Hall conductivity of Aubry-André system driven by rapidly oscillating magnetic field

Rajath Shashidhara
2012B5A7589P
rajath.shashidhara@gmail.com

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Under the supervision of

Dr. Tapomoy Guha Sarkar

Department of Physics
Birla Institute of Technology and Science, Pilani

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Abstract

Study of quantum hall effect in 2D square lattice driven by oscillatory magnetic field perpendicular to the plane. The time-dependent hamiltonian is solved in fourier space using Brillouin-Wigner perturbation theory, in the high frequency limit. Investigation of localization/delocalization transitions of time-averaged wavefunctions and calculation of hall conductivity of the system using TKNN invariant. Comparison of the results with the effective static hamiltonian picture carried out in the previous work.

Acknowledgements

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Declaration

I, Rajath Shashidhara, declare that this thesis titled, 'Hall conductivity of Aubry-André system driven by rapidly oscillating magnetic field' and the work presented in it is my original work. I affirm that all references are clearly attributed and any collaboration with others has been duly acknowledged.

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Certificate

This is to certify that the thesis titled, 'Hall conductivity of Aubry-André system driven by rapidly oscillating magnetic field' and submitted by Rajath Shashidhara 2012B5A7589P in partial fulfillment of the requirements of BITS F421T embodies the work done by him under my supervision.

Supervisor

Dr. Tapomoy Guha Sarkar
Department of Physics
Birla Institute of Technology and Science, Pilani

Date: November 16, 2016

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Appendix A

Brillouin-Wigner Perturbation Theory

When encountered with an analytically intractable quantum mechanical problem, perturbation techniques may be used to obtain approximate solutions. Formally, the problem can be stated as follows : Given a hamiltonian $\hat{H} = \hat{H}_0 + \hat{V}$ where $\hat{H}_0$ is exactly solvable and $\hat{V}$ is the perturbation term, and the eigendecomposition of $\hat{H}_0$ —

$$\hat{H}_0 \left|n\right\rangle = \epsilon_n \left|n\right\rangle$$
$$\langle m|n\rangle = \delta_{mn}$$
$$\sum_m \left|m\right\rangle \left\langle m\right| = 1$$

Brillouin-Wigner (BW) perturbation theory expresses $\{\left|\psi_n\right\rangle, \ldots\}$ and $\{E_n, \ldots\}$, such that $\hat{H}\left|\psi_n\right\rangle = E_n\left|\psi_n\right\rangle$ in terms of $\hat{V}$ and $\{\left|n\right\rangle, \ldots\}$. A simplistic exposition of BW theory is presented below.

To obtain the BW perturbative expansion, we begin with the eigenvalue equation.

$$\hat{H}\left|\psi_n\right\rangle = (\hat{H}_0 + \hat{V})\left|\psi_n\right\rangle = E_n\left|\psi_n\right\rangle$$

The wavefunctions $\left|\psi_n\right\rangle$ are normalized as $\langle n|\psi_n\rangle = 1$, as discussed in [1]. On contracting with $\left\langle n\right|$,

$$\langle n|(\hat{H}_0 + \hat{V})|\psi_n\rangle = E_n \langle n|\psi_n\rangle$$
$$\epsilon_n \langle n|\psi_n\rangle + \langle n|\hat{V}|\psi_n\rangle = E_n \langle n|\psi_n\rangle$$
$$E_n = \epsilon_n + \langle n|\hat{V}|\psi_n\rangle \tag{A.1}$$

Rewriting the eigenvalue equation as

$$\begin{aligned}
(E_n - \hat{H}_0)\left|\psi_n\right\rangle &= \hat{V}\left|\psi_n\right\rangle \\
&= 1\,\hat{V}\left|\psi_n\right\rangle \\
&= \sum_m \left|m\right\rangle\left\langle m\right|\hat{V}\left|\psi_n\right\rangle \\
&= \left|n\right\rangle\langle n|\hat{V}|\psi_n\rangle + (1 - \left|n\right\rangle\left\langle n\right|)\hat{V}\left|\psi_n\right\rangle \\
\text{Using Eq. A.1,}& \\
&= (E_n - \hat{H}_0)\left|n\right\rangle + (1 - \left|n\right\rangle\left\langle n\right|)\hat{V}\left|\psi_n\right\rangle \\
(E_n - \hat{H}_0)(\left|\psi_n\right\rangle - \left|n\right\rangle) &= (1 - \left|n\right\rangle\left\langle n\right|)\hat{V}\left|\psi_n\right\rangle \\
\left|\psi_n\right\rangle &= \left|n\right\rangle + (E_n - \hat{H}_0)^{-1}(1 - \left|n\right\rangle\left\langle n\right|)\hat{V}\left|\psi_n\right\rangle
\end{aligned}$$

Define resolvent operator as $\hat{R}_n = (E_n - \hat{H}_0)^{-1} = \sum_n |n\rangle (E_n - \epsilon_n)^{-1} \langle n|$,

$$|\psi_n\rangle = |n\rangle + \hat{R}_n(1 - |n\rangle \langle n|)\hat{V} |\psi_n\rangle \tag{A.2}$$

The above iterative equation is the main result of BW perturbation theory. Solving Eq. (A.2) self-consistently with Eq. (A.1), solutions to the eigenvalue equation are obtained. No approximation has been used until this point, and exact solution can be obtained if iterated infinitely. In practice, approximate solution is obtained by truncating the iteration.

Further, Eq. (A.2) can be simplified by expanding the recurrence relation

$$|\psi_n\rangle = |n\rangle + \hat{R}_n\hat{Q}_n\hat{V} |\psi_n\rangle$$

where $\hat{Q}_n = 1 - |n\rangle \langle n|$,

$$\begin{aligned} &= |n\rangle + \hat{R}_n\hat{Q}_n\hat{V} |n\rangle + \hat{R}_n\hat{Q}_n\hat{V}\hat{R}_n\hat{Q}_n\hat{V} |n\rangle + \dots \\ &= \sum_{k=0}^{\infty} \{\hat{R}_n\hat{Q}_n\hat{V}\}^k |n\rangle \\ &= (1 - \{\hat{R}_n\hat{Q}_n\hat{V}\})^{-1} |n\rangle \end{aligned} \tag{A.3}$$

When higher order term contributions are diminishingly small, truncating the series produces approximate solutions to the problem.

Unlike Rayleigh-Schrodinger (RS) perturbation theory, BW theory does not rely on power series expansion requiring strict analyticity and does not require seperate treatment of degenerate case. RS theory is an approximation to BW theory obtained by power series expansion of $(E_n - \epsilon_m)^{-1}$ in the resolvent operator [2]. [3] provides an excellent comparison between RS and BW perturbation theories including situations where BW perturbation technique is more applicable.

Recent efforts to extend the theory to many-body systems has led to systematization of BW theory in terms of model space and effective hamiltonian formalism. This new representation requires the introduction of a model space with respect to a set of reference states. Any complete set of orthonormal states of hilbert space is chosen as a set of reference states. Usually, eigenstates of the unperturbed hamiltonian $\hat{H}_0$ is chosen as a set of reference states. The hilbert space is partitioned into model space and orthogonal space, by choosing one state from the set of reference states as model state [1].

Let $R \equiv \{|\phi_n\rangle \dots\}$ is the set of reference states, $|\phi_0\rangle \in R$ is the model state, then $P = |\phi_0\rangle \langle\phi_0|$ is the corresponding projection operator of the model space and $Q = 1 - P$ is the projection operator corresponding to orthogonal space. A state $|\psi\rangle$ in the hilbert space can be projected onto the model space using operator P, $|\phi\rangle = P|\psi\rangle$ and a wavefunction $|\phi\rangle$ in model space can be reconstructed in hilbert space using the wave operator Ω as $|\psi\rangle = \Omega|\phi\rangle$. Provided the eigenvalue equation $\hat{H}|\psi\rangle = E|\psi\rangle$, then $|\phi\rangle = P|\psi\rangle$ satisfies the equation $\hat{H}_{eff}|\phi\rangle = E|\phi\rangle$, where

$$\hat{H}_{eff} = P\hat{H}\Omega P \tag{A.4}$$

[1] In this treatment, a single reference state is chosen as the model function. See [2] for multi-reference partitioning.

$$
\begin{aligned}
\hat{H}_{eff}\left|\phi\right\rangle &= P\hat{H}\Omega P\left|\phi\right\rangle \\
&= P\hat{H}\Omega PP\left|\psi\right\rangle \\
&= P\hat{H}\left|\psi\right\rangle \\
&= E\,P\left|\psi\right\rangle \\
&= E\left|\phi\right\rangle
\end{aligned}
$$

Therefore, the eigenvalues of the effective hamiltonian are equal to the eigenvalues of the original hamiltonian and the eigenfunctions of the original hamiltonian can be obtained by application of the wave operator Ω on the eigenfunctions of the effective hamiltonian $\left|\psi\right\rangle = \Omega\left|\phi\right\rangle$.

How do we obtain the wave operator Ω ? Operating Q on the eigenvalue equation,

$$
\begin{aligned}
Q\hat{H}\left|\psi\right\rangle &= E\,Q\left|\psi\right\rangle \\
Q\left|\psi\right\rangle &= \frac{Q\hat{H}}{E}\left|\psi\right\rangle
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
\left|\psi\right\rangle &= (P+Q)\left|\psi\right\rangle \\
&= P\left|\psi\right\rangle + Q\left|\psi\right\rangle \\
&= P\left|\psi\right\rangle + \frac{Q\hat{H}}{E}\left|\psi\right\rangle \qquad \text{(A.5)} \\
P\left|\psi\right\rangle &= \left(1-\frac{Q\hat{H}}{E}\right)\left|\psi\right\rangle \\
\left|\psi\right\rangle &= \left(1-\frac{Q\hat{H}}{E}\right)^{-1} P^2\left|\psi\right\rangle \qquad \text{(A.6)}
\end{aligned}
$$

Eq. (A.5) has the familiar iterative form as Eq. (A.2) with the choice of $P = \left|n\right\rangle\left\langle n\right|$. From Eq. (A.6),

$$
\Omega = \left(1-\frac{Q\hat{H}}{E}\right)^{-1} P \qquad \text{(A.7)}
$$

The inverse operation in the wave operator can be expanded to obtain the perturbative expansion.

Above presentation of partitioning and effective hamiltonian formalism of BW theory is described in detail in [2].

Bibliography

[1] J. J. Sakurai and J. Napolitano, *Modern quantum mechanics* (Addison-Wesley, 2011), pp. 306–309.

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