COMP 6651: Assignment 1

Submission through Moodle is **due by Feb 2nd at 23:59**

Instructions(!!!, apply to all assignments):

- **You are allowed to work with classmates, but you should write and submit your own solution.**
- **Each problem is worth the same number of points. For example, suppose some assignment has 6 problems, then each problem is worth** $100/6$ **points.**
- **Only pdf submissions are accepted, all other submissions will be ignored. Always double-check that you submitted the correct file.**
- **You can type or hand-write your solutions. You are responsible for making your writeup clearly readable. If the marker can't read your solution due to poor handwriting, the solution gets a grade of 0.**
- **Submit a file of reasonable size and quality. If you are handing in scanned images, they should have a good enough resolution, but not weigh dozens of megabytes.**
- **Your penalty for late submission is** $f(x) = \min(\frac{5}{144}x^2, 100)$**, where** x **is the number of hours by which your submission is late. Late for 5 minutes is considered as late for 1 hour. Thus, if your submission is late by 24 hours then the penalty is** $f(24) = \min(\frac{5}{144}24^2, 100) = 20$**, i.e., 20% penalty for 1 day delay. Note that** $f(54) = 100$**, i.e., penalty is 100% after 2 days and 6 hours.**

PROBLEMS.

1. Indicate, for each pair of expressions (A, B) in Table 1, whether A is O, o, Ω, ω, or Θ of B. Assume that $k \geq 1, \epsilon > 0$, and $c > 1$ are constants. Your answer should be in the form of the table with "yes" or "no" written in each box.

A	B	O	o	Ω	ω	Θ
$\log^k n$	n^ϵ					
n^k	c^n					
$\sqrt{n}$	$n \sin n$					
2^n	$2^{n/2}$					
$n^{\log n}$	$c^{\log n}$					
$\log(n!)$	$\log(n^n)$					

Table 1: Relative asymptotic growth

2. For $f, g : \mathbb{N} \to \mathbb{R}$ we say that f is asymptotically equal to g (notation $f(n) \sim g(n)$) if $\lim_{n\to\infty} \frac{f(n)}{g(n)} = 1$. Use Stirling's formula

$$n! \sim \left(\frac{n}{e}\right)^n \sqrt{2\pi n}.$$

to determine constants c and d such that $\frac{\binom{2n}{n}}{4^n} \sim cn^d$.

3. Show that $f(n) \ln f(n) = \Theta(n)$ implies $f(n) = \Theta(n/\ln n)$.

4. Consider the **searching problem**:

   Input: A sequence of n numbers $A = \{a_1, \ldots, a_n\}$ and a value v.

   Output: An index i such that $v = A[i]$, or the special value NIL if v does not appear in A.

   (1) Write pseudocode for **linear search:** which scans through the sequence, looking for v.

   (2) Use a loop invariant to prove that your algorithm is correct. Make sure that your loop invariant fulfills the three necessary properties.

   (3) What is the worse-case running time of linear search?

   Now, suppose the sequence A is sorted, then we can check the midpoint of the sequence against v and eliminate half of the sequence from further consideration. The **binary search** algorithm repeats this procedure, halving the size of the remaining portion of the sequence each time.

   (4) Write pseudocode for binary search.

   (5) Argue that the worst-case running time of binary search is $\Theta(\log n)$.

5. Use the divide-and-conquer integer multiplication algorithm to multiply two integers $X = 83645283$ and $Y = 75461934$. You are supposed to write down the execution of main steps.

6. Suppose you have k sorted arrays, each with n elements, i.e., $A_1[1..n], A_2[1..n], \ldots, A_k[1..n]$. You wish to merge them into a single sorted array of kn elements.

   (1) Here is one strategy: use the *Merge* procedure from the lecture to merge the first two arrays, then merge in the third, then merge in the fourth, and so on. What is the time complexity of this algorithm in terms of k and n?

   (2) Design a more efficient solution to this problem using divide and conquer. Provide a pseudocode, write down the recurrence and solve it.