Day	Outlook	Temp	Humidity	Wind	Play
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

Decision Tree Classifier Example.

Attribute : Outlook

Values : Sunny, Overcast, Rain

$S[9(\text{Yes}), 5(\text{No})]$ Entropy $(S) = -\frac{9}{14}\log_2\left(\frac{9}{14}\right) - \frac{5}{14}\log_2\left(\frac{5}{14}\right)$

$$= -\frac{9}{14}\frac{\log_{10}\left(\frac{9}{14}\right)}{\log_{10} 2} - \frac{5}{14}\frac{\log_{10}\left(\frac{5}{14}\right)}{\log_{10} 2}$$

$$= -(0.6429)\frac{-0.1919}{0.3010} - (0.3571)\frac{(-0.4472)}{0.3010}$$

$$= \frac{0.1234}{0.3010} + \frac{0.1597}{0.3010}$$

$$= \frac{0.2831}{0.3010} = 0.9405$$

Entropy $(S) = 0.9405$

$$\text{Entropy}(S_{\text{sunny}}) = -\frac{2}{5}\log_2\frac{2}{5} - \frac{3}{5}\log_2\frac{3}{5}$$
$$= 0.971$$

$$\text{Entropy}(S_{\text{overcast}}) = -\frac{4}{4}\log_2\frac{4}{4} - \frac{0}{4}\log_2\frac{0}{4}$$
$$= 0$$

$$\text{Entropy}(S_{\text{rain}}) = -\frac{3}{5}\log_2\frac{3}{5} - \frac{2}{5}\log_2\frac{2}{5}$$
$$= 0.971$$

$$\text{Information Gain}(S, \text{outlook}) = \text{Entropy}(S) - \sum_{V \in \{\text{sunny, overcast, rain}\}} \frac{|S_V|}{|S|}\text{Entropy}(S_V)$$

$$\therefore \text{Gain}(S, \text{outlook}) = \text{Entropy}(S) - \frac{5}{14}\text{Entropy}(S_{\text{sunny}}) - \frac{4}{14}\text{Entropy}(S_{\text{overcast}}) - \frac{5}{14}\text{Entropy}(S_{\text{rain}})$$

$$\Rightarrow 0.94 - \frac{5}{4}0.971 - \frac{4}{14}\times 0 - \frac{5}{14}0.971$$

$$\text{Gain}(S, \text{outlook}) \Rightarrow 0.2464.$$

Attribute : Temp Values = Hot, Mild, Cool

$$\text{Entropy}(S_{\text{Hot}}) = -\frac{2}{4}\log_2\frac{2}{4} - \frac{2}{4}\log_2\frac{2}{4} = 1.0$$

$$\text{Entropy}(S_{\text{Mild}}) = -\frac{4}{6}\log_2\frac{4}{6} - \frac{2}{6}\log_2\frac{2}{6} = 0.9183$$

$$\text{Entropy}(S_{\text{Cool}}) = -\frac{3}{4}\log_2\frac{3}{4} - \frac{1}{4}\log_2\frac{1}{4} = 0.8113$$

$$\text{Gain}(S, \text{Temp}) = \text{Entropy}(S) - \frac{4}{4}\text{Entropy}(S_{\text{Hot}}) - \frac{6}{14}\text{Entropy}(S_{\text{Mild}}) - \frac{4}{14}\text{Entropy}(S_{\text{Cool}})$$

$$Gain(S, Temp) = 0.94 - \frac{4}{14}1.0 - \frac{6}{14}0.9183 - \frac{4}{14}0.8113$$

$$= 0.0289$$

Attribute : Humidity Values : High, Normal.

$$Entropy(S_{High}) = -\frac{3}{7}\log_2\frac{3}{7} - \frac{4}{7}\log_2\frac{4}{7} = 0.9852$$

$$Entropy(S_{Normal}) = -\frac{6}{7}\log_2\frac{6}{7} - \frac{1}{7}\log_2\frac{1}{7} = 0.5916$$

$$Gain(S, Humidity) = Entropy(S) - \frac{7}{14}Entropy(S_{High}) - \frac{7}{14}Entropy(S_{Normal})$$

$$= 0.94 - \frac{7}{4}0.9852 - \frac{7}{14}0.5916$$

$$= 0.1516$$

Attribute (Wind) : Values : Strong, Weak.

$$Entropy(S_{Strong}) = -\frac{3}{3}\log_2\frac{3}{3} = 1.0$$

$$Entropy(S_{Weak}) = -\frac{6}{8}\log_2\frac{6}{8} - \frac{2}{8}\log_2\frac{2}{8} = 0.8113$$

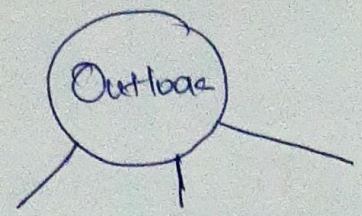
$$Gain(S, Wind) = 0.94 - \frac{6}{14} \times 1.0 - \frac{8}{14} \times 0.8113$$

$$= 0.0478$$

Attribute	Gain
Outlook	0.2464
Temp	0.0289
Humidity	0.1516
Wind	0.0478

Maximum Gain is 0.2464 for the attribute Outlook

Therefore this Outlook attribute comes as Root Node



Sunny Overcast Rain

Temp	Humidity	Wind	Play	
Hot	High	Weak	N	Yes
Hot	High	Strong	N	
Mild	High	Weak	N	
Cool	Normal	Weak	Y	
Mild	Normal	Strong	Y	

$$Entropy(S_{sunny}) = -\frac{2}{5}\log_2\frac{2}{5} - \frac{3}{5}\log\frac{3}{5}$$

$$= 0.97$$

Attribute: Temp Values: Hot Mild Cool.

$Entropy(S_{Hot}) = 0.0$

$Entropy(S_{Mild}) = 1.0$

$Entropy(S_{Cool}) = 0.0$

$$Gain(S_{sunny}, Temp) = Entropy(s) - \frac{2}{5}Entropy(S_{Hot}) - \frac{2}{5}Entropy(S_{Mild}) - \frac{1}{5}Entropy(S_{Cool})$$

$$\Rightarrow 0.97 - \frac{2}{5}\times 0.0 - \frac{2}{5}\times 1 - \frac{1}{5}\times 0$$

$$\Rightarrow 0.570$$

Attribute: Humidity Values: High, Normal

$Entropy(S_{High}) = 0.0$, $Entropy(S_{Normal}) = 0.0$

$$Gain(S_{sunny}, Humidity) = Entropy(S) - \frac{3}{5}Entropy(S_{High}) - Entropy(S_{Normal}) = 0.97$$

Attribute : Wind Values : Strong, Weak

Entropy (S_{strong}) = 1.0

Entropy (S_{weak}) = $\frac{1}{3}\log_2\frac{1}{3} - \frac{2}{3}\log_2\frac{2}{3}$

= 0.9183

Gain (S_{sunny}, Wind) = Entropy(s) − $\frac{2}{5}$ Entropy(S_{strong})

− $\frac{3}{5}$ Entropy(S_{weak})

= 0.97 − $\frac{2}{5}$ × 1.0 − $\frac{3}{5}$ × 0.918

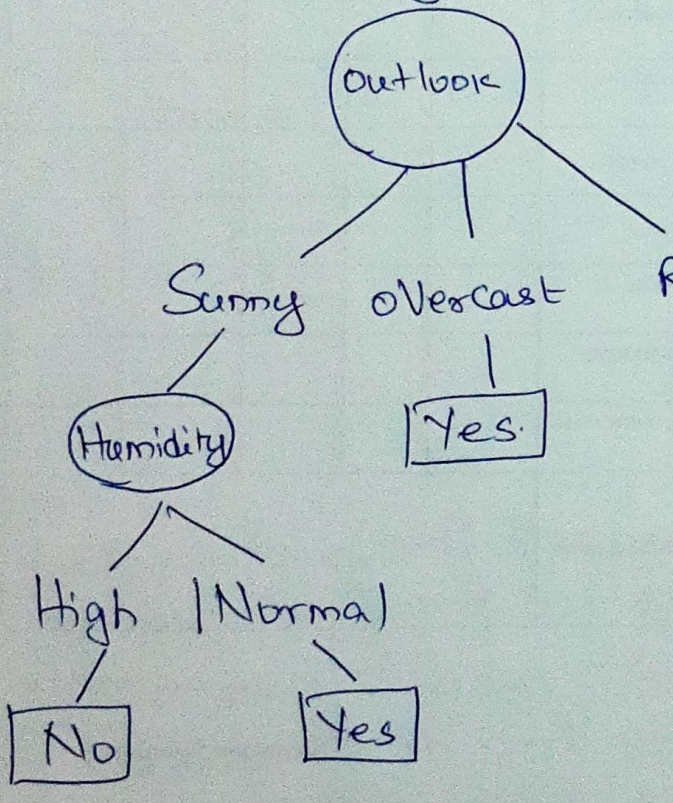
= 0.0192

Gain (S_{sunny}, Temp) = 0.570

Gain (S_{sunny}, Humidity) = 0.97

Gain (S_{sunny}, Wind) = 0.0192

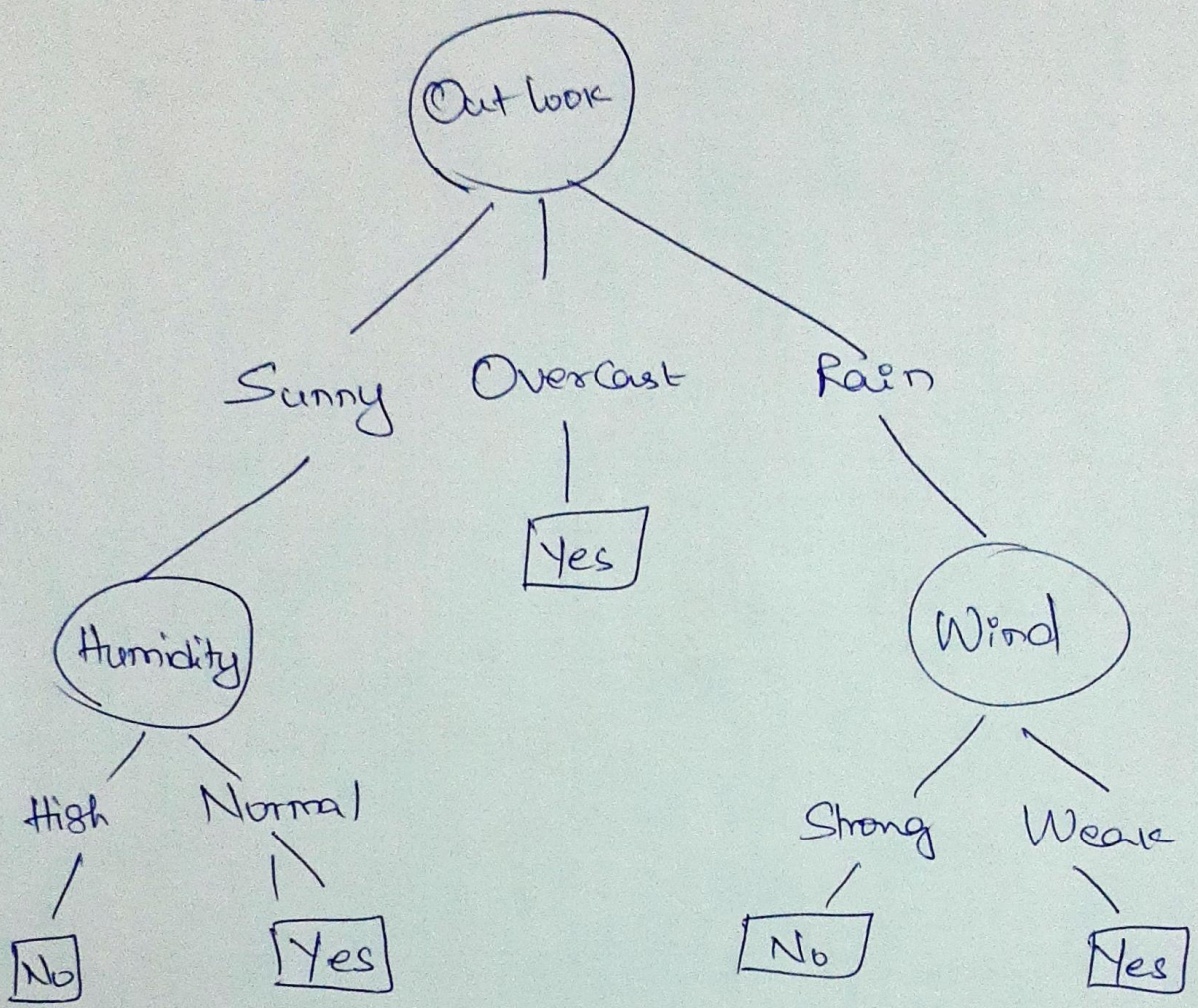
} maximum is Humidity



Day	Temp	Wind	Play
D_4	Mild	Weak	Y
D_5	Cool	Weak	Y
D_6	Cool	Strong	N
D_{10}	Mild	Weak	Y
D_{14}	Mild	Strong	N

Gain(S_{Rain}, Temp) = 0.0192
Gain(S_{Rain}, Humidity) = 0.0192
Gain(S_{Rain}, Wind) = 0.97

} Maximum is Wind



(Final Decision Tree).

Rules:

If Outlook = Sunny and Humidity = High then Play = No
If Outlook = Sunny and Humidity = Normal then Play = Yes
If Outlook = Overcast then Play = Yes
If Outlook = Rain and Wind = Strong then Play = No
If Outlook = Rain and Wind = Weak then Play = Yes.