Experiment

- An experiment is a device or a means of getting an answer to the problem under consideration.

- Absolute experiment deals with determining the absolute value of some characteristic like obtaining the average intelligent quotient of a group of people.

- Comparative experiment are designed to compare the effect of two or more objects on some population characteristic. Ex. Comparison of different kinds of varieties of crops.

Design of Experiment

- A design of experiment, or DOE, is a method used to plan and analyze a scientific experiment. It helps to ensure that the experiment is as efficient and effective as possible by carefully controlling the variables and measuring the results.
- DOEs are commonly used in fields such as manufacturing, engineering, and pharmaceuticals to improve processes and products.
- Suppose we are conducting an agriculture experiment to verify the truth to claim that the fertilizers increase the yield of wheat.
- Then there two variables involved directly here, the fertilizer and the yield of wheat. These two variable are called the experimental variables.
- In addition there are other variables also involved here : quality of seed, climate, nature of soil etc.,. These variables are called the extraneous variable.

Aim of Design of Experiment

The main aim of Design of experiment is the following:

- To control the extraneous variable,
- To minimize the experimental error.

Treatment

- Various objects of comparison in a comparative experiment are termed as a Treatment.
- Ex. In a field of experiment: different fertilizer, different varieties of crops, different method of cultivation.

Experimental units

- The smallest division of experimental method to which we apply the treatments and on which we make the observations of the variable under the study is termed as the experimental units.
- Ex. Plot of a land.

Basic principles of experimental design

- Randomisation: As it is not possible to eliminate completely the contribution of extraneous variables to the value of the response variable (the amount of yield of paddy), we try to control it by randomisation. The group of experimental units (plots of the same size) in which the manure is used is called the experimental group and the other group of plots in which the manure is not used and which will provide a basis for comparison is called the control group.
- Replication: In a comparative experiment, in which the effects of different manures on the yield are studied, each manure is used in more than one plot. In other words, we resort to replication which means repetition. It is essential to carry out more than one test on each manure in order to estimate the amount of the experimental error and hence to get some idea of the precision of the estimates of the manure effects.
- Local Control: To provide adequate control of extraneous variables, another essential principle used in the experimental design is the local control. This includes techniques such as grouping, blocking.

Grouping is a method used to divide the experimental units into groups based on some known or suspected source of variation. For example, in an agricultural experiment to test the effectiveness of a new fertilizer, the experimental units might be plots of land. The plots could be grouped based on soil type, to control for the fact that different types of soil may affect the growth of plants differently. By grouping the plots by soil type and applying the fertilizer to only one group of plots, the effects of soil type can be separated from the effects of the fertilizer

Blocking, on the other hand, is a method used to control for known or suspected sources of variation within the experimental units. For example, when testing a new drug on human subjects, the subjects are randomly divided into groups, or "blocks," based on some characteristic that might affect the outcome of the experiment, such as age, sex, or baseline health status. By randomly assigning subjects within each block to either the experimental group or the control group, the effects of the drug can be separated from the effects of the blocking variable.

Completely Randomized Design (CRD)

- CRD is the simplest of all the designs based on the principles of randomized and replication. In this design, treatments are allocated at random to the experimental units over the entire experimental material.
- Let us suppose we have k treatments. The i^{th} being replicated n_i times ($i = 1\ to\ k$), then the whole experimental material is divided into $\mathrm{N} = \sum n_i$ experimental units and the treatments are distributed completely at random over the units subject to the condition that the i^{th} treatment occurs n_i times.

Completely Randomized Design (CRD)

- Randomization assures that the extraneous factor does not continuously influence one factor.

Advantage of CRD

- There are complete flexibility in the model as the number of replication is not fixed.
- Analysis can be performed even if some observations are trussing.
- CRD results in the maximum use of experimental units since all the experimental material can be used.

Disadvantage of CRD

- The experimental errors are large as compared to the other designs since the homogeneity of the units are ignored.

ANOVA

ANOVA, or Analysis of Variance, is a statistical method used to test the differences between the means of two or more groups. It is used to determine whether there is a significant difference between the means of the groups, or whether any observed differences are due to chance.

ANOVA can be used with data from experiments that have a single factor, or independent variable, with more than two levels, or with data from experiments that have multiple factors. It is a powerful and widely used tool for analyzing data from designed experiments, and is often used in fields such as agriculture, biology, engineering, and psychology.

Statistical analysis of CRD

- ANOVA is a technique used to test the means of more than two samples. It divides the total variance in the group into parts, which are associated to different factors. The variation is split into two components as the variation within subgroup and a variation between subgroups.

Assumption and hypothesis of one way ANOVA

- It is assumed that the k (treatments) populations are independent and normally distributed with means $\mu_1, \mu_2, \ldots, \mu_k$ and common variance σ^2. We wish to derive appropriate methods for testing the hypothesis.
- $H_0 : \ \mu_1 = \mu_2 = \cdots = \mu_k$
- $H_1 :$ At least two of the means are not equal.

Statistical analysis of CRD

$Y_i = T_i$

Treatment:	**1**	**2**	$\cdots$	$\boldsymbol{i}$	$\cdots$	$\boldsymbol{k}$	
	y_{11}	y_{21}	$\cdots$	y_{i1}	$\cdots$	y_{k1}	
	y_{12}	y_{22}	$\cdots$	y_{i2}	$\cdots$	y_{k2}	
	$\vdots$	$\vdots$		$\vdots$		$\vdots$	
	y_{1n_1}	y_{2n_2}	$\cdots$	y_{in_i}	$\cdots$	y_{kn_k}	
Total	$Y_{1.}$	$Y_{2.}$	$\cdots$	$Y_{i.} = T_i$	$\cdots$	$Y_{k.}$	$Y_{..} = T$
Mean	$\bar{y}_{1.}$	$\bar{y}_{2.}$	$\cdots$	$\bar{y}_{i.}$	$\cdots$	$\bar{y}_{k.}$	$\bar{y}_{..}$

Statistical analysis of CRD

- Here y_{ij} denote the j^{th} observation from the i^{th} treatment.

$T_i =$
- Y_i is the total of all observations in the sample from the i^{th} treatment,
- $\overline{y_i}$ is the mean of all observations in the sample from the i^{th} treatment,

$T =$
- Y is the total of all $N = \sum n_i$observations,
- $\bar{y}$ is the mean of all N observations.

Model equation for one way ANOVA

- Each observation may be written as $Y_{ij} = \overbrace{\mu + \alpha_i}^{\mu_i} + \epsilon_{ij}$, $1 \leq i \leq k$, $1 \leq j \leq n_i$ where $\mu_i = \mu + \alpha_i$.
- α_i - Effect due to the i^{th} treatment such that $\sum \alpha_i = 0$
- ϵ_{ij} - Error effect.
- μ – Grand mean. Ie., $\mu = \frac{1}{k} \sum \mu_i$

Hypothesis of one way ANOVA

- The null hypothesis that the k population means are equal against the alternative that at least two of the means are unequal may now be replaced by the equivalent hypothesis.
- $H_0 : \alpha_1 = \alpha_2 = \cdots = \alpha_k = 0$
- H_1 : At least two of the α_i are not equal.

α_i - Effect due to the i^{th} treatment

ANOVA Table

ANOVA Table

Source of variation	Degrees of freedom	Sum of squares	Mean Square	F-ratio
Treatments	$k-1$	SST	$MST = \frac{SST}{k-1}$	$F = \frac{MST}{MSE}$; $MST > MSE$
Error	N-k	SSE	$MSE = \frac{SSE}{N-k}$	$\frac{MSE}{MST}$; $MSE > MST$
Total	N-1	$TSS = SST + SSE$		

One Way ANOVA

- Where k is the number of treatments(population).

  N total number of experimental units.

- Treatment Sum square : SST $= \sum_{i=1}^{k} \frac{T_i^2}{n_i} - C.F$ where

  $T_i = \sum_{j=1}^{n_i} y_{ij}$ and $C.F = \frac{T^2}{N}$, T = Grand Total = $\sum_{i=1}^{k} \sum_{j=1}^{n_i} y_{ij}$.

- Error Sum square : SSE =TSS-SST, where

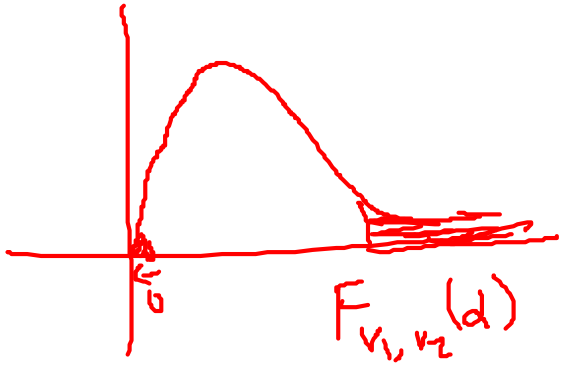
  TSS $= \sum_{i=1}^{k} \sum_{j=1}^{n_i} y_{ij}^2 - C.F$: total sum square.

Use of F test in ANOVA

- Test statistic:
- $F_{t.s} = \begin{cases} \frac{MST}{MSE} & ; if\ MST \geq MSE \\ \frac{MSE}{MST} & ; if\ MSE > MST \end{cases}$

Critical value

- $F_{v_1,v_2}(\alpha)$:
- Degrees of freedom :
- If $MST \geq MSE$ then $v_1 = k - 1,\ v_2 = N - k$.
- Otherwise $v_2 = k - 1,\ v_1 = N - k$.



Conclusion

- Make conclusion based on F test.

Problem 1

- A set of data involving four tropical feedstuffs A, B, C, D tried on 20 chicks is given below. All the twenty chicks are treated alike in all respects except the feeding treatments and each of the feeding treatment is given to 5 chicks. Analyze the data. Weight gain of baby chicks fed on different feeding materials composed of tropical feedstuffs.

Weight gain

A	55	49	42	21	52
B	61	112	30	89	63
C	42	97	81	95	92
D	169	137	169	85	154

① $H_0: \alpha_A = \alpha_B = \alpha_C = \alpha_D = 0$; feedstuffs are equal.

H_1: at least few of α_i's are non-zero.

② ANOVA (one way)

③ —

no of treatment

$i \le i \le 4$

To find ANOVA Table: $K=4$; $n_i = 5$, $N = 20$, T = Total freq = $\sum T_i = 1695$

$$C.F = \frac{T^2}{N} = \frac{1695^2}{20} = 143651.25$$

weight gain

Treat						T_i	T_i^2/n_i
A	55	49	42	21	52	219	$219^2/5$
B	61	112	30	89	63	355	$355^2/5$
C	42	97	81	95	92	407	.
D	169	137	169	85	154	714	.
						$T = 1695$	169886.2

$$SST = \sum \frac{T_i^2}{n_i} - C.F.$$
$$= 169886.2 - 143651.25$$
$$SST = \underline{26234.95}$$

$$TSS = \sum_i \sum_j y_{ij}^2 - C.F = 181445 - 143651.25$$
$$= \underline{37793.75}$$

$$SSE = TSS - SST = \underline{11558.8}$$

ANOVA

Source of variation	D.f.	S S	M.S	F-ratio
Treatment	$K-1=3$	$SST=26234.95$	$MST=\frac{SST}{K-1}=8744.983$	$F_{t.s}=\frac{MST}{MSE}=\frac{8744.983}{722.425}=12.105$
Error	$N-K=20-4=16$	$SSE=11558.8$	$MSE=\frac{SSE}{N-K}=722.425$	
Total	$N-1=19$	$TSS=37793.75$		

(4) $\alpha=5\%$. $V_1=3$, $V_2=16$

(5) $F_{V_1,V_2}(\alpha)=F_{3,16}(5\%)=3.24$

(6) $\because F_{t.s}>F_{V_1,V_2}(\alpha) \Rightarrow$ Reject H_0

(7) Treatments are not same.

Problem 2

- A completely randomized design experiment with 10 plots and 3 treatments gave the following results.

Plot No. :	1	2	3	4	5	6	7	8	9	10
Treatment :	A	B	C	A	C	C	A	B	A	B
Yield :	5	4	3	7	5	1	3	4	1	7

- Analyze the results for treatment effects.

Problem 3

- Set up an analysis of variance table for the following per acre production data for three varieties of wheat, each grown on 4 plots and state if the variety differences are significant.

Plot of land	Per acre production data		
	Variety of wheat		
	A	B	C
1	6	5	5
2	7	5	4
3	3	3	3
4	8	7	4

Problem 4

- The following table shows the lives in hour for four brands of electric lambs.
- Brand
- A : 1610, 1610, 1650, 1680, 1700, 1720, 1800
- B : 1580, 1640, 1640, 1700, 1750
- C : 1460, 1550, 1600, 1620, 1640, 1660, 1740, 1820
- D : 1510, 1520, 1520, 1570, 1600, 1680
- Perform the analysis of variance and test the homogeneity of mean lives of the four brand of lambs.

Randomized Block Design

- If the whole experimental area is not homogenous, then a simple method of controlling the variability of experimental material consists in grouping the whole area into relatively homogenous subgroups. The treatments can be applied in a random manner to relatively homogenous units within each subgroups/blocks and replicated over all the block. This design is known as RBD.

Layout

- Let us consider five treatments A,B,C,D and E each replicated four times. We divide the whole experimental area into four relatively homogenous blocks and each block into five units. Treatments are allocated at random to the plot of block.
- Blocks
- I A E B D C
- II E D C B A
- III C B A E D
- IV A D E C B

nu of blocks

		Blocks						Block Totals
		1	2		i		b	
Treatments	1	y_{11}	y_{21}	…	y_{i1}	…	y_{b1}	$B_1 = y_{o1}$
	2	y_{12}	y_{22}	…	y_{i2}	…	y_{b2}	$B_2 = y_{o2}$
	⋮	⋮	⋮		⋮		⋮	⋮
	j	y_{1j}	y_{2j}	…	y_{ij}	…	y_{bj}	$B_j = y_{oj}$
	⋮		⋮		⋮		⋮	⋮
nu of treat	v	y_{1v}	y_{2v}	…	y_{iv}	…	y_{bv}	$B_b = y_{ob}$
Treatment Totals		$T_1 = y_{1o}$	$T_2 = y_{2o}$	…	$T_i = y_{io}$	…	y_{vo}	Grand Total $G = y_{oo}$

$N = bv$

Statistical analysis of RBD

- y_{ij} - Individual measurements of j^{th} treatment in i^{th} block, $i = 1,2, \ldots, b, j = 1,2, \ldots, v$.
- y_{ij}'s are independently distributed following $N(\mu + \alpha_j + \beta_i, \sigma^2)$
- where μ : overall mean effect
  β_i : i^{th} block effect
  α_j : j^{th} treatment effect
  such that $\sum \alpha_j = 0 , \sum \beta_i = 0$

Assumption and hypothesis of two way ANOVA

- There are two null hypotheses to be tested.
- Related to the treatment effects :
- H_{0T} : $\alpha_1 = \alpha_2 = \cdots = \alpha_v = 0$
- H_{1T} : At least two of the α_j are not equal.
- Related to the block effects :
- H_{0B} : $\beta_1 = \beta_2 = \cdots = \beta_b = 0$
- H_{1B} : At least two of the β_i are not equal.

Model equation for two way ANOVA

- The linear model, in this case, is a two-way model as $Y_{ij} = \mu + \alpha_j + \beta_i + \epsilon_{ij}, 1 \leq i \leq b\,, 1 \leq j \leq v$
- ϵ_{ij} - are identically and independently distributed random errors following a normal distribution with mean 0 and variance σ^2.

ANOVA Table

ANOVA Table

Source of variation	Degrees of freedom	Sum of squares	M.S.	F
Treatments	$v_{.}-1$	SST	$MST = \frac{SST}{v_{.}-1}$	$F_T = \frac{MST}{MSE}$ (or) $\frac{MSE}{MST}$
Blocks	$b-1$	SSB	$MSB = \frac{SSB}{b-1}$	$F_B = \frac{MSB}{MSE}$ (or) $\frac{MSE}{MSB}$
Error (residual)	$(v_{.}-1)(b-1)$	SSE	$MSE = \frac{SSE}{(v_{.}-1)(b-1)}$	
Total	$vb-1$	TSS		

Two Way ANOVA

- $N = vb$ total number of experimental units.
- Correction factor $C.F = \frac{T^2}{N}$, T = Grand Total = sum of all y_{ij}.
- Treatment Sum square : SST $= \sum_{j=1}^{v} \frac{T_j^2}{b} - C.F$
  here T_j : j^{th} treatment total sum.
- Block Sum square : SSB $= \sum_{i=1}^{b} \frac{T_i^2}{v} - C.F$
  here T_i : i^{th} block total sum.
- Error Sum square : SSE =TSS-(SST+SSB), where

  TSS $= \sum_{i=1}^{k} \sum_{j=1}^{n_i} y_{ij}^2 - C.F$: total sum square.

Use of F test in ANOVA

- Test statistic:
- $F_{t.s}^{T} = \begin{cases} \frac{MST}{MSE} & ; if\ MST \geq MSE \\ \frac{MSE}{MST} & ; if\ MSE > MST \end{cases}$
- $F_{t.s}^{B} = \begin{cases} \frac{MSB}{MSE} & ; if\ MSB \geq MSE \\ \frac{MSE}{MSB} & ; if\ MSE > MSB \end{cases}$

Critical value

- $F^{T}_{v_1,v_2}(\alpha)$:
- Degrees of freedom :

If $MST \geq MSE$ then $v_1 = v - 1,\ v_2 = (v-1)(b-1)$.

Otherwise $v_2 = v - 1,\ v_1 = (v-1)(b-1)$.

- $F^{B}_{v_1,v_2}(\alpha)$:
- Degrees of freedom :

If $MSB \geq MSE$ then $v_1 = b - 1,\ v_2 = (v-1)(b-1)$.

Otherwise $v_2 = b - 1,\ v_1 = (v-1)(b-1)$.

Conclusion

- Make conclusion based on F test.

Problem 1

A, B, C

- Three verities of crops are tested in a randomized block design with four replications. The layout is given below. The yields are given in kilo gram analyze the significance difference.

(1)	(2)	(3)	(4)
C48	A51	B52	A49
A47	B49	C52	C51
B49	C53	A49	B50

(1) Treat: varities of crops block: replications

H_{0T}: The varities of crops donot diff significantly

H_{1T}: differ

H_{0B}: The blocks donot differe sig.

H_B: The blocks differe.

(2) 2-way anova:

(3)

Treat (crops)	Blocks (1)	(2)	(3)	(4)
A	47	51	49	49
B	49	49	52	50
C	48	53	52	51

$v = 3$

$b = 4$

$N = 12$

shift 50 →

	1	2	3	4	T_i	T_i^2/b
A	-3	+1	-1	-1	-4	16/4
B	-1	-1	2	0	0	0
C	-2	3	2	1	4	16/4
T_j	-6	3	3	0	$0 = T$	8
$T_j^2/3$	36/3	9/3	9/3	0		

$T = 0$

$$C.F = \frac{T^2}{N} = 0$$

$$SST = \sum_{i=1}^{3} \frac{T_i^2}{4} - C.F = 8 - 0 = 8$$

$$SSB = \sum \frac{T_j^2}{3} - C.F = 18 - 0 = 18$$

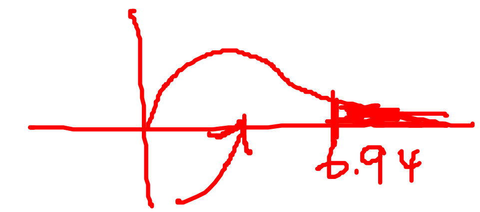
$$TSS = \sum\sum y_{ij}^2 - CF = 36 - 0 = 36$$

$$SSE = TSS - (SST + SSB) = 10$$

Two way ANOVA

SOV	d.f	S.S	M.S	F
Treat	$v-1=2$	$SST=8$	$MST=\frac{SST}{2}=4$	$F^{T}_{ts}=\frac{MST}{MSE}=\frac{4}{1.67}=2.4$
Block	$b-1=3$	$SSB=18$	$MSB=6$	$F^{B}_{ts}=\frac{MSB}{MSE}=3.6$
Error	$(v-1)(b-1)=6$	$SSE=10$	$MSE=\frac{10}{6}=1.67$	
Total	11	36		

T :



(4) $\alpha=5\%$ Treat: $V_1=2, V_2=6$; Block: $V_1=3, V_2=6$

(5) Critical val:

Treat: $F^{T}_{2,6}(5\%)=6.94$

Block: $F^{B}_{3,6}(5\%)=4.76$

(6) Treat: Accept H_{0T}

Block: Accept H_{0B}

Problem 2

- Consider the results given in the following table for an experiment involving six treatments in four randomized blocks. The treatments are indicated by numbers within parentheses. Analyze whether there is any significant difference in the treatments and blocks are homogenous.

Blocks	Treatments and yield					
1	(1) 24.7	(2) 27.7	(3) 20.6	(4) 16.2	(5) 16.2	(6) 24.9
2	(3) 22.9	(2) 28.8	(1) 27.3	(4) 15	(6) 22.5	(5) 17
3	(6) 26.3	(4) 19.6	(1) 38.5	(3) 36.8	(2) 39.5	(5) 15.4
4	(5) 17.7	(2) 31	(1) 28.5	(4) 14.1	(3) 34.9	(6) 22.6

Problem 3

- A tea company appoints four salesmen A, B, C, and D and observes their sales in three seasons. Summer, Winter and Monsoon. The figures (in lakhs) are given in the following table:

	Salesmen			
Seasons	A	B	C	D
Summer	: 36	36	21	35
Winter	: 28	29	31	32
Monsoon	: 26	28	29	29

Problem 4

- The following data represent the number of units of production per day turned out by 5 different workmen using different types of machines.
- i) Test whether the mean productivity is the same for the four different machine types.
- ii) Test whether 5 men differ with respect to mean productivity.

Workmen	Machine type			
	A	B	C	D
1	44	38	47	36
2	46	40	52	43
3	34	36	44	32
4	43	38	46	33
5	38	42	49	39

Problem 5

- The following table shows the experiment conducted between detergents A,B,C and D , and the three Engines 1,2 and 3
- Looking at the detergents as treatments and the engines as blocks, perform the two way analysis of variance test and comment on your results.

	Engines	2	3
Detergent A	45	43	51
Detergent B	47	46	52
Detergent C	48	50	55
Detergent D	42	37	49

Problem 6

- Four doctors each test four treatments for a certain disease and observe the number of days each persons takes to recover. The results are as follows.
- Discuses is there any difference between the (a) doctors (b) treatments.

	Treatment			
Doctor	1	2	3	4
A	10	14	19	20
B	11	15	17	21
C	9	12	16	19
D	8	13	17	20

Latin Square

- It is used eliminate the effect of two extraneous source of variability. An nxn Latin square is a square array of n distinct letters, with each letter appearing once in each row and each coloum.

Layout

- Let us consider four treatments A,B,C and D each applied once in each row and each coloum. Then the 4X4 Latin square is given below.
- A B C D
- B C D A
- C D A B
- D A B C

		Column					
		1	2	3	4	5	6
	1	A	B	C	D	E	F
	2	B	C	D	E	F	A
Rows	3	C	D	E	F	A	B
	4	D	E	F	A	B	C
	5	E	F	A	B	C	D
	6	F	A	B	C	D	E

ANOVA Table

ANOVA Table

Source of variation	Degrees of freedom	S.S	MS	F
Row	$m-1$	SSR	$MSR = \frac{SSR}{m-1}$	$F_R = MSR/MSE$
Column	$m-1$	SSC	$MSC = \frac{SSC}{m-1}$	$F_C = MSC/MSE$
Treatment	$m-1$	SST	$MST = \frac{SST}{m-1}$	$F_T : MST/MSE$
Error	$(m-1)(m-2)$	SSE	$MSE = \frac{SSE}{(m-1)(m-2)}$	
Total	m^2-1	TSS		

One Way ANOVA

- $N = m^2$, m=no. of rows=no. of coloums=no. of treatments.
- Correction factor $C.F = \frac{T^2}{N}$, T = Grand Total = sum of all y_{ij}.
- Row Sum square : SSR $= \sum_{i=1}^{m} \frac{R_i^2}{m} - C.F$
  here R_i : i^{th} total sum.
- Colum Sum square : SSC $= \sum_{k=1}^{m} \frac{C_k^2}{m} - C.F$
  here C_k : k^{th} total sum.
- Treatment Sum square : SST $= \sum_{j=1}^{m} \frac{T_j^2}{m} - C.F$
  here T_j : j^{th} treatment total sum.
- Error Sum square : SSE =TSS-(SST+SSR+SSC), where

  TSS $= \sum_{i=1}^{k} \sum_{j=1}^{n_i} y_{ij}^2 - C.F$: total sum square.

Use of F test in ANOVA

- Test statistic:
- $F_{t.s}^{T} = \begin{cases} \dfrac{MST}{MSE} & ;if\ MST \geq MSE \\ \dfrac{MSE}{MST} & ;if\ MSE > MST \end{cases}$

Problem 1

$m = 4$; $N = 4^2 = 16$

$T = 21.365$, $C.F = \frac{21.365^2}{16} = 28.53$

Example

Grain yield of three maize hybrids (A, B, and D) and a check (C).

Row	Column 1	Column 2	Column 3	Column 4	Row $\left(\sum R\right)$
1	1.640 (B)	1.210 (D)	1.425 (C)	1.345 (A)	5.620 $= R_1$
2	1.475 (C)	1.185 (A)	1.400 (D)	1.290 (B)	5.350 $= R_2$
3	1.670 (A)	0.710 (C)	1.665 (B)	1.180 (D)	5.225 $= R_3$
4	1.565 (D)	1.290 (B)	1.655 (A)	0.660 (C)	5.170 $= R_4$
Column total $\left(\sum C\right)$	6.350 C_1	4.395 C_2	6.145 C_3	4.475 C_4	21.365 $= T$

Treatment	Total
A	5.855 $- T_1$
B	5.885 $- T_2$
C	4.270 $- T_3$
D	5.355 $- T_4$

$SSR = \sum R_i^2/4 - CF = 0.0302$

$SSC = \sum \frac{C_i^2}{4} - CF = 0.8273$

$SST = \sum \frac{T_i^2}{4} - CF = 0.4268$

$TSS = \sum\sum y_{ij}^2 - CF = 1.4139$

$SSE = TSS - (R + C + T)$

$= 0.1296$

Solution 1

Anova

SOV	d.f	S.S	M.S.
Row	$m-1=3$	0.0302	$MSR = \frac{0.0302}{3}$
Col.	3	0.8273	$MSC = \frac{0.8273}{3}$
Treat	3	0.4268	$MST = \frac{0.4268}{3} = 0.142$
Error	$(m-1)(m-2)=6$	0.1296	$MSE = 0.0215$
Total	15	1.4139	

F-ratio

$F_{ts}^{R} =$

$F_{ts}^{C} =$

$F_{ts}^{T} = \frac{0.142}{0.0215} = 6.650$

T.

(4) $\alpha = 5\%$

(5) T: $F^{T}_{3,6}(5\%) = 4.76$, Reject H_{0T}

2. The sample data in the following Latin Square are the scores obtained by 9 college students of various ethnic backgrounds and various professional interests in an American history test. A, B, C are the three instructors by whom the 9 college students were taught the course in American history. Use 5% level of significance to analyze the design and test the following hypotheses. whether differences in

i) the ethnic background have no effect on the scores and

ii) professional interests have no effect on the scores.

	Ethnic background		
	Mexican	German	Polish
Law	A : 75	B : 86	C : 69
Medicine	B : 95	C : 79	A : 86
Engineering	C : 70	A : 83	B : 93