Control-Oriented Thermal Modeling

Mathematical Appendix

Warning

This document contains mathematical models, assumptions, and transfer function derivations. Reading this section is **optional** and not required to understand the system behavior or results.

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1 Solar Panel Modeling (Plant)

The photovoltaic panel is modeled as a lumped thermal system with uniform temperature.

Definitions

- $T(t)$: Panel temperature
- T_a: Ambient temperature
- m: Effective panel mass
- c_p: Specific heat capacity

The thermal capacitance is defined as:

$$C_{th} = mc_p \tag{1}$$

The thermal energy stored in the panel is:

$$E(t) = mc_p T(t) \tag{2}$$

Taking the time derivative:

$$\frac{dE(t)}{dt} = C_{th} \frac{dT(t)}{dt} \tag{3}$$

Heat Loss to Ambient

Heat transfer to ambient is modeled using Newton's law of cooling:

$$Q_{loss}(t) = hA\left(T(t) - T_a\right) \tag{4}$$

Define the thermal resistance:

$$R_{th} = \frac{1}{hA} \tag{5}$$

Thus:

$$Q_{loss}(t) = \frac{T(t) - T_a}{R_{th}} \tag{6}$$

Energy Balance

Applying conservation of energy:

$$C_{th}\frac{dT(t)}{dt} = Q_{in}(t) - \frac{T(t) - T_a}{R_{th}} \tag{7}$$

Define the temperature deviation variable:

$$\theta(t) = T(t) - T_a \tag{8}$$

Substituting:

$$\frac{d\theta(t)}{dt} + \frac{1}{R_{th}C_{th}}\theta(t) = \frac{1}{C_{th}}Q_{in}(t) \tag{9}$$

Transfer Function

Taking the Laplace transform:

$$\Theta(s) = \frac{R_{th}}{R_{th}C_{th}s + 1}Q_{in}(s) \tag{10}$$

Define:

$$K_{plant} = R_{th} \tag{11}$$

$$\tau_{plant} = R_{th}C_{th} \tag{12}$$

Final plant model:

$$G_{plant}(s) = \frac{K_{plant}}{\tau_{plant}s + 1} \tag{13}$$

2 Actuator Modeling (Ultrasonic Mister)

The misting actuator is modeled as a first-order dynamic system.

Definitions

- $u(t)$: Actuator command
- $q_m(t)$: Mist output
- K_{act}: Steady-state gain
- τ_{act}: Actuator time constant

Dynamic equation:

$$\frac{dq_m(t)}{dt} = \alpha u(t) - \beta q_m(t) \tag{14}$$

Rewriting:

$$\tau_{act}\frac{dq_m(t)}{dt} + q_m(t) = K_{act}u(t) \tag{15}$$

Transfer function:

$$G_{act}(s) = \frac{K_{act}}{\tau_{act}s + 1} \tag{16}$$

3 Sensor Modeling (NTC Thermistor)

The thermistor is modeled as a first-order thermal system.

Definitions

- $T_p(t)$: Panel surface temperature
- $T_s(t)$: Measured sensor temperature
- R_{sp}: Thermal resistance to panel
- C_s: Sensor thermal capacitance

Energy balance:

$$C_s\frac{dT_s(t)}{dt} = \frac{T_p(t) - T_s(t)}{R_{sp}} \tag{17}$$

Define:

$$\tau_{sensor} = R_{sp}C_s \tag{18}$$

Transfer function:

$$G_{sensor}(s) = \frac{1}{\tau_{sensor}s + 1} \tag{19}$$

4 Parameter Settings

Block	Parameter	Value
Actuator	K_{act}	-1400
Actuator	τ_{act}	5 s
Plant	K_{plant}	0.025
Plant	τ_{plant}	400 s
Sensor	τ_{sensor}	2 s
Environment	$T_{ambient}$	26°C
Reference	T_{target}	41°C
Logic	T_{high}	44°C