Control-Oriented Thermal Modeling

Mathematical Appendix

Warning

This document contains mathematical models, assumptions, and transfer function derivations. Reading this section is **optional** and not required to understand the system behavior or results.

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1 Controller Design and Closed-Loop Stability

Controller Design

Due to the slow thermal dynamics of the photovoltaic panel, a low-bandwidth controller was selected to avoid aggressive actuation and oscillatory behavior. A second-order controller with real, negative poles was chosen to ensure stability and monotonic convergence.

The controller transfer function is defined as:

$$C(s) = \frac{0.4s + 0.01}{20s^2 + 12s + 1} \tag{1}$$

The controller poles are located at:

$$s = -0.01, \quad s = -0.05$$

with a zero at:

$$s = -0.025$$

This pole–zero placement shapes the closed-loop response to align with the slow thermal time constant of the panel while maintaining adequate phase behavior.

Relay-Based Supervisory Logic

To reflect practical cooling constraints, relay and switch blocks were used to implement temperature-based actuation logic. The cooling system is activated when the panel temperature exceeds:

$$T_{\text{high}} = 44^{\circ}\text{C}$$

and deactivated once the temperature falls below:

$$T_{\text{low}} = 41^\circ\text{C}$$

This hysteresis-based logic prevents excessive actuator cycling and reflects realistic system operation.

2 Closed-Loop Transfer Function

The closed-loop system consists of the following components:

$$C(s) = \frac{0.4s + 0.01}{20s^2 + 12s + 1} \tag{2}$$

$$A(s) = \frac{1400}{5s + 1} \tag{3}$$

$$P(s) = \frac{0.025}{400s + 1} \tag{4}$$

$$H(s) = \frac{1}{2s + 1} \tag{5}$$

The forward-path transfer function is given by:

$$G(s) = C(s)A(s)P(s) \tag{6}$$

Substituting and simplifying:

$$G(s) = \frac{14s + 0.35}{40000s^4 + 32100s^3 + 6880s^2 + 417s + 1} \tag{7}$$

The closed-loop transfer function is obtained using standard unity feedback:

$$T(s) = \frac{G(s)}{1 + G(s)H(s)} \tag{8}$$

After substitution and simplification, the final closed-loop transfer function becomes:

$$T(s) = \frac{28s^2 + 14.7s + 0.35}{80000s^5 + 104200s^4 + 45860s^3 + 7714s^2 + 433s + 1.35} \tag{9}$$

3 Frequency-Domain Stability Analysis

The closed-loop frequency response was analyzed using a Bode plot.

The resulting stability margins are:

- Gain Margin (GM): 50.1 dB at 0.391 rad/s
- Phase Margin (PM): Infinite

The positive gain margin confirms closed-loop stability. The infinite phase margin indicates that the magnitude response does not cross the 0 dB threshold, implying unconditional stability under the current configuration.