Measurement Scales

Measurement Scales

- Purpose of performing mapping M is to be able to manipulate data in the numerical system and use the results to draw conclusions about the attribute in the empirical system.
- Not all mappings are the same and the differences among the mappings can restrict the type of analysis one can do
- The measurement mapping M, together with the empirical and numerical systems are a **measurement scale**.

Scale Types

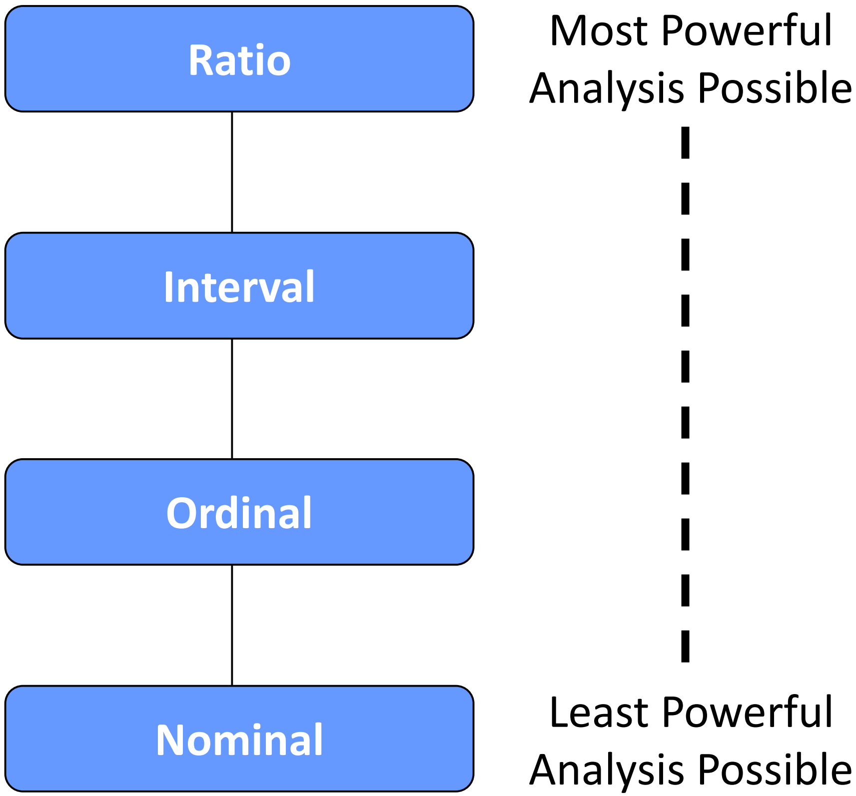
- Five major types
  - Nominal
  - Ordinal
  - Interval
  - Ratio
  - Absolute
- Knowing the characteristics of each type helps us to interpret the measures

Types of Measurement Scales

- Categorical (nominal)
  - Classification
  - Numbers are labels for categories
- Continuous (along a continuum)
  - Ordinal
  - Interval
  - Ratio

Measurement Scales Hierarchy

- Scales are hierarchical
- Each higher-level scale possesses all the properties of the lower ones
- A higher-level of measurement can be reduced to a lower one but not vice-versa



Scale types

- Nominal: just classification
- Ordinal: linear ordering (>)
- Interval: like ordinal, but interval between values is the same (so average has a meaning)
- Ratio: like interval, but there is a 0 (zero) (so A can be twice B)
- Absolute: counting number of occurrences

What statistics make sense

- Nominal – mode, median, percentiles
- **Ordinal – rank measures**
- **Interval – standard deviation, correlation**
- **Ratio – all stats**
- **Absolute – all stats**

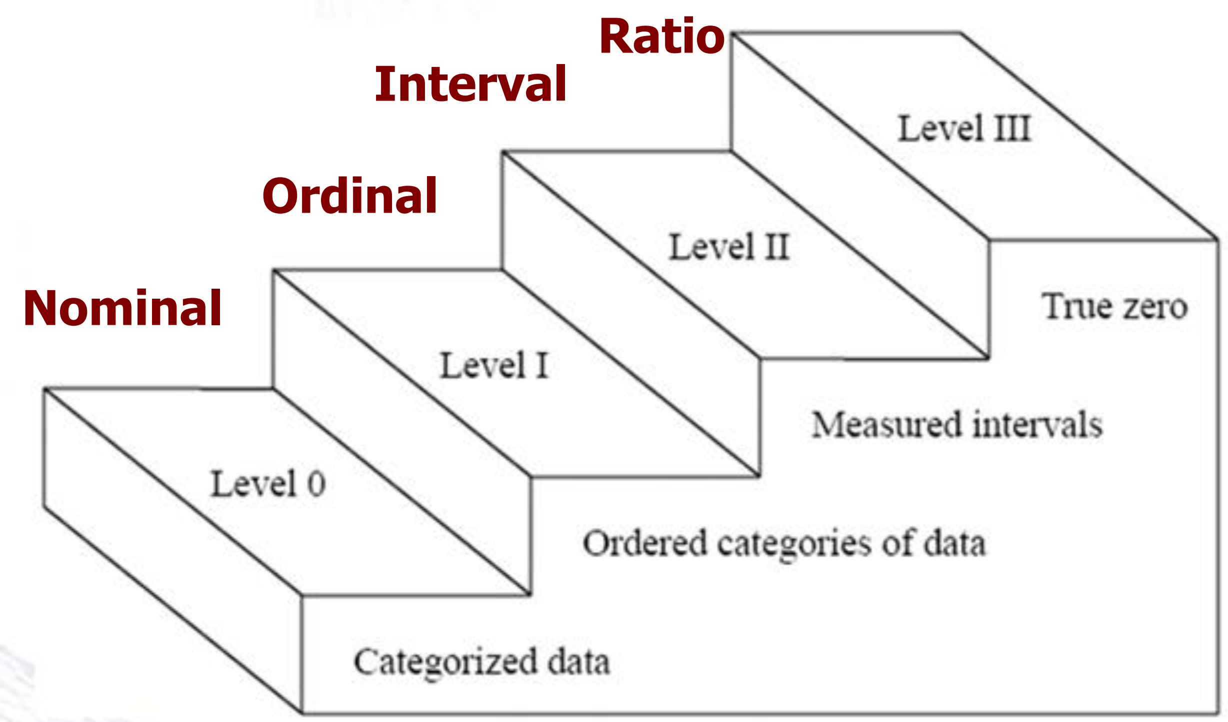
Nominal Scale

- Gender is a nominal scale

Male = 1

Female = 2

Scales (Summary)



Types of Measurement Scales (review)

- **Nominal**
  - scores ***only*** reflect the property of ***difference*** so the "numbers" have no numerical meaning.
  - Sometimes called categorical, discontinuous, or qualitative variables.
  - Mode is only appropriate measure of central tendency
    - E.g., biological sex, race, breed, etc…
- **Ordinal**
  - scores ***do*** indicate the property of ***magnitude***, but do ***not*** reflect ***equal intervals***.
  - the ***median*** is the appropriate measure of central tendency on ordinal scales
    - e.g., **"class rank"**
- **Interval**
  - possess both properties of difference and magnitude, but also have the property of ***equal intervals***.
    - e.g., intelligence or other test scores.
- **Ratio**
  - possess all of the properties of difference, magnitude, equal intervals, ***and*** an absolute, or ***true zero*** (a "0" means the complete absence of the thing being measured)
    - e.g., weight, height, distance, reaction time

Measurement Scales

- **Measurement scales** are "types" of data used in analysis.
- **There are five types of data:**

1. Nominal – symbols without any order.

For example, social security numbers.

Scales (Levels) of Measurement (cont.)

- ***1) Nominal Level :*** **"distinctiveness"**
  - **Simplest form of measurement**
  - **Classification by categories (similar to *mathematical equivalence class*):**
    - ***Exhaustive*** **– categories provide total coverage of all the elements with that attribute**
    - ***Mutually Exclusive*** **– every element belongs to only one category**
  - **NO relationship may be assumed among categories themselves**

  - **Examples (are these nominal ?) :**
    - **Software development activities by: requirements, design, code, test, integration**
    - **Software process models by : waterfall, spiral, iterative, prototyping, and other**
    - **Product Defect source by : design, coding, testing, integration, and packaging**
    - **Students in college by: major fields (include "undeclared" but not allowing double major) in a college**
    - **Students in a class by : birth places**
    - **Survey result answers by: excellent, good, neutral, poor, bad**
  - **Arithmetic operations or comparison may or may not be valid**
    - **What does it mean to talk about "best" place of birth?**
    - **What does it mean to talk about "average" defect source?**
    - **What does it mean to talk about "average" student major?**
    - **What does it mean to say software engineering is "better" than music**

Ordinal Scale

- Often useful to augment the nominal scale with information about an ordering of classes or categories
- Characteristics
  - The empirical relation system consists of classes that are ordered with respect to the attribute
  - Any mapping that preserve the ordering is acceptable
  - The numbers represent ranking only, so addition, subtraction, and any other arithmetic operations have no meaning

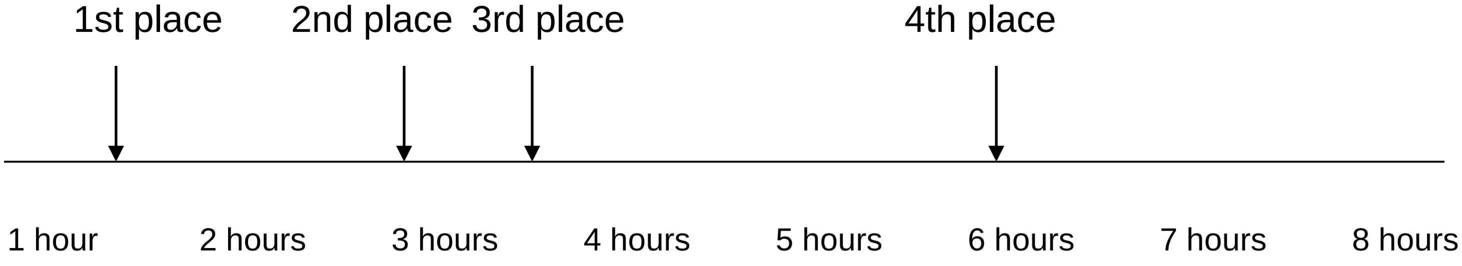
Ordinal Scale

- ERS: classes of entities that are ordered with respect to the attribute
- Mapping must preserve the ordering
- Numbers represent rankings only
  - Addition, subtraction, multiplication is meaningless
  - Ex: error severity
  - Transformation, must be order preserving (monotonic mappings)

Scales of Measurement: Ordinal Scale

- **Ordinal**: Designates an ordering: greater than, less than.
- Does not assume that the intervals between numbers are equal.
- Example:

  finishing place in a race (first place, second place)



Another Example of Ordinal Scale

How much pain did you have this past week?

1 None

2 Very mild

3 Mild

4 Moderate

5 Severe

6 Very severe

The Ordinal Scale (1/2)

Example: ***A degree-classification ordinal scale***

Joe	Michelle
Rachel	Christine
Michael	James
Clyde	Wendy

1st Class

2nd Class

Failed

3rd Class

Interval Scale

- Useful to augment the ordinal scale with information about the size of the intervals that separates the classes
- Characteristics
  - Preserve order
  - Preserve differences but not ratios. That is we know the difference between any two of the ordered classes in the range of the mapping, but computing the ratio of the two classes in the range does not make sense
  - Addition and subtraction are acceptable operations, but not multiplication and division

Scales of Measurement: Interval Scale

- **Interval**: designates an equal-interval ordering.
- The difference in temperature between 20 degrees F and 25 degrees F is the same as the difference between 76 degrees F and 81 degrees F.
- Examples: Temperature in Fahrenheit or Celsius is interval. Common IQ tests are *assumed* to use an interval metric.

Scales of Measurement: Interval Scale

- Likert scale: For each question below….
- 1 = Strongly Disagree
- 2 = Uncharacteristic
- 3 = Neutral
- 4 = Characteristic
- 5 = Strongly Agree

Scales (Levels) of Measurement (cont.)

- ***3) Interval : "equal difference"***
  - Categories are ordered and "equal" in size.
  - "Points" of measurement are equidistant".
  - Allows the arithmetical operation of "addition" and "subtraction" .
- **Example:**

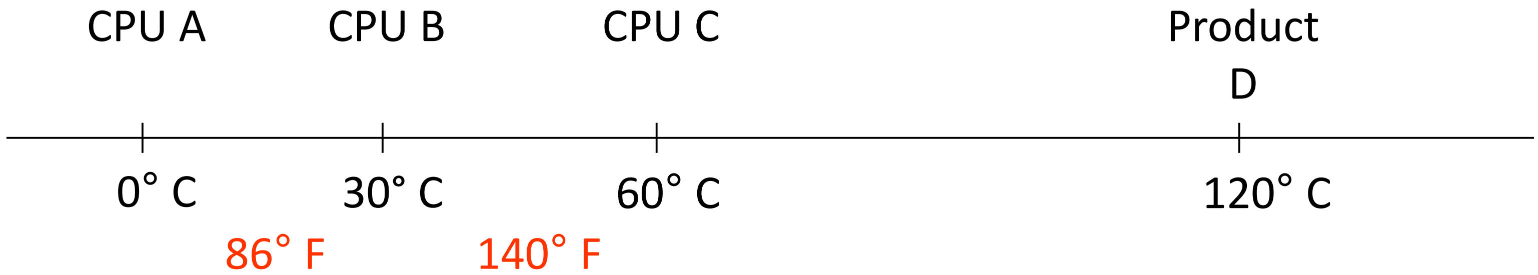
  On a product "usability" attribute scale of 1 to 10, product A is measured at 8 and product B is measured at 4; then we can say that A is 4 points higher, but not necessarily 2 times better. (This is because there is no "absolute or a meaningful" base such as a fixed "zero" or fixed "null" ---- the "bottom" is made up as '1'--- it could have been '2'.)

Interval Scale

- Order preserving
- Differences between measurement values are meaningful, but ratios are not
- Ex: Fahrenheit or centigrade of the form
  - $M2 = aM1 + b$ where a and b are constants

Interval Scale

- Indicates exact differences between measurement points
- Addition and subtraction can be applied
- Multiplication and Division **CANNOT** be applied
- We can say that product D has 8 more crashes per month but we cannot say that it has 3 times as more crashes



Temperature of Different CPUs

Ratio Scale

- The most useful scale of measurement, common in the physical science
- Characteristics
  - Preserving ordering, the size of interval between entities, and ratios between entities
  - There is a zero element, representing total lack of the attribute
  - Mapping must start at zero and increase at equal interval, known as units
  - All arithmetic can be meaningfully applied to the classes in the range of mapping

Scales of Measurement: Ratio Scale

- **Ratio**: Designates an equal-interval ordering with a true zero point (i.e., the zero implies an absence of the thing being measured).
- Examples:
  - Temperature in Kelvin (Zero is the absence of heat. Can't get colder).
  - Measurements of heights of students in this class (Zero means complete lack of height).
    - Someone 6 ft tall is twice as tall as someone 3 feet tall.

Types of Measurement Scales and Their Properties

	Property of Numbers		
Type of scale	Rank order	Equal interval	Absolute zero
Nominal	No	No	No
Ordinal	Yes	No	No
Interval	Yes	Yes	No
Ratio	Yes	Yes	Yes

Ratio Scales

Ratio scales allow for the identification of absolute differences between each scale point, and absolute comparisons between raw responses

Example 1.

Please circle the number of children **under** 18 years of age currently living in your household.

0 1 2 3 4 5 6 7 (if more than 7, please specify ___.)

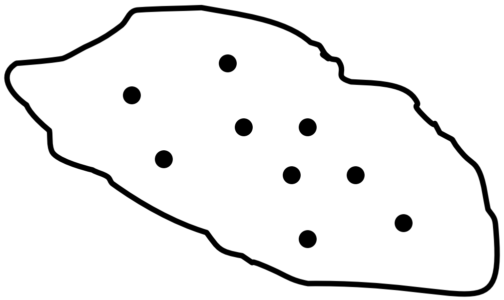
Ratio Scale

- Preserves ordering, interval sizes, and ratios.
  – ERS must capture ratios
- Has a meaningful zero value
- Ex. Kelvin temperature scale, length
- Transformation: M2 = aM!, where a is a constant

Pictorial Representations of Scaling Theory

Nominal level

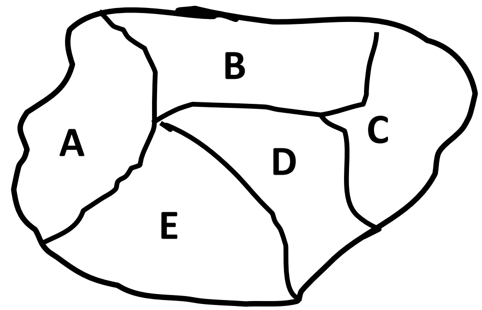
entities



measurement or mapping

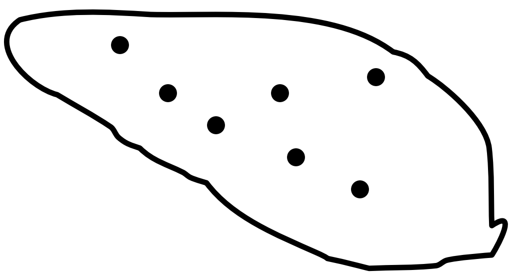
M (entity) = A or
B or
C or
D or
E

representation of measurement with partitioned data set



Ordinal level

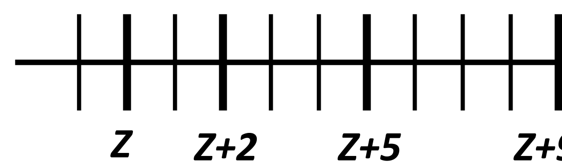
entities



measurement or mapping

M (entity) = Z or
Z+2 or
Z+5 or
Z+9
where Z+9 > Z+5 > Z+2 >Z

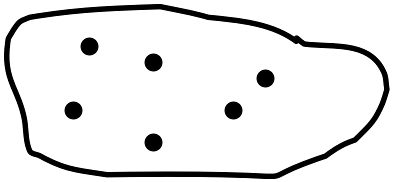
representation of measurement with <u>specific</u> and <u>ordered</u> integers



Pictorial Representations of Scaling Theory

Interval level

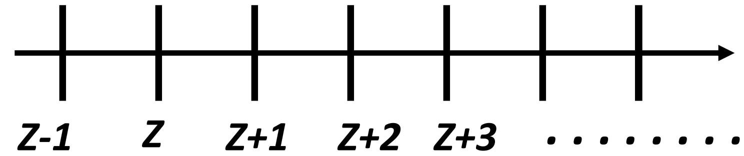
entities



measurement or mapping

M (entity) = Z or
Z+n
where n = 1
thru infinity

representation of measurement with integers ***starting at "some"Z***

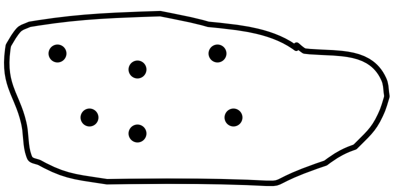


note : [(Z+5) – (Z+1)] = 4 = [(Z+22) – (Z+18)]
regardless of the value of Z

BUT: (Z+2)/(Z+1) = 2/1 = (Z+6)/(Z+3) if Z=0
(Z+2)/(Z+1) = 3/2 & (Z+6)/(Z+3) = 7/4 if Z=1

Ratio level

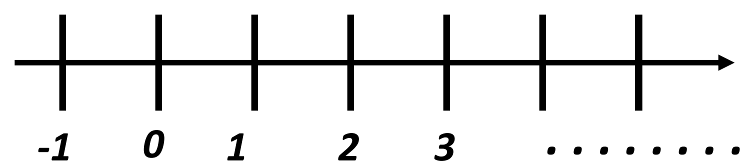
entities



measurement or mapping

M (entity) = 0, 1, 2 ---

representation of measurement with integers ***starting at Z=0***



note: with Z=0 fixed, 6/3 = 2/1 = - - - - -

Absolute Scale

- This is the most restrictive scale
- Characteristics
  - The measurement for an absolute scale is made simply by counting the number of elements in the entity set
  - The attribute always takes the form “number of occurrences of x in the entity”
  - There is only one possible measurement mapping, namely the actual count
  - All arithmetic analysis of the resulting counting is meaningful

Absolute Scale

- Measure by counting the attribute in the entity
- Only one unique mapping
- All arithmetic is meaningful
- Ex: number of classes, number of errors found during a system test
- Transformation: identity

Exercise 1

Determine the best "scale" for each of the following measurements using each of the Nominal, Ordinal, Interval and Ratio scales only once.

- Measuring execution time of a program **Ratio**
- Classification of objects based on their color **Nominal**
- Measuring duration of various phases of projects (Project scheduling) **Interval**
- Measuring complexity of many software modules by defining 4 complexity classes (Trivial, Simple, Moderate, Complex) **Ordinal**
- Measuring complexity of many software modules by defining cyclomatic complexity metrics

Exercise 1 (cont'd)

- Software products categorized according to their compatibility with the operating system (i.e. Windows, Linux, MacOS, DOS, etc.). **Nominal**
- Internet services categorized according to their relevant technologies (i.e. dial-up, DSL, high-speed, wireless, etc.) **Nominal**
- Measuring attitudes towards an Internet service (say, on an n-point rating, n = 0 to 10). **Ordinal**

Exercise 1 (cont'd)

- Measuring attitude towards Internet services, if the evaluation scores are numerically meaningful so the difference between a rate of 3 and a rate of 6 is exactly the same as the difference between a rate of 7 and a rate of 10. **Interval**
- Measuring attitude towards Internet services, if the differences between the data values are numerically meaningful and equal. In addition, a score of zero implies either the full absence of the service being evaluated or the full dissatisfaction with it. **Ratio**

Meaningfulness

- A statement about measurement is meaninful if its truth value is independent of all allowable transformation of scale types

Meaningful Statements

- Arnold is 76 in. tall; Dan is 38in tall, so Arnold is twice as tall as Danny
- Program A has twice as many lines of code as program B.

Meaningless Statements

- It is 98 degrees F in Calcutta and 49 degrees F in Colorado, so it’s twice as hot in Calcutta (Meaninglessness can be proved, just transform to centigrade)
- My program is twice as easy to use as yours

- **How to Convert Fahrenheit to Celsius**

  1. Take the temperature in Fahrenheit subtract 32.
  2. Divide by 1.8.
  3. The result is degrees Celsius.

- **How to Convert Celsius to Fahrenheit**

  1. Take the temperature in Celsius and multiply 1.8.
  2. Add 32 degrees.
  3. The result is degrees Fahrenheit.

Are the following statements meaningful?

1. Peter is twice as tall as Hermann
2. Peter's temperature is 10% higher than Hermann's
3. Defect X is more severe than defect Y
4. Defect X is twice as severe as defect Y
5. The cost for correcting defect X is twice as high as the cost for correcting defect Y
6. The average temperature of city A (15 ºC) is twice as high as the average temperature of city B (30 ºC)
7. Project Milestone 3 (end of coding) took ten times longer than Project Milestone 0 (project start)
8. Coding took as long as requirements analysis

Are the following statements meaningful?

1. interval* no*
2. interval* no*
3. ordinal yes
4. ordinal no
5. ratio yes
6. interval no
7. interval no
8. interval yes

1. "Peter is twice as tall as Hermann"
2. "Peter's temperature is 10% higher than Hermann's"
3. "Defect X is more severe than defect Y"
4. "Defect X is twice as severe as defect Y"
5. "The cost for correcting defect X is twice as high as the cost for correcting defect Y"
6. The average temperature of city A (30 ºC) is twice as high as the average temperature of city B (15 ºC)
7. "Project Milestone 3 (end of coding) took ten times longer than Project Milestone 0 (project start)"
8. "Coding took as long as requirements analysis"

Statistical operations on measures

- For statistical analysis use arithmetic operators.
- Two basic measures:
  - Measure of central tendency-average-tell middle(mean, median, mode)
  - Measure of dispersion-(maximum and minimum values)SD and variance

Central Tendency Indicators

- Nominal values; mode – the value of the most common item class
- Ordinal values; median – the middle value
- Interval values: mean, standard deviation
- Ratio values, mean, geometric mean, standard deviation, coefficient of variance

According to These Rules

- Parametric statistics can only be used if the values are on a ratio or absolute scale.
  - Parametric statistics also require a normal distribution of values.
- Non-parametric statistical analyses must be used for value on all of the other scales.
- Note: controversial position.

Objective & Subjective

- Strive for objective measures:
  - Different people/tools should produce same results
  - Subject measure can be useful
    - Measures can vary with the person measuring and reflect the judgment of the measurer
    - For example, severity ranking
    - Must be aware of inter-rate reliability

Measurement in extended number systems

- Many situation we cannot measure an attribute directly.
- Eg:-we wish to assess the quality of the different types of transport for travelling from our home to another city.

Options	Journey Time(Hours)	Cost pr mile(dollars)
Car	3	1.5
Train	5	2.0
Plane	3.5	3.5
Executive Coach	7	4.0

m(Transport)=(Cost per mile , Journey time)

M(car)=(3,1.5)

M(train)= (5,2)

M(plane)=(3.5,3.5)

M(Coach)=(7,4)

	Cost	Time
• m(Plane)<m(Train)	False	True
• m(Plane)>m(Train)	True	False
• m(Plane)=m(Train)	False	False

Meaningfulness of Indirect Measures

- Measuring a complex attribute in terms of simpler sub-attributes
- Scale for indirect measures are similar to direct ones.
- An indirect measure which is composed of direct measures has the scale type of the weakest component