CONCORDIA UNIVERSITY

DEPARTMENT OF COMPUTER SCIENCE AND SOFTWARE ENGINEERING

COMP 6651/4: Algorithm Design Techniques

Fall 2019

MidTerm - Close book exam - 2:30 hours

Instructor: Professor B. Jaumard

First Name	Last Name	ID#

- If you happen to use an algorithm we saw during one of the lectures, you need to cite it, but also to describe it in detail
- Analyze your algorithm means: provide a detailed complexity analysis of your algorithm
- Design an algorithm that ... You are required to put comments for your algorithm + justifications that the algorithm is exact/heuristic
- Any answer provided without any justification will not be considered

	a.	**b.**	**c.**	
Question 1				
Question 2				
Question 3				
Question 4				
Question 5				
Total				

Question 1. (20 points.)

Suppose we are given an array $A[1..n]$ with the special property that $A[1] \geq A[2]$ and $A[n-1] \leq A[n]$. We say that an element $A[x]$ is a local minimum if it is less than or equal to both its neighbors, or more formally, if $A[x-1] \geq A[x]$ and $A[x] \leq A[x+1]$. For example, there are five local minima in the following array:

9	7	7	2	1	3	7	5	4	7	3	3	4	8	6	9

We can obviously find a local minimum in $O(n)$ time by scanning through the array.

a. With the given boundary conditions, explain why the array must have at least one local minimum.

b. Describe and analyze an algorithm that finds a local minimum in $O(\log n)$ time. You need to provide the details of the algorithm, its justification and a detailed complexity analysis.

Question 2. (20 points.)

The **Widest Path Problem** is a problem of finding a path between two vertices of the graph maximizing the weight of the minimum-weight edge in the path, see Figure 1 in order to get the idea of the problem.

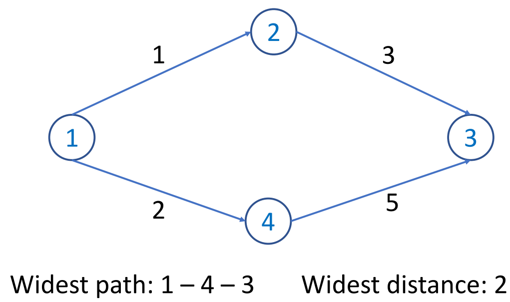

Figure 1: How to find a path between two nodes maximizing the weight of the minimum-weight edge in the path.

This problem can be seen as a graph with routers as its vertices and edges represent bandwidth between two vertices. Now if you want to find the maximum bandwidth path between two places in an Internet connection, then it corresponds to the Widest Path Problem.

The **Widest Path Problem** can be solved using Dijkstra's algorithm with a slight change.

a. Recall Dijkstra's algorithm (i.e., pseudo-code).

b. Recall the definition of a priority queue and how it can be use to implement efficiently Dijkstra's algorithm

c. Do a detailed complexity analysis of Dijkstra's algorithm, assuming you use a priority queue.

d. Explain how to modify the relaxation step of Dijkstra's algorithm in order to solve the Widest Path Problem.

e. Does it impact the complexity of the resulting algorithm? Justify your answer.

Question 3. (20 points)

Maximum Value Contiguous Subsequence. Given a sequence of n real numbers $A(1), A(2), \ldots, A(n)$, propose an algorithm that determines a contiguous subsequence $A(i), A(i+1), \ldots, A(j)$ for which the sum of elements in the subsequence is maximized. Illustrate your algorithm on the following sequence.

2	3	6	7	-2	9

Question 4. (20 points)

Suppose we perform a sequence of n operations on a data structure in which the i^{th} operation costs i if i is an exact power of 2, and 1 otherwise. Use aggregate analysis or accounting method to determine the amortized cost per operation.

Question 5. Recurrence relation. (20 points)

Solve the following recurrence relation using the technique with the characteristic equation. Provide the complete analytic expression of the solution.

$$d_1 = 1 \quad \text{and} \quad d_n = d_{n/2} + 1, \qquad (n \geq 2).$$