ME3040
Mechanical Vibrations

Course Project

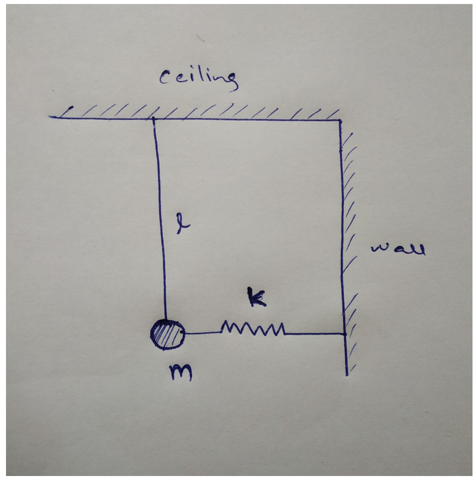
Topic:

Simple pendulum attached with horizontal spring

Rajath R
131801023
IIT Palakkad

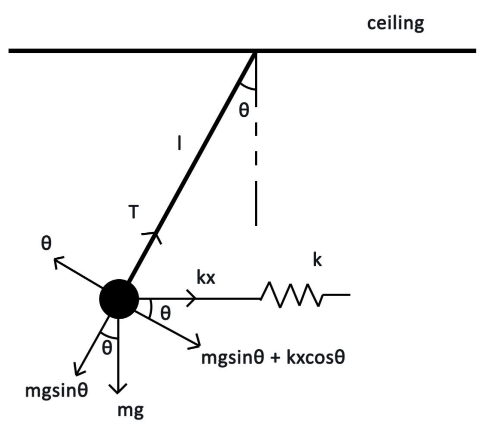
Simple pendulum attached with horizontal spring

This vibrating system consists of a simple pendulum hanging from a ceiling and a spring which is attached to it in the horizontal direction. The spring is attached to the bob and its other end is attached to the vertical wall. The forces acting on the bob will be Gravity, Tension and the spring force.



Here, we are taking the assumption that the **spring force is always in the horizontal direction**. We can consider this by taking a spring which is very long, such that the small deflection it has during the oscillation of the bob will not make much change in the direction of spring force.

FBD of the bob:



Pendulum rod of length l, bob of mass m and spring constant of stiffness k.
When rod is at an angular position θas shown, extension in spring= $x = Lsin\theta$

Acceleration in radial and axial directions,

$a_r = \overline{r} - r\widehat{\theta}, a_\theta = r\overline{\theta} + 2\widehat{r}\widehat{\theta}$

Where $\widehat{\theta} = \frac{d\theta}{dt}$, $\overline{\theta} = \frac{d^2\theta}{dt^2}$ and $\widehat{r} = \frac{dr}{dt}$, $\overline{r} = \frac{d^2r}{dt^2}$

Force balance in axial direction,
$\Sigma F_\theta = ma_\theta$

$- mgsin\theta - kxcos\theta = a_\theta = ml\overline{\theta}$

$- mgsin\theta - klsin\theta cos\theta = ml\overline{\theta}$

$\overline{\theta} =- \frac{g}{l}sin\theta - \frac{k}{m}sin\theta cos\theta$

$\frac{d^2\theta}{dt^2} =- \frac{g}{l}sin\theta - \frac{k}{m}sin\theta cos\theta$ -------Equation of motion

---

To solve the problem, the following values are taken:
m=1kg, l=1m, $\theta_{in} = \pi/3\ rad$, $\omega_{in} = 0$, k=200 N/m

To solve for the trajectory of the bob, numerical methods are used. (Euler's Method)
For a very small change in time Δt,we can write

$$\frac{d\theta(t_0+d\Delta t)}{dt} = \frac{d\theta(t_0)}{dt} + \Delta t\frac{d^2t_0}{dt^2}$$

$$\theta(t_0 + \Delta t) = \theta(t_0) + \Delta t\frac{d\theta(t_0)}{dt}$$

These equations are used to find θ at different times t.
A total of 30000 iterations are done and Δt is taken as 0.0005 seconds. So we simulate till t=15s from the starting position.
From this, using python libraries, we create images of the system at each θand by combining these, we get the final rendered video.

We also plot the angle θ with time to understand the exact mechanism.