6. Implement XOR function using McCulloch–Pitts neuron (consider binary data).

Solution: The truth table for XOR function is given in Table 3.

Table 3

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	0

In this case, the output is "ON" for only odd number of 1's. For the rest it is "OFF." XOR function cannot be represented by simple and single logic function; it is represented as

$$y = x_1\overline{x_2} + \overline{x_1}\,x_2$$

$$y = z_1 + z_2$$

where

$$z_1 = x_1\overline{x_2} \qquad \text{(function 1)}$$

$$z_2 = \overline{x_1}x_2 \qquad \text{(function 2)}$$

$$y = z_1 (\mathrm{OR}) z_2 \qquad \text{(function 3)}$$

A single-layer net is not sufficient to represent the function. An intermediate layer is necessary.

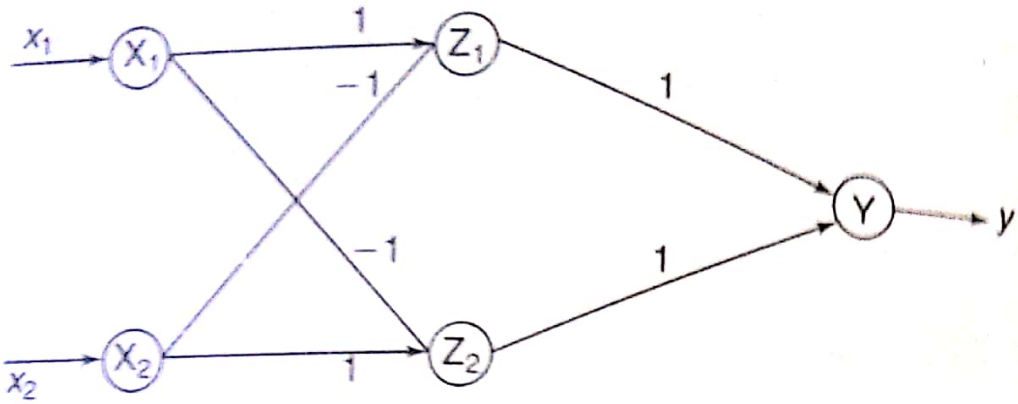

Figure 6 Neural net for XOR function (the weights
shown are obtained after analysis).

- First function ($z_1 = x_1\overline{x_2}$): The truth table for function z_1 is shown in Table 4.

Table 4

x_1	x_2	z_1
0	0	0
0	1	0
1	0	1
1	1	0

The net representation is given as
Case 1: Assume both weights as excitatory, i.e.,

$$w_{11} = w_{21} = 1$$

Calculate the net inputs. For inputs,

$$(0,0),\ z_{1in} = 0 \times 1 + 0 \times 1 = 0$$
$$(0,1),\ z_{1in} = 0 \times 1 + 1 \times 1 = 1$$
$$(1,0),\ z_{1in} = 1 \times 1 + 0 \times 1 = 1$$
$$(1,1),\ z_{1in} = 1 \times 1 + 1 \times 1 = 2$$

Hence, it is not possible to obtain function z_1 using these weights.

Case 2: Assume one weight as excitatory and the other as inhibitory, i.e.,

$$w_{11} = 1; \quad w_{21} = -1$$

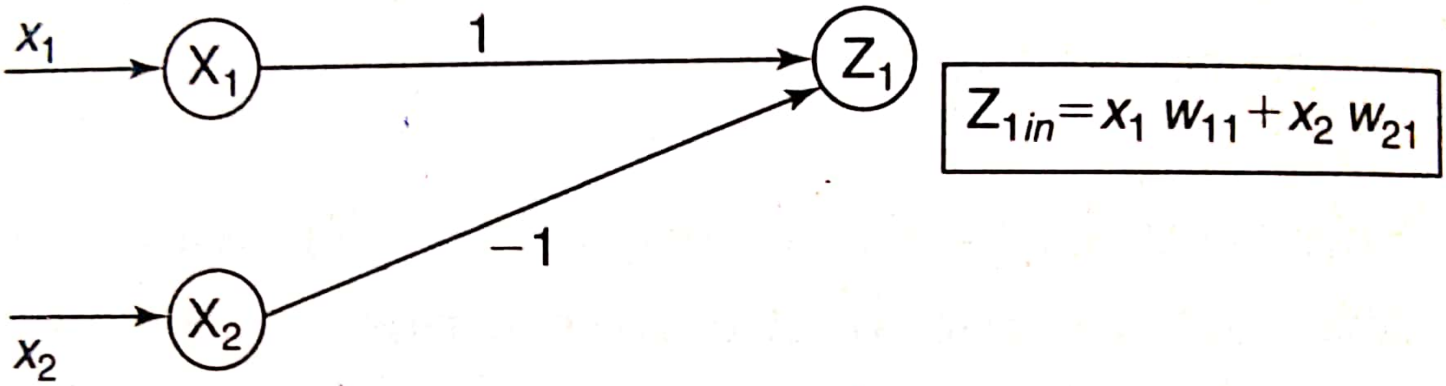

Figure 7 Neural net for Z_1.

Calculate the net inputs. For inputs

$$(0,0),\ z_{1in} = 0 \times 1 + 0 \times -1 = 0$$

$$(0,1),\ z_{1in} = 0 \times 1 + 1 \times -1 = -1$$

$$(1,0),\ z_{1in} = 1 \times 1 + 0 \times -1 = 1$$

$$(1,1),\ z_{1in} = 1 \times 1 + 1 \times -1 = 0$$

On the basis of this calculated net input, it is possible to get the required output. Hence,

$$w_{11} = 1$$

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$$w_{21} = -1$$

$$\theta \geq 1 \quad \text{for the } Z_1 \text{ neuron}$$

- **Second function** ($z_2 = \overline{x_1}x_2$): The truth table for function z_2 is shown in Table 5.

Table 5

x_1	x_2	z_2
0	0	0
0	1	1
1	0	0
1	1	0

The net representation is given as follows:
Case 1: Assume both weights as excitatory, i.e.,

$$w_{12} = w_{22} = 1$$

Now calculate the net inputs. For the inputs

$$(0, 0), \; z_{2in} = 0 \times 1 + 0 \times 1 = 0$$

$$(0, 1), \; z_{2in} = 0 \times 1 + 1 \times 1 = 1$$

$$(1, 0), \; z_{2in} = 1 \times 1 + 0 \times 1 = 1$$

$$(1, 1), \; z_{2in} = 1 \times 1 + 1 \times 1 = 2$$

Hence, it is not possible to obtain function z_2 using these weights.

Case 2: Assume one weight as excitatory and the other as inhibitory, i.e.,

$$w_{12} = -1; \quad w_{22} = 1$$

Now calculate the net inputs. For the inputs

$$(0, 0),\ z_{2in} = 0 \times -1 + 0 \times 1 = 0$$
$$(0, 1),\ z_{2in} = 0 \times -1 + 1 \times 1 = 1$$
$$(1, 0),\ z_{2in} = 1 \times -1 + 0 \times 1 = -1$$
$$(1, 1),\ z_{2in} = 1 \times -1 + 1 \times 1 = 0$$

Thus, based on this calculated net input, it is possible to get the required output, i.e.,

$$w_{12} = -1$$
$$w_{22} = 1$$
$$\theta \geqslant 1 \quad \text{for the } Z_2 \text{ neuron}$$

- **Third function** ($y = z_1$ OR z_2): The truth table for this function is shown in Table 6.

Table 6

x_1	x_2	y	z_1	z_2
0	0	0	0	0
0	1	1	0	1
1	0	1	1	0
1	1	0	0	0

Here the net input is calculated using

$$y_{in} = z_1 v_1 + z_2 v_2$$

Case 1: Assume both weights as excitatory, i.e.,

$$v_1 = v_2 = 1$$

Now calculate the net input. For inputs

$$(0, 0),\ y_{in} = 0 \times 1 + 0 \times 1 = 0$$
$$(0, 1),\ y_{in} = 0 \times 1 + 1 \times 1 = 1$$
$$(1, 0),\ y_{in} = 1 \times 1 + 0 \times 1 = 1$$
$$(1, 1),\ y_{in} = 0 \times 1 + 0 \times 1 = 0$$

(because for $x_1 = 1$ and $x_2 = 1$, $z_1 = 0$ and $z_2 = 0$)

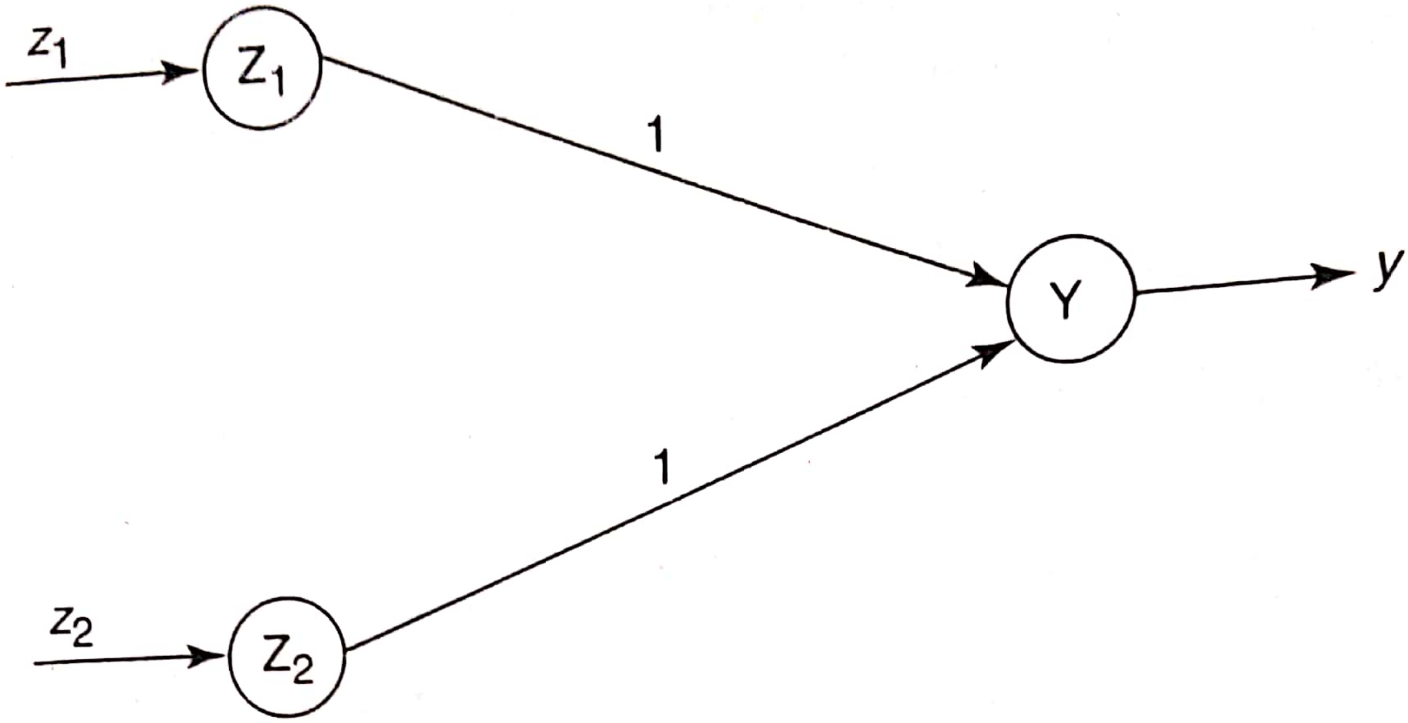

Figure 9 Neural net for $Y(Z_1 \text{ OR } Z_2)$.

Setting a threshold of $\theta \geq 1$, $v_1 = v_2 = 1$, which implies that the net is recognized. Therefore, the analysis is made for XOR function using M–P neurons. Thus for XOR function, the weights are obtained as

$$w_{11} = w_{22} = 1 \quad \text{(excitatory)}$$
$$w_{12} = w_{21} = -1 \quad \text{(inhibitory)}$$
$$v_1 = v_2 = 1 \quad \text{(excitatory)}$$