PROBLEM SHEET 2

Linear Algebra-CSD001P5M

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1. Show that $det(A) = 0$, where $A := \begin{bmatrix} sin\alpha & cos\alpha & sin(\alpha+\delta) \\ sin\beta & cos\beta & sin(\beta+\delta) \\ sin\gamma & cos\gamma & sin(\gamma+\delta) \end{bmatrix}$

2. If the entries in each row of matrix A add upto zero, then $det(A) = 0$.

3. Let A be a matrix of size $n \times n$ with integer entries. Prove that A is invertible, and that its inverse A^{-1} has integer entries if and only if $det(A) = \pm 1$.

4. Use the fact that 21375, 38798, 34162, 40223 and 79154 are all divisible by 19 to show that $det(A)$ is divisible by 19 , where $A := \begin{bmatrix} 2 & 1 & 3 & 7 & 5 \\ 3 & 8 & 7 & 9 & 8 \\ 3 & 4 & 1 & 6 & 2 \\ 4 & 0 & 2 & 2 & 3 \\ 7 & 9 & 1 & 5 & 4 \end{bmatrix}$ (Also you can use, if a prime number p divides a product ab of integers,then p divides a or p divides b).

5. Find the solution of the following system using Cramer's rule

$$\begin{cases} x_1 + 2x_2 - 3x_3 = -4 \\ 4x_1 - x_2 + 2x_3 = 8 \\ 2x_1 + 2x_2 - 3x_3 = -3 \end{cases}$$

6. Let A be a square matrix of size $n \times n$, then $det(adj(A)) = [det(A)]^{n-1}$.

7. Prove that $det \begin{bmatrix} A & B \\ 0 & D \end{bmatrix} = det(A)det(D)$, where A and D are square block matrices.