CONCORDIA UNIVERSITY

DEPARTMENT OF COMPUTER SCIENCE AND SOFTWARE ENGINEERING

COMP 6651/4: Algorithm Design Techniques - Fall 2013

Quiz # 2 - September 27, 2013

First Name	**Last Name**	**ID#**

Question 1. 10 points

Perform the complexity of the SELECT assuming that, instead of a division into $\lceil n/5 \rceil$ groups of 5 elements, we decide to divide the n elements into $\lceil n/7 \rceil$ group of 7 elements.

Algorithm SELECT

Step 1. Divide n elements into $\lceil n/7 \rceil$ group of 7 elements. Note that one group may have less than 7 elements.

Step 2. Find the median of each group by first insertion sorting the elements of each group, and then picking the median from the sorted list of group elements, and

Step 3. Use SELECT recursively to find the median x of the $\lceil n/7 \rceil$ medians found in Step 2.

Step 4. Partition the input array around the median-of-medians x using the modified version of PARTITION. Let k be one more than the number of elements on the low side of the partition, so that x is the kth smallest element and there are $n - k$ elements on the high side of the partition.

Step 5. If $i = k$, then return x. Otherwise, use SELECT recursively to find the ith smallest element on the low side if $i < k$, or the $(i - k)$th smallest elements on the high side if $i > k$.

What is the conclusion of your complexity analysis?

- Step 1. $O(n)$ $\rightsquigarrow$ 1 point
- Step 2. $O(n)$ $\rightsquigarrow$ 1 point
- Step 3. $T(\lceil n/7 \rceil)$ $\rightsquigarrow$ 1 point
- Step 4. $O(n)$ $\rightsquigarrow$ 1 point
- Step 5. $\leq T(5n/7 + 8)$ $\rightsquigarrow$ 2 points: 1 point for the result, 1 point for the explanations.

Overall complexity is given by the following relation:

$$T(n) \leq T(\lceil n/7 \rceil) + T(5n/7 + 8) + O(n).$$

$\rightsquigarrow$ 1 point

Let us show, using an induction proof, that: $T(n)cn$.
Base case: $T(1), T(2), \ldots, T(7)$ are smaller than cn Induction hypothesis: $\forall kleqn?1 \quad T(k)ck$. Induction step: Let us show that $T(n) \leq n$ $\rightsquigarrow$ 1 point

$$\begin{aligned}
T(n) &\leq c(\frac{n}{7} + 1) + c(\frac{5}{7}n + 8) + an \\
&\leq c \times \frac{6n}{7} + 9c + an \\
&\leq cn + (an + 9c - \frac{1}{7}cn)
\end{aligned}$$

Look for n such that

$$an + 9c - \frac{1}{7}cn = an + c \times (\frac{63 - n}{7}) \leq 0.$$

Consider $n \geq 64$, we then get:

$$64a \leq an \leq c \times \frac{1}{7}.$$

Take, e.g., $c = 7 \times 64$ and $a = 1$.

We then conclude that $T(n) = O(n)$.

$\rightsquigarrow$ 1 point

Question 2. **10 points**

Professor Midas drives an automobile from Newark to Reno along Interstate 80. His car's gas tank, when full, holds enough gas to travel n miles, and his map gives the distances between the gas stations on his route. The professor wishes to make as few gas stops as possible along the way. Give an efficient method by which Professor Midas can determine at which gas stations he should stop and prove that your algorithm provides an optimum solution.

See the Course Pack. 5 points for the algorithm, 5 points for the proof.