

Review of Data Structures

Virendra Singh

Professor, Indian Institute of Technology Bombay
And

Adjunct Professor, Indian Institute of Technology Jammu

<http://www.ee.iitb.ac.in/~viren/>

E-mail: viren@ee.iitb.ac.in, virendra.singh@iitjammu.ac.in

CSP L201: Data Organization & Retrieval



Lecture 4 A (23 September 2020)



भारतीय प्रौद्योगिकी
संस्थान जम्मू
INDIAN INSTITUTE OF
TECHNOLOGY JAMMU
विद्यामर्त्यं सर्वधनं ब्रह्मणः

Acknowledgement

- Prof. Sartaj Sahni, Uni. of Florida
- Prof. Hideo Fujiwara, NAIST
- Late Prof. A. Bhattacharya, SERC, IISc



Data Structures

- **Data structure** is a way to store and organize data in order to facilitate access and modification
- No single data structure works well for all purposes
- data object
- set or collection of instances
 - `integer = {0, +1, -1, +2, -2, +3, -3, ...}`
 - `daysOfWeek = {S,M,T,W,Th,F,Sa}`
- instances may or may not be related
 - `myDataObject = {apple, chair, 2, 5.2, red, green, Jack}`



Data Structure

Data object

relationships that exist among instances
and elements that comprise an instance

- Among instances of integer

$$369 < 370$$

$$280 + 4 = 284$$

- The relationships are usually specified by specifying operations on one or more instances.
 - add, subtract, predecessor, multiply



Linear (or Ordered) Lists

- instances are of the form
 - $(e_0, e_1, e_2, \dots, e_{n-1})$
 - where e_i denotes a list element
 - $n \geq 0$ is finite
 - list size is n
- $L = (e_0, e_1, e_2, e_3, \dots, e_{n-1})$
- relationships
 - e_0 is the zero' th (or front) element
 - e_{n-1} is the last element
 - e_i immediately precedes e_{i+1}



Linear List Examples/Instances

Students in EE717 =

(Deepak, Jaidev, , Amit, Abhishek, ..., Vijay)

Exams in EE717 =

(Test1, Midsem, Test 2, Test 3, Final)

Days of Week = (S, M, T, W, Th, F, Sa)

Months = (Jan, Feb, Mar, Apr, ..., Nov, Dec)



Linear List Operations

- **Size ()**
 - determine list size
 - $L = (a, b, c, d, e)$
 - size = 5
- **Get (theIndex)**
 - get element with given index
 - $L = (a, b, c, d, e)$
 - $get(0) = a$
 - $get(2) = c$
 - $get(-1) = \text{error}$
 - $get(9) = \text{error}$



Linear List Operations

- **IndexOf**(theElement)
 - determine the index of an element
 - $L = (a, b, d, b, a)$
 - $indexOf(d) = 2$
 - $indexOf(a) = 0$
 - $indexOf(z) = -1$
- **Remove**(theIndex)
 - remove and return element with given index
 - $L = (a, b, c, d, e, f, g)$
 - $remove(2)$ returns c
 - and L becomes (a, b, d, e, f, g)
 - index of d, e, f , and g decrease by 1



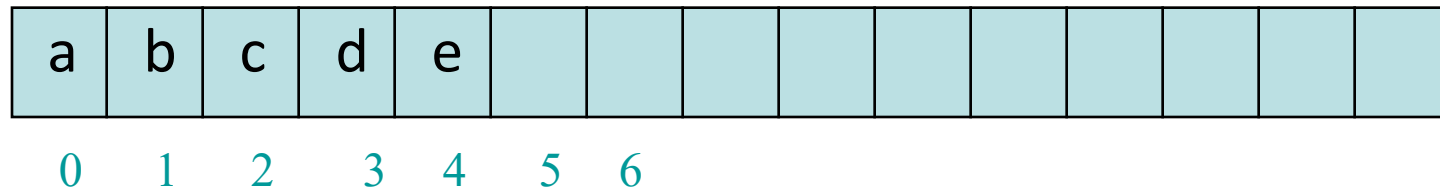
Linear List Operations

- **Add**(theIndex, theElement)
 - add an element so that the new element has a specified index
 - $L = (a, b, c, d, e, f, g)$
 - $add(0, h) \Rightarrow L = (h, a, b, c, d, e, f, g)$
 - index of a, b, c, d, e, f , and g increase by 1
 - $add(10, h) \Rightarrow$ error
 - $add(-6, h) \Rightarrow$ error



Linear List Array Representation

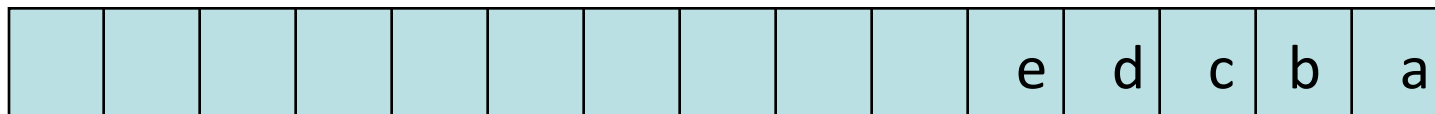
use a one-dimensional array `element[]`



$L = (a, b, c, d, e)$

Store element i of list in `element[i]`.

- Right to left mapping



Linear List Array Representation

- Mapping That Skips Every Other Position

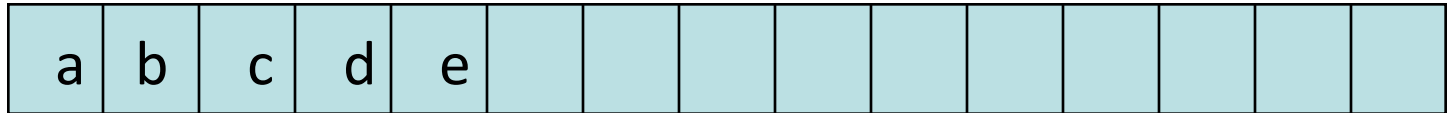


- Wrap Around Mapping



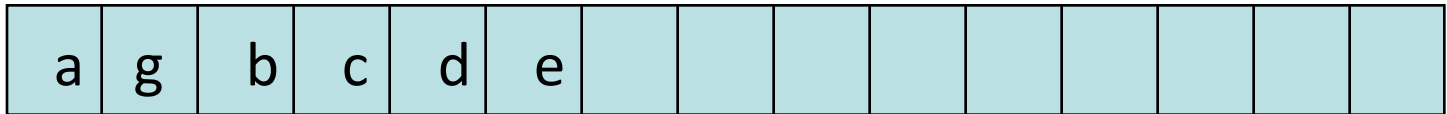
Add/Remove An Element

size = 5



add(1,g)

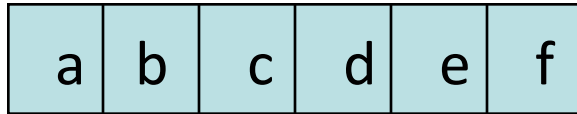
size = 6



Increasing Array Length

- Don't know how many elements will be in list.
- Must pick an initial length and dynamically increase as needed.

Length of array `element[]` is 6.



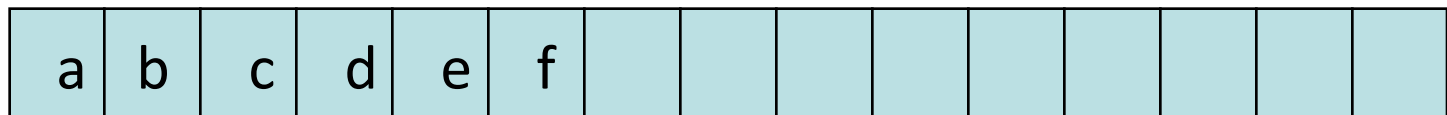
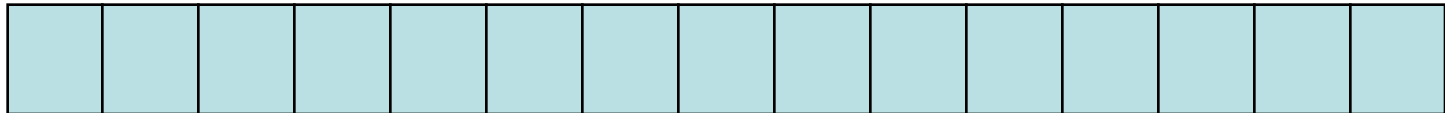
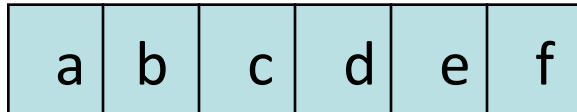
First create a new and larger array

```
newArray = new Object[15];
```



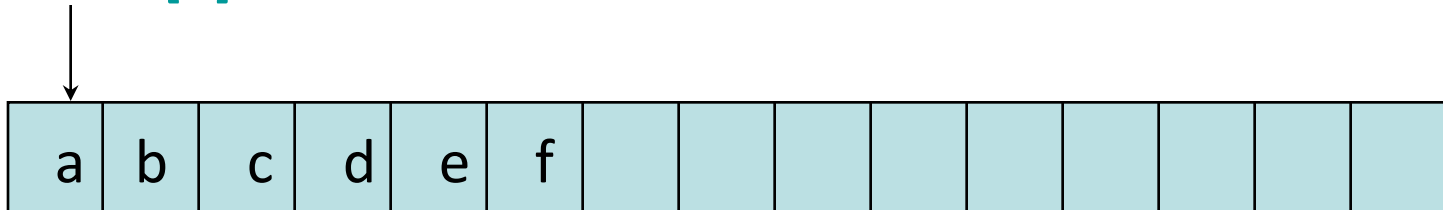
Increasing Array Length

Now copy elements from old array to new one.



Finally, rename new array.

element[0]



How Big Should The New Array Be?

At least **1** more than current array length.

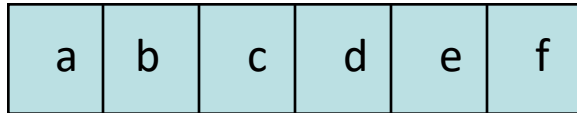
Cost of increasing array length is
Theta(new length)

Cost of **n** add operations done on an initially
empty linear list increases by
Theta(n^2)



Space Complexity

element[6]



newArray = new char[7];



$$\begin{aligned}\text{space needed} &= 2 * \text{newLength} - 1 \\ &= 2 * \text{maxListSize} - 1\end{aligned}$$

Array Doubling

Double the array length.

a	b	c	d	e	f
---	---	---	---	---	---

```
newArray = new char[12];
```

a	b	c	d	e	f						
---	---	---	---	---	---	--	--	--	--	--	--

Time for n adds goes up by $\Theta(n)$.

Space needed = $1.5 * \text{newLength}$.

Space needed $\leq 3 * \text{maxListSize} - 3$



How Big Should The New Array Be?

Resizing by any constant factor

$$\text{new length} = c * \text{old length}$$

increases the cost of n adds by $\Theta(n)$.

Resizing by an additive constant increases the cost of n add operations by $\Theta(n^2)$.



How Big Should The New Array Be?

Resizing by any constant factor

$$\text{new length} = c * \text{old length}$$

requires at most $(1+c) * (\text{maxListSize} - 1)$ space.

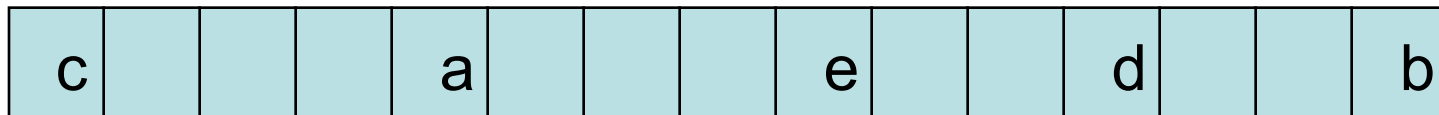
Resizing by an additive constant c requires
at most $(\text{maxListSize} - 1) + (\text{maxListSize} - 1 + c)$
 $= 2 * (\text{maxListSize} - 1) + c$ space.



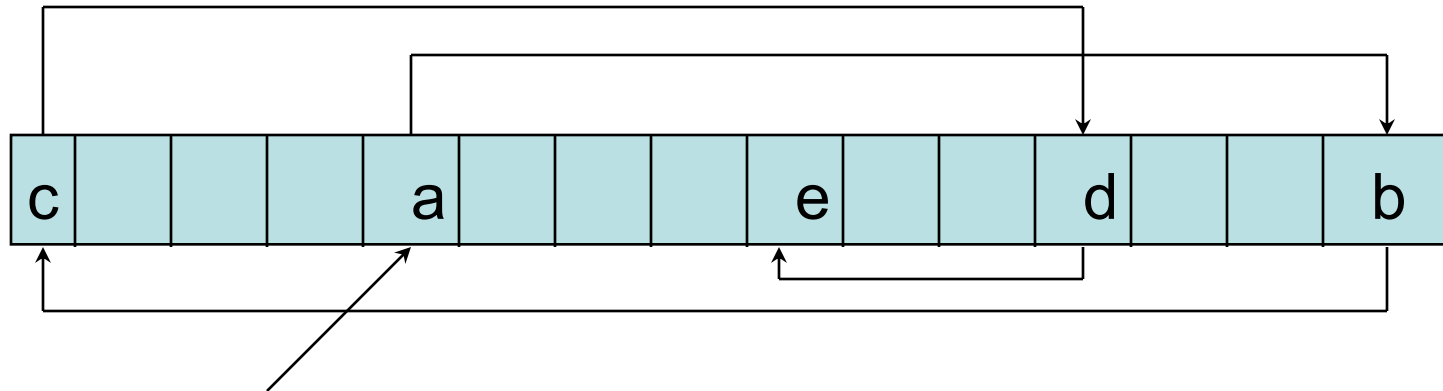
Linked Representation

- list elements are stored, in memory, in an **arbitrary order**
- explicit information (**called a link**) is used to go from one element to the next

A linked representation uses an arbitrary layout.



Linked Representation

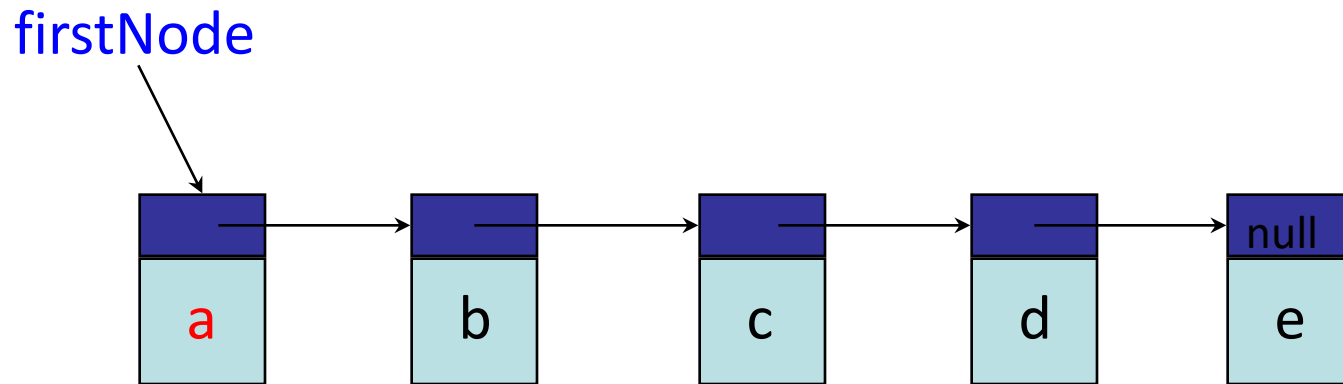


firstNode

- pointer (or link) in **e** is **null**

use a variable **firstNode** to get to the first element **a**

Normal Way To Draw A Linked List

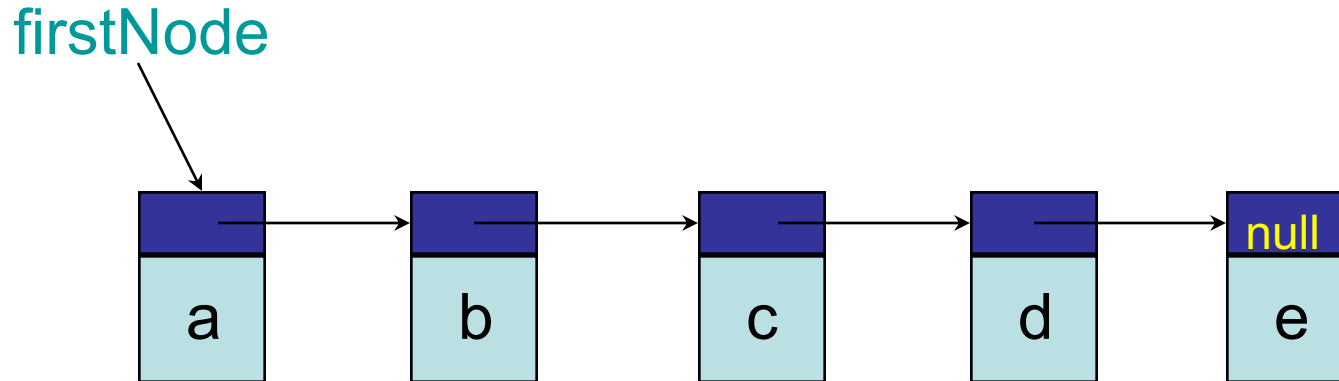


- link or pointer field of node



data field of node

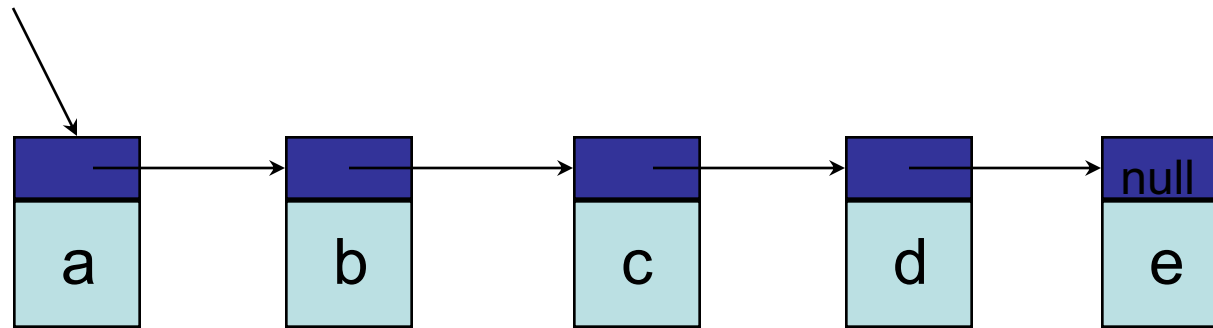
Chain



- A chain is a linked list in which each node represents one element.
- There is a link or pointer from one element to the next.
- The last node has a null pointer.

List Operations: get()

firstNode

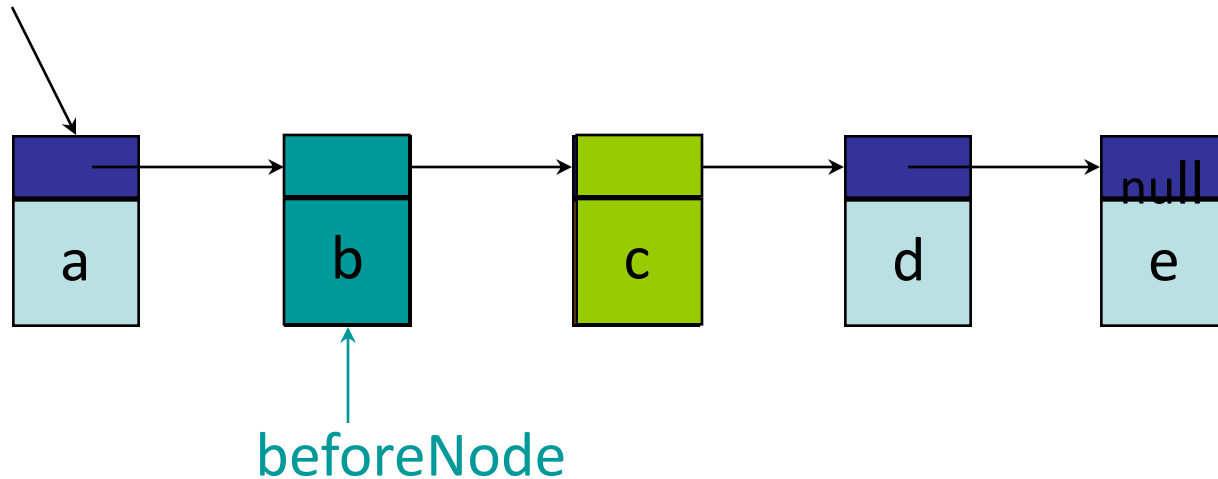


- `checkIndex(1);`
- `desiredNode = firstNode.next`
`return desiredNode.element;`

List Operations: Remove

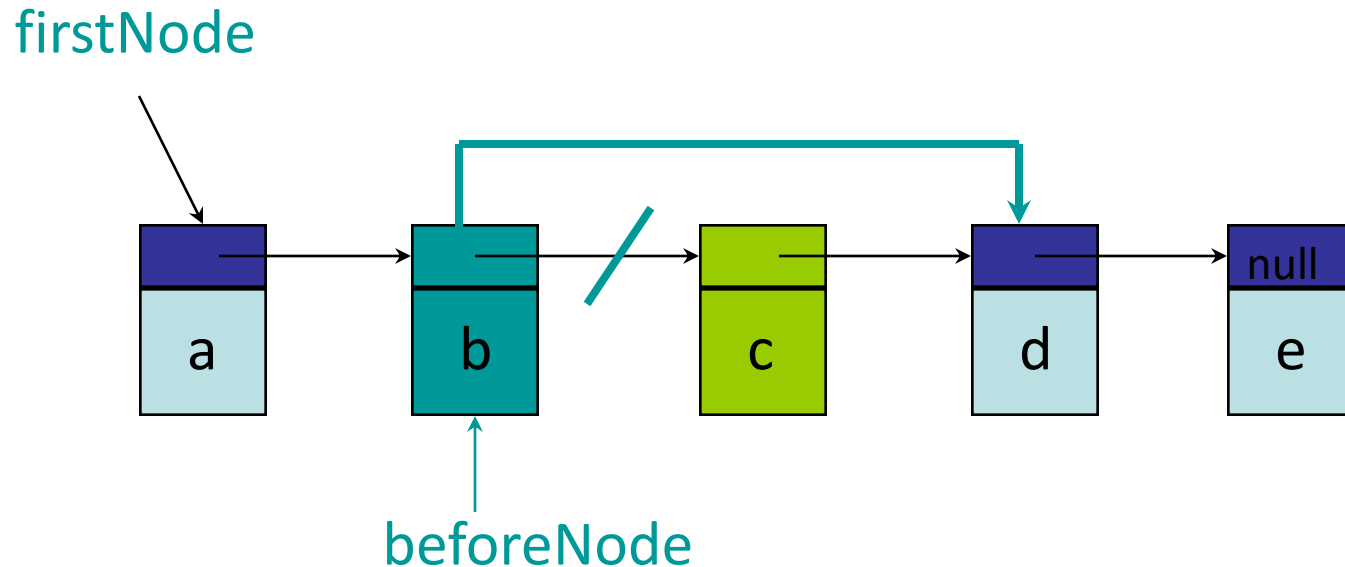
Remove (2)

firstNode



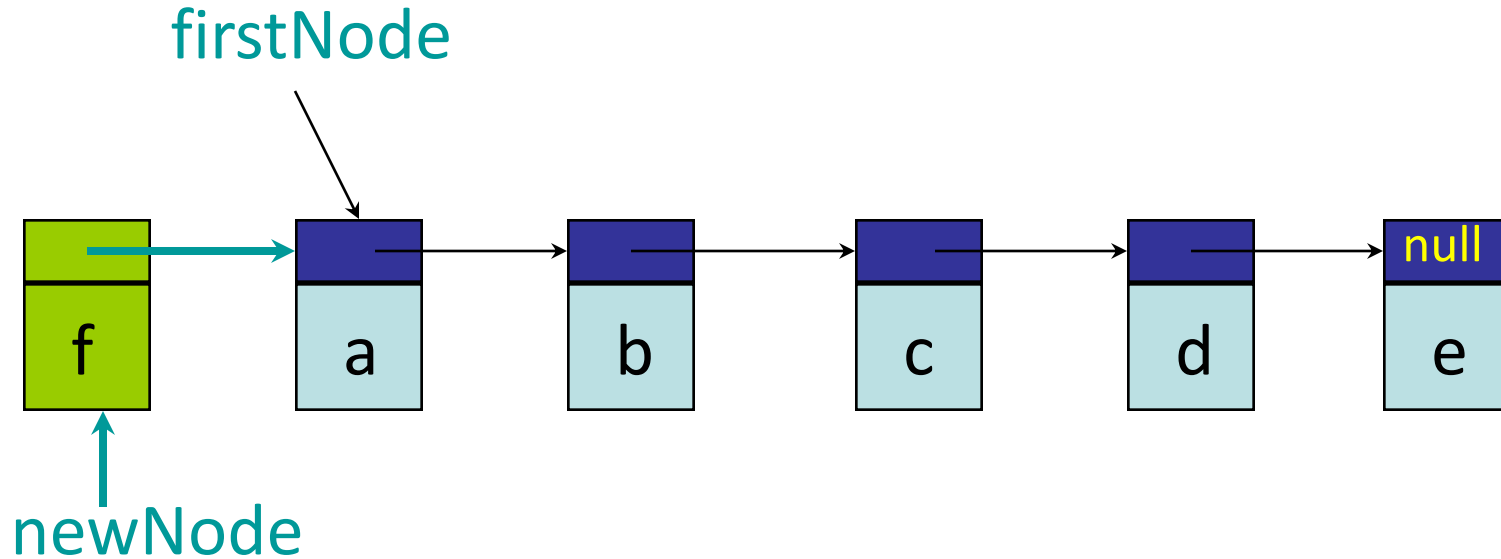
- first get to node just before node to be removed
- `beforeNode = firstNode.next;`

List Operations: Remove



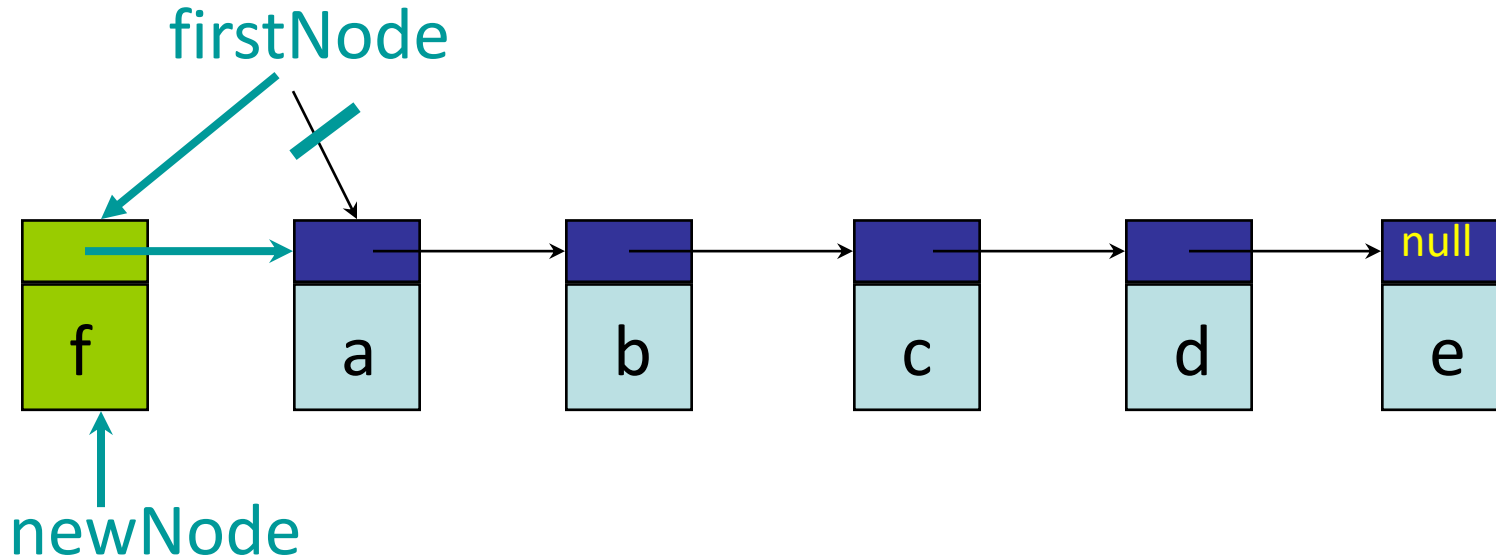
- now change pointer in beforeNode
-

List Operations: Add



Step 1: get a node, set its data and link fields

List Operations: Add



Step 2: update `firstNode`

`firstNode = newNode;`

Performance

- 40,000 operations of each type

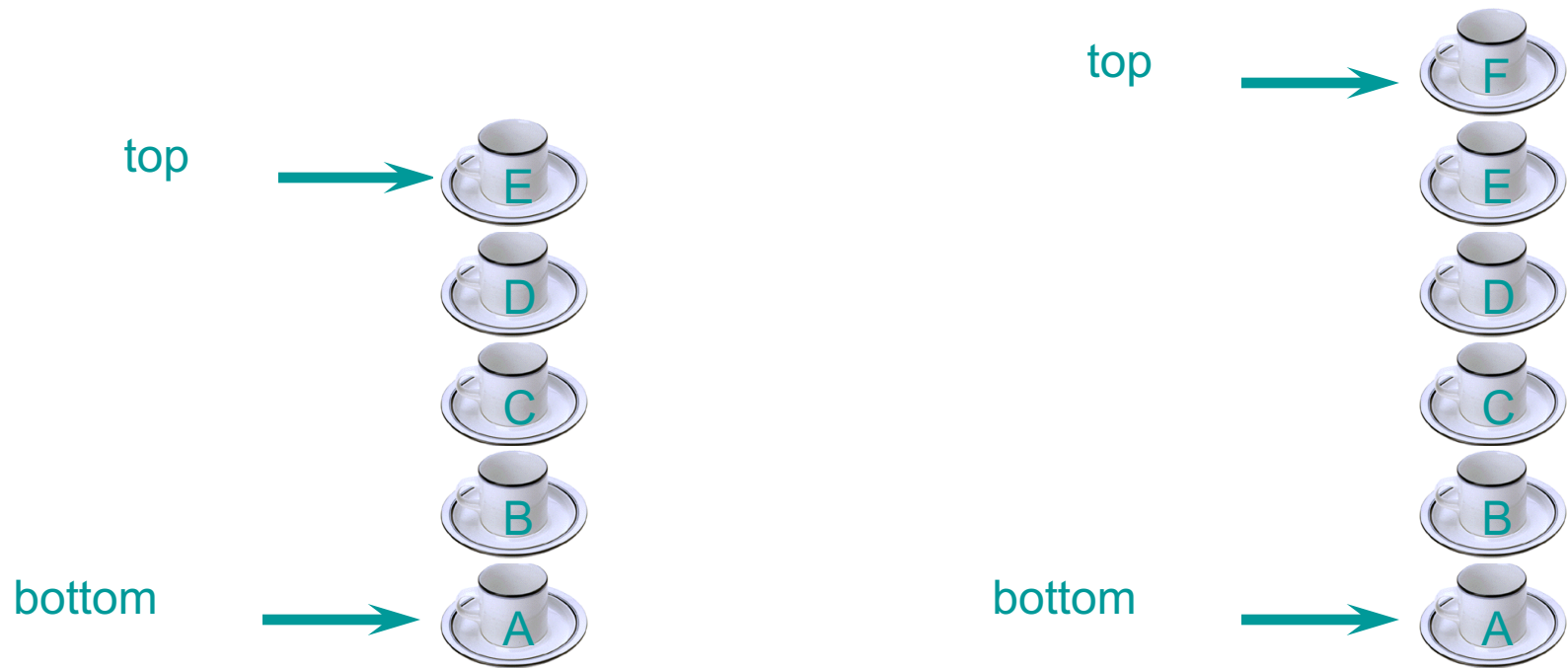
Operation	FastArray	LinearList	Chain
get	5.6ms	157sec	
best-case adds	31.2ms	304ms	
average adds	5.8sec	115sec	
worst-case adds	11.8sec	157sec	
best-case removes	8.6ms	13.2ms	
average removes	5.8sec	149sec	
worst-case removes	11.7sec	157sec	

Stacks

- Linear list.
- One end is called **top**.
- Other end is called **bottom**.
- Additions to and removals from the **top** end only.



Stack Of Cups



- Add a cup to the stack.
- Remove a cup from new stack.
- A stack is a **LIFO** list.

The Interface Stack

- ❖ `empty();`
- ❖ `peek();`
- ❖ `push(theObject);`
- ❖ `pop();`



Stack Applications

- Parentheses matching.
- Towers of Hanoi.
- Switchbox routing.
- Method invocation and return.
- Try-catch-throw implementation.

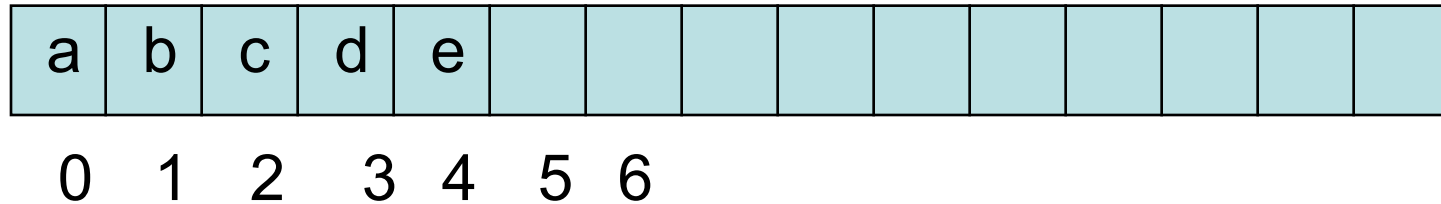


Derive From A Linear List

☐ ArrayList

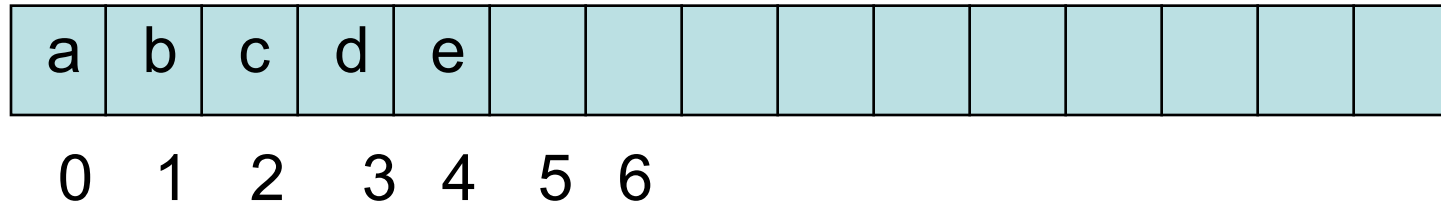
☐ Chain

Derive From ArrayList



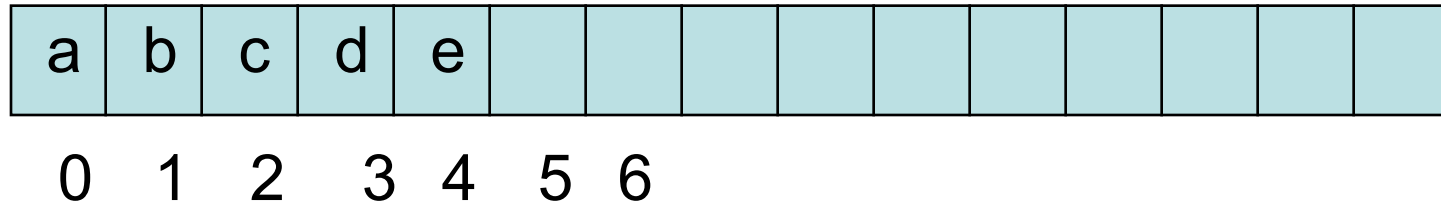
- stack top is either left end or right end of linear list
- `empty()` \Rightarrow `isEmpty()`
 - $O(1)$ time
- `peek()` \Rightarrow `get(0)` or `get(size() - 1)`
 - $O(1)$ time

Derive From ArrayList



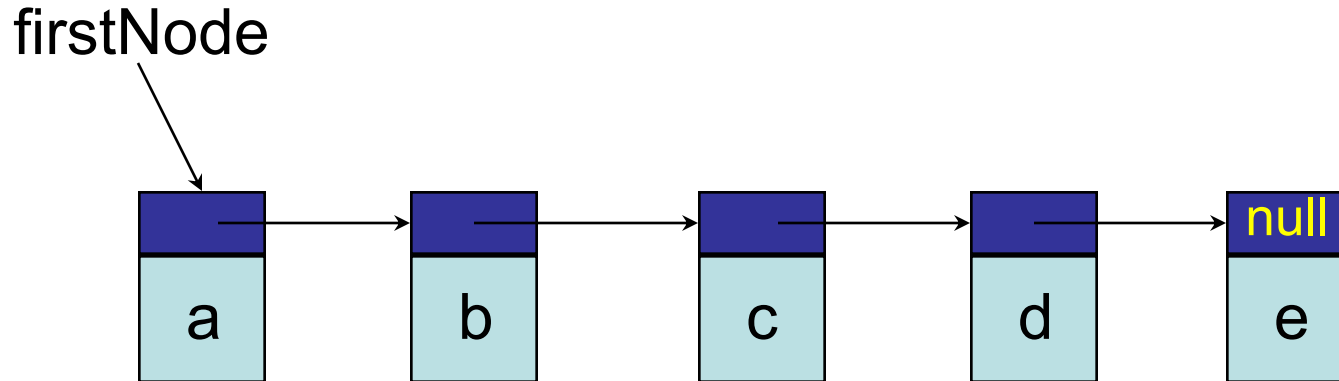
- when top is left end of linear list
 - `push(theObject) => add(0, theObject)`
 - $O(\text{size})$ time
 - `pop() => remove(0)`
 - $O(\text{size})$ time

Derive From ArrayList



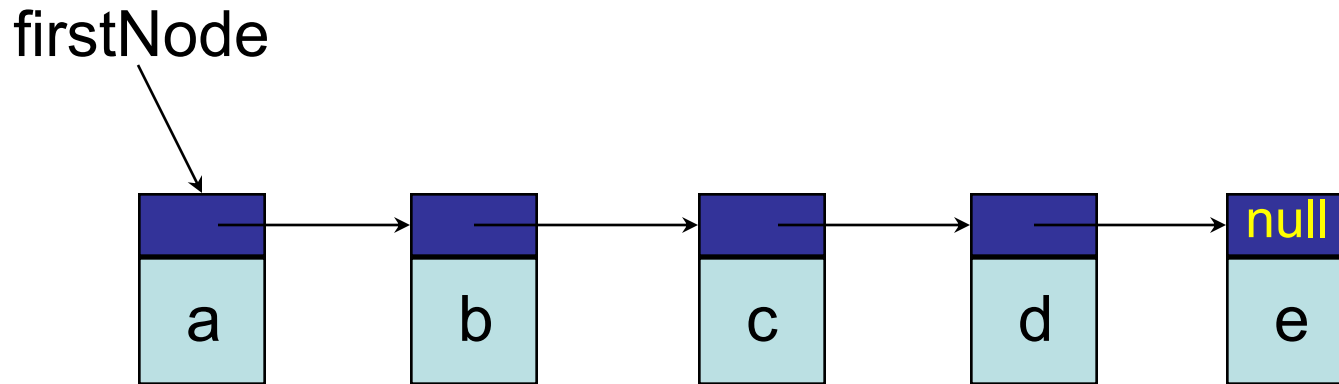
- when top is right end of linear list
 - `push(theObject) => add(size(), theObject)`
 - $O(1)$ time
 - `pop() => remove(size()-1)`
 - $O(1)$ time
- use right end of list as top of stack

Derive From Chain



- stack top is either left end or right end of linear list
- `empty() => isEmpty()`
 - $O(1)$ time

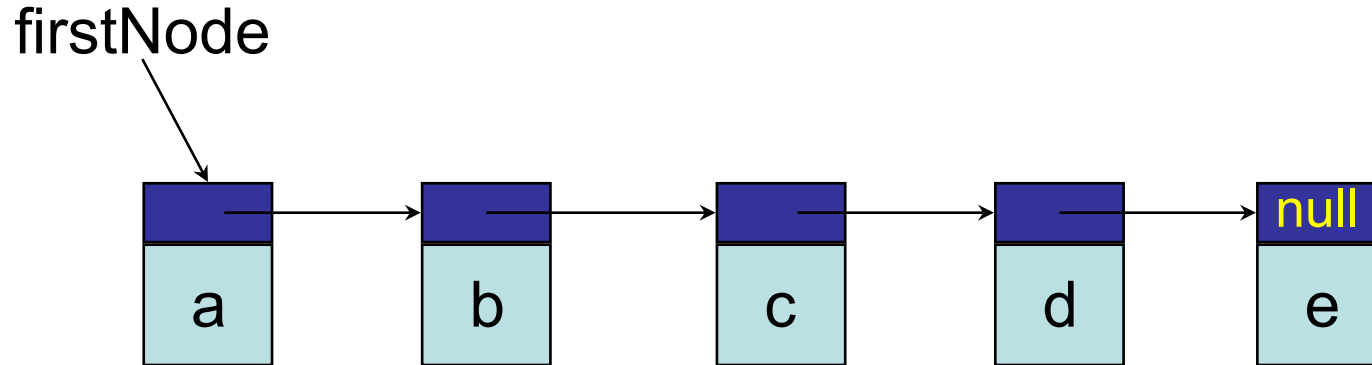
Derive From Chain



— when top is left end of linear list

- `peek() => get(0)`
- $O(1)$ time
- `push(theObject) => add(0, theObject)`
- $O(1)$ time
- `pop() => remove(0)`
- $O(1)$ time

Derive From Chain



— when top is right end of linear list

- $\text{peek()} \Rightarrow \text{get}(\text{size}() - 1)$
- $O(\text{size})$ time
- $\text{push}(\text{theObject}) \Rightarrow \text{add}(\text{size}(), \text{theObject})$
- $O(\text{size})$ time
- $\text{pop()} \Rightarrow \text{remove}(\text{size}()-1)$
- $O(\text{size})$ time

— use left end of list as top of stack

Queues

- Linear list.
- One end is called **front**.
- Other end is called **rear**.
- **Additions** are done at the **rear** only.
- **Removals** are made from the **front** only.

Bus Stop Queue



Bus Stop Queue



front

rear



Bus Stop Queue



front

rear



Bus Stop Queue



front

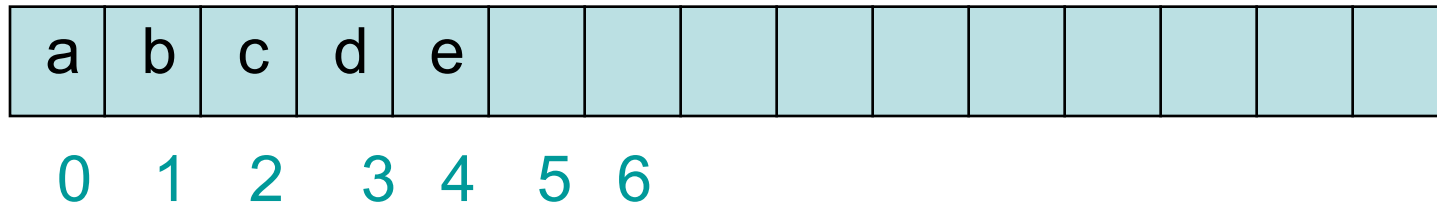
rear



The Interface Queue

- ❖ Empty();
- ❖ Element();
- ❖ getRearElement();
- ❖ put (theObject);
- ❖ remove();

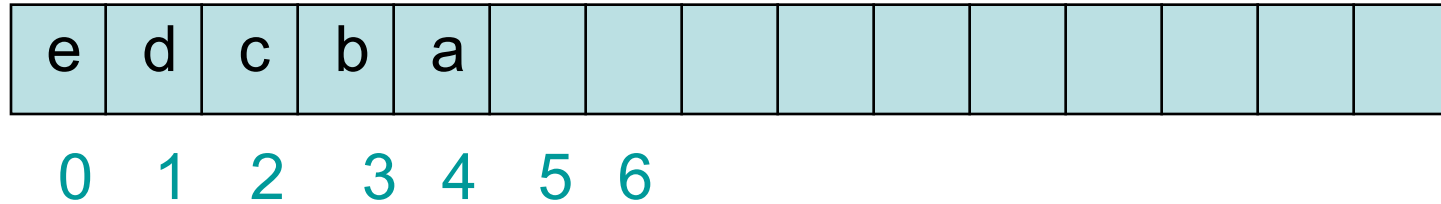
Derive From ArrayList



when front is left end of list and rear is right end

- `Queue.isEmpty() => super.isEmpty()`
 - $O(1)$ time
- `getFrontElement() => get(0)`
 - $O(1)$ time
- `getRearElement() => get(size() - 1)`
 - $O(1)$ time
- `put(theObject) => add(size(), theObject)`
 - $O(1)$ time
- `remove() => remove(0)`
 - $O(\text{size})$ time

Derive From ArrayList



– when rear is left end of list and front is right end

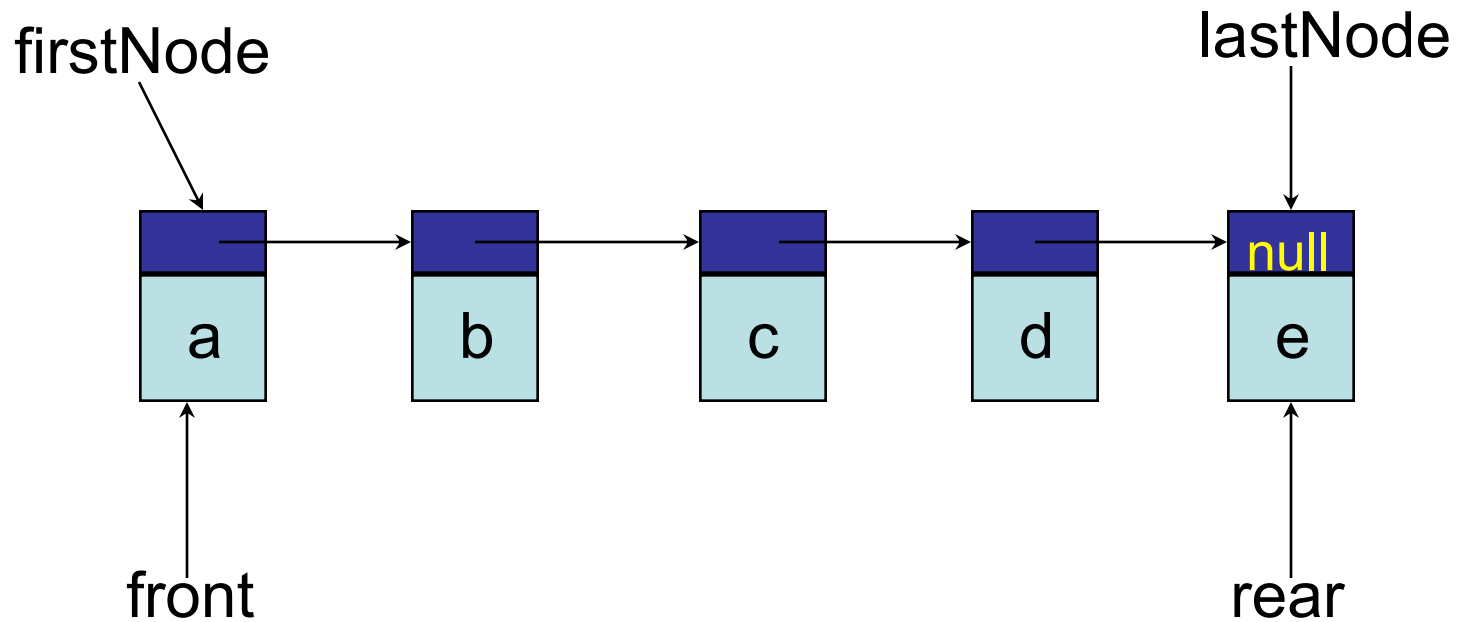
- `Queue.isEmpty() => super.isEmpty()`
 - $O(1)$ time
- `getFrontElement() => get(size() - 1)`
 - $O(1)$ time
- `getRearElement() => get(0)`
 - $O(1)$ time
- `put(theObject) => add(0, theObject)`
 - $O(\text{size})$ time
- `remove() => remove(size() - 1)`
 - $O(1)$ time

Derive From ArrayList

- to perform each operation in $O(1)$ time (excluding array doubling), we need a customized array representation.



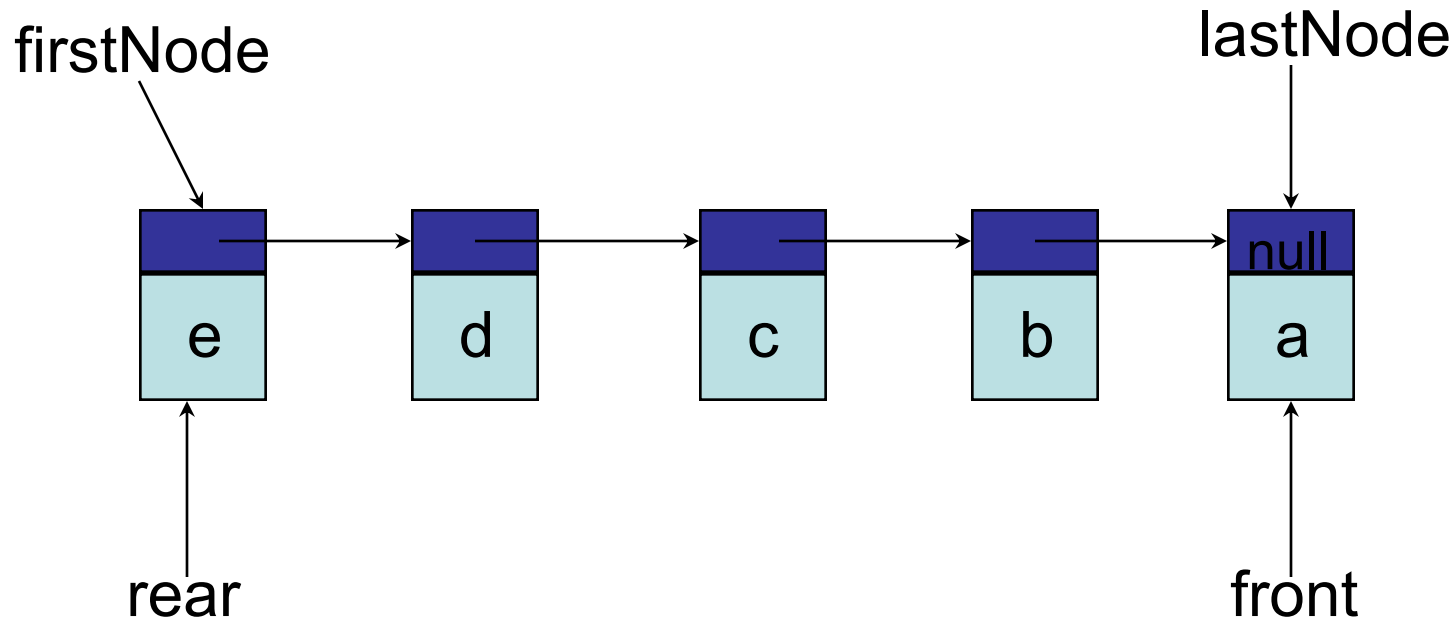
Derive From ExtendedChain



when front is left end of list and rear is right end

- `Queue.isEmpty()` => `super.isEmpty()`
 - $O(1)$ time
- `getFrontElement()` => `get(0)`
 - $O(1)$ time

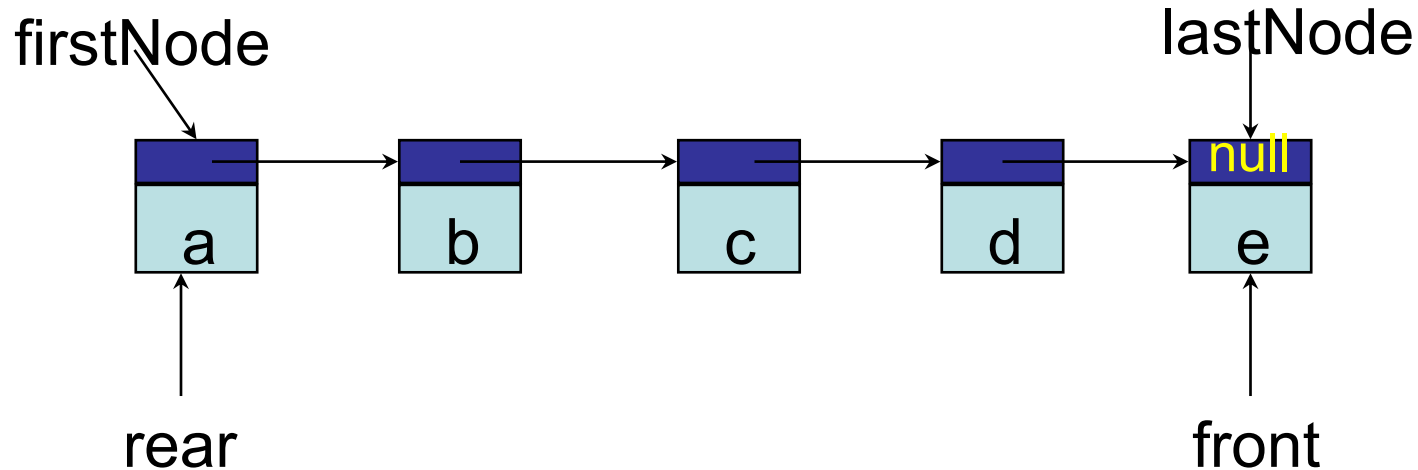
Derive From ExtendedChain



when front is right end of list and rear is left end

- `Queue.isEmpty() => super.isEmpty()`
 - $O(1)$ time
- `getFrontElement() => getLast()`
 - $O(1)$ time


Derive From ExtendedChain



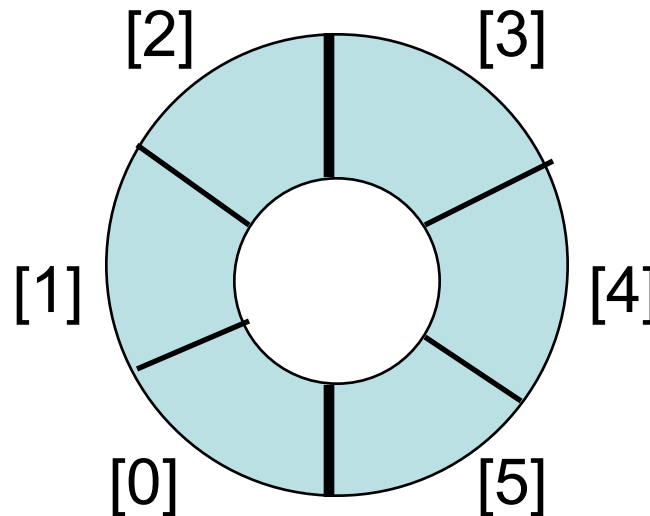
- `getRearElement()` \Rightarrow `get(0)`
 - $O(1)$ time
- `put(theObject)` \Rightarrow `add(0, theObject)`
 - $O(1)$ time
- `remove()` \Rightarrow `remove(size-1)`
 - $O(\text{size})$ time

Custom Array Queue

- Use a 1D array `queue`.

`queue[]` 

- Circular view of array.



Thank You



23 Sep 2020

DOR@IIT Jammu

54



भारतीय प्रौद्योगिकी
संस्थान जम्मू
INDIAN INSTITUTE OF
TECHNOLOGY JAMMU
विद्यामं सर्वधनं उपायम्