Question 1. (8 points)

Marking: correctly formulation of Radon transform = 2 points, correct use of symmetry to simplify the integral = 2 points, correct computation of the delta function using the SIFT theorem = 2 points, correct formulation for the Gaussian function =1 point, and correct final integration = 1 point

The integration of the Gaussian shape function is as follows:

$$g(\rho,\theta) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\delta(xcos\theta + ysin\theta - \rho)dxdy$$
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} Ae^{(-x^2-y^2)}\delta(xcos\theta + ysin\theta - \rho)dxdy$$

As the function is symmetric around the origin, the project in any direction is identical. Therefore, we can use the case of $\theta = 0$ to simplify the integration as:

$$g(\rho,\theta) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} Ae^{(-x^2-y^2)}\delta(x-\rho)dxdy = \int_{-\infty}^{\infty} f(\rho,y)dy = Ae^{-\rho^2}\int_{-\infty}^{\infty} e^{-y^2}dy$$
$$= Ae^{-\rho^2}\int_{-\infty}^{\infty} e^{-y^2}dy = Ae^{-\rho^2}\sqrt{\pi}\int_{-\infty}^{\infty}\frac{e^{-y^2}}{\frac{1}{\sqrt{2}}\sqrt{2\pi}}dy = Ae^{-\rho^2}\sqrt{\pi}$$

Note that $\frac{e^{-y^2}}{\frac{1}{\sqrt{2}}\sqrt{2\pi}}$ is a normal/Gaussian distribution function with a mean of 0 and standard deviation of $\frac{1}{\sqrt{2}}$. Thus, the integration of the function over $(-\infty, \infty)$ is 1.

Question 2 (10 points)

The spatial blurring for an arbitrary image $f(x,y)$ can be expressed as

$g(x,y) = \frac{1}{6}[2f(x,y+1) + 2f(x,y-1) + f(x+1,y) + f(x-1,y)]$ (**4 points**)

From Property 3 in Table 4.3, we have in the Fourier domain:

$G(u,v) = \frac{1}{6}\left[2e^{\frac{j2\pi v}{N}} + 2e^{\frac{-j2\pi v}{N}} + e^{\frac{j2\pi u}{M}} + e^{\frac{-j2\pi u}{M}}\right]F(u,v) = H(u,v)F(u,v)$ (**4 points**)

Where $H(u,v) = \frac{1}{3}[2\cos\left(\frac{2\pi v}{N}\right) + \cos\left(\frac{2\pi u}{M}\right)]$ is the filter function in the frequency domain (**2 points**).

Note that $\cos(z) = \frac{e^{jz}+e^{-jz}}{2}$

Question 3. (6 points)

Following the image coordinate convention in the book, vertical motion is in the x-direction and horizontal motion is in the y-direction. Then, the components of motion are as follows:

$$x_0(t)=\begin{cases} \frac{at}{T_1} & 0 \le t \le T_1 \\ a & T_1 < t \le T_1+T_2 \end{cases}$$

and

$$y_0(t)=\begin{cases} 0 & 0 \le t \le T_1 \\ \frac{b(t-T_1)}{T_1} & T_1 < t \le T_1+T_2. \end{cases}$$

Then, substituting these components of motion into Eq. (5.6-8) yields

$$\begin{aligned} H(u,v) &= \int_0^{T_1} e^{-j2\pi[uat/T_1]}dt + \int_{T_1}^{T_1+T_2} e^{-j2\pi[ua+vb(t-T_1)/T_2)}dt \\ &= \frac{T_1}{\pi ua}\sin(\pi ua)e^{-j\pi ua} + e^{-j2\pi ua}\int_{T_1}^{T_1+T_2} e^{-j2\pi vb(t-T_1)/T_2}dt \\ &= \frac{T_1}{\pi ua}\sin(\pi ua)e^{-j\pi ua} + e^{-j2\pi ua}\int_0^{T_2} e^{-j2\pi vb\tau/T_2}d\tau \\ &= \frac{T_1}{\pi ua}\sin(\pi ua)e^{-j\pi ua} + e^{-j2\pi ua}\frac{T_2}{\pi vb}\sin(\pi vb)e^{-j\pi vb} \end{aligned}$$

where in the third line we made the change of variables $\tau = t - T_1$. The blurred image is then

$$g(x,y)=\Im^{-1}[H(u,v)F(u,v)]$$

where $F(u,v)$ is the Fourier transform of the input image.

Marking: Correct plug-in of x0 and y0 into the integration function = 2 points, after the second line of the equation, each of 2 computational steps =2 points.

Part II: Programming

Q1 (18 points)

1. 4 points
2. 4 points
3. 4 points
4. 6 points: result comparison 4 points, appropriate comments 2 points

Q2 (8 points)

3 points for motion blurred image and 3 points for the magnitude image of the filter function. 2 points = the image filtering procedure is correct (Fourier transform -> filter in frequency domain -> inverse Fourier transform).