Competitive Edge – IIT Mathematics

First Edition

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Algebra

Progression and Series

MCQ-Single Correct

1. For any three positive real numbers a, b and c, $9\left(25a^2+b^2\right)+25\left(c^2-3ac\right)=15b\left(3a+c\right)$. Then :

(1) b, c and a are in G.P. (2) b, c and a are in A.P.
(3) a, b and c are in A.P. (4) a, b and c are in G.P. **[2017]**

2. If the 2nd, 5th and 9th terms of a non-constant A.P. are in G.P., then the common ratio of this G.P. is:

(1) $\frac{4}{3}$ (2) 1
(3) $\frac{7}{4}$ (4) $\frac{8}{5}$ **[2016]**

3. If the sum of first ten terms of the series $\left(1\frac{3}{5}\right)^2+\left(2\frac{2}{5}\right)^2+\left(3\frac{1}{5}\right)^2+4^2+\left(4\frac{4}{5}\right)^2+......$ is $\frac{16}{5}m$, then m is equal to :

(1) 101 (2) 100
(3) 99 (4) 102 **[2016]**

4. The sum of first 9 terms of the series $\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+..........$ is :

(1) 96 (2) 142
(3) 192 (4) 71 **[2015]**

5. If m is the A.M. of two distinct real numbers l and n $(\mathrm{l}, \mathrm{n}>1)$ and G_1, G_2 and G_3 are three geometric means between l and n, then $G_1^4+2G_2^4+G_3^4$ equals

(1) $4lm^2n$ (2) $4lmn^2$
(3) $4l^2m^2n^2$ (4) $4l^2mn$ **[2015]**

6. Three positive numbers from an increasing G.P. If the middle term in this G.P. is doubled, the new numbers are in A.P. Then the common ratio of the G.P. is

(1) $\sqrt{2}+\sqrt{3}$ (2) $3+\sqrt{2}$
(3) $2-\sqrt{3}$ (4) $2+\sqrt{3}$ **[2014]**

7. If $(10)^9+2(11)^1(10)^8+3(11)^2(10)^7+......+10(11)^9=k(10)^9$, then k is equal to

(1) $\frac{121}{10}$ (2) $\frac{441}{100}$
(3) 100 (4) 110 **[2014]**

8. The sum of first 20 terms of the sequence 0.7, 0.77, 0.777, …… , is

(1) $\frac{7}{9}\left(99-10^{-20}\right)$

(2) $\frac{7}{81}\left(179+10^{-20}\right)$

(3) $\frac{7}{9}\left(99+10^{-20}\right)$

(4) $\frac{7}{81}\left(179-10^{-20}\right)$ **[2013]**

9. If 100 times the 100^{th} term of an AP with non-zero common difference equals the 50 times its 50^{th} term, then the 150^{th} term of this AP is

(1) 150

(2) Zero

(3) -150

(4) 150 times its 50^{th} term. **[2012]**

10. Let a_n be the n^{th} term of an AP . If $\sum_{r=1}^{100} a_{2r}=\alpha$ and $\sum_{r=1}^{100} a_{2r-1}=\beta$, then the common difference of the A.P. is

(1) $\beta-\alpha$

(2) $\frac{\alpha-\beta}{200}$

(3) $\alpha-\beta$

(4) $\frac{\alpha-\beta}{100}$ **[2011]**

11. A person is to count 4500 currency notes. Let a_n denote the number of notes he counts in the n^{th} minute. If $a_1=a_2=.........=a_{10}=150$ and $a_{10,}a_{11},.....$ are in A.P. with common difference -2, then the time taken by him to count all notes is

(1) 34 minutes

(2) 125 minutes

(3) 135 minutes

(4) 24 minutes **[2010]**

12. The sum to the infinity of the series $1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+\frac{14}{3^4}+......$ is

(1) 2

(2) 3

(3) 4

(4) 6 **[2009]**

13. The first two terms of a geometric progression add up to 12. The sum of the third and the fourth terms is 48. If the terms of the geometric progression are alternately positive and negative, then the first term is

(1) -4

(2) -12

(3) 12

(4) 4 **[2008]**

14. In a geometric progression consisting of positive terms, each term equals the sum of the next two terms. Then the common ratio of this progression equals

(1) $\frac{1}{2}\left(1-\sqrt{5}\right)$

(2) $\frac{1}{2}\sqrt{5}$

(3) $\sqrt{5}$

(4) $\frac{1}{2}\left(\sqrt{5}-1\right)$ **[2007]**

15. If p and q are positive real numbers such that $p^2+q^2=1$, then the maximum value of $(\mathrm{p}+\mathrm{q})$ is

(1) 2

(2) ½

(3) $\frac{1}{\sqrt{2}}$ (4) $\sqrt{2}$ **[2007]**

16. Let $a_1, a_2, a_3, ...$ be terms of an A.P. If $\frac{a_1+a_2+......+a_p}{a_1+a_2+......+a_q} = \frac{p^2}{q^2}$, $p \neq q$, then $\frac{a_6}{a_{21}}$ equals

(1) $\frac{41}{11}$ (2) $\frac{7}{2}$

(3) $\frac{2}{7}$ (4) $\frac{11}{41}$ **[2006]**

17. If $a_1, a_2, ..., a_n$ are in H.P., then the expression $a_1a_2 + a_2a_3 + ... + a_{n-1}a_n$ is equal to

(1) $n(a_1 - a_n)$ (2) $(n-1)(a_1 - a_n)$

(3) $n a_1 a_n$ (3) $(n-1)a_1 a_n$

[2006]

18. If the coefficients of r^{th}, $(r+1)^{th}$ and $(r+2)^{th}$ terms in the binomial expansion of $(1+y)^m$ are in A.P. , then m and r satisfy the equation

(1) $m^2 - m(4r-1) + 4r^2 - 2 = 0$ (2) $m^2 - m(4r+1) + 4r^2 + 2 = 0$

(3) $m^2 - m(4r+1) + 4r^2 - 2 = 0$ (4) $m^2 - m(4r-1) + 4r^2 + 2 = 0$ **[2005]**

19. If $x = \sum_{n=0}^{\infty} a^n$, $y = \sum_{n=0}^{\infty} b^n$, $z = \sum_{n=0}^{\infty} c^n$ where a,b,c are in A.P. and $|a| < 1, |b| < 1, |c| < 1$, then x, y , z are in

(1) G.P. (2) A.P.

(3) Arithmatic-Geometric Progression (4) H.P. **[2005]**

20. If in a triangle ABC, the altitudes from the vertices A,B,C on opposite sides are in H.P., then sin A, sin B, sin C are in

(1) G.P. (2) A.P.

(3) Arithmatic-Geometric Progression (4) H.P. **[2005]**

21. Let two numbers have arithmetic mean 9 and geometric mean 4. Then these numbers are the roots of the quadratic equation

(1) $x^2 + 18x + 16 = 0$ (2) $x^2 - 18x - 16 = 0$

(3) $x^2 + 18x - 16 = 0$ (4) $x^2 - 18x + 16 = 0$ **[2004]**

22. Let T_r be the r^{th} term of an A.P. whose first term is a and common difference is d. If for some positive integers $m, n, m \neq n$, $T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$, then a – d equals

(1) 0 (2) 1

(3) $\frac{1}{mn}$ (4) $\frac{1}{m} + \frac{1}{n}$ **[2004]**

23. The sum of the first n terms of the series $1^2+2.2^2+3^2+2.4^2+5^2+2.6^2+...$ is $\frac{n(n+1)^2}{2}$ when n is even. When n is odd the sum is

(1) $\frac{3n(n+1)}{2}$ (2) $\frac{n^2(n+1)}{2}$

(3) $\frac{n(n+1)^2}{4}$ (4) $\left[\frac{n(n+1)}{2}\right]^2$ **[2004]**

24. Let f(x) be a polynomial function of second degree. If f(1)=f(-1) and a,b,c are in A.P., then f'(a), f'(b) and f'(c) are in

(1) A.P. (2) G.P.

(3) H.P. (4) Arithmatic-Geometric Progression **[2003]**

25. If x_1, x_2, x_3 and y_1, y_2, y_3 are both in G.P. with the same common ratio, then the points (x_1, y_1), (x_2, y_2) and (x_3, y_3)

(1) lie on a straight line (2) lie on an ellipse

(3) lie on a circle (4) are vertices of a triangle **[2003]**

26. The real number x when added to its inverse gives the minimum value of the sum of x equal to

(1) 2 (2) 1

(3) -1 (4) -2 **[2003]**

27. Let R_1 and R_2 respectively be the maximum ranges up and down an inclined plane and R be the maximum range on the horizontal plane. Then R_1, R, R_2 are in

(1) arithmetic-geometric progression (2) A.P.

(3) G.P. (4) H.P. **[2003]**

28. If 1, $\log_9(3^{1-x}+2)$, $\log_3[4.3^x-1]$ are in A.P. , then x equals

(1) $\log_3 4$ (2) $1-\log_3 4$

(3) $1-\log_4 3$ (4) $\log_4 3$ **[2002]**

29. $1^3-2^3+3^3-4^3+......+9^3$ is equal to

(1) 425 (2) -425

(3) 475 (4) -475 **[2002]**

30. Sum of infinite number of terms in G.P. is 20 and sum of their squares is 100. The common ratio of G.P. is

(1) 5 (2) $\frac{3}{5}$

(3) $\frac{8}{5}$ (4) $\frac{1}{5}$ **[2002]**

31. The value of $2^{1/4}.4^{1/n}.8^{1/6}.....\infty$ is

(1) 1 (2) 2

(3) 3/2 (4) 4 **[2002]**

32. Fifth term of a G.P. is 2, then the product of its 9 terms is

(1) 256 (2) 512

(3) 1024 (4) none of these **[2002]**

33. If a,b,c are distinct positive real numbers and $a^2+b^2+c^2=1$, then $ab+bc+ca$ is

(1) less than 1 (2) equal to 1

(3) greater than 1 (4) any real number **[2002]**

Assertion-Reason Type

(1) Statement-I is True; Statement-II is true; Statement-II is **not** a correct explaination of Statement-I.

(2) Statement-I is True; Statement-II is False.

(3) Statement-I is False; Statement-II is true

(4) Statement-I is True; Statement-II is true; Statement-II is a **correct** explaination of Statement-I.

1. **Statement-I** : The sum of the series 1+ (1 +2 +4) + (4 + 6 + 9) + (9 + 12 + 16) + ... + (361 + 380 + 400) is 8000.

Statement-II : $\sum_{k=1}^{n}\left(k^3-(k-1)^3\right)=n^3$ for any natural number n. **[2012]**

Quadratic Equations

MCQ-Single Correct

1. If, for a positive integer n, the quadratic equation, x(x + 1) + (x + 1)(x + 2) + ...+(x + $\overline{n-1}$)(x + n) = 10n has two consequitive integral solutions, then n is equal to :

(1) 12 (2) 9

(3) 10 (4) 11 **[2017]**

2. The sum of all real values of x satisfying the equation $\left(x^2-5x+5\right)^{x^2+4x-60}=1$

(1) -4 (2) 6

(3) 5 (4) 3 **[2016]**

3. Let α and β be the roots of equation $x^2-6x-2=0$. If $a_n=\alpha^n-\beta^n$, for $n\ge 1$, then the value of $\frac{a_{10}-2a_8}{2a_9}$ is equal to :

(1) -6 (2) 3

(3) -3 (4) 6 **[2015]**

4. Let α and β be the roots of the equation $px^2+qx+r=0$, p ≠ 0. If p, q , r are in A.P. and $\frac{1}{\alpha}+\frac{1}{\beta}=4$, then the value of |α - β| is

(1) $\frac{\sqrt{61}}{9}$ (2) $\frac{2\sqrt{17}}{9}$

(3) $\frac{\sqrt{34}}{9}$ (4) $\frac{2\sqrt{13}}{9}$ **[2014]**

5. If the equations $x^2 + 2x + 3 = 0$ and $ax^2 + bx + c = 0$, a , b, c ε R, have a common root , then a : b : c is

(1) 3 : 2 : 1 (2) 1 : 3 : 2

(3) 3 : 1 : 2 (4) 1 : 2: 3 **[2013]**

6. The equation $e^{\sin x}-e^{-\sin x}-4=0$ has

(1) infinite number of real roots (2) exactly one real root

(3) no real roots | (4) exactly four real roots. **[2012]**

7. Let for $a \neq a_1 \neq 0,\ f(x) = ax^2 + bx + c,\ g(x) = a_1x^2 + b_1x + c_1$ and $p(x) = f(x) - g(x)$. If , p(x) =0 only for x = -1 and p(-2) = 2, then the value of p(2) is

(1) 6 (2) 18

(3) 3 (4) 9 **[2011]**

8. Sachin and Rahul attempted to solve a quadratic equation. Sachin made a mistake in writing down the constant term and ended up in roots (4,3) . Rahul made a mistake in writing down coefficient of x to get roots (3,2). The correct roots of equation are

(1) -6,-1 (2) -4,-3

(3) 6,1 (4) 4,3 **[2011]**

9. If α and β are the roots of the equation $x^2 - x + 1 = 0,$ then $\alpha^{2009} + \beta^{2009}$ =

(1) -1 (2) 1

(3) 2 (4) -2 **[2010]**

10. If the roots of the equation $bx^2 + cx + a = 0$ be imaginary, then for all real values of x, the expression $3b^2x^2 + 6bcx + 2c^2$ is

(1) greater than 4ab (2) less than 4ab

(3) greater than -4ab (4) less than -4ab **[2009]**

11. The quadratic equations $x^2 - 6x + a = 0$ and $x^2 - cx + 6 = 0$ have one root in common. The other roots of the first and second equations are integers in the ratio 4 : 3. Then the common root is

(1) 1 (2) 4

(3) 3 (4) 2 **[2008]**

12. If the roots of the quadratic equation $x^2 + px + q = 0$ are $\tan 30^\circ$ and $\tan 15^\circ$, respectively then the value of 2 + q – p is

(1) 2 (2) 3

(3) 0 (4) 1 **[2006]**

13. All the values of m for which both roots of the equations $x^2 - 2mx + m^2 - 1 = 0$ are greater than -2 but less than 4, lie in the interval

(1) -2 < m < 0 (2) m > 3

(3) -1 < m < 3 (4) 1 < m < 4 **[2006]**

14. If x is real, the maximum value of $\frac{3x^2+9x+17}{3x^2+9x+7}$ is

(1) ¼ (2) 41

(3) 1 (4) 17/7 **[2006]**

15. The value of α for which the sum of the square of roots of the $x^2-(a-2)x-a-1=0$ assume the least value is

(1) 1 (2) 0

(3) 3 (4) 2 **[2005]**

16. If roots of the equation $x^2 - bx + c = 0$ be the consecutive integers, then $b^2 -4c$ equals

(1) -2 (2) 3

(3) 2 (4) 1 **[2005]**

17. If both the roots of the quadratic equation $x^2 - 2kx + k^2 + k - 5 = 0$ are less than 5, then k lies in the interval

(1) (5,6] (2) (6,∞)

(3) (-∞,4) (4) [4,5] **[2005]**

18. If (1 – p) is a root of quadratic equation $x^2 + px + (1 - p) = 0$, then its roots are

(1) 0,1 (2) -1,2

(3) 0,-1 (4) -1,1 **[2004]**

19. If one root of the equation x2 + px + 12 = 0 is 4, while the equation x2 + px + q = 0 has equal roots, then the value of 'q' is

(1) $\frac{49}{4}$ (2) 4

(3) 3 (4) 12 **[2004]**

20. If the sum of the roots of the quadratic equation $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals, then $\frac{a}{c}$, $\frac{b}{a}$ and $\frac{c}{b}$ are in

(1) arithmetic progression | (2) geometric progression

(3) harmonic progression | (4) arithmetic-geometric-progression **[2003]**

21. The number of real solutions of the equation $x^2 - 3\ |x| + 2 = 0$ is

(1) 2 (2) 4

(3) 1 (4) 3 **[2003]**

22. The value of 'a' for which one root of the quadratic equation $\left(a^2 - 5a + 3\right)x^2 + (3a-1)x + 2 = 0$ is twice as large as the other , is

(1) 2/3 (2) -2/3

(3) 1/3 (4) -1/3 **[2003]**

23. If $\alpha \neq \beta$ but $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$, then the equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ is

(1) $3x^2 - 25x + 3 = 0$ (2) $x^2 + 5x - 3 = 0$

(3) $x^2 - 5x + 3 = 0$ (4) $3x^2 - 19x + 3 = 0$ **[2002]**

24. Difference between the corresponding roots of $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ is same and $a \neq b$, then

(1) a + b + 4 = 0 (2) a + b - 4 = 0

(3) a – b – 4 = 0 (4) a – b + 4 = 0 **[2002]**

25. If p and q are the roots of the equation $x^2 + px + q = 0$, then

(1) p = 1 , q = -2 (2) p = 0 , q = 1

(3) p = -2 , q = 0 (4) p = - 2 , q = 1 **[2002]**

26. Product of real roots of the equation $t^2x^2 + |x| + 9 = 0$

(1) is always positive (2) is always negative

(3) does not exist (4) none of these **[2002]**

MCQ-Single Correct

1. A man X has 7 friends, 4 of them are ladies and 3 are men. His wife Y also has 7 friends, 3 of them are ladies and 4 are men. Assume X and Y have no common friends. Then the total number of ways in which X and Y together can throw a party inviting 3 ladies and 3 men, so that 3 friends of each of X and Y are in this party, is :

(1) 485 (2) 468

(3) 469 (4) 484 **[2017]**

2. If all the words (with or without meaning) having five letters, formed using the letters of the word SMALL and arranged as in a dictionary; then the position of the word SMALL is :

(1) 59^{th} (2) 52^{nd}

(3) 58^{th} (4) 46^{th} **[2016]**

3. The number of points, having both co-ordinates as integers, that lie in the interior of the triangle with vertices (0,0) , (0,41) and (41,0), is :

(1) 861 (2) 820

(3) 780 (4) 901 **[2015]**

4. The number of integers greater than 6,000 that can be formed, using the digits 3, 5, 6, 7 and 8, without repetition, is :

(1) 192 (2) 120

(3) 72 (4) 216 **[2015]**

5. Let T_n be the number of all possible triangles formed by joining vertices of an n-sided regular polygon. If $T_{n+1} - T_n = 10$, then the value of n is

(1) 5 (2) 10

(3) 8 (4) 7 **[2013]**

6. Assuming the balls to be identical except for difference in colours, the number of ways in which one or more balls can be selected from 10 white, 9 green and 7 black balls is

(1) 630 (2) 879

(3) 880 (4) 629 **[2012]**

7. Let X = {1, 2, 3, 4, 5}. The number of different ordered pairs (Y,Z) that can be formed such that $Y \subseteq X,\ Z \subseteq X,$ and $Y \cap Z$ is empty, is

(1) 2^5 (2) 5^3

(3) 5^2 (4) 3^5. **[2012]**

8. There are 10 points in a plane, out of these 6 are collinear. If N is the number of triangles formed by joining these points, then

(1) $140 < N \leq 190$ (2) $N > 190$

(3) $N \leq 100$ (4) $100 < N \leq 140$ **[2011]**

9. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on the shelf so that the dictionary is always in the middle. Then the number of such arrangements is

(1) less than 500 (2) at least 500 but less than 750

(3) at least 750 but less than 1000 (4) at least 1000 **[2009]**

10. How many different words can be formed by jumbling the letters in the word MISSISSIPPI in which no two S are adjacent?

(1) $8.{}^6C_4.{}^7C_4$ (2) $6.7.{}^8C_4$

(3) $6.8.{}^7C_4$ (4) $7.{}^6C_4.{}^8C_4$ **[2008]**

11. The set S = {1, 2, 3, …., 12} is to be partitioned into three sets A, B, C of equal size. Thus, $A \cup B \cup C = S$, $A \cap B = B \cap C = A \cap C = \phi$. The number of ways to partition S is

(1) $\frac{12!}{3!(4!)^3}$ (2) $\frac{12!}{3!(3!)^4}$

(3) $\frac{12!}{(4!)^3}$ (4) $\frac{12!}{(3!)^4}$ **[2007]**

12. At an election, a voter may vote for any number of candidates, not greater than the number to be elected. There are 10 candidates and 4 are to be elected. If a voter votes for at least one candidate, then the number of ways in which he can vote is

(1) 5040 (2) 6210

(3) 385 (4) 1110 **[2006]**

13. If the letters of word SACHIN are arranged in all possible ways and these words are written out as in dictionary, then the word SACHIN appears at serial number

(1) 601 (2) 600

(3) 603 (4) 602 **[2005]**

14. How many ways are there to arrange the letters in the word GARDEN with the vowels in alphabetic order?

(1) 120 (2) 480

(3) 360 (4) 240 **[2004]**

15. The number of ways of distributing 8 identical balls in 3 distinct boxes so that none of the boxes is empty is

(1) 5 (2) ${}^{8}C_{3}$

(3) 3^{8} (4) 21 **[2004]**

16. A student is to answer 10 out of 13 questions in an examination such that he must choose at least 4 from the first five questions. The number of choices available to him is

(1) 140 (2) 196

(3) 280 (4) 346 **[2003]**

17. The number of ways in which 6 men and 5 women can dine at a round table if no two women are to sit together is given by

(1) 6!×5! (2) 30

(3) 5!×4! (4) 7!×5! **[2003]**

18. If ${}^{n}C_{r}$ denotes the number of combinations of n things taken r at a time, then the expression ${}^{n}C_{r+1} + {}^{n}C_{r-1} + 2\times {}^{n}C_{r}$ equals

(1) ${}^{n+2}C_{r}$ (2) ${}^{n+2}C_{r+1}$

(3) ${}^{n+1}C_{r}$ (4) ${}^{n+1}C_{r+1}$ **[2003]**

19. If f(x) = x^n, then the value of $f(1) - \frac{f'(1)}{1!} + \frac{f''(1)}{2!} - \frac{f'''(1)}{3!} + \ldots + \frac{(-1)^n f^n(1)}{n!}$ is

(1) 2^n (2) 2^{n-1}

(3) 0 (4) 1 **[2003]**

20. Number of numbers greater than 1000 but less than 4000 formed using the digits 0, 2, 3, 4 with repetition allowed is

(1) 125 (2) 105

(3) 128 (4) 625 **[2002]**

21. Five digit number divisible by 3 is formed using 0, 1, 2, 3, 4, 6 and 7 without repetition. Total number of such numbers are

(1) 312 (2) 3125

(3) 120 (4) 216 **[2002]**

22. The sum of integers from 1 to 100 that are divisible by 2 or 5 is

(1) 3000 (2) 3050

(3) 3600 (4) 3250 **[2002]**

23. Total number of four digit odd numbers that can be formed using 0, 1, 2, 3, 5, 7 are

(1) 216 (2) 375

(3) 400 (4) 720 **[2002]**

Assertion – Reason Type

1. In a shop there are five types of ice-creams available. A child buys six ice-creams.

   Statement – I : The number of different ways the child can buy the six ice-creams is ${}^{10}C_5$.

   Statement – II : The number of different ways the child can buy the six ice-creams is equal to the number of different ways of arranging 6 A's and 4 B's in a row.

Binomial Theorem

MCQ-Single Correct

1. The value of $\left({}^{21}C_1 - {}^{10}C_1\right)$ + $\left({}^{21}C_2 - {}^{10}C_2\right)$ + $\left({}^{21}C_3 - {}^{10}C_3\right)$ + $\left({}^{21}C_4 - {}^{10}C_4\right)$ + ... + $\left({}^{21}C_{10} - {}^{10}C_{10}\right)$ is :

(1) $2^{21} - 2^{11}$ (2) $2^{21} - 2^{10}$

(3) $2^{20} - 2^{9}$ (4) $2^{20} - 2^{10}$ **[2017]**

2. If the number of terms in the expansion of $\left(1 - \frac{2}{x} + \frac{4}{x^2}\right)^n$, x ≠ 0, is 28, then the sum of the coefficients of all the terms in this expansion, is :

(1) 2187 (2) 243

(3) 729 (4) 64 **[2016]**

3. The sum of coefficients of integral powers of x in the binomial expansion of $\left(1 - 2\sqrt{x}\right)^{50}$ is :

(1) $\frac{1}{2}\left(3^{50}\right)$ (2) $\frac{1}{2}\left(3^{50} - 1\right)$

(3) $\frac{1}{2}\left(2^{50} + 1\right)$ (4) $\frac{1}{2}\left(3^{50} + 1\right)$ **[2015]**

4. If the coefficients of x^3 and x^4 in the expansion of $(1 + ax + bx^2)(1 - 2x)^{18}$ in powers of x are both zero, then (a,b) is equal to

(1) $\left(16, \frac{251}{3}\right)$ (2) $\left(14, \frac{251}{3}\right)$

(3) $\left(14, \frac{272}{3}\right)$ (4) $\left(16, \frac{272}{3}\right)$ **[2014]**

5. The term independent of x in the expansion of $\left(\frac{x+1}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x - x^{1/2}}\right)^{10}$ is

(1) 120 (2) 210

(3) 310 (4) 4 **[2013]**

6. If n is a positive integer, then $\left(\sqrt{3}+1\right)^{2n}-\left(\sqrt{3}-1\right)^{2n}$ is

(1) an even positive integer.

(2) a rational number other than positive integers.

(3) an irrational number.

(4) an odd positive integer. **[2012]**

7. The remainder left out when $8^{2n}-(62)^{2n+1}$ is divided by 9 is

(1) 0 (2) 2

(3) 7 (4) 8 **[2009]**

8. In a binomial distribution B$\left(n, p=\frac{1}{4}\right)$, if the probability of at least one success is greater than or equal to $\frac{9}{10}$, then n is greater than

(1) $\frac{1}{\log_{10}4-\log_{10}3}$ (2) $\frac{1}{\log_{10}4+\log_{10}3}$

(3) $\frac{9}{\log_{10}4-\log_{10}3}$ (4) $\frac{4}{\log_{10}4-\log_{10}3}$ **[2009]**

9. The sum of the series ${}^{20}C_0-{}^{20}C_1+{}^{20}C_2-{}^{20}C_3+....+{}^{20}C_{10}$ is

(1) $-{}^{20}C_{10}$ (2) $\left(\frac{1}{2}\right){}^{20}C_{10}$

(3) 0 (4) ${}^{20}C_{10}$ **[2007]**

10. In the binomial expansion of $(a-b)^n$, n ≥ 5 the sum of 5^{th} and 6^{th} terms is zero, then a/b equals

(1) $\frac{5}{n-4}$ (2) $\frac{6}{n-5}$

(3) $\frac{n-5}{6}$ (4) $\frac{n-4}{5}$ **[2007]**

11. For natural numbers m, n if $(1-y)^m (1+y)^n = 1 + a_1 y + a_2 y^2 + ...$, and $a_1 = a_2 = 10$, then (m,n) is

(1) (20,45) (2) (35,20)

(3) (45,35) (4) (35,45) **[2006]**

12. The value of ${}^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3$ is

(1) ${}^{55}C_4$ (2) ${}^{55}C_3$

(3) ${}^{56}C_3$ (4) ${}^{56}C_4$ **[2005]**

13. If the coefficient of x^7 in $\left[ax^2 + \left(\frac{1}{bx}\right)\right]^{11}$ equals the coefficient of x^{-7} in $\left[ax^2 - \left(\frac{1}{bx}\right)\right]^{11}$, then a and b satisfy the relation

(1) a – b = 1 (2) a + b = 1

(3) $\frac{a}{b} = 1$ (4) ab = 1 **[2005]**

14. If x is so small that x^3 and higher powers of x may be neglected, then $\frac{(1+x)^{3/2} - (1+x/2)^3}{(1-x)^{1/2}}$ may be approximated as

(1) $1 - \frac{3}{8}x^2$ (2) $3x + \frac{3}{8}x^2$

(3) $-\frac{3}{8}x^2$ (4) $\frac{x}{2} - \frac{3}{8}x^2$ **[2005]**

15. The coefficient of the middle term in the binomial expansion in powers of x of $(1+\alpha x)^4$ and of $(1-\alpha x)^6$ is the same if α equals

(1) -5/3 (2) 3/5

(3) -3/10 (4) 10/3 **[2004]**

16. The coefficient of x^n in expansion of $(1 + x)(1 – x)^n$ is

(1) (n – 1) (2) $(-1)^n(1-n)$

(3) $(-1)^{n-1}(n-1)^2$ (4) $(-1)^{n-1}n$ **[2004]**

17. If $S_n = \sum_{r=0}^{n} \frac{1}{{}^nC_r}$ and $t_n = \sum_{r=0}^{n} \frac{r}{{}^nC_r}$, then $\frac{t_n}{S_n}$ is equal to

(1) n/2 (2) n/2 -1

(3) n -1 (4) n – ½ **[2004]**

18. The number of integral terms in the expansion of $\left(\sqrt{3}+\sqrt[8]{5}\right)^{256}$ is

(1) 32 (2) 33

(3) 34 (4) 35 **[2003]**

19. If x is positive, the first negative term in the expansion of $(1 + x)^{27/5}$ is

(1) 7^{th} term (2) 5^{th} term

(3) 8^{th} term (4) 6^{th} term **[2003]**

20. The positive integer just greater than $(1 + .0001)^{1000}$ is

(1) 4 (2) 5

(3) 2 (4) 3 **[2002]**

21. r and n are positive integers r > 1, n > 2 and coeffient of $(r + 2)^{th}$ term and $3r^{th}$ term in the expansion of $(1 + x)^{2n}$ are equal, then n equals

(1) 3r (2) 3r + 1

(3) 2r (4) 2r + 1 **[2002]**

22. The coefficients of x^p and x^q in the expansion of $(1 + x)^{p+q}$ are

(1) equal (2) equal with opposite signs

(3) reciprocals of each other (4) none of these **[2002]**

23. If the sum of the coefficients in the expansion of $(a+b)^n$ is 4096, then the greatest coefficient in the expansion is

(1) 1594 (2) 792

(3) 924 (4) 2924 **[2002]**

Assertion – Reason Type

1. Let $S_1 = \sum_{j=1}^{10} j(j-1)\,{}^{10}C_j$, $S_2 = \sum_{j=1}^{10} j\,{}^{10}C_j$ and $S_3 = \sum_{j=1}^{10} j^2\,{}^{10}C_j$ **[2010]**

Statement – I : $S_3 = 55 \times 2^9$

Statement – II : $S_1 = 90\times2^8$ and $S_2 = 10\times2^8$

2. **Statement-I :** $\sum_{r=0}^{n}(r+1)\,{}^nC_r = (n+2)2^{n-1}$. **[2008]**

Statement-II : $\sum_{r=0}^{n}(r+1)\,{}^nC_r x^r = (1+x)^n + nx(1+x)^{n-1}$

Matrices

MCQ-Single Correct

1. If A = $\begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$, then $adj(3A^2+12A)$ is equal to:

(1) $\begin{bmatrix} 72 & -84 \\ -63 & 51 \end{bmatrix}$ (2) $\begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$

(3) $\begin{bmatrix} 51 & 84 \\ 63 & 72 \end{bmatrix}$ (4) $\begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$ **[2017]**

2. If A = $\begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and $A\,adj\,A = AA^T$, then 5a +b is equal to :

(1) 5 (2) 4

(3) 13 (4) -1 **[2016]**

3. If A = $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ is a matrix satisfying the equation $AA^T = 9I$, where I is 3×3 identity matrix, then ordered pair (a,b) is equal to :

(1) (-2,1) (2) (2,1)

(3) (-2,-1) (4) (2,-1) **[2015]**

4. If A is a 3×3 non-singular matrix such that AA'=A'A and $B = A^{-1}A$, then BB' equals

(1) I + B (2) I

(3) B^{-1} (4) $\left(B^{-1}\right)'$ **[2014]**

5. If P = $\begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is the adjoint of a 3×3 matrix A and |A|=4, then α is equal to

(1) 11 (2) 5

(3) 0 (4) 4 **[2013]**

6. Let A = $\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$. If u_1 and u_2 are column matrices such that $Au_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $Au_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, then $u_1 + u_2$ is equal to

(1) $\begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$ (2) $\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$

(3) $\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ (4) $\begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$ **[2012]**

7. If $\omega \neq 1$ is the complex cube root of unity and matrix H = $\begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix}$, then H^{70} is equal to

(1) H^2 (2) H

(3) 0 (4) -H **[2011]**

8. The number of 3×3 non-singular matrices, with four entries as 1 and all other entries as 0, is

(1) 5 (2) 6

(3) at least 7 (4) less than 4 **[2010]**

9. Let A be a square matrix all of whose entries are integers. Then which one of the following is true?

(1) If detA = ±1 , then A^{-1} exists but all its entries are not necessarily integers

(2) If detA = ±1 , then A^{-1} exists and all its entries are non-integers

(3) If detA = ±1 , then A^{-1} exists and all its entries are integers

(4) If detA = ±1 , then A^{-1} need not exist **[2008]**

10. Let A = $\begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$. If | A^2 | = 25, then $|\alpha|$ equals

(1) 5^2 (2) 1

(3) 1/5 (4) 5 **[2007]**

11. If A and B are square matrices of size $n\times n$ such that $A^2 - B^2 = (A-B)(A+B)$, then which of the following will be always true?

(1) A = B

(2) AB = BA

(3) Either of A or B is a zero matrix

(4) Either of A or B is an identity matrix **[2006]**

12. Let A = $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and B = $\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$, a , b $\in$ N. Then

(1) there cannot exist any B such that AB = BA

(2) there exist more than one but finite number of B's such that AB = BA

(3) there exists exactly one B such that AB = BA

(4) there exist infinitely many B's such that AB = BA **[2006]**

13. If $A^2 - A + I = 0$, then the inverse of A is

(1) A + I (2) A

(3) A – I

(4) I – A

14. If A = $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then which one of the following holds for all $n \geq 1$, by the principle of mathematical induction

(1) $A^n = nA - (n-1)I$

(2) $A^n = 2^{n-1}A - (n-1)I$

(3) $A^n = nA + (n-1)I$

(4) $A^n = 2^{n-1}A + (n-1)I$ **[2005]**

15. Let $A = \begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$. The only correct statement about the matrix A is

(1) A is a zero matrix

(2) $A^2 = I$

(3) A^{-1} does not exist

(4) A = (-1)I, where I is a unit matrix

[2004]

16. Let $A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{pmatrix}$ (10) and $B = \begin{pmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{pmatrix}$. If B is the inverse of matrix A , then α is

(1) -2

(2) 5

(3) 2

(4) -1 **[2004]**

17. If $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ and $A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$, then

(1) $\alpha = a^2 + b^2$, $\beta = ab$

(2) $\alpha = a^2 + b^2$, $\beta = 2ab$

(3) $\alpha = a^2 + b^2$, $\beta = a^2 - b^2$

(4) $\alpha = 2ab$ $\beta = a^2 + b^2$ **[2003]**

Assertion-Reason type

(1) Statement-I is True; Statement-II is true; Statement-II is **not** a correct explaination of Statement-I.

(2) Statement-I is True; Statement-II is False.

(3) Statement-I is False; Statement-II is true

(4) Statement-I is True; Statement-II is true; Statement-II is a **correct** explaination of Statement-I.

1. Consider the following relation **R** on the set of real square matrices of order 3. **[2011]**

R = {(A,B)|A = $P^{-1}BP$ for some invertible matrix P}.

Statement – I : R is an equivalence relation.

Statement-II : For any two invertible 3×3 matrices M and N, $(MN)^{-1} = N^{-1}M^{-1}$.

2. Let A be a 2×2 matrix with non-zero entries and let $A^2 = I$, where I is 2×2 identity matrix. Define Tr(A) = sum of diagonal elements of A and |A| = determinant of matrix A.**[2010]**

Statement-I : Tr(A) = 0

Statement-II : |A|=1

3. Let A be a 2×2 matrix **[2009]**

Statement-I : adj(adj A) = A

Statement-II : |adj A| = |A|

4. Let A be a 2×2 matrix with real entries. Let I be the 2×2 identity matrix. Denote by tr(A), the sum of diagonal entries of A. Assume that $A^2 = I$ **[2008]**

Statement-I : If $A \neq I$ and $A \neq -I$, then det A = -1.

Statement-II : If $A \neq I$ and $A \neq -I$, then $tr(A) \neq 0$

Solutions

1. $A^2 = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} 4+12 & -6-3 \\ -8-4 & 12+1 \end{bmatrix} = \begin{bmatrix} 16 & -9 \\ -12 & 13 \end{bmatrix}$

$3A^2 + 12A = \begin{bmatrix} 48+24 & -27-36 \\ -36-48 & 39+12 \end{bmatrix} = \begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$

$adj\left(3A^2 + 12A\right) = \begin{bmatrix} 51 & 84 \\ 63 & 72 \end{bmatrix}$

Option (3) , is the correct answer.

2. $adj\ A = \begin{bmatrix} 2 & -3 \\ b & 5a \end{bmatrix}$, $A^T = \begin{bmatrix} 5a & 3 \\ -b & 2 \end{bmatrix}$

$A\,adj(A) = AA^T$

$\Rightarrow A\,adj\ A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ b & 5a \end{bmatrix} = \begin{bmatrix} 10a-b^2 & -15a-5ab \\ 6+2b & -9+10a \end{bmatrix}$

$\Rightarrow AA^T = [\ \ ]$

Coordinate Geometry

<u>Straight Line</u>

MCQ-Single Correct

1. Let k be an integer such that the triangle with vertices (k,-3k), (5,k) and (-k,2) has area 28 sq units. Then the orthocentre of this triangle is at the point :

(1) $\left(2,-\frac{1}{2}\right)$ (2) $\left(1,\frac{3}{4}\right)$

(3) $\left(1,-\frac{3}{4}\right)$ (4) $\left(2,\frac{1}{2}\right)$ **[2017]**

2. Two sides of a rhombus are along the lines, $x-y+1=0$ and $7x-y-5=0$. If its diagonals intersect at (-1,-2), then which one of the following is a vertex of this rhombus?

(1) (-3,-8) (2) $\left(\frac{1}{3},-\frac{8}{3}\right)$

(3) $\left(-\frac{10}{3},-\frac{7}{3}\right)$ (4) (-3,-9) **[2016]**

3. Locus of the image of the point (2,3) in the line $(2x-3y+4)+k(x-2y+3)=0$, $k\in R$, is a :

(1) straight line parallel to y-axis (2) circle of radius $\sqrt{2}$.

(3) circle of radius $\sqrt{3}$ (4) straight line parallel to x-axis. **[2015]**

4. Let a, b, c and d be non-zero numbers. If the point of intersection of the lines $4ax+2ay+c=0$ and $5bx+2by+d=0$ lies in the fourth quadrant and is equidistant from the two axes then

(1) $2bc-3ad=0$ (2) $2bc+3ad=0$

(3) $3bc-2ad=0$ (4) $3bc+2ad=0$ **[2014]**

5. Let PS be the median of the triangle with vertices P(2,2), Q(6,-1) and R(7,3). The equation of the line passing through (1,-1) and parallel to PS is

(1) $4x-7y-11=0$ (2) $2x+9y+7=0$

(3) $4x+7y+3=0$ (4) $2x-9y-11=0$ **[2014]**

6. The x-coordinate of the incentre of the triangle that has the coordinates of mid points of its sides as (0,1) (1,1) and (1,0) is

(1) $2-\sqrt{2}$ (2) $1+\sqrt{2}$

(3) $1-\sqrt{2}$ (4) $2+\sqrt{2}$ **[2013]**

7. A ray of light along $x+\sqrt{3}y=\sqrt{3}$ gets reflected upon reaching x-axis, the equation of the reflected rays is

(1) $\sqrt{3}y=x-\sqrt{3}$ (2) $y=\sqrt{3}x-\sqrt{3}$

(3) $\sqrt{3}y = x-1$

(4) $y = x+\sqrt{3}$ **[2013]**

8. If the line $2x+y=k$ passes through the point which divides the line segment joining the points (1,1) and (2,4) in the ration 3:2, then k equal

(1) 6

(2) 11/5

(3) 29/5

(4) 5 **[2012]**

9. A line is drawn through the point (1,2) to meet the coordinate axes at P and Q such that it forms a triangle OPQ, where O is the origin. If the area of the triangle OPQ is least, then the slope of the line PQ is

(1) -2

(2) -1/2

(3) -1/4

(4) -4 **[2012]**

10. The lines $x+y=|a|$ and $ax-y=1$ intersect each other in the first quadrant. Then the set of all possible values of a is the interval

(1) $(-1,\infty)$

(2) $(-1,1]$

(3) $(0,\infty)$

(4) $[1,\infty)$ **[2011]**

11. If A(2,-3) and B(-2,1) are two vertices of a triangle and third vertex moves on the line $2x+3y=9$, then the locus of the centroid of the triangle is

(1) $2x+3y=3$

(2) $2x-3y=1$

(3) $x-y=1$

(4) $2x+3y=1$

[2011]

12. The line L given by $\frac{x}{5}+\frac{y}{b}=1$ passes through the point (13,32). The line K is parallel to L and has the equation $\frac{x}{c}+\frac{y}{3}=1$. Then the distance between L and K is

(1) $\sqrt{17}$

(2) $\frac{17}{\sqrt{15}}$

(3) $\frac{23}{\sqrt{17}}$

(4) $\frac{23}{\sqrt{15}}$ **[2010]**

13. Three distinct points A, B and C are given in the 2-dimensional coordinate plane such that the ratio of the distance of any one of them from the point (1,0) to the distance from the point (-1,0) is equal to $\frac{1}{3}$. Then the circumcentre of the triangle ABC is at the point

(1) (0,0)

(2) $\left(\frac{5}{4},0\right)$

(3) $\left(\frac{5}{2},0\right)$

(4) $\left(\frac{5}{3},0\right)$ **[2009]**

14. The lines $p\left(p^2+1\right)x-y+q=0$ and $\left(p^2+1\right)^2 x+\left(p^2+1\right)y+2q=0$ are perpendicular to a common line for

(1) no value of p
(2) exactly one value of p
(3) exactly two values of p
(4) more than two values of p **[2009]**

15. The perpendicular bisector of the line segment joining P(1,4) and Q(k,3) has y-intercept -4. Then a possible value of k is

(1) 1
(2) 2
(3) -2
(4) -4 **[2008]**

16. A straight line through the point A(3,4) is such that its intercept between the axes is bisected at A. Its equation is

(1) $x+y=7$
(2) $3x-4y+7=0$
(3) $4x+3y=24$
(4) $3x+4y=25$ **[2006]**

17. The two lines $x=ay+b,\ z=cy+d$; and $x=a'y+b',\ z=c'y+d'$ are perpendicular to each other if

(1) $aa'+cc'=-1$
(2) $aa'+cc'=1$
(3) $\frac{a}{a'}+\frac{c}{c'}=-1$
(4) $\frac{a}{a'}+\frac{c}{c'}=1$ **[2006]**

18. If $\left(a,a^2\right)$ falls inside the angle made by the lines $y=\frac{x}{2}$, $x>0$ and $y=3x$, x>0, then a belongs to

(1) $\left(0,\frac{1}{2}\right)$
(2) $\left(3,\infty\right)$
(3) $\left(\frac{1}{2},3\right)$
(4) $\left(-3,-\frac{1}{2}\right)$ **[2006]**

19. The line parallel to the x-axis and passing through the intersection of the lines $ax+2by+3b=0$ and $bx-2ay-3a=0$, where $\left(a,b\right)\neq\left(0,0\right)$ is

(1) below the x-axis at a distance of $\frac{3}{2}$ from it
(2) below the x-axis at a distance of $\frac{2}{3}$ from it
(3) above the x-axis at a distance of $\frac{3}{2}$ from it
(4) above the x-axis at a distance of $\frac{2}{3}$ from it **[2005]**

20. If non-zero numbers a, b , c are in H.P. , then the straight line $\frac{x}{a}+\frac{y}{b}+\frac{1}{c}=0$ always passes through a fixed point. The point is

(1) (-1,2) (2) (-1,-2)

(3) $(1,-2)$ (4) $\left(1,-\frac{1}{2}\right)$ **[2005]**

21. If a vertex of a triangle is (1,1) and the mid-points of two sides through this vertex are (-1,2) and (3,2), then the centroid of the triangle is

(1) $\left(-1,\frac{7}{3}\right)$ (2) $\left(-\frac{1}{3},\frac{7}{3}\right)$

(1) $\left(1,\frac{7}{3}\right)$ (2) $\left(\frac{1}{3},\frac{7}{3}\right)$ **[2005]**

22. If the pair of lines $ax^2+2(a+b)xy+by^2=0$ lie along diameters of a circle and divide the circle into four sectors such that the area of one of the sectors is thrice the areas of another sector then

(1) $3a^2-10ab+3b^2=0$ (2) $3a^2-2ab+3b^2=0$

(3) $3a^2+10ab+3b^2=0$ (4) $3a^2+2ab+3b^2=0$ **[2005]**

23. Let A(2,-3) and B(-2,1) be vertices of a triangle ABC. If the centroid of this triangle moves on the line $2x+3y=1$, then the locus of the vertex C is the line

(1) $2x+3y=9$ (2) $2x-3y=7$

(3) $3x+2y=5$ (4) $3x-2y=3$ **[2004]**

24. The equation of the straight line passing through the point (4,3) and making intercepts on the co-ordinate axes whose sum is -1 is

(1) $\frac{x}{2}+\frac{y}{3}=-1$ and $\frac{x}{-2}+\frac{y}{1}=-1$ (2) $\frac{x}{2}-\frac{y}{3}=-1$ and $\frac{x}{-2}+\frac{y}{1}=-1$

(3) $\frac{x}{2}+\frac{y}{3}=1$ and $\frac{x}{2}+\frac{y}{1}=1$ (3) $\frac{x}{2}-\frac{y}{3}=1$ and $\frac{x}{-2}+\frac{y}{1}=1$ **[2004]**

25. If the sum of the slopes of the lines given by $x^2-2cxy-7y^2=0$ is four times their product, then c has the value

(1) 1 (2) -1

(3) 2 (4) -2 **[2004]**

26. If one of the lines given by $6x^2-xy+4cy^2=0$ is $3x+4y=0$, then c equals

(1) 1 (2) -1

(3) 3 (4) -3 **[2004]**

27. If the equation of the locus of a point equidistant from the points (a_1, b_1) and (a_2, b_2) is $(a_1 - a_2)x + (b_1 - b_2)y + c = 0$, then the value of 'c' is

(1) $\frac{1}{2}(a_2^2 + b_2^2 - a_1^2 - b_1^2)$ (2) $a_1^2 + a_2^2 + b_1^2 - b_2^2$

(3) $\frac{1}{2}(a_1^2 + a_2^2 - b_1^2 - b_2^2)$ (4) $\sqrt{a_1^2 + b_1^2 - a_2^2 - b_2^2}$ **[2003]**

28. Locus of centroid of the triangle whose vertices are $(a\cos t, a\sin t)$, $(b\sin t, -b\cos t)$ and (1,0), where t is a parameter , is

(1) $(3x-1)^2 + (3y)^2 = a^2 - b^2$ (2) $(3x-1)^2 + (3y)^2 = a^2 + b^2$

(3) $(3x+1)^2 + (3y)^2 = a^2 + b^2$ (4) $(3x+1)^2 + (3y)^2 = a^2 - b^2$ **[2003]**

29. If the pair of straight lines $x^2 - 2pxy - y^2 = 0$ and $x^2 - 2qxy - y^2 = 0$ be such that each pair bisects the angle between the other pair, then

(1) p = q (2) p = -q

(3) pq = 1 (4) pq = -1 **[2003]**

30. A square of side a lies above the x-axis and has one vertex at the origin. The side passing through the origin makes an angle α $\left(0 < \alpha < \frac{\pi}{4}\right)$ with the positive direction of x-axis. The equation of its diagonal not passing through the origin is

(1) $y(\cos\alpha - \sin\alpha) - x(\sin\alpha - \cos\alpha) = a$

(2) $y(\cos\alpha + \sin\alpha) + x(\sin\alpha - \cos\alpha) = a$

(3) $y(\cos\alpha + \sin\alpha) + x(\sin\alpha + \cos\alpha) = a$

(4) $y(\cos\alpha + \sin\alpha) + x(\cos\alpha - \sin\alpha) = a$ **[2003]**

31. If the pair of lines $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ intersect on the y-axis then

(1) $2fgh = bg^2 + ch^2$ (2) $bg^2 \neq ch^2$

(3) abc = 2fgh (4) none of these **[2002]**

32. Lines represented by $3ax^2 + 5xy + \left(a^2 - 2\right)y^2 = 0$ are □ to each other for

(1) two values of a (2) $\forall$ a

(3) for one value of a (4) for no values of a **[2002]**

33. Locus of mid-point of the portion between the axes of $x\cos\alpha + y\sin\alpha = p$, where p is constant, is

(1) $x^2 + y^2 = \frac{4}{p^2}$ (2) $x^2 + y^2 = 4p^2$

(3) $\frac{1}{x^2} + \frac{1}{y^2} = \frac{2}{p^2}$ (4) $\frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{p^2}$ **[2002]**

34. A triangle with vertices (4,0) , (-1,-1), (3,5) is

(1) isosceles and right angled (2) isosceles but not right angled

(3) right angled but not isosceles (4) neither right angled nor isosceles **[2002]**

Circle

MCQ Single Correct

1. The radius of a circle, having minimum area, which touches the curve $y = 4 - x^2$ and the lines $y = |x|$ is :

(1) $2\left(\sqrt{2}+1\right)$ (2) $2\left(\sqrt{2}-1\right)$

(3) $4\left(\sqrt{2}-1\right)$ (4) $4\left(\sqrt{2}+1\right)$ **[2017]**

2. If one of the diameters of the circle , given by the equation , $x^2 + y^2 - 4x + 6y - 12 = 0$, is a chord of the circle S , whose centre is at (-3,2), then the radius of S is :

(1) $5\sqrt{3}$ (2) 5

(3) 10 (4) $5\sqrt{2}$ **[2016]**

3. The number of common tangents to the circles $x^2 + y^2 - 4x - 6y - 12 = 0$ and $x^2 + y^2 + 6x + 18y + 26 = 0$, is :

(1) 2 (2) 3

(3) 4 (4) 1 **[2015]**

4. Let C be the circle with centre at (1,1) and radius =1. If T is the circle centred at (0,y) , passing through origin and touching the circle C externally, then the radius of T is equal to

(1) $\frac{\sqrt{3}}{\sqrt{2}}$ (2) $\frac{\sqrt{3}}{2}$

(3) $\frac{1}{2}$ (4) $\frac{1}{4}$ **[2014]**

5. The circle passing through (1,-2) and touching the axis of x at $(3,0)$ also passes through the point

(1) $(2,-5)$ (2) $(5,-2)$

(3) $(-2,5)$ (4) $(-5,2)$ **[2013]**

6. The length of the diameter of the circle which touches the x-axis at the point (1,0) and passes through the point (2,3)

(1) 6/5 (2) 5/3

(3) 10/3 (4) 3/5 **[2012]**

7. The equation of the circle passing through the points (1,0) and (0,1) and having the smallest radius is

(1) $x^2+y^2+2x+2y-7=0$ (2) $x^2+y^2+x+y-2=0$

(3) $x^2+y^2-2x-2y+1=0$ (4) $x^2+y^2-x-y=0$ **[2011]**

8. The circle $x^2+y^2=4x+8y+5$ intersects the line $3x-4y=m$ at two distinct points if

(1) $-35<m<15$ (2) $15<m<65$

(3) $35<m<85$ (4) $-85<m<-35$ **[2010]**

9. If P and Q are the points of intersection of the circles $x^2+y^2+3x+7y+2p-5=0$ and $x^2+y^2+2x+2y-p^2=0$, then there is a circle passing through P, Q and (1,1) for

(1) all values of p (2) all except one value of p

(3) all except two values of p (4) exactly one value of p **[2009]**

10. The point diametrically opposite to the point P (1,0) on the circle $x^2+y^2+2x+4y-3=0$ is

(1) (3,-4) (2) (-3,4)

(3) (-3,-4) (4) (3,4) **[2008]**

11. Consider a family of circles which are passing through the point (-1,1) and are tangent to x-axis. If (h,k) are the co-ordinates of the centre of the circles, then the set of values of k is given by the interval

(1) $0<k<\frac{1}{2}$ (2) $k\geq\frac{1}{2}$

(3) $-\frac{1}{2}\leq k\leq\frac{1}{2}$ (4) $k\leq\frac{1}{2}$ **[2007]**

12. If the lines $3x-4y-7=0$ and $2x-3y-5=0$ are two diameters of a circle of area 49π square units, the equation of the circle is

(1) $x^2+y^2+2x-2y-47=0$ (2) $x^2+y^2+2x-2y-62=0$

(3) $x^2+y^2-2x+2y-62=0$ (4) $x^2+y^2-2x+2y-47=0$ **[2006]**

13. Let C be the circle with centre (0,0) and radius 3 units. The equation of the locus of the mid points of the chords of the circle C that subtend an angle of $\frac{2\pi}{3}$ at its centre is

(1) $x^2+y^2=\frac{3}{2}$ (2) $x^2+y^2=1$

(3) $x^2+y^2=\frac{27}{4}$ (4) $x^2+y^2=\frac{9}{4}$ **[2006]**

14. If the circles $x^2+y^2+2ax+cy+a=0$ and $x^2+y^2-3ax+dy-1=0$ intersect in two distinct points P and Q then the line $5x+by-a=0$ passes through P and Q for

(1) exactly one value of a (2) no value of a

(3) infinitely many values of a (4) exactly two values of a **[2005]**

15. A circle touches the x-axis and also touches the circle with centre at (0,3) and radius 2. The locus of the centre of the circle is

(1) an ellipse (2) a circle

(3) a hyperbola (4) a parabola **[2005]**

16. If a circle passes through the point (a,b) and cuts the circle $x^2+y^2=p^2$ orthogonally, then the equation of the locus of its centre is **[2005]**

(1) $x^2+y^2-3ax-4by+\left(a^2+b^2-p^2\right)=0$ (2) $2ax+2by-\left(a^2-b^2+p^2\right)=0$

(3) $x^2+y^2-2ax-3by+\left(a^2-b^2-p^2\right)=0$ (4) $2ax+2by-\left(a^2+b^2+p^2\right)=0$

17. If a circle passes through the point (a,b) and cuts the circle $x^2+y^2=4$ orthogonally, then the locus of its centre is **[2004]**

(1) $2ax+2by+\left(a^2+b^2+4\right)=0$ (2) $2ax+2by-\left(a^2+b^2+4\right)=0$

(3) $2ax-2by+\left(a^2+b^2+4\right)=0$ (4) $2ax-2by-\left(a^2+b^2+4\right)=0$

18. A variable circle passes through the fixed point A (p,q) and touches x-axis. The locus of the other end of the diameter through A is

(1) $(x-p)^2 = 4qy$

(2) $(x-q)^2 = 4py$

(3) $(y-p)^2 = 4qx$

(4) $(y-q)^2 = 4px$ **[2004]**

19. If the lines $2x+3y+1=0$ and $3x-y-4=0$ lie along diameters of a circle of circumference 10π, then the equation of the circle is

(1) $x^2+y^2-2x+2y-23=0$

(2) $x^2+y^2-2x-2y-23=0$

(3) $x^2+y^2+2x+2y-23=0$

(4) $x^2+y^2+2x-2y-23=0$ **[2004]**

20. The intercept on the line y = x by the circle $x^2+y^2-2x=0$ is AB. Equation of the circle on AB as a diameter is

(1) $x^2+y^2-x-y=0$

(2) $x^2+y^2-x+y=0$

(3) $x^2+y^2+x+y=0$

(4) $x^2+y^2+x-y=0$ **[2004]**

21. If the two circles $(x-1)^2+(y-3)^2=r^2$ and $x^2+y^2-8x+2y+8=0$ intersect in two distinct points, then

(1) $2<r<8$

(2) $r<2$

(3) r = 2

(4) $r>2$ **[2003]**

22. The lines $2x-3y=5$ and $3x-4y=7$ are diameters of a circle having area as 154 sq units. Then the equation of the circle is

(1) $x^2+y^2+2x-2y=62$

(2) $x^2+y^2+2x-2y=47$

(3) $x^2+y^2-2x+2y=47$

(4) $x^2+y^2-2x+2y=62$ **[2003]**

23. If the chord $y=mx+1$ of the circle $x^2+y^2=1$ subtends an angle of measure 45^o at the major segment of the circle then the value of m is

(1) $2\pm\sqrt{2}$

(2) $-2\pm\sqrt{2}$

(3) $-1\pm\sqrt{2}$

(4) none of these **[2002]**

24. The centres of a set of circles, each of radius 3, lie on the circle $x^2+y^2=25$. The locus of any point in the set is

(1) $4 \le x^2 + y^2 \le 64$

(2) $x^2 + y^2 \le 25$

(3) $x^2 + y^2 \ge 25$

(4) $3 \le x^2 + y^2 \le 9$ **[2002]**

25. The centre of the circle passing through (0,0) and (1,0) and touching the circle $y^2 = 9$ is

(1) $\left(\frac{1}{2}, \frac{1}{2}\right)$

(2) $\left(\frac{1}{2}, -\sqrt{2}\right)$

(3) $\left(\frac{3}{2}, \frac{1}{2}\right)$

(4) $\left(\frac{1}{2}, \frac{3}{2}\right)$ **[2002]**

26. The equation of a circle with origin as a centre and passing through equilateral triangle whose median is of length 3a is

(1) $x^2 + y^2 = 9a^2$

(2) $x^2 + y^2 = 16a^2$

(3) $x^2 + y^2 = 4a^2$

(4) $x^2 + y^2 = a^2$ **[2002]**

Parabola

MCQ-Single Correct

1. Let P be the point on the parabola, $y^2 = 8x$ which is at a minimum distance from the centre C of the circle, $x^2 + (y+6)^2 = 1$. Then the equation of the circle, passing through C and having its centre at P is :

(1) $x^2 + y^2 - x + 4y - 12 = 0$ (2) $n = \frac{C_P - C}{C - C_V}$

(3) $x^2 + y^2 - 4x + 9y + 18 = 0$ (4) $x^2 + y^2 - 4x + 8y + 12 = 0$ **[2016]**

2. The centres of those circles which touch the circle, $x^2 + y^2 - 8x - 8y - 4 = 0$, externally and also touch the x-axis, lie on :

(1) an ellipse which is not a circle. (2) a hyperbola.

(3) a parabola. (4) a circle **[2016]**

3. Let O be the vertex and Q be any point on the parabola, $x^2 = 8y$. If the point P divides the line segment OQ internally in the ratio $1:3$, then the locus of P is :

(1) $y^2 = x$ (2) $y^2 = 2x$

(3) $x^2 = 2y$ (4) $x^2 = y$ **[2015]**

4. The slope of the line touching both the parabolas $y^2 = 4x$ and $x^2 = -32y$ is

(1) $\frac{1}{2}$ (2) $\frac{3}{2}$

(3) $\frac{1}{8}$ (4) $\frac{2}{3}$ **[2014]**

5. If two tangents drawn from a point P to the parabola $y^2 = 4x$ are at right angles, then the locus of P is

(1) $2x + 1 = 0$ (2) $x = -1$

(3) $2x - 1 = 0$ (4) $x = 1$ **[2010]**

6. The area of the region bounded by the parabola $(y-2)^2 = x-1$, the tangent to the parabola at the point (2,3) and the x-axis is

(1) 3 (2) 6

(3) 9 (4) 12 **[2009]**

7. A parabola has the origin as its focus and the line x = 2 as the directrix. Then the vertex of the parabola is at

(1) (0,2) (2) (1,0)

(3) (0,1) (4) (2,0) **[2008]**

8. The locus of the vertices of the family of parabolas $y = \frac{a^3x^2}{3} + \frac{a^2x}{2} - 2a$ is

(1) $xy = \frac{105}{64}$ (2) $xy = \frac{3}{4}$

(3) $xy = \frac{35}{16}$ (4) $xy = \frac{64}{105}$ **[2006]**

9. Let P be the point (1,0) and Q a point on the locus $y^2 = 8x$. The locus of mid point of PQ is

(1) $y^2 - 4x + 2 = 0$ (2) $y^2 + 4x + 2 = 0$

(3) $x^2 + 4y + 2 = 0$ (4) $x^2 - 4y + 2 = 0$ **[2005]**

10. If $a \neq 0$ and the line $2bx + 3cy + 4d = 0$ passes through the points of intersection of the parabolas $y^2 = 4ax$ and $x^2 = 4ay$, then

(1) $d^2 + (2b+3c)^2 = 0$ (2) $d^2 + (3b+2c)^2 = 0$

(3) $d^2 + (2b-3c)^2 = 0$ (4) $d^2 + (3b-2c)^2 = 0$ **[2004]**

11. The normal at the point $(bt_1^2, 2bt_1)$ on a parabola meets the parabola again in the point $(bt_2^2, 2bt_2)$, then

(1) $t_2 = -t_1 - \frac{2}{t_1}$ (2) $t_2 = -t_1 + \frac{2}{t_1}$

(3) $t_2 = t_1 - \frac{2}{t_1}$

(4) $t_2 = t_1 + \frac{2}{t_1}$ **[2003]**

12. Two common tangents to the circle $x^2 + y^2 = 2a^2$ and parabola $y^2 = 8ax$ are

(1) $x = \pm(y + 2a)$

(2) $y = \pm(x + 2a)$

(3) $x = \pm(y + a)$

(4) $y = \pm(x + a)$ **[2002]**

Assertion-Reason Type

(1) Statement-I is True; Statement-II is true; Statement-II is **not** a correct explaination of Statement-I.
(2) Statement-I is True; Statement-II is False.
(3) Statement-I is False; Statement-II is true
(4) Statement-I is True; Statement-II is true; Statement-II is a **correct** explaination of Statement-I.

1. Given : A circle, $2x^2+2y^2=5$ and a parabola, $y^2=4\sqrt{5}x$.

Statement-I : An equation of a common tangent to these curves is $y=x+\sqrt{5}$

Statement-II : If the line, $y=mx+\dfrac{\sqrt{5}}{m}$ $(m\neq 0)$ is their common tangent, then m satisfies $m^4-3m^2+2=0$

2. **Statement-I** : An equation of a common tangent to the parabola $y^2=16\sqrt{3}x$ and the ellipse $2x^2+y^2=4$ is $y=2x+2\sqrt{3}$.

Statement-II : If the line $y=mx+\dfrac{4\sqrt{3}}{m}$, $(m\neq 0)$ is a common tangent to the parabola $y^2=16\sqrt{3}x$ and ellipse $2x^2+y^2=4$, then m satisfies $m^4+2m^2=24$.

3. Let the tangent to the parabola be $y=mx+\dfrac{\sqrt{5}}{m}$, $(m\neq 0)$.

Now, its distance from the centre of the circle must be equal to the radius of the circle.

So, $\left|\dfrac{\sqrt{5}}{m}\right|=\dfrac{\sqrt{5}}{\sqrt{2}}\sqrt{1+m^2}\Rightarrow(1+m^2)m^2=2\Rightarrow m^4+m^2-2=0$

$\Rightarrow(m^2-1)(m^2+2)=0\Rightarrow m=\pm 1$

So, the common tangents are $y=x+\sqrt{5}$ and $y=-x-\sqrt{5}$.

<u>Ellipse</u>

MCQ-Single Correct

1. The eccentricity of an ellipse whose centre is at the origin is $\frac{1}{2}$. If one of its directrices is x = -4, then the equation of the normal to it at $\left(1, \frac{3}{2}\right)$ is :

(1) $2y - x = 2$ (2) $4x - 2y = 1$

(3) $4x + 2y = 7$ (4) $x + 2y = 4$ **[2017]**

2. The area (in sq. units) of the quadrilateral formed by the tangents at the end points of the latera recta to the ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$, is :

(1) 18 (2) $\frac{27}{2}$

(3) 27 (4) $\frac{27}{4}$ **[2015]**

3. The locus of the foot of perpendicular drawn from the centre of the ellipse $x^2 + 3y^2 = 6$ on any tangent to it is

(1) $\left(x^2 - y^2\right)^2 = 6x^2 + 2y^2$ (2) $\left(x^2 - y^2\right)^2 = 6x^2 - 2y^2$

(3) $\left(x^2 + y^2\right)^2 = 6x^2 + 2y^2$ (4) $\left(x^2 + y^2\right)^2 = 6x^2 - 2y^2$ **[2014]**

4. The equation of the circle passing through the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$, and having centre at (0,3) is

(1) $x^2 + y^2 - 6y + 7 = 0$ (2) $x^2 + y^2 - 6y - 5 = 0$

(3) $x^2 + y^2 - 6y + 5 = 0$ (4) $x^2 + y^2 - 6y - 7 = 0$ **[2013]**

5. An ellipse is drawn by taking a diameter of the circle $(x-1)^2 + y^2 = 1$, as its semi-minor axis and a diameter of the circle $x^2 + (y-2)^2 = 4$ as its semi-major axis. If the centre of the ellipse is at the origin and its axes are the coordinate axes, then the equation of the ellipse is

(1) $4x^2+y^2=8$

(2) $x^2+4y^2=16$

(3) $4x^2+y^2=4$

(4) $x^2+4y^2=8$ **[2012]**

6. The ellipse $x^2+4y^2=4$ is inscribed in a rectangle aligned with the coordinate axes, which in turn is inscribed in another ellipse that passes through the point (4,0). Then the equation of the ellipse is

(1) $x^2+16y^2=16$

(2) $x^2+12y^2=16$

(3) $4x^2+48y^2=-48$

(4) $4x^2+64y^2=48$ **[2009]**

7. A focus of an ellipse is at the origin. The directrix is the line x = 4 and the eccentricity is ½. Then the length of the semi-major axis is

(1) $\frac{8}{3}$

(2) $\frac{2}{3}$

(3) $\cot\left(\operatorname{cosec}^{-1}\frac{5}{3}+\tan^{-1}\frac{2}{3}\right)$

(4) $\frac{6}{17}$ **[2008]**

8. In an ellipse, the distance between its foci is 6 and minor axis is 8. Then its eccentricity is

(1) $\frac{3}{5}$

(2) $\frac{1}{2}$

(3) $\frac{4}{5}$

(4) $\frac{1}{\sqrt{5}}$ **[2006]**

9. An ellipse has OB as semi minor axis, F and F’ its focii and the angle FBF’ is a right angle. Then the eccentricity of the ellipse is

(1) $\frac{1}{\sqrt{2}}$

(2) $\frac{1}{2}$

(3) $\frac{1}{4}$

(4) $\frac{1}{\sqrt{5}}$ **[2005]**

10. The eccentricity of an ellipse, with its centre at the origin, is $\frac{1}{2}$. If one of the directrices is x = 4, then the equation of the ellipse is

(1) $3x^2+4y^2=1$

(2) $3x^2+4y^2=12$

(3) $4x^2+3y^2=12$ (4) $4x^2+3y^2=1$ **[2004]**

11. The foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{b^2}=1$ and the hyperbola $\frac{x^2}{144}-\frac{y^2}{81}=\frac{1}{25}$ coincide. Then the value of b^2 is

(1) 1 (2) 5

(3) 7 (4) 9 **[2003]**

<u>Hyperbola</u>

MCQ-Single Correct

1. A hyperbola passes through the point $P\left(\sqrt{2},\sqrt{3}\right)$ and has foci at $\left(\pm 2,0\right)$. Then the tangent to this hyperbola at P also passes through the point :

(1) $\left(3\sqrt{2},2\sqrt{3}\right)$ (2) $\left(2\sqrt{2},3\sqrt{3}\right)$

(3) $\left(\sqrt{3},\sqrt{2}\right)$ (4) $\left(-\sqrt{2},-\sqrt{3}\right)$ **[2017]**

2. The eccentricity of the hyperbola whose length of the latus rectum is equal to 8 and the length of its conjugate axis is equal half of the distance between its foci, is :

(1) $\frac{4}{\sqrt{3}}$ (2) $\frac{2}{\sqrt{3}}$

(3) $\sqrt{3}$ (4) $\frac{4}{3}$ **[2016]**

3. The equation of the hyperbola whose foci are (-2,0) and (2,0) and eccentricity is 2 is given by

(1) $-x^2+3y^2=3$ (2) $-3x^2+y^2=3$

(3) $x^2-3y^2=3$ (4) $3x^2-y^2=3$ **[2011]**

4. For the hyperbola $\frac{x^2}{\cos^2\alpha}-\frac{y^2}{\sin^2\alpha}=1$, which of the following remains constant when α varies?

(1) eccentricity (2) directrix

(3) abscissae of vertices (4) abscissae of foci **[2007]**

5. The locus of point $P(\alpha,\beta)$ moving under the condition that the line $y=\alpha x+\beta$ is a tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is

(1) an ellipse (2) a circle

(3) a parabola (4) a hyperbola **[2005]**

Calculus

Sets and Relations

MCQ-Single Correct

1. Let A and B be two sets containing four and two elements respectively. Then the number of subsets of the set $A \times B$, each having at least three elements is:

(1) 256 (2) 275

(3) 510 (4) 219 **[2015]**

2. If $X = \{4^n - 3n - 1 : n \in N\}$ and $Y = \{9(n-1) : n \in N\}$, where N is the set of natural numbers, then $X \cup Y$ is equal to

(1) N (2) Y – X

(3) X (4) Y **[2014]**

3. Let A and B be two sets containing 2 elements and 4 elements respectively. The number of subsets of $A \times B$ having 3 or more elements is

(1) 220 (2) 219

(3) 211 (4) 256 **[2013]**

4. Let S be a non-empty subset of R. Consider the following statement:

P: There is a rational number $x \in S$ such that x > 0.

Which of the following statements is the negation of the statement P?

(1) There is no rational number $x \in S$ such that $x \le 0$
(2) Every rational number $x \in S$ satisfies $x \le 0$
(3) $x \in S$ and $x \le 0 \Rightarrow x$ is not rational
(4) There is a rational number $x \in S$ such that $x \le 0$ **[2010]**

5. Consider the following relations:

R = {(x,y)|x,y are real numbers and x=wy for some rational number w};

S = $\left\{ \left(\frac{m}{n}, \frac{p}{q} \right) | \, m, n, p \, and \, q \, are \, \text{int} \, egers \, such \, n, q \ne 0 \, and \, qm = pn \right\}$. Then

(1) neither R nor S is an equivalence relation
(2) S is an equivalence relation but R is not an equivalence relation

(3) R and S both are equivalence relations
(4) R is an equivalence relation but S is not an equivalence relation

6. If A, B nd C are three sets such that $A \cap B = A \cap C$ and $A \cup B = A \cup C$, then

(1) A = B (2) A = C

(3) B = C (4) $A \cap B = \phi$ **[2009]**

7. Let R be the real line. Consider the following subsets of the plane $R \times R$.

S = {(x,y): y = x + 1 and 0 < x <2 }, T = {(x,y):x – y is an integer}. Which one of the following is true?

(1) neither S nor T is an equivalence relation on R
(2) Both S and T are equivalence relations on R
(3) S is an equivalence relation on R but T is not
(4) T is an equivalence relation on R but S is not **[2008]**

8. Let W denote the words in the English dictionary. Define the relation R by : **[2006]**

R = {(x,y)ε$W \times W$ | the words x and y have at least one letter in common}. Then R is

(1) Not reflexive, symmetric and transitive (2) reflexive, symmetric and not transitive

(3) reflexive, symmetric and transitive (4) reflexive, not symmetric and transitive

9. Let R = {(3,3), (6,6), (9,9),(12,12),(6,12),(3,9),(3,12),(3,6)} be a relation on the set A = {3,6,9,12} be a relation on the set A = {3,6,9,12}. The relation is

(1) reflexive and transitive only (2) reflexive only

(3) an equivalence relation (4) reflexive and symmetric only **[2005]**

10. Let R = {(1,3),(4,2),(2,4),(2,3),(3,1)} be a relation on the set A = {1,2,3,4}. The relation R is

(1) a function (2) reflexive

(3) not symmetric (4) transitive **[2004]**

__**Functions**__

MCQ-Single Correct

1. The function f : $R \to \left[-\frac{1}{2},\frac{1}{2}\right]$ defined as $f(x)=\frac{x}{1+x^2}$, is :

(1) invertible. (2) injective but not surjective.

(3) surjective but not injective. (4) neither injective nor surjective. **[2017]**

2. Let a , b , c ε R. If f(x) = ax^2+bx+c is such that a + b + c = 3 and f(x+y) = f(x) +f(y) +xy $\forall$ x , y ε R, then $\sum_{n=1}^{10} f(n)$ is equal to :

(1) 330 (2) 165

(3) 190 (4) 255 **[2017]**

3. If $f(x)+2f\left(\frac{1}{x}\right)=3x, x\neq 0$ and $S=\{x\in R: f(x)=f(-x)\}$; then S:

(1) contains exactly one element. (2) contains exactly two elements.

(3) contains more than two elements (4) is an empty set **[2016]**

4. If $a\in R$ and the equation $-3(x-[x])^2+2(x-[x])+a^2=0$ (where [x] denotes the greatest integer ≤ x) has no integral solution, then all possible values of a lie in the interval

(1) $(-1,0)\cup(0,1)$ (2) (1,2)

(3) $(-2,-1)$

(3) $\frac{1}{1+\{g(x)\}^5}$ (4) $1+\{g(x)\}^5$ **[2014]**

6. For real x, let $f(x)=x^3+5x+1$, then

(1) f is one-one but not onto R (2) f is onto R but not one-one

(3) f is one-one and onto R (4) f is neither one-one nor onto R **[2009]**

7. Let f : $N\to Y$ be a function defined as f(x) = 4x +3, where Y = {y ε N : y = 4x +3 for some xεN }. Show that f is invertible and its inverse is

(1) g(y) = $\dfrac{3y+4}{4}$ (2) g(y) = $4+\dfrac{y+3}{4}$

(3) g(y) = $\dfrac{y+3}{4}$ (4) g(y) = $\log\left|\sin\left(x-\dfrac{\pi}{4}\right)\right|+c$ **[2008]**

8. The largest interval lying in $\left(-\dfrac{\pi}{2},\dfrac{\pi}{2}\right)$ for which the function

$[f(x)=4^{-x^2}+\cos^{-1}\left(\dfrac{x}{2}-1\right)+\log(\cos x)]$ is defined , is

(1) $[0,\pi]$ (2) $\left(-\dfrac{\pi}{2},\dfrac{\pi}{2}\right)$

(3) $\left[-\dfrac{\pi}{4},\dfrac{\pi}{2}\right)$ (4) $\left[0,\dfrac{\pi}{2}\right)$ **[2007]**

9. Let f : $(-1,1)\rightarrow B$, be a function defined by $f(x)=\tan^{-1}\left(\dfrac{2x}{1-x^2}\right)$, then f is both one-one and onto when B is the interval

(1) $\left(0,\dfrac{\pi}{2}\right)$ (2) $\left[0,\dfrac{\pi}{2}\right)$

(3) $\left[-\dfrac{\pi}{2},\dfrac{\pi}{2}\right]$ (4) $\left(-\dfrac{\pi}{2},\dfrac{\pi}{2}\right)$ **[2005]**

10. A real valued function f(x) satisfies the functional equation f(x-y) = f(x) f(y) – f(a – x) f(a + y) where a is a given constant and f(0) = 1, f(2a – x) is equal to

(1) -f(x) (2) f(x)

(3) f(a) + f(a-x) (4) f(-x) **[2006]**

11. The range of the function f(x) = ${}^{7-x}P_{x-3}$ is

(1) { 1, 2, 3} (2) {1,2,3,4,5}

(3) {1,2,3,4} (4) {1,2,3,4,5,6} **[2004]**

12. If f : $R\rightarrow S$, defined by $f(x)=\sin x-\sqrt{3}\cos x+1$, is onto, then the interval of S is

(1) [0,3] (2) [-1,1]

(3) [0,1] (4) [-1,-3] **[2004]**

13. The graph of the function $y = f(x)$ is symmetrical about the line x = 2 , then

(1) f(x + 2) = f(x -2) (2) f(2+x) = f(2-x)

(3) f(x) = f(-x) (4) f(x) = -f(-x) **[2004]**

14. The domain of the function $f(x) = \frac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$ is

(1) [2,3] (2) [2,3)

(3) [1,2] (4) [1,2) **[2004]**

15. A function f from the set of natural numbers to integers defined by

f(n) = $\begin{cases} \frac{n-1}{2}, & when\, n\, is\, odd \\ -\frac{n}{2}, & when\, n\, is\, even \end{cases}$ is

(1) one-one but not onto (2) onto but not one-one

(3) one-one and onto both (4) neither one-one nor onto **[2003]**

16. If $f : R \to R$ satisfies f(x+y) = f(x) + f(y), for all x,y ε R and f(1) = 7, then $\sum_{r=1}^{n} f(r)$ is

(1) $\frac{7n}{2}$ (2) $\frac{7(n+1)}{2}$

(3) 7n(n+1) (4) $\frac{7n(n+1)}{2}$ **[2003]**

17. Domain of definition of the function f(x) = $\frac{3}{4-x^2} + \log_{10}\left(x^3 - x\right)$, is

(1) (1,2) (2) $(-1,0) \cup (1,2)$

(3) $(1,2) \cup (2,\infty)$ (4) $(-1,0) \cup (1,2) \cup (2,\infty)$ **[2003]**

18. The function f(x) = $\log\left(x + \sqrt{x^2+1}\right)$, is

(1) an even function

(2) an odd function

(3) a periodic function

(4) neither an even nor an odd function **[2003]**

19. The period of $\sin^2\theta$ is

(1) π^2

(2) π

(3) π^3

(4) $\pi/2$ **[2002]**

20. Which one is not periodic

(1) $|\sin 3x| + \sin^2 x$

(2) $\cos\sqrt{x} + \cos^2 x$

(3) cos4x + $\tan^2 x$

(4) $\cos 2x + \sin x$ **[2002]**

21. If f(x+y) = f(x).f(y) $\forall$ x,y and f(5) = 2, f'(0) = 3, then f'(5) is

(1) 0

(2) 1

(3) 6

(4) 2 **[2002]**

22. The domain of $\sin^{-1}\left[\log_3\left(\frac{x}{3}\right)\right]$ is

(1) [1,9]

(2) [-1,9]

(3) [-9,1]

(4) [-9,-1] **[2002]**

Assertion – Reason Type

1. Let f be a function defined by $f\left(x\right)=\left(x-1\right)^2+1,\left(x\geq 1\right)$

   Statement – I : The set { x : f(x) = $f^{-1}(x)$ } = {1,2}.

   Statement – II : f is bijection and $f^{-1}\left(x\right)= 1+\sqrt{x-1}$, $x\geq 1.$

2. Let f(x) = $\left(x+1\right)^2-1,\ x\geq -1$

   Statement – I : The set { x : f(x) = $f^{-1}(x)$ } = {0,-1}

   Statement – II : f is a bijection.

Limits Continuity and Differentiability

MCQ-Single Correct

1. If for $x \in \left(0, \frac{1}{4}\right)$, the derivative of $\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$ is $\sqrt{x}$. g(x), then g(x) equals :

(1) $\frac{9}{1+9x^3}$ (2) $\frac{3x\sqrt{x}}{1-9x^3}$

(3) $\frac{3x}{1-9x^3}$ (4) $\frac{3}{1+9x^3}$ **[2017]**

2. $\lim\limits_{x \to \frac{\pi}{2}} \frac{\cot x - \cos x}{(\pi - 2x)^3}$ equals :

(1) $\frac{1}{24}$ (2) $\frac{1}{16}$

(3) $C_6H_5CH = CHC_6H_5$ (4) $\frac{1}{4}$ **[2017]**

3. Let $p = \lim\limits_{x \to 0+} \left(1 + \tan^2 \sqrt{x}\right)^{\frac{1}{2x}}$ then log p is equal to :

(1) 1 (2) ½

(3) ¼ (4) 2 **[2016]**

4. For $x \in R$, $f(x) = |\log 2 - \sin x|$ and g(x) = f(f(x)), then :

(1) g’(0) = cos(log2)

(2) g’(0) = -cos(log2)

(3) g is differentiable at x = 0 and g’(0) = - sin(log2)

(4) g is not differentiable at x = 0 **[2016]**

5. $\lim\limits_{x \to 0} \frac{(1-\cos 2x)(3+\cos x)}{x \tan 4x}$ is equal to :

(1) 3 (2) 2

(3) ½ (4) 4 **[2015]**

6. If the function, g(x) = $\begin{cases} k\sqrt{x+1} & ,0 \le x \le 3 \\ mx+2 & ,3 < x \le 5 \end{cases}$ is differentiable, then the value of k + m is :

(1) 16/5 (2) 10/3

(3) 4 (4) 2 **[2015]**

7. $\lim_{x \to 0} \frac{\sin\left(\pi \cos^2 x\right)}{x^2}$ is equal to

(1) $\pi / 2$ (2) 1

(3) - π (4) π **[2014]**

8. If $y = \sec\left(\tan^{-1} x\right)$, then $\frac{dy}{dx}$ at x = 1 is equal to

(1) ½ (2) 1

(3) $\sqrt{2}$ (4) $\frac{1}{\sqrt{2}}$ **[2013]**

9. $\lim_{x \to 0} \frac{(1-\cos 2x)(3+\cos x)}{x \tan 4x}$ is equal to :

(1) 1/2 (2) 1

(3) 2 (4) -1/4 **[2013]**

10. Let f: $R \to [0, \infty)$ be such that $\lim_{x \to 5} f(x)$ exists and $\lim_{x \to 5} \frac{\left(f(x)\right)^2 - 9}{\sqrt{|x-5|}} = 0$, then $\lim_{x \to 5} f(x)$ equals

(1) 2 (2) 3

(3) 0 (4) 1 **[2011]**

11. If function f(x) is differentiable at x = a, then $\lim_{x \to a} \frac{x^2 f(a) - a^2 f(x)}{x-a}$ is

(1) $2af(a) - a^2 f'(a)$ (2) $2af(a) + a^2 f(a)$

(3) $-a^2 f'(a)$ (4) $af(a) - a^2 f'(a)$ **[2011]**

12. Let f : $R \to R$ be a positive increasing function with $\lim_{x\to\infty}\frac{f(3x)}{f(x)}=1$. Then $\lim_{x\to\infty}\frac{f(2x)}{f(x)}=$

(1) 2/3 (2) 3/2

(3) 3 (4) 1 **[2010]**

13. Let f : $(-1,1)\to R$ be a differentiable function with f(0) = -1 and f'(0) =1. Let $\mathrm{g}(\mathrm{x})=[\mathrm{f}(2\mathrm{f}(\mathrm{x})+2]^2$. Then g'(0) =

(1) -4 (2) 0

(3) -2 (4) 4 **[2010]**

14. Let y be an implicit function of x defined by $x^{2x}-2x^x\cot y-1=0$. Then y'(1) equals

(1) -1 (2) 1

(3) log 2 (4) - log 2 **[2009]**

15. Let f(x) = $\begin{cases}(x-1)\sin\left(\frac{1}{x-1}\right), & if\ x\neq 1\\ 0, & if\ x=1\end{cases}$. Then which one of the following is true?

(1) f is neither differentiable at x = 0 nor at x = 1

(2) f is differentiable at x = 0 and at x = 1

(3) f is differentiable at x = 0 but not at x = 1

(4) f is differentiable at x = 1 but not at x = 0 **[2008]**

16. The set of points where $f(x)=\frac{x}{1+|x|}$ is differentiable is

(1) $(-\infty,0)\cup(0,\infty)$

(2) $(-\infty,-1)\cup(-1,\infty)$

(3) $(-\infty,\infty)$

(4) $(0,\infty)$ **[2006]**

17. If $x^m.y^n = (x+y)^{m+n}$, then $\frac{dy}{dx}$ is

(1) $\frac{y}{x}$ (2) $\frac{x+y}{xy}$

(3) xy (4) $\frac{x}{y}$ **[2006]**

18. Let α and β be the distant roots of $ax^2+bx+c=0$, then $\lim_{x\to\alpha}\frac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2}$ is equal to

(1) $\frac{a^2}{2}(\alpha-\beta)^2$ (2) 0

(3) $-\frac{a^2}{2}(\alpha-\beta)^2$ (4) $\frac{1}{2}(\alpha-\beta)^2$ **[2005]**

19. Suppose f(x) is differentiable x = 1 and $\lim_{h\to 0}\frac{1}{h}f(1+h)=5,$ then f'(1) equals

(1) 3 (2) 4

(3) 5 (4) 6 **[2005]**

20. Let f be differentiable for all x. If f(1) = -2 and $f'(x)\geq 2$ for $x\in[1,6]$, then

(1) $f(6)\geq 8$ (2) $f(6)<8$

(3) f(6) < 5 (4) f(6) = 5 **[2005]**

21. If f is a real-valued differentiable function satisfying $|f(x)-f(y)|\leq(x-y)^2$, x , y ε R and f(0) = 0 , then f(1) equals

(1) -1 (2) 0

(3) 2 (4) 1 **[2005]**

22. If $\lim_{x\to\infty}\left(1+\frac{a}{x}+\frac{b}{x^2}\right)^{2x}=e^2$, then the values of a and b, are

(1) $a\in R, b\in R$ (2) a = 1, $b\in R$

(3) $a \in R, b = 2$ (4) a = 1 and b = 2 **[2004]**

23. Let $f(x) = \dfrac{1-\tan x}{4x-\pi}$, $x \neq \dfrac{\pi}{4}$, $x \in \left[0, \dfrac{\pi}{2}\right]$. If f(x) is continuous in $\left[0, \dfrac{\pi}{2}\right]$, then $f\left(\dfrac{\pi}{4}\right)$ is

(1) 1 (2) ½

(3) -1/2 (4) -1 **[2004]**

24. If $x = e^{y+e^{y+....to\infty}}$, x > 0 , then $\dfrac{dy}{dx}$ is

(1) $\dfrac{x}{1+x}$ (2) $\dfrac{1}{x}$

(3) $\dfrac{1-x}{x}$ (4) $\dfrac{1+x}{x}$ **[2004]**

25. $\lim\limits_{x\to\pi/2} \dfrac{\left[1-\tan\left(\dfrac{x}{2}\right)\right]\left[1-\sin x\right]}{\left[1+\tan\left(\dfrac{x}{2}\right)\right]\left[\pi-2\pi\right]^3}$ is

(1) 1/8 (2) 0

(3) 1/32 (4) ∞ **[2003]**

26. If $\lim\limits_{x\to0} \dfrac{\log(3+x)-\log(3-x)}{x} = k$, the value of k is

(1) 0 (2) -1/3

(3) 2/3 (4) -2/3 **[2003]**

27. Let f (a) = g(a) = k and their n^{th} derivatives $f^{n}(a),\ g^{n}(a)$ exist and are not equal for some n.

Further if $\lim\limits_{x\to a} \dfrac{f(a)g(x)-f(a)-g(a)f(x)+g(a)}{g(x)-f(x)} = 4,$ then the value of k is

(1) 4 (2) 2

(3) 1 (4) 0 **[2003]**

28. If f(x) = $\begin{cases} xe^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)} & x \neq 0 \\ 0 & x = 0 \end{cases}$

then f(x) is

(1) Continuous as well as differentiable for all x

(2) Continuous for all x but not differentiable at x = 0

(3) Neither differentiable nor continuous at x = 0

(4) Discontinuous everywhere **[2003]**

29. $\lim_{n\to\infty}\frac{1+2^4+3^4+.....+n^4}{n^5}-\lim_{n\to\infty}\frac{1+2^3+3^3+......+n^3}{n^5}$

is

(1) $\frac{1}{30}$ (2) zero

(3) $\frac{1}{4}$ (4) $\frac{1}{5}$ **[2003]**

30. $\lim_{x\to 0}\frac{\log x^n-[x]}{[x]}$, $n \in N$, ([x] denotes greatest integer less than or equal to x)

(1) has value -1 (2) has value 0

(3) has value 1 (4) does not exist **[2002]**

31. $\lim_{n\to\infty}\frac{1^p+2^p+3^p+.....+n^p}{n^{p+1}}$ is

(1) $\frac{1}{p+1}$ (2) $\frac{1}{p-1}$

(3) $\frac{1}{p}-\frac{1}{p-1}$ (4) $\frac{1}{p+2}$ **[2002]**

32. f is defined in [-5,5] as f(x) = x, if x is rational and = -x, if x is irrational. Then

(1) f(x) is continuous at every x, except x = 0

(2) f(x) is discontinuous at every x, except x = 0

(3) f(x) is continuous everywhere

(4) f(x) is discontinuous everywhere

33. If $y=\left(x+\sqrt{1+x^2}\right)^x$, then $\left(1+x^2\right)\frac{d^2y}{dx^2}+x\frac{dy}{dx}$ is

(1) n^2y (2) $-n^2y$

(3) - y (4) $2n^2y$ **[2002]**

34. If f(1) = 1, f'(1) = 2, then $\lim\limits_{x\to1}\frac{\sqrt{f(x)}-1}{\sqrt{x}-1}$ is

(1) 2 (2) 4

(3) 1 (4) ½ **[2002]**

35. $\lim\limits_{x\to0}\frac{\sqrt{1-\cos 2x}}{\sqrt{2}x}$ is

(1) 1 (2) -1

(3) 0 (4) does not exist **[2002]**

36. $\lim\limits_{x\to\infty}\left(\frac{x^2+5x+3}{x^2+x+3}\right)^{1/x}$

(1) e^4 (2) e^2

(3) e^3 (4) 1 **[2002]**

37. Let f(x) = 4 and f'(x) = 4 , then $\lim\limits_{x\to2}\frac{xf(2)-2f(x)}{x-2}$ equals

(1) 2 (2) -2

(3) -4 (4) 3 **[2002]**

Assertion – Reason Type

1. Define F (x) as the product of two real functions $f_1(x) = x$, $x \in R$ and **[2011]**

$$f_2(x) = \begin{cases} \sin 1/x, & if\ x \neq 0 \\ 0, & if\ x = 0 \end{cases}$$ as follows

F (x) = $\begin{cases} f_1(x).f_2(x), & if\ x \neq 0 \\ 0, & if\ x = 0 \end{cases}$

Statement-I : F(x) is continuous on R

Statement-II : $f_1(x)$ and $f_2(x)$ are continuous on R

2. Let $f : R \to R$ be a continuous function defined by $f(x) = \dfrac{1}{e^x + 2e^{-x}}$, **[2010]**

Statement-I : $f(c) = \dfrac{1}{3}$, for some $c \in R$

Statement-II : $0 < f(x) \leq \dfrac{1}{2\sqrt{2}}$, for all $x \in R$

3. Let f(x) = x|x| and g(x) = sin x **[2009]**

Statement-I : gof is differentiable at x = 0 and its derivative is continuous at that point.

Statement-II : gof is twice differentiable at x = 0.

Applications of Derivatives

MCQ-Single Correct

1. Twenty meters of wire is available for fencing off a flower-bed in the form of a circular sector. Then the maximum area (in sq. m) of the flower bed is

(1) 12.5 (2) 10

(3) 25 (4) 30 **[2017]**

2. The normal to the curve $y(x-2)(x-3)=x+6$ at the point where the curve intersects the y-axis passes through the point :

(1) $\left(-\frac{1}{2},-\frac{1}{2}\right)$ (2) $\left(\frac{1}{2},\frac{1}{2}\right)$

(3) $\left(\frac{1}{2},-\frac{1}{3}\right)$ (4) $\left(\frac{1}{2},\frac{1}{3}\right)$ **[2017]**

3. A wire of length 2 units is cut into two parts which are bent respectively to form to form a square of side = x units and a circle of radius = r units. If the sum of the areas of the square and the circle so formed is minimum, then :

(1) $(4-\pi)x=\pi r$ (2) x = 2r

(3) 2x = r (4) 2x = $(\pi+4)r$ **[2016]**

4. Consider f(x) = $\tan^{-1}\left(\sqrt{\frac{1+\sin x}{1-\sin x}}\right)$, $x\in\left(0,\frac{\pi}{2}\right)$. A normal to y = f(x) at $x=\frac{\pi}{6}$ also passes through the point :

(1) $\left(0,\frac{2\pi}{3}\right)$ (2) $\left(\frac{\pi}{6},0\right)$

(3) $\left(\frac{\pi}{4},0\right)$ (4) (0,0) **[2016]**

5. Let f(x) be a polynomial of degree four having extreme values at x = 1 and x = 2 . If $\lim_{x\to 0}\left[1+\frac{f(x)}{x^2}\right]=3$, then f(2) is equal to :

(1) -4 (2) 0

(3) 4 (4) -8 **[2015]**

6. The normal to the curve, $x^2+2xy-3y^2=0$, at (1,1) :

(1) meets the curve again in the second quadrant.

(2) meets the curve again in the third quadrant.

(3) meets the curve again in the fourth quadrant.

(4) does not meet the curve again. **[2015]**

7. If x = -1 and x = 2 are extreme points of f(x) = $\alpha\log|x|$ + βx^2+x, then

(1) $\alpha = -6, \beta = \frac{1}{2}$
(2) $\alpha = -6, \beta = -\frac{1}{2}$
(3) $\alpha = 2, \beta = -\frac{1}{2}$
(4) $\alpha = 2, \beta = \frac{1}{2}$ **[2014]**

8. If f and g are differentiable functions in [0,1] satisfying f(0) = 2 = g(1), g(0) = 0 and f(1) = 6, then for some $c \in \left]0,1\right[$

(1) 2f'(c) = g'(c)
(2) 2 f'(c) = 3 g'(c)
(3) f'(c) = g'(c)
(4) f'(c) = 2 g'(c) **[2014]**

9. The real number k for which the equation, $2x^3 + 3x + k = 0$ has two distinct real roots in [0,1]

(1) lies between 2 and 3
(2) lies between -1 and 0
(3) does not exist
(4) lies between 1 and 2 **[2013]**

10. The intercepts on x-axis made by tangents to the curve, $y = \int_0^x |t| dt$, $x \in R$, which are parallel to the line y = 2x , are equal to

(1) ± 2
(2) ± 3
(3) ± 4
(4) ± 1 **[2013]**

11. The curve that passes through the point (2,3) , and has the property that the segment of any tangent to it lying between the coordinate axes is bisected by the point of contact, is given by

(1) $x^2 + y^2 = 13$
(2) $\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = 2$
(3) 2y -3x = 0
(4) $y = \frac{6}{x}$ **[2011]**

12. Let f : $R \to R$ be defined by f(x) = $\begin{cases} k - 2x, & if\ x \le -1 \\ 2x + 3, & if\ x > -1 \end{cases}$. If f has a local minimum at x = -1 , then a possible value of k is

(1) 0
(2) -1/2
(3) -1
(4) 1 **[2010]**

13. The equation of the tangent to the curve $y = x + \frac{4}{x^2}$, that is parallel to the x-axis, is

(1) y = 1
(2) y = 2
(3) y = 3
(4) y = 0 **[2010]**

14. Given P(x) = $x^4 + ax^3 + bx^2 + cx + d$ such that x = 0 is the only real root of P'(x) = 0 . If P(-1) < P(1) , then in the interval [-1,1]

(1) P(-1) is the minimum and P(1) is the maximum of P
(2) P(-1) is not minimum but P(1) is the maximum of P
(3) P(-1) is the minimum and P(1) is not the maximum of P
(4) neither P(-1) is the minimum nor P(1) is the maximum of P **[2009]**

15. The shortest distance between the line $y - x = 1$ and the curve $x = y^2$ is

(1) $\frac{3\sqrt{2}}{8}$ (2) $\frac{2\sqrt{3}}{8}$

(3) $\frac{3\sqrt{2}}{5}$ (4) $\frac{\sqrt{3}}{4}$ **[2009]**

16. How many real solutions does the equation $x^7 + 14x^5 + 16x^3 + 30x - 560 = 0$ have?

(1) 7 (2) 1

(3) 3 (4) 5 **[2008]**

17. Suppose the cube $x^3 - px + q$ has three distinct real roots where p > 0 and q > 0. Then which one of the following holds?

(1) The cubic has minima at $\sqrt{\frac{\rho}{3}}$ and maxima at $-\sqrt{\frac{\rho}{3}}$

(2) The cubic has minima at $-\sqrt{\frac{\rho}{3}}$ and maxima at $\sqrt{\frac{\rho}{3}}$

(3) The cubic has minima at both $\sqrt{\frac{\rho}{3}}$ and $-\sqrt{\frac{\rho}{3}}$

(4) The cubic has maxima at both $\sqrt{\frac{\rho}{3}}$ and $-\sqrt{\frac{\rho}{3}}$ **[2008]**

18. A value of C for which the conclusion of Mean Value Theorm holds for the function f(x) = $\log_e x$ on the interval [1,3] is

(1) $2\log_3 e$ (2) $\frac{1}{2}\log_e 3$

(3) $\log_3 e$ (4) $\log_e 3$ **[2007]**

19. The function f(x) = $\tan^{-1}(\sin x + \cos x)$ is an increasing function in

(1) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ (2) $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$

(3) $\left(0, \frac{\pi}{2}\right)$ (4) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ **[2007]**

20. A body falling from rest under gravity passes a certain point P. It was at a distance of 400m from P, 4s prior to passing through P. If g = 10 m/s^2 , then the height above the point P from where the body began to fall is

(1) 720m (2) 900m

(3) 320m (4) 680m **[2006]**

21. The function $f(x) = \frac{x}{2} + \frac{2}{x}$ has a local minimum at

(1) x = 2 (2) x = -2

(3) x = 0 (4) x = 1 **[2006]**

22. Angle between the tangents to the curve $y = x^2 - 5x + 6$ at the points (2,0) and (3,0) is

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{2}$

(3) $\frac{\pi}{6}$ (4) $\frac{\pi}{4}$ **[2006]**

23. Area of the greatest rectangle that can be inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

(1) 2ab (2) ab

(3) $\sqrt{ab}$ (4) $\frac{a}{b}$ **[2005]**

24. The normal to the curve $x=a(\cos\theta+\theta\sin\theta)$, y = $a(\sin\theta-\theta\cos\theta)$ at any point 'θ' is such that

(1) If passes through the origin

(2) If makes angle $\frac{\pi}{2}+\theta$ with the x – axis

(3) It passes through $\left(a\frac{\pi}{2},-a\right)$

(4) It is at a constant distance from the origin **[2005]**

25. A function is matched below against an interval where it is supposed to be increasing. Which of the following pairs is incorrectly matched?

	Interval	**Function**
(1)	$(-\infty,\infty)$	x^3-3x^2+3x+3
(2)	$[2,\infty)$	$2x^3-3x^2-12x+6$
(3)	$\left(-\infty,\frac{1}{3}\right]$	$3x^2-2x+1$
(4)	$[-\infty,-4]$	x^3+6x^2+6

[2005]

26. A spherical iron ball 10 cm in radius is coated with a layer of ice of uniform thickness, then mels at a rate of 50 cm^3/min. When the thickness of ice is 5 cm, then the rate at which the thickness of ice decreases , is

(1) $\frac{1}{36\pi}cm/\min$ (2) $\frac{1}{18\pi}cm/\min$

(3) $\frac{1}{54\pi}cm/\min$ (4) $\frac{5}{6\pi}cm/\min$ **[2005]**

27. A lizard, at an initial distance of 21 cm behind an insect, moves from rest with an acceleration of 2 cm/s^2 and pursues the insect which is crawling uniformly along a straight line at a speed of 20 cm/s. Then the lizard will catch the insect after

(1) 20 s (2) 1 s

(3) 21 s (4) 24 s **[2005]**

28. Two points A and B move from rest along a straight line with constant acceleration f and f' respectively. If A takes m sec. more than B and describes 'n' units more than B in acquiring the same speed then

(1) $(f-f')m^2 = ff'n$ (2) $(f + f')m^2 = ff'n$

(3) $\frac{1}{2}(f+f')m = ff'n^2$ (4) $(f' - f)n = \frac{1}{2} ff'm^2$ **[2005]**

29. A particle is projected from a point O with velocity u at an angle of 60^o with the horizontal. When it is moving in a direction at right angles to its direction at O, its velocity then is given by

(1) u/3 (2) u/2

(3) 2u/3 (4) $u/\sqrt{3}$ **[2005]**

30. If the equation

$a_n x^n + a_{n-1}x^{n-1} + \ldots\ldots + a_1 x = 0$, $a1 \neq 0$, $n \geq 2$, has a positive root $x = \alpha$, then the equation $na_n x^{n-1} + (n-1)a_{n-1}x^{n-2} + \ldots\ldots + a_1 = 0$ has a positive root, which is

(1) Greater than α (2) smaller than α

(3) greater than or equal to α (4) equal to α **[2005]**

31. A point on the parabola $y^2 = 18x$ at which the ordinate increases at twice the rate of the abscissa is

(1) (2,4) (2) (2,-4)

(3) $\left(\frac{-9}{8}, \frac{9}{2}\right)$ (4) $\left(\frac{9}{8}, \frac{9}{2}\right)$ **[2004]**

32. A function y = f(x) has a second order derivative f''(x) = 6(x-1). If its graph passes through the point (2,1) and at that point the tangent to the graph is y = 3x – 5, then the function is

(1) $(x-1)^2$ (2) $(x-1)^3$

(3) $(x+1)^3$ (4) $(x+1)^2$ **[2004]**

33. The normal to the curve x = a(1 + cosθ), y = a sinθ at 'θ' always passes through the fixed point

(1) (a,0) (2) (0,a)

(3) (0,0) (4) (a,a) **[2004]**

34. If 2a + 3b + 6c = 0, then at least one root of the equation $ax^2 + bx + c = 0$ lies in the interval

(1) (0,1) (2) (1,2)

(3) (2,3) (4) (1,3) **[2004]**

35. If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$, where a > 0, attains its maximum and minimum at p and q respectively such that $p^2 = q$, then a equals

(1) 3 (2) 1

(3) 2 (4) ½ **[2003]**

36. Two particles start simultaneously from the same point and move along two straight lines, one with uniform velocity $\vec{u}$ and the other from rest with uniform acceleration $\vec{f}$. Let α be the angle between their directions of motion. The relative velocity of the second particle with respect to the first is least after a time

(1) $\dfrac{u\sin\alpha}{f}$ (2) $\dfrac{f\cos\alpha}{u}$

(3) u sin α (4) $\dfrac{u\cos\alpha}{f}$ **[2003]**

37. Two stones are projected from the top of a cliff h meters high, with the same speed u so as to hit the ground at the same spot. If one of the stones is projected horizontally and the other is projected at an angle θ to the horizontal then tan θ equals

(1) $\sqrt{\dfrac{2u}{gh}}$ (2) $2g\sqrt{\dfrac{u}{h}}$

(3) $2h\sqrt{\dfrac{u}{g}}$ (4) $u\sqrt{\dfrac{2}{gh}}$ **[2003]**

38. A body travels a distance s in t seconds. It starts from rest and ends at rest. In the first part of the journey, it moves with constant acceleration f and in the second part with constant retardation r. The value of t is given by

(1) $2s\left(\dfrac{1}{f}+\dfrac{1}{r}\right)$ (2) $\dfrac{2s}{\dfrac{1}{f}+\dfrac{1}{r}}$

(3) $\sqrt{2s(f+r)}$ (4) $\sqrt{2s\left(\dfrac{1}{f}+\dfrac{1}{r}\right)}$ **[2003]**

39. If 2a + 3b + 6c = 0 (a, b, c ε R), then the quadratic equation $ax^2+bx+c = 0$ has

(1) atleast one root in [0,1] (2) atleast one root in [2,3]

(3) atleast one root in [4,5] (4) none of these **[2002]**

40. f(x) and g(x) are two differentiable functions on [0,2] such that f''(x) – g''(x) = 0 , f'(1) = 2g'(1) = 4, f(2) = 3 g(2) = 9 , then f(x) – g(x) at x = 3/2 is

(1) 0 (2) 2

(3) 10 (4) 5 **[2002]**

41. The maximum distance from origin of a point on the curve x = a sin t – b sin $\left(\dfrac{at}{b}\right)$, y = cos t – b cos $\left(\dfrac{at}{b}\right)$, both a, b > 0 is

(1) a – b (2) a + b

(3) $\sqrt{a^2+b^2}$ (4) $\sqrt{a^2-b^2}$ **[2002]**

Assertion – Reason Type

1. Consider the function, f(x) = |x-2|+ |x-5|, $x \in R$

   Statement – I : f'(4) = 0

   Statement – **II** : f is continuous in [2,5], differentiable in (2,5) and f(2) = f(5)

2. Let a, b ε R be such that the function f given by f(x) = ln |x| + bx^2 +ax, $x \neq 0$ has extreme values at x = -1 and x = 2.

   Statement – I : f has local maximum at x = -1 and x = 2

   Statement – **II :** a = ½ and b = -1/4.

3. Let f be a function defined by f(x) = $\begin{cases} \dfrac{\tan x}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$

   Statement-I : x = 0 is point of minima of f.

   Statement – II : f'(0) = 0.

<u>Definite Integrals</u>

MCQ-Single Correct

1. The integral $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1+\cos x}$ is equal to

(1) -2 (2) 2

(3) 4 (4) -1 **[2017]**

2. $\lim_{n\to\infty}\left(\frac{(n+1)(n+2)......3n}{n^{2n}}\right)^{1/n}$ is equal to :

(1) $\frac{27}{e^2}$ (2) $\frac{9}{e^2}$

(3) 3log3 -2 (4) $\frac{18}{e^4}$ **[2016]**

3. The integral $\int_2^4 \frac{\log x^2}{\log x^2+\log\left(36-12x+x^2\right)}dx$ is equal to :

(1) 4 (2) 1

(3) 6 (4) 2 **[2015]**

4. The integral $\int_0^{\pi}\sqrt{1+4\sin^2\frac{x}{2}-4\sin\frac{x}{2}}dx$ equals

(1) $\pi-4$ (2) $\frac{2\pi}{3}-4-4\sqrt{3}$

(3) $4\sqrt{3}-4$ (4) $4\sqrt{3}-4-\frac{\pi}{3}$ **[2014]**

5. If g(x) = $\int_0^x \cos 4t dt$ then g(x + π) equals

(1) g(x) (2) g(x) . g(π)

(3) $\frac{g(x)}{g(\pi)}$ (4) g(x) + g(π) **[2012]**

6. Let [.] denotes the greatest integer function, then the value of $\int_0^{1.5} x\left[x^2\right]dx$ is

(1) ¾ (2) 5/4

(3) 0 (4) 3/2 **[2011]**

7. Let p(x) be a function defined on R such that p'(x) = p'(1-x), for all x ε [0,1] , p(0) = 1 and p(1) = 41 . Then $\int_0^1 p(x)dx$ equals

(1) 21 (2) 41

(3) 42 (4) $\sqrt{41}$ **[2010]**

8. $\int_0^\pi [\cot x]dx$, [*] denotes the greatest integer function, is equal to

(1) $\frac{\pi}{2}$ (2) 1

(3) -1 (4) $-\frac{\pi}{2}$ **[2009]**

9. Let I = $\int_0^1 \frac{\sin x}{\sqrt{x}}dx$ and J = $\int_0^1 \frac{\cos x}{\sqrt{x}}dx$. Then which one of the following is true?

(1) I > 2/3 and J > 2 (2) I < 2/3 and J < 2

(3) I < 2/3 and J > 2 (4) I > 2/3 and J < 2 **[2008]**

10. The value of the integral, $\int_3^6 \frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}}dx$ is

(1) ½ (2) 3/2

(3) 2 (4) 1 **[2006]**

11. $\int_0^\pi xf(\sin x)dx$ is equal to **[2006]**

(1) $\pi\int_0^\pi f(\cos x)dx$ (2) $\pi\int_0^\pi f(\sin x)dx$

(3) $\frac{\pi}{2}\int_0^{\pi/2} f(\sin x)dx$

(4) $\pi\int_0^{\pi/2} f(\cos x)dx$

12. $\int_{-3\pi/2}^{-\pi/2}\left[(x+\pi)^3+\cos^2(x+3\pi)\right]dx$ is equal to

(1) $\frac{\pi^4}{32}$

(2) $\frac{\pi^4}{32}+\frac{\pi}{2}$

(3) $\frac{\pi}{2}$

(4) $\frac{\pi}{4}-1$ **[2006]**

13. The value of $\int_1^a [x]\, f'(x)dx$, a > 1, where [x] denotes the greatest integer not exceeding x is

(1) af(a) – {f(1) +f(2)+……+f([a])}

(2) [a] f(a) – {f(1) +f(2) + ….. +f([a])}

(3) [a]f([a]) – {f(1) + f(2) + ….+f(a)}

(4) af([a]) – {f(1) + f(2) + …..+f(a)}**[2006]**

14. $\lim_{n\to\infty}[\frac{1}{n^2}\sec^2\frac{1}{n^2}+\frac{2}{n^2}\sec^2\frac{4}{n^2}+.....+\frac{1}{n^2}\sec^2 1]$ equals

(1) $\frac{1}{2}\sec 1$

(2) $\frac{1}{2}\cos ec 1$

(3) tan1

(4) $\frac{1}{2}\tan 1$ **[2005]**

15. If $I_1=\int_0^1 2^{x^2}dx$, $I_2=\int_0^1 2^{x^3}dx$, $I_3=\int_1^2 2^{x^2}dx$ and $I_4=\int_1^2 2^{x^3}dx$ then

(1) $I_2 > I_1$

(2) $I_1 > I_2$

(3) $I_3 = I_4$

(4) $I_3 > I_4$ **[2005]**

16. Let f : $R\to R$ be a differentiable function having f(2) = 6 , f′(2) = $\left(\frac{1}{48}\right)$. Then $\lim_{x\to 2}\int_6^{f(x)}\frac{4t^3}{x-2}dt$ equals

(1) 24

(2) 36

(3) 12

(4) 18 **[2005]**

17. The value of $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx$, a > 0 , is

(1) aπ (2) $\frac{\pi}{2}$

(3) $\frac{\pi}{a}$ (4) 2π **[2005]**

18. $\lim_{n\to\infty} \sum_{r-1}^{n} \frac{1}{n} e^{\frac{r}{n}}$ is

(1) e (2) e - 1

(3) 1-e (4) e + 1 **[2004]**

19. The value of $\int_{-2}^{3} \left|1-x^2\right| dx$ is

(1) $\frac{28}{3}$ (2) $\frac{14}{3}$

(3) $\frac{7}{3}$ (4) $\frac{1}{3}$ **[2004]**

20. The value of $I = \int_{0}^{\pi/2} \frac{(\sin x + \cos x)^2}{\sqrt{1+\sin 2x}} dx$ is

(1) 0 (2) 1

(3) 2 (4) 3 **[2004]**

21. If $\int_{0}^{\pi} x f(\sin x) dx = A \int_{0}^{\pi/2} f(\sin x) dx$, then A is

(1) 0 (2) π

(3) π/4 (4) 2π **[2004]**

22. If $f(x) = \frac{e^x}{1+e^x}$, I_1 = $\int_{f(-a)}^{f(a)} x g\{x(1-x)\} dx$ and I_2 = $\int_{f(-a)}^{f(a)} g\{x(1-x)\} dx$ then the value of $\frac{I_2}{I_1}$ is

(1) 2 (2) -3

(3) -1 (4) 1 **[2004]**

23. If f(y) = e^y , g(y) = y ; y > 0 and F(t) = $\int_0^t f(t-y)g(y)dy$, then

(1) F(t) = 1-e^{-t}(1+t) (2) F(t) = e^t – (1+t)

(3) F(t) = te^t (4) F(t) = te^{-t} **[2003]**

24. If f(a + b –x) = f(x), then $\int_a^b xf\left(x\right)dx$ is equal to

(1) $\frac{a+b}{2}\int_a^b f\left(b-x\right)dx$ (2) $\frac{a+b}{2}\int_a^b f(x)dx$

(3) $\frac{b-a}{2}\int_a^b f(x)dx$ (4) $\frac{a+b}{2}\int_a^b f\left(a+b-x\right)dx$ **[2003]**

25. The value of $\lim\limits_{x\to 0}\frac{\int_0^{x^2}\sec^2 t\,dt}{x\sin x}$ is

(1) 3 (2) 2

(3) 1 (4) 0 **[2003]**

26. The value of the integral $I=\int_0^1 x\left(1-x\right)^n dx$ is

(1) $\frac{1}{n+1}$ (2) $\frac{1}{n+2}$

(3) $\frac{1}{n+1}-\frac{1}{n+2}$ (4) $\frac{1}{n+1}+\frac{1}{n+2}$ **[2003]**

27. Let $\frac{d}{dx}F(x)=\left(\frac{e^{\sin x}}{x}\right), x>0$. If $\int_1^4 \frac{3}{x}e^{\sin x^3}dx=F(k)-F(1)$, then one of the possible values of k, is

(1) 15 (2) 16

(3) 63 (4) 64 **[2003]**

28. Let f(x) be a function satisfying f'(x) = f(x) with f(0) = 1 and g(x) be a function that satisfies f(x) + g(x) = x^2 . Then the value of the integral $\int_0^1 f(x)g(x)dx$, is

(1) $e-\frac{e^2}{2}-\frac{5}{2}$

(2) $e+\frac{e^2}{2}-\frac{3}{2}$

(3) $e-\frac{e^2}{2}-\frac{3}{2}$

(4) $e+\frac{e^2}{2}+\frac{5}{2}$ **[2003]**

29. $\int_0^{\sqrt{2}} \left[x^2\right]dx$ is

(1) 2 - $\sqrt{2}$

(2) $2+\sqrt{2}$

(3) $\sqrt{2}-1$

(4) $\sqrt{2}-2$ **[2002]**

30. I_n = $\int_0^{\pi/4} \tan^n x dx$, then $\lim\limits_{n\to\infty}\left[I_n+I_{n-2}\right]$ equals (1) ½

(2) 1

(3) ∞

(4) 0 **[2002]**

31. $\int_{\pi}^{10\pi} |\sin x|\, dx$ is

(1) 20

(2) 8

(3) 10

(4) 18 **[2002]**

32. If y = f(x) makes positive intercept of 2 and 0 unit in x and y axes and encloses an area of ¾ square units with the axes then $\int_0^2 f'(x)dx$ is

(1) 3/2

(2) 1

(3) 5/4

(4) -3/4 **[2002]**

33. $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x}dx$ is

(1) $\frac{\pi^2}{4}$

(2) π^2

(3) 0

(4) $\frac{\pi}{2}$

[2002]

Assertion – Reason Type

1. **Statement – I** : The value of the integral $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$ is equal to $\frac{\pi}{6}$. **[2013]**

Statement – II : $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$.

Indefinite Integrals

MCQ-Single Correct

1. Let $I_n = \int \tan^n x dx$, (n > 1). If $I_4 + I_6 = a \tan^6 x + bx^5 + C$, where C is a constant of integration, then the ordered pair (a,b) is equal to :

(1) $\left(-\frac{1}{5},1\right)$ (2) $\left(\frac{1}{5},0\right)$

(3) $\left(\frac{1}{5},-1\right)$ (4) $\left(-\frac{1}{5},0\right)$ **[2017]**

2. The integral $\frac{\pi^2}{16}$ is equal to :

(1) $\frac{x^{10}}{2\left(x^5+x^3+1\right)^2}+C$ (2) $\frac{x^5}{2\left(x^5+x^3+1\right)^2}+C$

(3) $\frac{-x^{10}}{2\left(x^5+x^3+1\right)^2}+C$ (4) $\frac{-x^5}{\left(x^5+x^3+1\right)^2}+C$ **[2016]**

3. The integral $\int \frac{dx}{x^2\left(x^4+1\right)^{3/4}}$ equals :

(1) $\left(x^4+1\right)^{1/4}+c$ (2) $-\left(x^4+1\right)^{1/4}+c$

(3) $-\left(\frac{x^4+1}{x^4}\right)^{1/4}+c$ (4) $\left(\frac{x^4+1}{x^4}\right)^{1/4}+c$ **[2015]**

4. The integral $\int\left(1+x-\frac{1}{x}\right)e^{x+\frac{1}{x}}dx$ is equal to

(1) $(x-1)e^{x+\frac{1}{x}}+c$ (2) $xe^{x+\frac{1}{x}}+c$

(3) $(x+1)e^{x+\frac{1}{x}}+c$ (4) $-xe^{x+\frac{1}{x}}+c$ **[2014]**

5. If $\int f(x)dx=\Psi(x)$, then $\int x^5 f\left(x^3\right)dx$ is equal to **[2013]**

(1) $\frac{1}{3}x^3\Psi\left(x^3\right)-3\int x^3\Psi\left(x^3\right)dx+C$

(2) $\frac{1}{3}x^3\Psi\left(x^3\right)-\int x^2\Psi\left(x^3\right)dx+C$

(3) $\frac{1}{3}\left[x^3\Psi\left(x^3\right)-\int x^3\Psi\left(x^3\right)dx\right]+C$

(4) $\frac{1}{3}\left[x^3\Psi\left(x^3\right)-\int x^2\Psi\left(x^3\right)dx\right]+C$

6. The value of $\sqrt{2}\int\frac{\sin x dx}{\sin\left(x-\frac{\pi}{4}\right)}$ is

(1) $x+\log\left|\cos\left(x-\frac{\pi}{4}\right)\right|+c$

(2) $x-\log\left|\sin\left(x-\frac{\pi}{4}\right)\right|+c$

(3) $x+\log\left|\sin\left(x-\frac{\pi}{4}\right)\right|+c$

(4) $x-\log\left|\cos\left(x-\frac{\pi}{4}\right)\right|+c$ **[2008]**

7. $\int\left\{\frac{(\log x-1)}{1+(\log x)^2}\right\}^2 dx$ is equal to

(1) $\frac{\log x}{(\log x)^2+1}+C$

(2) $\frac{x}{x^2+1}+C$

(3) $\frac{xe^x}{1+x^2}+C$

(4) $\frac{x}{(\log x)^2+1}+C$ **[2005]**

8. If $\int\frac{\sin x}{\sin(x-\alpha)}dx=Ax+B\log\sin(x-\alpha)+C$, then the value of (A,B) is

(1) (sinα, cosα)

(2) (cosα, sinα)

(3) (-sinα, cosα)

(4) (-cosα, sinα) **[2004]**

9. $\int\frac{dx}{\cos x-\sin x}$ is equal to

(1) $\frac{1}{\sqrt{2}}\log|\tan\left(\frac{x}{2}-\frac{\pi}{8}\right)|+C$

(2) $\frac{1}{\sqrt{2}}\log\left|\cot\left(\frac{x}{2}\right)\right|+C$

(3) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}-\frac{3\pi}{8}\right)\right|+C$

(4) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}+\frac{3\pi}{8}\right)\right|+C$ **[2004]**

Area

MCQ – Single Correct

1. The area (in sq. units) of the region $\{(x,y): x \geq 0,\ x + y \leq 3,\ x^2 \leq 4y$ and $y \leq 1 + \sqrt{x}\}$ is :

(1) $\frac{59}{12}$ (2) $\frac{3}{2}$

(3) $\frac{7}{3}$ (4) $\frac{5}{2}$ **[2017]**

2. The area (in sq. units) of the region $\{(x,y) : y^2 \geq 2x$ and $x^2 + y^2 \leq 4x,\ x \geq 0,\ y \geq 0\}$ is :

(1) $\pi - \frac{8}{3}$ (2) $\pi - \frac{4\sqrt{2}}{3}$

(3) $\frac{\pi}{2} - \frac{2\sqrt{2}}{3}$ (4) $\pi - \frac{4}{3}$ **[2016]**

3. The area (in square units) of the region described by $\{(x,y) :\ y^2 \leq 2x$ and $y \geq 4x - 1\}$ is :

(1) $\frac{5}{64}$ (2) $\frac{15}{64}$

(3) $\frac{9}{32}$ (4) $\frac{7}{32}$ **[2015]**

4. The area of the region described by $A = \{(x,y) : x^2 + y^2 \leq 1$ and $y^2 \leq 1 - x\}$ is **[2014]**

(1) $\frac{\pi}{2} + \frac{4}{3}$ (2) $\frac{\pi}{2} - \frac{4}{3}$

(3) $\frac{\pi}{2} - \frac{2}{3}$ (4) $\frac{\pi}{2} + \frac{2}{3}$

5. The area (in square units) bounded by the curves $y = \sqrt{x},\ 2y - x + 3 = 0,$ x-axis, and lying in the first quadrant is

(1) 36 (2) 18

(3) $\frac{27}{4}$ (4) 9 **[2013]**

6. The area bounded between the parabola $x^2 = \frac{y}{4}$ and $x^2 = 9y$ and the straight line y = 2 is

(1) $\frac{20\sqrt{2}}{3}$ (2) $10\sqrt{2}$

(3) $20\sqrt{2}$ (4) $\frac{10\sqrt{2}}{3}$ **[2012]**

7. The area bounded by the curves $y^2 = 4x$ and $x^2 = 4y$ is

(1) 8/3 (2) 0

(3) 32/3 (4) 16/3 **[2011]**

8. The area bounded by the curves y = cos x and y = sin x between the ordinates x = 0 and x = 3π/2 is

(1) $4\sqrt{2}+2$ (2) $4\sqrt{2}-1$

(3) $4\sqrt{2}+1$ (4) $4\sqrt{2}-2$ **[2010]**

9. The area of the plane region bounded by the curves $x + 2y^2 = 0$ and $x + 3y^2 = 1$ is equal to

(1) 5/3 (2) 1/3

(3) 2/3 (4) 4/3 **[2008]**

10. The area enclosed between the curve $y = \log_e(x + e)$ and the coordinate axes is

(1) 1 (2) 2

(3) 3 (4) 4 **[2005]**

11. The parabolas $y^2 = 4x$ and $x^2 = 4y$ divide the square region bounded by the lines x = 4 , y = 4 and the coordinate axes. If S_1 , S_2 , S_3 are respectively the areas of these parts numbered from top to bottom; then $S_1 : S_2 : S_3$ is

(1) 1 : 2 : 1 (2) 1 : 2 :3

(3) 2 : 1 : 2 (4) 1 : 1 : 1 **[2005]**

12. Let f(x) be a non-negative continuous function such that the area bounded by the curve y = f (x), x-axis and the ordinates x = 3 : $\sqrt{2}$ and x = β > π/4 is $\left(\beta\sin\beta+\frac{\pi}{4}\cos\beta+\sqrt{2}\beta\right)$. Then f(π/2) is

(1) $\left(\frac{\pi}{4}+\sqrt{2}-1\right)$ (2) $\left(\frac{\pi}{4}-\sqrt{2}+1\right)$

(3) $1+\frac{1}{4.2!}+\frac{1}{16.4!}+\frac{1}{64.6!}+.........$ (4) $\left(1-\frac{\pi}{4}+\sqrt{2}\right)$ **[2005]**

13. The area of the region bounded by the curves y = |x-2|, x = 1, x = 3 and the x-axis is

(1) 1 (2) 2

(3) 3 (4) 4 **[2004]**

14. The area of the region bounded by the curves y = |x - 1| and y = 3 - |x| is

(1) 2 sq units (2) 3 sq units

(3) 4 sq units (4) 6 sq units **[2003]**

15. The area bounded by the curves y = ln x, y = ln |x| , y = |ln x| and y = |ln| |x| is

(1) 4 sq. units (2) 6 sq. units

(3) 10 sq. units (4) none of these **[2002]**

Differential Equations

MCQ-Single Correct

1. If $(2+\sin x)\frac{dy}{dx}+(y+1)\cos x=0$ and y(0) = 1 , then $y\left(\frac{\pi}{2}\right)$ is equal to :

(1) 1/3 (2) -2/3

(3) -1/3 (4) 4/3 **[2017]**

2. If a curve y = f(x) passes through the point (1 , -1) and satisfies the differential equation,

y(1 + xy) dx = x dy , then $f\left(-\frac{1}{2}\right)$ is equal to :

(1) - 4/5 (2) 2/5

(3) 4/5 (4) -2/5 **[2016]**

3. Let y(x) be the solution of the differential equation $(x\log x)\frac{dy}{dx}+y=2x\log x$, $(x\ge 1)$. Then y(e) is equal to :

(1) 0 (2) 2

(3) 2e (4) e **[2015]**

4. Let the population of rabbits surviving at a time t be governed by the differential equation $\frac{dp(t)}{dt}=\frac{1}{2}p(t)-200.$ If p(0) = 100, then p(t) equals

(1) $400 – 300e^{t/2}$ (2) $300 – 200e^{-t/2}$

(3) $600 – 500e^{t/2}$ (4) $400 – 300e^{-t/2}$ **[2014]**

5. At present, a firm is manufacturing 2000 items. It is estimated that the rate of change of production P w.r.t. additional number of workers x is given by $\frac{dP}{dx}=100-12\sqrt{x}$. If the firm employs 25 more workers, then the new level of production of items is

(1) 3000 (2) 3500

(3) 4500 (4) 2500 **[2013]**

6. A spherical baloon is filled with 4500 π cubic meters of helium gas. If a leak in the balloon causes the gas to escape at the rate of 72 π cubic metres per minute, then the rate (in meters per minute) at which the radius of the balloon decreases 49 minutes after the leakage began is

(1) 2/9 (2) 9/2

(3) 9/7 (4) 7/9 **[2012]**

7. The population p(t) at time t of a certain mouse species satisfies the differential equation $\frac{dp(t)}{dt} = 0.5p(t) - 450.$ If p(0) = 850 , then the time at which the population becomes zero is

(1) $\frac{1}{2}\ln 18$ (2) ln 18

(3) 2 ln 18 (4) ln 9. **[2012]**

8. Consider the differential equation $y^2dx + \left(x - \frac{1}{y}\right)dy = 0$. If y(1) = 1, then x is given by

(1) $1 + \frac{1}{y} - \frac{e^{1/y}}{e}$ (2) $1 - \frac{1}{y} + \frac{e^{1/y}}{e}$

(3) $4 - \frac{2}{y} - \frac{e^{1/y}}{e}$ (4) $3 - \frac{1}{y} + \frac{e^{1/y}}{e}$ **[2011]**

9. Solution of the differential equation cos x dy = y(sin x – y) dx, $0 < x < \frac{\pi}{2}$ is

(1) y sec x = tan x + c (2) y tan x = sec x + c

(3) tan x = (sec x + c)y (4) sec x = (tan x +c)y **[2010]**

10. The differential equation which represents the family of curves $y = c_1e^{c_2x}$, where c_1 and c_2 are arbitrary constants is

(1) $y' = y^2$ (2) $y'' = y'y$

(3) $yy'' = y'$ (4) $yy'' = (y')^2$ **[2009]**

11. The solution of the differential equation $\frac{dy}{dx} = \frac{x+y}{x}$ satisfying the condition y(1) = 1 is

(1) $y = \ln x + x$ (2) $y = x\ln x + x^2$

(3) $y = xe^{(x-1)}$ (4) $y = x \ln x + x$ **[2008]**

12. The differential equation of the family of circles with fixed radius 5 units and centre on the line y = 2 is

(1) $(x-2)y'^2 = 25 - (y-2)^2$ (2) $(y-2)y'^2 = 25 - (y-2)^2$

(3) $(y-2)^2y'^2 = 25 - (y-2)^2$ (4) $(x-2)^2y'^2 = 25 - (y-2)^2$ **[2008]**

13. The normal to a curve at P(x,y) meets the x-axis at G. If the distance of G from the origin is twice the abscissa of P, then the curve is

(1) an ellipse (2) a parabola

(3) a circle (4) a hyperbola **[2007]**

14. The differential equation of all circles passing through the origin and having their centres on the x-axis is

(1) $x^2 = y^2 + xy\dfrac{dy}{dx}$ (2) $x^2 = y^2 + 3xy\dfrac{dy}{dx}$

(3) $y^2 = x^2 + 2xy\dfrac{dy}{dx}$ (4) $y^2 = x^2 - 2xy\dfrac{dy}{dx}$ **[2007]**

15. The differential equation whose solution is $Ax^2 + By^2 = 1$, where A and B are arbitrary constants is of

(1) second order and second degree (2) first order and second degree

(3) first order and first degree (4) second order and first degree **[2006]**

16. The differential equation representing the family of curves $y^2 = 2c(x + \sqrt{c})$, where c > 0, is a parameter, is of order and degree as follows:

(1) order 1, degree 2 (2) order 1, degree 1

(3) order 1, degree 3 (4) order 2, degree 2 **[2005]**

17. If $x\dfrac{dy}{dx} = y(\log y - \log x + 1)$, then the solution of the equation is

(1) $y\log\left(\dfrac{x}{y}\right) = cx$ (2) $x\log\left(\dfrac{y}{x}\right) = cy$

(3) $\log\left(\frac{y}{x}\right)=cx$ (4) $\log\left(\frac{x}{y}\right)=cy$ **[2005]**

18. The differetnatial equation for the family of curves $x^2 + y^2 - 2ay = 0$, where a is an arbitrary constant is

(1) $2(x^2 - y^2)y' = xy$ (2) $2(x^2 + y^2)y' = xy$

(3) $(x^2 - y^2)y' = 2xy$ (4) $(x^2 + y^2)y' = 2xy$ **[2004]**

19. The solution of the differential equation $y\,dx + (x + x^2y)\,dy = 0$ is

(1) $-\frac{1}{xy}=C$ (2) $-\frac{1}{xy}+\log y=C$

(3) $\frac{1}{xy}+\log y=C$ (4) $\log y = Cx$ **[2004]**

20. The degree and order of the differential equation of the family of all parabolas whose axis is x-axis, are respectively

(1) 2,1 (2) 1,2

(3) 3,2 (4) 2,3 **[2003]**

21. The solution of the differential equation $(1+y^2)+\left(x-e^{\tan^{-1}y}\right)\frac{dy}{dx}=0$, is

(1) $(x-2)=ke^{-\tan^{-1}y}$ (2) $2xe^{2\tan^{-1}y}=e^{2\tan^{-1}y}+k$

(3) $xe^{\tan^{-1}y}=\tan^{-1}y+k$ (4) $xe^{2\tan^{-1}y}=e^{\tan^{-1}y}+k$ **[2003]**

22. The solution of the equation $\frac{d^2y}{dx^2}=e^{-2x}$

(1) $\frac{1}{4}e^{-2x}$ (2) $\frac{1}{4}e^{-2x}+cx+d$

(3) $\frac{1}{4}e^{-2x}+cx^2+d$ (4) $\frac{1}{4}e^{-2x}+c+d$ **[2002]**

23. The order and degree of the differential equation $\left(1+3\frac{dy}{dx}\right)^{2/3}=4\frac{d^3y}{dx^3}$ are

(1) 1, 2/3 (2) 3,1

(3) 3,3 (4) 1,2 **[2002]**