Advice for Studying TCS and Math

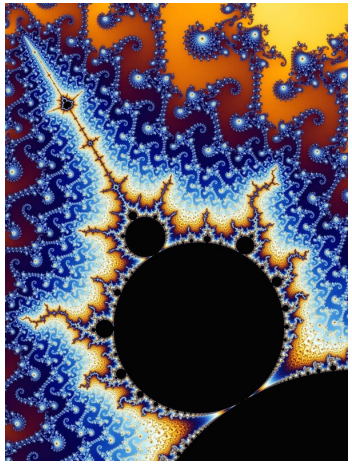
Some aspects of TCS

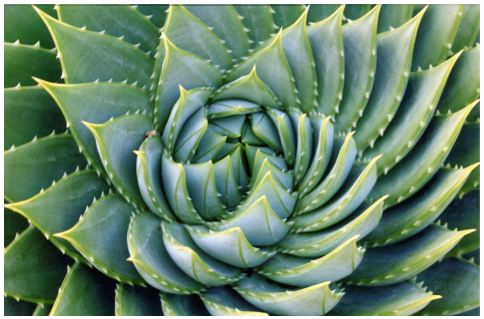
Beauty

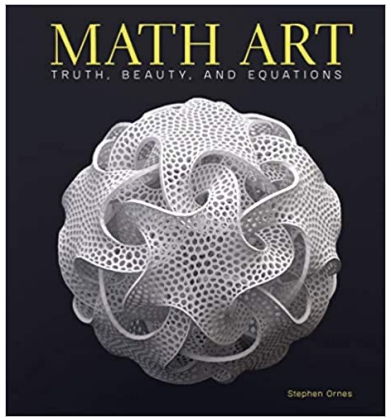
Universality

Challenge

Beauty

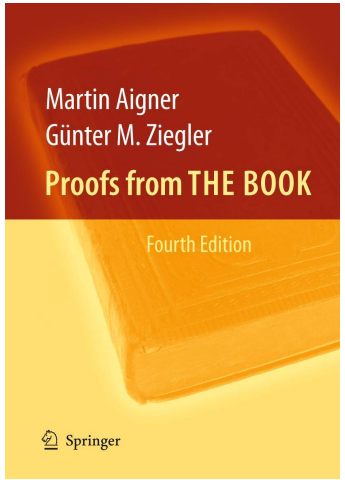




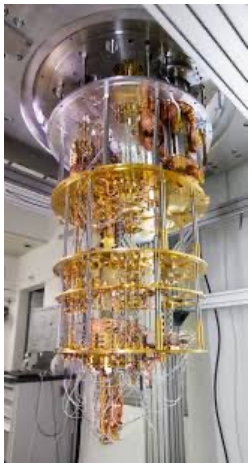


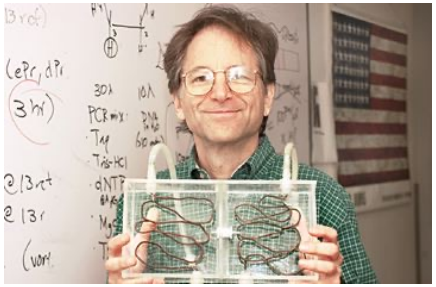
- "The Book" notion of Paul Erdős

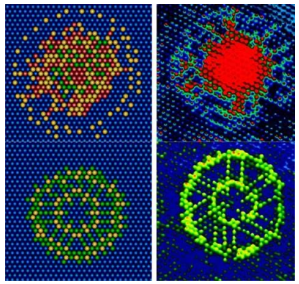
- TED talk by Cedric Villani "What's so sexy about math?" https://youtu.be/Kc0Kthyo0hU

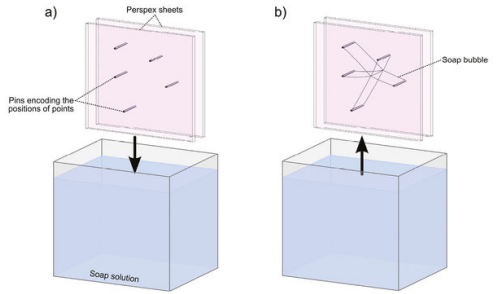


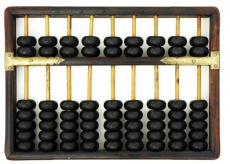
Universality











Challenge

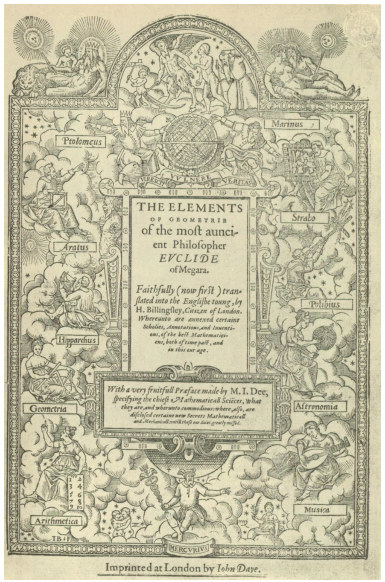
- US $1,000,000 Millenium Prize Problem: P vs NP

http://www.claymath.org/millennium-problems/p-vs-np-problem

- Can integer factoring be solved efficiently by classical computers?
- Most efficient algorithm for matrix multiplication?
- Can all-pairs shortest path be computed in subcubic time?
- Is P = BPP, or, in words, is randomness important for polynomial time computation?
- More on https://en.wikipedia.org/wiki/List_of_unsolved_problems_in_computer_science

Modern Mathematical Thinking

- Dates back to Euclid's Elements 300 BC
- Part 1: building blocks – axioms, definition, etc.
- Part 2: buildings – theorems, claims, (counter)examples, etc.



Examples: Fermat Primes

- Definitions:
- **Prime number**: a natural number ≥ 2 that is divisible only by itself and 1. E.g., $2, 3, 5, 7, 11, 13, 17$
- **Fermat number**: $F_n = 2^{2^n} + 1$ for $n \geq 0$. E.g., $F_0 = 3, F_1 = 5, F_2 = 17, F_3 = 257, F_4 = 65537$
- Conjecture:
- F_n is prime for $n \geq 0$
- False: $F_5 = 641 \times 6700417$

TCS courses

- Definitions: languages, DFAs, NFAs, PDAs, Turing machines,…
- Results: theorems, lemmas, claims, examples, and so on

Study obstacles and counter-measures

Obstacle

- Wrong motivation
- Wrong study habits
- Wrong problem solving
- Wrong attitude

Counter-measure

- Right motivation
- Right study habits
- Right problem solving
- Right attitude

What is wrong motivation?

- It is a required course
- It fits the schedule
- It may be helpful in work/graduate school
- It can be used to boost GPA
- It looks interesting

What is right motivation?

- It can improve the mind

What makes wrong motivation wrong and right motivation right?

Human Brain

On average human brain…

- …weighs 2% of total weight of the body*, and
- …consumes 20% of total energy (at rest)*.
- When brain is more active it consumes a lot more energy: up to 6,000 calories per day during chess tournament (standard diet is about 2,000 calories per day)**.

*https://www.scientificamerican.com/article/thinking-hard-calories/

**https://www.espn.com/espn/story/_/id/27593253/why-grandmasters-magnus-carlsen-fabiano-caruana-lose-weight-playing-chess

What are wrong study habits?

- Not giving the material enough time and attention
- Spending too much time as an ”observer” of mathematics
- Spending too little time as a “doer” of mathematics
- Not asking questions
- Not trying to answer questions
- Relying too much on memorization alone
- Relying too much on understanding alone

What are right study habits?

- Giving the material enough time and attention
- The “inverted view of the textbook” principle
- Asking questions, coming to office hours
- Trying to answer questions, participating in discussions
- Finding a balance between memorization and understanding

Ineffective study habit scenario 1

- Alice wishes to study material of Chapter 2
- She doesn’t feel like reading the book, she decides instead to watch pre-recorded videos
- She watches the video and finds some explanations confusing
- She decides to find other video explanations on Youtube
- These videos use another notation, and assume slightly different background
- She spends a lot of time trying to reconcile differences between two videos
- She decides to go over the slides again
- After several hours she insists it was enough studying for one day
- Meanwhile, Chapter 2 material was not studied much in spite of all the effort and time…

Same effort more results:

- Alice can increase active thinking tasks: taking a piece of paper and filling in details, reading and comprehending
- Alice can reduce passive thinking tasks: watching explanations, searching for more explanations

Ineffective study habit scenario 2

- Bob went to all lectures, watched all videos, and read all chapters
- Bob comes to the midterm and sees the question:
- “State and prove the Pumping Lemma for regular languages.”
- Bob saw the statement of the lemma at least 10 times, and even used it to solve problems.
- Bob remembers that there were some variables, like x, u, v, and a constant m that was very important. But what was their order and relationship? Was it for all i? Or there exists i? Hm…
- Bob is confused why after all the studying he cannot answer this question.

Why Does it Happen and How to fix it?

When professional skater skates, it looks easy

People do not typically assume that by watching professional skaters they can learn to skate

The same is true for mathematics: one cannot learn to do math by simply watching professional mathematician do math

Attending lectures/watching pre-recorded videos is important

Reading textbook, reproving theorems, solving problems is essential

Memorization vs Understanding Principle

- Middle ground: some memorization is required and some understanding is required
- Definitions need to be memorized first
- Understanding gets developed through problem solving

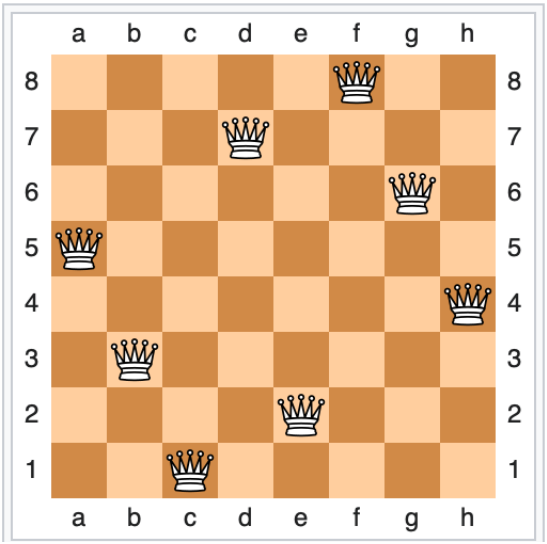
- 20/80 rule: 20% of effort is spent on memorization and 80% of effort is spent on developing understanding

What is wrong problem solving?

- Not understanding the statement of the problem
- Placing the cart before the horse
- Getting stuck
- Not knowing when and how to get help
- Not knowing how to check the answer

Placing the cart in front of the horse example

- The 8-queens puzzle:
- Is it possible to position 8 queens on the standard chess board so that none of the queens attack each other?
- Answering this question without knowing the rules of chess is "placing the cart before the horse"



Why does it happen?

What is not knowing how to check the answer?

- Beware: solution addiction!

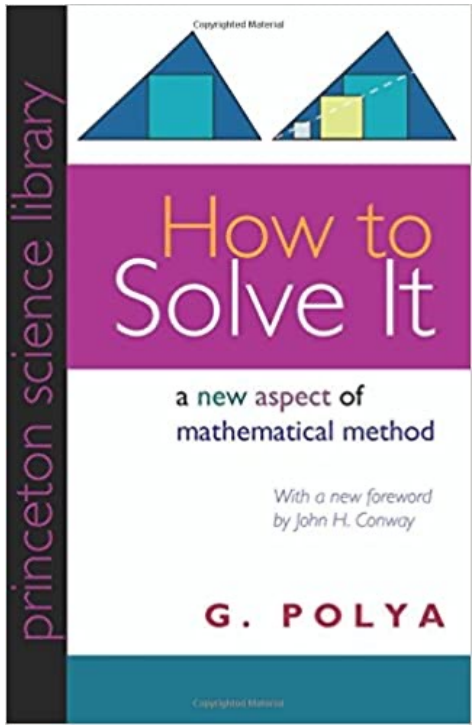
What is right problem solving?

- Understanding the statement of the problem
- Knowing how to get unstuck
- Knowing when and how to get help
- Knowing how to check the answer

How not to get stuck on a problem

1. Understand the problem
2. Devise a plan
3. Carry out the plan
4. Look back

If you get stuck, find a way to modify the problem, for example, simplify it.
Keep doing it until you find a version you can solve.
Then examine what you learnt about the simpler version of the problem to see if it helps with the original problem.



How not to get stuck on a problem

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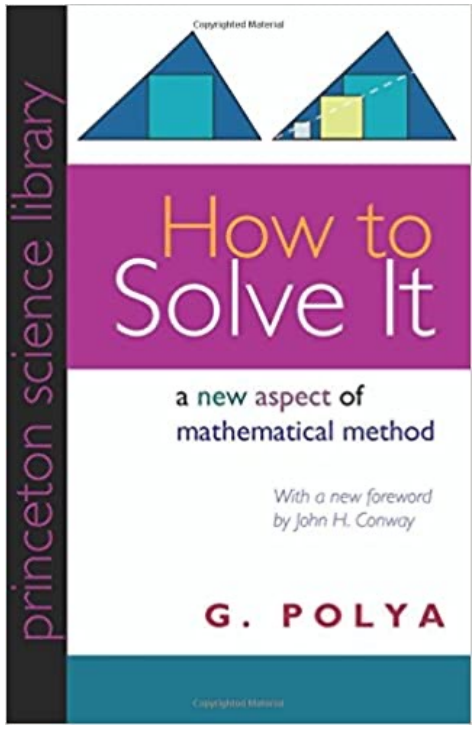
Keep attacking the problem (relentlessly) from different angles:

1. Try small special cases
2. Try induction
3. Try contradiction
4. Try contrapositive
5. Try applying pigeonhole principle



How not to get stuck on a problem

1. Understand the problem
2. Devise a plan
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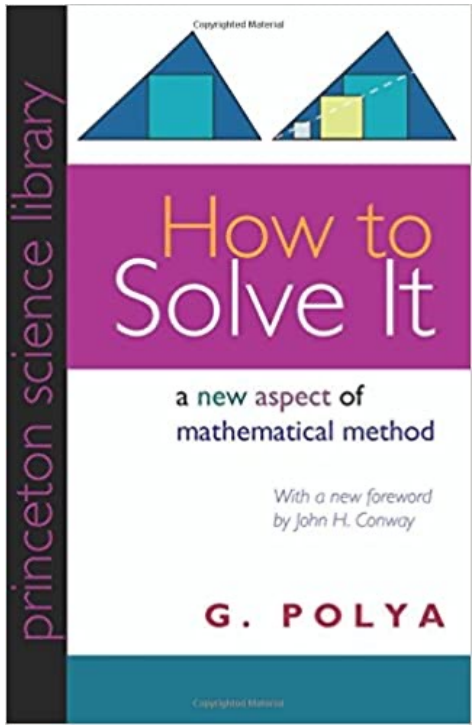
Keep asking questions:

1. What is the closest problem to this one that I have seen before?
2. Can I use the solution to the similar problem?
3. Can I use the ideas/technique used in a similar problem?



What is wrong attitude?

- This course is difficult
- This course is boring
- This course is useless

What is right attitude?

- This course is fun

How to enjoy the course?

- Treat problems as you would treat puzzles, e.g., sudoku, wordle, etc.
- Organize study groups
- Discover connections between this course and other areas of life

Common misunderstanding: ”quantity must eventually turn into quality; therefore, if I just go to lectures/watch videos/attend tutorials/read textbook and solutions, I will eventually become really good at solving problems.”

Better view: “all of the above activities are essentially observing someone else do math; no amount of watching someone else do math will make you good at doing math yourself. You get good at math by practicing yourself.”

Common misunderstanding: “main value of the textbook is in the text of chapters and exercises are supplementary.”

Better view: “main value of the textbook is in the exercises and the text of chapters is supplementary”

Common misunderstanding: “if only I get more solutions, I would then learn how to solve problems.”

Better view: “you get better at solving problems by solving problems. Being stuck and trying to overcome it and if it doesn’t work asking for help is most efficient way to learn that I know”

Common misunderstanding: “I don’t want to ask a question during lecture, because it probably has a simple answer and I will figure it out myself later.”

Better view: “If I don’t use this opportunity to ask a question during the lecture and figure out what’s bothering right now, I probably won’t get to it myself later, and this misunderstanding can snowball out of proportion.”