Moments

The rth moment of a variable x about any point $x = A$, usually denoted by μ_r' is given by

$$\mu_r' = \frac{1}{N} \sum_i f_i (x_i - A)^r, \quad \sum_i f_i = N$$

$$= \frac{1}{N} \sum_i f_i d_i^r,$$

where $d_i = x_i - A$.

$$\mu_r' = \frac{1}{N} \sum_{i=1}^{n} f_i (x_i - A)^r$$

$$A = \bar{x}$$

The rth moment of a variable about the mean $\bar{x}$, usually denoted by μ_r is given by

$$\mu_r = \frac{1}{N} \sum_i f_i (x_i - \bar{x})^r = \frac{1}{N} \sum_i f_i z_i^r$$

where $z_i = x_i - \bar{x}$.

$$\mu_r = \frac{1}{N} \sum_{i=1}^{n} f_i (x_i - \bar{x})^r$$

Particular Cases

In particular

$$\mu_o = \frac{1}{N}\sum_i f_i (x_i - \overline{x})^o = \frac{1}{N}\sum_i f_i = 1$$

and $\mu_1 = \frac{1}{N}\sum_i f_i (x_i - \overline{x}) = 0$, being the algebraic sum of deviations from the mean. Also

$$\mu_2 = \frac{1}{N}\sum_i f_i (x_i - \overline{x})^2 = \sigma^2$$

These results, viz., $\mu_o = 1$, $\mu_1 = 0$, and $\mu_2 = \sigma^2$, are of fundamental importance and should be committed to memory.