PROBLEM SHEET 1

Linear Algebra-CSD001P

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1.Prove that product of two upper triangular matrices is an upper triangular matrix.

2. Consider three systems of linear equations (S.L.E.-I), (S.L.E.-II) and (S.L.E.-III) in n unknowns such that (S.L.E.-I) and (S.L.E.-II) are equivalent and (S.L.E.-II) and (S.L.E.-III) are equivalent. Prove that (S.L.E.-I) and (S.L.E.-III) are equivalent.

3.If possible, find the matrices B of size 3×3 such that
(a). $BA = 2A$ for every A.
(b). $BA = 2B$ for every A.

4. Find the value of c in the following inverse matirx of size $n \times n$ such that if
$A := \begin{bmatrix} n & -1 & -1\dots & -1 \\ -1 & n & -1\dots & -1 \\ & \vdots & \vdots & \\ -1 & -1 & -1\dots & n \end{bmatrix}$, then $A^{-1} := \frac{1}{n+1} \begin{bmatrix} c & 1 & 1\dots & 1 \\ 1 & c & 1\dots & 1 \\ & \vdots & \vdots & \\ 1 & 1 & 1\dots & c \end{bmatrix}$.

5. show that if A is a square matrix such that $A^k = 0$ for some positive integer k, then the matrix $I - A$ is invertible.

6. Under what conditions on the entries is A invertible?
$A := \begin{bmatrix} a & b & c \\ d & e & 0 \\ f & 0 & 0 \end{bmatrix}$

7.Let

$$A := \begin{bmatrix} 3 & -6 & 2 & -1 \\ -2 & 4 & 1 & 3 \\ 0 & 0 & 1 & 1 \\ 1 & -2 & 1 & 0 \end{bmatrix}$$

for which $(y_1, y_2, y_3, y_4, y_5)$ does the system of equations $Ax = y$ have a solution?

8. If A is a matrix of size $m \times n$, B is a matrix of size $n \times m$ and $n < m$, then AB is not invertible.

9. Show that
$$\begin{bmatrix} 0 & 0 & 0 & a & 0 \\ 0 & 0 & b & 0 & c \\ 0 & d & 0 & e & 0 \\ f & 0 & g & 0 & 0 \\ 0 & h & 0 & 0 & 0 \end{bmatrix}$$
is not invertible for any values of the entries a, b, c, d, e, f, g, h.