$$\rho(x,y,x) \;=\; \{constant,\ x^2+y^2+z^2 \le 1; 0,\ otherwise\}$$

$$\sigma(x,y) = \int_{-\sqrt{1-x^2-y^2}}^{\sqrt{1-x^2-y^2}} \rho(x,y,z)\,dz \;=\; 2\rho\sqrt{1-x^2-y^2} = 2\rho\sqrt{1-r^2}\ \textit{inside the circle of radius } 1$$

axially symmetric system —– denoting the position of light ray as (ξ,θ) *and source as* (β,ϕ).

$$\alpha_x(\xi,\theta) = \int_0^{2\pi}\int_0^1 2\rho\frac{(\xi cos\theta - rcos\Phi)}{(\xi cos\theta - rcos\Phi)^2+(\xi sin\theta - rsin\Phi)^2}\sqrt{1-r^2}\ rdrd\Phi$$

$$\alpha_y(\xi,\theta) = \int_0^{2\pi}\int_0^1 2\rho\frac{(\xi sin\theta - rsin\Phi)}{(\xi cos\theta - rcos\Phi)^2+(\xi sin\theta - rsin\Phi)^2}\sqrt{1-r^2}\ rdrd\Phi$$

because it is axially symmetric we can choose $\theta = 0$
solving the above integral as a complex integral, (refer. Meneghetti eq. 2.12; also, there in the MOND-non-spherical-lensing notes page 10).

$$\alpha_x(\xi,\theta) = \int_0^{2\pi}\int_0^1 2\rho\frac{(\xi cos\theta - rcos\Phi)}{(\xi cos\theta - rcos\Phi)^2+(\xi sin\theta - rsin\Phi)^2}\sqrt{1-r^2}\ rdrd\Phi = \frac{4\pi\rho}{3\xi}(1-(1-\xi^2)^{\frac{3}{2}})\ for\ |\xi| < 1$$

$$= \frac{4\pi\rho}{3\xi}\ for\ |\xi|\ > 1$$

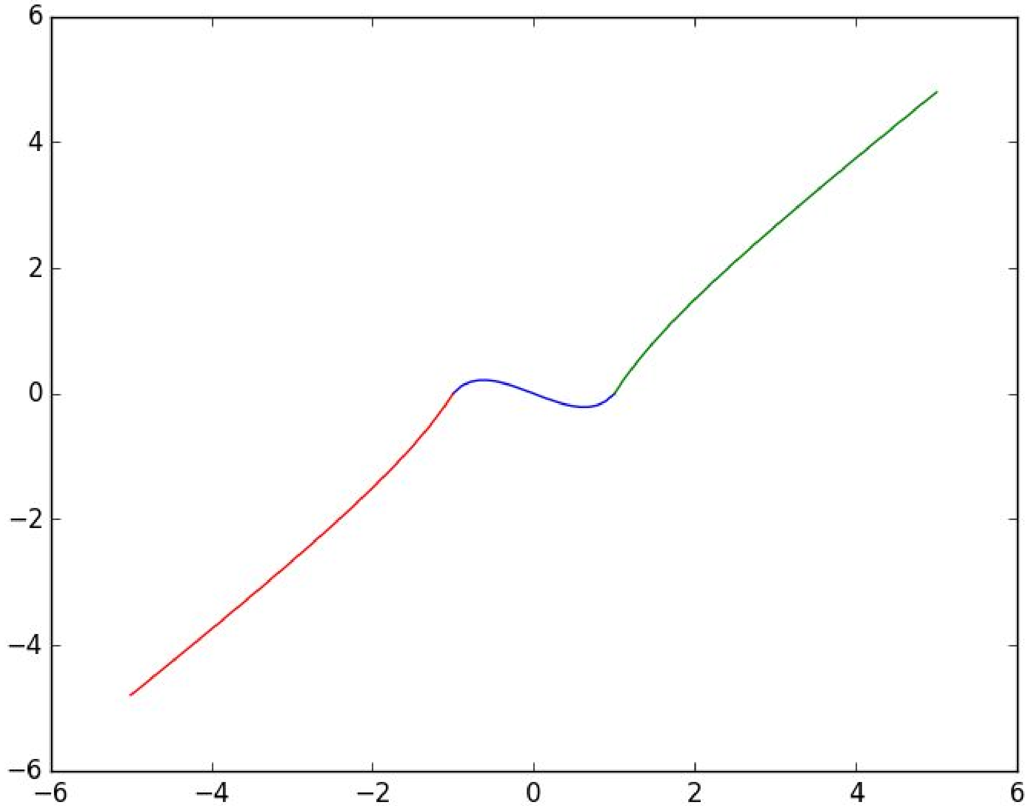
$$\alpha_y(\xi) = \int_0^{2\pi}\int_0^1 2\rho\frac{(-rsin\Phi)}{(\xi cos\theta - rcos\Phi)^2+(\xi sin\theta - rsin\Phi)^2}\sqrt{1-r^2}\ rdrd\Phi \;=\; 0$$

therefore,

$$\beta = \xi - \frac{4\pi\rho}{3\xi}(1-(1-\xi^2)^{\frac{3}{2}})\ for\ |\xi| < 1$$

$$= \xi \;-\; \frac{4\pi\rho}{3\xi}\ for\ |\xi|\ > 1$$

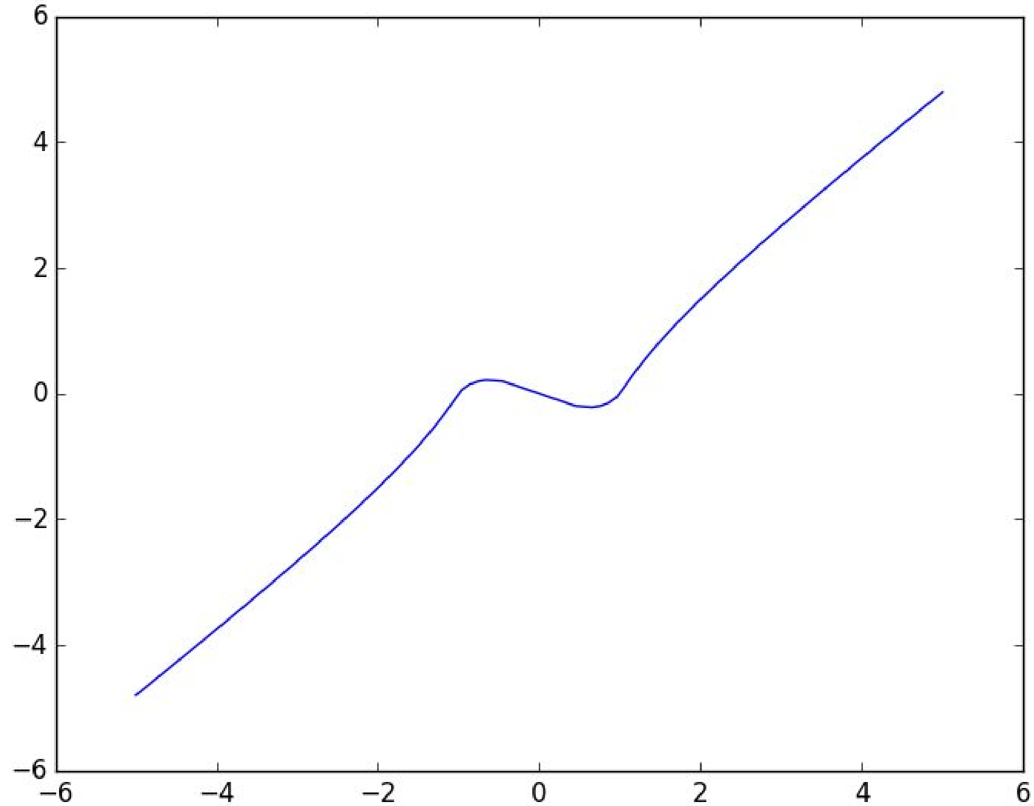
plot of the above function for $\xi = (-5.0, 5.0)\ for\ \rho = \frac{3}{4\pi}$



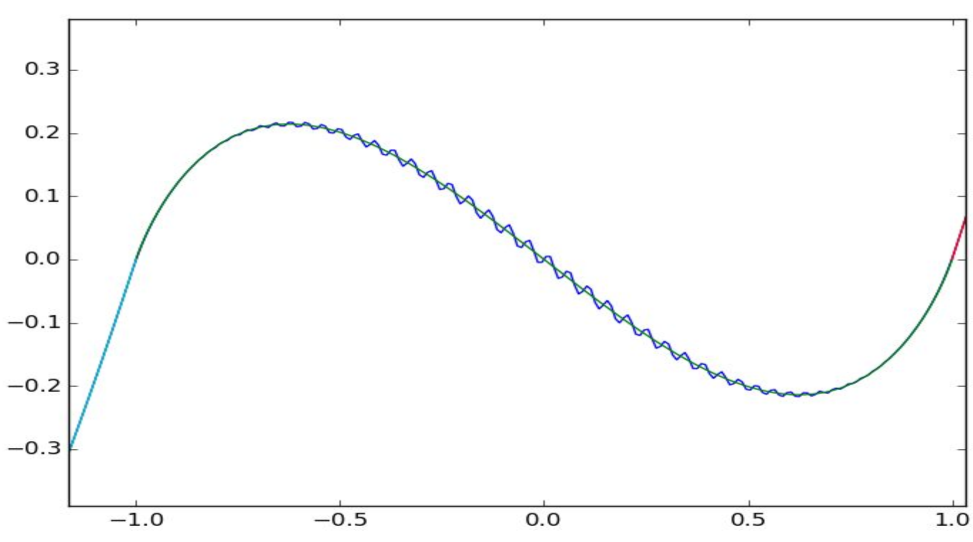
There are 3 images for $\beta\ \varepsilon\ (-0.22, 0.22)$ *and* 1 *image for* $\beta\ \varepsilon\ \Re - (-0.22, 0.22)$

The function

β *obtained from numerical integration*(*gauss* − *legendre*) *in my code is* :



When zoomed in on the bend region:



The error due to numerical integration is oscillatory. This leads to detection of wrong number of images due to a given source. A horizontal line intersects these tiny peaks at at least 2 points increasing the number of images by 2 or more.