

Simulating Quantum Many Body Dynamics on a current Quantum Computer

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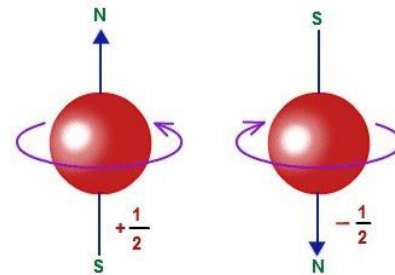
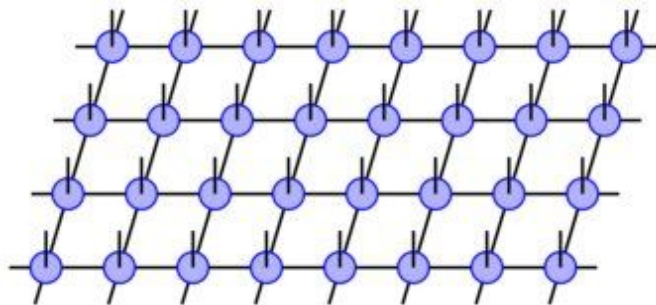
Computational Science and Engineering

Advised by: Richard Milbradt

Content

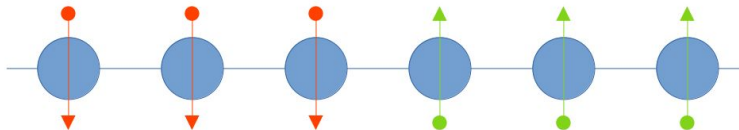
- Quantum Many Body Dynamics
- Setup
- Implementation
- Results
- Summary

Quantum Many Body Dynamics (1) - Introduction



Goal of this study:

To simulate evolution of a 1D spin- $\frac{1}{2}$ chain on a current quantum computer.



Setup (1)

- Governing equation: Schrödinger equation

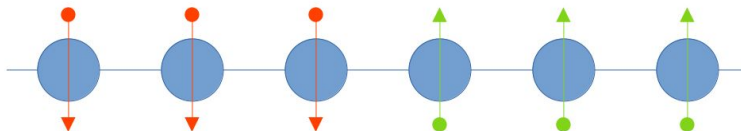
$$i\hbar \frac{d}{dt} |\psi(t)\rangle = H |\psi(t)\rangle \quad |\psi(t)\rangle = U_t |\psi(0)\rangle \quad \text{with} \quad U_t = e^{-iHt/\hbar}.$$

- General Hamiltonian for quantum many body physics

$$\hat{H} = -J \sum_{j=1}^{N-1} \left(\hat{\sigma}_j^x \hat{\sigma}_{j+1}^x + \hat{\sigma}_j^y \hat{\sigma}_{j+1}^y \right) + U \sum_{j=1}^{N-1} \hat{\sigma}_j^z \hat{\sigma}_{j+1}^z + \sum_{j=1}^N h_j \hat{\sigma}_j^z,$$

Setup (2)

- 2 distinct cases for Hamiltonian parameters
 - Case (i) - XX chain - $U = 0, h_j = 0$
 - Case (ii) - Disordered XX chain - $U = 0, h_j$ sampled from $[-h, +h]$
- Initial State: 6 particle domain wall - $|111000\rangle$



- Observables
 - Local Magnetization $M_j(t) = \langle \psi(t) | \hat{\sigma}_j^z | \psi(t) \rangle$.
 -

$$N_{\text{half}}(t) = \sum_{j=1}^{N/2} \langle \psi(t) | \frac{\hat{\sigma}_j^z + 1}{2} | \psi(t) \rangle$$

Implementation - (1)

- Trotter decomposition of Unitary time evolution operator $U(t) = e^{-i\hat{H}t}$
- Idea: Break down complex operator to a sequence of simpler operators

$$\hat{H} = \hat{A} + \hat{B},$$

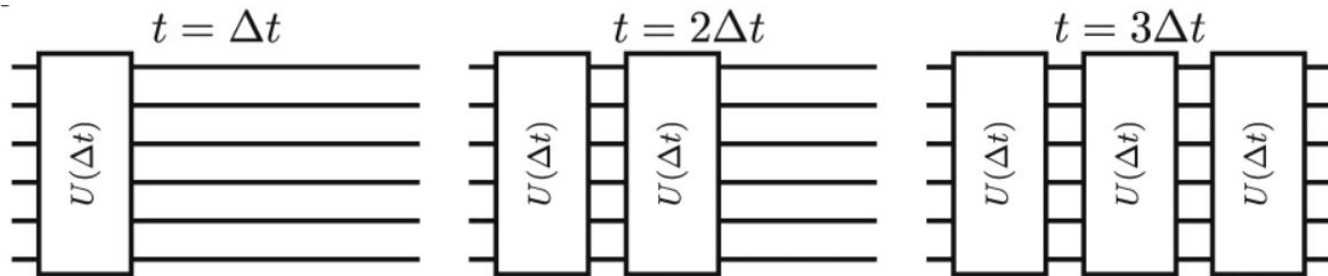
$$e^{-i\hat{H}t} = e^{-i\hat{A}t}e^{-i\hat{B}t} + \mathcal{O}(t^2),$$

- Baker–Campbell–Hausdorff formula,

$$e^X e^Y = e^Z \quad Z = X + Y + \frac{1}{2}[X, Y] + \frac{1}{12}[X, [X, Y]] - \frac{1}{12}[Y, [X, Y]] + \dots$$

Implementation - (2)

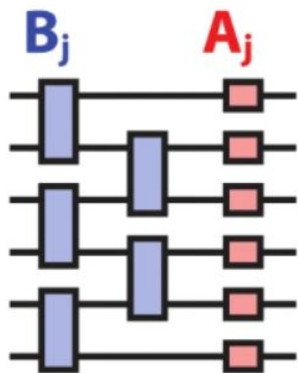
- **Step 1:** Split operator into discrete evolution operators $e^{-i\hat{H}t} = \left(e^{-i\hat{H}\Delta t}\right)^M$



Implementation - (3)

$$\hat{H} = -J \sum_{j=1}^{N-1} \left(\hat{\sigma}_j^x \hat{\sigma}_{j+1}^x + \hat{\sigma}_j^y \hat{\sigma}_{j+1}^y \right) + U \sum_{j=1}^{N-1} \hat{\sigma}_j^z \hat{\sigma}_{j+1}^z + \sum_{j=1}^N h_j \hat{\sigma}_j^z,$$

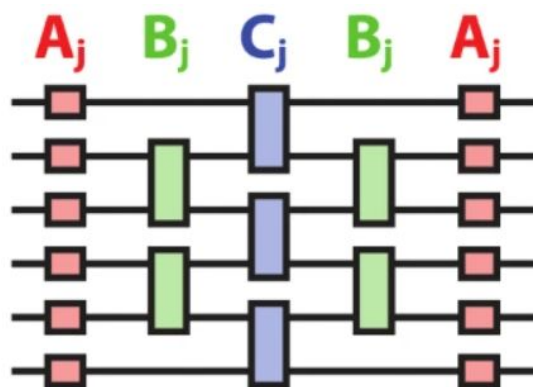
- **Step 2:** Approximate the discrete evolution operators - 2 options:



Basic

If $H = A + B$

$$e^{-iH\Delta t} = e^{-iA\Delta t} e^{-iB\Delta t} + O(\Delta t^2)$$



Symmetric

If $H = A + B$

$$e^{-iH\Delta t} = e^{-iA\Delta t/2} e^{-iB\Delta t} e^{-iA\Delta t/2} + O(\Delta t^3)$$

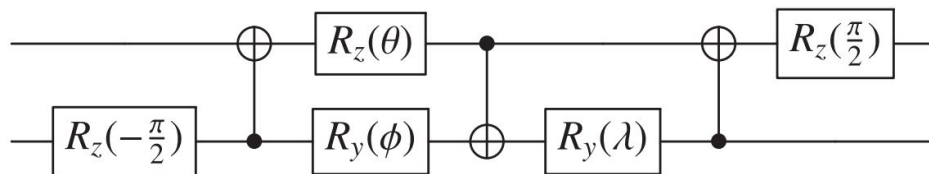
Implementation (4)

- **Step 3:** implement A_j , B_j and C_j
 - For A_j , use the R_z gate.

$$\boxed{R_z(\theta)} = e^{-i\frac{\theta}{2}Z} = \begin{pmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\frac{\theta}{2}} \end{pmatrix}$$

- For B_j and C_j :

$$N(\alpha, \beta, \gamma) = \exp[i(\alpha\sigma^x \otimes \sigma^x + \beta\sigma^y \otimes \sigma^y + \gamma\sigma^z \otimes \sigma^z)].$$



$$\theta = \frac{\pi}{2} - 2\gamma, \phi = 2\alpha - \frac{\pi}{2}, \text{ and } \lambda = \frac{\pi}{2} - 2\beta.$$

Implementation (5)



Qiskit/qiskit-aer

Aer is a high performance simulator for quantum circuits that includes noise models



85 Contributors 38 Issues 356 Stars 310 Forks



ibm_nairobi OpenQASM 3

Details

7
Qubits

32
QV

2.6K
CLOPS

Status: ● Online - Queue paused

Total pending jobs: **433 jobs**

Processor type ⓘ: **Falcon r5.11H**

Version: **1.3.3**

Basis gates: **CX, ID, RZ, SX, X**

Your usage: **213 jobs**

Median CNOT Error: **1.124e-2**

Median SX Error: **3.420e-4**

Median Readout Error: **2.880e-2**

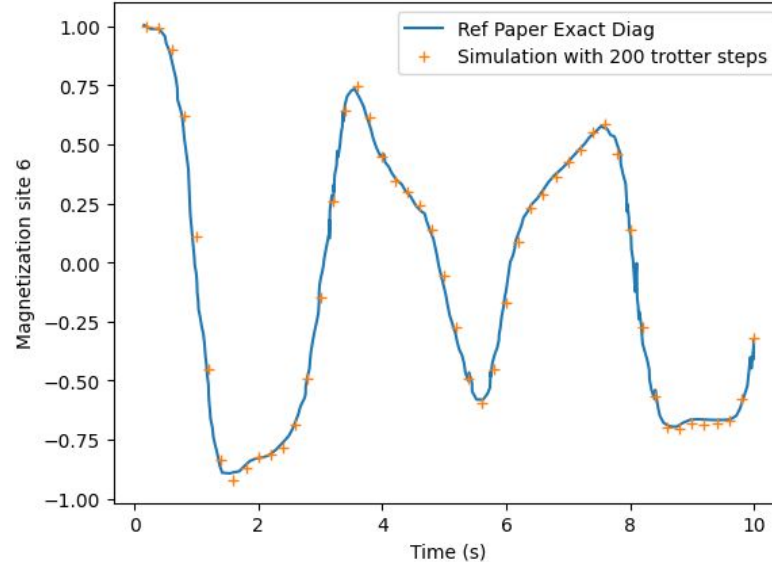
Median T1: **107.57 us**

Median T2: **65.85 us**

Instances with access: **1 Instances** ↓

IBM Quantum								
Your resources All Systems All Simulators								
You have access to the following systems with your IBM Quantum account.								
Search by system or simulator name								Your systems & simulators
Name	Qubits	↓	QV	CLOPS	Status	Total pending jobs	Processor type	Plan
ibmq_perth	7		32	2.9K	● Online - Reserved	226	Falcon r5.11H	open
ibmq_lagos	7		32	2.7K	● Online - Reserved	306	Falcon r5.11H	open
ibmq_nairobi	7		32	2.6K	● Online - Queue paused	435	Falcon r5.11H	open
ibmq_jakarta	7		16	2.4K	● Online - Reserved	77	Falcon r5.11H	open
ibmq_manila	5		32	2.8K	● Online	367	Falcon r5.11L	open
ibmq_quito	5		16	2.5K	● Online	41	Falcon r4T	open
ibmq_belem	5		16	2.5K	● Online	17	Falcon r4T	open
ibmq_lima	5		8	2.7K	● Online	14	Falcon r4T	open
simulator_stabilizer	5000		-	-	● Online	0	Clifford simulator	open

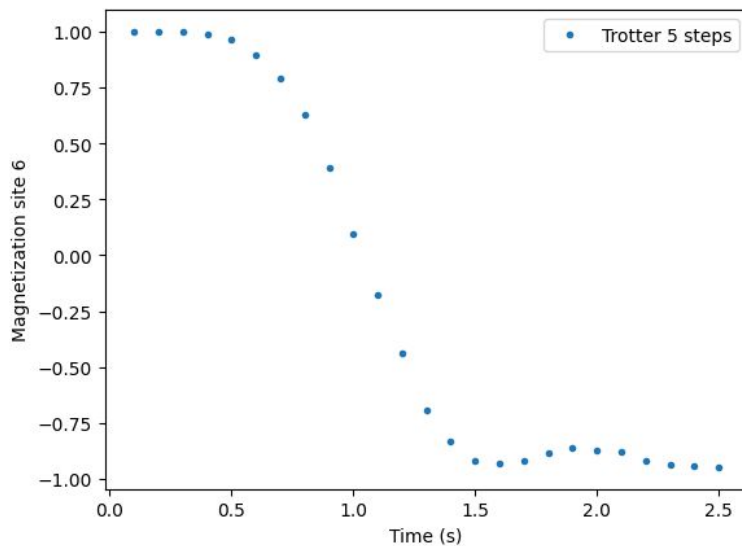
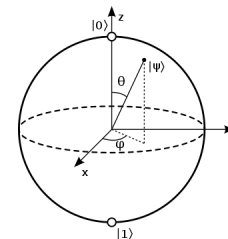
Results (1) - Verification of results with reference



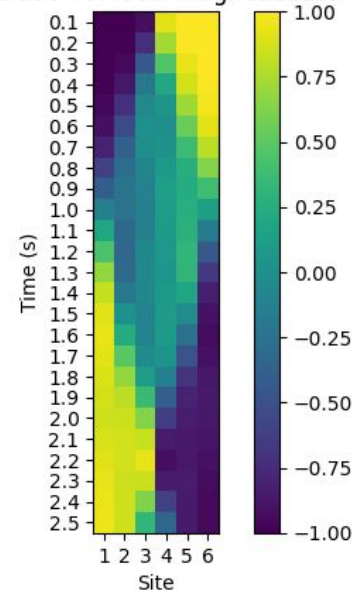
Comparison of simulator and Reference Paper results

Note: For each final time value, a fixed number of trotter steps is done in this analysis.

Results (2) - XX chain - Simulator



Evolution of local magnetization

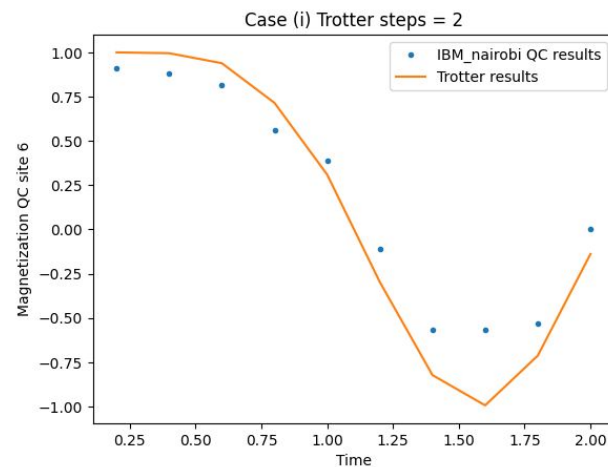
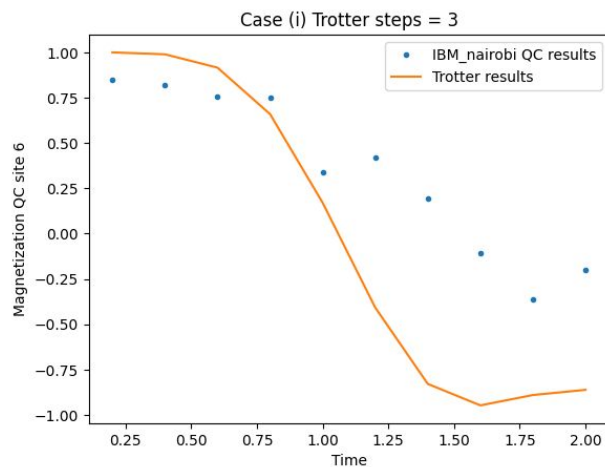
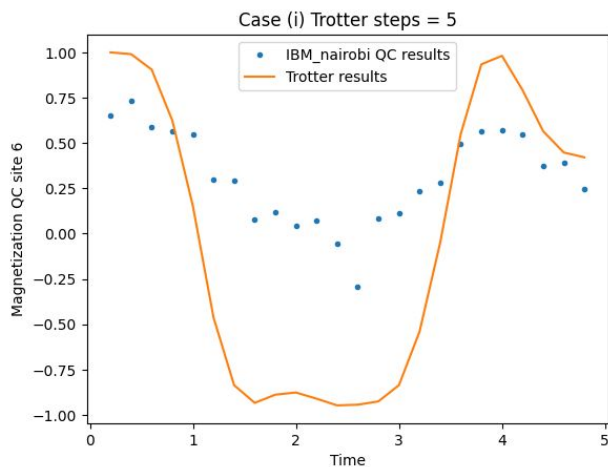


Local Magnetization at site 6 run on simulator

$$\hat{H} = -J \sum_{j=1}^{N-1} (\hat{\sigma}_j^x \hat{\sigma}_{j+1}^x + \hat{\sigma}_j^y \hat{\sigma}_{j+1}^y) + U \sum_{j=1}^{N-1} \hat{\sigma}_j^z \hat{\sigma}_{j+1}^z + \sum_{j=1}^N h_j \hat{\sigma}_j^z,$$

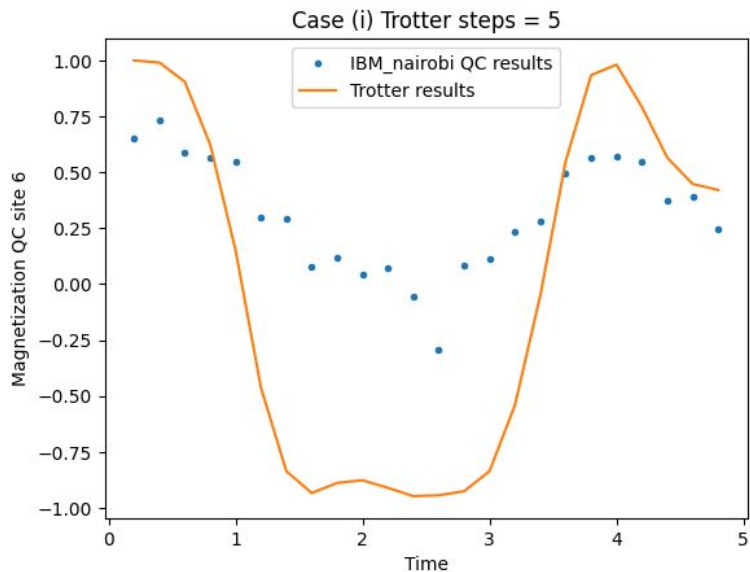
Results (3) - XX chain - IBM 6 Qubit QC

- Influence of Trotter steps
 - Circuit with low depth -> More accurate

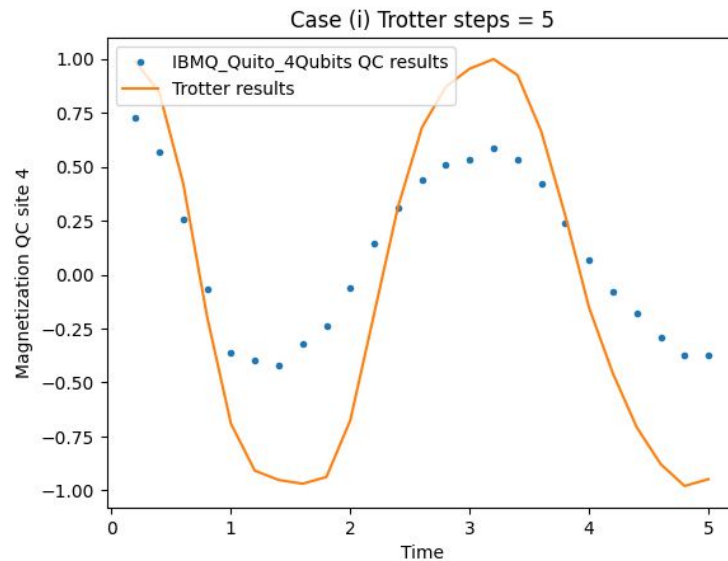


Results (3) - XX chain - IBM QC

- Influence of number of qubits

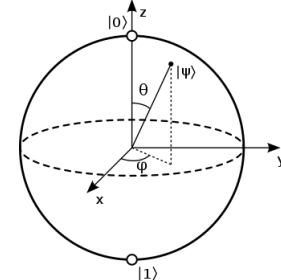


6 qubit system

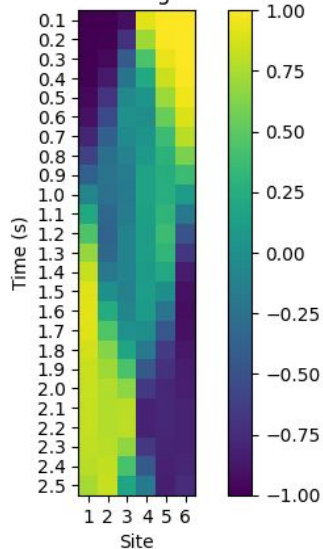


4 qubit system

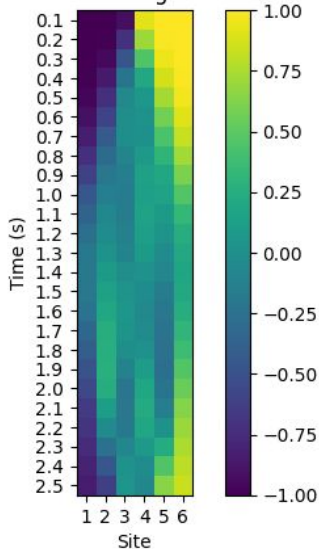
Results (4) - Disordered XX - (1)



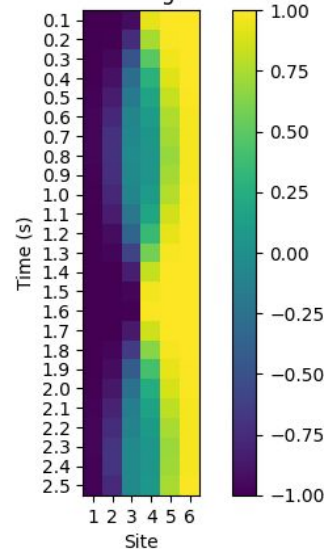
Evolution of local magnetization $h = 0$



Evolution of local magnetization $h = 2$



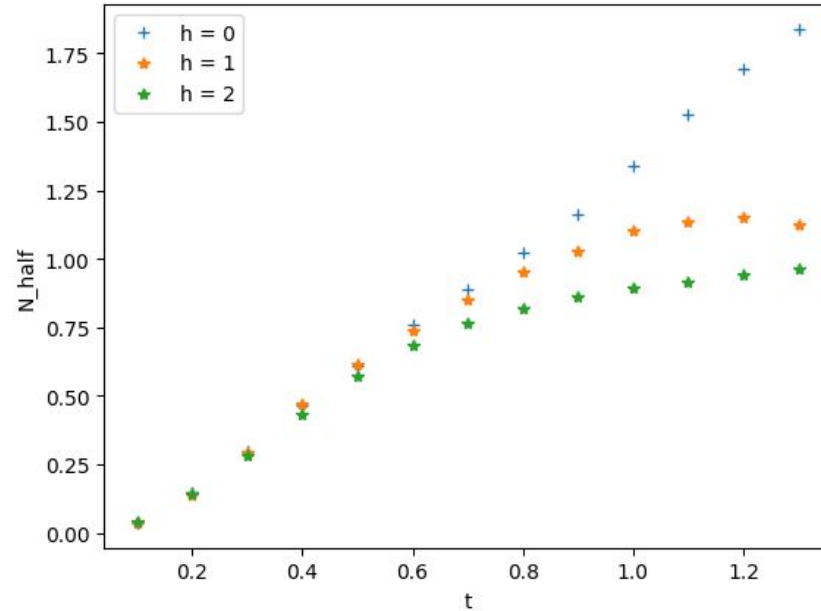
Evolution of local magnetization $h = 5$



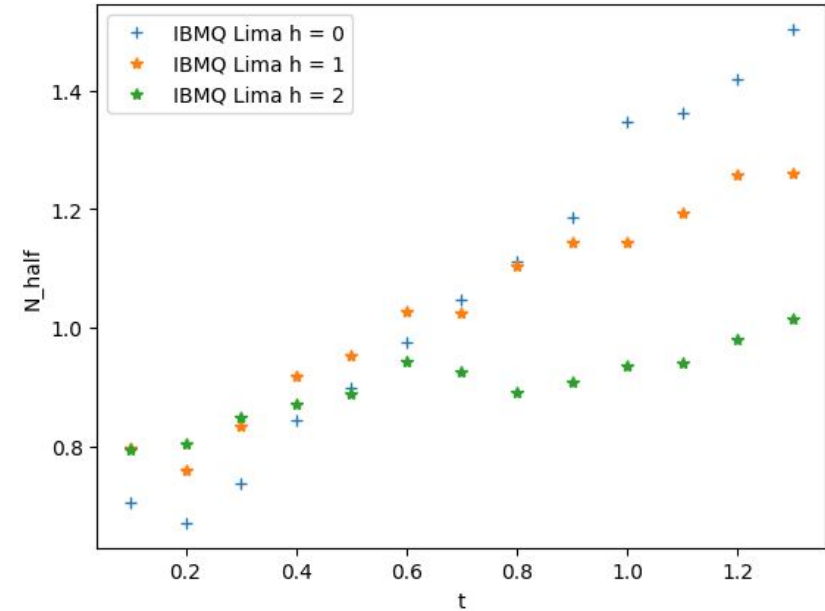
$$\hat{H} = -J \sum_{j=1}^{N-1} (\hat{\sigma}_j^x \hat{\sigma}_{j+1}^x + \hat{\sigma}_j^y \hat{\sigma}_{j+1}^y) + U \sum_{j=1}^{N-1} \hat{\sigma}_j^z \hat{\sigma}_{j+1}^z + \sum_{j=1}^N h_j \hat{\sigma}_j^z,$$

$$N_{\text{half}}(t) = \sum_{j=1}^{N/2} \langle \psi(t) | \frac{\hat{\sigma}_j^z + 1}{2} | \psi(t) \rangle$$

Results (4) - Disordered XX - (2)



Simulator 4 qubits



IBM 4 qubit Quantum Computer

Summary

- IBM quantum computers have low quantitative accuracy
- Accuracy improved with reduced circuit depths (lower evolution times) and lesser qubits (smaller many body system)
- However, they can still show qualitative physical behavior
- Need for improved quality of machines over increasing number of qubits.

References

- 1) **Smith, A., Kim, M.S., Pollmann, F. et al. *Simulating quantum many-body dynamics on a current digital quantum computer. npj Quantum Inf 5, 106 (2019). <https://doi.org/10.1038/s41534-019-0217-0>***
- 2) *Vatan, F. & Williams, C. Optimal quantum circuits for general two-qubit gates. Phys. Rev. A 69, 032315 (2004)*
- 3) *Hatano, N. & Suzuki, M. Finding exponential product formulas of higher orders. Lect.Notes Phys 679, 37–68 (2005)*
- 4) *IBM Quantum. <https://quantum-computing.ibm.com/>, 2023*

THANK YOU.
ANY QUESTIONS?