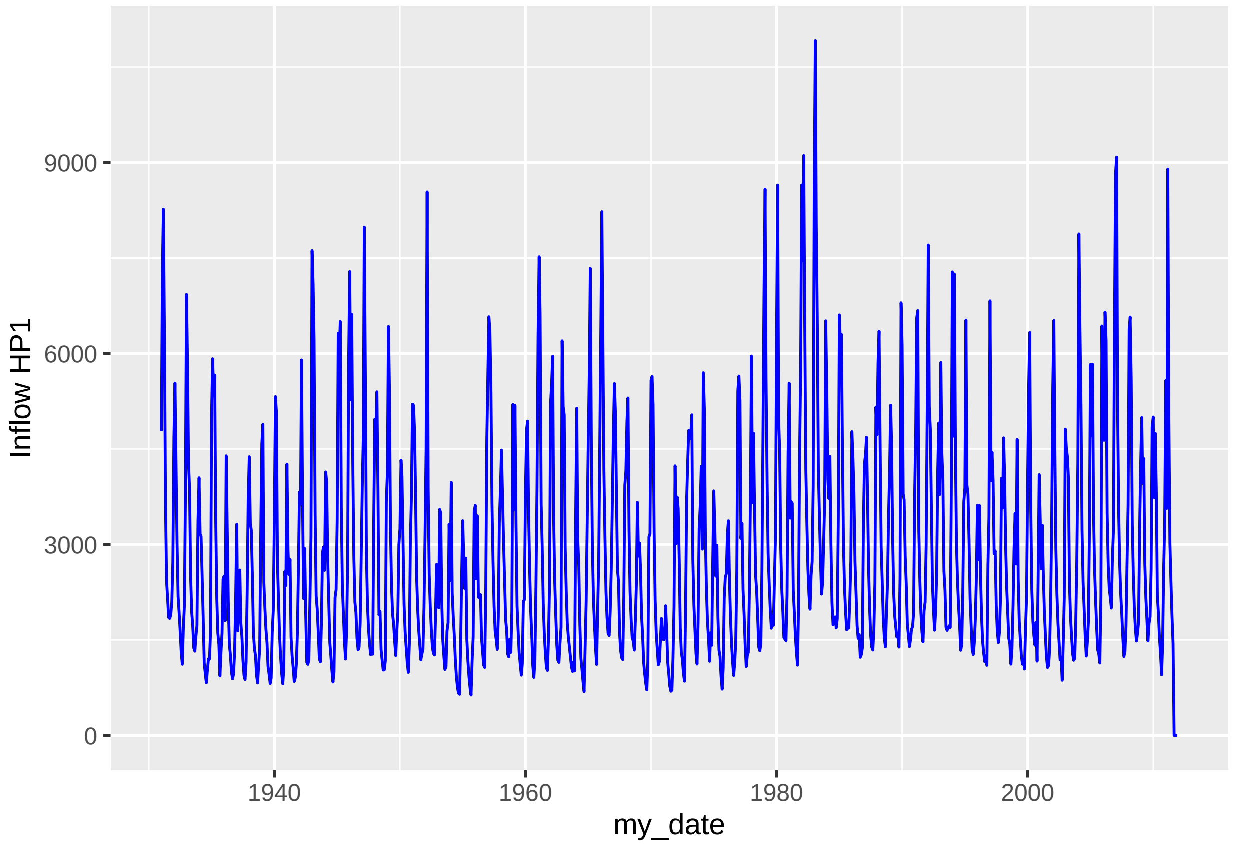
```
#add that to inflow_data and store in a new data frame
inflow_data <- cbind(my_date,raw_inflow_data[,3:(3+nhydro-1)])
head(inflow_data)
```

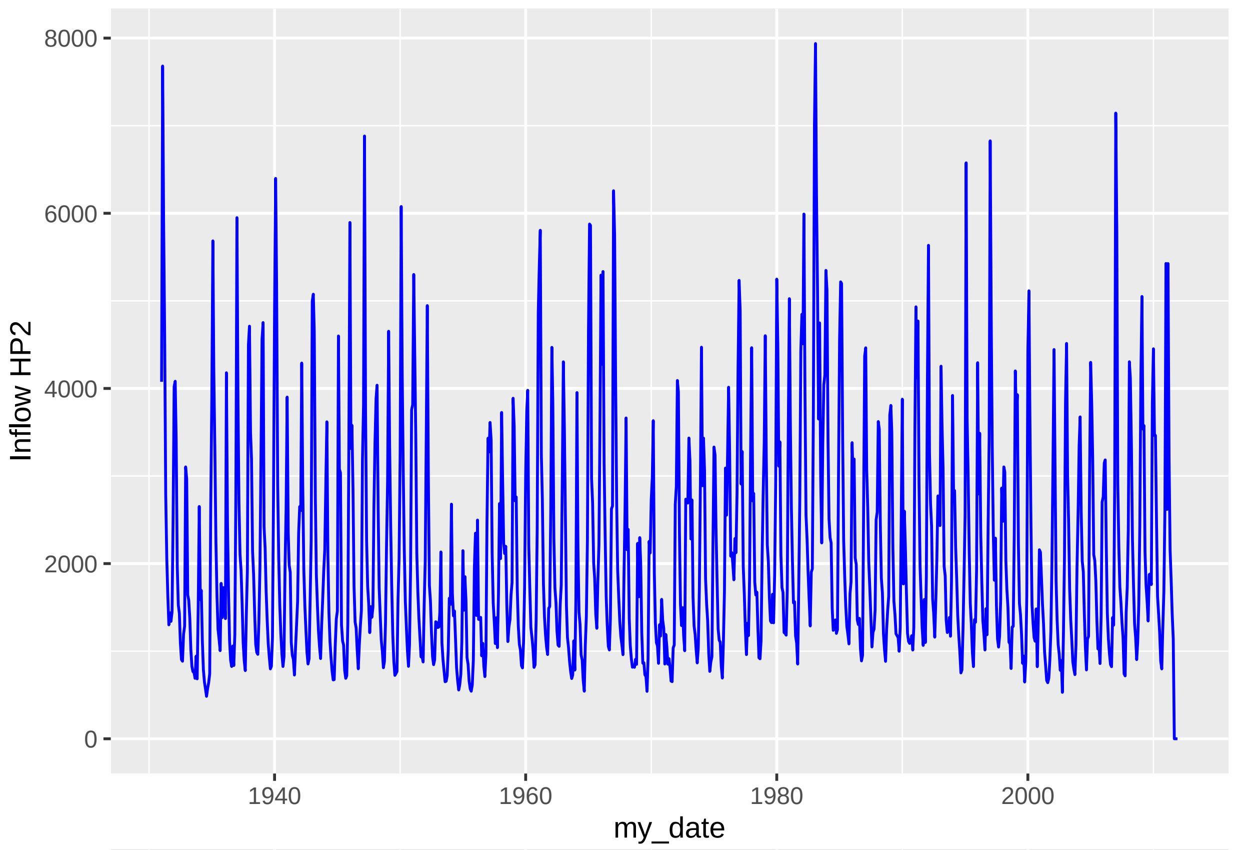
```
##      my_date  HP1  HP2  HP3  HP4  HP5  HP6 HP7  HP8 HP9 HP10 HP11 HP12 HP13
## 1 1931-01-01 4782 4076 2518 2450 2649 1462 450  968 246 2636  452 4870  452
## 2 1931-02-01 7323 7681 4188  150 2401  758 554  219  74 4158  457 4550  796
## 3 1931-03-01 8266 5921 3253 2389 3261  707 615  333 123 3847  631 6537  804
## 4 1931-04-01 6247 4600 2449 1253 2006  469 474  297 113 3291  510 7298  644
## 5 1931-05-01 3642 2789 1651 2374 2454 3167 378 3295 938 1956  276 4942  421
## 6 1931-06-01 2425 2062 1270 2672 2433 3236 301 2547 951 1371  201 2478  305
##    HP14  HP15
## 1 17342 31270
## 2 21530 43827
## 3 33299 49884
## 4 34674 43962
## 5 15184 35156
## 6  8611 25764
```

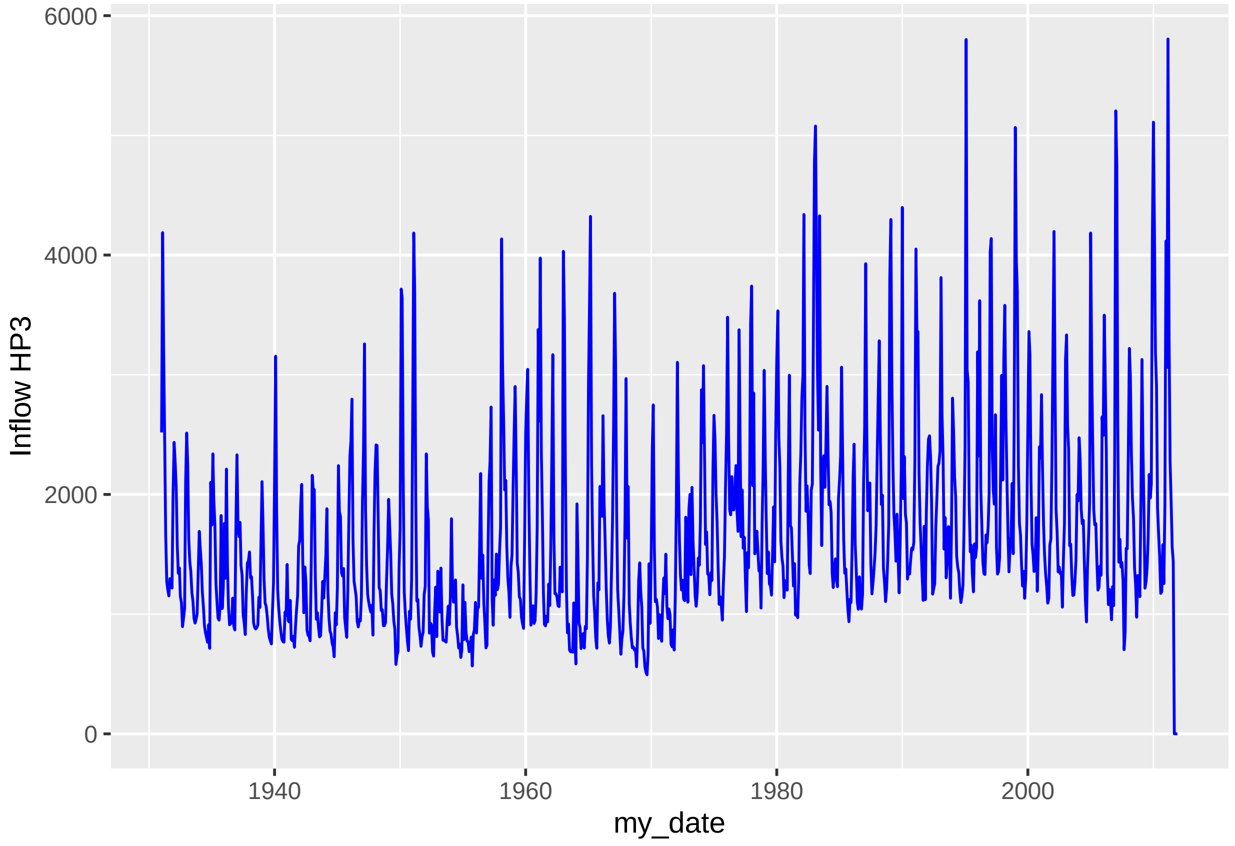
Initial Plots

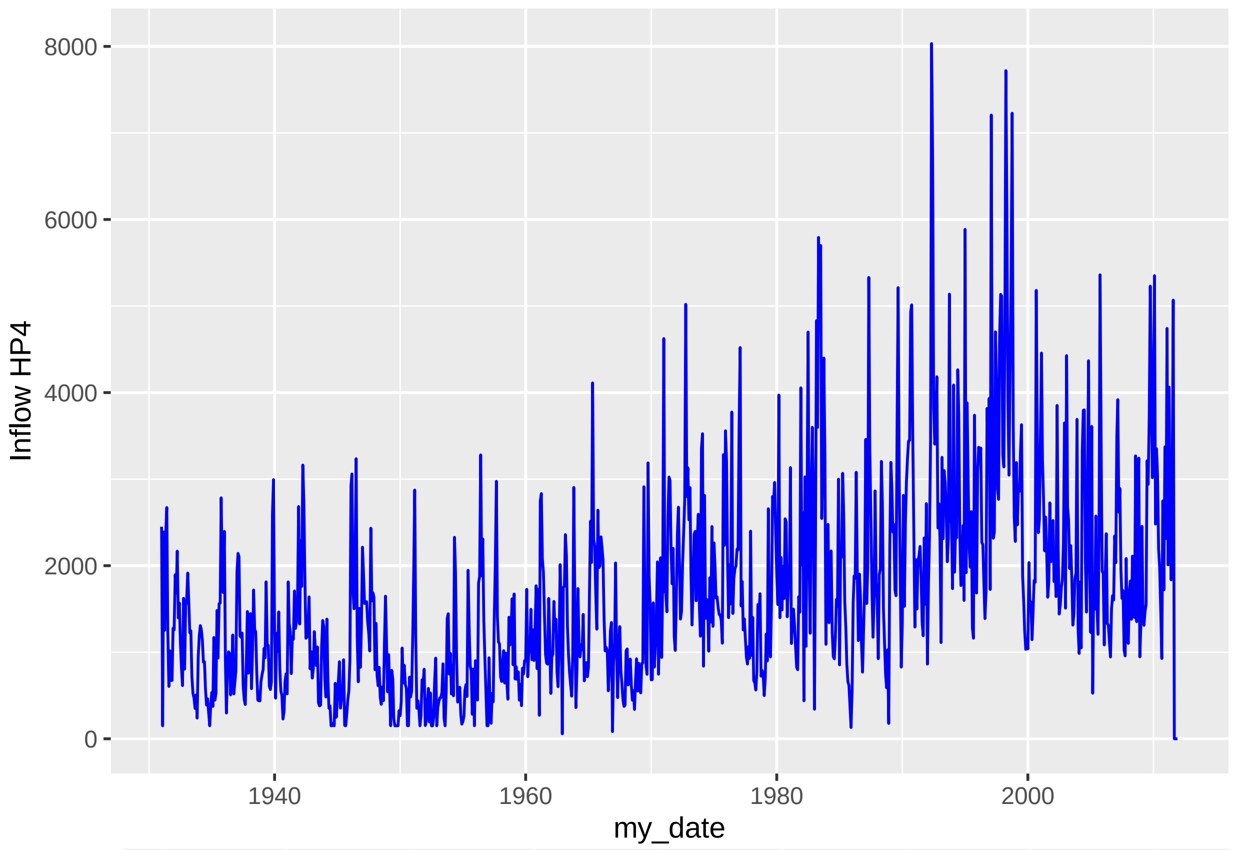
Initial time series plot.

```
#using package ggplot2
for(i in 1:nhydro){
  print(ggplot(inflow_data, aes(x=my_date, y=inflow_data[,(1+i)])) +
            geom_line(color="blue") +
            ylab(paste0("Inflow ",colnames(inflow_data)[(1+i)],sep=""))
        )
}
```

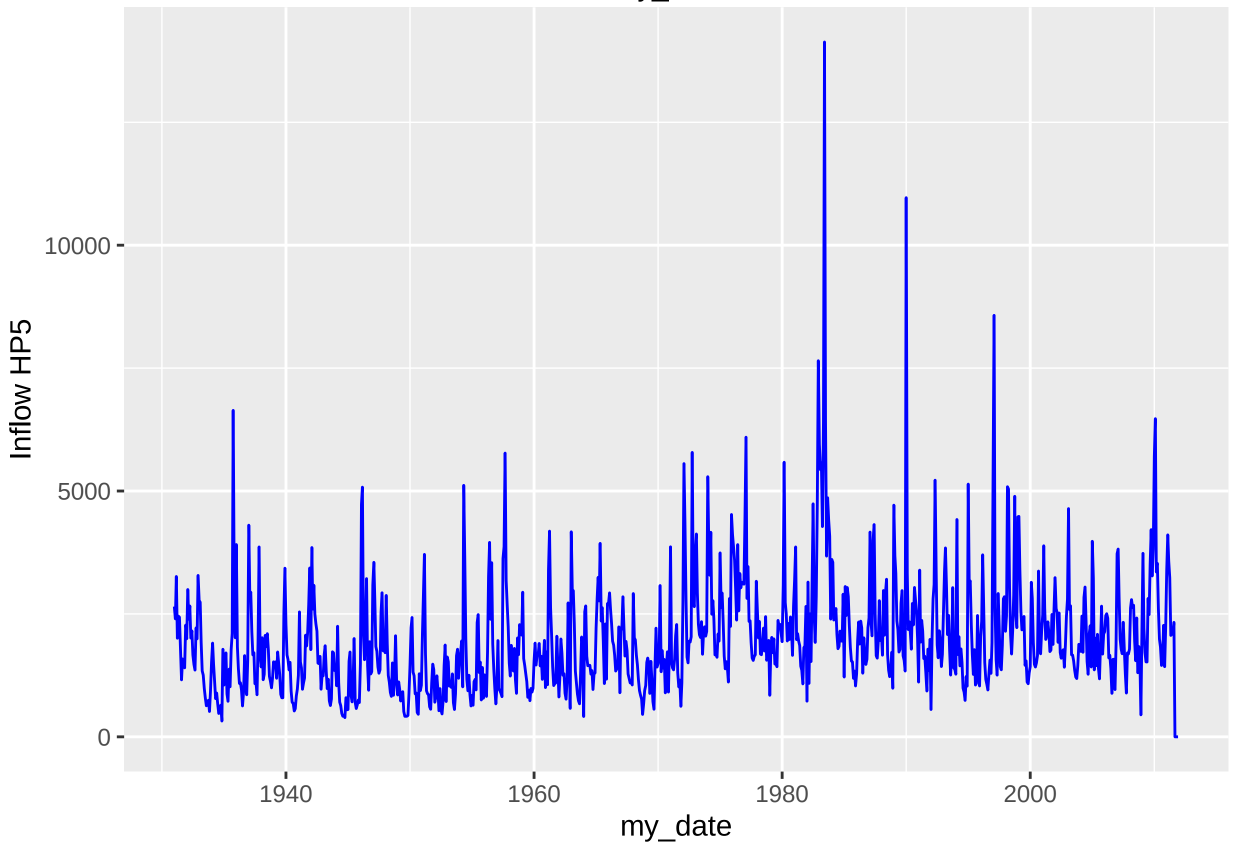



Inflow HP2
8000
6000
4000
2000
0
1940
1960
1980
2000
my_date

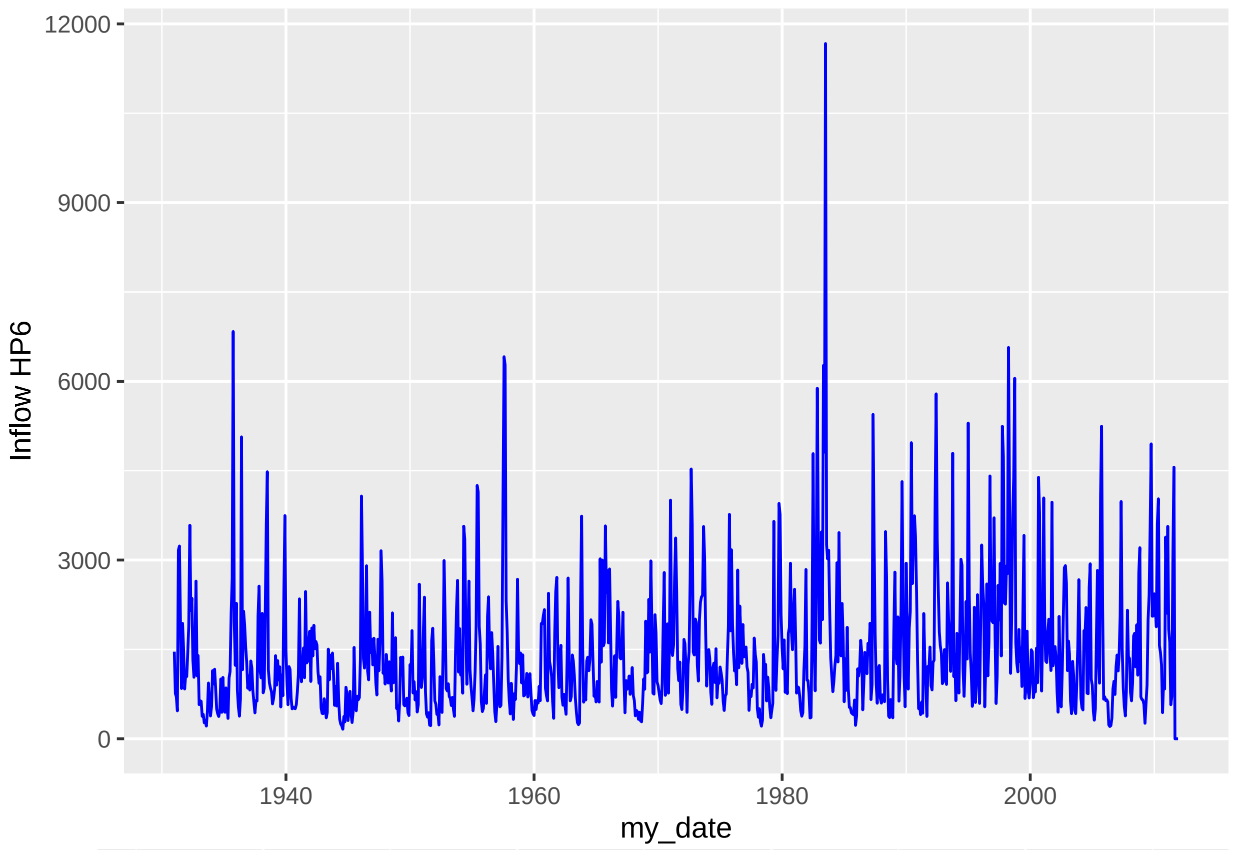

Inflow HP3
6000
4000
2000
0
1940
1960
1980
2000
my_date

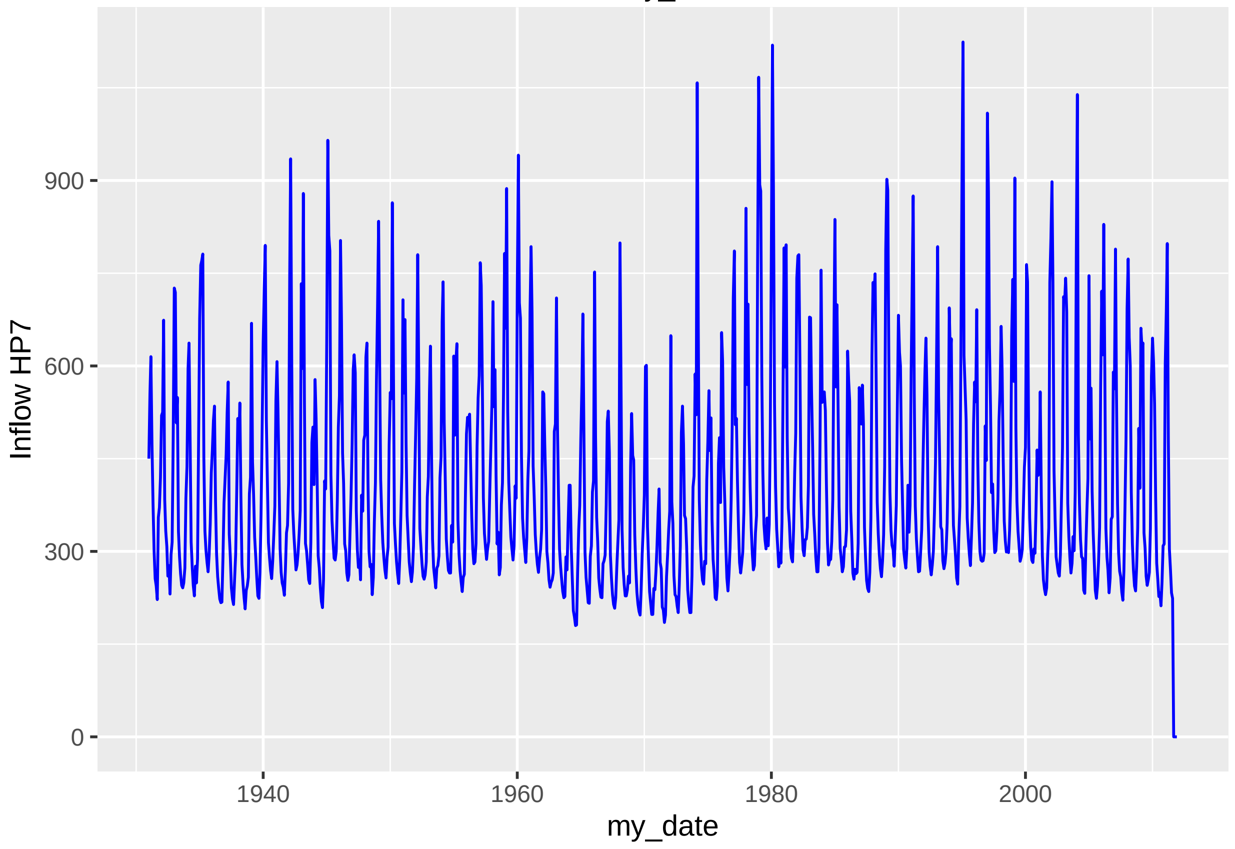


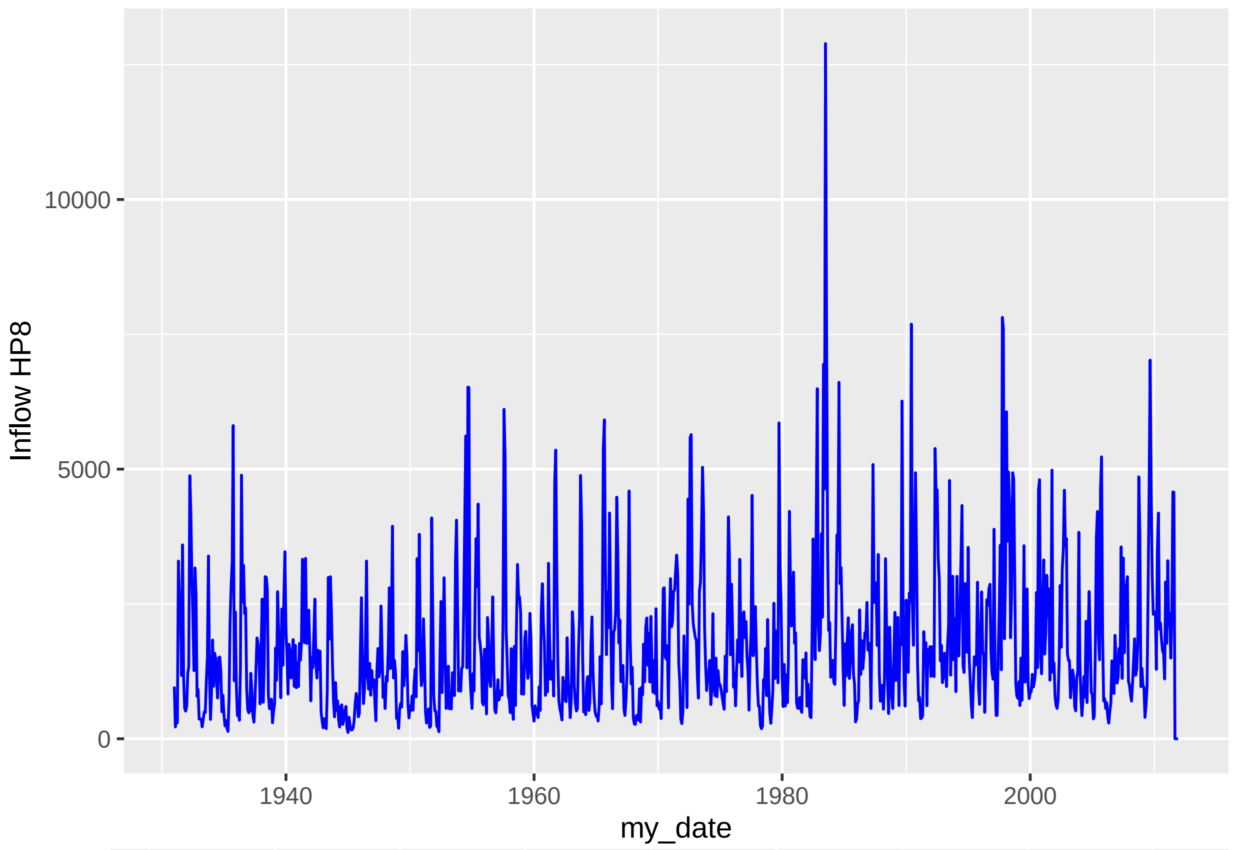
Inflow HP4
8000
6000
4000
2000
0
1940
1960
1980
2000
my_date

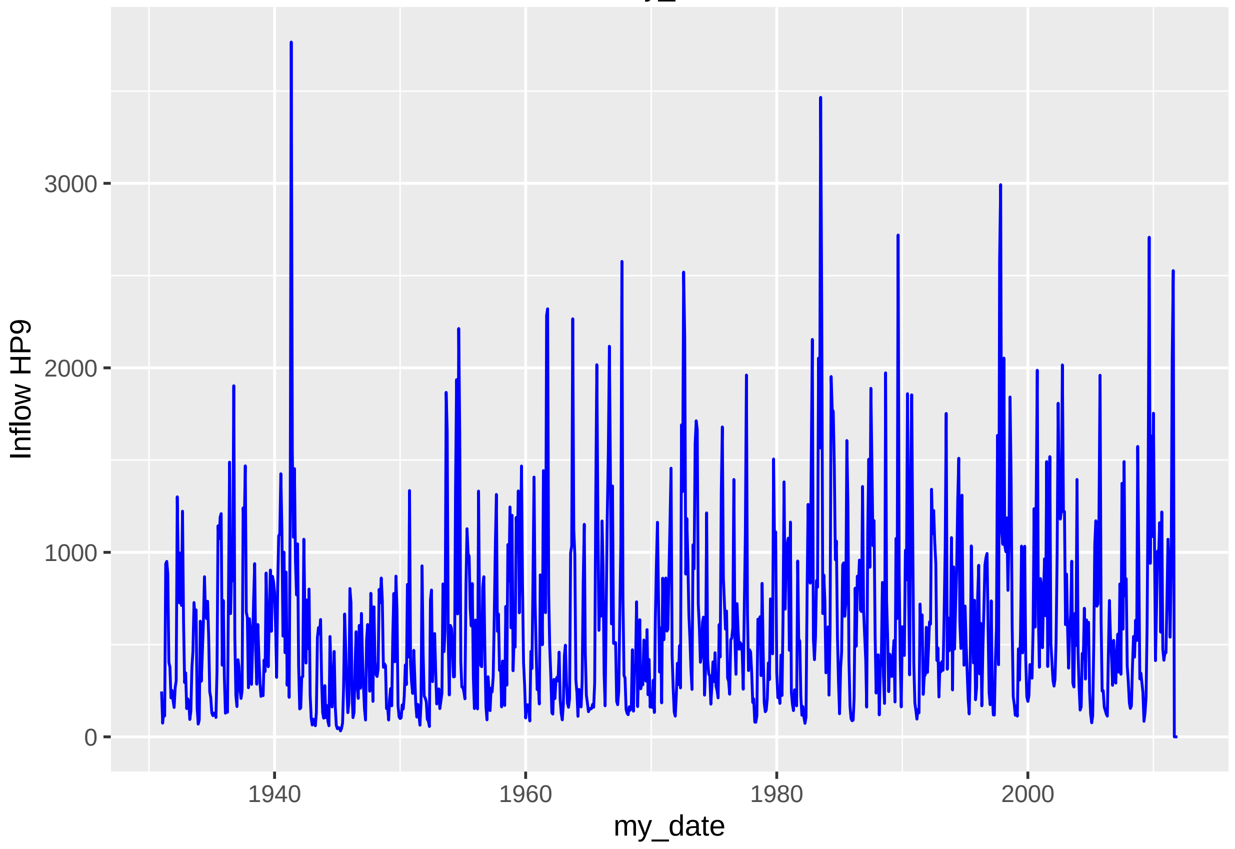


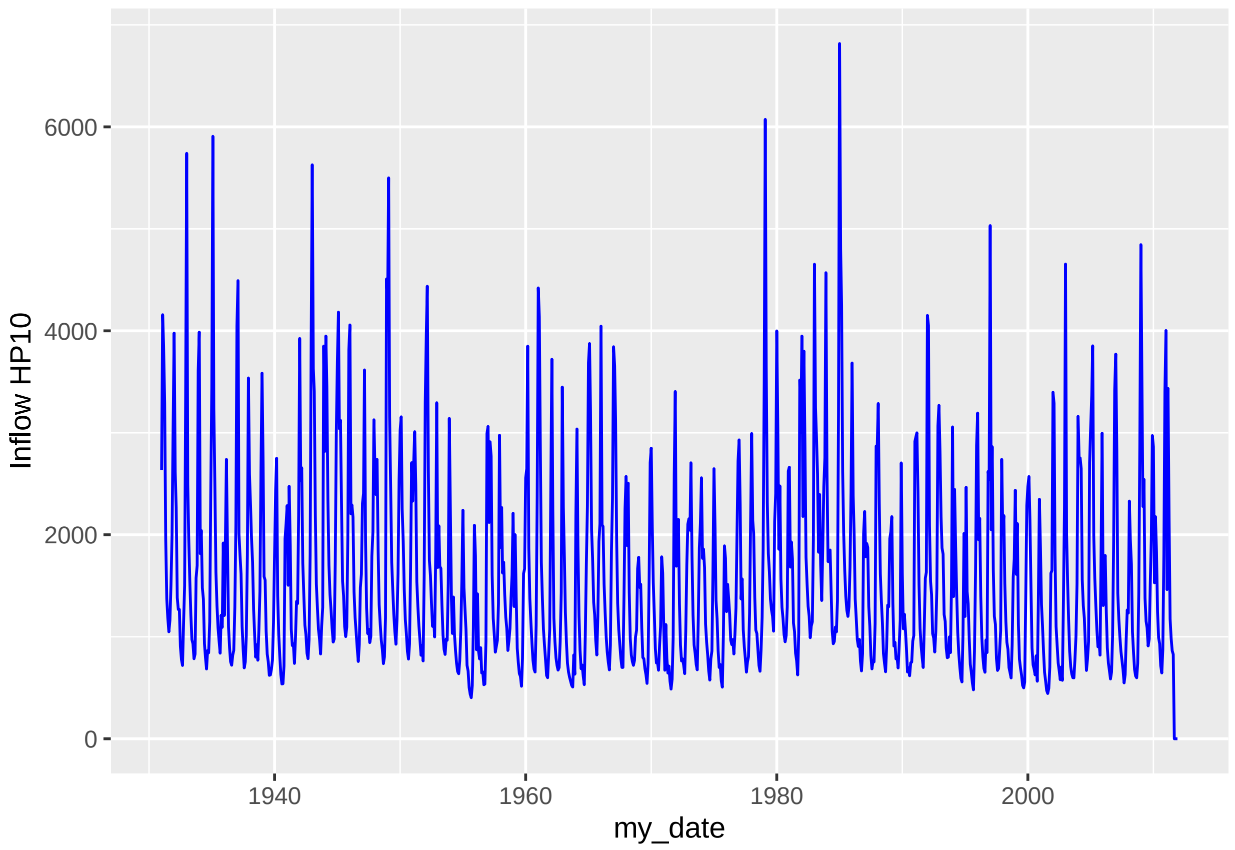
Inflow HP5
10000
5000
0
1940
1960
1980
2000
my_date

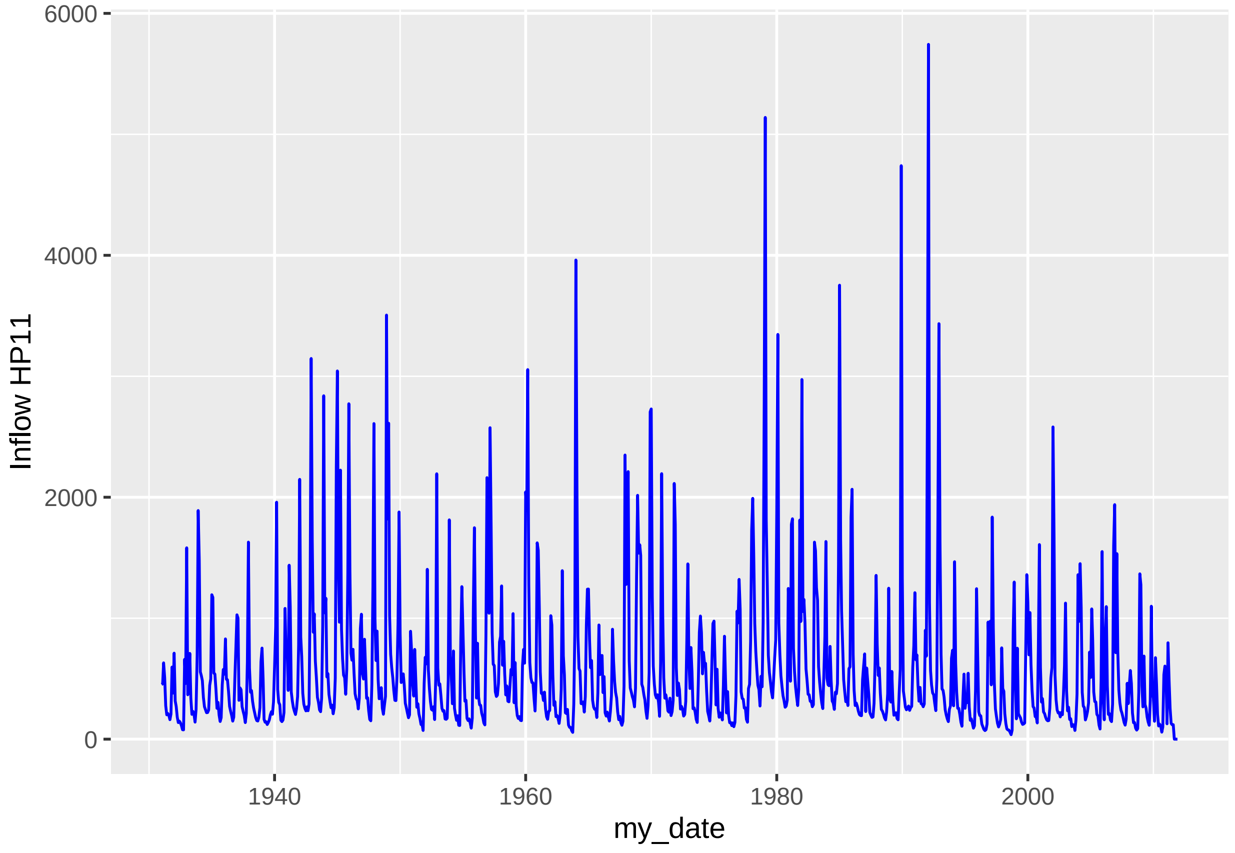

12000
9000
6000
3000
0
Inflow HP6
1940
1960
1980
2000
my_date

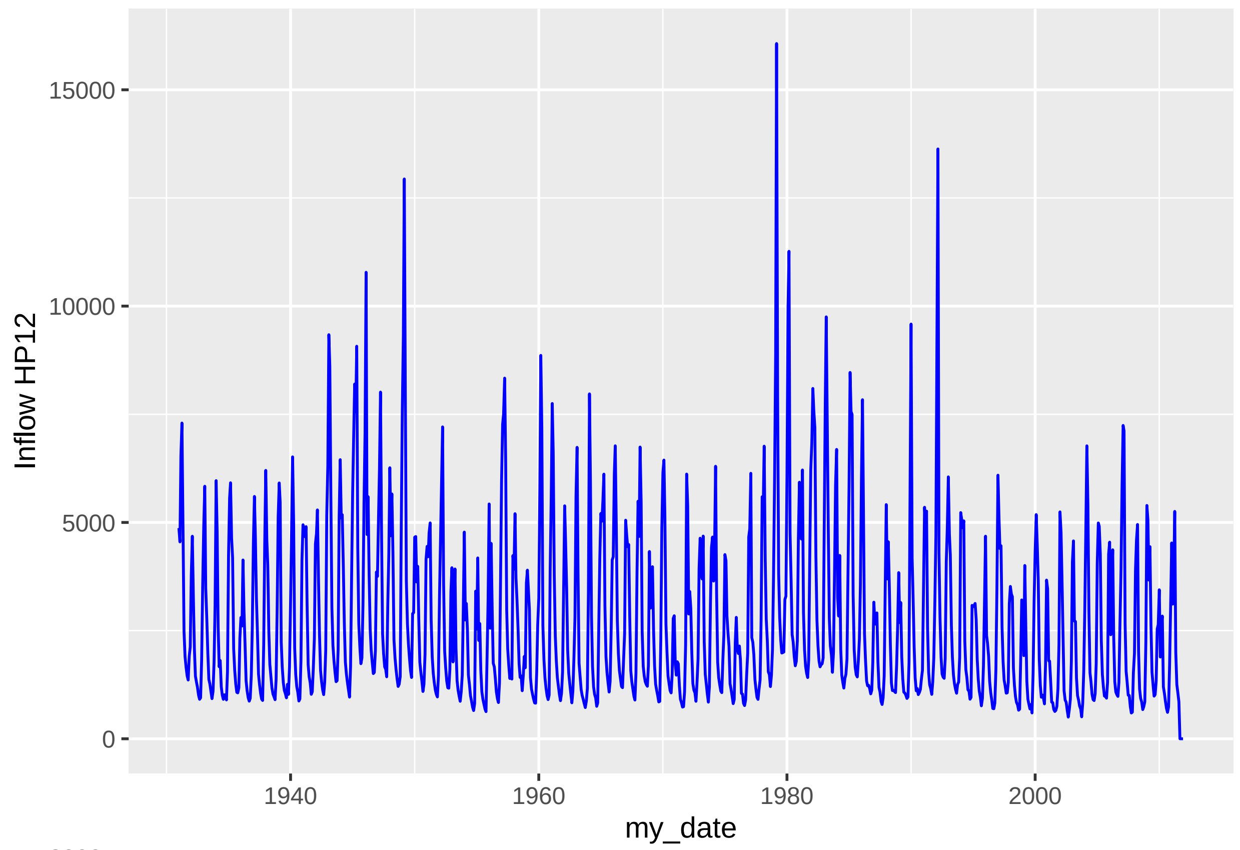
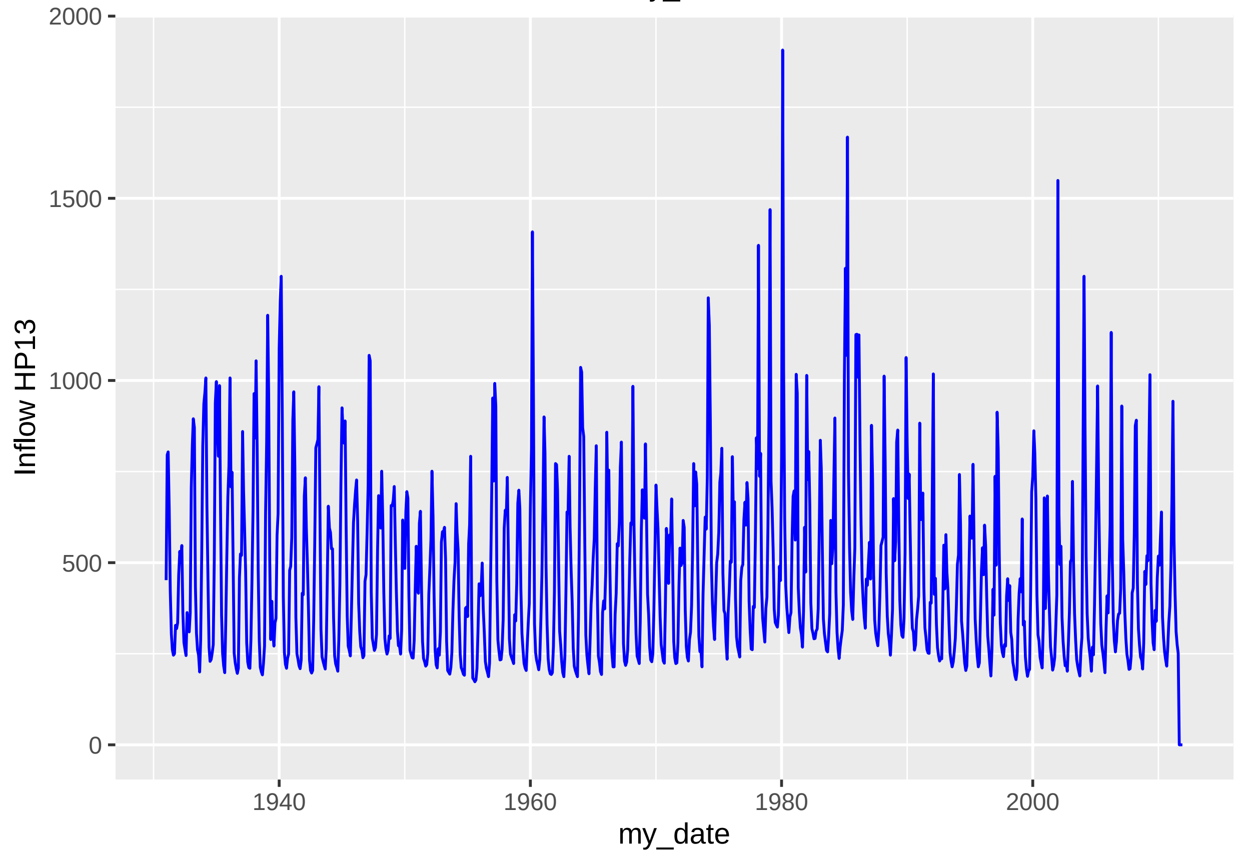

900
600
300
0
Inflow HP7
1940
1960
1980
2000
my_date

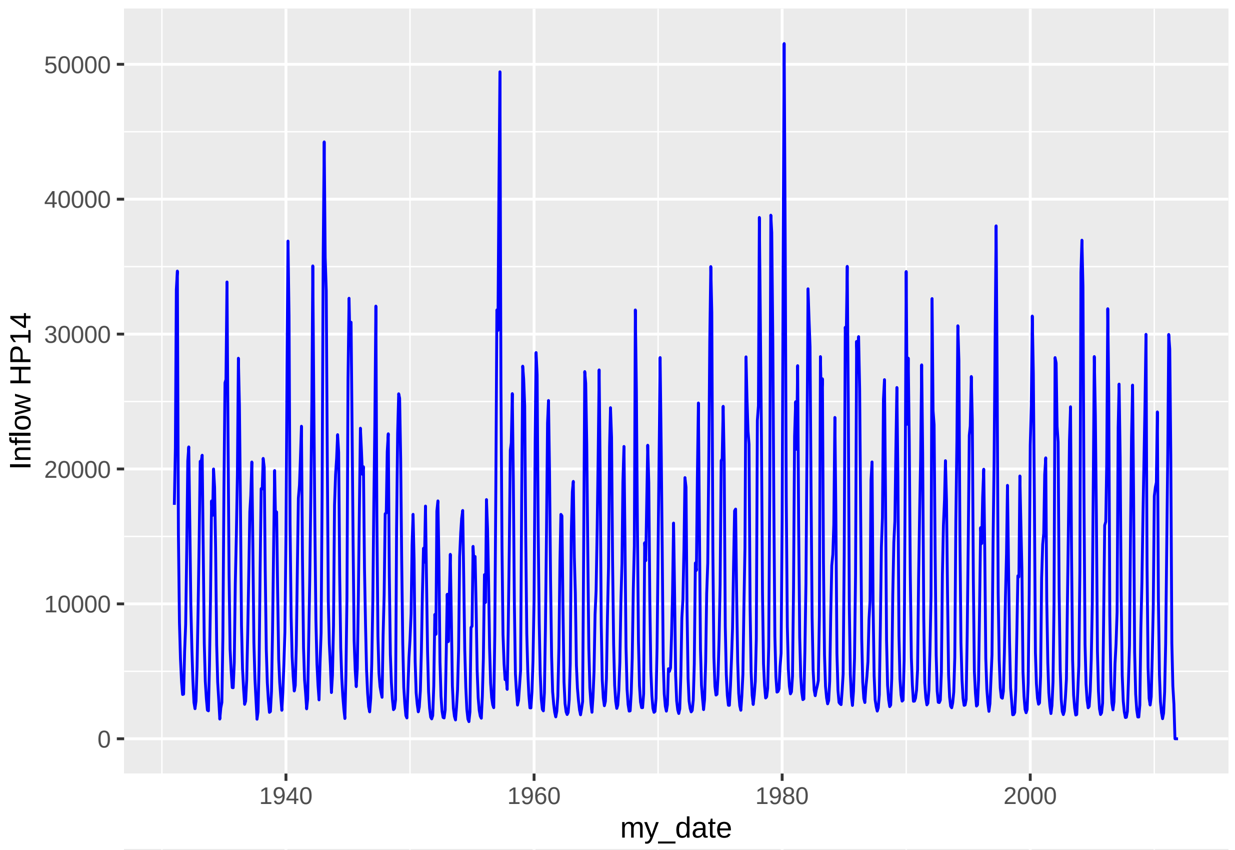

Inflow HP8
10000
5000
0
1940
1960
1980
2000
my_date

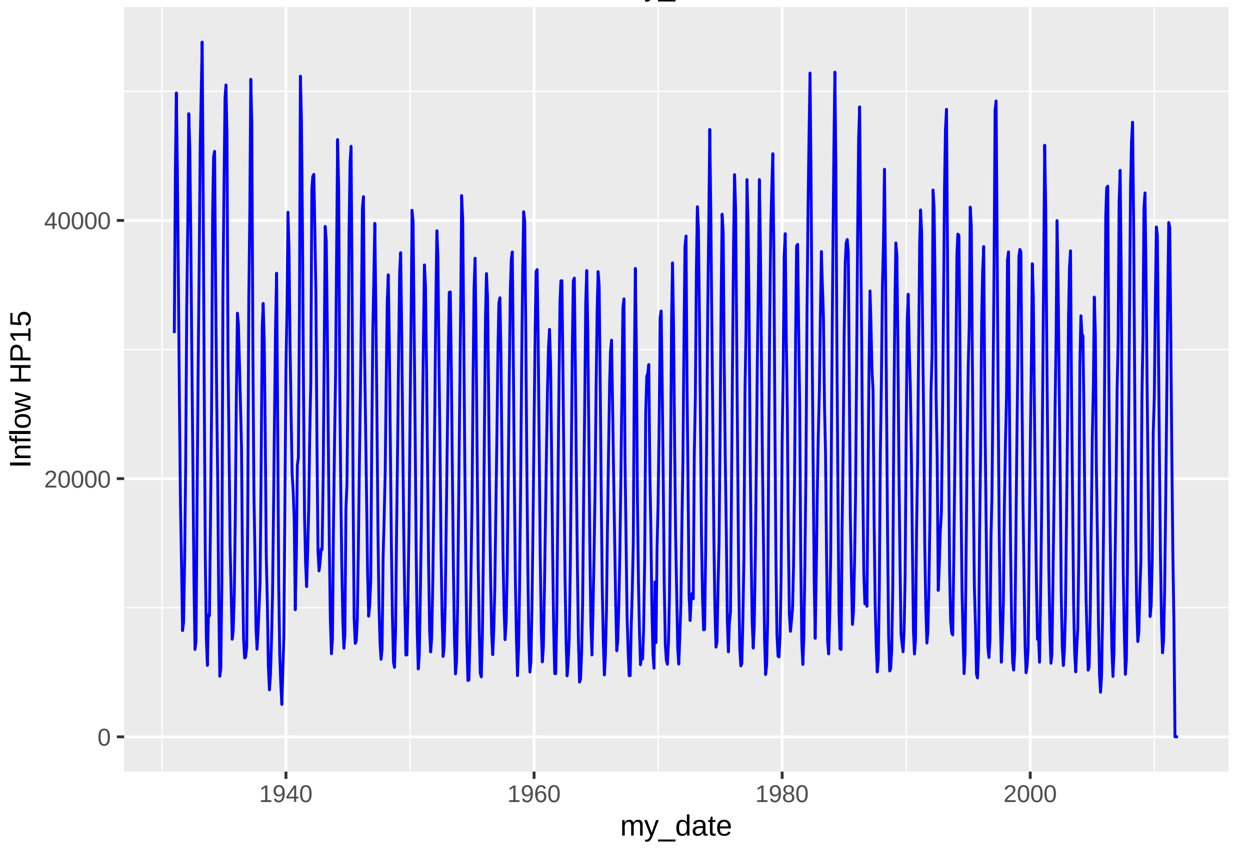

Inflow HP9
3000
2000
1000
0
1940
1960
1980
2000
my_date


6000
4000
2000
0
Inflow HP10
1940
1960
1980
2000
my_date


6000
4000
2000
0
Inflow HP11
1940
1960
1980
2000
my_date


Inflow HP12
15000
10000
5000
0
1940
1960
1980
2000
my_date

Inflow HP13
2000
1500
1000
500
0
1940
1960
1980
2000
my_date





## Zeros in the end on data

The initial plots showed that we have zeros in the end of the data set. It could be missing observation or observation that haven't been observed yet. Use the tail() to find out how many zeros you have and how many lines you will need to remove.

```r
#check the final obs on data
tail(inflow_data)
```

```
##        my_date  HP1  HP2  HP3  HP4  HP5  HP6 HP7  HP8  HP9 HP10 HP11 HP12 HP13
## 967 2011-07-01 1883 1426 1560 2930 2105 2988 233 4578 2045  864  119 1068  275
## 968 2011-08-01 1444 1139 1441 5069 2328 4559 224 4573 2527  827  120  854  251
## 969 2011-09-01    0    0    0    0    0    0   0    0    0    0    0    0    0
## 970 2011-10-01    0    0    0    0    0    0   0    0    0    0    0    0    0
## 971 2011-11-01    0    0    0    0    0    0   0    0    0    0    0    0    0
## 972 2011-12-01    0    0    0    0    0    0   0    0    0    0    0    0    0
##     HP14  HP15
## 967 3910 14162
## 968 2561  8896
## 969    0     0
## 970    0     0
## 971    0     0
## 972    0     0
```

Note our last observation is from August 2011 but the data file was filled with zeros. Let's remove the last four rows of our data set.

```r
#Remove last for rows by replacing current data frame
inflow_data <- inflow_data[1:(nobs-4),]

#update object with number of observations
nobs <- nobs-4

#Tail again to check if the rows were correctly removed
tail(inflow_data)
```

```
##        my_date  HP1  HP2  HP3  HP4  HP5  HP6 HP7  HP8  HP9 HP10 HP11 HP12 HP13
## 963 2011-03-01 8897 5426 5805 2009 3576 1834 798 2097 1071 3435  797 3693  943
## 964 2011-04-01 4991 3207 3323 4063 3235 1620 481 2325  902 2173  493 5255  563
## 965 2011-05-01 3025 2156 2274 2351 2063  572 304 1496  540 1175  254 1998  415
## 966 2011-06-01 2415 1813 1936 1836 2087  713 270 2294  898  985  130 1256  311
## 967 2011-07-01 1883 1426 1560 2930 2105 2988 233 4578 2045  864  119 1068  275
## 968 2011-08-01 1444 1139 1441 5069 2328 4559 224 4573 2527  827  120  854  251
##      HP14  HP15
## 963 29976 39843
## 964 28892 39441
## 965 20978 31023
## 966  7081 21840
## 967  3910 14162
## 968  2561  8896
```

Fixed!

Transforming data into time series object

Many of the functions we will use require a time series object. You can transform your data in a time series using the function *ts()*.

```
ts_inflow_data <- ts(inflow_data[,2:(2+nhydro-1)],frequency=12)
#note that we are only transforming columns with inflow data, not the date columns  #start=my_date[1],e
head(ts_inflow_data,15)
```

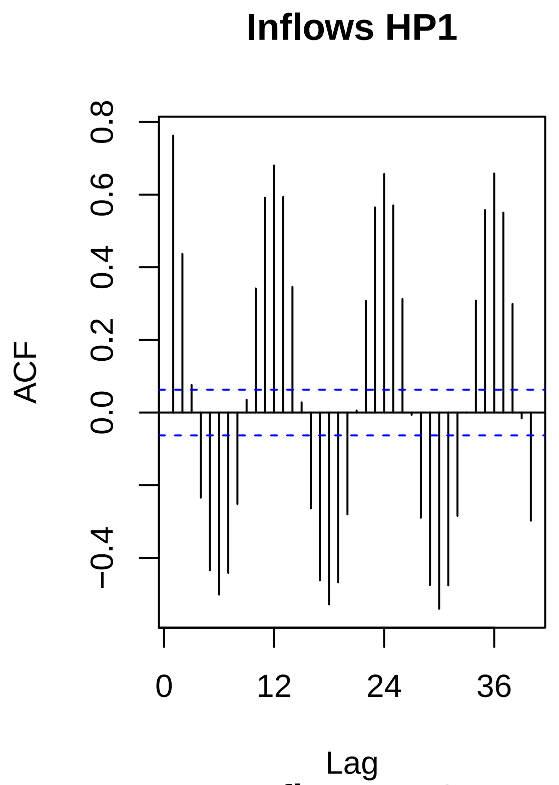
```
##        HP1  HP2  HP3  HP4  HP5  HP6 HP7  HP8 HP9 HP10 HP11 HP12 HP13  HP14
## Jan 1 4782 4076 2518 2450 2649 1462 450  968 246 2636  452 4870  452 17342
## Feb 1 7323 7681 4188  150 2401  758 554  219  74 4158  457 4550  796 21530
## Mar 1 8266 5921 3253 2389 3261  707 615  333 123 3847  631 6537  804 33299
## Apr 1 6247 4600 2449 1253 2006  469 474  297 113 3291  510 7298  644 34674
## May 1 3642 2789 1651 2374 2454 3167 378 3295 938 1956  276 4942  421 15184
## Jun 1 2425 2062 1270 2672 2433 3236 301 2547 951 1371  201 2478  305  8611
## Jul 1 2158 1644 1204 1238 1798 1957 256 2585 883 1186  213 1905  261  5939
## Aug 1 1854 1301 1152  605 1160  844 244 1173 404 1049  196 1647  246  4259
## Sep 1 1839 1439 1297 1016 1584 1937 222 3596 378 1162  161 1453  250  3282
## Oct 1 1896 1340 1259  674 1563 1484 355 1140 211 1507  208 1358  328  3305
## Nov 1 2095 1447 1218  674 1404  835 371  563 252 1996  596 1905  319  6500
## Dec 1 2725 2479 2013 1278 2272 1073 419  512 197 3015  381 2121  335  8461
## Jan 2 4679 4021 2435 1259 1995 1044 520  609 159 3978  711 3811  467 14002
## Feb 2 5535 4082 2262 1895 2996 1454 525 1219 268 2615  316 4681  531 20596
## Mar 2 4310 3398 2065 1686 2392 1888 674 1332 304 2269  271 3329  501 21638
##        HP15
## Jan 1 31270
## Feb 1 43827
## Mar 1 49884
## Apr 1 43962
## May 1 35156
## Jun 1 25764
## Jul 1 18109
## Aug 1 13320
## Sep 1  8225
## Oct 1  8900
## Nov 1 13766
## Dec 1 20880
## Jan 2 33160
## Feb 2 39791
## Mar 2 48274
```

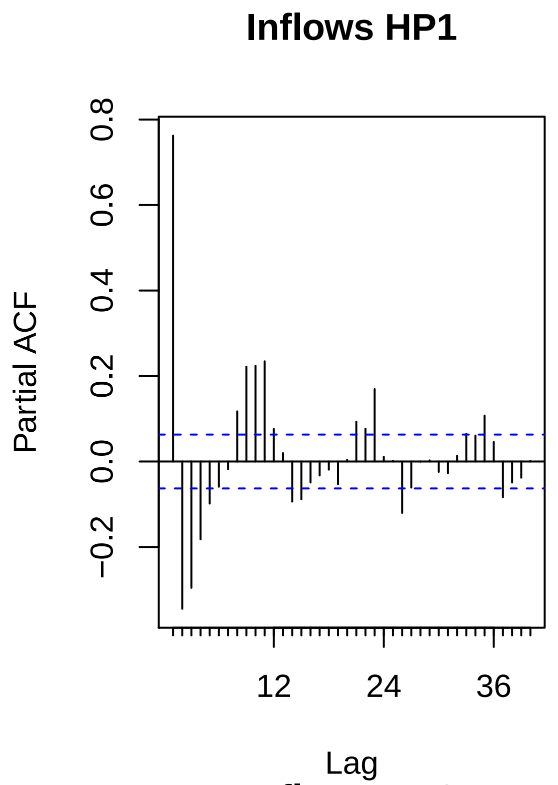
Note that ts_inflow_data has information on start, end and frequency.

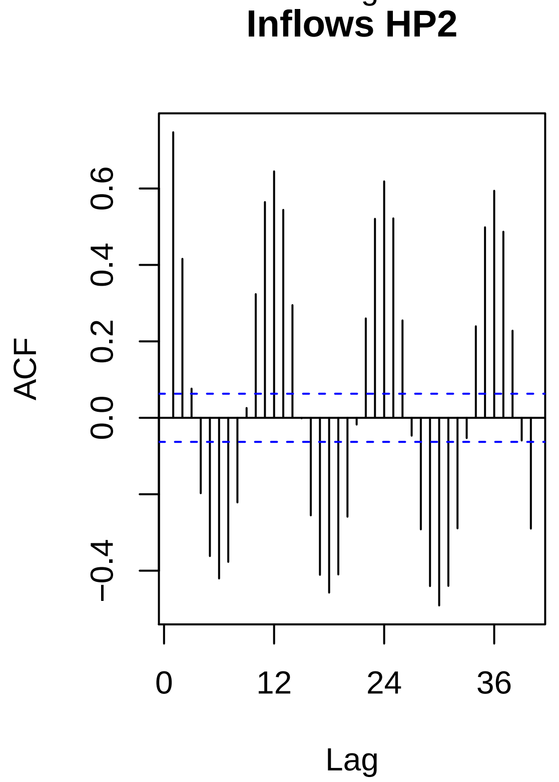
Plotting ACF and PACF

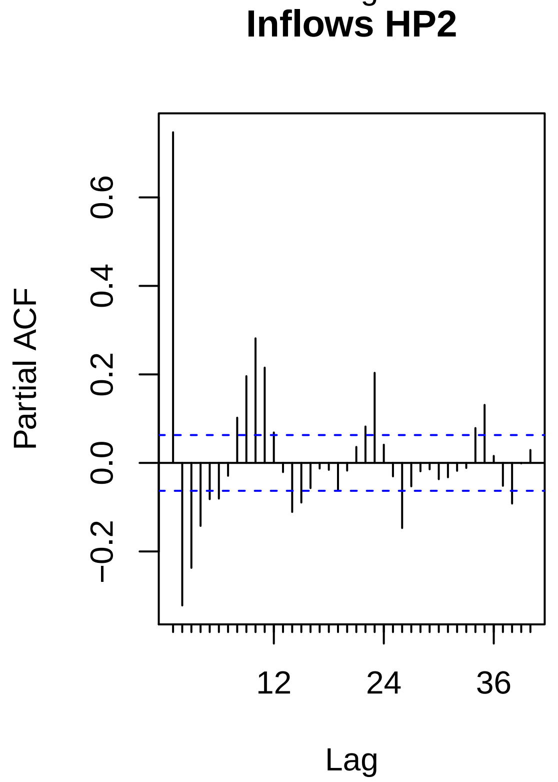
Let's use functions Acf() and Pacf() from package "forecast".

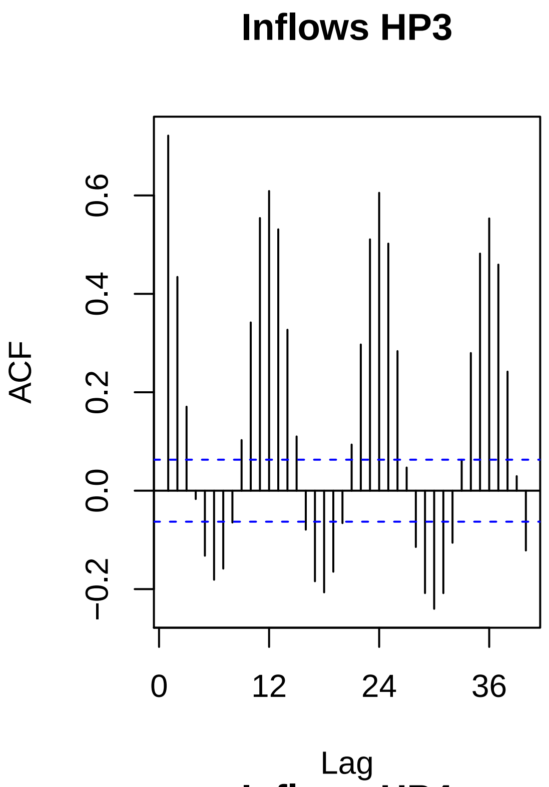
```
#Acf and Pacf for HP1
for(i in 1:nhydro){
  par(mfrow=c(1,2))  #place plot side by side
  Acf(ts_inflow_data[,i],lag.max=40,main=paste("Inflows HP",i,sep=""))
  # because I am not storing Acf() into any object, I don't need to specify plot=TRUE
  Pacf(ts_inflow_data[,i],lag.max=40,main=paste("Inflows HP",i,sep=""))
}
```

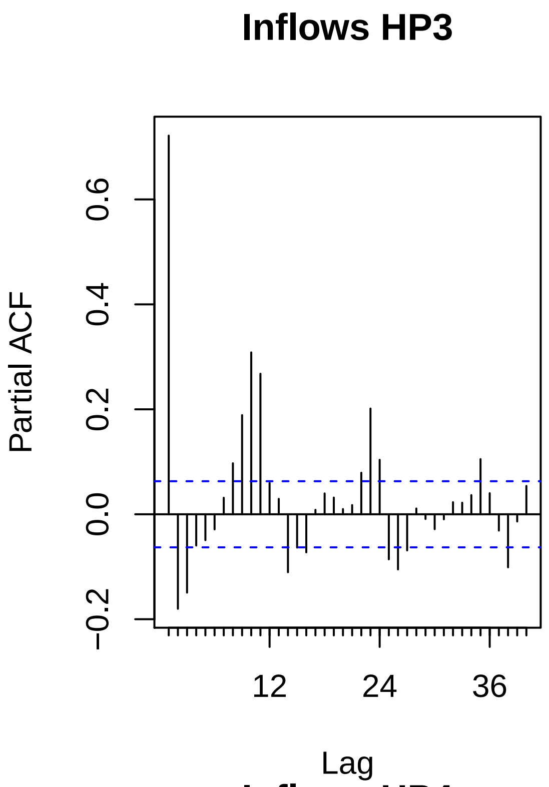

Inflows HP1
ACF
0.8
0.6
0.4
0.2
0.0
−0.4
0
12
24
36
Lag

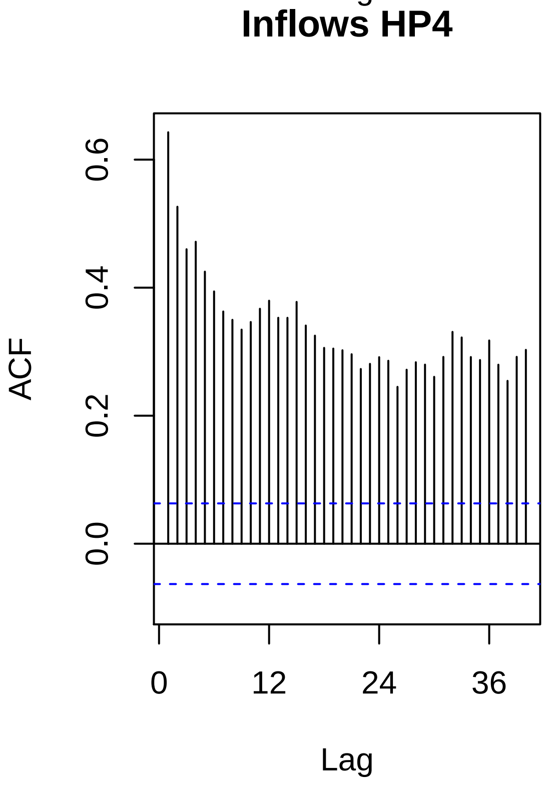

Inflows HP1
Partial ACF
0.8
0.6
0.4
0.2
0.0
−0.2
12
24
36
Lag

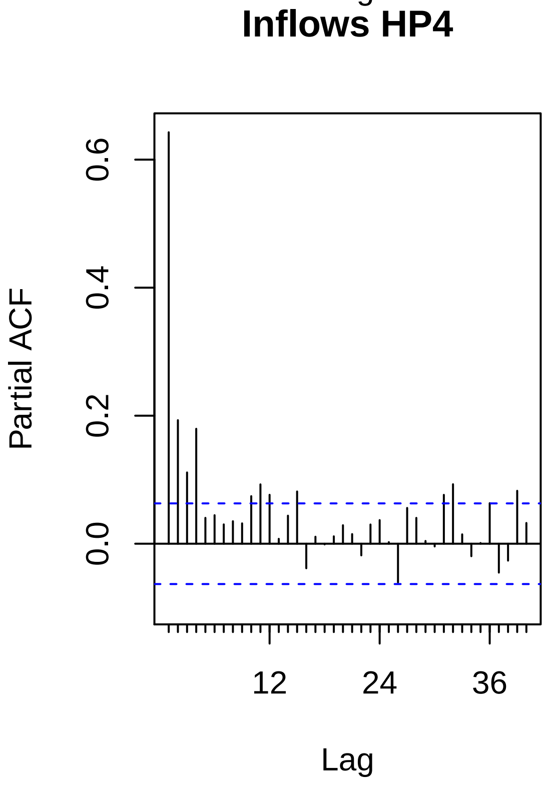

Inflows HP2
ACF
0.6
0.4
0.2
0.0
−0.4
0
12
24
36
Lag

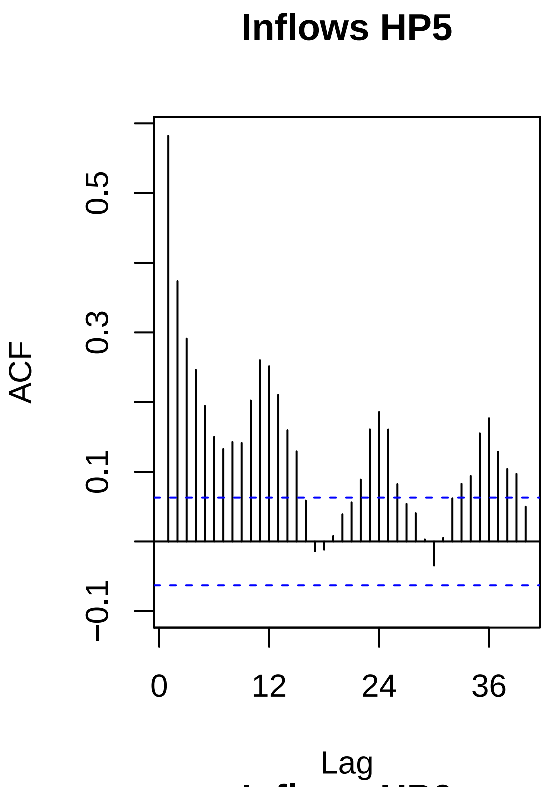

Inflows HP2
Partial ACF
0.6
0.4
0.2
0.0
−0.2
12
24
36
Lag

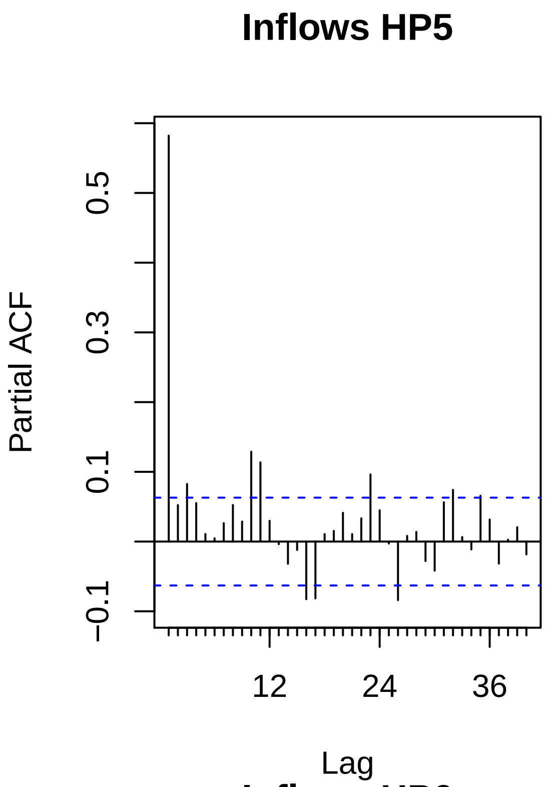

Inflows HP3
ACF
0.6
0.4
0.2
0.0
−0.2
0
12
24
36
Lag

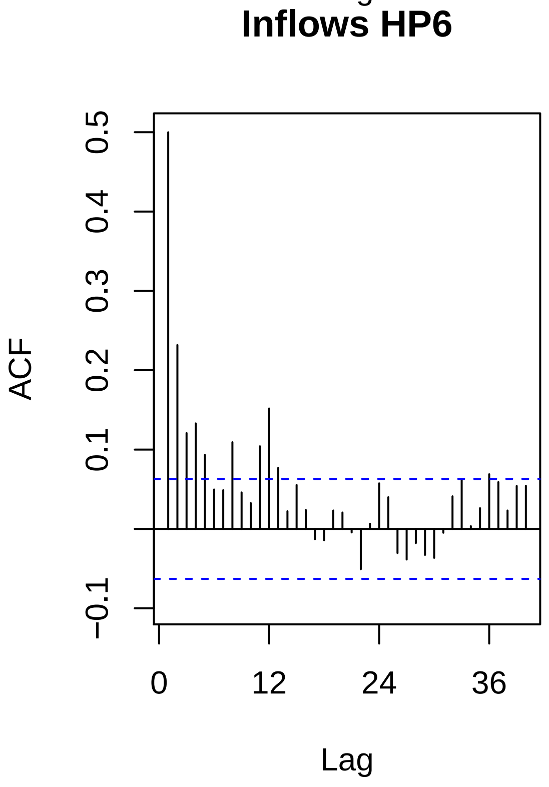

Inflows HP3
Partial ACF
0.6
0.4
0.2
0.0
−0.2
12
24
36
Lag

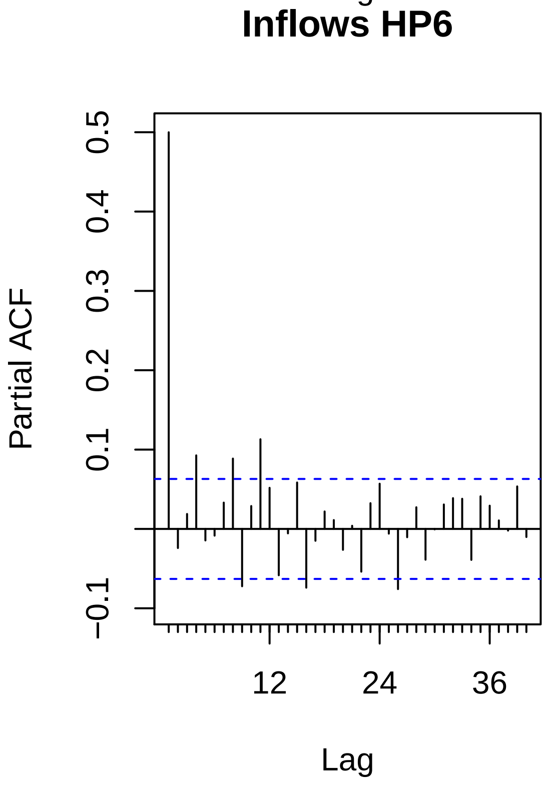

Inflows HP4
ACF
0.6
0.4
0.2
0.0
0
12
24
36
Lag

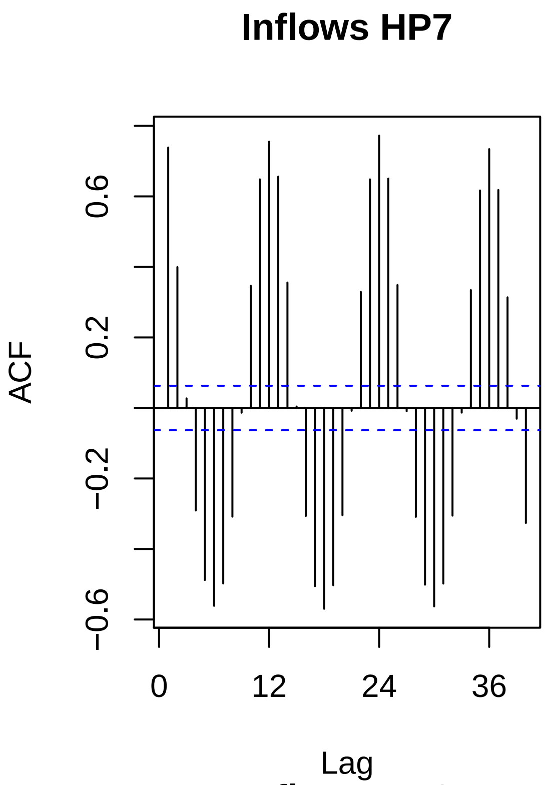

Inflows HP4
Partial ACF
0.6
0.4
0.2
0.0
12
24
36
Lag

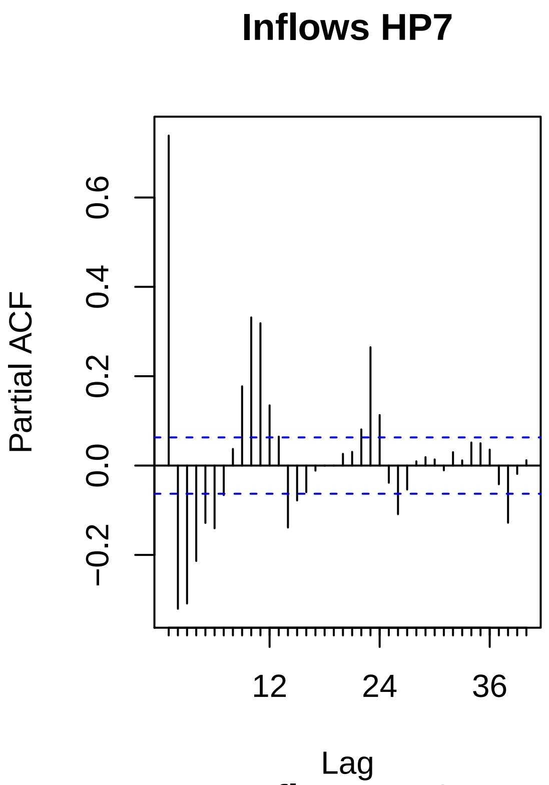

Inflows HP5
ACF
0.5
0.3
0.1
−0.1
0
12
24
36
Lag

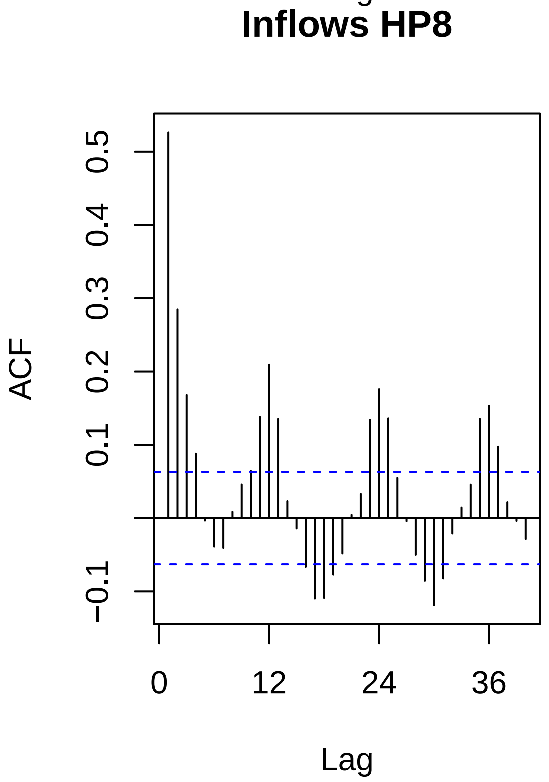

Inflows HP5
Partial ACF
0.5
0.3
0.1
−0.1
12
24
36
Lag

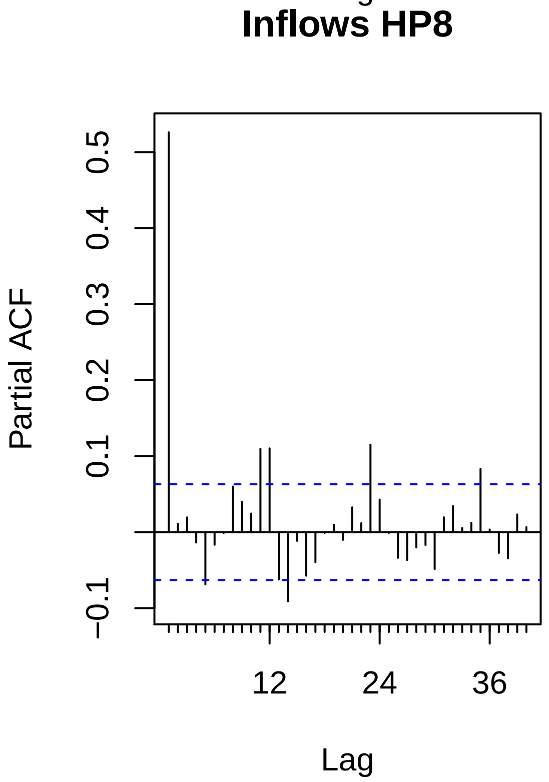

Inflows HP6
ACF
0.5
0.4
0.3
0.2
0.1
−0.1
0
12
24
36
Lag

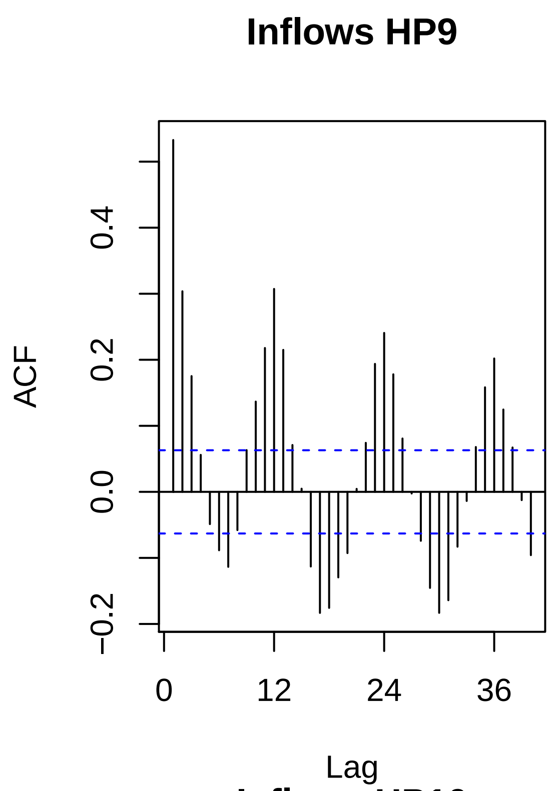

Inflows HP6
Partial ACF
0.5
0.4
0.3
0.2
0.1
−0.1
12
24
36
Lag

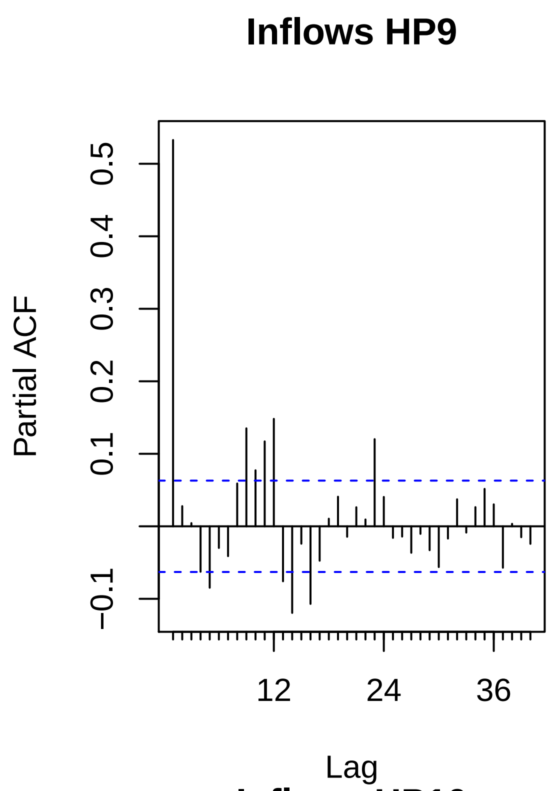

Inflows HP7
ACF
0.6
0.2
−0.2
−0.6
0
12
24
36
Lag

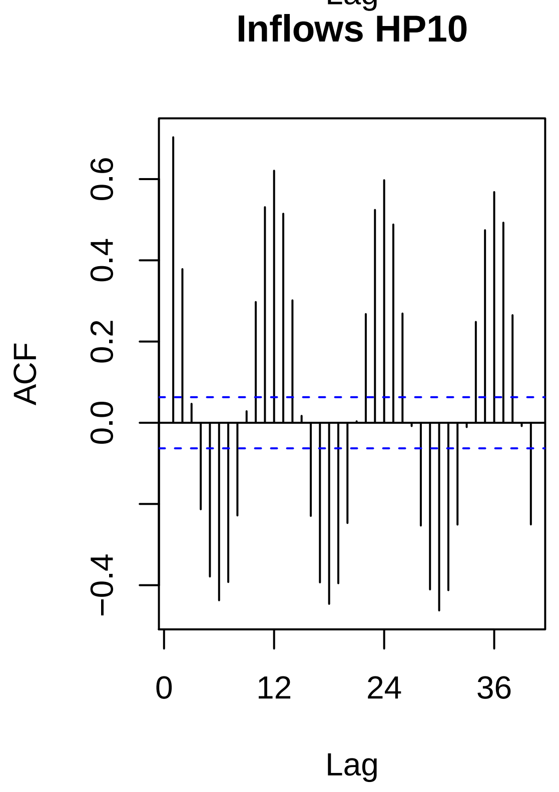

Inflows HP7
Partial ACF
0.6
0.4
0.2
0.0
−0.2
12
24
36
Lag

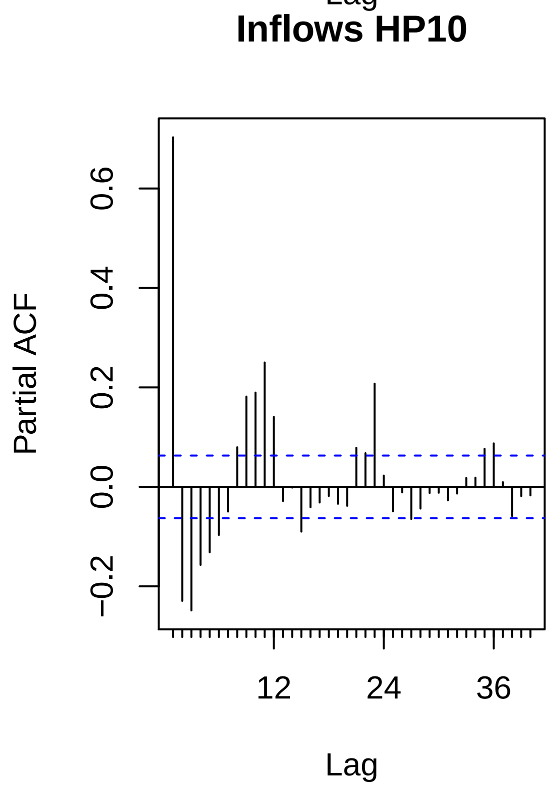

Inflows HP8
ACF
0.5
0.4
0.3
0.2
0.1
−0.1
0
12
24
36
Lag

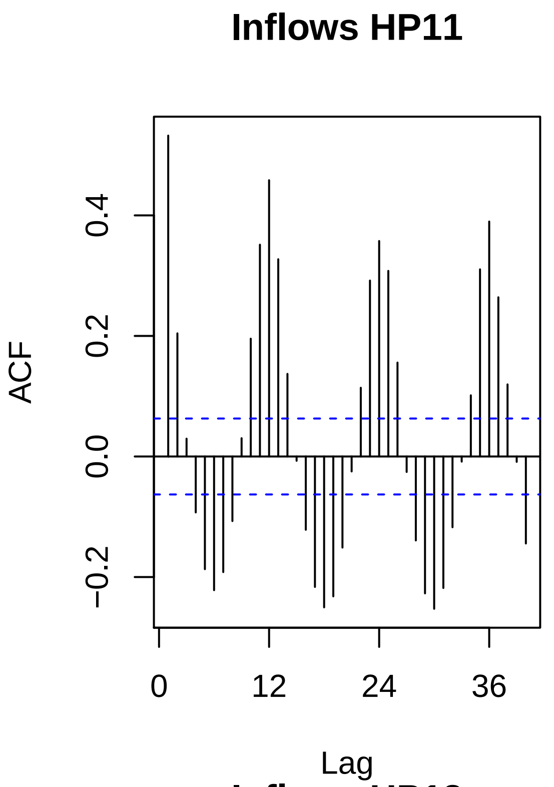

Inflows HP8
Partial ACF
0.5
0.4
0.3
0.2
0.1
−0.1
12
24
36
Lag

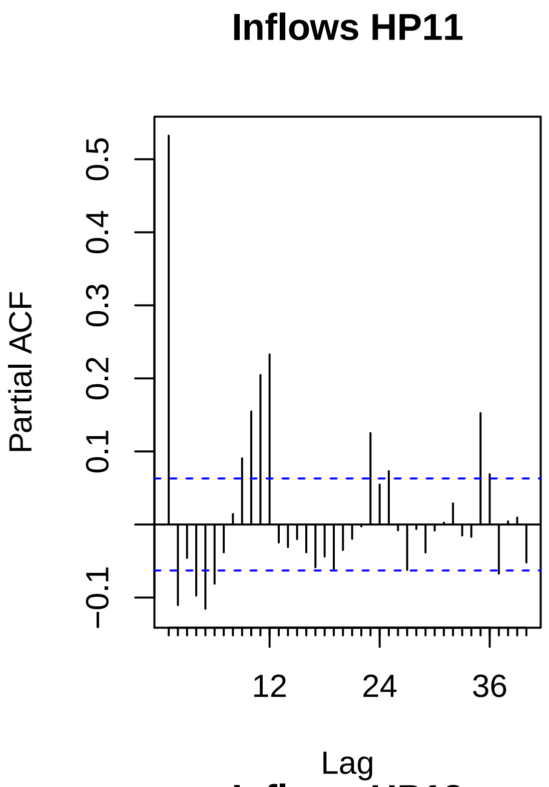

Inflows HP9
ACF
−0.2
0.0
0.2
0.4
0
12
24
36
Lag

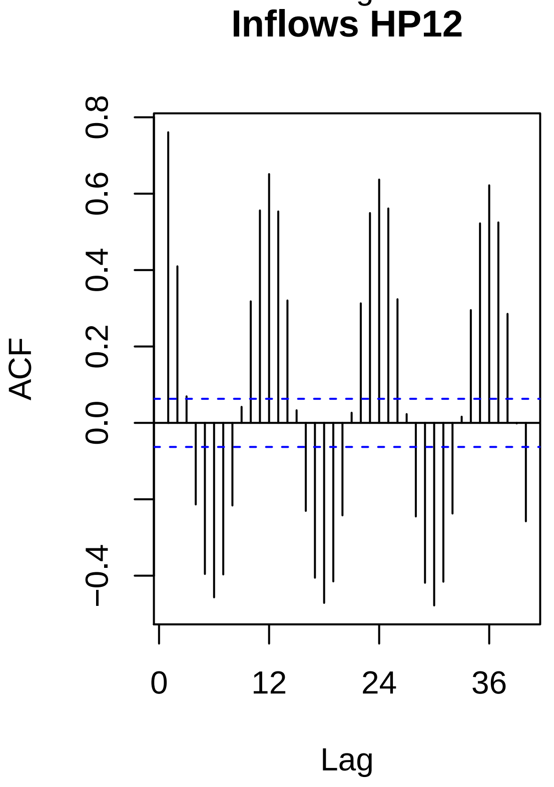

Inflows HP9
Partial ACF
−0.1
0.1
0.2
0.3
0.4
0.5
12
24
36
Lag

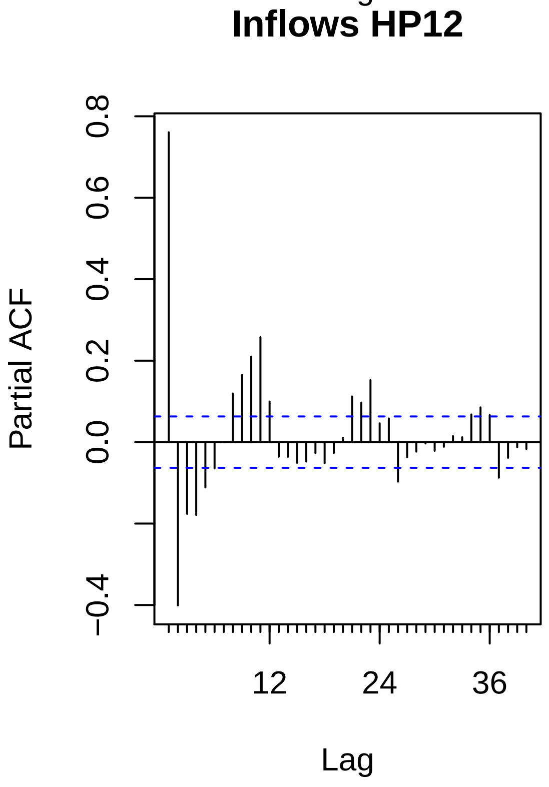

Inflows HP10
ACF
−0.4
0.0
0.2
0.4
0.6
0
12
24
36
Lag

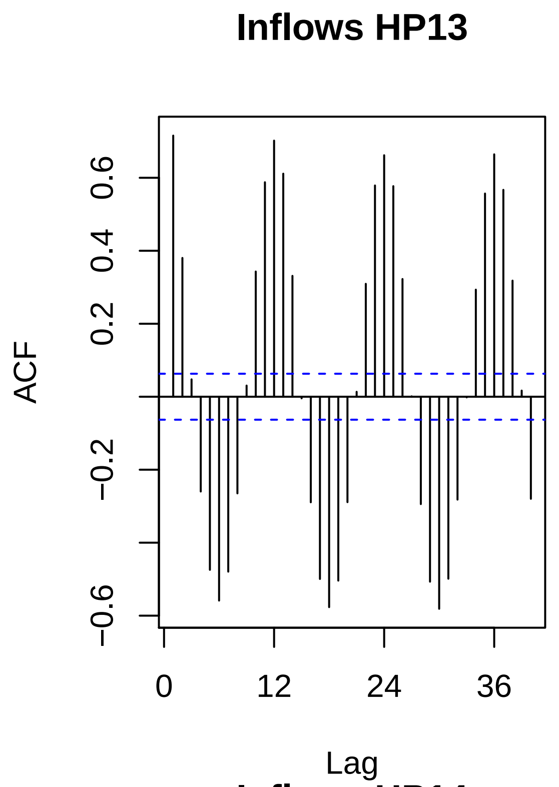

Inflows HP10
Partial ACF
−0.2
0.0
0.2
0.4
0.6
12
24
36
Lag

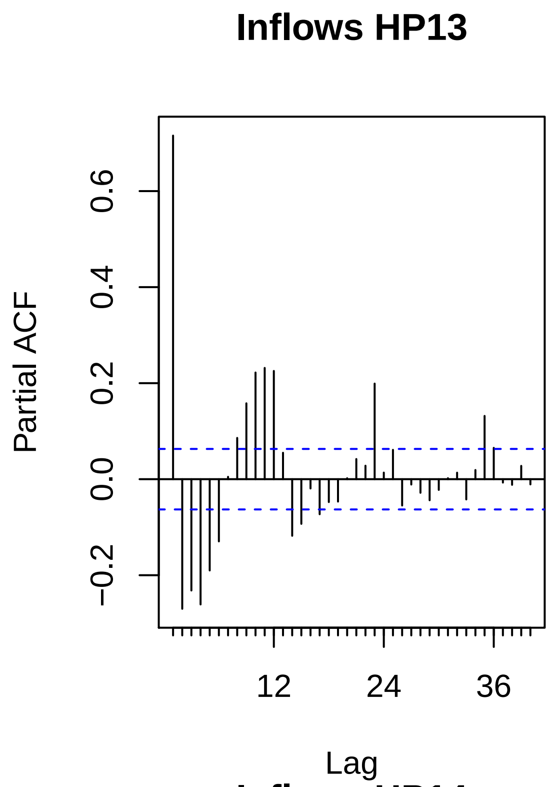

Inflows HP11
ACF
Lag
0
12
24
36
−0.2
0.0
0.2
0.4

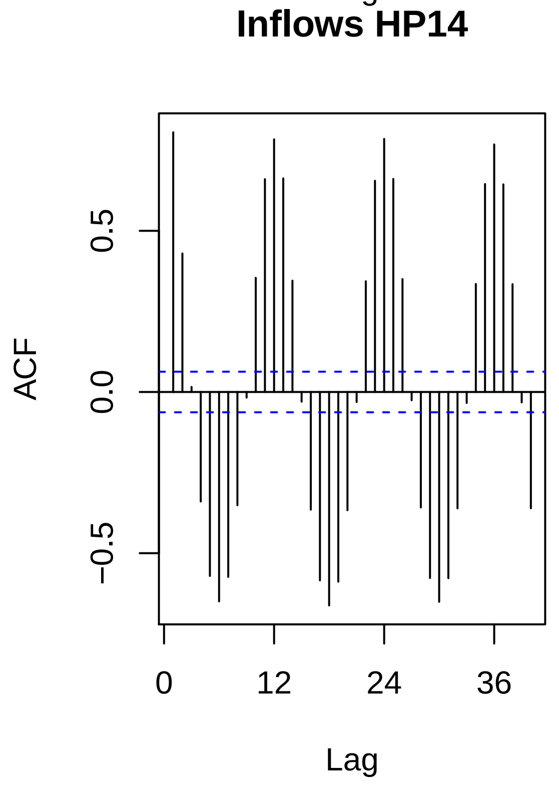

Inflows HP11
Partial ACF
Lag
12
24
36
−0.1
0.1
0.2
0.3
0.4
0.5

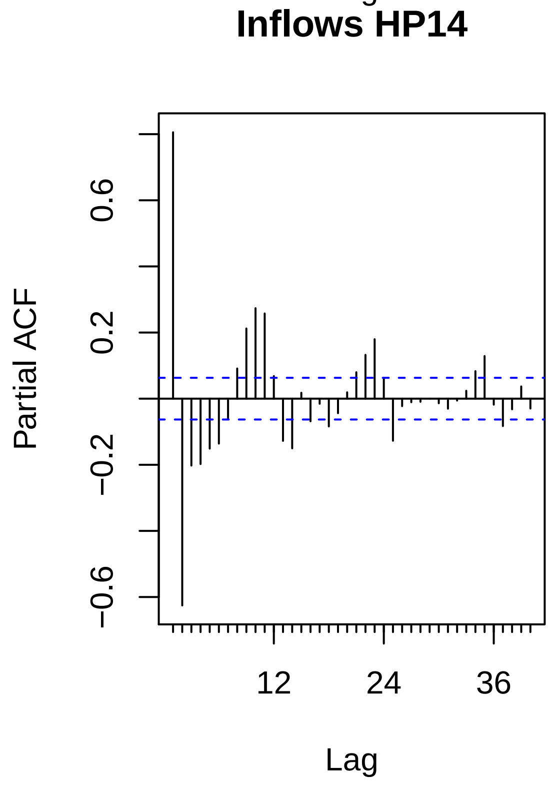

Inflows HP12
ACF
Lag
0
12
24
36
−0.4
0.0
0.2
0.4
0.6
0.8

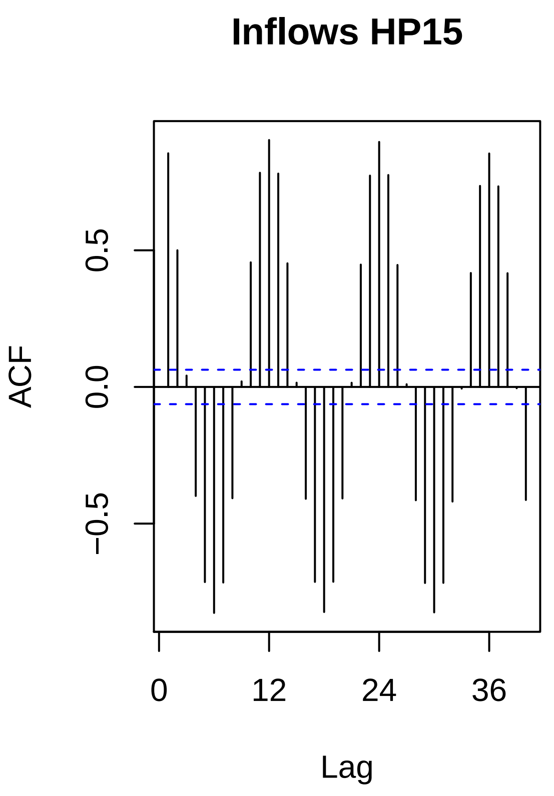

Inflows HP12
Partial ACF
Lag
12
24
36
−0.4
0.0
0.2
0.4
0.6
0.8

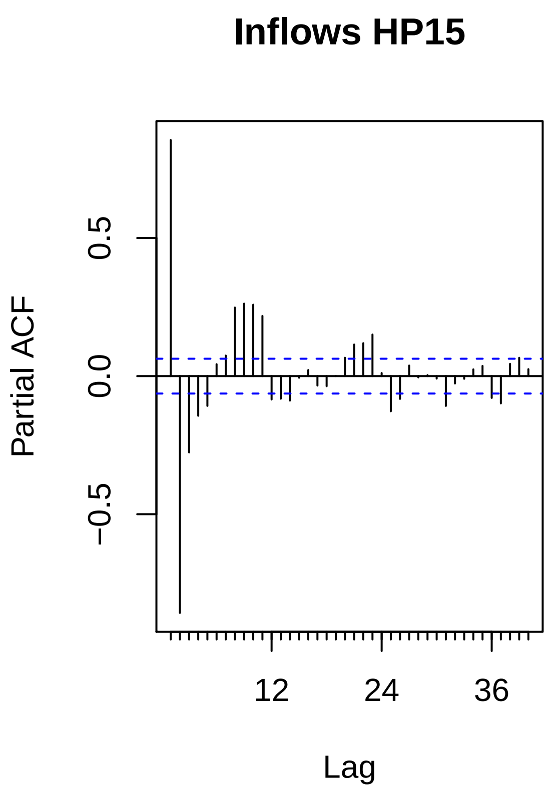

Inflows HP13
ACF
0.6
0.4
0.2
−0.2
−0.6
0
12
24
36
Lag


Inflows HP13
Partial ACF
0.6
0.4
0.2
0.0
−0.2
12
24
36
Lag


Inflows HP14
ACF
0.5
0.0
−0.5
0
12
24
36
Lag


Inflows HP14
Partial ACF
0.6
0.2
−0.2
−0.6
12
24
36
Lag





Trend Component

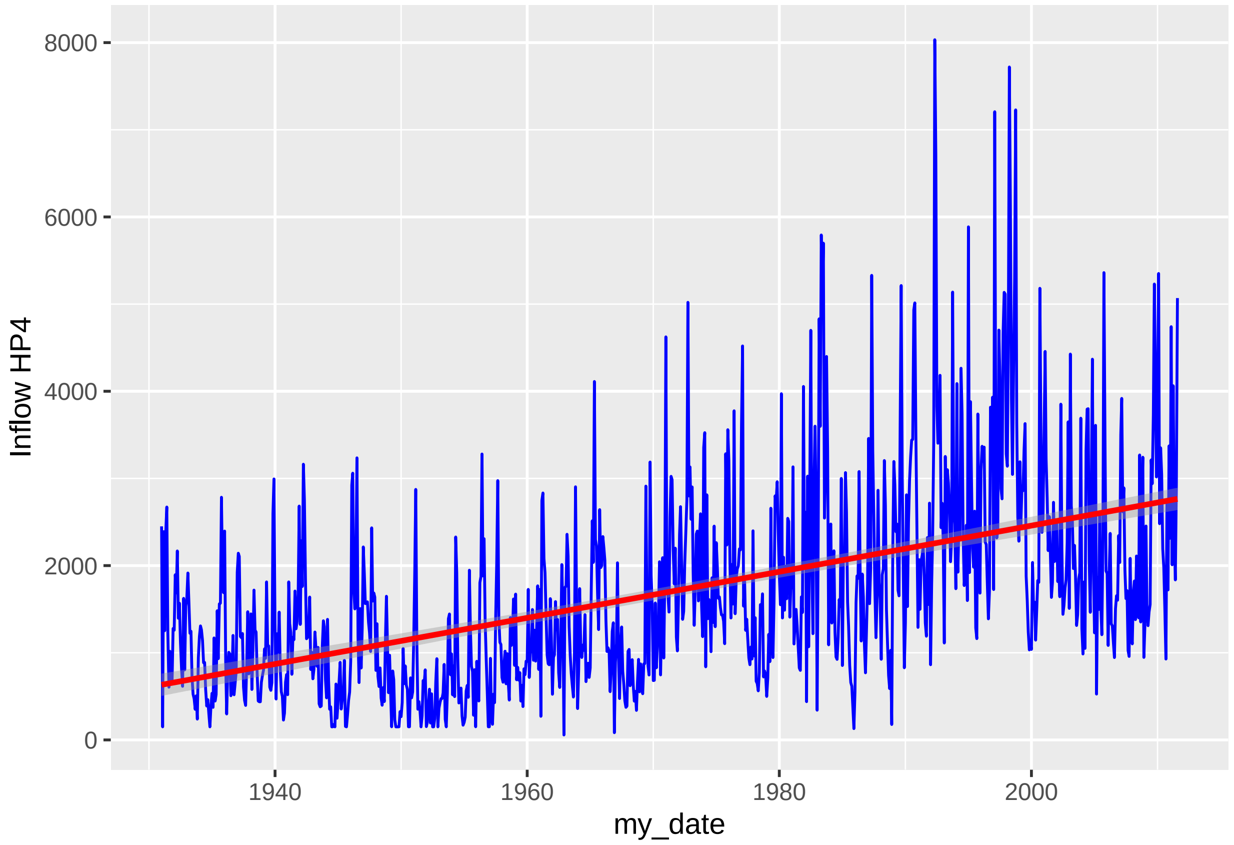
Let's identify and remove trend component like we leaned on the recorded videos for M4. You start by fitting a linear model to $Y_t = \beta_0 + \beta_1 * t + \epsilon_t$.

```
#Create vector t
t <- c(1:nobs)

#Choose one hydro plant to study, as an exercise try to generalize this routine for all 15 HP
#from the plot HP4 seems to have a trend so let's play with that column
iHP=4  #change this to chekc other HP
#prep_data <- data.frame("Inflow"=inflow_data[,iHP],"Time"=t)

#Fit a linear trend to TS of iHP
linear_trend_model=lm(inflow_data[,iHP+1]~t)
summary(linear_trend_model)
```

```
##
## Call:
## lm(formula = inflow_data[, iHP + 1] ~ t)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -2069.3  -695.1  -220.5   505.6  5777.2
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 630.6526    65.3049   9.657   <2e-16 ***
## t             2.2050     0.1168  18.885   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1015 on 966 degrees of freedom
## Multiple R-squared:  0.2696, Adjusted R-squared:  0.2689
## F-statistic: 356.7 on 1 and 966 DF,  p-value: < 2.2e-16
```

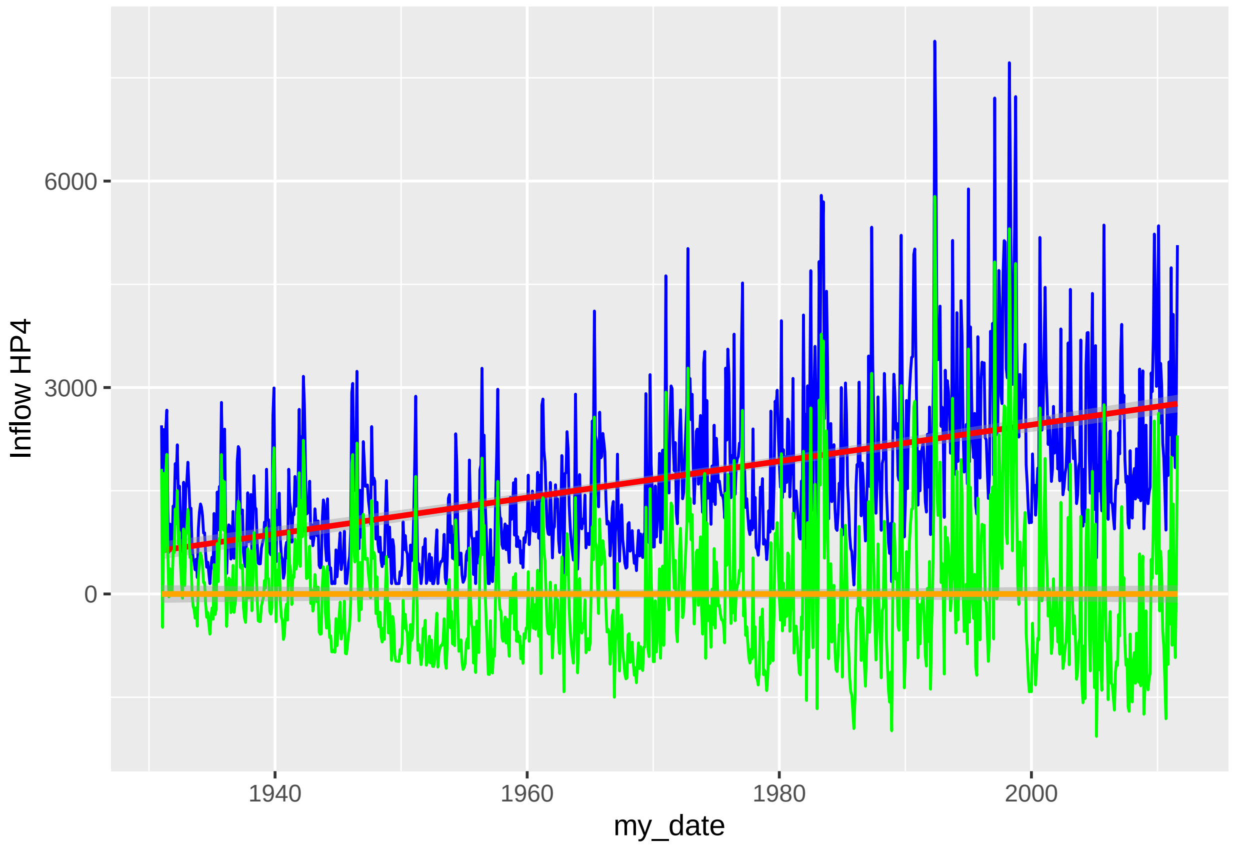
```
beta0=as.numeric(linear_trend_model$coefficients[1])  #first coefficient is the intercept term or beta0
beta1=as.numeric(linear_trend_model$coefficients[2])  #second coefficient is the slope or beta1

#Let's plot the time series with its trend line
ggplot(inflow_data, aes(x=my_date, y=inflow_data[,(1+iHP)])) +
            geom_line(color="blue") +
            ylab(paste0("Inflow ",colnames(inflow_data)[(1+iHP)],sep="")) +
            #geom_abline(intercept = beta0, slope = beta1, color="red")
            geom_smooth(color="red",method="lm")
```

```
## `geom_smooth()` using formula 'y ~ x'
```



```
#remove the trend from series
detrend_inflow_data <- inflow_data[,(iHP+1)]-(beta0+beta1*t)

#Understanding what we did
ggplot(inflow_data, aes(x=my_date, y=inflow_data[,(1+iHP)])) +
            geom_line(color="blue") +
            ylab(paste0("Inflow ",colnames(inflow_data)[(1+iHP)],sep="")) +
            #geom_abline(intercept = beta0, slope = beta1, color="red")
            geom_smooth(color="red",method="lm") +
            geom_line(aes(y=detrend_inflow_data), col="green")+
            geom_smooth(aes(y=detrend_inflow_data),color="orange",method="lm")
```

```
## `geom_smooth()` using formula 'y ~ x'
## `geom_smooth()` using formula 'y ~ x'
```



Note that blue line is our original series, red lien is our trend, green line is our original series minus the trend or in other words the detrended series. And in orange is the trend line for the detrended series which has slope 0 meaning we were able to effectively eliminate the trend with a linear model.

Seasonal Component

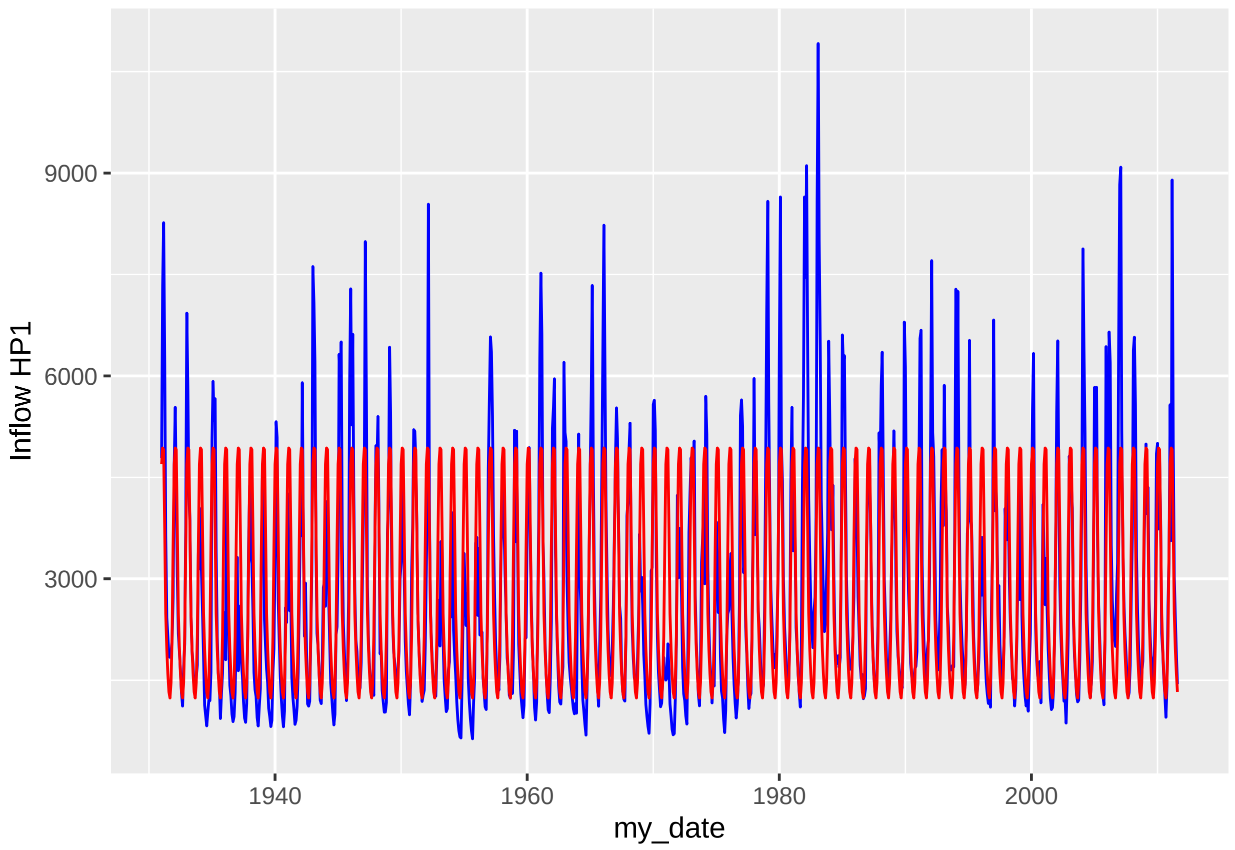
Now let's shift attention to the seasonal component.

```
#Let's choose another HP
iHP=1

#Use seasonal means model
#First create the seasonal dummies
dummies <- seasonaldummy(ts_inflow_data[,iHP])
#this function only accepts ts object, no need to add one here because date
#object is not a column

#Then fit a linear model to the seasonal dummies
seas_means_model=lm(inflow_data[,(iHP+1)]~dummies)
summary(seas_means_model)
```

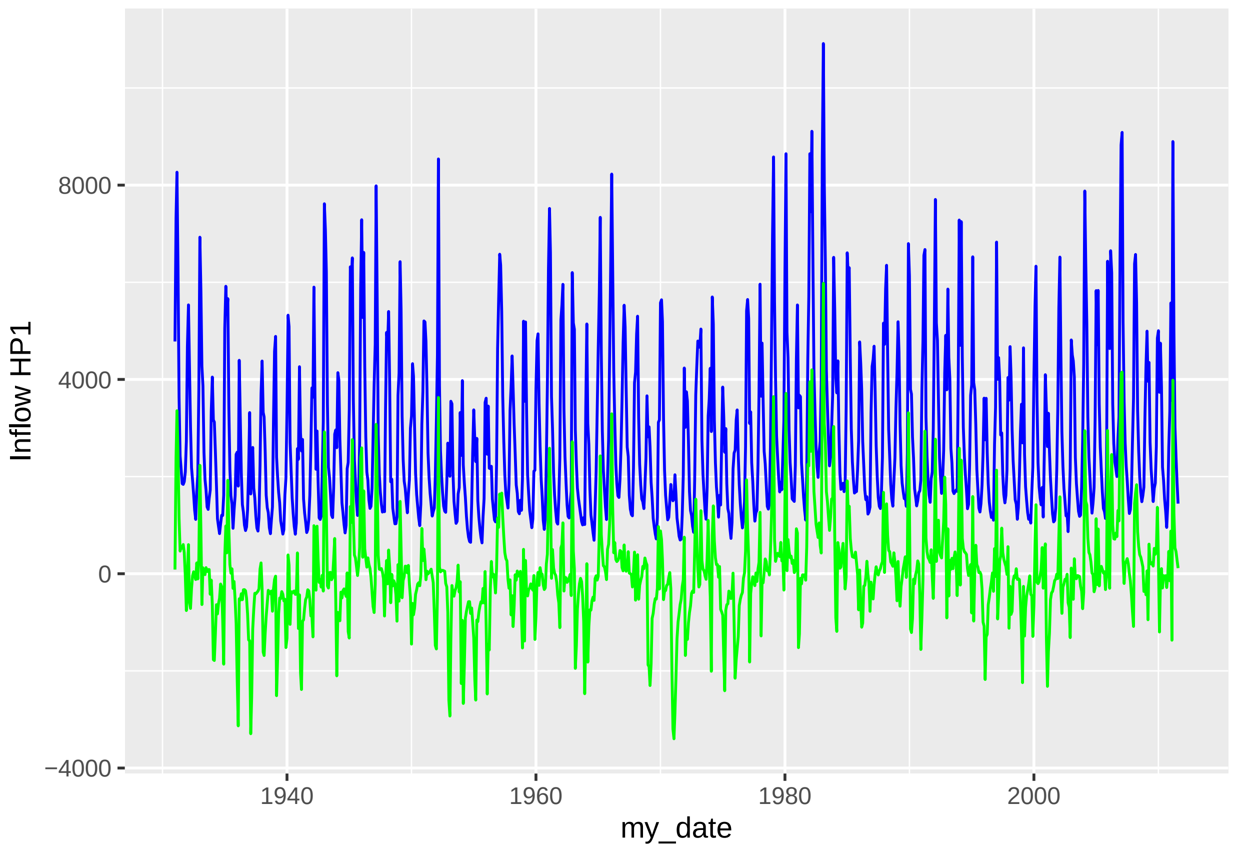
```
##
## Call:
## lm(formula = inflow_data[, (iHP + 1)] ~ dummies)
```

```
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -3397.3  -456.5   -43.1   340.9  5979.7
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   3482.7      115.2  30.229  < 2e-16 ***
## dummiesJan    1213.3      162.4   7.470 1.81e-13 ***
## dummiesFeb    1452.6      162.4   8.943  < 2e-16 ***
## dummiesMar    1427.6      162.4   8.789  < 2e-16 ***
## dummiesApr     257.4      162.4   1.585    0.113
## dummiesMay    -992.8      162.4  -6.112 1.43e-09 ***
## dummiesJun   -1524.0      162.4  -9.382  < 2e-16 ***
## dummiesJul   -1883.6      162.4 -11.597  < 2e-16 ***
## dummiesAug   -2154.3      162.4 -13.263  < 2e-16 ***
## dummiesSep   -2245.3      162.9 -13.781  < 2e-16 ***
## dummiesOct   -2018.1      162.9 -12.386  < 2e-16 ***
## dummiesNov   -1335.4      162.9  -8.196 7.95e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1030 on 956 degrees of freedom
## Multiple R-squared:  0.6467, Adjusted R-squared:  0.6426
## F-statistic: 159.1 on 11 and 956 DF,  p-value: < 2.2e-16
```

```
#Look at the regression coefficient. These will be the values of Beta

#Store regression coefficients
beta_int=seas_means_model$coefficients[1]
beta_coeff=seas_means_model$coefficients[2:12]

#compute seasonal component
inflow_seas_comp=array(0,nobs)
for(i in 1:nobs){
  inflow_seas_comp[i]=(beta_int+beta_coeff%*%dummies[i,])
}

#Understanding what we did
ggplot(inflow_data, aes(x=my_date, y=inflow_data[,(1+iHP)])) +
            geom_line(color="blue") +
            ylab(paste0("Inflow ",colnames(inflow_data)[(1+iHP)],sep="")) +
            geom_line(aes(y=inflow_seas_comp), col="red")
```



```r
#Removing seasonal component
deseason_inflow_data <- inflow_data[,(1+iHP)]-inflow_seas_comp

#Understanding what we did
ggplot(inflow_data, aes(x=my_date, y=inflow_data[,(1+iHP)])) +
            geom_line(color="blue") +
            ylab(paste0("Inflow ",colnames(inflow_data)[(1+iHP)],sep="")) +
            geom_line(aes(y=deseason_inflow_data), col="green")
```



Inflow HP1
8000
4000
0
−4000
1940
1960
1980
2000
my_date