

TC1

$$\forall x F(x) \\ \sim y(a)$$

$$\forall x [F(x) \rightarrow y(x)]$$

$$\sim y(a)$$

$$\rightarrow F(a) \rightarrow y(a)$$

$$\sim F(a)$$

$$y(a)$$

$$\forall x [F(x)]$$

$$\rightarrow F(a)$$

x

TC2

Σ

$$\frac{\exists n(F(n) \rightarrow g(n))}{\quad}$$

$$\exists n g(n) \quad F(a)$$

$$\rightarrow F(a)$$

$$F(a) \rightarrow g(a)$$

$$\frac{\neg F(a)}{\times}$$

$$\frac{g(a)}{\frac{\exists n g(n)}{g(a)} \frac{g(b)}{g(c)}}$$

TC3

$$\forall n [F(n) \rightarrow g(n)]$$

$$\exists n (F(n) \wedge \neg g(n))$$

$$\sim \exists n (F(n) \wedge \neg g(n))$$

$$\exists n (F(n) \wedge \neg g(n))$$

$$F(a)$$

$$\frac{F(a)}{H(a)}$$

$$\neg \exists n (F(n) \wedge \neg g(n))$$

$$\neg (F(a) \wedge \neg g(a))$$

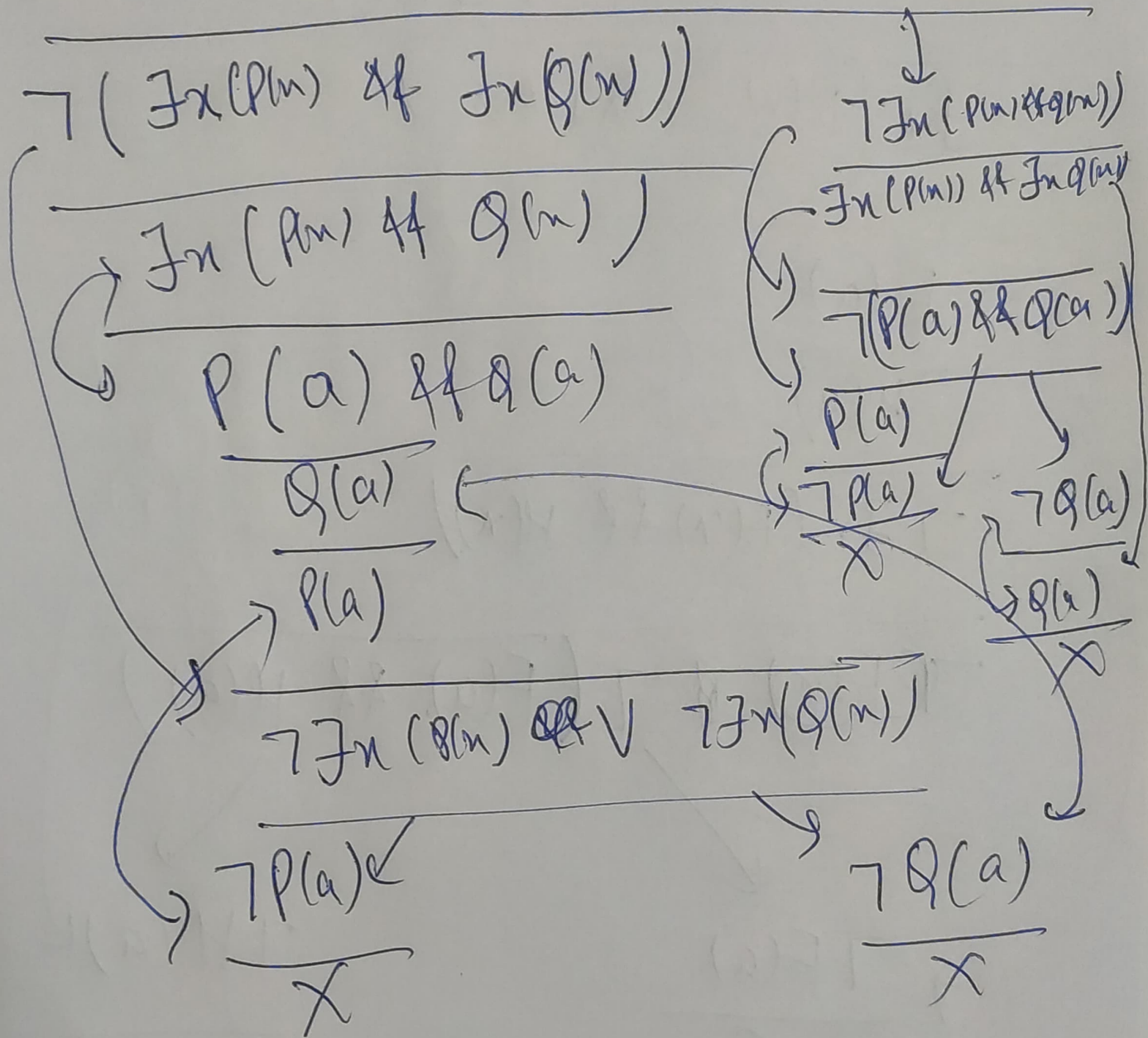
$$\neg F(a)$$

X

$$\neg H(a)$$

X

$$\text{TC4} \quad \neg \left(\exists x (P(x) \wedge Q(x)) \right) \Leftrightarrow \left(\exists x (P(x)) \wedge \neg \exists x (Q(x)) \right)$$



$$\frac{TC6}{\exists x (\forall y L(x, y)) \not\equiv \sim \forall x (\exists y \neg L(y, x))}$$

$$\begin{array}{c}
 \exists x (\forall y L(x, y)) \\
 \hline
 \sim \forall x (\exists y \neg L(y, x)) \\
 \downarrow \quad \downarrow \\
 \begin{array}{c}
 \sim L(a, b) \\
 \hline
 L(a, b)
 \end{array}
 \end{array}
 \quad \text{Contradiction}$$

X

$$P1 \quad \underline{\underline{TC7}}$$

\approx

$$P1 \quad \forall x (P(x) \rightarrow \exists y (P(y) \wedge q(y, x)))$$

$$P2 \quad \forall x (P(x) \rightarrow \neg(x, a))$$

$$P2 \vdash P(x) \rightarrow (P(y) \rightarrow (\neg(y, x) \rightarrow \sim q(x, y)))$$

$$P3 \quad \circ) \quad \forall x (\forall y (P(x) \rightarrow \dots))$$

$$P1 \vdash \neg P(x) \vee \neg P(y) \wedge \neg \neg(x, y) \vee \neg q(x, y)$$

\approx

$$P3 \quad \forall x \forall y (\neg P(x) \vee \neg P(y) \vee \neg \neg(x, y) \vee \neg q(x, y))$$

$$\text{arg} \quad P1 \wedge P2 \wedge P3 \wedge P(a)$$

TCF
 $\forall n (P(n) \rightarrow \exists y (P(y) \wedge q(y, n)))$

$\forall n (P(n) \rightarrow \neg \neg (n, a))$

$\forall n (\neg \exists y (\neg P(y) \vee \neg q(y, n)) \vee \neg \neg \neg (n, a))$

$P(a)$

$P(a) \rightarrow \exists y (P(y) \wedge q(y, a))$

$\neg P(a)$

$\frac{\neg P(a)}{X}$

$\exists y (P(y) \wedge q(y, a))$

$P(a)$

$q(a, a)$

$\forall n (P(n) \rightarrow \neg \neg (n, a))$

$P(a) \rightarrow \neg \neg (a, a)$

$\neg P(a)$

$\neg \neg (a, a)$

$\frac{\neg P(a)}{X} \vee \frac{\neg P(a)}{X} \vee \frac{\neg \neg (a, a)}{X} \vee \frac{\neg \neg (a, a)}{X}$