SOFT COMPUTING

Module 5: FUZZY LOGIC AND APPROXIMATE REASONING

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SYLLABUS

- Fuzzy truth values
- Fuzzy propositions
- Fuzzy rules:
- Formation of rules
- Decomposition of rules
- Aggregation of rules
- Fuzzy reasoning
- FIS
- Fuzzy Decision Making

LOGIC

- It is an **analytical theory of art of reasoning**
- **Goal** is to **systematize and codify principles of valid reasoning**

- **Father of Logic:** Greek philosopher , Aristotle(384-322 B.C.)

- The **modern symbolic logic** started with the book “Begriff ss chrift (1879)”
- by Gottlob Frege (1848 – 1925)(**Father of Modern Logic**)
- who **developed a system of logic** for use in his **study of the foundations of arithmetic**

LOGICAL CONNECTIVES

- Consider the proposition p and q whose truth values belong to the truth value set {0, 1}. The meaning of the logical connectives is given by definitions and expressed by equations in which p and q stand for the truth values of the propositions p and q.
-
- Negation we have $\sim p = 1 - p$
- Conjunction we have $p \wedge q = \min(p, q)$
- Disjunction we have $p \vee q = \max(p, q)$
- Implication we have $p \to q = \min(1, 1 + q - p)$
- Equivalence we have $p \leftrightarrow q = 1 - |p - q|$

SOME IMPORTANT PROPERTIES

- CONTRADICTION: $p \wedge \overline{p}$
- TAUTOLOGY: $p \vee \overline{p}$
- MODUS PONENS: $p \wedge (p \rightarrow q) \rightarrow q$
- MODUS TOLENS: $\overline{q} \wedge (p \rightarrow q) \rightarrow \overline{p}$

THREE VALUED LOGIC

- One reason for questioning the above principle is the difficulty arising with estimating truth values of propositions expressing future events
- Example: Mr. Smith will win the elections
- Future events are not yet true or false
- Their truth values are unknown
- It will be determined when the events happen
- Several three valued logics have been established
- In all these logics, the truth, falsity and indeterminacy are represented by truth values 1, 0 and ½ respectively

THREE VALUD LOGIC (Contd…)

- There are five best known three valued logics
- Due to Lukasiewicz, Bochvar, Kleene, Heyting and Reichenbach
- Popular one is:
- Put forth by Lukasiewicz in the early 1930s

- Quasi Contradiction: A compound proposition that does not assume the truth value 1 under all possible truth values for its simple propositions (i.e. it never assumes the value 1)

- Quasi Tautology: A compound proposition that does not assume the truth value 0 under all possible truth values for its simple propositions. (i.e. it never assumes the value 0)

TRUTH TABLE FOR LUKASIEWICZ LOGIC

p	q	$\overline{p}$	$\overline{q}$	$p \wedge q$	$p \vee q$	$p \rightarrow q$	$p \leftrightarrow q$
1	1	0	0	1	1	1	1
1	½	0	½	½	1	½	½
1	0	0	1	0	1	0	0
½	1	½	0	½	1	1	½
½	½	½	½	½	½	1	1
½	0	½	1	0	½	½	½
0	1	1	0	0	1	1	0
0	½	1	½	0	½	1	½
0	0	1	1	0	0	1	1

N-VALUED LOGIC

- Once the three valued logics were accepted as meaningful and useful, it became desirable to explore generalizations to n–valued logics for an arbitrary number of truth values .

- For any given n, the truth values in these generalized logics are usually labeled by rational numbers in the unit internal [0, 1].

- These values are obtained by evenly dividing the interval between 0 and 1 exclusive.

- The set of truth values of an n–valued logic is thus defined as

$$T_n = \left\{ 0 = \frac{0}{n-1}, \frac{1}{n-1}, \frac{2}{n-1}, \ldots, \frac{n-2}{n-1}, \frac{n-1}{n-1} = 1 \right\}$$

- These values can be interpreted as degree of truth.

INFINITE VALUED LOGIC

- If the truth values are represented by all real numbers in [0, 1], the many–valued logic is called the infinite–valued logic
- Sometimes, it is referred as the standard Lukasiewicz logic (L_1).
- There is a correspondence between the fuzzy set theory and the infinite – valued logic

Fuzzy set	Infinite valued Logic
– Complementation	Negation
– Intersection	Conjunction
– Union	Disjunction

FUZZY LOGIC

- Founder is L.A.Zadeh (started in 1973)
- Introduced the idea of Linguistic variables and compositional rules of inference

- Fuzzy logic as the name suggests, is the logic underlying models of reasoning which are approximate rather than exact

- Fuzzy logic provides an effective conceptual framework for dealing with the problem of knowledge representation in an environment of uncertainty and imprecision

FUZZY LOGIC BASIC (CONTD....)

- A precise definition telling us what fuzzy logic is does not exist
- Although there is not a unique system of knowledge called fuzzy logic its meaning can be explained
- There is a correspondence (isomorphism) between classical sets and classical logic
- Fuzzy sets are a generalization of classical sets and infinite-valued logic is a generalization of classical logic.
- There is a correspondence (isomorphism) between these two areas.

FUZZY LOGIC BASICS (CONTD…..)

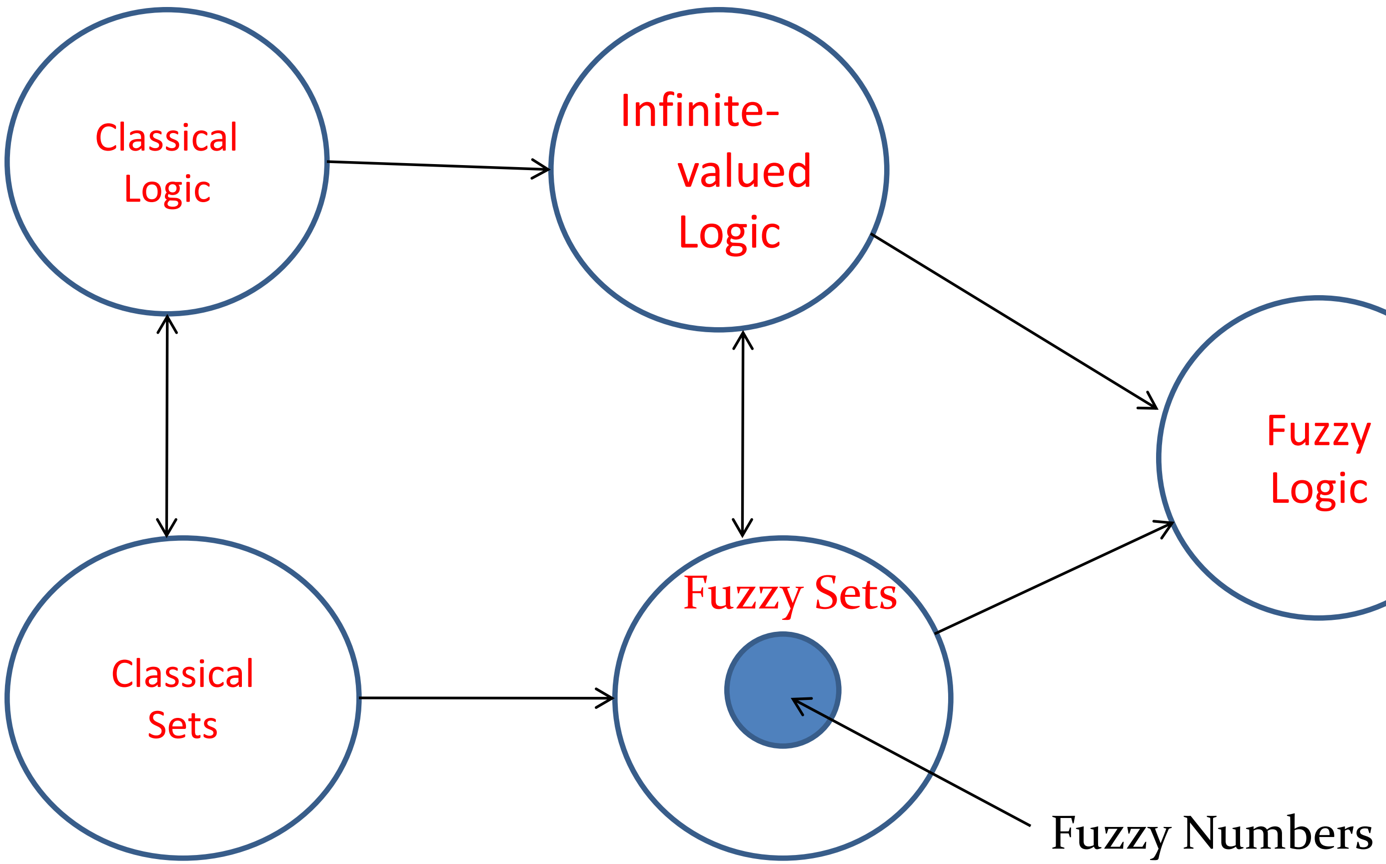
- Fuzzy logic uses as a major tool fuzzy set theory

- Basic mathematical ideas for fuzzy logic evolve from infinite-valued logic, thus there is a link between the two logics

- Fuzzy logic can be considered as an extension of infinite-valued logic in the sense of incorporating fuzzy sets and fuzzy relations into the system of infinite –valued logic.

- FUZZY LOGIC FOCUSES ON LINGUISTIC VARIABLES IN NATURAL LANGUAGES AND AIMS TO PROVIDE FOUNDATIONS FOR APPROXIMATE REASONING WITH IMPRECISE PROPOSITIONS

Evolvement of Fuzzy Logic



DOMAIN OF FUZZY LOGIC

- Fuzzy logic deals with:

1. **Linguistic variables**
2. **Linguistic modifiers**
3. **Propositional fuzzy logic**
4. **Inferential rules**
5. **Approximate reasoning**

LINGUISTIC VARIABLES

- **Linguistic Variables: Those variables whose values are words or sentences in natural or artificial languages**
- Example: The word 'Age' used in natural language
- Age is a linguistic variable taking values very young, young, middle age, old and very old, called the terms of the linguistic variable age.
- **The terms are fuzzy sets**
- **Each term is defined by a membership function.**
- Consider the universal set U = [0, 100]

MEMBERSHIP FUNCTIONS OF TERMS FOR LINGUISTIC VARIABLE "OLD"

$$\mu_{very\ young}(x) = 1 \qquad for\ 0 \le x \le 5,$$
$$= (30 - x)/25 \qquad for\ 5 \le x \le 30.$$

$$\mu_{young}(x) = (x-5)/25 \qquad for\ 5 \le x \le 30,$$
$$= (50 - x)/20 \qquad for\ 30 \le x \le 50$$

$$\mu_{middle\ age}(x) = (x-30)/20 \qquad for\ 30 \le x \le 50,$$
$$= (70 - x)/20 \qquad for\ 50 \le x \le 70.$$

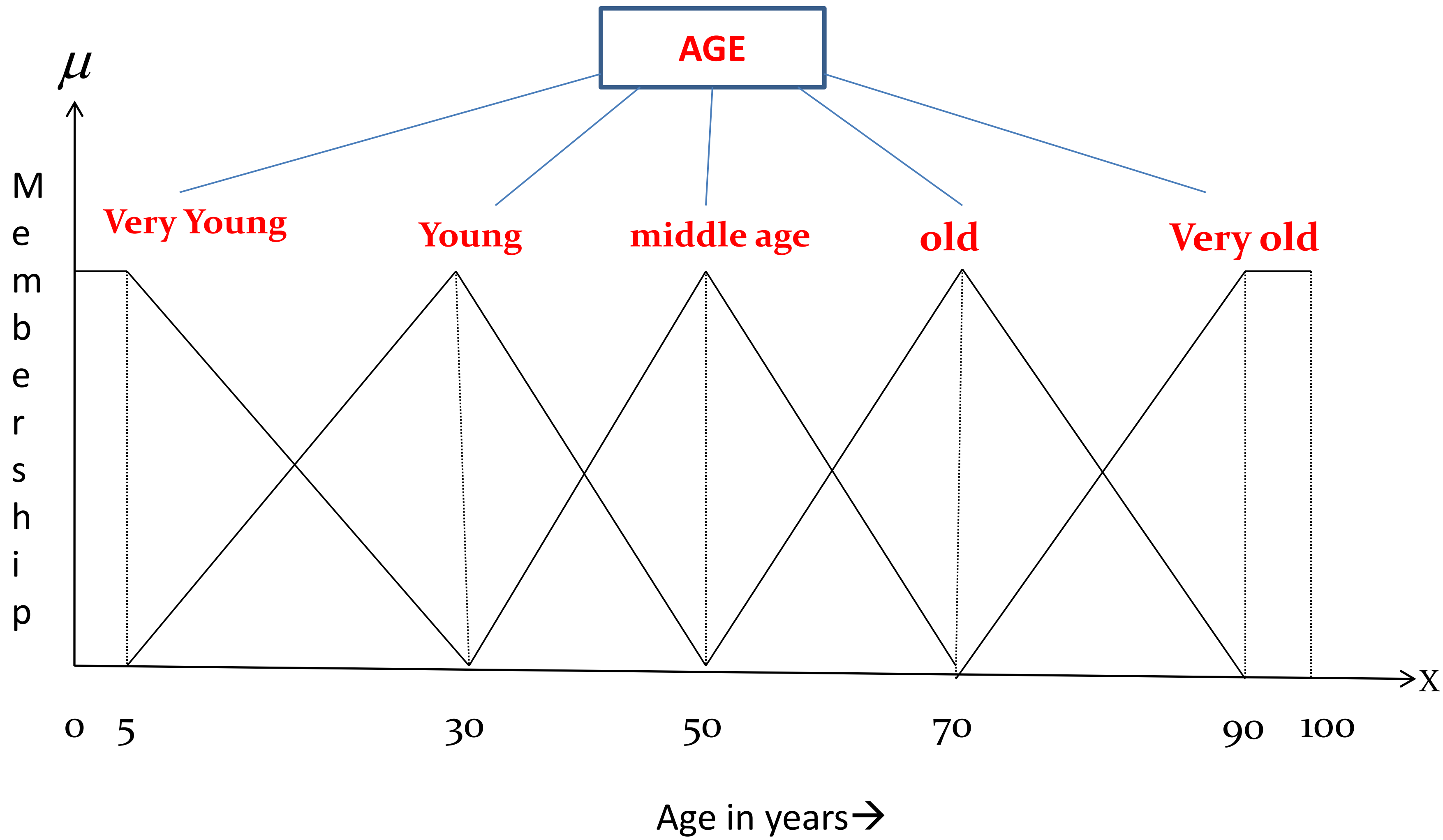
MEMBERSHIP FUNCTIONS OF TERMS FOR LINGUISTIC VARIABLE "OLD" CONTD…

- Continuing

$$\begin{aligned}\mu_{old}(x) &= (x-50)/20 \qquad for\ 50 \le x \le 70, \\ &= (90-x)/20 \qquad for\ 70 \le x \le 90.\end{aligned}$$

$$\begin{aligned}\mu_{very\ old}(x) &= (x-70)/20 \qquad for\ 70 \le x \le 90, \\ &= 1 \qquad\qquad\qquad for\ 90 \le x \le 100.\end{aligned}$$

Terms of the linguistic variable *age*



LINGUISTIC MODIFIERS

- Let $x \in U$ and A is a fuzzy set with membership function $\mu_A(x)$
- We denote by m a **linguistic modifier**, for instance **very, not fairly** etc.
- Then by mA we mean a modified fuzzy set by m whose membership function $\mu_{mA}(x)$ is a **composition of a suitable function f(x)** and $\mu_A(x)$

  expressed as $\mu_{mA}(x) = f(\mu_A(x))$.
- Examples:
- Not: f(x) = 1 – x, $\mu_{notA}(x) = 1 - \mu_A(x)$,
- Very: f(x)= $(x)^2$ $\mu_{veryA}(x) = [\mu_A(x)]^2$
- Fairly: f(x)= $(x)^{1/2}$ $\mu_{fairlyA}(x) = [\mu_A(x)]^{1/2}$

TRUTH (Baldwin, 1979)

- **The most important linguistic variable**
- It is **described by a fuzzy set** with membership function
- Truth and its terms have been **defined differently** in fuzzy logic. The **simplest definition** is

$$\mu_{true}(x) = \mathrm{x},\ \forall x \in [0,1]$$

- The modifiers applied to

$$true = \{(x, \mu_{true}(x)) \big| x \in [0,1], \mu_{true}(x) = x,\ x \in [0,1]\}.$$

$$\mu_{not\ true}(x) = \mu_{false}(x) = 1 - x$$

$$\mu_{very\ true}(x) = [\mu_{true}(x)]^2 = x^2.$$

$$\mu_{fairly\ true}(x) = [\mu_{true}(x)]^{1/2} = x^{1/2}.$$

APPROXIMATE REASONING

- **Approximate reasoning** uses **fuzzy sets** and **fuzzy logic** to **model human reasoning**.
- It **lacks the precision** of the exact reasoning in classical logic but it is **more effective** dealing with **complex** and **ill-defined systems.**
- It is an **active area of research** with some topics still under discussion and debate:

  1.Qualification & Quantification rules

  2.Compositional rules for inference

  3.Truth modifications

EXAMPLE

- Classical Modus ponens: $p \wedge (p \rightarrow q) \rightarrow q$
- This is an **implication tautology**
- In order to assign inference validity to **modus ponens** we consider it as a **procedure for deriving true propositions**
- Such a rule expressed in symbolic form with the scheme called **syllogism**
  - Premise 1 p
  - Premise 2 $p \rightarrow q$
  - Conclusion q

EXAMPLE (CONTD….)

- More detail form:

  Premise 1: x is A

  Premise 2: If x is A then y is B

  Conclusion: y is B

- (Say) x = Socrates, A= a man, B = mortal
- Premise 1: Socrates is a man
- Premise 2: If Socrates is a man then Socrates is mortal
- Conclusion: Socrates is mortal

TAUTOLOGY

- A statement is said to be a **tautology** iff it assumes the truth value **TRUE** for **all possible assignment of truth values** to its **primary components**
- **Modus Ponens is a Tautology**
- We have the following **tautological equivalences**:

$$F \vee p \Leftrightarrow p$$

$$F \wedge p \Leftrightarrow F$$

$$T \vee p \Leftrightarrow T$$

$$T \wedge p \Leftrightarrow p$$

RULE BASED SYSTEMS

- **Rules:** Expressions of the type IF **Antecedent** THEN **Consequent**
- **Rule Based System:** A system which represents knowledge using such rules

- **Newell and Simon** developed one of the **most popular rule based systems in 1972**
- **Book-> Human Problem Solving:** Allen Newell, Herbert Alexander Simon, Prentice-Hall, 1972 - Education - 920 pages

- In which rules were used to model human problem solving behaviour

RULE BASED SYSTEMS

- **Two major motivations** behind the development of rule based systems:
  - Creation of programs that **act to reproduce human behaviour**
  - The production of **expert systems**

- The **components** of a rule based system:

1. A working memory

2. A rule base

3. An inference engine

WORKING MEMORY (WM)

- It is the **storage medium** in a rule based system.
- It comprises of **a set of facts** known about a domain
- Its **contents** represent the **current state** of the system
- Its **components** are called the **working memory elements (WMEs)**
- With the help of WM the **rules communicate with each other**
- The **initial WM** contains the **starting status** of a rule based system
- The rules are **fired** when the **conditions in the antecedent are satisfied.**
- The firing of a rule **changes the status** of the rule based system

WORKING MEMORY CONTD…

- This **may lead to the change in the working memory** by generating new WMEs and may also generate new rules.

- **The firing of a rule may**
  - **Generate new elements**
  - **Modify existing elements**
  - **Delete existing elements**

RULE BASE (RB)

- A RB **contains several rules** of the form given above
- **The RB provides the power of reasoning** to the system
- The **most general structure** of a rule may be of the form:
- IF (**Condition 1 AND Condition 2 AND**… **Condition m**) THEN (**Action 1 OR Action 2 OR**…**Action k**)
- The **reasoning is carried out by firing one or more rules** in the system
- **At a time** the current state of the working memory **may make several of the rules** to be capable of being fired
- A situation when **only one such rule** can be fired **creates no problem**

CONFLICT RESOLUTION STRATEGIES

- **When more than one rule can be fired, it** leads to a conflict among the rules
- **Conflict resolution policies** are used to solve such a situation
- Some of the conflict resolution strategies are:
- **Order**
- **Specificity**
- **Recency**
- **Refractoriness**
- In **order strategy**, the first rule in order of presentation is fired. This strategy is one of the most common ones. In PROLOG this strategy is followed.

SPECIFICITY

- Among the rules whose antecedents are satisfied, **select the one which has the most specific conditions**. For example, if there are rule sets like
- IF **(elephant)** THEN add colour (grey)
- IF **(elephant) AND (royal)** THEN add colour (white)
- IF **(elephant) AND (African)** THEN add ears (large)
- It is clear that the **last two rules are more specific** than the first rule. So, these two rules are given preference over the first rule.

RECENCY

- Under recency, there are **several options**
- Sometimes the rule that matches on the **most recently created working memory element is fired**
- In the other extreme, the rule which was **least recently used is fired**

REFRACTORINESS

- Under refractoriness, the **rule which was fired with the same set of values of the variables is not selected**
- The advantage of this approach is that **it helps in avoiding the looping of reasoning**, which occurs as a consequence of firing a rule repeatedly
- **IT MAY BE NOTED THT OFTEN A COMBINATION OF THE ABOVE PRINCIPLES ARE USED TO RESOLVE CONFLICT**

INFERENCE ENGINE

- The inference engine **is a process**
- It **uses the rules in a rule based system** and the **information stored in a working memory** to **derive new information** about a given problem
- Given the contents of the working memory, the inference engine **determines a set of rules which can be fired**
- These are those **rules whose antecedents are satisfied** under the present scenario
- As mentioned above, if the **rule set contains only one element then that rule is fired**

INFERENCE ENGINE CONTD…

- If this rule set contains more than one rule, one or more of the **conflict resolution methods are used** to determine the specific rule which can be fired
- **If a rule is fired then its consequent is carried out**

- This may **lead to addition of a working memory element** or **modify such a working memory element** or **add one or more new rules** to the rule base

- After one such firing **again matching is carried out** to find a fresh set of rules which can be fired.
- **This process is continued until no other rule can be fired**

FUZZY RULE BASED SYSTEMS

- There are numerous **complex phenomena**, which can only be approximately described
- There are **ill-defined** phenomena also, which cannot be analysed through crisp or conventional approaches
- **These phenomena can be expressed through variables**, which do not have their values as numbers but words or sentences in a natural or artificial language
- **Examples:**
- Tajmahal is **beautiful**
- Einstein was a highly **talented** man
- Too **much** of rain brings in flood in rivers

FUZZY RULE BASED SYSTEMS

- **Highlighted words** in the above sentences are **values of linguistic variables**
- **These words can be represented as fuzzy sets** and their **values are labels of the corresponding fuzzy set**
- **Quantified variables** using conventional approaches may provide an accurate value being assigned
- On the other hand, **linguistic variables** provide **less precise** but **more realistic means** of system analysis
- Thus the **applicability of linguistic variables in societal systems is much more realistic than the quantified variables**

FUZZY RULE BASED SYSTEMS

- **Traditional approaches** to analyse social systems were dependent upon **well-structured mathematical models** insist on **precision** rather than permitting approximation

- In order to handle the situations in social systems and analyze them **we need to sacrifice exactness** and more precisely numerical values by the way **permitting linguistic variables**

- This may **provide approximate characterization of a social system** but will provide a **more realistic analysis and reasoning process** through the **use of fuzzy logic** in which the **truth values are Linguistic instead of being numeric**

FUZZY RULE BASED SYSTEMS

- There are many **special linguistic terms** like **very, more or less, extremely or fairly**, which are **used to modify other linguistic terms**. Such terms are called **linguistic hedges**.
- Hedges are also used to **modify fuzzy predicates**.
- This in turn helps in **modifying fuzzy truth values** and also **fuzzy probabilities**.
- The **fundamental difference** between **classical propositions** and **fuzzy propositions** is in the **range of their truth values**
- While each proposition (**crisp**) is required to be either **true or false**, the **truth or falsity of fuzzy propositions is a matter of degree**

FUZZY RULE BASED SYSTEMS

- **Degree of truth** of each fuzzy proposition is expressed by a **number in the unit interval [0, 1]**
- Fuzzy proposition **p: x is F**
- can be **modified to** another fuzzy proposition '**Hp**'
- by introducing a **linguistic hedge H** in it as '**x is HF**'
- The unary operations that represent linguistic hedges are called **modifiers**
- Given a fuzzy predicate F on a domain X and any $x \in X$, the modified fuzzy predicate HF for a linguistic hedge H is given by
- HF(x) = h(F(x)),
- where h is the unary operation corresponding to H

FUZZY RULE BASED SYSTEMS

- A **linguistic variable** is characterised by a unique tuple
- (X, T(X), U, G, M), where

- X: The **name of the variable**
- T(X): The term-set of X, that is, **the collection of its linguistic values**
- U: is the **universe of discourse**
- G: is a **syntactic rule** for generating the terms in T(X)
- M(X): is a **semantic rule** which associates with **each term in T(X) its meaning M(X)**

EXAMPLE

- **linguistic variable** named **Height**
- Height = tall + very tall + not tall + very very tall + not very tall +…+short + very short + not short + …+ not very tall and not very short + … extremely tall + … + more or less tall
- U(Tall) = [0, 200], **measured in cm**
- **A value** of height, that is, **tall** may be viewed as **a name of a fuzzy subset of U** which is characterised by **its compatibility function c: U $\longrightarrow$ [0, 1], with c(u)** representing the compatibility of a **numerical height u** with the **label tall**
- For example, the compatibilities of the numerical height 190, 150 and 40 with tall might be 1, 0.7 and 0, respectively

FUZZY RULE BASED SYSTEMS

- In the expression for height in the previous slide, among the terms **we have the primary terms**, for example, **tall** and **short**
- The **meanings** of these terms **are both subjective** and **context-dependent** and hence **must be defined a priori**
- We have used the **modifier** 'not', two **sentential connectives** 'and' and 'or' and also **linguistic hedges** like 'very', 'more', 'less', 'extremely' and 'quite'.
- Some of the **standard methods** to define the **compatibility functions** of **linguistic variable** X modified by **using hedges** 'very' and 'more or less' are presented below.

$$c_{veryX}(u) = (c_X(u))^2 \qquad c_{more\,or\,less\,X}(u) = (c_X(u))^{1/2}$$

FUZZY RULE BASED SYSTEMS

$$c_{not\ X}(u) = 1 - c_X(u)$$

$$c_{XandY}(u) = c_X(u) \wedge c_Y(u)$$

$$c_{XorY}(u) = c_X(u) \vee c_Y(u)$$

- If $c_{tall}(70) = 0.2$ and $c_{short}(70) = 0.1$ then

$c_{very\,tall}(70) = 0.04$ $c_{very\,short}(70) = 0.01$

$c_{not\,very\,tall}(70) = 0.96$ $c_{not\,very\,short}(70) = 0.99$

$$c_{not\,very\,tall\,and\,not\,very\,short}(70) = 0.96$$

FORMATION OF FUZZY RULES

- The general way of representing human knowledge is by forming natural language expressions given by
- IF antecedent TEN consequent
- There are tree general forms that exist for any linguistic variable
- 1. Assignment statements
- 2. Conditional statements
- 3. Unconditional statements
- **ASSIGNMENT STATEMENTS:**
- These are
- Y = small

FORMATION OF FUZZY RULES CONTD…

- Orange colour = orange
- a = s
- Climate = autumn
- Outside temperature = normal
- **CONDITIONAL STATEMENTS:**
- IF y is very cool THEN stop
- IF A is high THEN B is low ELSE B is not low
- IF temperature is high THEN climate is hot
- **UNCONDITIONAL STATEMENTS:**
- Goto sum

FORMATION OF FUZZY RULES CONTD…

- Stop
- Turn the pressure low
- Generally both unconditional and conditional statements place some restrictions on the consequent of the rule based process
- The restriction statements, irrespective of conditional or unconditional statements are usually connected by linguistic connectives such as “and”, “or” or “else”.
- The restrictions denoted as R1, R2, … Rn apply to the consequent of the rules

DECOMPOSITION OF COMPOUND RULES

- A compound rule is a collection of many simple rules combined together
- Any compound rule structure may be decomposed and reduced to a number of simple canonical rule forms
- The rules are generally based on natural language representations
- The methods:
- 1. **Multiple conjunctive antecedents**
- 2. **Multiple disjunctive antecedents**
- **3.Conditional statements (with ELSE and UNLESS)**
- 4. **Nested IF-THEN rules**

MULTIPLE CONJUNCTIVE ANTECEDENTS

- IF x is A1, A2, … An THEN y is Bm
- Assume a new fuzzy subset Am is defined as
- $A_m = A_1 \cap A_2 \cap \ldots \cap A_n$ and is expressed by their membership function as
- $\mu_{A_m}(x) = \min[\mu_{A_1}(x), \mu_{A_2}(x), \ldots \mu_{A_n}(x)]$
- In view of the fuzzy intersection operation, the compound rule may be rewritten as
- IF Am THEN Bm

MULTIPLE DISJUNCTIVE ANTECEDENTS

- IF x is A1 OR A2… OR An THEN y is Bm
- This can be written as
- IF x is An then y is Bm
- Where the fuzzy set Am is defined as
- $A_m = A_1 \cup A_2 \cup \ldots A_n$ is defined by
- $\mu_{A_m}(x) = \max[\mu_{A_1}(x), \mu_{A_2}(x), \ldots \mu_{A_n}(x)]$

CONDITIONAL STATEMENTS (WITH ELSE and UNLESS)

- Statements of the kind
- I. IF A1 THEN (B1 ELSE B2)
- Can be decomposed into two simple canonical rule forms, connected by “OR”
- (IF A1 THEN B1) OR (IF NOT A1 THEN B2

- II. IF A1 (THEN B1) UNLESS A2
- Can be decomposed as
- (IF A1 THEN B1) OR IF A1 THEN NOT B1

CONDITIONAL STATEMENTS (WITH ELSE and UNLESS)

- III. IF A1 THEN B1 ELSE IF A2 THEN B2
- (IF A1 THEN B1) OR
- IF NOT A1 AND IF A2 THEN B2

NESTED_IF_THEN RULES

- The Rule
- IF A1 THEN [IF A2 THEN B1] can be written in the form
- IF A1 AND A2 THEN B1
- So, based upon all the rules above compound rules can be decomposed into series of canonical simple rules

AGGREGATION OF FUZZY RULES

- The rule based system involves more than one rule
- **Aggregation of rules** is the process of obtaining the overall consequents from the individual consequents provided by each rule
- The methods adopted are:
- I. **Conjunctive system of rules**
- II. **Disjunctive system of rules**

CONJUNCTIVE SYSTEM OF RULES

- Applicable for a system of rules to be jointly satisfied
- The rules are connected by “and” connectives
- Here the aggregated output ‘y’ is determined by the fuzzy intersection of all the individual consequents; yi, I = 1, 2, …n
- y = y1 and y2…..and yn
- $y = y_1 \cap y_2 \ldots \cap y_n$
- The aggregated output can be defined by the membership function
- $\mu_y(y) = \min[\mu_{y_1}(y), \mu_{y_2}(y), \ldots \mu_{y_n}(y)], \forall y \in Y$

DISJUNCTIVE SYSTEM OF RULES

- Applicable when at least one of the rules is required
- The rules are connected by "or" connectives
- Here the aggregated output 'y' is determined by the fuzzy union of all the individual consequents; yi, I = 1, 2, …n
- y = y1 or y2…..or yn
- $y = y_1 \cup y_2 \cup \ldots y_n$
- The membership function of the aggregate output is given by
- $\mu_y(y) = \max[\mu_{y_1}(y), \mu_{y_2}(y), \ldots \mu_{y_n}(y)]$

FUZZY INFERENCE SYSTEM

- A fuzzy inference system uses “IF…THEN” rules along with connections “OR” or “AND” for making necessary decision rules
- The inputs may be crisp or fuzzy
- The output is always a fuzzy set
- Only when FIS is used as a controller its outputs should be crisp
- This necessitates the inclusion of a defuzzification unit for converting fuzzy variables into crisp variables along FIS

CONSTRUCTION OF A FIS

- Its functional blocks are:

1. A rule base that contains numerous fuzzy IF-THEN rules

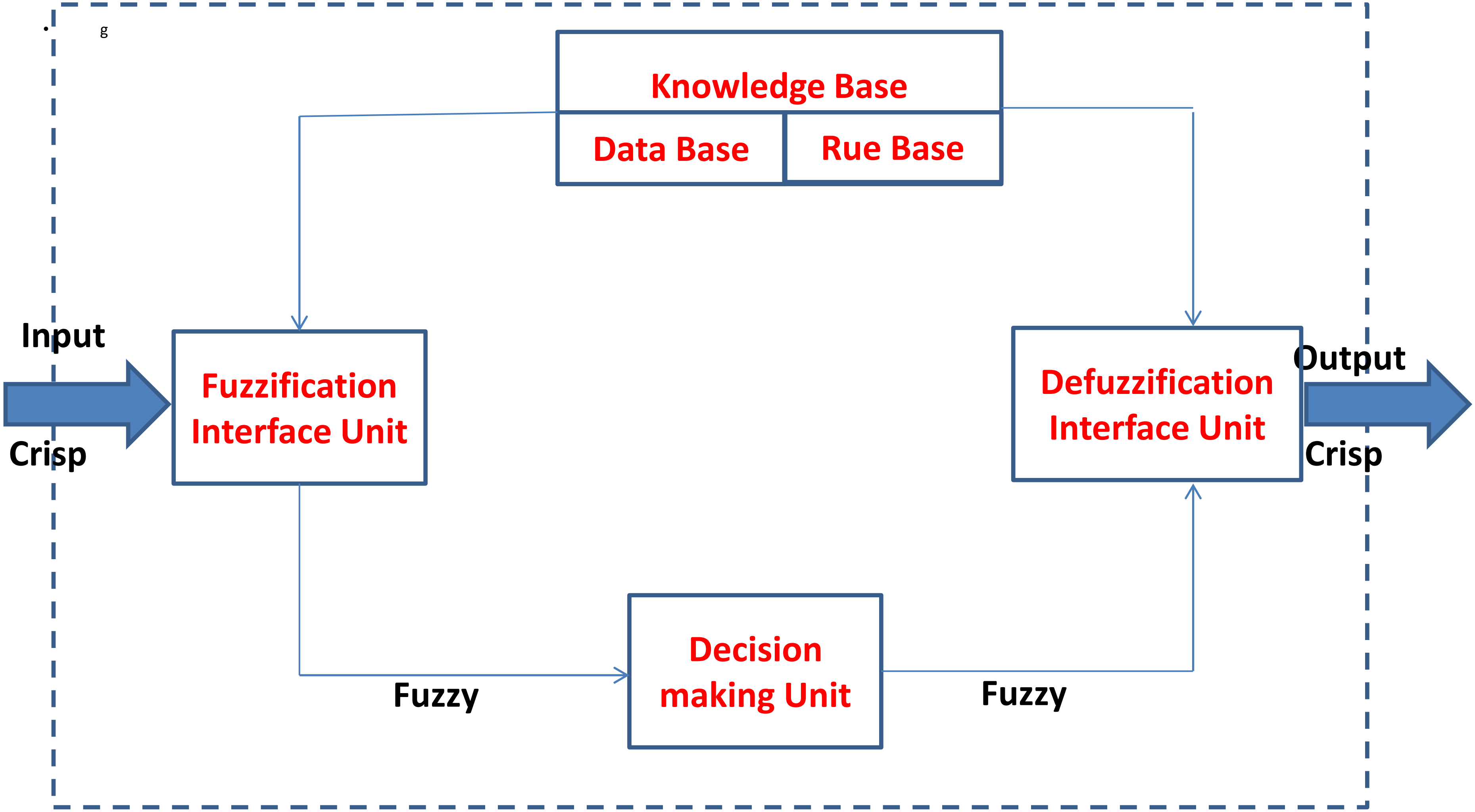
2. A database that defines the membership functions of fuzzy sets used in fuzzy rules

3. A decision-making unit that performs operation on the rules

4. Fuzzification interface unit that converts the crisp quantities into fuzzy quantities

5. A defuzzification interface unit that converts the fuzzy quantities into crisp units

BLOCK DIAGRAM OF FIS



WORKING METHODOLOGY

- **STEP-1:** The crisp inputs are converted into a fuzzy input through a fuzzy interface unit
- **STEP-2:** Database and rule base are called together as knowledge base. It is framed
- **STEP-3:** Defuzzification process is carried out to produce crisp output
- **STEP-4:** The fuzzy rules are formed in the rule base and suitable decisions are made in the decision making unit

MAMDANI FIS (1975)

- Fuzzy sets are used as rule consequents
- It was first proposed to control a steam engine and a boiler combination by synthesizing (combine elements of several sources—to help you make a point) a set of fuzzy rules obtained from people working on the system
- **ALGORITHMIC STEPS:**
- **STEP-1:** Determine a set of fuzzy rules
- **STEP-2:** Make the inputs fuzzy by using input membership functions
- **STEP-3:** Combine the fuzzified inputs according to the fuzzy rules for establishing a rule strength

MAMDANI FIS CONTD…

- **STEP-4:** Determine the consequent of the rule by combining the rule strength and the output membership function
- **STEP-5:** Combine all the consequents to get an output distribution
- **STEP-6:** Finally, a defuzzified output distribution is obtained

DECISION MAKING

- Making decision is undoubtedly **one of the most fundamental activities of human beings**

- We all are **faced in our daily life** with **varieties of alternative actions to take**

- The **beginnings of decision making**, as a subject of study, can be traced, presumably, to the **late 18th century**, when various **studies were made in France** regarding **methods of election** and **social choice**

DECISION MAKING CONTD…

- Since these **initial studies**, decision making is based largely on
- **Theories** and
- **Methods developed in this century is enormous**

- The **subject of decision making** is
- **The study of how decisions are actually made** and
- **How they can be made better and more successfully**

DECISION MAKING CONTD…

- **Much of the focus in developing** the field has been
- **In the area of management**
- In which the decision-making process is of key importance
- Functions such as
- **Inventory control**
- Investment
- personnel actions
- new product development
- **allocation of resources** and
- many others

DECISION MAKING CONTD…

- Decision making is an **integral part of management planning**, **organizing**, **controlling** and **motivation processes**.

- The **selection of one strategy** over others **depends on some criteria**, like **utility, sales, cost, return** etc.

- The **decision should be made** whenever the **organisation or an individual faces a problem of decision making** or **dissatisfied with the existing decisions** or when **alternative decisions are specified.**

DECISION MAKING CONTD…

- **Classical decision making generally deals with**
- A **set of alternative states** of nature
- A **set of alternative actions** that are available to the decision maker
- A **relation indicating the state or outcome** to be expected from each alternative action
- A **utility or objective function**, which orders the outcomes according to their desirability

DECISION MAKING CONTD…

- A decision **is said to be made under conditions of certainty** when the **outcome for each action can be determined and ordered precisely**

- The **alternative that leads to the outcome yielding the highest utility is chosen**
- That is, the **decision making problem becomes an optimization problem**, the problem of maximizing the utility function

CLASSES OF DECISION MAKING PROBLEMS

- **Several classes** of decision-making problems are usually recognized.
- These classifications **depend upon their criteria for doing so**
- These are:
- **Individual decision making** and **Multiperson decision making**
- This classification depends upon the **number of decision makers**
- Whether it is only one person or a group of decision makers involved in the process

CLASSES OF DECISION MAKING PROBLEMS

- **Single criterion decision making** and **Multicriteria decision making**
- This characterisation depends on
- Whether a **simple optimisation of a utility function** is done under constraints or an **optimization is done under multiple objective criteria**
- **Single stage decision making** and **multistage decision making**
- This classification is decided on whether **decision making can be done at a single stage** or **can be done iteratively in several stages**

FUZZY DECISION MAKING

- In the conventional approach to decision making, the principal ingredients of a decision process are
- A **set of alternatives**
- A **set of constraints** on the choice between **different alternatives**
- A **performance function** which associates with each alternative the **gain (or loss) resulting from the choice of that alternative**

FUZZY DECISION MAKING

- Much of the decision-making in the real world takes place in an environment in which
- **The goals, the constraints and the consequences of possible actions are not known precisely**
- To deal quantitatively with imprecision, we usually employ the concepts and techniques of probability theory and information theory
- In doing so, **we are tacitly accepting the premise that imprecision** – whatever its nature- **can be equated with randomness**
- However, **randomness and fuzziness can be differentiated**
- By **fuzziness we mean** a **type of imprecision which is associated with fuzzy sets**

FUZZY DECISION MAKING

- Essentially, **randomness has to do with uncertainty concerning membership or non-membership of an object in a crisp set**
- Whereas fuzziness deals with classes in which there may be grades of membership of objects in a fuzzy set
- This distinction between the two concepts of randomness and fuzziness we can say that
- **Fuzziness is quite different from probability theory**
- The concept of fuzzy decision making was introduced and studied by Bellman and Zadeh in their seminal paper in 1970

FUZZY DECISION MAKING

- Decision processes in which fuzziness enters in one way or another can be studied from many points of view
- This leads to the **introduction of the three basic concepts**
- **Fuzzy goal**
- **Fuzzy constraint**
- **Fuzzy decision**
- Roughly speaking:
- **A fuzzy goal is an objective**
- which **can be characterised as a fuzzy set in an appropriate space**

AN EXAMPLE

- Suppose, we have the set of alternatives X be **Z, the set of integers.**
- Then a **fuzzy goal G may be expressed in terms of its membership function defined over X**
- It can be like
- The alternate x should be much smaller than 100
- The concept of fuzzy goal provides significant advantages to a fuzzy decision making frame work than the performance function used in the case of a normal decision process

FUZZY DECISION MAKING

- A **fuzzy decision situation** in this model **is characterised by** the following components:
- A **set of possible actions**: A
- A **set of goals**
- Each goal is **expressed in terms of a fuzzy set defined on A**
- A **set of constraints**
- Each of which is also **expressed by a fuzzy set defined on A**
- Thus, in this process of decision making,
- **Relevant goals and constraints are expressed in terms of fuzzy sets**
- A **decision is determined by an appropriate aggregation of these fuzzy sets**