Contents

Chapter 1

Minkowski space

For spacetime graphics, see Minkowski diagram. For Minkowski space associated to a number field, see Minkowski space (number field).

In mathematical physics, **Minkowski space** or **Minkowski spacetime** (named after the mathematician Hermann Minkowski) is the mathematical space setting in which Einstein's theory of special relativity is most conveniently formulated. In this setting the three ordinary dimensions of space are combined with a single dimension of time to form a four-dimensional manifold for representing a spacetime.

In theoretical physics, Minkowski space is often contrasted with Euclidean space. While a Euclidean space has only spacelike dimensions, a Minkowski space also has one timelike dimension. The isometry group of a Euclidean space is the Euclidean group and for a Minkowski space it is the Poincaré group.

The spacetime interval between two events in Minkowski space is either space-like, light-like ('null') or time-like.

1.1 History

In 1905 (published 1906) it was noted by Henri Poincaré that, by taking time to be the imaginary part of the fourth spacetime coordinate $\sqrt{-1}\ ct$, a Lorentz transformation can be regarded as a rotation of coordinates in a four-dimensional Euclidean space with three real coordinates representing space, and one imaginary coordinate, representing time, as the fourth dimension. Since the space is then a pseudo-Euclidean space, the rotation is a representation of a hyperbolic rotation, although Poincaré did not give this interpretation, his purpose being only to explain the Lorentz transformation in terms of the familiar Euclidean rotation.[1] This idea was elaborated by Hermann Minkowski,[2] who used it to restate the Maxwell equations in four dimensions, showing directly their invariance under the Lorentz transformation. He further reformulated in four dimensions the then-recent theory of special relativity of Einstein. From this he concluded that time and space should be treated equally, and so arose his concept of events taking place in a unified four-dimensional space-time continuum. In a further development,[3] he gave an alternative formulation of this idea that did not use the imaginary time coordinate, but represented the four variables (x, y, z, t) of space and time in coordinate form in a four dimensional affine space. Points in this space correspond to events in space-time. In this space, there is a defined light-cone associated with each point (see diagram above), and events not on the light-cone are classified by their relation to the apex as *space-like* or *time-like*. It is principally this view of space-time that is current nowadays, although the older view involving imaginary time has also influenced special relativity. Minkowski, aware of the fundamental restatement of the theory which he had made, said:

> The views of space and time which I wish to lay before you have sprung from the soil of experimental physics, and therein lies their strength. They are radical. Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality. – Hermann Minkowski, 1908

For further historical information see references Galison (1979), Corry (1997), Walter (1999).

1.2 Structure

Formally, Minkowski space is a four-dimensional real vector space equipped with a nondegenerate, symmetric bilinear form with signature (−,+,+,+) (Some may also prefer the alternative signature (+,−,−,−); in general, mathematicians and general relativists prefer the former while particle physicists tend to use the latter.) In other words, Minkowski space is a pseudo-Euclidean space with $n = 4$ and $n - k = 1$ (in a broader definition any $n > 1$ is allowed). Elements of Minkowski space are called *events* or four-vectors. Minkowski space is often denoted $\mathbf{R}^{1,3}$ to emphasize the signature, although it is also denoted M^4 or simply M. It is perhaps the simplest example of a pseudo-Riemannian manifold.

1.2.1 The Minkowski inner product

This inner product is similar to the usual Euclidean inner product, but is used to describe a different geometry; the geometry is usually associated with relativity. Let M be a 4-dimensional real vector space. The Minkowski inner product is a map $\eta\colon M \times M \to \mathbf{R}$ (i.e. given any two vectors v, w in M we define $\eta(v,w)$ as a real number) which satisfies properties (1), (2), and (3) listed here, as well as property (4) given below:

Note that this is not an inner product in the usual sense, since it is not positive-definite, i.e. the quadratic form $\eta(v,v)$ need not be positive for nonzero v. The positive-definite condition has been replaced by the weaker condition of nondegeneracy (every positive-definite form is nondegenerate but not vice-versa). The inner product is said to be *indefinite*. These misnomers, "Minkowski inner product" and "Minkowski metric," conflict with the standard meanings of inner product and metric in pure mathematics; as with many other misnomers, the usage of these terms is due to similarity to the mathematical structure.

Just as in Euclidean space, two vectors v and w are said to be *orthogonal* if $\eta(v,w) = 0$. Minkowski space differs by including hyperbolic-orthogonal events in case v and w span a plane where η takes negative values. This difference is clarified by comparing the Euclidean structure of the ordinary complex number plane to the structure of the plane of split-complex numbers. The **Minkowski norm** of a vector v is defined by

$$\|v\| = \sqrt{|\eta(v,v)|}.$$

This is not a norm in the usual sense because it fails to be subadditive, but it does define a useful generalization of the notion of length to Minkowski space. In particular, a vector v is called a *unit vector* if $\|v\| = 1$ (i.e., $\eta(v,v) = \pm1$). A basis for M consisting of mutually orthogonal unit vectors is called an *orthonormal basis*.

By the Gram–Schmidt process, any inner product space satisfying conditions (1), (2), and (3) above always has an orthonormal basis. Furthermore, the number of positive and negative unit vectors in any such basis is a fixed pair of numbers, equal to the *signature* of the inner product. This is Sylvester's law of inertia.

Then the fourth condition on η can be stated:

Which signature is used is a matter of convention. Both are fairly common. See sign convention.

1.2.2 Standard basis

A standard basis for Minkowski space is a set of four mutually orthogonal vectors $\{e_0,e_1,e_2,e_3\}$ such that

$$-(e_0)^2 = (e_1)^2 = (e_2)^2 = (e_3)^2 = 1$$

These conditions can be written compactly in the following form:

$$\langle e_\mu, e_\nu \rangle = \eta_{\mu\nu}$$

where μ and ν run over the values (0, 1, 2, 3) and the matrix η is given by

$$\eta = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(As was previously noted, sometimes the opposite sign convention is preferred.)

This tensor is frequently called the "Minkowski tensor". Relative to a standard basis, the components of a vector v are written (v^0,v^1,v^2,v^3) and we use the Einstein notation to write $v = v^\mu e_\mu$. The component v^0 is called the **timelike component** of v while the other three components are called the **spatial components**.

In terms of components, the inner product between two vectors v and w is given by

$$\langle v, w \rangle = \eta_{\mu\nu} v^\mu w^\nu = -v^0 w^0 + v^1 w^1 + v^2 w^2 + v^3 w^3$$

and the norm-squared of a vector v is

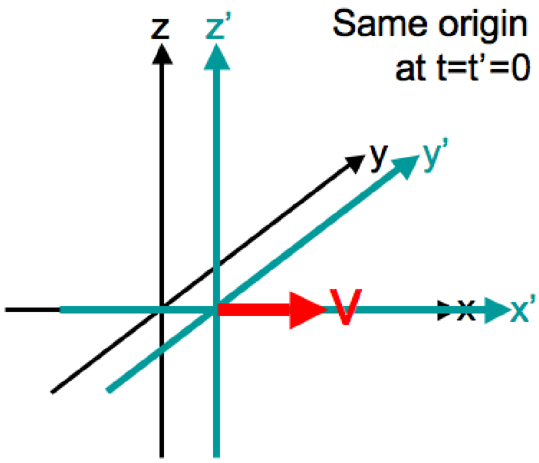
$$v^2 = \eta_{\mu\nu} v^\mu v^\nu = -(v^0)^2 + (v^1)^2 + (v^2)^2 + (v^3)^2$$

1.3 Alternative definition

The section above defines Minkowski space as a vector space. There is an alternative definition of Minkowski space as an affine space which views Minkowski space as a homogeneous space of the Poincaré group with the Lorentz group as the stabilizer. See Erlangen program.

Note also that the term "Minkowski space" is also used for analogues in any dimension: if $n \geq 2$, n-dimensional Minkowski space is a vector space or affine space of real dimension n on which there is an inner product or pseudo-Riemannian metric of signature $(n-1,1)$, i.e., in the above terminology, $n-1$ "pluses" and one "minus".

1.4 Lorentz transformations and symmetry



Standard configuration of coordinate systems for Lorentz transformations.

The Poincaré group is the group of all isometries of Minkowski spacetime including boosts, rotations, and translations. The Lorentz group is the subgroup of isometries which leave the origin fixed and includes the boosts and rotations; members of this subgroup are called Lorentz transformations. Among the simplest Lorentz transformations is a Lorentz boost. The archetypal Lorentz boost is

$$\begin{bmatrix} U'_0 \\ U'_1 \\ U'_2 \\ U'_3 \end{bmatrix} = \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} U_0 \\ U_1 \\ U_2 \\ U_3 \end{bmatrix}$$

where

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

is the Lorentz factor, and

$$\beta = \frac{v}{c}.$$

All four-vectors in Minkowski space transform according to the same formula under Lorentz transformations. Minkowski diagrams illustrate Lorentz transformations.

1.5 Causal structure

Main article: Causal structure

Vectors are classified according to the sign of η(*v*,*v*). When the standard signature (−,+,+,+) is used, a vector *v* is:

This terminology comes from the use of Minkowski space in the theory of relativity. The set of all null vectors at an event of Minkowski space constitutes the light cone of that event. Note that all these notions are independent of the frame of reference. Given a timelike vector *v*, there is a worldline of constant velocity associated with it. The set { *w* : η(*w*,*v*) = 0 } corresponds to the **simultaneous hyperplane** at the origin of this worldline. Minkowski space exhibits relativity of simultaneity since this hyperplane depends on *v*. In the plane spanned by *v* and such a *w* in the hyperplane, the relation of *w* to *v* is hyperbolic-orthogonal.

Once a direction of time is chosen, timelike and null vectors can be further decomposed into various classes. For timelike vectors we have

1. *future directed timelike* vectors whose first component is positive, and
2. *past directed timelike* vectors whose first component is negative.

Null vectors fall into three classes:

1. the *zero vector*, whose components in any basis are (0,0,0,0),
2. *future directed null* vectors whose first component is positive, and
3. *past directed null* vectors whose first component is negative.

Together with spacelike vectors there are 6 classes in all.

An orthonormal basis for Minkowski space necessarily consists of one timelike and three spacelike unit vectors. If one wishes to work with non-orthonormal bases it is possible to have other combinations of vectors. For example, one can easily construct a (non-orthonormal) basis consisting entirely of null vectors, called a **null basis**. Over the reals, if two null vectors are orthogonal (zero inner product), then they must be proportional. However, allowing complex numbers, one can obtain a null tetrad which is a basis consisting of null vectors, some of which are orthogonal to each other.

Vector fields are called timelike, spacelike or null if the associated vectors are timelike, spacelike or null at each point where the field is defined.

1.5.1 Causality relations

Let $x, y \in M$. We say that

1. x ***chronologically precedes*** y if $y - x$ is future directed timelike.
2. x ***causally precedes*** y if $y - x$ is future directed null or future directed timelike

1.6 Reversed triangle inequality

If v and w are two equally directed timelike four-vectors, then

$$|v + w| \geq |v| + |w|,$$

where

$$|v| := \sqrt{-\eta_{\mu\nu} v^\mu v^\nu}.$$

1.7 Locally flat spacetime

Strictly speaking, the use of the Minkowski space to describe physical systems over finite distances applies only in systems without significant gravitation. In the case of significant gravitation, spacetime becomes curved and one must abandon special relativity in favor of the full theory of general relativity.

Nevertheless, even in such cases, Minkowski space is still a good description in an infinitesimal region surrounding any point (barring gravitational singularities). More abstractly, we say that in the presence of gravity spacetime is described by a curved 4-dimensional manifold for which the tangent space to any point is a 4-dimensional Minkowski space. Thus, the structure of Minkowski space is still essential in the description of general relativity.

In the realm of weak gravity, spacetime becomes flat and looks globally, not just locally, like Minkowski space. For this reason Minkowski space is often referred to as *flat spacetime*.

1.8 See also

1.9 References

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1.10 External links

Media related to Minkowski diagrams at Wikimedia Commons

- Animation clip on YouTube visualizing Minkowski space in the context of special relativity.

Chapter 2

Spacetime

For other uses of this term, see Spacetime (disambiguation).

In physics, **spacetime** (also **space–time**, **space time** or **space–time continuum**) is any mathematical model that combines space and time into a single interwoven continuum. The spacetime of our universe is usually interpreted from a Euclidean space perspective, which regards space as consisting of three dimensions, and time as consisting of one dimension, the 'fourth dimension'. By combining space and time into a single manifold called Minkowski space, physicists have significantly simplified a large number of physical theories, as well as described in a more uniform way the workings of the universe at both the supergalactic and subatomic levels.

In non-relativistic classical mechanics, the use of Euclidean space instead of spacetime is appropriate, as time is treated as universal and constant, being independent of the state of motion of an observer. In relativistic contexts, time cannot be separated from the three dimensions of space, because the observed rate at which time passes for an object depends on the object's velocity relative to the observer and also on the strength of gravitational fields, which can slow the passage of time for an object as seen by an observer outside the field.

In cosmology, the concept of spacetime combines space and time to a single abstract universe. Mathematically it is a manifold consisting of "events" which are described by some type of coordinate system. Typically **three spatial dimensions** (length, width, height), and one **temporal dimension** (time) are required. Dimensions are independent components of a coordinate grid needed to locate a point in a certain defined "space". For example, on the globe the latitude and longitude are two independent coordinates which together uniquely determine a location. In spacetime, a coordinate grid that spans the 3+1 dimensions locates events (rather than just points in space), i.e., time is added as another dimension to the coordinate grid. This way the coordinates specify *where* and *when* events occur. However, the unified nature of spacetime and the freedom of coordinate choice it allows imply that to express the temporal coordinate in one coordinate system requires both temporal and spatial coordinates in another coordinate system. Unlike in normal spatial coordinates, there are still restrictions for how measurements can be made spatially and temporally (see Spacetime intervals). These restrictions correspond roughly to a particular mathematical model which differs from Euclidean space in its manifest symmetry.

Until the beginning of the 20th century, time was believed to be independent of motion, progressing at a fixed rate in all reference frames; however, later experiments revealed that time slows at higher speeds of the reference frame relative to another reference frame. Such slowing, called time dilation, is explained in special relativity theory. Many experiments have confirmed time dilation, such as the relativistic decay of muons from cosmic ray showers and the slowing of atomic clocks aboard a Space Shuttle relative to synchronized Earth-bound inertial clocks. The duration of time can therefore vary according to events and reference frames.

When dimensions are understood as mere components of the grid system, rather than physical attributes of space, it is easier to understand the alternate dimensional views as being simply the result of coordinate transformations.

The term *spacetime* has taken on a generalized meaning beyond treating spacetime events with the normal 3+1 dimensions. It is really the combination of space and time. Other proposed spacetime theories include additional dimensions—normally spatial but there exist some speculative theories that include additional temporal dimensions and even some that include dimensions that are neither temporal nor spatial (e.g., superspace). How many dimensions are needed to describe the universe is still an open question. Speculative theories such as string theory predict 10 or 26 dimensions (with M-theory predicting 11 dimensions: 10 spatial and 1 temporal), but the existence of more than four dimensions would only appear to make a difference at the subatomic level.[1]

2.1 Spacetime in literature

Incas regarded space and time as a single concept, referred to as ***pacha*** (Quechua: *pacha*, Aymara: *pacha*).[2][3] The peoples of the Andes maintain a similar understanding.[4]

Arthur Schopenhauer wrote in §18 of *On the Fourfold Root of the Principle of Sufficient Reason* (1813): “the representation of coexistence is impossible in Time alone; it depends, for its completion, upon the representation of Space; because, in mere Time, all things follow one another, and in mere Space all things are side by side; it is accordingly only by the combination of Time and Space that the representation of coexistence arises”.

The idea of a unified spacetime is stated by Edgar Allan Poe in his essay on cosmology titled *Eureka* (1848) that “Space and duration are one”. In 1895, in his novel *The Time Machine*, H. G. Wells wrote, “There is no difference between time and any of the three dimensions of space except that our consciousness moves along it”, and that “any real body must have extension in four directions: it must have Length, Breadth, Thickness, and Duration”.

Marcel Proust, in his novel *Swann’s Way* (published 1913), describes the village church of his childhood’s Combray as “a building which occupied, so to speak, four dimensions of space—the name of the fourth being Time”.

2.1.1 Mathematical concept

In Encyclopedie under the term *dimension* Jean le Rond d’Alembert speculated that duration (time) might be considered a fourth dimension if the idea was not too novel.[5]

Another early venture was by Joseph Louis Lagrange in his *Theory of Analytic Functions* (1797, 1813). He said, “One may view mechanics as a geometry of four dimensions, and mechanical analysis as an extension of geometric analysis”.[6]

The ancient idea of the cosmos gradually was described mathematically with differential equations, differential geometry, and abstract algebra. These mathematical articulations blossomed in the nineteenth century as electrical technology stimulated men like Michael Faraday and James Clerk Maxwell to describe the reciprocal relations of electric and magnetic fields. Daniel Siegel phrased Maxwell’s role in relativity as follows:

> [...] the idea of the propagation of forces at the velocity of light through the electromagnetic field as described by Maxwell’s equations – rather than instantaneously at a distance –formed the necessary basis for relativity theory.[7]

Maxwell used vortex models in his papers on On Physical Lines of Force, but ultimately gave up on any substance but the electromagnetic field. Pierre Duhem wrote:

> [Maxwell] was not able to create the theory that he envisaged except by giving up the use of any model, and by extending by means of analogy the abstract system of electrodynamics to displacement currents.[8]

In Siegel’s estimation, “this very abstract view of the electromagnetic fields, involving no visualizable picture of what is going on out there in the field, is Maxwell’s legacy.”[9] Describing the behaviour of electric fields and magnetic fields led Maxwell to a unified view of an electromagnetic field. Being functions, these fields took values on a domain, a piece of spacetime. It is the intermingling of electric and magnetic manifestations, described by Maxwell’s equations that give spacetime its structure. In particular, the rate of motion of an observer determines the electric and magnetic profiles of the electromagnetic field. The propagation of the field is determined by the electromagnetic wave equation which also requires spacetime for description.

Spacetime was described as an affine space with quadratic form in Minkowski space of 1908.[10] In his 1914 textbook *The Theory of Relativity*, Ludwik Silberstein used biquaternions to represent events in Minkowski space. He also exhibited the Lorentz transformations between observers of differing velocities as biquaternion mappings. Biquaternions were described in 1853 by W. R. Hamilton, so while the physical interpretation was new, the mathematics was well known in English literature, making relativity an instance of applied mathematics.

The 1926 thirteenth edition of the *Encyclopædia Britannica* included an article by Einstein titled “Space–Time”.[11])

The first inkling of general relativity in spacetime was articulated by W. K. Clifford. Description of the effect of gravitation on space and time was found to be most easily visualized as a “warp” or stretching in the geometrical fabric of space and time, in a smooth and continuous way that changed smoothly from point-to-point along the spacetime fabric.

2.2 Basic concepts

Spacetimes are the arenas in which all physical events take place—an event is a point in spacetime specified by its time and place. For example, the motion of planets around the sun may be described in a particular type of spacetime, or the motion of light around a rotating star may be described in another type of spacetime. The basic elements of spacetime are events. In any given spacetime, an event is a unique position at a unique time. Because events are spacetime points, an example of an event in classical relativistic physics is (x, y, z, t) , the location of an elementary (point-like) particle at a particular time. A spacetime itself can be viewed as the union of all events in the same way that a line is the union of all of its points, formally organized into a manifold, a space which can be described at small scales using coordinate systems.

A spacetime is independent of any observer.[12] However, in describing physical phenomena (which occur at certain moments of time in a given region of space), each observer chooses a convenient metrical coordinate system. Events are specified by four real numbers in any such coordinate system. The trajectories of elementary (point-like) particles through space and time are thus a continuum of events called the world line of the particle. Extended or composite objects (consisting of many elementary particles) are thus a union of many world lines twisted together by virtue of their interactions through spacetime into a "world-braid".

However, in physics, it is common to treat an extended object as a "particle" or "field" with its own unique (e.g., center of mass) position at any given time, so that the world line of a particle or light beam is the path that this particle or beam takes in the spacetime and represents the history of the particle or beam. The world line of the orbit of the Earth (in such a description) is depicted in two spatial dimensions *x* and *y* (the plane of the Earth's orbit) and a time dimension orthogonal to *x* and *y*. The orbit of the Earth is an ellipse in space alone, but its world line is a helix in spacetime.[13]

The unification of space and time is exemplified by the common practice of selecting a metric (the measure that specifies the interval between two events in spacetime) such that all four dimensions are measured in terms of units of distance: representing an event as $(x_0, x_1, x_2, x_3) = (ct, x, y, z)$ (in the Lorentz metric) or $(x_1, x_2, x_3, x_4) = (x, y, z, ict)$ (in the original Minkowski metric) where c is the speed of light.[14] The metrical descriptions of Minkowski Space and spacelike, lightlike, and timelike intervals given below follow this convention, as do the conventional formulations of the Lorentz transformation.

2.2.1 Spacetime intervals

In a Euclidean space, the separation between two points is measured by the distance between the two points. The distance is purely spatial, and is always positive. In spacetime, the separation between two events is measured by the *invariant interval* between the two events, which takes into account not only the spatial separation between the events, but also their temporal separation. The interval, s^2, between two events is defined as:

$$s^2 = \Delta r^2 - c^2 \Delta t^2 \text{ (spacetime interval)},$$

where *c* is the speed of light, and Δ*r* and Δ*t* denote differences of the space and time coordinates, respectively, between the events. The choice of signs for s^2 above follows the space-like convention (−+++). The reason s^2 is called the interval and not s is that s^2 can be positive, zero or negative.

Spacetime intervals may be classified into three distinct types, based on whether the temporal separation ($c^2\Delta t^2$) or the spatial separation (Δr^2) of the two events is greater: time-like, light-like or space-like.

Certain types of world lines are called geodesics of the spacetime – straight lines in the case of Minkowski space and their closest equivalent in the curved spacetime of general relativity. In the case of purely time-like paths, geodesics are (locally) the paths of greatest separation (spacetime interval) as measured along the path between two events, whereas in Euclidean space and Riemannian manifolds, geodesics are paths of shortest distance between two points.[15][16] The concept of geodesics becomes central in general relativity, since geodesic motion may be thought of as "pure motion" (inertial motion) in spacetime, that is, free from any external influences.

Time-like interval

$$c^2\Delta t^2 > \Delta r^2$$
$$s^2 < 0$$

For two events separated by a time-like interval, enough time passes between them that there could be a cause–effect relationship between the two events. For a particle traveling through space at less than the speed of light, any two events which occur to or by the particle must be separated by a time-like interval. Event pairs with time-like separation define a negative squared spacetime interval ($s^2 < 0$) and may be said to occur in each other's future or past. There exists a reference frame such that the two events are observed to occur in the same spatial location, but there is no reference frame in which the two events can occur at the same time.

The measure of a time-like spacetime interval is described by the proper time, $\Delta\tau$:

$$\Delta\tau = \sqrt{\Delta t^2 - \frac{\Delta r^2}{c^2}} \text{ (proper time).}$$

The proper time interval would be measured by an observer with a clock traveling between the two events in an inertial reference frame, when the observer's path intersects each event as that event occurs. (The proper time defines a real number, since the interior of the square root is positive.)

Light-like interval

$$c^2\Delta t^2 = \Delta r^2$$
$$s^2 = 0$$

In a light-like interval, the spatial distance between two events is exactly balanced by the time between the two events. The events define a squared spacetime interval of zero ($s^2 = 0$). Light-like intervals are also known as “null” intervals.

Events which occur to or are initiated by a photon along its path (i.e., while traveling at c , the speed of light) all have light-like separation. Given one event, all those events which follow at light-like intervals define the propagation of a light cone, and all the events which preceded from a light-like interval define a second (graphically inverted, which is to say "*pastward*") light cone.

Space-like interval

$$c^2\Delta t^2 < \Delta r^2$$
$$s^2 > 0$$

When a space-like interval separates two events, not enough time passes between their occurrences for there to exist a causal relationship crossing the spatial distance between the two events at the speed of light or slower. Generally, the events are considered not to occur in each other’s future or past. There exists a reference frame such that the two events are observed to occur at the same time, but there is no reference frame in which the two events can occur in the same spatial location.

For these space-like event pairs with a positive squared spacetime interval ($s^2 > 0$), the measurement of space-like separation is the proper distance, $\Delta\sigma$:

$$\Delta\sigma = \sqrt{s^2} = \sqrt{\Delta r^2 - c^2\Delta t^2} \text{ (proper distance).}$$

Like the proper time of time-like intervals, the proper distance of space-like spacetime intervals is a real number value.

2.3 Mathematics of spacetimes

For physical reasons, a spacetime continuum is mathematically defined as a four-dimensional, smooth, connected Lorentzian manifold (M, g) . This means the smooth Lorentz metric g has signature $(3, 1)$. The metric determines the geometry of spacetime, as well as determining the geodesics of particles and light beams. About each point (event) on this manifold, coordinate charts are used to represent observers in reference frames. Usually, Cartesian coordinates (x, y, z, t) are used. Moreover, for simplicity’s sake, units of measurement are usually chosen such that the speed of light c is equal to 1.

A reference frame (observer) can be identified with one of these coordinate charts; any such observer can describe any event p . Another reference frame may be identified by a second coordinate chart about p . Two observers (one in each reference frame) may describe the same event p but obtain different descriptions.

Usually, many overlapping coordinate charts are needed to cover a manifold. Given two coordinate charts, one containing p (representing an observer) and another containing q (representing another observer), the intersection of the charts represents the region of spacetime in which both observers can measure physical quantities and hence compare results. The relation between the two sets of measurements is given by a non-singular coordinate transformation on this intersection. The idea of coordinate charts as local observers who can perform measurements in their vicinity also makes good physical sense, as this is how one actually collects physical data—locally.

For example, two observers, one of whom is on Earth, but the other one who is on a fast rocket to Jupiter, may observe a comet crashing into Jupiter (this is the event p). In general, they will disagree about the exact location and timing of this impact, i.e., they will have different 4-tuples (x, y, z, t) (as they are using different coordinate systems). Although their kinematic descriptions will differ, dynamical (physical) laws, such as momentum conservation and the first law of thermodynamics, will still hold. In fact, relativity theory requires more than this in the sense that it stipulates these (and all other physical) laws must take the same form in all coordinate systems. This introduces tensors into relativity, by which all physical quantities are represented.

Geodesics are said to be time-like, null, or space-like if the tangent vector to one point of the geodesic is of this nature. Paths of particles and light beams in spacetime are represented by time-like and null (light-like) geodesics, respectively.

2.3.1 Topology

Main article: Spacetime topology

The assumptions contained in the definition of a spacetime are usually justified by the following considerations.

The connectedness assumption serves two main purposes. First, different observers making measurements (represented by coordinate charts) should be able to compare their observations on the non-empty intersection of the charts. If the connectedness assumption were dropped, this would not be possible. Second, for a manifold, the properties of connectedness and path-connectedness are equivalent, and one requires the existence of paths (in particular, geodesics) in the spacetime to represent the motion of particles and radiation.

Every spacetime is paracompact. This property, allied with the smoothness of the spacetime, gives rise to a smooth linear connection, an important structure in general relativity. Some important theorems on constructing spacetimes from compact and non-compact manifolds include the following:

- A compact manifold can be turned into a spacetime if, and only if, its Euler characteristic is 0. (Proof idea: the existence of a Lorentzian metric is shown to be equivalent to the existence of a nonvanishing vector field.)
- Any non-compact 4-manifold can be turned into a spacetime.[17]

2.3.2 Spacetime symmetries

Main article: Spacetime symmetries

Often in relativity, spacetimes that have some form of symmetry are studied. As well as helping to classify spacetimes, these symmetries usually serve as a simplifying assumption in specialized work. Some of the most popular ones include:

- Axisymmetric spacetimes
- Spherically symmetric spacetimes
- Static spacetimes
- Stationary spacetimes

2.3.3 Causal structure

Main article: Causal structure
See also: Causality (physics) and Causality

The causal structure of a spacetime describes causal relationships between pairs of points in the spacetime based on the existence of certain types of curves joining the points.

2.4 Spacetime in special relativity

Main article: Minkowski space

The geometry of spacetime in special relativity is described by the Minkowski metric on R^4. This spacetime is called Minkowski space. The Minkowski metric is usually denoted by η and can be written as a four-by-four matrix:

$$\eta_{ab} = \mathrm{diag}(1, -1, -1, -1)$$

where the Landau–Lifshitz space-like convention is being used. A basic assumption of relativity is that coordinate transformations must leave spacetime intervals invariant. Intervals are invariant under Lorentz transformations. This invariance property leads to the use of four-vectors (and other tensors) in describing physics.

Strictly speaking, one can also consider events in Newtonian physics as a single spacetime. This is Galilean–Newtonian relativity, and the coordinate systems are related by Galilean transformations. However, since these preserve spatial and temporal distances independently, such a spacetime can be decomposed into spatial coordinates plus temporal coordinates, which is not possible in the general case.

2.5 Spacetime in general relativity

In general relativity, it is assumed that spacetime is curved by the presence of matter (energy), this curvature being represented by the Riemann tensor. In special relativity, the Riemann tensor is identically zero, and so this concept of "non-curvedness" is sometimes expressed by the statement *Minkowski spacetime is flat.*

The earlier discussed notions of time-like, light-like and space-like intervals in special relativity can similarly be used to classify one-dimensional curves through curved spacetime. A time-like curve can be understood as one where the interval between any two infinitesimally close events on the curve is time-like, and likewise for light-like and space-like curves. Technically the three types of curves are usually defined in terms of whether the tangent vector at each point on the curve is time-like, light-like or space-like. The world line of a slower-than-light object will always be a time-like curve, the world line of a massless particle such as a photon will be a light-like curve, and a space-like curve could be the world line of a hypothetical tachyon. In the local neighborhood of any event, time-like curves that pass through the event will remain inside that event's past and future light cones, light-like curves that pass through the event will be on the surface of the light cones, and space-like curves that pass through the event will be outside the light cones. One can also define the notion of a three-dimensional "spacelike hypersurface", a continuous three-dimensional "slice" through the four-dimensional property with the property that every curve that is contained entirely within this hypersurface is a space-like curve.[18]

Many spacetime continua have physical interpretations which most physicists would consider bizarre or unsettling. For example, a compact spacetime has closed timelike curves, which violate our usual ideas of causality (that is, future events could affect past ones). For this reason, mathematical physicists usually consider only restricted subsets of all the possible spacetimes. One way to do this is to study "realistic" solutions of the equations of general relativity. Another way is to add some additional "physically reasonable" but still fairly general geometric restrictions and try to prove interesting things about the resulting spacetimes. The latter approach has led to some important results, most notably the Penrose–Hawking singularity theorems.

2.6 Quantized spacetime

Main article: Quantum spacetime

In general relativity, spacetime is assumed to be smooth and continuous—and not just in the mathematical sense. In the theory of quantum mechanics, there is an inherent discreteness present in physics. In attempting to reconcile these two theories, it is sometimes postulated that spacetime should be quantized at the very smallest scales. Current theory is focused on the nature of spacetime at the Planck scale. Causal sets, loop quantum gravity, string theory, causal dynamical triangulation, and black hole thermodynamics all predict a quantized spacetime with agreement on the order of magnitude. Loop quantum gravity makes precise predictions about the geometry of spacetime at the Planck scale.

2.7 See also

- Anthropic_principle § Applications of the principle §§ Spacetime
- Basic introduction to the mathematics of curved spacetime
- Four-vector
- Frame-dragging
- Global spacetime structure
- Hole argument
- List of mathematical topics in relativity
- Local spacetime structure
- Lorentz invariance
- Manifold
- Mathematics of general relativity
- Metric space
- Philosophy of space and time
- Relativity of simultaneity
- Strip photography
- World manifold

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[18] See "Quantum Spacetime and the Problem of Time in Quantum Gravity" by Leszek M. Sokolowski, where on this page he writes "Each of these hypersurfaces is spacelike, in the sense that every curve, which entirely lies on one of such hypersurfaces, is a spacelike curve." More

commonly a space-like hypersurface is defined technically as a surface such that the normal vector at every point is time-like, but the definition above may be somewhat more intuitive.

2.9 External links

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