ASSIGNMENT 2

Linear Algebra-CSD001P5M

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1. Let A be a square matrix such that A^k is the zero matrix for some positive integer k, what is the possible value of $det(A)$? Is this matrix A invertible? Is the matrix $I - A$ invertible? [1 marks]

2. Find the values of k for which A is invertible, $A := \begin{bmatrix} 2 & 1 & 0 \\ k & 3 & k \\ 4+k & 5 & k \end{bmatrix}$ [2 marks]

3. Let F_n be the 1, 1, -1 tridiagonal matrix of size $n \times n$:
for $i = 2, 3, \ldots, n-1$,

$$(F_n)_{ij} = \begin{cases} 1 & j = i, \\ 1 & j = i-1 \\ -1 & j = i+1 \end{cases}$$

$$F_n = det \begin{bmatrix} 1 & -1 & 0 & \cdots & 0 & 0 \\ 1 & 1 & -1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & -1 \\ 0 & 0 & 0 & \cdots & 1 & 1 \end{bmatrix},$$

show that $det(F_n) = det(F_{n-1}) + det(F_{n-2})$. [2 marks]

4. Let $A := \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$. If $det(A) = 1$ then find the determinant values of the following matrices

$$\begin{bmatrix} a & c & b \\ d & f & e \\ g & i & h \end{bmatrix}, \begin{bmatrix} c & b & a \\ f & e & d \\ i & h & g \end{bmatrix}, \begin{bmatrix} 3a & 3b & 3c \\ d & e & f \\ g & h & i \end{bmatrix}, \begin{bmatrix} 3a & 3b & c \\ 3d & 3e & f \\ 3g & 3h & i \end{bmatrix}, \begin{bmatrix} 3a & 3b & 3c \\ 3d & 3e & 3f \\ 3g & 3h & 3i \end{bmatrix}, \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}, \begin{bmatrix} b & c & a \\ e & f & d \\ h & i & g \end{bmatrix}.$$

[3 marks]

5. What is the maximum number of zeros that a matrix of size 4×4 can have without having a zero determinant? [2 marks]