Part I: Theoretical questions

Q1 (6 points)

First, we need to obtain the histogram equalization transformation (i.e., the CDF) of p_r:

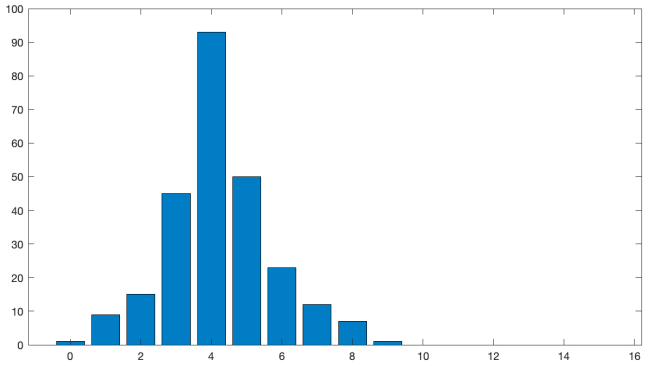
$$s = T(r) = \int_0^r p_r(w)\,dw = \int_0^r (-2w + 2)\,dw = -r^2 + 2r \quad \textbf{(2 points)}$$

Then, we need to find the CDF of p_z:

$$v = G(z) = \int_0^z p_z(w)\,dw = \int_0^z 2w\,dw = z^2 \quad \textbf{(2 points)}$$

Finally, $z = G^{-1}(v) = \pm\sqrt{v}$
However, only positive intensity levels are allowed, so $z = \sqrt{v}$ (**1 point**). Then, we replace v and with s, which gives us $z = \sqrt{-r^2 + 2r}$ (**1 point**).

Q2 (20 points)



(b) **(10 points): (i) and (ii) are 5 points each.**
i) Calculating s_k and histogram equalization

Here L = 16 and n =256

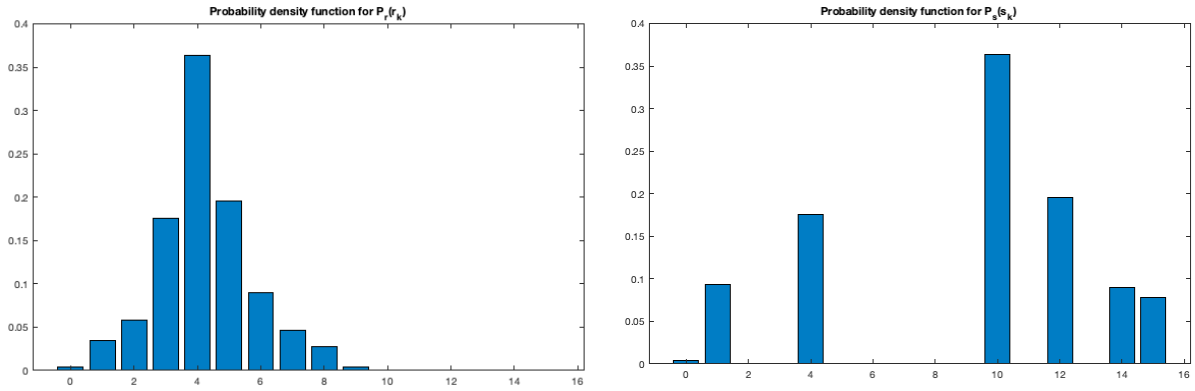
Grey level r_k	**# of pixels n_k**	$\boldsymbol{p_r(r_k) = \frac{n_k}{n}}$	$\boldsymbol{\widetilde{s_k} = \frac{L-1}{n}\sum_{j=0}^{k} n_j}$	**Discrete values s_k**
0	1	0.004	0.06	0
1	9	0.035	0.59	1
2	15	0.059	1.46	1
3	45	0.176	4.10	4
4	93	0.363	9.55	10
5	50	0.195	12.48	12
6	23	0.090	13.83	14
7	12	0.047	14.53	15
8	7	0.027	14.94	15

9	1	0.004	15.00	15
10	0	0.000	15.00	15
11	0	0.000	15.00	15
12	0	0.000	15.00	15
13	0	0.000	15.00	15
14	0	0.000	15.00	15
15	0	0.000	15.00	15

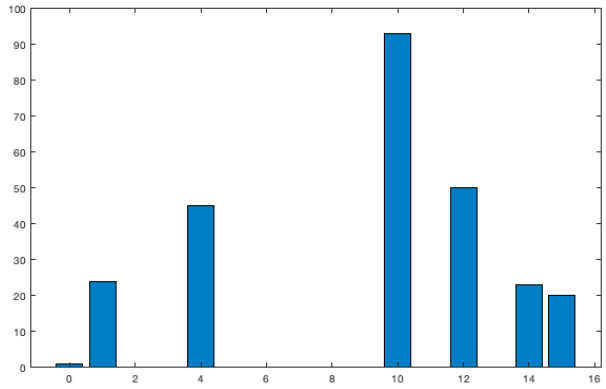
For histogram equalization, we map the grey level values by S=T(r) transformation, which yields the following table.

Grey level s_k	**# of pixels n_{s_k}**	$\boldsymbol{p_s(s_k) = \frac{n_{s_k}}{n}}$
0	1	0.004
1	24	0.094
2	0	0.000
3	0	0.000
4	45	0.176
5	0	0.000
6	0	0.000
7	0	0.000
8	0	0.000
9	0	0.000
10	93	0.363
11	0	0.000
12	50	0.195
13	0	0.000
14	23	0.090
15	20	0.078

ii) Probability density function for $\boldsymbol{p_r(r_k)}$ **and** $\boldsymbol{p_s(s_k)}$



(c) **(5 points)** Histogram of image after histogram equalization



Q3 (4 points)

If we use h_s to represent the histogram of $s(x,y) = f(x,y) - g(x,y)$, then the intensity value of $s(x,y)$ is $u_k - c$. In this case, the height of each bin from h_s is the same as h_f (**2 points**), but the bin locations are shifted to the left by the amount of c (**2 points**).

Q4 (6 points)

First, we need to convert the intensity value of each pixel into binary:

$$\begin{matrix} 0 & 1 & 8 & 6 \\ 2 & 2 & 1 & 1 \\ 1 & 15 & 14 & 12 \\ 3 & 6 & 9 & 10 \end{matrix} \rightarrow \begin{matrix} 0000 & 0001 & 1000 & 0110 \\ 0010 & 0010 & 0001 & 0001 \\ 0001 & 1111 & 1110 & 1100 \\ 0011 & 0110 & 1001 & 1010 \end{matrix}$$ **(2 points)**

Then the 4 bit maps from the highest bit to the lowest are (each bit map = **1 point**):

$$\begin{matrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{matrix} \quad , \quad \begin{matrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{matrix} \quad , \quad \begin{matrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{matrix} \quad , \quad \begin{matrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{matrix}$$

Part II: programming questions

1- 1 point
2- 3 points
3- 2 points
4- 3 points
5- 1 point