Errors in Row Reduction Sheet (Linear Algebra)

We can prove previous theorem another way:

Suppose for each $i = 1, 2, \ldots, k$, E_i is a corresponding elementary matrix for the elementary row operation e_i. We have $E_k E_{k-1} \ldots E_1[A|b] = [C|d]$, $F := E_k E_{k-1} \cdots E_1$ is an invertible matrix as each E_i is invertible. Therefore $[FA|Fb] = F[A|b] = [C|d]$ and $FA = C$ and $Fb = d$.

Now consider the systems of linear equations $Ax = b$ (S.L.E-I) and $Cx = d$ (S.L.E.-II). Suppose y is a solution for the system (S.L.E-I), then $Ay = b$. If we multiply the matrix equation $Ay = b$ by F from left, we have $FAy = Fby$. Hence $Cy = d$ and this shows that y is also a solution for the system $Cx = d$. Therefore the set of solutions for the system $Ax = b$ is a subset of the set of solutions for the system $Cx = d$.

Now if z is a solution for the system (S.L.E.-II), then $Cz = d$. If we multiply the matrix equation $Cy = d$ by F^{-1} from left, we have $F^{-1}Cy = F^{-1}d$. Hence $Az = b$ and this shows that z is also a solution for the system $Ax = b$. Therefore the set of solutions for the system $Cx = d$ is a subset of the set of solutions for the system $Ax = b$. If one system does not have any solution, then another system also does have any solution. Hence the set of solutions for the system $Cx = d$ is same as the set of solutions for the system $Ax = b$.

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① "Now consider the systems of linear equations $Ax=b$ (SLE I) and $Cx=d$ (SLE-II). Suppose y is a solution for the systems (SLE I), then $Ay=b$. If we multiply the matrix equation $Ay=b$ by F from left, we have $\boxed{FAy = Fby}$

Error:- It should be $\boxed{FAy = Fb}$

If we multiply $Ay=b$ by F from left $\boxed{FAy=Fb}$ only.

Some examples of row-reduced echelon matrices:

$$1.\ \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\ 2.\ \begin{bmatrix}1&0&2&0&1\\0&1&0&0&1\\0&0&0&1&9\\0&0&0&0&0\end{bmatrix}\ 3.\ \begin{bmatrix}0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\end{bmatrix}\ 4.\ \begin{bmatrix}0&1&5&0&0\\0&0&0&1&1\end{bmatrix}$$

$$5.\ \begin{bmatrix}0&0&0&0&1&0&0\\0&0&0&0&0&1&0\\0&0&0&0&0&0&0\end{bmatrix}\ 6.\ \begin{bmatrix}1&0&0&0\\0&1&1&0\\0&0&0&1\end{bmatrix}\ 7.\ \begin{bmatrix}0&1&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix}$$

Some examples of matrices which are not row-reduced echelon matrices:

$$1.\ \begin{bmatrix}1&1&0\\0&0&0\\0&1&0\end{bmatrix}\ 2.\ \begin{bmatrix}1&5&7&0\\0&1&0&1\\0&0&0&0\end{bmatrix}\ 3.\ \begin{bmatrix}0&1\\1&0\end{bmatrix}\ 4.\ \begin{bmatrix}1&6&3&0\\0&1&0&0\\1&0&1&1\end{bmatrix}\ 5.\ \begin{bmatrix}1&2&8&0&3\end{bmatrix}$$

$$6.\ \begin{bmatrix}1&0&0&0\\0&2&0&0\\0&0&0&0\end{bmatrix}\ 7.\ \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&2\\0&0&0&1\end{bmatrix}$$

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Example No -(5) $[1\ \ 2\ \ 8\ \ 0\ \ 3]$

It should be row-reduced Echelon matrix

conditions :-

A matrix R of size $m \times n$ is called, row reduce Echelon matrix –

(i) R is row reduced

(ii) Every row of R which has all its entries 0 occurs below every row which has a non-zero entry.

(iii) if rows $1 - - r$, are the non zero entry of row i occurs in columns k_i, $i = 1 - r$ then $k_1 < k_2 < - - k_r$.

$[1\ \ 2\ \ 8\ \ 0\ \ 3]$ is satisfying all. I guess.

Theorem 0.8. *If A is a matrix of size $m \times n$ and $m < n$, then the homogeneous system of linear equations $Ax = 0$ has a non-zero solution.*

Proof. Let R be a row-reduced echelon matrix which is row-equivalent to A. Then the systems of linear equations $Ax = 0$ and $Rx = 0$ are equivalent and they have same set of solutions. Suppose r is the number of non-zero rows in R, then we have $r \leq m < n$. Let the leading non-zero entry of i-th row of R occurs in column k_i for $i = 1, 2, \ldots, r$. There are r non-zero equations of the system $Rx = 0$, where the unknown x_{k_i} will only appear with non-zero coefficient 1 in the i-th equation. Here $n - r$ is positive integer, let $u_1, u_2, \ldots, u_{n-r}$ denote the $n-r$ unknowns (free-variables), other than $x_{k_1}, x_{k_2}, \ldots, x_{k_r}$. Then the r non-zero equations in $Rx = 0$ are of the form

$$(S.L.E, Rx = 0)\begin{cases} x_{k_1} + c_{11}u_1 + c_{12}u_2 + \ldots + c_{1n-r}u_j = 0 \\ x_{k_2} + c_{21}u_1 + c_{22}u_2 + \ldots + c_{2n-r}u_j = 0 \\ \ldots \\ x_{k_r} + c_{r1}u_1 + c_{r2}u_2 + \ldots + c_{rn-r}u_j = 0 \end{cases}$$

All the solutions of the system of equations $Rx = 0$ are obtained by assigning any values of $u_1, u_2, \ldots, u_{n-r}$, then find the values of $x_{k_1}, x_{k_r}, \ldots, x_{k_r}$ from (S.L.E, Rx=0). Hence the system $Ax = 0$ has solutions other than zero. □

In view of proof of Theorem 0.8, if $r < n$ then the system $Ax = 0$ has non-zero solution.

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Theorem - 0.8

U_{n-r}

$$(SLE, Rx=0) = \begin{cases} x_{k_1} + c_{11}u_1 + c_{12}u_2 + \ldots + c_{1n-r}u_j = 0 \\ x_{k_2} + c_{21}u_1 + c_{22}u_2 + \ldots + c_{2n-r}u_j = 0 \\ \vdots \\ x_{k_r} + c_{r1}u_1 + c_{r2}u_2 + \ldots + c_{rn-r}u_j = 0 \end{cases}$$

It should be U_{n-r}. according to theorem parameters.

Theorem 0.11. *A square matrix A that has either a left or a right inverse that means there exists a matrix B such that $AB = I$ or $BA = I$. Then A is invertible.*

Proof. Let us assume $AB = I$. Consider the system of linear equations $Bx = 0$, if we multiply the equation by the matrix A from left then we have $x = 0$. This shows that $Bx = 0$ has only zero solution. Hence B is invertible and $A = B^{-1}$. As inverse of an invertible matrix is also invertible, A is invertible.

Now for the case $BA = I$, we have the system of linear equations $Ax = 0$ has only zero solution. Hence A is invertible. $\square$

Let $k \geq 2$ be a positive integer and $A_1, A_2, \ldots, A_k$ be square matrices of same size. If each A_i is invertible then the product $A_1 A_2 \cdots A_k$ is invertible, whereas the converse is not true.

** One application for the system of linear equations: click here .

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let $k \geq 2$ be a positive integer and $A_1, A_2 -- A_k$ be square matrices of same size. If each A_i is invertible then the product $A_1, A_2 -- A_k$ is invertible. whereas the converse is not true.

According to class discussion – Converse of this condition is also true,

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