Partitions:

These are the values which divided the series into a number of equal parts.

Quartiles: The three points which divided the series in to four equal parts are called quartiles. It is denoted by Q_1, Q_2, Q_3.

Deciles: The nine points which divided the series in to ten equal parts are called deciles. It is denoted by $D_1, D_2, \cdots, D_9$.

Percentiles: The ninety-nine points which divided the series in to hundred equal parts are called percentiles. It is denoted by $P_1, P_2, \cdots, P_{99}$.

Case A Raw data:

Suppose a data set contains n values say $x_1, x_2 \ldots\ldots x_n$. Arrange the data into either ascending order or descending order. The partition values can be defined as follows.

(i)n is an odd number:

Quartiles	Deciles	Percentiles
$Q_1 = \left(\frac{n+1}{4}\right)^{th}$ term value	$D_1 = \left(\frac{n+1}{10}\right)^{th}$ term value	$P_1 = \left(\frac{n+1}{100}\right)^{th}$ term value
$Q_2 = \left(\frac{2(n+1)}{4}\right)^{th}$ term value	$D_2 = \left(\frac{2(n+1)}{10}\right)^{th}$ term value	$P_2 = \left(\frac{2(n+1)}{100}\right)^{th}$ term value
$Q_3 = \left(\frac{3(n+1)}{4}\right)^{th}$ term value	$D_9 = \left(\frac{9(n+1)}{10}\right)^{th}$ term value	$P_{99} = \left(\frac{99(n+1)}{100}\right)^{th}$ term value

(ii)n is an even number:

Quartiles	Deciles	Percentiles
$Q_1 = Next\ to\left(\frac{n}{4}\right)^{th}$ term value	$D_1 = Next\ to\left(\frac{n}{10}\right)^{th}$ term value	$P_1 = Next\ to\left(\frac{n}{100}\right)^{th}$ term value
$Q_2 = Next\ to\left(\frac{2n}{4}\right)^{th}$ term value	$D_2 = Next\ to\left(\frac{2n}{10}\right)^{th}$ term value	$P_2 = Next\ to\left(\frac{2n}{100}\right)^{th}$ term value
$Q_3 = Next\ to\left(\frac{3n}{4}\right)^{th}$ term value	$D_9 = Next\ to\left(\frac{9n}{10}\right)^{th}$ term value	$P_{99} = Next\ to\left(\frac{99n}{100}\right)^{th}$ term value

Case b: For discrete frequency distribution:

Quartiles:- $Q_k : \frac{k(N+1)}{4}$; the corresponding variable is the quartile value. Here, $k = 1, 2, 3$.

Deciles:- $D_k : \frac{k(N+1)}{10}$; the corresponding variable is the decile value. Here, $k = 1, 2, 3, 4, 5, 6, 7, 8, 9$.

Percentiles:- $P_k : \frac{k(N+1)}{100}$; the corresponding variable is the percentile value. Here, $k = 1, 2, \cdots, 99$.

Case c: For continuous frequency distribution:

Quartiles:- Step-1: Find quartile class by:

Compute $\frac{kN}{4}$; Identify the same value in cf list, otherwise find cf just greater than $\frac{kN}{4}$.

Step-2: Use the formula

$$Q_k = l + \frac{h}{f}\left(\frac{kN}{4} - c\right); k = 1, 2, 3.$$

Deciles:- Step-1: Find decile class by:

Compute $\frac{kN}{10}$; Identify the same value in cf list, otherwise find cf just greater than $\frac{kN}{10}$.

Step-2: Use the formula

$$D_k = l + \frac{h}{f}\left(\frac{kN}{10} - c\right); k = 1, 2, \cdots, 9.$$

Percentiles:- Step-1: Find percentile class by:

Compute $\frac{kN}{100}$; Identify the same value in cf list, otherwise find cf just greater than $\frac{kN}{100}$.

Step-2: Use the formula

$$P_k = l + \frac{h}{f}\left(\frac{kN}{100} - c\right); k = 1, 2, \cdots, 99.$$

Quartile Deviation

Definition: Average amount by which the two quartiles differ from the median.

$$\textbf{Quartile Deviation (Q.D.)} = \frac{Q_3 - Q_1}{2}$$

- The Median ± Q.D. covers exactly 50 per cent of the observations.

- When Q.D. is very small, it describes high uniformity or small variation of the central 50% items, and a high Q.D. means that the variation among the central items is large.

Relative measure of Q.D.

$$\textbf{Coefficient of Q.D.} = \frac{Q_3 - Q_1}{Q_3 + Q_1}$$

It can be used to compare the degree of variation in different distributions.

Example 3: Calculate the value of Q.D. and its coefficient of Q.D. from the following data.

Roll No.	1	2	3	4	5	6	7
Marks	20	28	40	12	30	15	50

Solution: Marks in ascending order 12 15 20 28 30 40 50

Q_1 = Size of $\frac{N+1}{4}th$ item = Size of $\frac{7+1}{4}$ = 2nd item.

Size of 2nd item is 15. Hence $\boldsymbol{Q_1 = 15}$

Q_3 = Size of $3\left(\frac{N+1}{4}\right)th$ item = Size of $3\left(\frac{7+1}{4}\right)$ = 6th item.

Size of 6th item is 40. Hence $\boldsymbol{Q_3 = 40.}$

$$\therefore \boldsymbol{Q.D.} = \frac{Q_3 - Q_1}{2} = \frac{40-15}{2} = \mathbf{12.5}$$

$$\textit{Coefficient of } \boldsymbol{Q.D.} = \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{40-15}{40+15} = \mathbf{0.455}$$

Example 4: Compute the value of Q.D. and its coefficient from the following data.

Marks	10	20	30	40	50	60
No. of Students	4	7	15	8	7	2

Solution:

Marks	*Frequency*	*cumulative frequency*
10	4	4
20	7	11
30	15	26
40	8	34
50	7	41
60	2	43

Q_1 = Size of $\frac{N+1}{4}$ th item = Size of 11^{th} item.

Size of 11^{th} item is 20. Hence $\boldsymbol{Q_1 = 20}$

Q_3 = Size of $3\left(\frac{N+1}{4}\right)$ th item = Size of $3\left(\frac{43+1}{4}\right) = 33^{rd}$ item.

Size of 33^{rd} item is 40. Hence $\boldsymbol{Q_3 = 40.}$

$$\therefore \boldsymbol{Q.D.} = \frac{Q_3 - Q_1}{2} = \frac{40-20}{2} = \mathbf{10}$$

$$\textit{Coefficient of } \boldsymbol{Q.D.} = \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{40-20}{40+20} = \mathbf{0.333}$$

Example 4: Compute the value of Q.D. and coefficient of Q.D. from the following data

C.I.	10-20	20-30	30-40	40-50	50-60	60-70	70-80
f	12	19	5	10	9	6	6

Solution:

Marks	*Frequency*	*Cumulative Frequency*
10-20	12	12
20-30	19	31
30-40	5	36
40-50	10	46
50-60	9	55
60-70	6	61
70-80	6	67
	$N = 67$	

Q_1 = Size of $\frac{N}{4}$ *th* item = Size of $\frac{67}{4}$ = 16.75th item.

Q_1 lies in the interval **20-30**

$$Q_1 = l + \frac{\frac{N}{4} - c.f.}{f} \times i \qquad l = 20,\ N/4 = 16.75,\ c.f. = 12 \quad f = 19,\ \ i = 10$$

$$Q_1 = 20 + \frac{\frac{67}{4} - 12}{19} \times 10 = 20 + 2.5 = 22.5$$

Hence $\mathbf{Q_1 = 22.5}$

Q_3 = Size of $\frac{3N}{4}$ th item = Size of $\frac{3\times67}{4}$ = 50.25th item.

Q_3 lies in the class **50-60**.

$$Q_3 = l + \frac{\frac{3N}{4} - c.f.}{f} \times i \qquad l = 50,\ 3N/4 = 50.25,\ c.f. = 46\ \ f = 9,\ \ i = 10$$

$$Q_3 = 50 + \frac{50.25-46}{9} \times 10 = 50+4.72 = 54.72$$

Hence $\mathbf{Q_3 = 54.72}$

$$\boldsymbol{Q.D.} = \frac{Q_3-Q_1}{2} = \frac{54.72-22.5}{2} = \boldsymbol{16.11}$$

$$\textbf{\textit{Coefficient of Q.D.}} = \frac{Q_3-Q_1}{Q_3+Q_1} = \frac{54.72-22.5}{54.72+22.5} = \boldsymbol{0.4172}$$