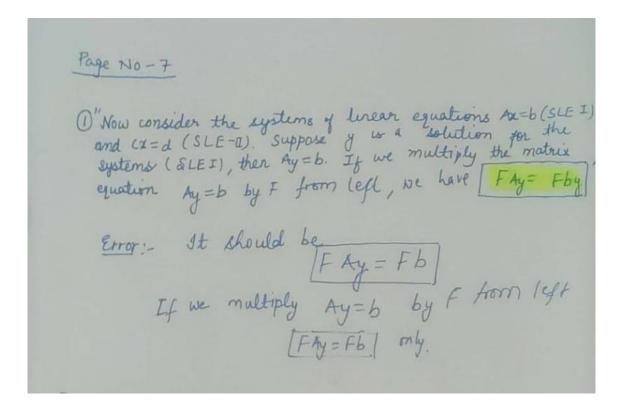
Errors in Row Reduction Sheet (Linear Algebra)

We can prove previous theorem another way:

Suppose for each i = 1, 2, ..., k, E_i is a corresponding elementary matrix for the elementary row operation e_i . We have $E_k E_{k-1} ... E_1[A|b] = [C|d]$, $F := E_k E_{k-1} ... E_1$ is an invertible matrix as each E_i is invertible. Therefore [FA|Fb] = F[A|b] = [C|d] and FA = C and Fb = d.

Now consider the systems of linear equations Ax = b (S.L.E-I) and Cx = d (S.L.E.-II). Suppose y is a solution for the system (S.L.E-I), then Ay = b. If we multiply the matrix equation Ay = b by F from left, we have FAy = Fby. Hence Cy = d and this shows that y is also a solution for the system Cx = d. Therefore the set of solutions for the system Ax = b is a subset of the set of solutions for the system Cx = d.

Now if z is a solution for the system (S.L.E.-II), then Cz = d. If we multiply the matrix equation Cy = d by F^{-1} from left, we have $F^{-1}Cy = F^{-1}d$. Hence Az = b and this shows that z is also a solution for the system Ax = b. Therefore the set of solutions for the system Cx = d is a subset of the set of solutions for the system Ax = b. If one system does not have any solution, then another system also does have any solution. Hence the set of solutions for the system Cx = d is same as the set of solutions for the system Ax = b.



Some examples of row-reduced echelon matrices:

Some examples of matrices which are not row-reduced echelon matrices:

$$1.\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} 2.\begin{bmatrix} 1 & 5 & 7 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} 3.\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} 4.\begin{bmatrix} 1 & 6 & 3 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix} 5.\begin{bmatrix} 1 & 2 & 8 & 0 & 3 \end{bmatrix}$$

$$6.\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} 7.\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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Example NO-(5) [1 2 8 0 3]

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Conditions:

A matrix & of size mxn is called, now reduce

Echelon matrix—

(i) & is now reduced

(ii) Every Now of & which has all its entries

O occurs below every how which has a non-zero entry.

(iii) & cows I— r, are the non zero entry of

Pow i occurs in columns &; i=1— r then

[1 2 8 0 3] is satisfying all. I guess.

Theorem 0.8. If A is a matrix of size $m \times n$ and m < n, then the homogeneous system of linear equations Ax = 0 has a non-zero solution.

Proof. Let R be a row-reduced echelon matrix which is row-equivalent to A. Then the systems of linear equations Ax=0 and Rx=0 are equivalent and they have same set of solutions. Suppose r is the number of non-zero rows in R, then we have $r \leq m < n$. Let the leading non-zero entry of i-th row of R occurs in column k_i for $i=1,2,\ldots,r$. There are r non-zero equations of the system Rx=0, where the unknown x_{k_i} will only appear with non-zero coefficient 1 in the i-th equation. Here n-r is positive integer, let u_1,u_2,\ldots,u_{n-r} denote the n-r unknowns (free-variables), other than $x_{k_1},x_{k_2},\ldots,x_{k_r}$. Then the r non-zero equations in Rx=0 are of the form

$$(S.L.E, Rx = 0) \begin{cases} x_{k_1} + c_{11}u_1 + c_{12}u_2 + \dots + c_{1n-r}u_j = 0 \\ x_{k_2} + c_{21}u_1 + c_{22}u_2 + \dots + c_{2n-r}u_j = 0 \\ \dots \\ x_{k_r} + c_{r1}u_1 + c_{r2}u_2 + \dots + c_{rn-r}u_j = 0 \end{cases}$$

All the solutions of the system of equations Rx=0 are obtained by assigning any values of u_1,u_2,\ldots,u_{n-r} , then find the values of $x_{k_1},x_{k_r},\ldots,x_{k_r}$ from (S.L.E, Rx=0). Hence the system Ax=0 has solutions other than zero.

In view of proof of Theorem 0.8, if r < n then the system Ax = 0 has non-zero solution.

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Theorem- 0.8

$$(SLE, kx=0) = \begin{cases} \chi_{k_1} + \zeta_1 u_1 + \zeta_2 u_2 + \dots + \zeta_{1n-r} u_j = 6 \\ \chi_{k_2} + \zeta_1 u_1 + \zeta_2 u_k + \dots + \zeta_{n-r} u_j = 6 \end{cases}$$

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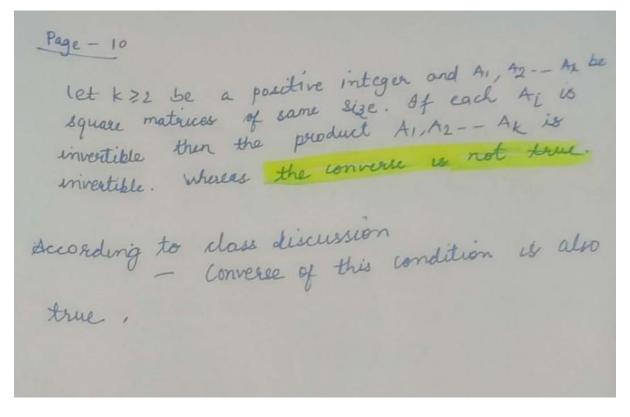
$$\chi_{k_1} + \zeta_1 u_1 + \zeta_2 u_1$$

Theorem 0.11. A square matrix A that has either a left or a right inverse that means there exists a matrix B such that AB = I or BA = I. Then A is invertible.

Proof. Let us assume AB = I. Consider the system of linear equations Bx = 0, if we multiply the equation by the matrix A from left then we have x = 0. This shows that Bx = 0 has only zero solution. Hence B is invertible and $A = B^{-1}$. As inverse of an invertible matrix is also invertible, A is invertible.

Now for the case BA = I, we have the system of linear equations Ax = 0 has only zero solution. Hence A is invertible.

Let $k \geq 2$ be a positive integer and A_1, A_2, \ldots, A_k be square matrices of same size. If each A_i is invertible then the product $A_1 A_2 \cdots A_k$ is invertible, whereas the converse is not true. ** One application for the system of linear equations: click here.



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