1 Formulation

Notations:

- I: {1, 2, 3.. n, n+1} set of n nodes, where n+1 is copy of depot node
- K = number of drones
- $dist_{ij}$: euclidean distance from all the nodes to every node in I

Decision variables:

- x_{ij}: $\begin{cases} 1 \; if \; arc \; from \; i \; to \; j \; is \; selected \; by \; the \; truck \\ 0 \; otherwise \end{cases} \quad \forall \; i, j \in I$
- y_{ij}: $\begin{cases} 1 \; if \; arc \; from \; i \; to \; j \; is \; selected \; by \; the \; drone \\ 0 \; otherwise \end{cases} \quad \forall \; i, j \in I$
- pos_i: position of node i if it comes in route of the truck
- C_t: cost of running the truck for per unit distance, in dollars
- C_d: 2 times cost of running a drone for per unit distance (as it has to come back to launch site), in dollars

Objective Function:

- $$\sum_{i \in I} \sum_{j \in I} x_{ij} * dist_{ij} * C_t + \sum_{i \in I} \sum_{j \in I} y_{ij} * dist_{ij} * C_d$$

Constraints:

Capacity Constraint:

- $\sum_{i \in I} x_{ij} + \sum_{i \in I} y_{ij} = 1 \;\; \forall \; j \in I \backslash \{1, n+1\}$

Truck Constraint:

- $x_{ii} = 0 \;\; \forall \; i \in I$
- $\sum_{j \in I} x_{ij} <= 1 \;\; \forall \; i \in I$
- $\sum_{j \in I} x_{ji} <= 1 \;\; \forall \; i \in I$
- $\sum_{j \in I} x_{ij} = \sum_{j \in I} x_{ji} \; \forall \; i \in I \backslash \{1, n+1\}$
- $x_{ij} + x_{ji} <= 1 \;\; \forall \; i, j \in I$

- $\sum_{j \in I} x_{1j} = 1$
- $\sum_{j \in I} x_{j1} = 0$
- $\sum_{j \in I} x_{j(n+1)} = 1$
- $\sum_{j \in I} x_{(n+1)j} = 0$

Drone Constraint:

- $y_{ii} = 0 \ \ \forall\ i \in I$
- $K * \sum_{i \in I} x_{ij} >= \sum_{i \in I} y_{ji} \ \ \forall\ j \in I$
- $y_{ij} + y_{ji} <= 1 \ \ \forall\ i, j \in I$
- $\sum_{j \in I} y_{1j} = 0$
- $\sum_{j \in I} y_{j1} = 0$
- $\sum_{j \in I} y_{j(n+1)} = 0$
- $\sum_{j \in I} y_{(n+1)j} = 0$

Position Constraint:

- $pos_i <= i_{(n+1)} \ \ \forall\ i \in I$
- $pos_i >= 1 \ \ \forall\ i \in I$
- $pos_j >= pos_i + 1 - M(1 - x_{ij})\ \forall\ i, j \in I$

Variable Constraint:

- $pos_i \in Z^+ \ \ \forall\ i \in I$
- $x_{ij} \in \{0, 1\} \ \ \forall\ i, j \in I$
- $y_{ij} \in \{0, 1\} \ \ \forall\ i, j \in I$

Model:

$$min\ (\sum_{i \in I} \sum_{j \in I} x_{ij} * dist_{ij} * C_t + \sum_{i \in I} \sum_{j \in I} y_{ij} * dist_{ij} * C_d)$$