NATIONAL INSTITUE OF TECHNOLOGY KARANATAKA, SURATHKAL

ASSIGNMENT 8

APPLIED COMPUTATIONAL METHODS IN MECHANICAL SCIENCES

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ASSIGNMENT ON BOUNDARY VALUE PROBLEM

Answer

Given function:

$$\gamma \frac{d^2\emptyset}{dx^2} + \frac{d\emptyset}{dx} - \emptyset = 0$$

With boundary conditions

$$\emptyset = \begin{cases} 1, & x = 0 \\ 0, & x = 1 \end{cases}$$

Discretization using Central difference scheme:

$$\left[1 + \frac{\Delta x}{2\gamma}\right]\emptyset_{i+1} + \left[-2 - \frac{(\Delta x)^2}{\gamma}\right]\emptyset_i + \left[1 - \frac{\Delta x}{2\gamma}\right]\emptyset_{i-1} = 0$$

Discretization using Forward difference scheme:

$$\left[1 + \frac{\Delta x}{\gamma}\right]\emptyset_{i+1} + \left[-2 - \frac{(\Delta x)^2}{\gamma} - \frac{(\Delta x)}{\gamma}\right]\emptyset_i + \emptyset_{i-1} = 0$$

Discretization using Backward difference scheme:

$$\emptyset_{i+1} + \left[-2 - \frac{(\Delta x)^2}{\gamma} + \frac{(\Delta x)}{\gamma}\right]\emptyset_i + \left[1 - \frac{\Delta x}{\gamma}\right]\emptyset_{i-1} = 0$$

Analytical solution:

Assume solution as $\emptyset = Ae^{r_1 x} + Be^{r_2 x}$

$$r_{1,2} = \frac{-1 \mp \sqrt{1 + 4\gamma}}{2\gamma}$$

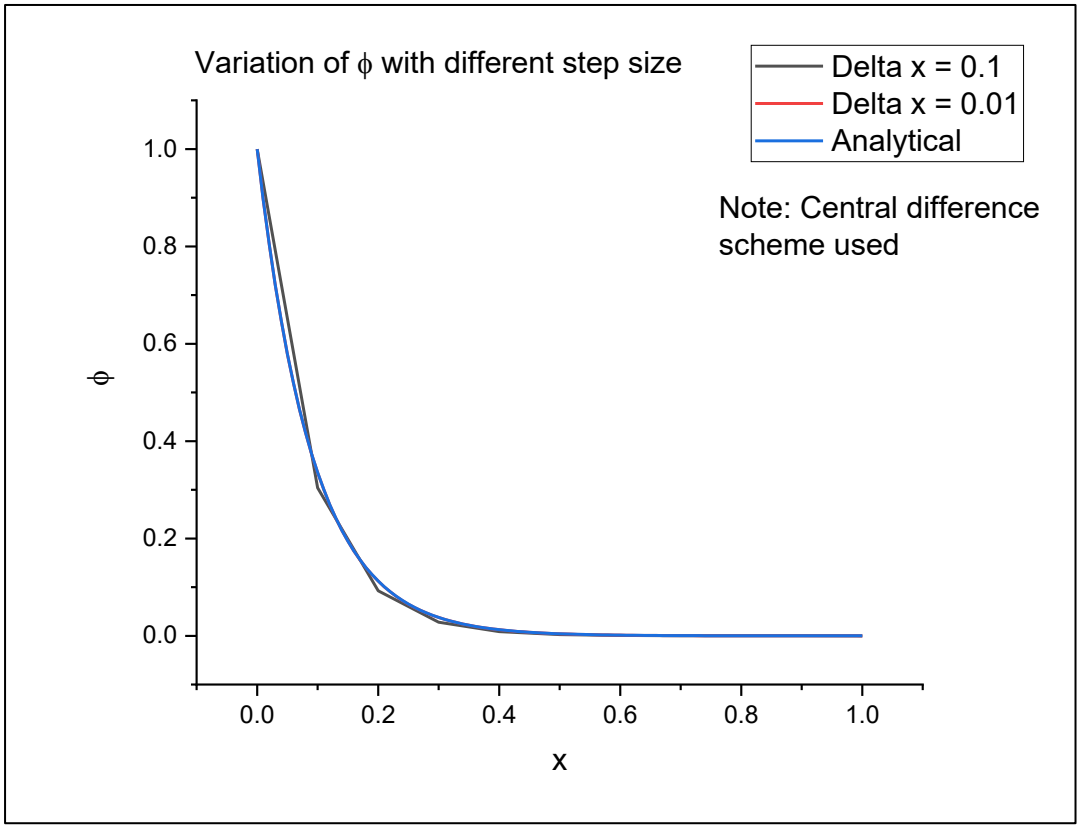
Applying the boundary conditions, $A = 1, B = 0$

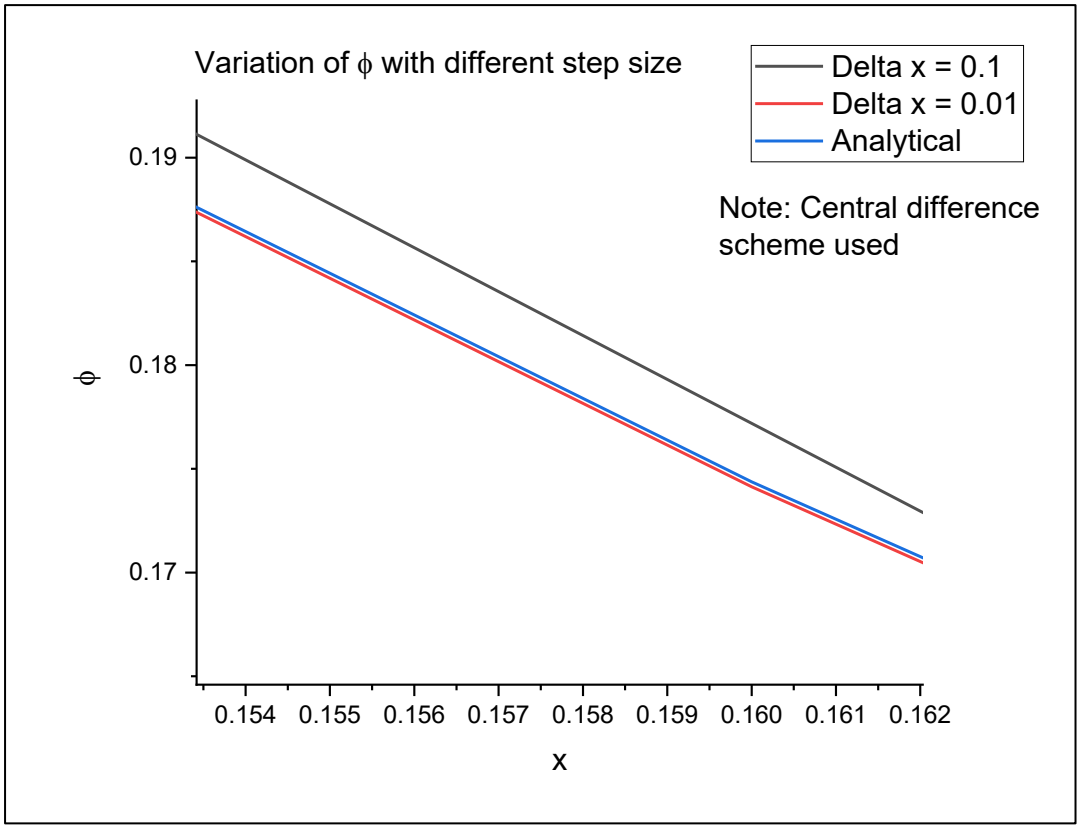
$$\emptyset = e^{-10.916x}$$

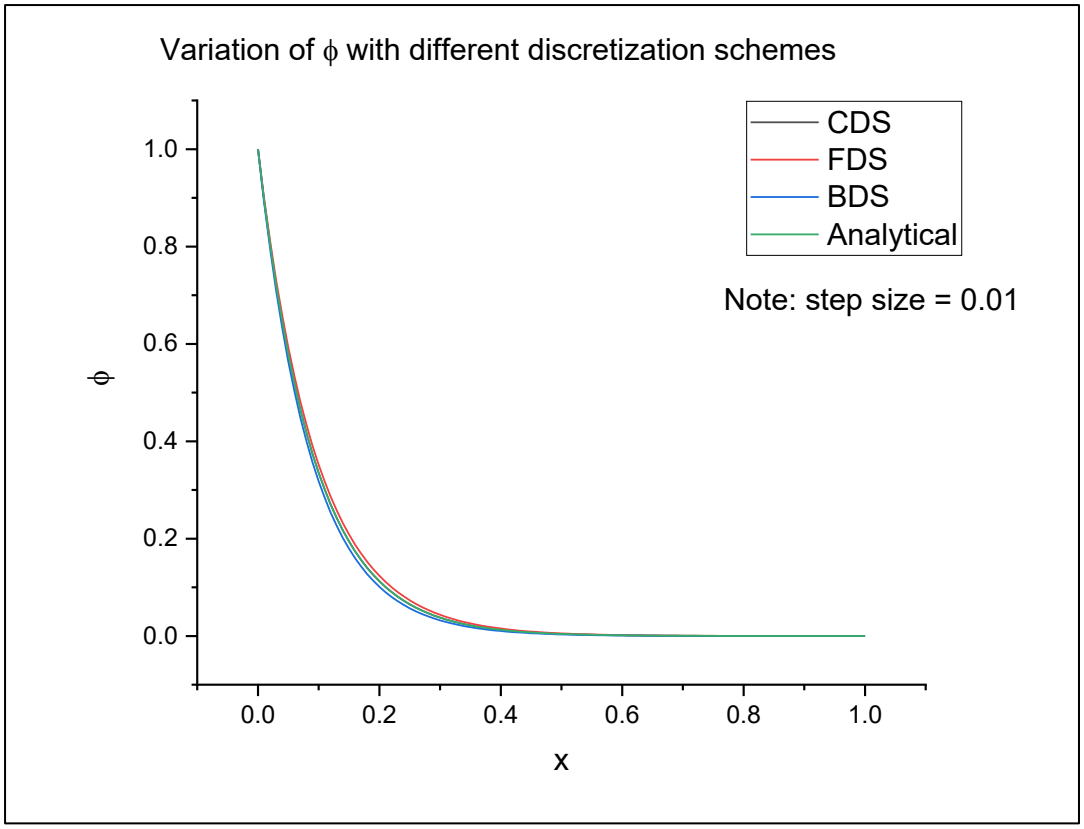
Code(C++)

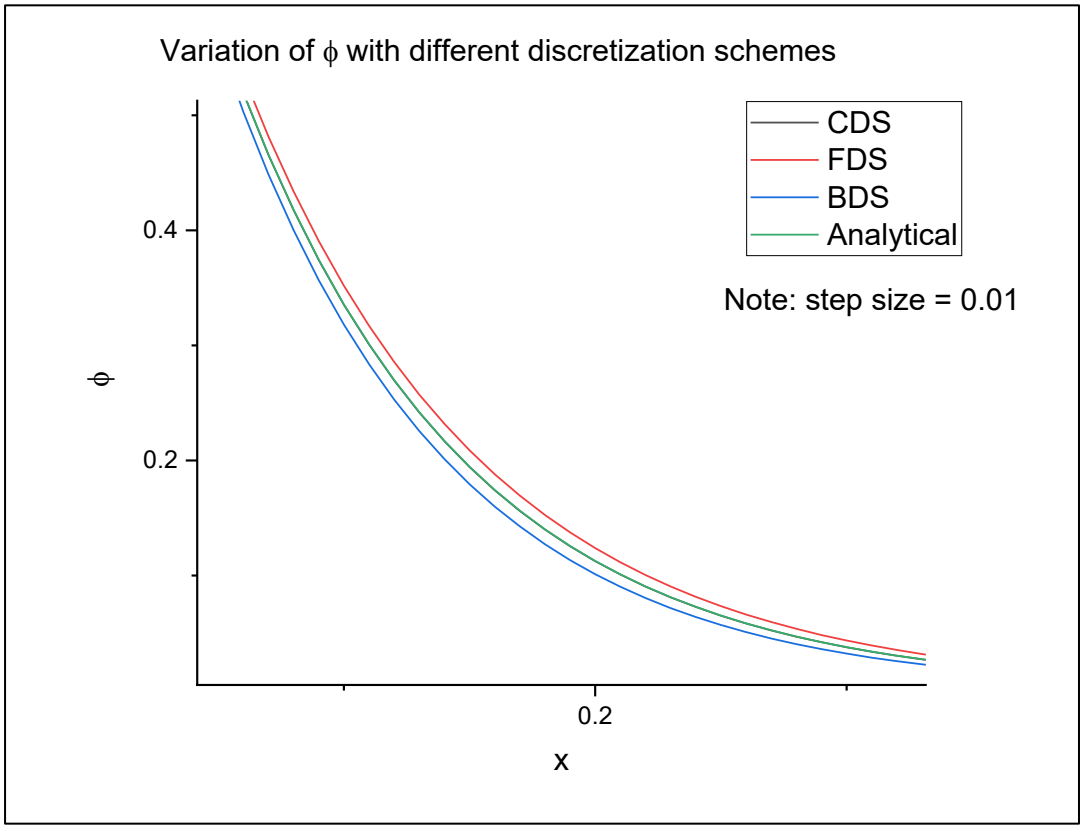
```cpp
#include<iostream>
#include<fstream>
#include<cmath>
using namespace std;
main()
{
    float gamma=0.1,dx=0.01,l=1,x;
    int N=int(l/dx)+1,choice=1,i;
    float phi[N],a,b_value,c,b[N-2],factor,phi_anly[N],B[N-2]={ };
    //Boundary conditions
    phi[0]=1;
    phi[N-1]=0;
    //Central difference scheme
    if(choice==1)
    {
        a=1+dx/(2*gamma);
        b_value=-(2+dx*dx/gamma);
        c=1-dx/(2*gamma);
    }
    //Forward difference scheme
    else if(choice==2)
    {
        a=1+dx/gamma;
        b_value=-(2+dx/gamma*(1+dx));
        c=1;
    }
    //Backward difference scheme
    else
    {
        a=1;
        b_value=-(2+dx/gamma*(-1+dx));
        c=1-dx/gamma;
    }
    B[0]=-c*phi[0];
    B[N-3]=-a*phi[N-1];
    //Thomas algorithm
    for(i=0;i<N-2;++i)
        b[i]=b_value;
    for(i=1;i<N-2;++i)
    {
        factor = c/b[i-1];
        b[i]=b[i]-factor*a;
        B[i]=B[i]-factor*B[i-1];
    }
    phi[N-2]=B[N-3]/b[N-3];
    //Numerical solution
    for(i=N-3;i>0;--i)
        phi[i]=(B[i-1]-a*phi[i+1])/b[i-1];
    //Analytical solution
    for(i=0;i<N;++i)
    {
        x=i*dx;
        phi_anly[i]=exp(-10.916*x);
    }
    //Writing to Data file
    fstream f;
    f.open("DATA.txt",ios::out);
    for(i=0;i<N;++i)
        f<<i*dx<<" "<<phi[i]<<" "<<phi_anly[i]<<"\n";
    f.close();
}
```

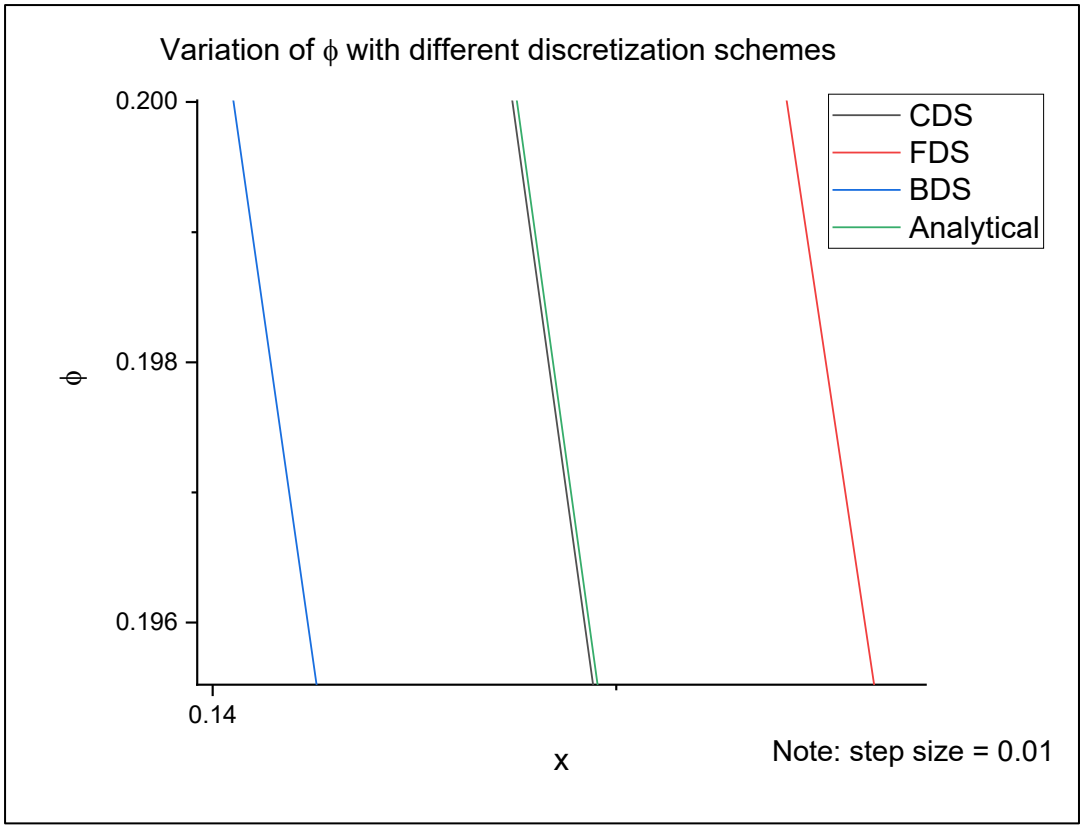
Output


Variation of ϕ with different step size
Delta x = 0.1
Delta x = 0.01
Analytical
Note: Central difference scheme used
ϕ
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
x


Variation of ϕ with different step size
Delta x = 0.1
Delta x = 0.01
Analytical
Note: Central difference scheme used
ϕ
0.19
0.18
0.17
0.154
0.155
0.156
0.157
0.158
0.159
0.160
0.161
0.162
x


Variation of ϕ with different discretization schemes
CDS
FDS
BDS
Analytical
Note: step size = 0.01
ϕ
x
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0


Variation of ϕ with different discretization schemes
CDS
FDS
BDS
Analytical
Note: step size = 0.01
ϕ
x
0.4
0.2
0.2


Variation of ϕ with different discretization schemes
0.200
0.198
0.196
ϕ
0.14
x
CDS
FDS
BDS
Analytical
Note: step size = 0.01