CONCORDIA UNIVERSITY

DEPARTMENT OF COMPUTER SCIENCE AND SOFTWARE ENGINEERING

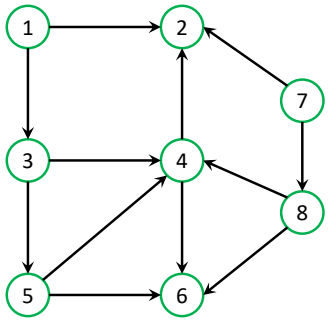
COMP 6651: Algorithm Design Techniques Winter 2022

Quiz # 9

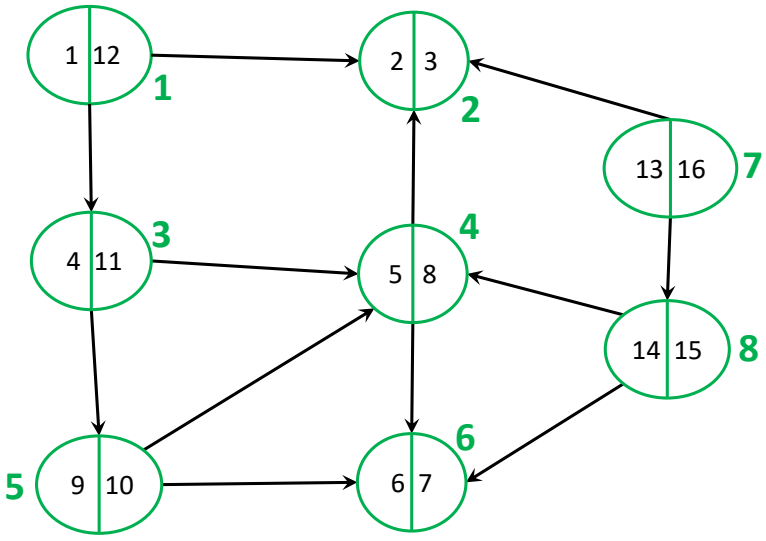
First Name	Last Name	ID#

Question 1. 4 points.

Provide the start and finish times of the depth first search algorithm on the graph below, assuming nodes are read in the input file following the increasing order or their indices, and the adjacency lists are ordered as indicated below.



ADJ(1) = (2,3)
ADJ(2) = ∅
ADJ(3) = (4,5)
ADJ(4) = (2,6)
ADJ(5) = (4,6)
ADJ(6) = ∅
ADJ(7) = (2,8)
ADJ(8) = (4,6)



Question 2

Consider an undirected graph for which you want to compute a matching.

(2a) **1 point.** Recall the formal definition of an augmented path (a figure is not a formal definition)

> An augmented path with respect to a matching M is an alternating path with first and last vertices exposed An exposed node is a node that is not matched, i.e., it is not an endpoint of one of the edges in the matching

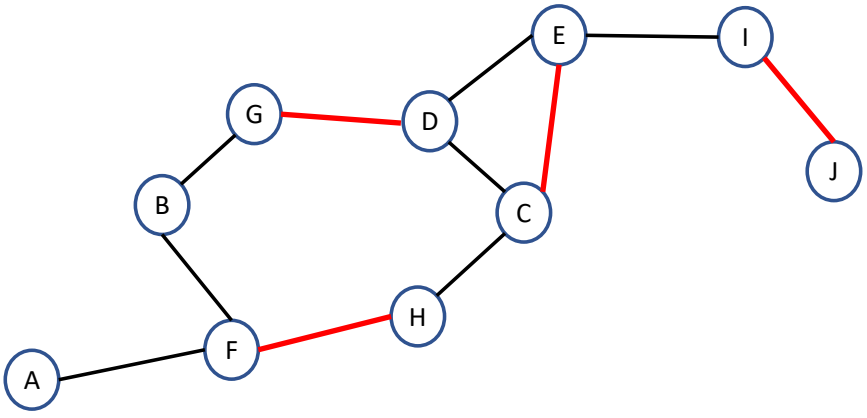
(2b) **1 point.** Recall the formal definition of flower

> A flower in a graph $G = (V, E)$ w.r.t. a matching M and a (free) root node r, is a subgraph with two components:
>
> **A stem** is an **even** length alternating path that starts at the root node r (**exposed** node) and terminates at some node w (**matched** ndoe). We permit the possibility that r = w (empty stem).
>
> **A blossom** is an **odd** length alternating cycle that starts and terminates at the terminal node w of a stem and has no other node in common with the stem. w is called the base of the blossom

(2c) **1 point.** Recall the formal definition of an augmenting flower

> Augmenting flower: If after the blossom shrinking, one can find an augmented path going through the node associated with the skrinked blossom

(2d) **3 points.** Is the following matching optimal? You need to justify your answer.



> Some preliminary comments as some of you distinguished optimal vs. optimum.
>
> Optimal and optimum both mean "best possible" or "most favorable."

However, in mathematics, there is a difference between maximum/maximal and minimum/minimal.

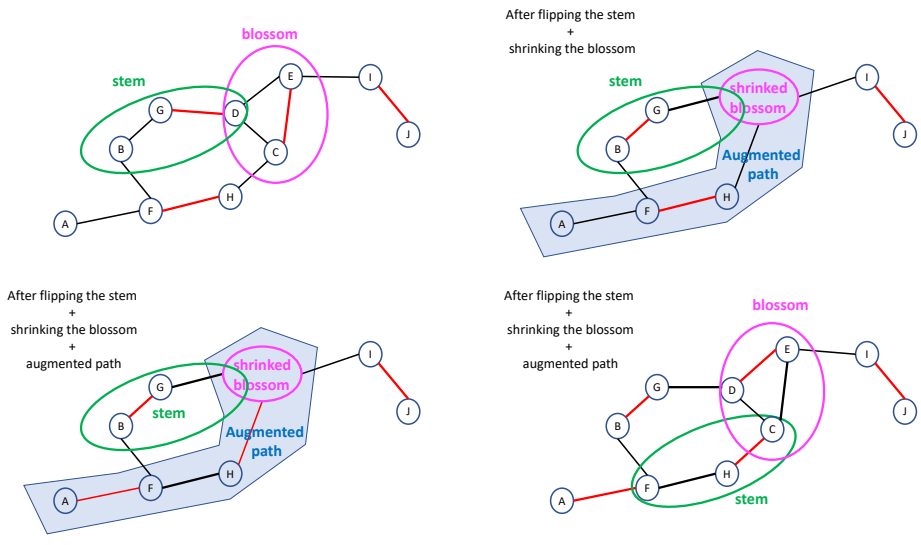
Maximum and minimum relate to absolute values — there is nothing higher than the maximum and nothing lower than the minimum.

Maximal and minimal, however, can be more vague, and relate to a solution that cannot be easily improved with respect to a given solution neighborhood.

Example: the matching that is represented below is not of maximum cardinality as there exists another matching of largest cardinality. However, it can be viewed as a matching of maximal cardinality as there is no "easy" way to increase its size, i.e., adding one more edge in the matching without modifying the current matching.

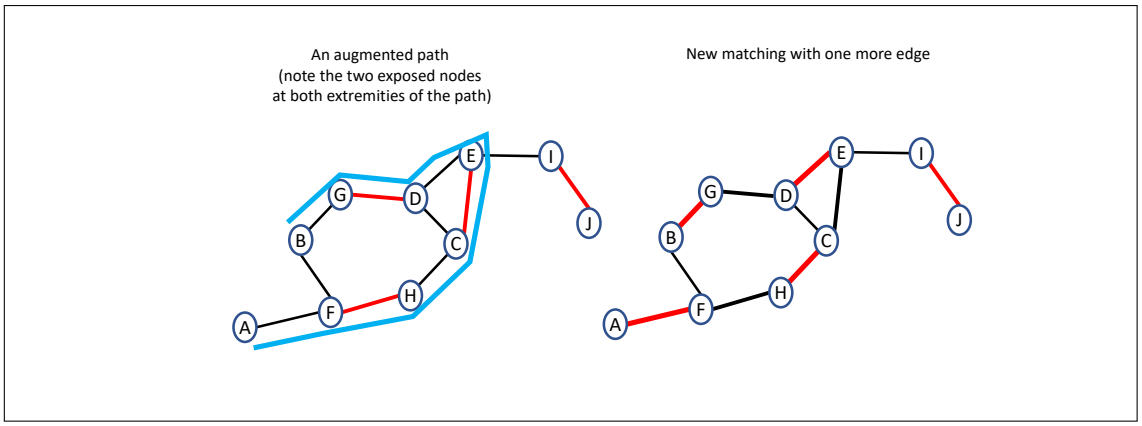
Both interpretations were accepted in the grading of that question, as long as justification of the answer was provided. In other words, if you said that the matching is optimal and you explained the difference between maximal/maximum, you get 3 points. If you answered that the matching is not optimal, then we assume you did not know about the difference between maximal/maximum, and grading is done assuming you interpreted optimal by matching of maximum cardinality. Providing the matching of maximum cardinality without any justification does not provide any point: you need to show that you have understood the algorithm presented during the lecture and that you know how to use it: a suggestion (suggested exercices of last week) was made with respect to that practice.

The proposed matching is not of maximum cardinality and the subsequent figures explain how to find an augmented path after the identification of a flower and its blossom.



Resulting new matching has 5 edges.

There was another solution with the usage of an augmented path, which is illustrated below.


An augmented path
(note the two exposed nodes
at both extremities of the path)
New matching with one more edge
A
B
C
D
E
F
G
H
I
J
A
B
C
D
E
F
G
H
I
J