COMP 6651: Assignment 9

Fall 2020

Submission through Moodle is due by November 22nd at 23:55

1. Recall that a language L belongs to the class $\mathcal{NP}$ if there exists a polynomial time verification algorithm A and a constant d such that for every $x \in \{0,1\}^*$

   - if $x \in L$ then there exists y such that $|y| \leq |x|^d$ and $A(x,y) = 1$;
   - if $x \notin L$ then for every y with $|y| \leq |x|^d$ we have $A(x,y) = 0$.

   (a) On a test, Alex mistakenly forgot to specify the second item (if $x \notin L \ldots$). Alex's definition defines another class of languages, $Alex - \mathcal{NP}$. Determine the class $Alex - \mathcal{NP}$. Give a simple description of this class. Prove your answer.

   (b) Another student, Joana, mistakenly forgot to specify the first item (if $x \in L \ldots$). Joana's definition defines another class of languages, $Joana - \mathcal{NP}$. Determine the class $Joana - \mathcal{NP}$. Give a simple description of this class. Prove your answer.

   (c) Another student, Steve, replaced both items with the following: for every $x \in \{0,1\}^*$ there exists y such that $x \in L$ if and only if $|y| \leq |x|^d$ and $A(x,y) = 1$. This is again an incorrect definition of $\mathcal{NP}$. This definitions gives another class of languages, $Steve - \mathcal{NP}$. Determine the class $Steve - \mathcal{NP}$. Give a simple description of this class. Prove your answer.

2. Let $G = (V,E)$ be a simple undirected graph. Recall that a set $S \subseteq V$ is called a *vertex cover* if every edge has at least one endpoint in S, i.e., $\forall \{u,v\} \in E$ we have $\{u,v\} \cap S \neq \emptyset$. The Minimum Vertex Cover problem (MVC) asks to find a vertex cover of minimum size for a given graph G.

   (a) State the optimization version, decision version, and search version of MVC.

   (b) Let $MVC - DEC$ denote the decision version of MVC. Recall that $CLIQUE = \{\langle G, k\rangle :$ G has a clique of size at least $k\}$ is the decision version of the Maximum Clique problem. Show that $CLIQUE \leq_p MVC - DEC$. Give a reduction and prove its correctness and that it runs in polynomial time.

3. Recall that a simple undirected graph $G = (V,E)$ is k-colorable if there exists a coloring $c : V \to \{1, 2, \ldots, k\}$ such that for every edge $\{u,v\} \in E$ we have $c(u) \neq c(v)$. Define $k - COL = \{\langle G\rangle :$ G is k-colorable$\}$. Prove that $3 - COL \leq_p 4 - COL$. Give a reduction and prove its correctness and that it runs in polynomial time.