CS 5350/6350: Machine Learining Fall 2018

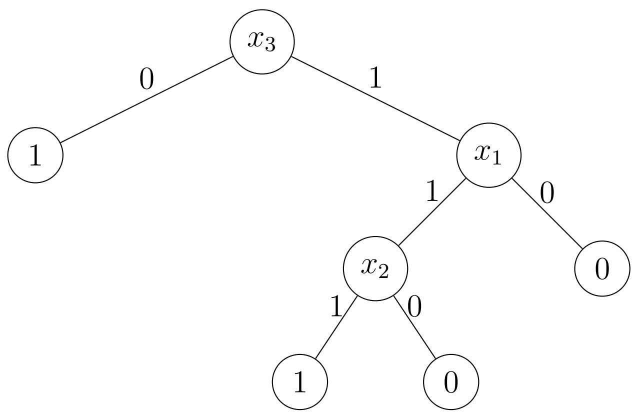
Homework 1

Handed out: 28 August, 2018
Due date: 11 September, 2018

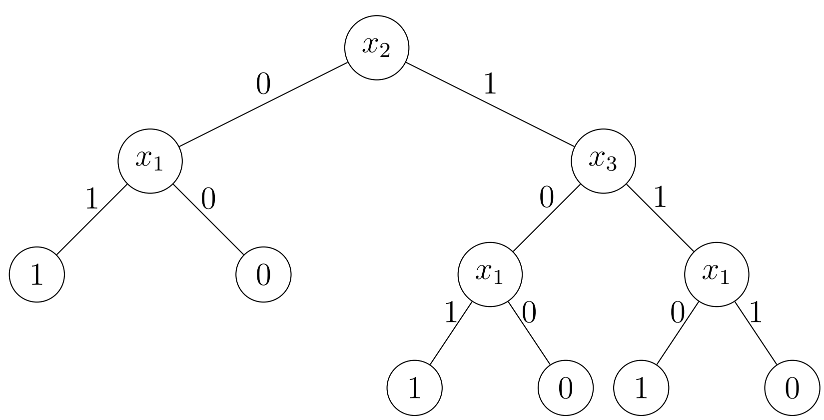
1 Decision Trees

1. [6 points] In this warm-up question, you will be drawing decision trees for boolean functions.

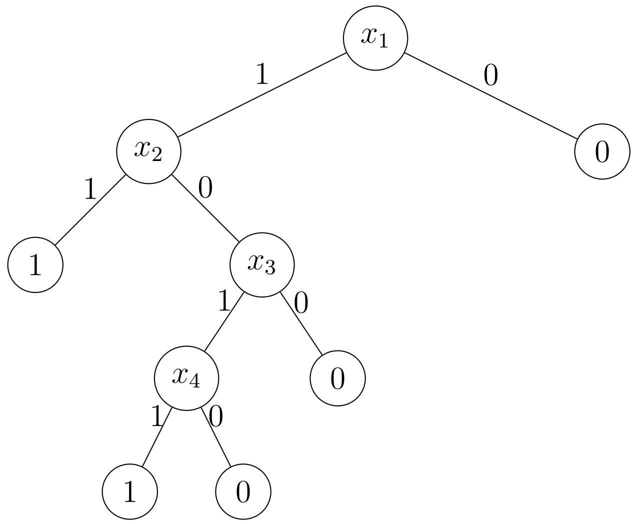
   (a) $(x_1 \wedge x_2) \vee \neg x_3$

   (b) x_1 xor $(x_2 \wedge x_3)$

   (c) $x_1 \wedge (x_2 \vee (x_3 \wedge x_4))$



2. [24 points] Mark loves mangoes. Unfortunately he is lousy at picking ripe mangoes at the grocery. He needs your help. You need to build a decision tree that will help Mark decide if a mango is ripe or not. You need to make the decision based on four features described below:

   (a) **Variety** (*Alphonso, Keitt or Haden*): Describes the variety of the mango.

   (b) **Color** (*Red, Yellow or Green*): Describes the color of the mango.

   (c) **Smell** (*Sweet or None*): Describes the smell of the mango.

   (d) **Time** (*One or Two*): Number of weeks since the mango was plucked.

   You are given the following dataset which contains data for 8 different mangoes. For each mango, the values of the above four features are listed. The label of whether the mango was ripe or not is also provided.

Variety	Color	Smell	Time	Ripe?
Alphonso	Red	None	Two	False
Keitt	Red	None	One	True
Alphonso	Yellow	Sweet	Two	True
Keitt	Green	None	Two	False
Haden	Green	Sweet	One	True
Alphonso	Yellow	None	Two	False
Keitt	Yellow	Sweet	One	False
Alphonso	Red	Sweet	Two	True

Table 1: Training data for the mango prediction problem.

   (a) [5 points] How many possible functions are there to map these four features to a boolean decision? How many functions are consistent with the given training dataset?

Ans: 3 * 3 * 2 * 2 = 36 labels
Total Functions: 2^{36}
We have 8 lines of known data hence there are 28 other combinations which may have differing labels. Hence 2^{28} functions are possible which are consistent with the given training set.

(b) [3 points] What is the entropy of the labels in this data? When calculating entropy, the base of the logarithm should be base 2.

$$Entropy = -\sum_i p_i log\, p_i$$

Ans: Total Entropy (Ripe?):
4/8 True, 4/8 False; Entropy = 1

(c) [4 points] Compute the information gain of each feature and enter it into Table 2. Specify upto 3 decimal places.

Feature	Information Gain
Variety	0.156
Color	0.0615
Smell	0.1887
Time	0.0489

Table 2: Information gain for each feature.

- Variety:
  Alphonso - 2/4 False, 2/4 True; Entropy = -1/2 log (1/2) - 1/2 log (1/2) = 1
  Keitt - 1/3 False, 2/3 True; Entropy = -1/3 log (1/3) - 2/3 log(2/3) = -1/3 * (-1.585) - 2/3 * (-0.585) = 0.918
  Haden - 1 - True 0 - False; Entropy = 0
  Expected Entropy = 4/8 * 1 + 3/8 * 0.918 + 1/8 * 0 = 0.844
  Gain (Variety) = 1 - 0.844 = 0.156

- Color:
  Red - 1/3 False, 2/3 True; Entropy = -1/3 log(1/3) - 2/3 log(2/3) = 0.918
  Yellow - 2/3 False, 1/3 True; Entropy = 0.918
  Green - 1/2 False, 1/2 True; Entropy = 1
  Expected Entropy = 3/8 * 0.918 + 3/8 * 0.918 + 2/8 * 1 = 0.9385
  Gain (Color) = 1 - 0.9385 = 0.0615

- Smell:
  None - 3/4 False, 1/4 True; Entropy = -3/4 log(3/4) - 1/4 log(1/4) = -3/4 (-0.415) - 1/4 (-2) = 0.8113
  Sweet - 1/4 False, 3/4 True; Entropy = -1/4 log(1/4) - 3/4 log(3/4) = 0.8113
  Expected Entropy = 4/8 * 0.8113 + 4/8 * 0.8113 = 0.8113

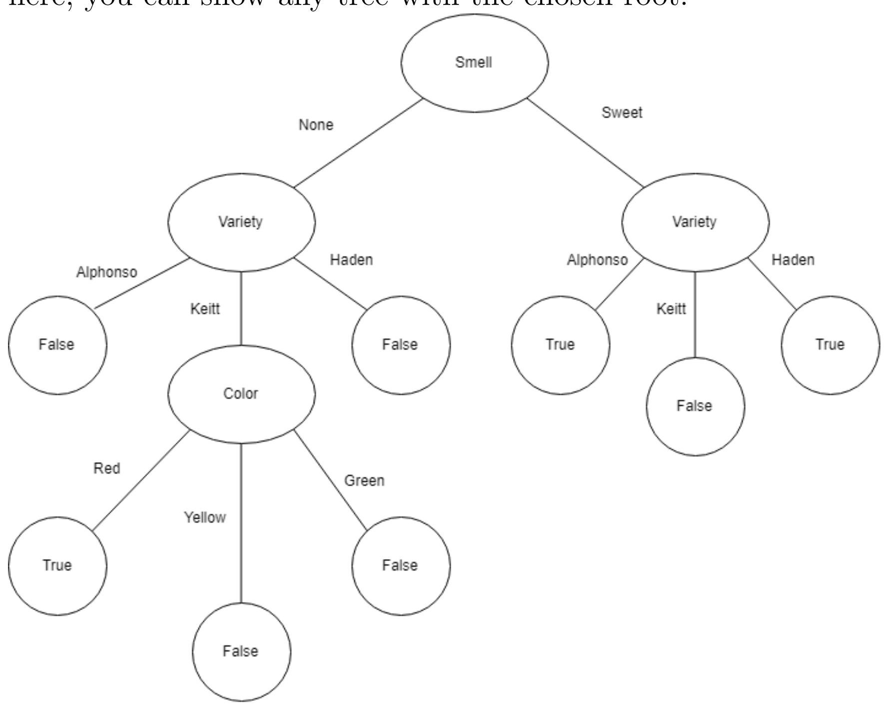
Gain (Smell) = 1 - 0.8113 = 0.1887

- Time:
  Two - 3/5 False, 2/5 True; Entropy = -3/5 log(3/5) - 2/5log(2/5) = -3/5 * (-0.737) - 2/5 * (-1.322) = 0.971 One - 1/3 False, 2/3 True; Entropy = -1/3 log(1/3) - 2/3 log(2/3) = 0.918
  Expected Entropy = 5/8 * 0.971 + 3/8 * 0.918 = 0.9511
  Gain (Time) = 1 - 0.9511 = 0.0489

(d) [1 points] Which attribute will you use to construct the root of the tree using the information gain heuristic of the ID3 algorithm?

Ans: The attribute which has the highest information gain: here "Smell".

(e) [8 points] Using the root that you selected in the previous question, construct a decision tree that represents the data. You do not have to use the ID3 algorithm here, you can show any tree with the chosen root.



(f) [3 points] Suppose you are given three more examples, listed in Table 3. Use your decision tree to predict the label for each example. Also report the accuracy of the classifier that you have learned.
Ans: **Accuracy** on Test Data Set = 2/3 = **66.67%**

3. [10 points] Recall that in the ID3 algorithm, we want to identify the best attribute that splits the examples that are relatively pure in one label. Aside from entropy, which we saw in class and you used in the previous question, there are other methods to measure impurity.

Variety	Color	Smell	Time	Ripe?	**Prediction**
Alphonso	Green	Sweet	Two	True	**True**
Keitt	Red	Sweet	One	False	**False**
Haden	Yellow	None	Two	True	**False**

Table 3: Test data for mango prediction problem

We will now develop a variant of the ID3 algorithm that does not use entropy. If, at some node, we stopped growing the tree and assign the most common label of the remaining examples at that node, then the empirical error on the training set at that node will be

$$MajorityError = 1 - \max_i p_i$$

where, p_i is the fraction of examples that are labeled with the i^{th} label.

(a) [2 points] Notice that $MajorityError$ can be thought of as a measure of impurity just like entropy. Just like we used entropy to define information gain, we can define a new version of information gain that uses $MajorityError$ in place of entropy. Write down an expression that defines a new version of information gain that uses $MajorityError$ in place of entropy.

Ans: For Entropy:

$$InformationGain = H(S) - \sum_{v \in Values(Attributes)} (|S_v|/|S|) * entropy(S_v)$$

For Majority Error:

$$NewInformationGain = MajorityError(S) - \sum_{v \in Values(Attributes)} (|S_v|/|S|) * MajorityError(S_v)$$

(b) [6 points] Calculate the value of your newly defined information gain from the previous question for the four features in the mango dataset from 1. Use 3 significant digits. Enter the information gain into Table 4.

Feature	Information Gain (using majority error)
Variety	0.125
Color	0.125
Smell	0.25
Time	0.125

Table 4: Information gain for each feature.

Ans: MajorityError(S, L=True) = 1 - 4/8 = 1/2

- Variety
  MajErr(Alphonso) = 1 - 2/4 = 1/2
  MajErr(Keitt) = 1 - 2/3 = 1/3
  MajErr(Haden) = 1 - 1 = 0
  MajErr(Variety) = 1/2 * 1/2 + 3/8 * 1/3 + 0 = 1/4 + 1/8 = 3/8 = 0.375
  Info Gain = 0.5 - 0.375 = 0.125
- Color
  MajErr(Red) = 1 - 2/3 = 1/3
  MajErr(Yellow) = 1 - 2/3 = 1/3
  MajErr(Green) = 1 - 1/2 = 1/2
  MajErr(Color) = 3/8 * 1/3 + 3/8 * 1/3 + 1/4 * 1/2 = 1/8 + 1/8 + 1/8 = 3/8 = 0.375
  Info Gain = 0.5 - 0.375 = 0.125
- Smell
  MajErr(None) = 1 - 3/4 = 1/4
  MajErr(Sweet) = 1 - 3/4 = 1/4
  MajErr(Smell) = 1/2 * 1/4 + 1/2 * 1/4 = 1/4 = 0.25
  Info Gain = 0.5 - 0.25 = 0.25
- Time
  MajErr(One) = 1 - 2/3 = 1/3
  MajErr(Two) = 1 - 3/5 = 2/5
  MajErr(Time) = 3/8 * 1/3 + 5/8 * 2/5 = 1/8 + 1/4 = 3/8 = 0.375
  Info Gain = 0.5 - 0.375 = 0.125

(c) [2 points] According to your results in the last question, which attribute should be the root for the decision tree? Do these two measures (entropy and majority error) lead to the same tree?
Ans: According to Information Gain on majority error, the root node should be "Smell".
No the two measures might lead to the different tree, since values on other attributes are all the same and anything can be picked. This doesnt enforce the tree to be same as the entropy tree.

2 Linear Classifier

In the questions in this section, we have four features x_1, x_2, x_3 and x_4 and the label is represented by o.

1. [5 points] Find a linear classifier that correctly classifies the given dataset. You dont need to run any learning algorithm here. Recall that a linear classifier is described by a weight vector $\mathbf{w}$ and a bias b. The classifier predicts 1 if $\mathbf{w}^T\mathbf{x} + b \geq 0$ and -1 otherwise. For this problem, the dataset has 4 features, so the weight vector has 4 components w_1, w_2, w_3 and w_4. So the linear classifier will predict 1 if $w_1x_1+w_2x_2+w_3x_3+w_4x_4+b \geq 0$ and -1 otherwise. Specify the values of w_1, w_2, w_3, w_4 and b that correctly predicts the label for the dataset below.

x_1	x_2	x_3	x_4	o
1	1	1	1	1
0	1	1	1	1
1	1	0	0	-1
1	0	0	0	-1

Ans: One set of classifiers:
w_1 = 1, w_2 = 1, w_3 = 1, w_4 = 1 and b = -3
$x_1 + x_2 + x_3 + x_4 - 3 \geq 0$
Another set of classifiers:
w_1 = -1, w_2 = -1, w_3 = 1, w_4 = 1 and b = 0
$-x_1 - x_2 + x_3 + x_4 \geq 0$

2. [6 points] Suppose the dataset below is an extension of the above dataset. Check if your classifier from the previous question correctly classifies the dataset. Report its accuracy.

x1	x2	x3	x4	o	Classifier 1	Classifier 2
0	0	0	0	-1	-1	1
0	0	0	1	-1	-1	1
0	0	1	0	-1	-1	1
0	0	1	1	-1	-1	1
1	0	1	1	1	1	1
1	1	0	1	1	1	-1

Ans: For the given test dataset:
Classifier 1: 100% accuracy
Classifier 2: 1/6 = 16.67% accuracy

3. [9 points] Given the remaining missing data points of the above dataset in the table below, find a linear classifier that correctly classifies the whole dataset (all three tables together). Specify the values of w_1, w_2, w_3, w_4 and b for the classifier.

x1	x2	x3	x4	o
0	1	0	0	-1
0	1	0	1	-1
0	1	1	0	-1
1	0	0	1	-1
1	0	1	0	-1
1	1	1	0	1

Ans: Using Classifier 1: we still get 100% accuracy for the total set of examples. So Classifier 1 with w_1, w_2, w_3, w_4, b = 1, 1, 1, 1, -3 satisfies the entire test/learning data set.

3 Experiments

Cross-Validation

1. [20 points] **Implementation**

   For this problem, your will be using the data in **data** folder. This folder contains two files: **train.csv** and **test.csv**. You will train your algorithm on the training file. Remember that you should not look at or use your testing file until your training is complete.

   (a) [15 points] Implement the decision tree data structure and the ID3 algorithm for your decision tree (Remember that the decision tree need not be a binary tree!). For debugging your implementation, you can use the previous toy examples like the mango data from Table 1. Discuss what approaches and design choices you had to make for your implementation and what data structures you used.

   Ans: I used the concept of classes while creating nodes of the tree and building the decision tree. My tree node has 3 informations: The node type (label or node), based on the attribute type- attribute value (gives the attribute or final label) and dictionary of choices available to that attribute mapped to node objects.
   - Building tree was using the recursive algorithm ID3. The node selection was based on entropy of the attributes on the remaining dataset and information gain for the attributes at each level.
   - Labels were placed based on maximum occurrence of a particular label at that level.
   - Additional usage was on the copy library to do a deep copy of the dictionary in order to be passed down the decision tree. So that any modifications to this data set is not affected in the parent nodes.
   - Handling unknown choices - At every level of the tree I have another branch on the node called "others" along with all other possible choices on that node attribute. This is of type Label node. And assumes the max frequent label value in the dataset at that level. This handles unknown choices while traversing down the tree (if there is no choice among the choice list return the value at "others" branch).

   (b) [1 points] Report the error of your decision tree on the **data/train.csv** file.

   Ans: The accuracy on the decision tree for the training set was 100%.

   (c) [1 points] Report the error of your decision tree on the **data/test.csv** file.

   Ans: The accuracy on the decision tree for the testing set was 100%. This is a weird case but it means training set accounted for all possible outcomes which might appear for future cases.

   (d) [3 points] Report the maximum depth of your decision tree.

Ans: The maximum depth of the decision tree built was 6. Assuming root node to be counted as depth of 0. Used a recursive algorithm to traverse till the leaf nodes. Return 0 on leaf nodes and at every level return max depth among its children + 1.

2. [20 points] **Limiting Depth**

In this section, you will be using 5-fold cross-validation in order to limit the depth of your decision tree, effectively pruning the tree to avoid overfitting. You will be using the 5 cross-validation files for this section, titled data/CVfolds/foldX.csv where X is a number between 1 and 5 (inclusive)

(a) [10 points] Run 5-fold cross-validation using the specified files. Experiment with depths in the set $\{1, 2, 3, 4, 5, 10, 15\}$, reporting the average cross-validation accuracy and standard deviation for each depth. Explicitly specify which depth should be chosen as the best, and explain why.

```
Depth       Accuracy            Std. Dev
depth1 : 0.767228177641654 : 0.13453882989269683
depth2 : 0.9595712098009189 : 0.03868519610283838
depth3 : 0.9784073506891271 : 0.02287576008314949
depth4 : 0.9796324655436447 : 0.01749143762967826
depth5 : 0.9843797856049005 : 0.012827192399002712
depth10 : 0.9848392036753445 : 0.013327523872609227
depth15 : 0.9848392036753445 : 0.013327523872609227
```

Looking at the values on Accuracy and standard deviation, it looks like there is not much accuracy improvement from depth 5 to depth 10. And also Standard deviation on depth 5 is the least among all the depths. So picking depth 5 is the best.

(b) [5 points] Using the depth with the greatest cross-validation accuracy from your experiments: train your decision tree on the **data/train.csv** file. Report the accuracy of your decision tree on the **data/test.csv** file.

Ans: Accuracy on train.csv = 99.72% and accuracy on test.csv = 99.62 %

(c) [5 points] Discuss the performance of the depth limited tree as compared to the full decision tree. Do you think limiting depth is a good idea? Why?

Ans: Though there is a slight drop in accuracy of 0.2%, limiting tree is mostly a good idea. This helps minimize over fitting the decision tree to the given training set. There by helps minimize fitting the decision tree to include the noise in the given training data set. Also limiting the tree depth helps speed up learning phase to some extent.

4 CS 6350 only: Decision Trees with Attribute Costs

[10 points] Sometimes, we may encounter situations where the features in our learning problem are associated with costs. For example, if we are building a classifier in a medical scenario, features may correspond to the results of different tests that are performed on a patient. Some tests may be inexpensive (or inflict no harm), such as measuring the patient's body temperature or weight. Some other tests may be expensive (or may cause discomfort to the patient), such as blood tests or radiographs.

In this question, we will explore the problem of learning decision trees in such a scenario. We prefer decision trees that use features associated with low costs at the top of the tree and only use higher cost features if needed at the bottom of the trees. In order to impose this preference, we can modify the information gain heuristic that selects attributes at the root of a tree to penalize costly attributes.

In this question, we will explore different such variations. Suppose we denote $Gain(S, A)$ as the information gain of an attribute A for a dataset S (using the original version of information from ID3). Let $Cost(A)$ denote the cost of the attribute A. We can define two cost-sensitive information gain criteria for attributes as:

1. $Gain_T(S, A) = \frac{Gain(S,A)^2}{Cost(A)}$
2. $Gain_N(S, A) = \frac{2^{Gain(S,A)}-1}{\sqrt{Cost(A)+1}}$

In both cases, note that attributes with higher costs are penalized and so will get chosen only if the information gain is really high.

To evaluate these two methods for root selection, we will use the following training set:

Shape	Color	Size	Material	Label
square	red	big	metal	+
square	blue	small	plastic	+
triangle	yellow	medium	metal	+
triangle	pink	big	leather	-
square	pink	medium	leather	-
circle	red	small	plastic	-
circle	blue	small	metal	-
ellipse	yellow	small	plastic	-
ellipse	blue	big	leather	+
ellipse	pink	medium	wood	+
circle	blue	big	wood	+
triangle	blue	medium	plastic	+

Suppose we know the following costs of the attributes:

Attribute	Cost
Shape	10
Color	30
Size	50
Material	100

1. [8 points] Compute the modified gains $Gain_T$ and $Gain_S$ for each attribute using these costs. Fill in your results in the table below. (upto 3 decimal places)

Attribute	$Gain_T$	$Gain_N$
Shape	0.00037	0.013
Color	0.00045	0.015
Size	0.00057	0.0174
Material	0.00035	0.0138

Expected Entropy = -7/12 log (7/12) - 5/12 log (5/12) = -0.583*-0.778 - 0.417 * -1.261 - = 0.454 + 0.526 = 0.98

- Shape
  Entropy(square) = - 2/3 log (2/3) - 1/3 log (1/3) = -0.667*-0.584 - 0.333 * -1.586 = 0.39 + 0.529 = 0.919
  Entropy(triangle) = -2/3 log (2/3) - 1/3 log (1/3) = 0.919
  Entropy(circle) = -1/3 log (1/3) - 2/3 log (2/3) = 0.919
  Entropy(ellipse) = -1/3 log (1/3) - 2/3 log (2/3) = 0.919
  Entropy(Shape) = 1/4 * 0.919 * 4 = 0.919
  Information Gain = 0.98 - 0.919 = 0.061
  $Gain_T = 0.061^2/10 = 0.00037$
  $Gain_N = (2^{0.061} - 1)/\sqrt{11} = 0.013$
- Color
  Entropy(red) = - 1/2 log (1/2) - 1/2 log (1/2) = 1
  Entropy(blue) = - 4/5 log (4/5) - 1/5 log (1/5) = 0.2576 + 0.4644 = 0.722
  Entropy(yellow) = -1/2 log (1/2) - 1/2 log (1/2) = 1
  Entropy(pink) = - 2/3 log (2/3) - 1/3 log (1/3) = 0.919
  Entropy(Color) = 1/6 * 1 + 5/12 * 0.722 + 1/6 * 1 + 1/4 * 0.919 = 0.333 + 0.301 + 0.23 = 0.864
  Information Gain = 0.98 - 0.864 = 0.116
  $Gain_T = 0.116^2/30 = 0.00045$
  $Gain_N = (2^{0.116} - 1)/\sqrt{31} = 0.015$
- Size
  Entropy(big) = -3/4 log (3/4) - 1/4 log (1/4) = 0.31125 + 0.5 = 0.81125
  Entropy(medium) = -3/4 log (3/4) - 1/4 log (1/4) = 0.81125
  Entropy(small) = -3/4 log (3/4) - 1/4 log (1/4) = 0.81125
  Entropy(Size) = 0.81125
  Information Gain = 0.98 - 0.81125 = 0.169
  $Gain_T = 0.169^2/50 = 0.00057$
  $Gain_N = (2^{0.169} - 1)/\sqrt{51} = 0.0174$
- Material
  Entropy(metal) = -2/3 log (2/3) - 1/3 log (1/3) = 0.919
  Entropy(plastic) = -1/2 log (1/2) - 1/2 log (1/2) = 1
  Entropy(leather) = -2/3 log (2/3) - 1/3 log (1/3) = 0.919

Entropy(wood) = 0
Entropy(Material) = 1/4 * 0.919 + 1/3*1 + 1/4 * 0.919 = 0.333 + 0.4595 = 0.7925
Information Gain = 0.98 - 0.7925 = 0.1875
$Gain_T = 0.1875^2/100 = 0.00035$
$Gain_N = (2^{0.1875} - 1)/\sqrt{101} = 0.0138$

2. [2 points] For each variant of gain, which feature would you choose as the root?
Ans: For both variants Size has the maximum Gain. So "Size" will be the root node.