6.1
Take 1 pill from Bottle 1, 2 pills from Bottle 2, and so on. We'll expect (1 + 2 + … + 20) = 210 gram of pills.
The answer would be {(weight – 210 grams) / 0.1} where weight is the weight of the bottles on the scale.

6.2
The probability of winning Game #1 = p.
Probability of winning Game #2 = P(2,3) + P(3,3)
where,
P(2,3) = probability to make 2 of 3 shots, and
P(3,3) = probability to make 3 of 3 shots.

2 of 3 shots can be taken in C(3,2) ways: (YYN, YNY, NYY). Consider YYN, since these shots are independent, we can simply multiply their probabilities to find the probability of getting YYN.

Therefore, P(YYN) = p*p*(1-p).

It's obvious that the probability of other sequences(YNY, NYY) is also p*p*(1-p). So the total probability:

P(2,3) = 3*p*p*(1-p).

3 of 3 shots can be tken in one way, therefor:

P(3,3) = p*p*p.

So, probability of winning Game #2 = P(2,3) +P(3,3) = 3*p*p - 2*p*p*p.

Now, if P(Game #1) > P(Game #2):

p > 3*p*p - 2*p*p*p,

on solving this equation, we get,

p < .5

So, play Game #1 when 'p' < .5 otherwise play Game #2.

6.3
Both removed squares have identical color. So their removal leaves us with 30 squares of one color and 32 squares of the other color. Since a domino always covers one black and one white squares, the 62 remaining squares cannot be tiled.

6.4

Think of the question in reverse, how many cases are there in which the ants will not collide? There will never be a collision between the ants if they choose to walk in the same direction (clockwise or counter-clockwise). Now that we know that there are only two scenarios where the ants will not collide, we have to ask ourselves how many different ways are there for the ants to move on the sides of the triangle? Well, each ant can move in 2 different directions. Because there are 3 ants, this means that there are 2^3 (which equals eight) possible ways that the ants can move. And since we already know that there are only 2 ways in which the ants can avoid collision entirely, this means that there are 6 scenarios where the ants will collide. And 6 out of 8 possible scenarios, means that the probability of collision is 6/8, which equals 3/4 or .75.

Similarly, for an n-vertex polygon, the total no. of ways in which ants can move is 2^n and the no. of ways in which they won't collide still remains the same which is 2. So, the probability that they will collide is $2/2^n$ or $1/2^{n-1}$

6.5

Since we can't hold 4 quarts in the 3 quart jug, we have to look to filling up the 5 quart jug with exactly 4 quarts. Lets count the steps as we move along:

1. Fill 3 quart jug (5p – 0, 3p – 3)

2. Transfer to 5 quart jug (5p – 3, 3p – 0)

3. Fill 3 quart jug (5p – 3, 3p – 3)

4. Transfer to 5 quart jug (5p – 5, 3p – 1)

5. Empty 5 quart jug (5p – 0, 3p – 1)

6. Transfer to 5 quart jug (5p – 1, 3p – 0)

7. Fill 3 quart jug (5p – 1, 3p – 3)

8. Transfer to 5 quart jug (5p – 4, 3p – 0)

6.6

If we have 1 blue-eyed person:

As he see everyone else is black-eyed, he immediately know himself is blue-eyed, so he will leave the island the same day

If we have 2 blue-eyed person:

day 1: the 2 blue-eyed person are not sure whether there are 1 or 2 blue-eyed person, and not sure if he himself is the blue-eyed, so both will stay

day 2: the 2 blue-eyed person will see the other blue-eyed person does not leave, so they both know that they are blue-eyed, both will leave.

If we have 3 blue-eyed person:

day 1: The 3 blue-eyed person are not sure there are 2 or 3 blue-eyed person on island, so no one will leave

day 2: If there are 2 blue-eyed person, these 2 person will leave on day 2.

day 3: If one blue-eyed person is still there, each blue-eyed person knows that there are 3 blue-eyed person, every blue-eyed person will leave

If we have n blue-eyed person, it will take n days for all blue-eyed person to leave

6.8

We can try a method similar to binary search. By first dropping the egg from 50th Floor. If it does not break then try from 75 and so on. But, if it breaks at 50, then we have just one egg left and thus would have to try all the floors from 1 to 49. This is not an optimal solution, as the worst case scenario takes a lot of steps.

If we drop 1st Egg from X th floor initially, then for the 2nd Egg we should try X + (X-1)th floor(to keep the worst case number same).

Thus we can say,

X + (X-1) + (X-2)...1 = 100

=> X(X+1)/2 = 100 (By using, S(n)= n(n+1)/2 formula)

=> X=14.

So we should drop the 1st Egg from 14th, then 27th, then 39th and so on. When it breaks, we can start with the 2nd Egg sequentially. This way the worst case scenario will need 14 counts.

6.9

Since all doors begin from closed state, the number of toggles determine whether each of them will be open or closed after we are done. If a door is toggled for odd times, it will end up with open. If even time, the door will be closed at the end. Also, after nth pass, door number <= n will not be toggled anymore.

For door #1, it will be toggled once therefore it will end up open.

For door #2, it will be toggled twice (in pass 1 and 2) and is end up closed.

For door #3, it is toggled twice (pass 1, 3) and ends up closed.

For door #4, it is toggled 3 times (pass 1, 2, 4) and ends up open.

For door #5, it is toggled twice (pass 1, 5) and ends up closed.

For door #6, it is toggled four times (pass 1,2,3,6) and ends up closed.

For door #7, it is toggled twice (pass 1,7) and ends up closed.

For door #8, it is toggled four times (pass 1,2,4,8) and ends up closed.

For door #9, it is toggled thrice (pass 1,3,9) and ends up open.

....

For door #100, it is toggled 9 times (pass 1, 2, 4, 5, 10, 20, 25, 50 and 100) and ends up open.

We can see that doors 1, 4, 9,...,100 are left open. These are perfect squares. From 1 to 100, there are 1, 4, 9, 16, 25, 36, 49, 64, 81, 100 perfect squares or 10 open doors.