Module 4

Hypothesis Testing I

- Introduction
- Types of errors
- critical region
- procedure of testing hypothesis
- Large sample tests
- Z test for Single Proportion
- Difference of Proportion
- mean and difference of means.

Introduction

The method of hypothesis testing uses tests of significance to determine the likelihood that a statement (often related to the mean or variance of a given distribution) is true, and at what likelihood we would, as statisticians, accept the statement as true.

A hypothesis should be specific, clear and precise. It should state as far as possible in mostly single terms so that the same is easily understood by all. It should state the relationship between variables.

While understanding the mathematical concepts that go into the formulation of these tests is important, knowledge of how to appropriately use each test (and when to use which test) is equally important.

A Statistical hypothesis is a conjecture about a population parameter. This conjecture may or may not be true.

The null hypothesis, symbolized by H_0, is a statistical hypothesis that states that there is no difference between a parameter and a specific value or that there is no difference between two parameters.

The alternative hypothesis, symbolized by H_1, is a statistical hypothesis that states a specific difference between a parameter and a specific value or states that there is a difference between two parameters.

In other words, we can say H_1 is complementary to H_0.

Type of Tests

Two-Tailed Test:

A medical researcher is interested in finding out whether a new medication will have any undesirable side effects. The researcher is particularly concerned with the pulse rate of the patients who take the medication.

What are the hypotheses to test whether the pulse rate will be different from the mean pulse rate of 82 beats per minute?

$H_0 :\ \mu = 82$ and $H_1 : \mu \neq 82$:

This is a Two-Tailed test.

Right-Tailed Test:

A chemist invents an additive to increase the life of an automobile battery. If the mean lifetime of the battery is 36 months, then his hypotheses are

$H_0 : \mu = 36$ and $H_1 : \mu > 36$

Left-Tailed Test:

A contractor wishes to lower heating bills by using a special type of insulation in houses. If the average of the monthly heating bills is Rs.78, her hypotheses about heating costs will be

$H_0 : \mu$ = Rs.78 and $H_1 : \mu$ < Rs. 78

A test statistic is computed after stating the null hypothesis. It is based on the appropriate probability distribution.

A test statistics uses the data obtained from a sample to make a decision about whether or not the null hypothesis should be rejected.

The numerical value obtained from a test statistic is called the calculated value.

Errors in Hypothesis Testing:

A Type I error occurs if one rejects the null hypothesis when it is true. This is similar to a good product being rejected by the consumer and hence Type I error is also known as producer's risk.

The level of significance is an important concept in hypothesis testing. It is always some percentage. The level of significance is the maximum probability of rejecting a null hypothesis when it is true and is denoted by α. The probability of making a correct decision is $1-\alpha$. The level of significance may be taken as 1% or 5% or 10% (i.e., α = 0.01 or 0.05 or 0.1). If we fix the level of significance at 5%, then the probability of making type-I error is 0.05. This also means that we are 95% confident of making a correct decision. When no level of significance is mentioned, it is taken as = 0.05.

A Type II error occurs if one does not reject the null hypothesis when it is false. As this error is similar to that of accepting a product of inferior quality, it is known as consumer's risk. The probability of committing Type II error is denoted by β.

Types of Errors

	Study reports **NO** difference (Do not reject H_0)	Study reports **IS** a difference (Reject H_0)
H_0 is true Difference Does **NOT** exist in population	✔	X Type I Error
H_A is true Difference **DOES** exist in population	X Type II Error	✔

Important Tests of Hypothesis

For the purpose of testing a hypothesis, several tests of hypothesis were developed.

They can be classified as

Parametric test

Non-parametric test

The important parametric tests are

Z-test (for Large Samples)

t-test (for Small Samples)

F-test (for Small Samples)

Z-test

If the sample size n is greater than or equal to 30 ($n \geq 30$), the sample is called a Large Sample.
The z-test is a statistical test for the mean of a population. It can be used for large sample or when the population is normally distributed and σ is known.

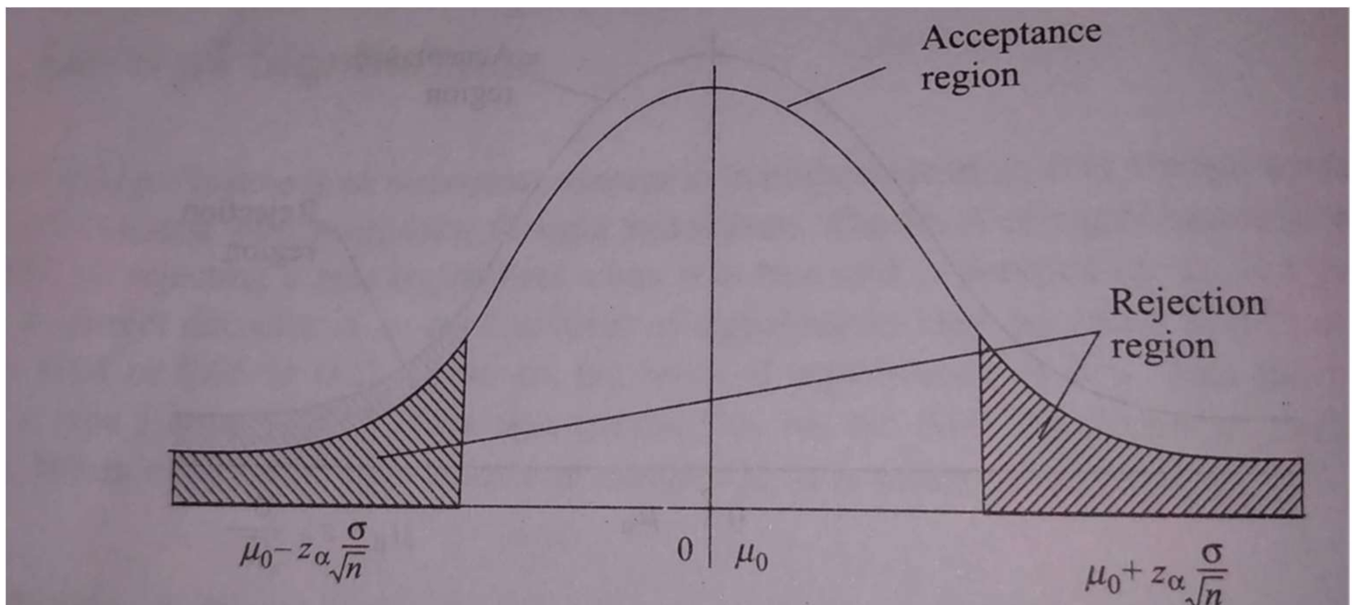
The critical values for some standard LOS's are given in the following table:

Table 1:

Type of Test	$\alpha = 1\%(0.01)$	$\alpha = 2\%(0.02)$	$\alpha = 5\%(0.05)$	$\alpha = 10\%(0.1)$
Two-Tailed	$\lvert z_\alpha \rvert = 2.58$	$\lvert z_\alpha \rvert = 2.33$	$\lvert z_\alpha \rvert = 1.96$	$\lvert z_\alpha \rvert = 1.645$
Right-Tailed	$z_\alpha = 2.33$	$z_\alpha = 2.055$	$z_\alpha = 1.645$	$z_\alpha = 1.28$
Left-Tailed	$z_\alpha = -2.33$	$z_\alpha = -2.055$	$z_\alpha = -1.645$	$z_\alpha = -1.28$

Critical Region

A region corresponding to a statistics which amounts to rejection of the null hypothesis H_0 is known as the critical region. It is also called as the region of rejection. The critical region is the region of the standard normal curve corresponding to a predetermined level of significance. The region under the normal curve which is not shaded is known as the acceptance region.



Two-tailed test: Rejection and acceptance region

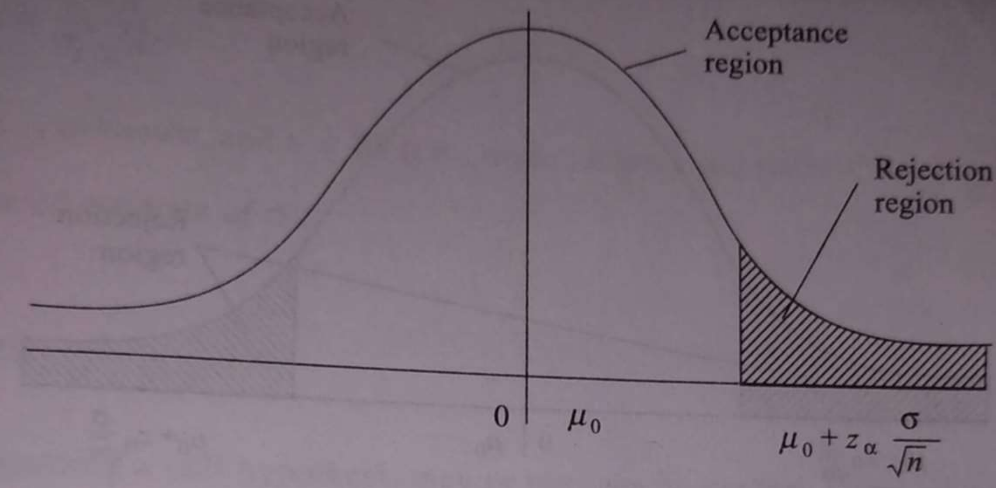


Fig. 6.1 *Right-tailed test: Rejection and acceptance regions*

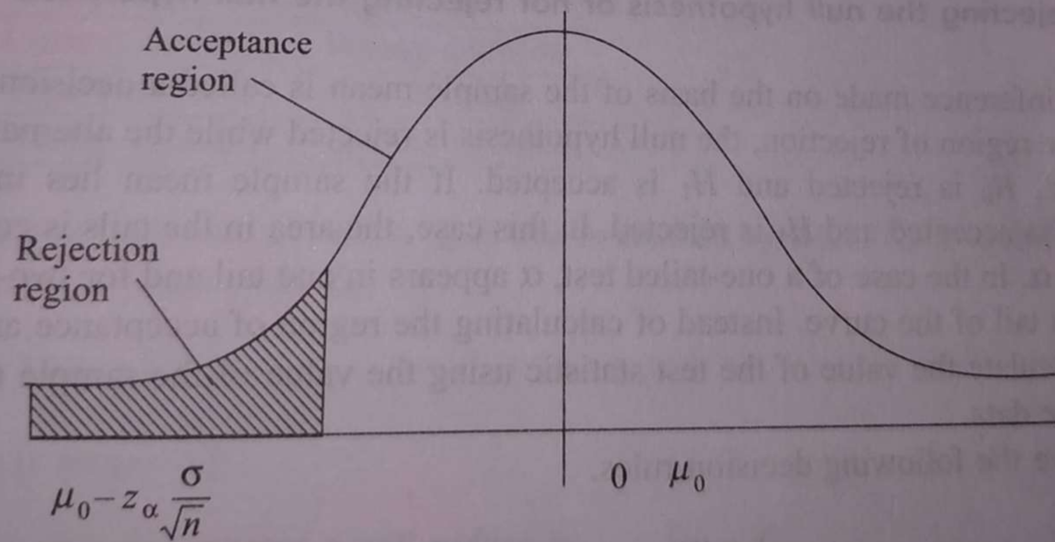


Fig. 6.2 *Left-tailed test: Rejection and acceptance region*

Procedure for Hypothesis Testing

The main question in hypothesis testing is whether to accept the null hypothesis or not to accept the null hypothesis. The following tests are involved in hypothesis testing.

Step 1: State the Null (H_0) and Alternative (H_1) Hypotheses
Step 2: Decide the nature of test (one-tailed or two-tailed based on H_1)
Step 3: Obtain z_α value which depends upon α value and the nature of test.
Step 4: For large samples, when population n's standard deviation is known, the test statistics is Z $=\frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$. The corresponding distribution is normal.
Step 5: Comparison and Conclusion.
- If $|z| < |z_\alpha|, H_0$ is accepted or H_1 is rejected, that is, there is no significant difference at α % Level of Significance.
- If $|z| > |z_\alpha|, H_0$ is rejected or H_1 is accepted, that is, there is significant difference at α % LOS.