COMP 6651: Assignment 10

Fall 2020

Submission through Moodle is due by November 29th at 23:55

1. Consider the following language

$$SPECIAL - CLIQUE = \{\langle G\rangle \mid G = (V, E) \text{ has a clique of size at least } |V|/2\}$$

Prove that $SPECIAL - CLIQUE$ is $\mathcal{NP}$-complete.

2. Consider the following language

$$L_{\text{COL}} = \{\langle G, k\rangle \mid \ G \text{ is } k\text{-colorable}\}.$$

It is known that L_{COL} is $\mathcal{NP}$-complete. Use this fact to prove that the language corresponding to the following scheduling decision problem is $\mathcal{NP}$-complete.

There are k final exams $F_1, \ldots, F_k$ to be scheduled, and ℓ students. For each student i, you are also given the subset of exams $S_i \subseteq \{F_1, \ldots, F_k\}$ that the student is taking. In addition you are given a natural number h. You must schedule the exams into time slots so that no student is required to take two exams in the same slot. The problem is to determine if such schedule exists that uses at most h time slots. State this problem as a language and prove that it is $\mathcal{NP}$-complete.

3. Let G be an undirected graph. Consider the following version of the game Minesweeper. Each node in G is either empty or contains a single hidden mine. If the player clicks on a node with a mine, the player loses the game. If the player clicks on a node without a mine, the node is labelled with the number of mines contained in the adjacent (neighbouring) nodes. The regular Minesweeper game is a special case played on the grid graph.

Now, consider *mine labelling feasibility problem*. You are given a graph G, in which some nodes are labelled with numbers and other nodes are unlabelled. The goal is to decide if it is possible to place mines on some of the unlabelled nodes such that all labels are correct, that is if node v is labelled with number k then it has exactly k neighbours with mines.

   (a) Formulate mine labelling feasibility problem as a language.

   (b) Show that $3SAT$ reduces to this language.

   (c) Derive the corollary that mine labelling feasibility problem is $\mathcal{NP}$-complete.