Probability

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Chapter 1

Introduction

Probability is the branch of mathematics concerning numerical descriptions of how likely an event is to occur, or how likely it is that a proposition is true. The probability of an event is a number between 0 and 1, where, roughly speaking, 0 indicates impossibility of the event and 1 indicates certainty.

1.1 Sample Space

In probability theory, the sample space of an experiment or random trial is the set of all possible outcomes or results of that experiment. A sample space is usually denoted using set notation, and the possible ordered outcomes, or sample points, are listed as elements in the set.

1.1.1 Examples of a Sample Space

1.1.1.1 Toss a coin

While tossing a fair coin, the Sample Space would be H-Heads and T-Tails.

$$S = H, T$$

Similarly , in tossing two identical coins or a coin twice , the Sample Space would be just the cross product of single coin toss

i.e. $S = HH, HT, TH, TT$

Extending it furthur, In tossing 3 identical coins or a coin thrice

$S = HHH, HHT, HTH, THH, HTT, THT, TTH, TTT$

Q: However , If we have all different coins, the case would be much more surprising

In this case, we see that the first leftmost element can be filled in 6 ways. Now in each case we have only two coins with 2 sides are available for the rest of 2 positions which can be filled in $2 \times 2 \times 2$ ways.

$S = H_1H_2H_3, H_2T_3, T_3H_2, T_2T_3 and swapped$

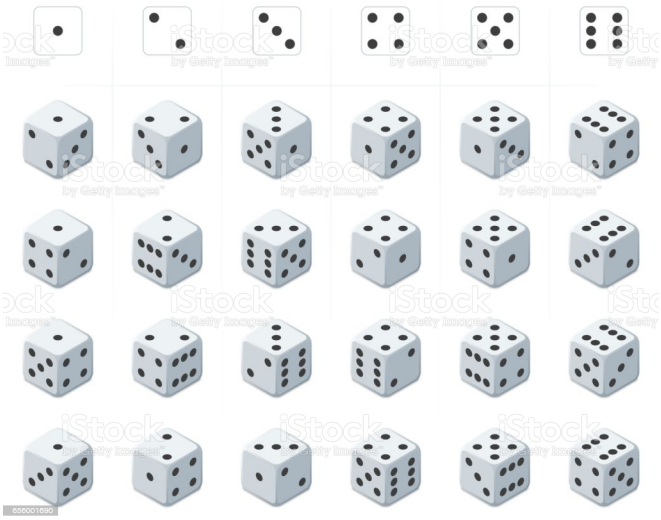
So, 48 total ways.

We see that first place can be filled in 6 ways, second place in 4 ways and third place in 2 ways.

1.1.1.2 Throw a dice

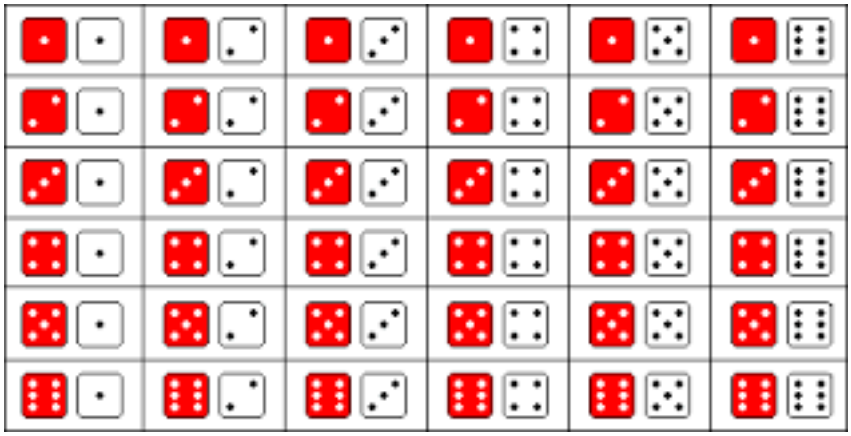
In a throw of a dice, the Sample Space would be

$S = 1, 2, 3, 4, 5, 6$



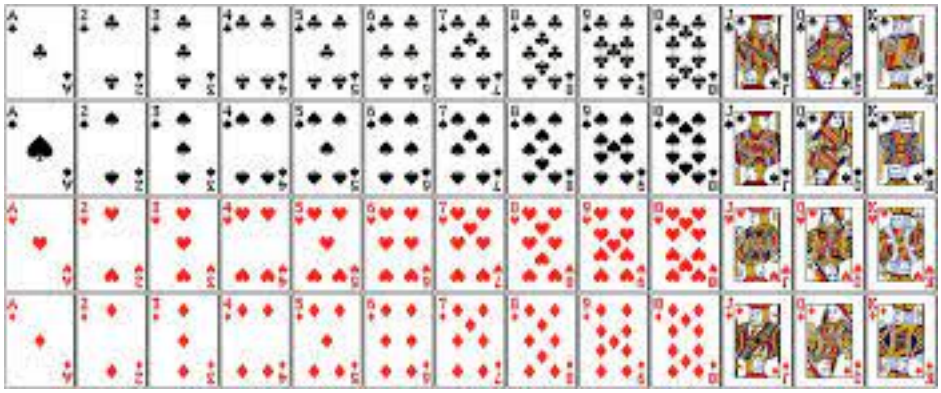
Further as above , if we throw two identical dice or a dice twice, the Sample Space would be simply the cross-product

$$\begin{array}{cccccc} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{array}$$



1.1.1.3 Pack of Cards

Clubs, Hearts, Diamonds, Spades.



1.2 Event

Chapter 2

Probability

2.0.0.1 Matrix Match Type Problems

Matrix 1: A student goes to exam on a bicycle or scooter or car or on foot. The probabilities of his using bicycle, scooter car or being on foot are $\frac{1}{10}$, $\frac{1}{5}$, $\frac{3}{10}$ or $\frac{2}{5}$ respectively. The probability of reaching late at the examination centre by using these modes are $\frac{1}{3}, \frac{1}{4}, \frac{1}{12}$ and $\frac{1}{24}$ respectively. The student reached the exam centre on time. Under Column I, the modes of travel are given and under Column II, the probabilies that the student used the particular mode of transport provided that he reached the exam centre on time. Match the corresponding entries.

Column I	Column II
(P) Bicycle	(A) $\frac{2}{5}$
(Q)Scooter	(B) $\frac{1}{5}$
(R)Car	(C) $\frac{2}{15}$
(S)On Foot	(D) $\frac{4}{15}$

2.0.0.2 Linked Comprehension Type Questions

Comprehension 1: There are n biased coins and n boxes numbered 1 to n. The coins are tossed and each put into its corresponding box. For the k^{th} coin, the odds in favour of appearance of head are k and the odds against are $2n-k$. Let E_n denote the event of selecting the k^{th} box and H denote the event that the selected box contains a Head.

Q1: If $P(E_k) = c\forall k$, then $P(E_m|H)$ where $m\epsilon\{1,2,3,.....n\}$ is

A) $\frac{m}{n(n+1)}$

B) $\frac{2m}{n(n+1)}$

C) $\frac{c}{n+1}$

D) $\frac{2}{n+1}$

Q2: If $P(E_k) \propto k$ for $k = 1,2,3.....n$. Find $P(H)$

A) $\frac{2n+1}{6n}$

B) $\frac{2n+1}{4n}$

C) $\frac{2}{3}$

D) $\frac{1}{3}$

Q3: Evaluate the probability P that only the m^{th} box contains a head .Its value for $m = 3, n = 5$ is

A) 0.12

B) 0.0832

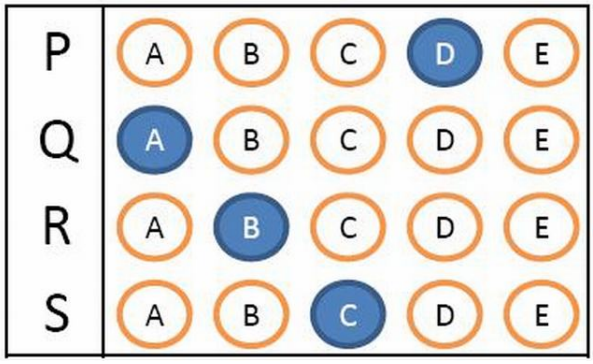
C) 0.648

D) None of these

2.0.0.3 Hints and Solutions

Matrix Match Type Problems

Matrix 1:Answers



Linked Comprehension Type Questions

Comprehension 1 Answers Q1) B Q2) A Q3) C