COMP 6651: Assignment 8

Fall 2020

Submission through Moodle is due by November 15th at 23:55

1. Alice, Bob, and Charlie, as a hobby, are making 3 types of beer: light, regular, and strong. They sell their beer at a farmer's market. A gallon of light beer gives $30 of profit. A gallon of regular beer gives $10 of profit, and a gallon of strong beer gives $30 of profit. The process of making the beer consists of 3 steps: (1) brewing preparation, (2) testing, (3) packaging (we exclude the brewing itself since it does not involve human labour). A gallon of light beer requires 2 minutes of brewing preparation, 1 minute of testing, and 2 minutes of packaging. A gallon of regular beer requires 1 minute of brewing preparation, 2 minutes of testing, and 2 minutes of packaging. A gallon of strong beer requires 1 minute of brewing preparation, 3 minutes of testing, and 1 minute of packaging. Alice is responsible for brewing preparation and can spend at most 10 minutes per day on this hobby. Bob is responsible for testing the beer and can spend at most 25 minutes per day on this hobby. Charlie is responsible for packaging and can spend at most 30 minutes per day on this hobby. How much of each type of beer should the three friends choose to make in order to maximize their profits? What is their maximum daily profit?

   (a) Write down the linear program corresponding to this problem in standard form.

   (b) Solve the linear program from the previous part using simplex. Show all steps. In particular, for each pivot operation state the entering variable, leaving variable, basic feasible solution, and the new slack form of the LP.

   (c) Write down the dual of the LP.

2. Give (very) short proofs of the following claims:

   (a) If an LP (linear program) has more than one feasible solution then it has infinitely many feasible solutions.

   (b) There exists an IP (integer program) whose dual optimal value is not equal to its primal optimal value.

   (c) There exists an LP whose feasible region is unbounded, while the optimal value of the LP is bounded.

   (d) The variable that becomes non-basic in one iteration of the simplex cannot become basic in the next iteration.