A Simple example showing the implementation of K-means clustering algorithm (using K=2) K-no.of clusters

S.No.	V_1	V_2	
1	1.0	1.0	x
2	1.5	2.0	
3	3.0	4.0	
4	5.0	7.0	x
5	3.5	5.0	
6	4.5	5.0	
7	3.5	4.5	

Step 1: Randomly choose following two centroids for 2 clusters

$m1 = (1.0, 1.0)$

$m2 = (5.0, 7.0)$

Find distance between every individual Point and the centroids.

$d(m_1, 2) = \sqrt{|1.0-1.5|^2 + |1.0-2.0|^2} = 1.12$

$d(m_2, 2) = \sqrt{|5.0-1.5|^2 + |7.0-2.0|^2} = 6.10$

$d(m_1, 3) = \sqrt{|1.0-3.0|^2 + |1.0-4.0|^2} = 3.61$

$d(m_2, 3) = \sqrt{|5.0-3.0|^2 + |7.0-4.0|^2} = 3.61$

$d(m_1, 5) = \sqrt{|1.0-3.5|^2 + |1.0-5.0|^2} = 4.72$

$d(m_2, 5) = \sqrt{|5.0-3.5|^2 + |7.0-5.0|^2} = 2.5$

$d(m_1, 4) = \sqrt{|1.0-5.0|^2 + |1.0-7.0|^2} = 7.21$

$d(m_1, 6) = 5.31$ $\quad d(m_2, 6) = 2.06$

$d(m_1, 7) = 4.3$ $\quad d(m_2, 7) = 2.52$

S.No	Centroid 1 m_1	Centroid 2 m_2
1	0	7.21
2	1.12	6.10
3	3.61	3.61
4	7.21	0
5	4.72	2.5
6	5.31	2.06
7	4.30	2.52

Pts closer to m_1 => 2, 3, 1

Pts closer to m_2 => 5, 6, 7, 4

Now find the new centroids

$m_1 = \frac{1}{3}(1.0+1.5+3.0), \frac{1}{3}(1.0+2.0+4.0$

$= (1.83, 2.33)$

$m_2 = \frac{1}{4}(5.0+3.5+4.5+3.5) + \frac{1}{4}(7.0+5.0+5.0$
$+4.5$

$= (4.12, 5.38)$

$m_1 = (1.83, 2.33)$ $m_2 = (4.12, 5.38)$

Step 3: Now Using the new centroids Compute the Euclidean distance between the individual pts & centroids

$1 = (1.0, 1.0)$

$$d(m_1, 1) = \sqrt{|1.83 - 1.0|^2 + (2.33 - 1.0)^2}$$

$$= \sqrt{(.83)^2 + (1.33)^2} = 1.568$$

$$d(m_2, 1) = \sqrt{(4.12 - 1.0)^2 + (5.38 - 1.0)^2} = 5.38$$

$2 = (1.5, 2.0)$

$$d(m_1, 2) = \sqrt{(1.83 - 1.5)^2 + (2.33 - 2.0)^2} = 0.47$$

$$d(m_2, 2) = \sqrt{(4.12 - 1.5)^2 + (5.38 - 2.0)^2} = 4.28$$

$3 = (3.0, 4.0)$

$$d(m_1, 3) = \sqrt{(1.83 - 3.0)^2 + (2.33 - 4.0)^2} = 2.04$$

$$d(m_2, 3) = \sqrt{(4.12 - 3.0)^2 + (5.38 - 4.0)^2} = 1.78$$

$d(m_1, 4) = 5.64$ $d(m_2, 4) = 1.84$

$d(m_1, 5) = 3.15$ $d(m_2, 5) = 0.73$

$d(m_1, 6) = 3.78$ $d(m_2, 6) = 0.54$

$d(m_1, 7) = 2.74$ $d(m_2, 7) = 1.08$

S.No	Centroid 1	Centroid 2
1	1.57 ✓	5.38
2	0.47 ✓	4.28
3	2.04	1.78 ×
4	5.64	1.84 ×
5	3.15	0.73 ×
6	3.78	0.54 ×
7	2.74	1.08 ×

The clusters obtained are $\{1, 2\}$ & $\{3, 4, 5, 6, 7\}$

Next Centroids are $m_1 = \frac{1}{2}(1.0 + 1.5), \frac{1}{2}(1.0 + 2.0)$

$$= (1.25, 1.5)$$

$$m_2 = \frac{1}{5}(3.0 + 5.0 + 3.5 + 4.5 + 3.5),$$

$$\frac{1}{5}(4.0 + 7.0 + 5.0 + 5.0 + 4.5)$$

$$= (3.9, 5.1)$$

Step3: Now Using the new centroids Compute the distance between the individual pts & Centroids

$d(m_1,1) = 0.58$ $\left[\sqrt{(1.0-1.25)^2+(1.0-1.5)^2} = 0.58\right)$

$d(m_2,1) = 5.02$ $\left[\sqrt{(1.0-3.9)^2+(1.0-5.1)^2} = 5.02\right)$

$d(m_1,2) = 0.58$ $d(m_2,2) = 3.92$

$d(m_1,3) = 3.05$ $d(m_2,3) = 1.42$

$d(m_1,4) = 6.66$ $d(m_2,4) = 2.20$

$d(m_1,5) = 4.16$ $d(m_2,5) = 0.41$

$d(m_1,6) = 4.78$ $d(m_2,6) = 0.61$

$d(m_1,7) = 3.75$ $d(m_2,7) = 0.72$

$m_1 = (1.25, 1.5)$

$m_2 = (3.9, 5.1)$

S.No	Centroid 1	Centroid 2
1	0.58 ✓	5.02
2	0.58 ✓	3.92
3	3.05	1.42 ×
4	6.66	2.20 ×
5	4.16	0.41 ×
6	4.78	0.61 ×
7	3.75	0.72 ×

The clusters formed are

$\{1,2\}$ $\{3,4,5,6,7\}$

There is no change in the cluster formation and the algorithm comes to a halt.
