CONCORDIA UNIVERSITY

DEPARTMENT OF COMPUTER SCIENCE AND SOFTWARE ENGINEERING

COMP 6651: Algorithm Design Techniques

Fall 2019

Quiz # 1

2.5 per question for questions 1 to 4: .5 for each correct answer + 0.5 if all answers for a given question are correct.

1. Complexity of $1^3 + 2^3 + 3^3 + + n^3$

   a. $\Omega(n^3)$ ✓

   b. $\Theta(n^4)$ ✓

   c. $O(n^2)$ ✗

   d. $\Omega(n^2)$ ✓

2. $e^{\log_2 n}$ is

   a. $O(n^2)$ ✓

   b. $O(n)$ ✗

   c. $O(2^n)$ ✓

   d. $O(\log n)$ ✗

3. An array of n numbers is given, where n is an even number. The maximum as well as the minimum of these n numbers needs to be determined. Which of the following is TRUE about the minimum number of comparisons needed?

   a. At most $1.5n - 2$ comparisons are needed. ✓

   b. At least $n \log_2 n$ comparisons are needed. ✗

   c. At least $2n - c$ comparisons, for some constant c, are needed. ✗

   d. None of the above ✗

4. Complexity of $\log n!$

   a. $O(2^n)$ ✓

   b. $O(n \log n)$ ✓

   c. $O(2^{\log n})$ ✗

   d. $O(n!)$ ✓

5. Solve the following recurrence equation

$$\begin{array}{ll} t_n = 2t_{n-2} - t_{n-4} & n \geq 4 \\ t_n = n & 0 \leq n \leq 3. \end{array}$$

Express your solution with the simplest expression using the Θ notation.

Characteristic equation : $x^4 - 2x^2 + 1 = 0$ **(2 points)**
$\rightsquigarrow (x^2-1)^2$ because of the identity:

$$(a-b)^2 = a^2 + b^2 - 2ab.$$

Then $x^2 - 1$ is a difference of two squared terms, we use the identity

$$a^2 - b^2 = (a-b)(a+b).$$

It leads to: $x^2 - 1 = (x-1)(x+1)$.
Characteristic equation is then equivalent to:

$$(x-1)^2(x+1)^2 = 0. \qquad \textbf{(2 points)}$$

Both two roots 1 and -1 are of multiplicity two. **(2 points)**

$G(n) = (C_1 + C_2 n)(1)^n + (C_3 + C_4 n)(-1)^n = C_1 + C_2 n + (C_3 + C_4 n)(-1)^n$ **(2 points)**

$n = 2k$**:** $G(2k) = C_1 + C_3 + n(C_2 + C_4) = A_1 + B_1 n$

$n = 2k+1$**:** $G(2k+1) = C_1 - C_3 + n(C_2 - C_4) = A_2 + B_2 n$

It follows:

$$C_1 = \frac{A_1 + A_2}{2}, \quad C_2 = \frac{B_1 + B_2}{2}, \quad C_3 = \frac{A_1 - A_2}{2}, \quad C_4 = \frac{B_1 - B_2}{2}.$$

Clearly, the coefficient of n is not zero, as otherwise t_n = constant and then the recurrence solution is not satisfied unless $t_n = 0$, consequently

$$t_n = \Theta(n). \qquad \textbf{(2 points)}$$