CONCORDIA UNIVERSITY

DEPARTMENT OF COMPUTER SCIENCE AND SOFTWARE ENGINEERING

COMP 6651/4: Algorithm Design Techniques

Fall 2017

MidTerm - Close book exam - 2:30 hours

Instructor: Professor B. Jaumard

First Name	**Last Name**	**ID#**

- If you happen to use an algorithm we saw during one of the lectures, you need to cite it, but also to describe it in detail
- Analyze your algorithm means: provide a detailed complexity analysis of your algorithm
- Design an algorithm that ... You are required to put comments for your algorithm + justifications that the algorithm is exact/heuristic
- Any answer provided without any justifications will not be considered

	a.	**b.**	**c.**	
Question 1				
Question 2				
Question 3				
Question 4				
Question 5				
Total				

Question 1. (20 points.)

Suppose we are given an array $A[1..n]$ with the special property that $A[1] \geq A[2]$ and $A[n-1] \leq A[n]$. We say that an element $A[x]$ is a local minimum if it is less than or equal to both its neighbors, or more formally, if $A[x-1] \geq A[x]$ and $A[x] \leq A[x+1]$. For example, there are five local minima in the following array:

9	7	7	2	1	3	7	5	4	7	3	3	4	8	6	9

We can obviously find a local minimum in $O(n)$ time by scanning through the array.

a. With the given boundary conditions, explain why the array must have at least one local minimum.

b. Describe and analyze an algorithm that finds a local minimum in $O(\log n)$ time. You need to provide the details of the algorithm, its justification and a detailed complexity analysis.

Question 2. (20 points.)

Let $G = (V, L)$ be a flow network with source s and sink t, and suppose that each link $\ell \in L$ has capacity $c_\ell = 1$. Assume also, for convenience, that $|E| = \Omega(V)$.

(a) Suppose that we implement the Ford-Fulkerson maximum-flow algorithm by using depth-first search to find augmenting paths in the residual graph G_R.

(a1) Provide the detailed description of a depth-first algorithm that searches for an augmenting path in the residual graph from s (source) to t (sink). Do not forget to indicate the description of the required data structure and the impact it has (next question) on the complexity analysis.

(a2) What is the worst-case running time of the depth first search algorithm in G_R?

(a3) What is the worst-case running time of the Ford-Fulkerson algorithm in G?

(b) Suppose that a maximum flow for G has been computed using Ford-Fulkerson, and a new edge with unit capacity is added to E. Describe how the maximum flow can be efficiently updated. (Note: It is not the value of the flow that must be updated, but the flow itself.) Analyze the complexity of your algorithm.

(c) Suppose that a maximum flow for G has been computed using Ford-Fulkerson, but an edge is now removed from E. Describe how the maximum flow can be efficiently updated. Analyze the complexity of your algorithm.

Question 3. (20 points)

Professor Olay is consulting for an oil company, which is planning a large pipeline running east to west through an oil field of n wells. From each well, a spur pipeline is to be connected directly to the main pipeline along a shortest path (either north or south), as shown in Figure 1. Given $x-$ and $y-$ coordinates of the wells, how should the professor pick the optimal location of the main pipeline (the one that minimizes the total length of the spurs)? Show that the optimal location can be determined in linear time. If you re-use an algorithm from the class, you need to recall the details of its steps, plus you need to justify why the problem of Professor Olay can be reduced to one of the algorithms we saw in class.

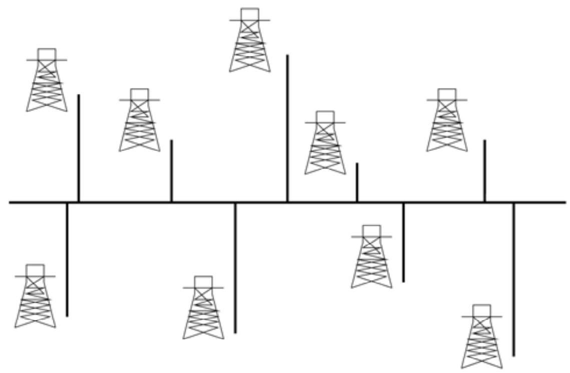

Figure 1: Professor Olay needs to determine the position of the east-west oil pipeline that minimizes the total length of the north-south spurs.

Question 4. (20 points)

Suppose you are managing the construction of billboards on the Stephen Daedalus Memorial Highway, a heavily traveled stretch of road that runs west-east for M miles. The possible sites for billboards are given by numbers $x_l, x_2, \ldots, x_n$, each in the interval $[0, M]$ (specifying their position along the highway, measured in miles from its western end). If you place a billboard at location x_i, you receive a revenue of $r_i > 0$. Regulations imposed by the countys Highway Department require that no two of the billboards be within less than or equal to 5 miles of each other. Youd like to place billboards at a subset of the sites so as to maximize your total revenue, subject to this restriction.

Example. Suppose $M = 20, n = 4, \{x_1, x_2, x_3, x_4\} = \{6, 7, 12, 14\}$, and $\{r_l, r_2, r_3, r_4\} = \{5, 6, 5, 1\}$. Then the optimal solution would be to place billboards at x_l and x_3, for a total revenue of 10.

Give an algorithm that takes an instance of this problem as input and returns the maximum total revenue that can be obtained from any valid subset of sites. The running time of the algorithm should be polynomial in n.

Question 5. Recurrence relation. (20 points)

Solve the following recurrence relation using the technique with the characteristic equation. Provide the complete analytic expression of the solution.

$$d_1 = 1 \quad \text{and} \quad d_n = d_{n/2} + 1, \qquad (n \geq 2).$$