Simulating Quantum Many Body Dynamics on a current Quantum Computer

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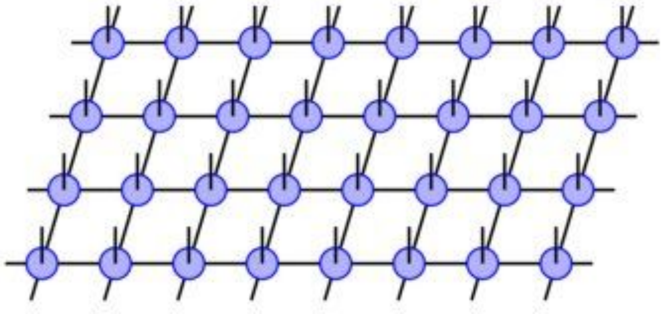
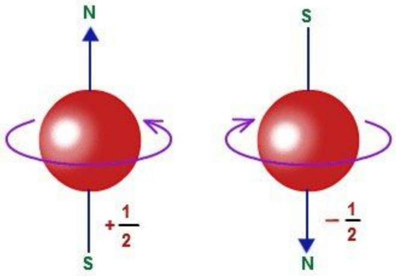
Content

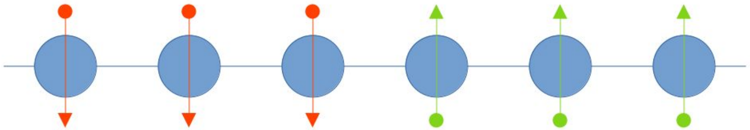
Quantum Many Body Dynamics (1) - Introduction





Goal of this study:

To simulate evolution of a 1D spin-½ chain on a current quantum computer.



Setup (1)

- Governing equation: Schrödinger equation

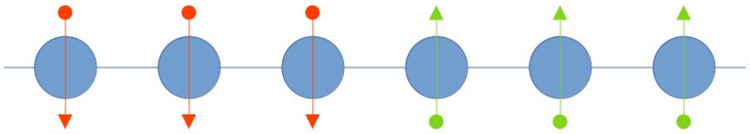
$$i\hbar\frac{\mathrm{d}}{\mathrm{d}t}|\psi(t)\rangle = H|\psi(t)\rangle \qquad\qquad |\psi(t)\rangle = U_t|\psi(0)\rangle \quad \text{with} \quad U_t = \mathrm{e}^{-iHt/\hbar}.$$

- General Hamiltonian for quantum many body physics

$$\hat{H} = -J\sum_{j=1}^{N-1}\left(\hat{\sigma}_j^x\hat{\sigma}_{j+1}^x + \hat{\sigma}_j^y\hat{\sigma}_{j+1}^y\right) + U\sum_{j=1}^{N-1}\hat{\sigma}_j^z\hat{\sigma}_{j+1}^z + \sum_{j=1}^{N} h_j\hat{\sigma}_j^z,$$

Setup (2)

- 2 distinct cases for Hamiltonian parameters
  - Case (i) - XX chain - U = 0, $h_j = 0$
  - Case (ii) - Disordered XX chain - U = 0, h_j sampled from [-h, +h]
- Initial State: 6 particle domain wall - | 1 1 1 0 0 0 >



- Observables
  - Local Magnetization $M_j(t) = \langle \psi(t) | \hat{\sigma}_j^z | \psi(t) \rangle.$
  - $$N_{\text{half}}(t) = \sum_{j=1}^{N/2} \langle \psi(t) | \frac{\hat{\sigma}_j^z + 1}{2} | \psi(t) \rangle$$

Implementation - (1)

- Trotter decomposition of Unitary time evolution operator $U(t) = e^{-iHt}$
- Idea: Break down complex operator to a sequence of simpler operators

$$\hat{H} = \hat{A} + \hat{B},$$

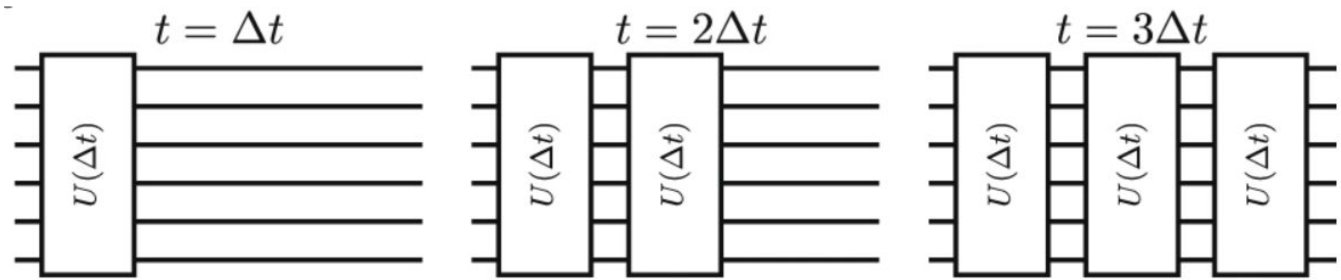
$$e^{-i\hat{H}t} = e^{-i\hat{A}t} e^{-i\hat{B}t} + \mathcal{O}(t^2),$$

- Baker–Campbell–Hausdorff formula,

$$e^X e^Y = e^Z \qquad Z = X + Y + \frac{1}{2}[X,Y] + \frac{1}{12}[X,[X,Y]] - \frac{1}{12}[Y,[X,Y]] + \cdots$$

Implementation - (2)

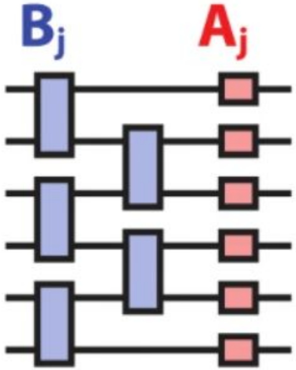
- **Step 1**: Split operator into discrete evolution operators $e^{-i\hat{H}t} = \left(e^{-i\hat{H}\Delta t}\right)^M$



Implementation - (3)

$$\hat{H} = -J\sum_{j=1}^{N-1}\left(\hat{\sigma}_j^x\hat{\sigma}_{j+1}^x + \hat{\sigma}_j^y\hat{\sigma}_{j+1}^y\right) + U\sum_{j=1}^{N-1}\hat{\sigma}_j^z\hat{\sigma}_{j+1}^z + \sum_{j=1}^{N} h_j\hat{\sigma}_j^z,$$

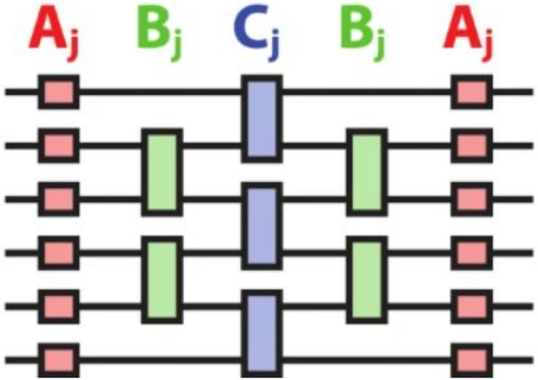
- **Step 2**: Approximate the discrete evolution operators - 2 options:



Basic

If H = A + B

$$e^{-iH\Delta t} = e^{-iA\Delta t}\, e^{-iB\Delta t} + O(\Delta t^2)$$



Symmetric

If H = A + B

$$e^{-iH\Delta t} = e^{-iA\Delta t/2}\, e^{-iB\Delta t}\, e^{-iA\Delta t/2} + O(\Delta t^3)$$

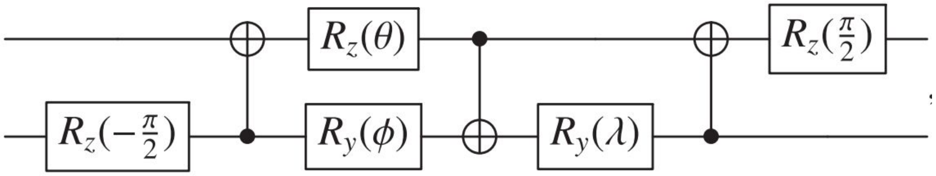
Implementation (4)

- **Step 3**: implement A_j, B_j and C_j
  - For A_j, use the R_z gate.

$$\boxed{R_z(\theta)} = e^{-i\frac{\theta}{2}Z} = \begin{pmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\frac{\theta}{2}} \end{pmatrix}$$

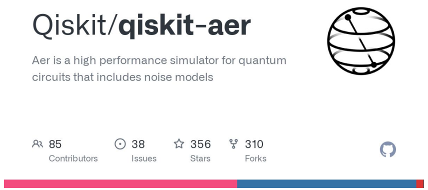
  - For B_j and C_j:

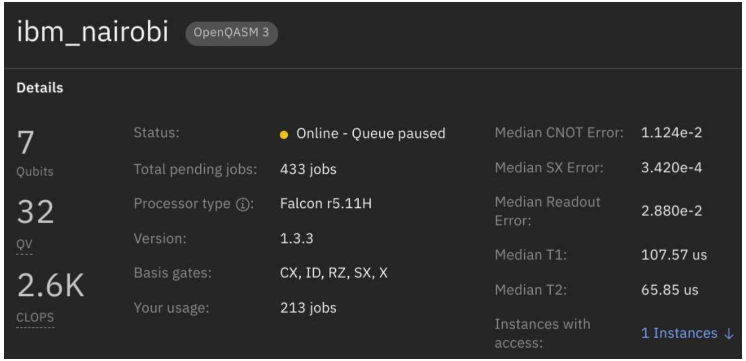
$$N(\alpha, \beta, \gamma) = \exp[i\,(\alpha\sigma^x \otimes \sigma^x + \beta\sigma^y \otimes \sigma^y + \gamma\sigma^z \otimes \sigma^z)].$$

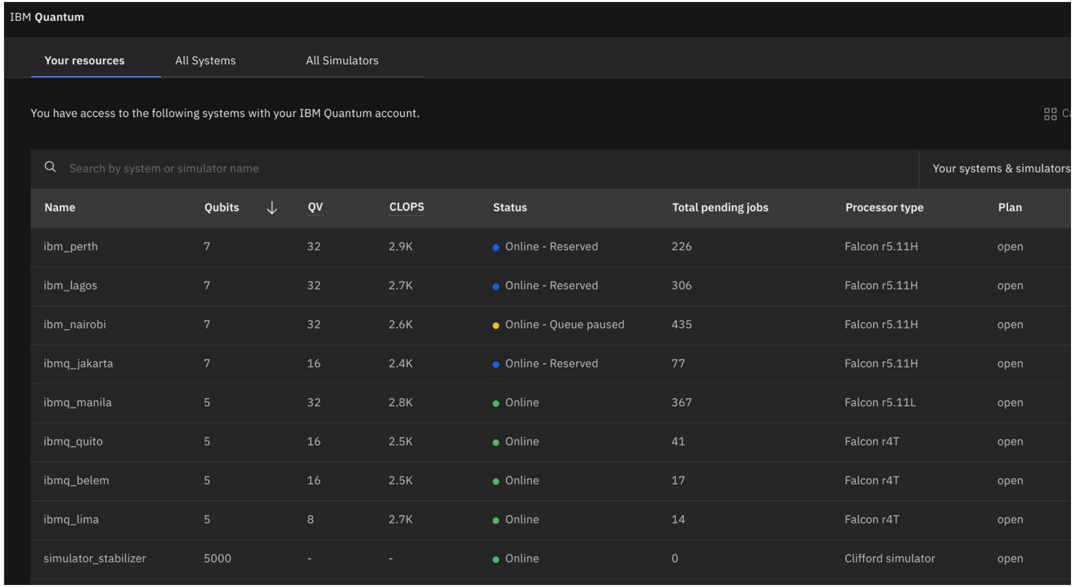


$$\theta = \frac{\pi}{2} - 2\gamma,\ \phi = 2\alpha - \frac{\pi}{2},\ \text{and}\ \lambda = \frac{\pi}{2} - 2\beta.$$

Implementation (5)

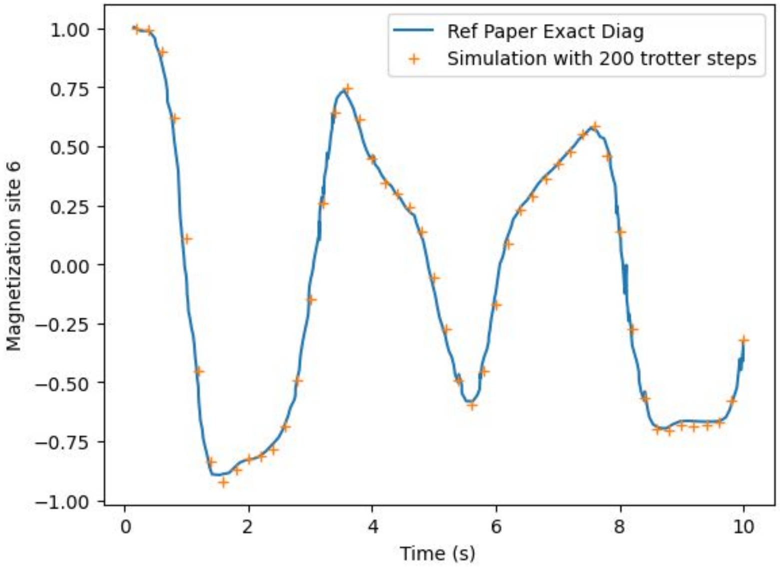






Name	Qubits	QV	CLOPS	Status	Total pending jobs	Processor type	Plan
ibm_perth	7	32	2.9K	Online - Reserved	226	Falcon r5.11H	open
ibm_lagos	7	32	2.7K	Online - Reserved	306	Falcon r5.11H	open
ibm_nairobi	7	32	2.6K	Online - Queue paused	435	Falcon r5.11H	open
ibmq_jakarta	7	16	2.4K	Online - Reserved	77	Falcon r5.11H	open
ibmq_manila	5	32	2.8K	Online	367	Falcon r5.11L	open
ibmq_quito	5	16	2.5K	Online	41	Falcon r4T	open
ibmq_belem	5	16	2.5K	Online	17	Falcon r4T	open
ibmq_lima	5	8	2.7K	Online	14	Falcon r4T	open
simulator_stabilizer	5000	-	-	Online	0	Clifford simulator	open

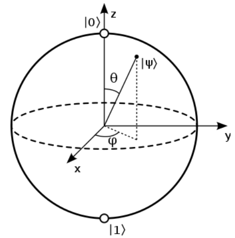
Results (1) - Verification of results with reference

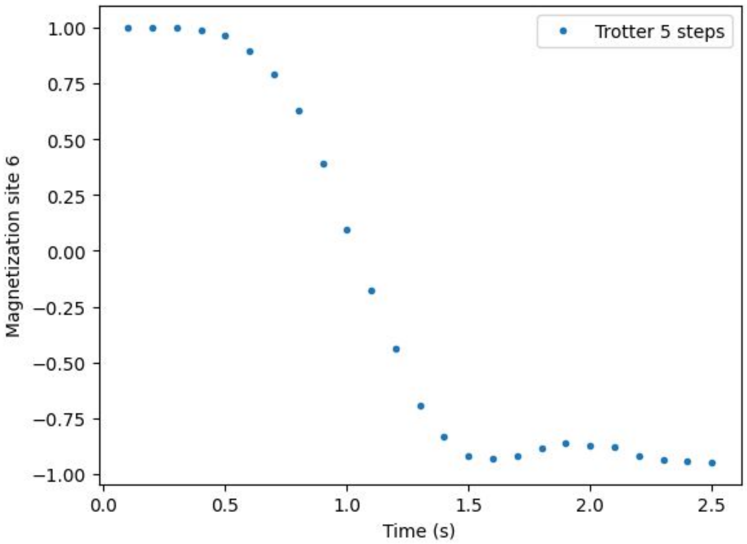


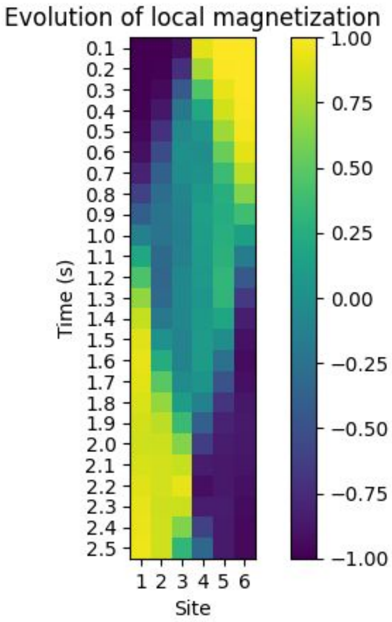
Comparison of simulator and Reference Paper results

Note: For each final time value, a fixed number of trotter steps is done in this analysis.

Results (2) - XX chain - Simulator





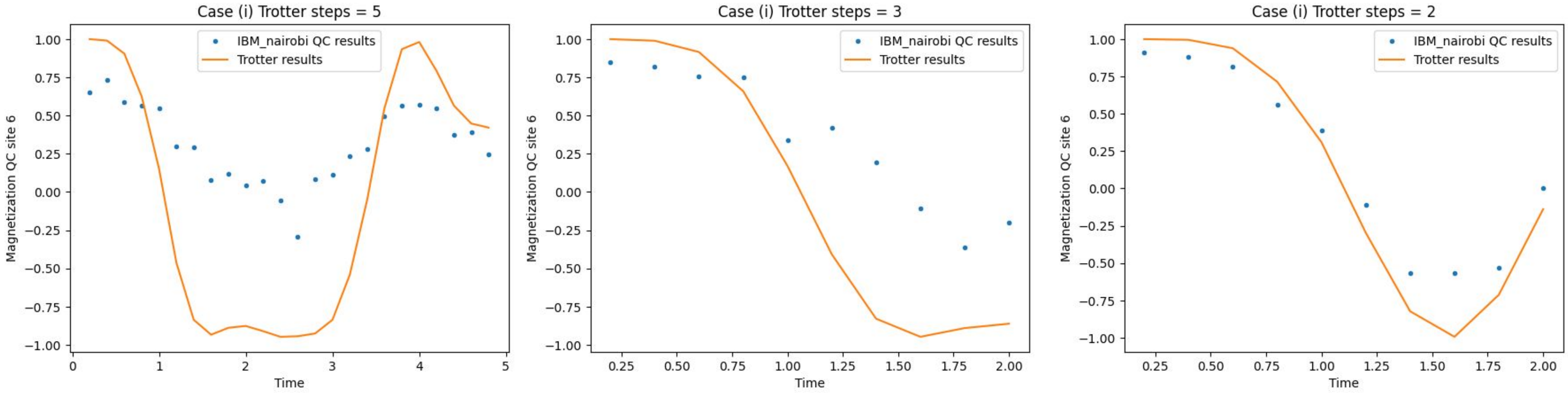


Local Magnetization at site 6 run on simulator

$$\hat{H} = -J\sum_{j=1}^{N-1}\left(\hat{\sigma}_j^x\hat{\sigma}_{j+1}^x + \hat{\sigma}_j^y\hat{\sigma}_{j+1}^y\right) + \cancel{U\sum_{j=1}^{N-1}\hat{\sigma}_j^z\hat{\sigma}_{j+1}^z} + \cancel{\sum_{j=1}^{N} h_j\hat{\sigma}_j^z},$$

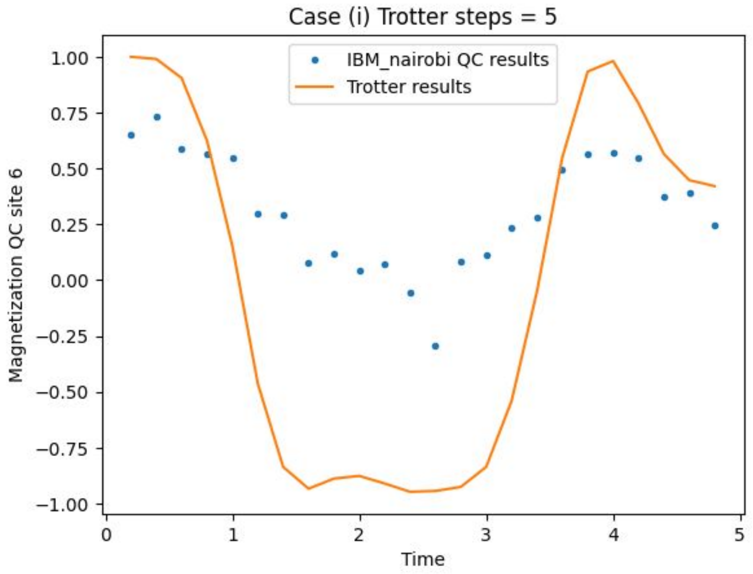
Results (3) - XX chain - IBM 6 Qubit QC

- Influence of Trotter steps
    - Circuit with low depth -> More accurate

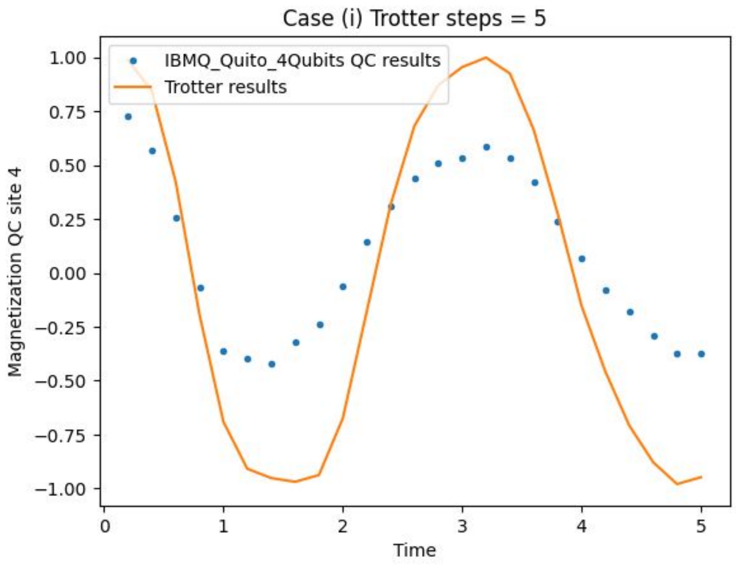


Results (3) - XX chain - IBM QC

- Influence of number of qubits

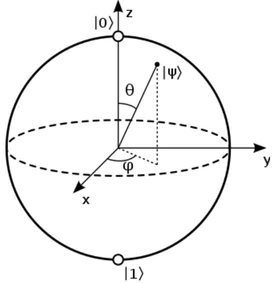


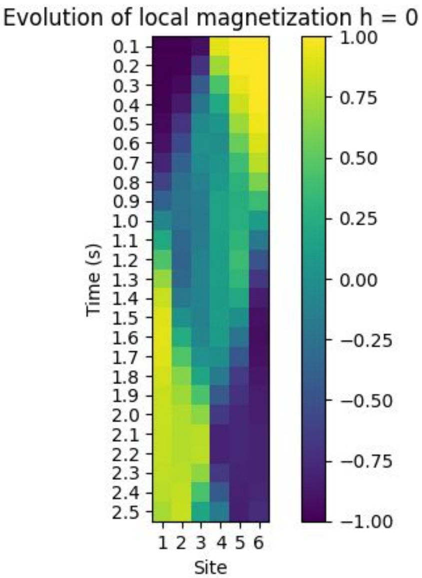
6 qubit system

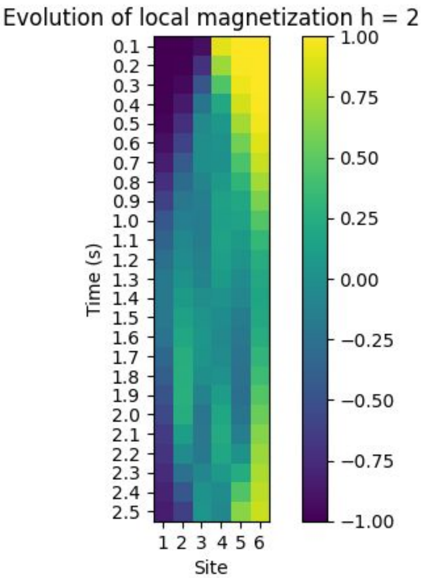


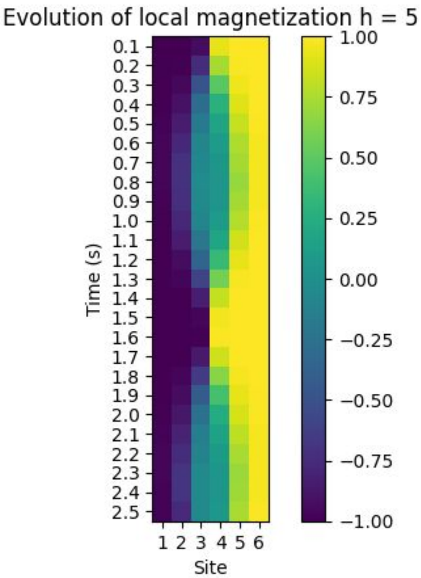
4 qubit system

Results (4) - Disordered XX - (1)





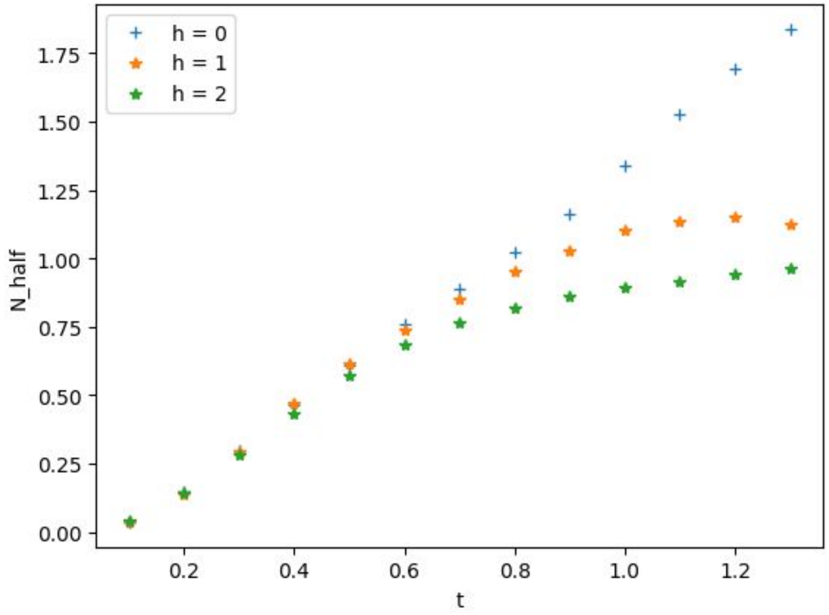




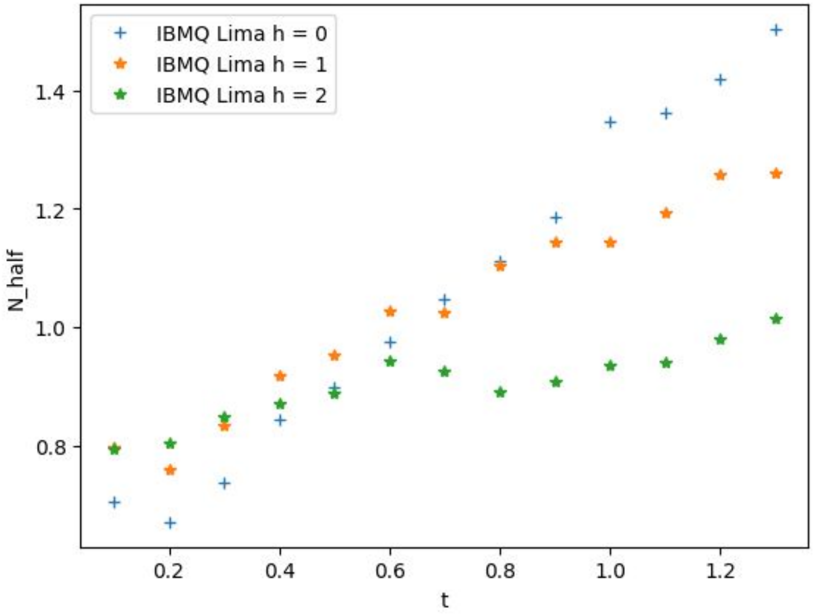
$$\hat{H} = -J \sum_{j=1}^{N-1} \left(\hat{\sigma}_j^x \hat{\sigma}_{j+1}^x + \hat{\sigma}_j^y \hat{\sigma}_{j+1}^y \right) + U \cancel{\sum_{j=1}^{N-1} \hat{\sigma}_j^z \hat{\sigma}_{j+1}^z} + \sum_{j=1}^{N} h_j \hat{\sigma}_j^z,$$

$$N_{\mathrm{half}}(t) = \sum_{j=1}^{N/2} \langle \psi(t) | \frac{\hat{\sigma}_j^z + 1}{2} | \psi(t) \rangle$$

Results (4) - Disordered XX - (2)



Simulator 4 qubits



IBM 4 qubit Quantum Computer

Summary

- IBM quantum computers have low quantitative accuracy
- Accuracy improved with reduced circuit depths (lower evolution times) and lesser qubits (smaller many body system)
- However, they can still show qualitative physical behavior
- Need for improved quality of machines over increasing number of qubits.

References

1) ***Smith, A., Kim, M.S., Pollmann, F. et al. Simulating quantum many-body dynamics on a current digital quantum computer. npj Quantum Inf 5, 106 (2019). https://doi.org/10.1038/s41534-019-0217-0***
2) *Vatan, F. & Williams, C. Optimal quantum circuits for general two-qubit gates. Phys. Rev. A 69, 032315 (2004)*
3) *Hatano, N. & Suzuki, M. Finding exponential product formulas of higher orders. Lect.Notes Phys 679, 37–68 (2005)*
4) *IBM Quantum. https://quantum-computing.ibm.com/, 2023*

THANK YOU.
ANY QUESTIONS?