Concordia University

Department of Computer Science & Software Engineering

COMP 478/6771 Image Processing

Assignment 3 - Due Date: *Nov 14, 2023*

Part I: Theoretical questions

1. (**8 points**) Prove the validity of the following properties of the Radon transform:

$$g(\rho, \theta) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x, y)\delta(x\cos\theta + y\sin\theta - \rho)\,dx\,dy$$

Show that the Radon transform of the Gaussian shape $f(x, y) = Ae^{(-x^2-y^2)}$ is given by $g(\rho, \theta) = A\sqrt{\pi}e^{-\rho^2}$. (**Hint**: Refer to Example 5.15 in the textbook [see the last page of this document], where we used symmetry to simplify integration, and remember that the integration of a Gaussian distribution function is 1.)

2. (**10 points**) Given a 3x3 spatial mask that performs image blurring as below:

$$h = \frac{1}{6}\begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 2 \\ 0 & 1 & 0 \end{bmatrix}$$

Find the equivalent filter *H(u,v)* in the frequency domain (use the coordinate system defined in the lecture). For this question, please use Property 3 of Table 4.4 in the textbook, which summarizes the property of spatial translation for an image:

3)	Translation (general)	$f(x, y)e^{j2\pi(u_0x/M + v_0y/N)} \Leftrightarrow F(u - u_0, v - v_0)$
		$f(x - x_0, y - y_0) \Leftrightarrow F(u, v)e^{-j2\pi(ux_0/M + vy_0/N)}$

3. (**6 points**) Simple linear motion blur of an image $f(x, y)$ can be modelled as integration of linear shifts over time *T*: $g(x, y) = \int_0^T f[x - x_0(t), y - y_0(t)]dt$, where $g(x, y)$ is the blurred image. Thus, the Fourier transform of the filter function is defined as

$$H(u,v) = \int_0^T e^{-j2\pi[ux_0(t) + vy_0(t)]}\,dt \qquad (5\text{-}74)$$

For a particular motion blur, we have:

$$x_0(t) = \begin{cases} \frac{at}{T_1} & 0 \le t \le T_1 \\ a & T_1 < t \le T_1 + T_2 \end{cases} \quad \text{and} \quad y_0(t) = \begin{cases} 0 & 0 \le t \le T_1 \\ \frac{b(t-T_1)}{T_1} & T_1 < t \le T_1 + T_2 \end{cases}$$

By using Equation 5-74, please derive the corresponding transfer function H(u,v) in the Fourier domain.

Part II: Programming questions

1. (**18 points**) Download the image "*cameraman.tif*" from the assignment package then perform edge detection using existing MATLAB functions (with the parameter choices of your own) for:
   a) Laplacian of Gaussian (Marr-Hildreth) edge detector
   b) Canny edge detector

   1) (**4 points**) Briefly list the steps involved in implementing the edge detectors.
   2) (**4 points**) Explain how edge linking (the final step of the **Canny algorithm**) was implemented. Does the first method need this step?
   3) (**4 points**) List the parameters that determine the performance of the algorithms. What parameter values did you use and why?
   4) (**6 points**) Show and compare the results obtained by the two methods (give some comments).

2. (**8 points**) Please continue to use the same image as Q1, "*cameraman.tif*". In class, we have learnt the filter function of motion blur in Fourier domain as (see Page 356 of the textbook):

$$H(u,v) = \frac{T}{\pi(ua + vb)}\sin[\pi(ua + vb)]e^{-j\pi(ua + vb)} \qquad (5\text{-}77)$$

   To generate a discrete filter transfer function of size $M \times N$, we sample this equation for $u = 0, 1, 2, \ldots, M-1$ and $v = 0, 1, 2, \ldots, N-1$.

   Simulate motion blur for the input image with the parameters of T=1, a=0.1, and b=0.1. Please showcase the images before and after the motion degradation, as well as the magnitude image of the filter function in the Fourier domain.

Hint:

- To use Equation 5-77 to generate the filter function in the Fourier domain, it should be symmetric, so the ranges of $u = 0, 1, 2, \ldots, M-1$ and $v = 0, 1, 2, \ldots, N-1$ are not ideal in implementation.
- After using fft2(), the lower frequencies are shifted to the four corners of the image field, and can be "recentered" using the function fftshift().

EXAMPLE 5.15: Using the Radon transform to obtain the projection of a circular region.

Before proceeding, we illustrate how to use the Radon transform to obtain an analytical expression for the projection of the circular object in Fig. 5.38(a):

$$f(x,y) = \begin{cases} A & x^2 + y^2 \le r^2 \\ 0 & \text{otherwise} \end{cases}$$

where A is a constant and r is the radius of the object. We assume that the circle is centered on the origin of the xy-plane. Because the object is circularly symmetric, its projections are the same for all angles, so all we have to do is obtain the projection for $\theta = 0°$. Equation (5-102) then becomes

$$\begin{aligned} g(\rho, \theta) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\delta(x-\rho)\,dx\,dy \\ &= \int_{-\infty}^{\infty} f(\rho, y)\,dy \end{aligned}$$

where the second expression follows from Eq. (4-13). As noted earlier, this is a line integral (along the line $L(\rho, 0)$ in this case). Also, note that $g(\rho, \theta) = 0$ when $|\rho| > r$. When $|\rho| \le r$ the integral is evaluated from $y = -(r^2 - \rho^2)^{1/2}$ to $y = (r^2 - \rho^2)^{1/2}$. Therefore,

$$\begin{aligned} g(\rho, \theta) &= \int_{-\sqrt{r^2-\rho^2}}^{\sqrt{r^2-\rho^2}} f(\rho, y)\,dy \\ &= \int_{-\sqrt{r^2-\rho^2}}^{\sqrt{r^2-\rho^2}} A\,dy \end{aligned}$$

Carrying out the integration yields

$$g(\rho,\theta) = g(\rho) = \begin{cases} 2A\sqrt{r^2 - \rho^2} & |\rho| \le r \\ 0 & \text{otherwise} \end{cases}$$

where we used the fact that $g(\rho, \theta) = 0$ when $|\rho| > r$. Figure 5.38(b) shows a plot of this result. Note that $g(\rho, \theta) = g(\rho)$; that is, g is independent of θ because the object is symmetric about the origin.