A Concise Course in Graphs of Physics

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Contents

Preface

Earlier larger volume books on graphs were written by us but then we thought that probably the student might lose interst in them. So we thought of writing a very concise and to the point book for Physics Graphs. And the outcome of this advent is the present book.

Chapter 1

Introduction to Graphs

1.1 Theory

1.1.1 Introduction 2016

A graph is an accurate pictorial representation of data. The accuracy of data in physics requires that graphs be made on good quality graph paper. Nearly all graphs in physics are smooth line graphs; broken line (connect the dots) graphs and bar graphs are seldom appropriate.

The style and format of a graph will depend upon its intended purpose. Three types are common in physics

1. PICTORIAL GRAPHS. These are the kind found in mathematics and physics textbooks. Their purpose is to simply and clearly illustrate a mathematical relation. No attempt is made to show data points or errors on such a graph.

2. DISPLAY GRAPHS. These present the data from an experiment. They are found in laboratory reports, research journals, and sometimes in textbooks. They show the data points as well as a smooth line representing the mathematical relation.

3. COMPUTATIONAL GRAPHS. These are drawn for the purpose of extracting a numerical result from the data. An example is the calculation of the slope of a straight line graph, or its intercepts.

1.1.2 Elements of a good Graph

Certain informational and stylistic features are required in all graphs:

1. The graph must have a descriptive title or caption, clearly stating what the graph illustrates.

2. Data points are plotted as small dots with a sharp pencil, or as pinpricks. Some method should be used to emphasize the location of the points, for example, a neat circle drawn around each point.

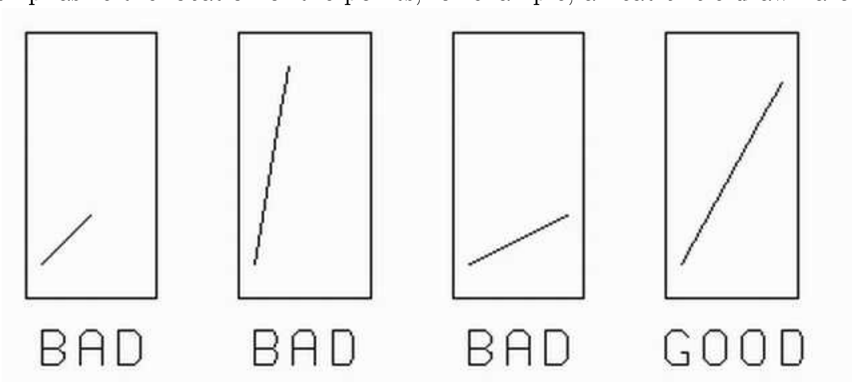


Figure :Good and bad choices of geometric slope of a graph.

3. Curves drawn through the points should be smooth (use [1] French curves if your hand is not steady). The curve should stand out clearly.

4. Choose scales that are convenient to plot and easy to read.

[1] A French curve is a template usually made from metal, wood or plastic composed of many different curves. It is used in manual drafting to draw smooth curves of varying radii. The shapes are segments of the Euler spiral or clothoid curve. The curve is placed on the drawing material, and a pencil, knife or other implement is traced around its curves to produce the desired result.

Modern computer-aided design (CAD) systems use vector-based graphics to achieve a precise radius, so no template is required. Digital computers can also be used to generate a set of coordinates that accurately describe an arbitrary curve, and the points can be connected with line segments to approximate the curve with a high degree of accuracy. Some computer-graphics systems make use of Bézier curves, which allow a curve to be bent in real time on a display screen to follow a set of coordinates, much in the way a French curve would be placed on a set of three or four points on paper.

5. Choose scales such that the graph occupies most of the page. The two scales need not have the same size units. Also, the scales need not begin at zero.

6. Indicate the name, letter symbol and units of each variable plotted on each axis.

7. All text (title, labels, etc.) should be printed.

PHYSICAL SLOPE AND GEOMETRIC SLOPE

Slope. When textbooks refer to the "slope" of a plotted graph line we mean the "physical slope"

physical slope $= \dfrac{\Delta y}{\Delta x}$

where Δy and Δx are expressed in the physical units of the x and y axes. This slope has physical significance in describing the physical data.

Geometric slope. A line which makes a 45° angle with an axis will not necessarily have a physical slope of size 1. Some authors introduce the term "geometric slope" to describe the tilt of the line on the page. This is a ratio of lengths of the legs of the triangle, without reference to the units plotted on the axes.

There is seldom (probably never) any need to calculate the geometric slope of a line on a graph. The idea is only useful when describing the appearance of the graph on the page. One rule of graph construction states that the graph should occupy most of the page. For square graph paper this suggests a geometric slope of 45°. See Fig for examples of good and bad choices of geometric slope.

THE APPEARANCE OF THE GRAPH

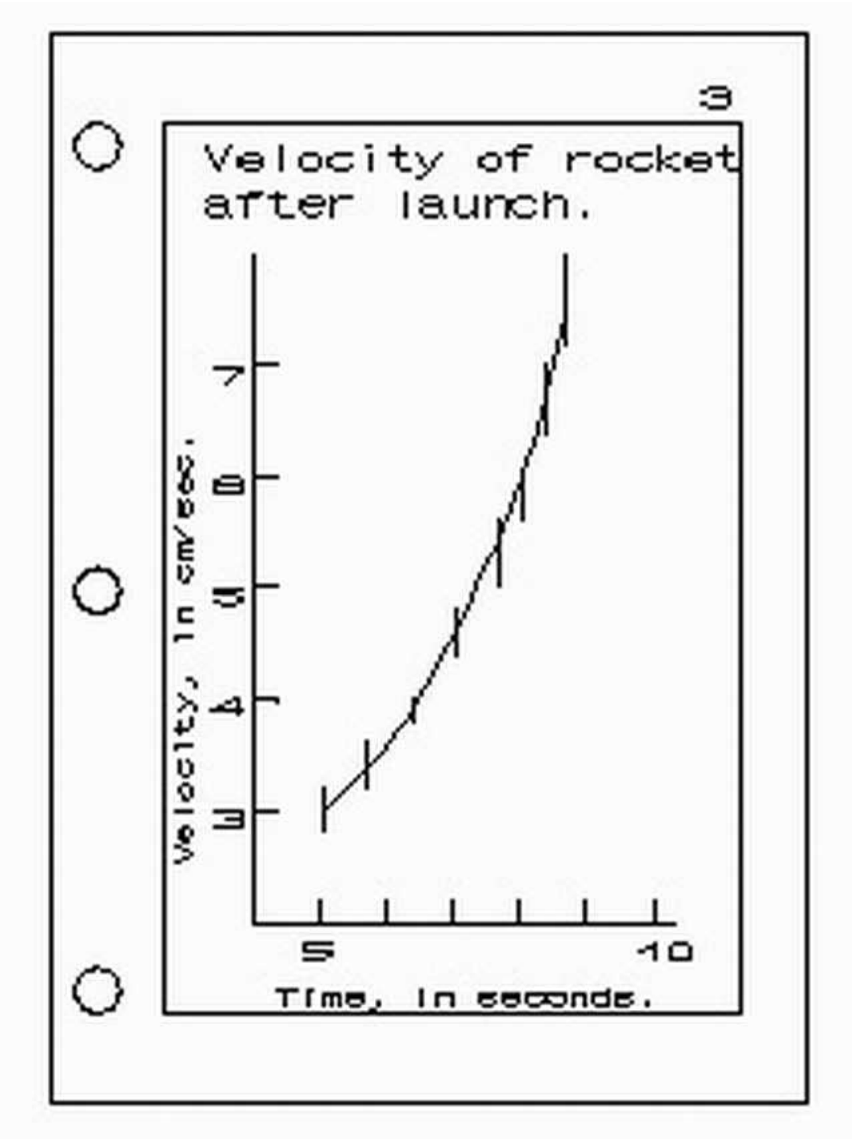


Figure : Elements of a graph.

Use quality graph paper, size 8.5 by 11 inches only.

The left margin is largest, for binding or stapling.

Nothing should be in the white margins except a page number. Axes, lettering and labeling should all be within the printed grid area. [The grid lines serve as guide lines for neat, uniform printed lettering.]

The title must be descriptive.

Both axes are labeled with the full name of the quantity (not merely its symbol), and its units.

The plotted points and curve should occupy most (more than half) of the area of the graph paper.

Sometimes a small sketch of the experimental situation may be included, located where it will not confuse the interpretation of the graph. In the same manner an equation, or short explanatory comment, may be included.

1.1.3 Graphical Representation of Uncertainties

Display graphs and computational graphs should clearly show the size of the experimental uncertainties (errors) in each plotted point. There are several conventional ways to do this, the commonest being the use of error bars illustrated below:

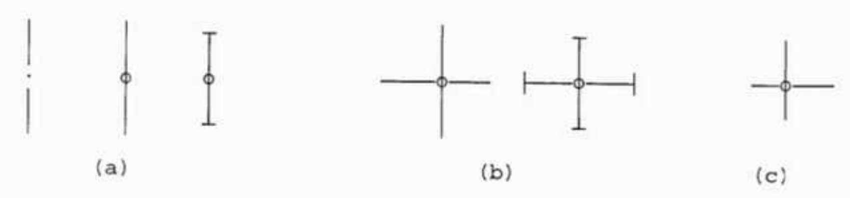


Figure : Various styles of error bars.

The plotted point is represented as a dot, and the range of uncertainty is shown by the extent of the bars on either side. The types shown in (a) are suitable where the error is entirely in one variable, or where the errors in both variables have been lumped together. The types shown in (b) are preferred where it is necessary to show the error in each variable explicitly.

When the uncertainties have a symmetric distribution about the mean, the error bars extend equally on either side of the points. If the data distributions are not symmetric, the plotted points will not be centrally located in the range of uncertainty and the error bars might look like those in Figure. part (c).

Error bars may not be necessary when the data points are so numerous that their scatter is clearly shows the uncertainty. In these cases error bars would clutter the graph making it difficult to interpret. Another situation where error bars are inappropriate is when the scale of the graph is such that the bars would be very small. In this case, it may be possible to indicate the uncertainty by the size of the circle or rectangle surrounding each point.

1.1.4 Curve Fitting

The curve drawn through plotted data need not pass exactly through every data point. But usually the curve should pass within the uncertainty range of each point, that is, within the error bars, if the bars represent limits of error.

One principle of curve fitting is also a fundamental rule of science itself:

Assume the simplest relation consistent with the data. We are not justified in assuming a more complex relation than can be demonstrated by the data. If a curve were drawn with detail smaller than the data uncertainty, that detail would be only a guess.

This rule of simplicity may also be expressed mathematically. The mathematical relations encountered in physics may often be represented by power series such as

$y = A + Bx + Cx^2 + Dx^3 + Ex^4 + ...$

where A, B, C, ... are constants.

For very "wiggly" curves, many terms of this equation, including high powers of x, might be required to express the equation of the relation. The simplest relations are those which contain the smallest powers of x. The simplest relations of all are

y = a or y = a + bx

which describe straight lines. Many relations in physics are, fortunately, of this form. Others only include the x^2 term, describing a parabolic curve. Note that double valued curves, sometimes encountered in physics, cannot be represented by Equation.

When sizable amounts of data are taken, standard mathematical methods are available which generate the equation of the simplest curve which statistically "best fits" the data.

The student may wonder how one can be certain that the curve fitted to the data is the "correct" curve. The answer is that relations are never known with certainty. The uncertainty of available data always limits the certainty of the results. Someday someone may obtain more accurate data and be able to show that the old relations are slightly incorrect, and provide us with better ones. As data improves, so does our understanding of relations—this is the way of scientific progress. But we never should claim to know a relation better than the data allows.

1.1.5 Uncertainty in a Slope

One use of a computational graph is to determine the slope of a straight line. This is illustrated in Figure. Eight data points are shown with error bars on each. If these bars represent maximum error, any line drawn to represent this data should pass within all bars.

If the error bars represent error estimates smaller than the maximum (average deviation, standard deviation, etc.), then the fitted curve need not pass within all of the error bars, just most of them.

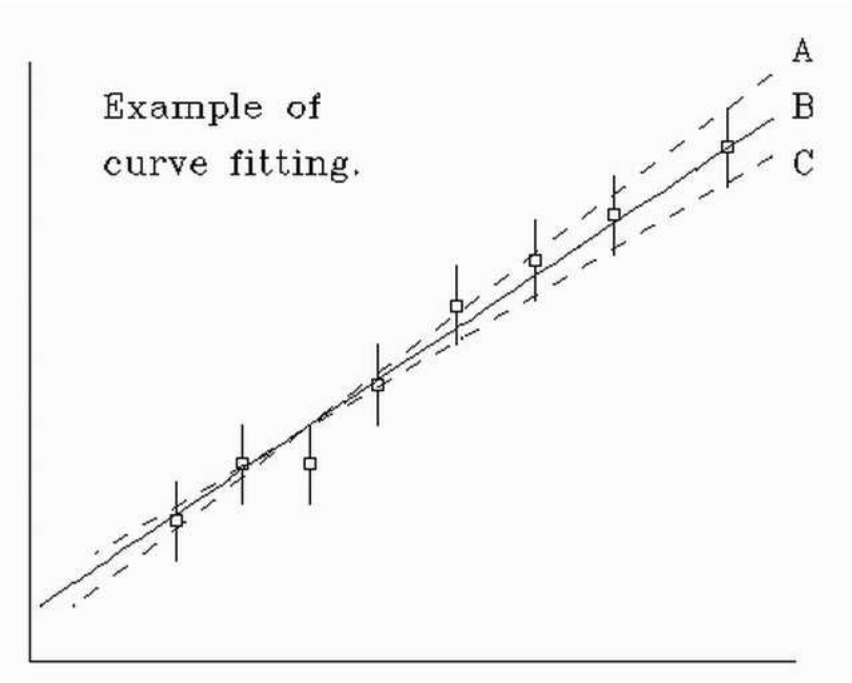


Figure : Fitting a curve.

Even a simple "manual" curve fit with a ruler can reveal the uncertainties in the slope resulting from uncertainties in the data. Figure. illustrates this process.

The dotted lines A and C fall within the error bars, and represent the maximum and minimum slope one could justify from this data. The "best" value of slope might be that of solid line B.

The third point from the left seems to limit the slope the most, and would appear to be "suspect." But one ought not to "throw it out" without better reason, based on further investigation.

1.1.6 Graphical Analysis of Data

Graphs can be a valuable tool for determining or verifying functional relations between variables. Many special types of graph paper are available for handling the most frequently encountered relations. You are probably already familiar with linear graph paper and polar coordinate paper.

You may have purchased a packet of graph paper for this course. It includes samples of graph papers you will use in this course, and a few other types. As you read the material below, examine the corresponding papers from your packet.

LINEAR RELATIONS are those which satisfy the equation

y = mx + b

where the variables are x and y, and m and b are constants. When y is plotted against x on ordinary Cartesian (linear) graph paper, the points fall on a straight line with slope m and a y-intercept b, as shown in Fig. 1.5.

The slope of an experimental relation is often physically significant. It is obtained by choosing two well-separated points on the line (x_1, y_1) and (x_2, y_2). From Eq. 7-3:

$y_1 = mx_1 + b$

and $y_2 = mx_2 + b$

Subtract the first from the second.

$(y_2 - y_1) = m(x_2 - x_1)$.

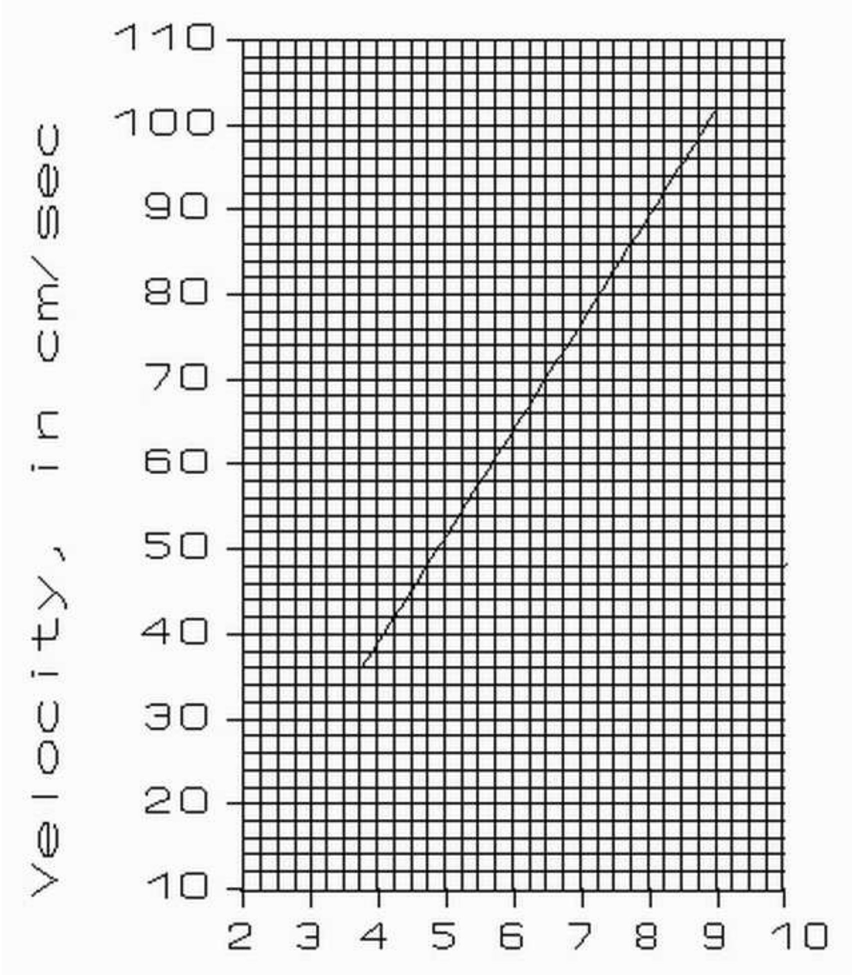


Figure : Measuring a slope on linear graph paper.

Therefore,

m = $\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{\Delta y}{\Delta x}$

The slope of a straight line is a ratio of the "lengths" of two legs of a right triangle constructed with the legs parallel to the graph axes and with the graph line along the hypotenuse. Figure. illustrates this, the slope being Δy/Δx.

So far this discussion has been strictly mathematical. Now let's consider a fairly realistic physical example. Figure. shows the curve from measurements of the velocity of a moving body as a function of time.

If we use letters v for velocity and t for time, we'd expect this curve to be described by the relation:

v = v_o + at

Here the constant a (acceleration) is the slope of the line, while v_o plays the role of b in Equation. These two constants are physically significant, and we wish to find their values from the graph.

We choose two points on the line at t = 4.25 and 8.75, with corresponding values of velocity: 42 cm/s and 98 cm/s. Mark these points on Figure. , to confirm these values. The slope of the line is therefore:

$a = \dfrac{(98cm/s - 42cm/s)}{8.75s - 4.25s} = \dfrac{56}{4.5}\ cm/s^2 = 56/4.5\ cm/s^2 = 12.44cm/s^2$

When calculating this ratio do not use ruler-measured lengths. The lengths are expressed in the units marked on the graph axes. The calculated slope is therefore independent of the particular choice of units, of the way you choose to label the graph scale divisions, and is also independent of the size of the graph paper.

INTERCEPTS: The values of the intercepts are often physically significant. They can be simply read from the graph—if the $x = 0$ and $y = 0$ axes happen to be within the graph's boundaries. In the equation y = mx + b, the y intercept is b.

The v intercept of Figure. is the value of v when t = 0. It has the same units and dimensions as y. If, as in this case, the v intercept does not lie within the area of the graph, it may be calculated using the slope and one value taken from a point on the fitted line. Take the point v = 98 cm/s when t = 8.75 sec.

v = v_o + at , in our case, v = v_o + 12.44 t

so,

v = v_o - 12.44 t = 98 - 12.44(8.75) = -10.89 cm/s A check of the graph, Figure. , shows that this looks reasonable.

STRAIGHTENING A CURVE. When it is possible to convert an experimental relation to a straight line graph it is usually useful to do so. Look for such opportunities. For example, when studying gases at constant temperature we find that

PV = C

where P is pressure, V is the gas volume and C is constant. The graph of P vs. V is a branch of an hyperbola. But if we graph P vs. 1/V, or V vs. 1/P, the data would fall on a straight line.

$$P = \left(\frac{1}{V}\right) C$$

One reason for doing this is that it is easier to fit the experimental data with a ruler-drawn straight line, than to draw the best hyperbola on a PV graph. Another advantage is that the P vs. 1/V graph has a slope

$$C = \frac{\triangle P}{\triangle \left(\frac{1}{V}\right)}$$

Therefore the constant C is easily determined from the straight line. This constant was not evident, nor was it easy to determine from the PV graph!

Inexpensive electronic calculators make it so easy to manipulate data that there is no good excuse to pass up an opportunity to "linearize" experimental graphs.

1.1.7 EXERCISES.

In each case state how you could plot (x,y) data on linear paper to obtain a straight line graph. What quantity in the equation is determinable from the slope of the straight line? What quantity is determinable from an intercept?

(1.1) x (y + 1) = 3

(1.2) 1/x + 1/y = 5

(1.3) $y = Ae^{-x}$

(1.4) $y = \sqrt{(A - x)}$

(1.5) $y^2 + x^2 = 7$

1.2 Common Graph Forms in Physics

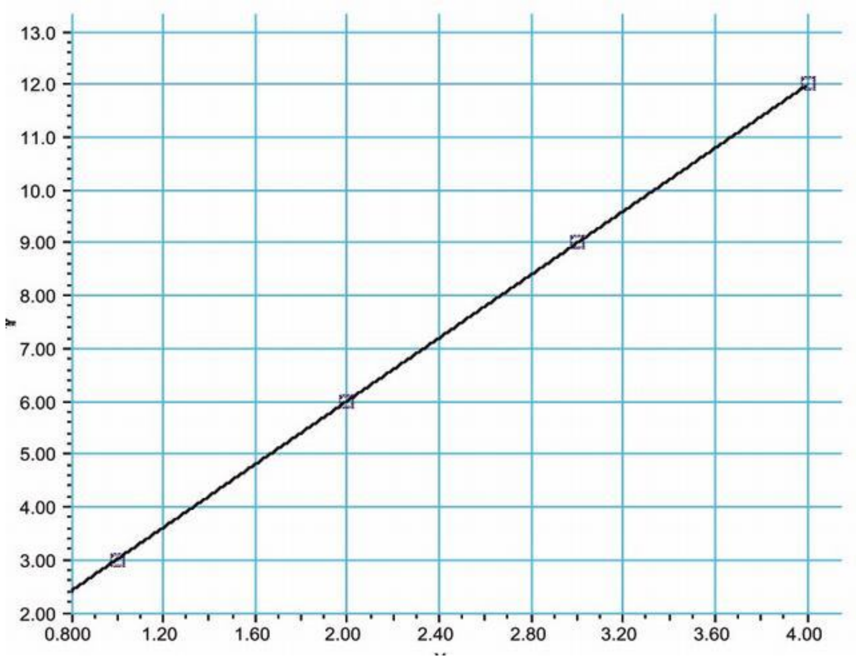
Working with graphs – interpreting, creating, and employing – is an essential skill in the sciences, and especially in physics where relationships need to be derived. As an introductory physics student you should be familiar with the typical forms of graphs that appear in physics. Below are a number of typical physical relationships exhibited graphically using standard X-Y coordinates (e.g., no logarithmic, power, trigonometric, or inverse plots, etc.). Study the forms of the graphs carefully, and be prepared to use the program Graphical Analysis to formulate relationships between variables by using appropriate curve-fitting strategies. Note that all non-linear forms of graphs can be made to appear linear by “linearizing” the data. Linearization consists of such things as plotting X versus Y^2 or X versus 1/Y or Y versus log (X), etc. Note: While a 5th order polynomial might give you a better fit to the data, it might not represent the simplest model.

1.2.1 Linear Relationship

What happens if you get a graph of data that looks like this? How does one relate the X variable to the Y variable? It’s simple, Y = A + BX where B is the slope of the line and A is the Y-intercept. This is characteristic of Newton’s second law of motion and of Charles’ law:

F= ma

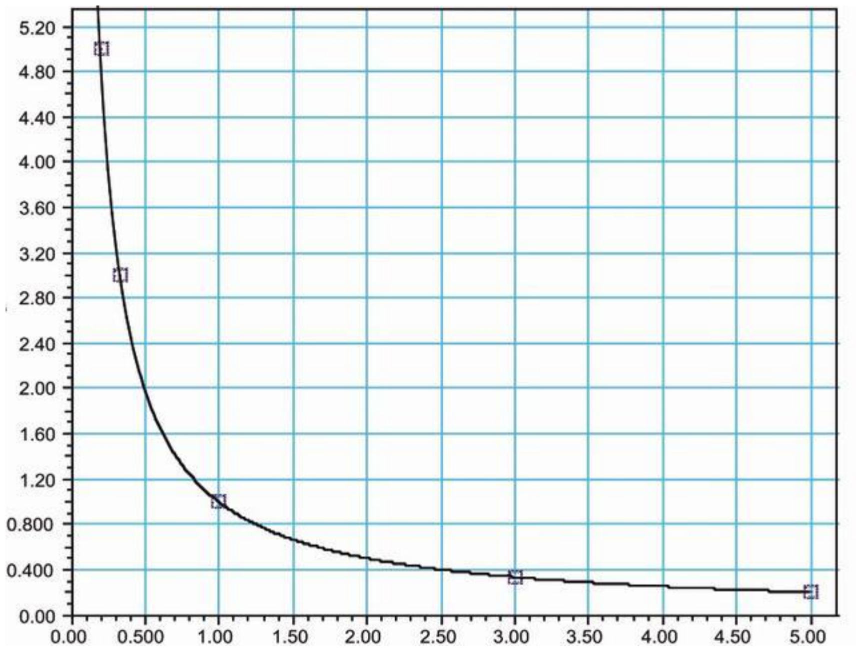
P/T = const.



1.2.2 Inverse Relationship

This might be a graph of the pressure and temperature for a changing volume constant temperature gas. How would you find this relationship short of using a computer package? The answer is to simplify the plot by manipulating the data. Plot the Y variable versus the inverse of the X variable. The graph becomes a straight line. The resulting formula will be Y = A/X or XY = A. This is typical of Boyle's law:

PV = const

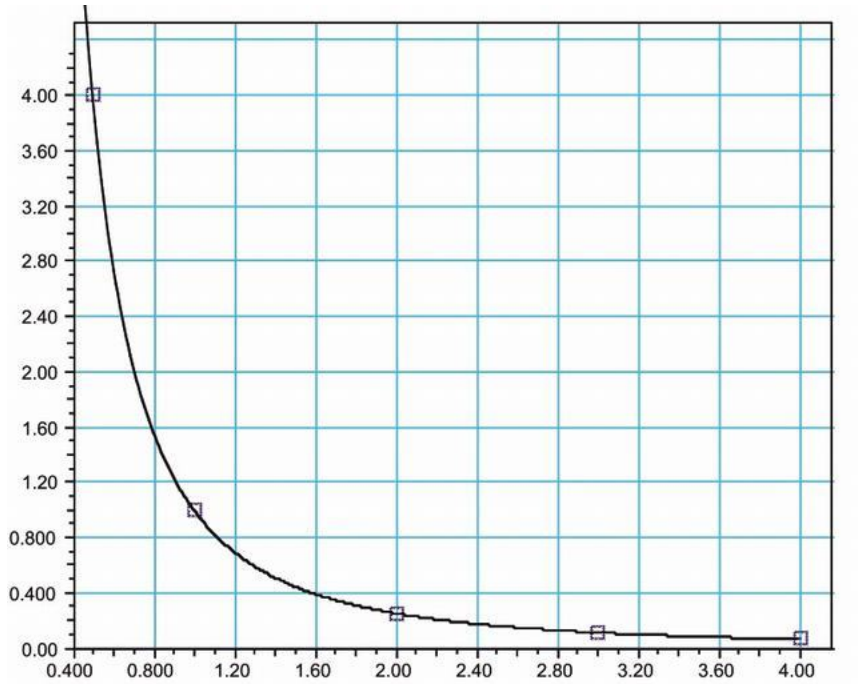


1.2.3 Inverse-Square Relationship

Of the form $Y = A/X^2$. Characteristic of Newton's law of universal gravitation, and the electrostatic force law:

$$F = \frac{Gm_1m_2}{r^2}$$

$$F = \frac{kq_1q_2}{r^2}$$



In the latter two examples above there are only subtle differences in form. Many common graph forms in physics appear quite similar. Only by looking at the "RMSE" (root mean square error provided in Graphical Analysis) can one conclude whether one fit is better than another. The better fit is the one with the smaller RMSE. See below for more examples of common graph forms in physics.

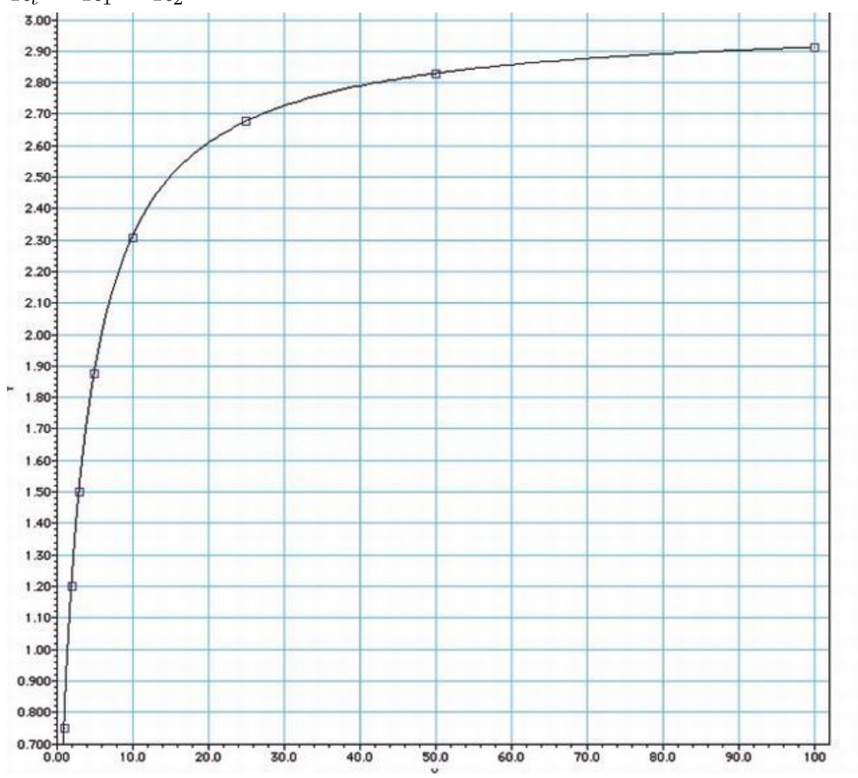
1.2.4 Double-Inverse Relationship

Of the form 1/Y = 1/X + 1/A. Most readily identified by the presence of an asymptotic boundary (y = A) within the graph. This form is characteristic of the thin lens and parallel resistance formulas.

$$\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}$$

and

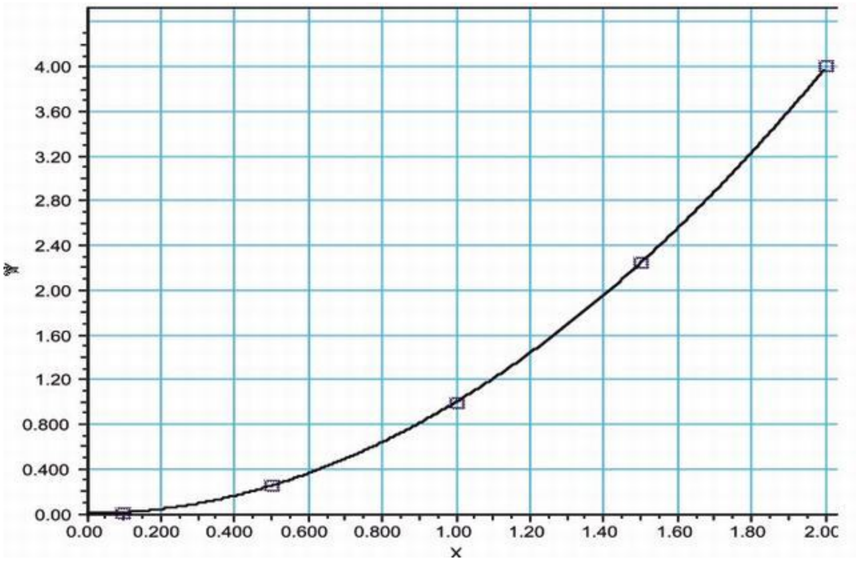
$$\frac{1}{R_t} = \frac{1}{R_1} + \frac{1}{R_2}$$



1.2.5 Power Relationship

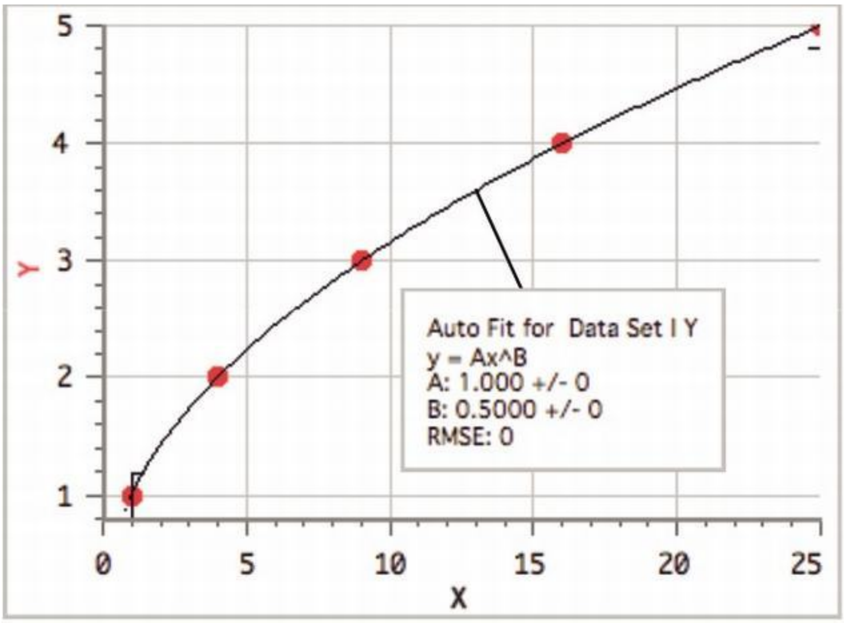
Top opening parabola. Of the form $Y = AX^2$. Typical of the distance-time relationship:

$$d = \frac{1}{2}at^2$$



1.2.6 Power Relationship 2

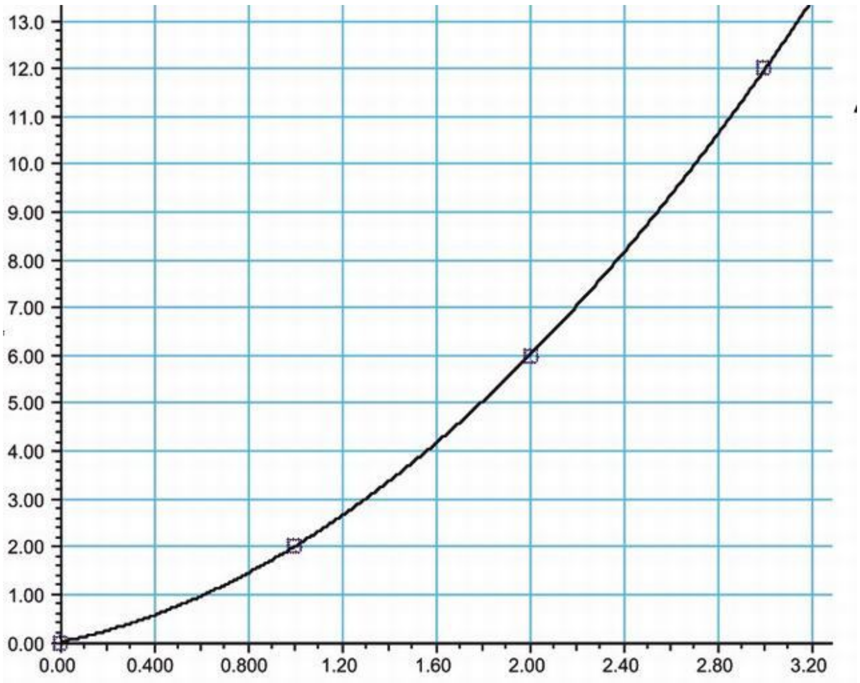
Side opening parabola. Of the form $Y^2 = A_1X$ or $Y = A_2X^{1/2}$. Typical of the simple pendulum relationship: $P^2 = k_1l$ or $P = k_2\sqrt{l}$



1.2.7 Polynomial of Second Degree

Of the form $Y = AX + BX^2$. Typical of the kinematics equation:

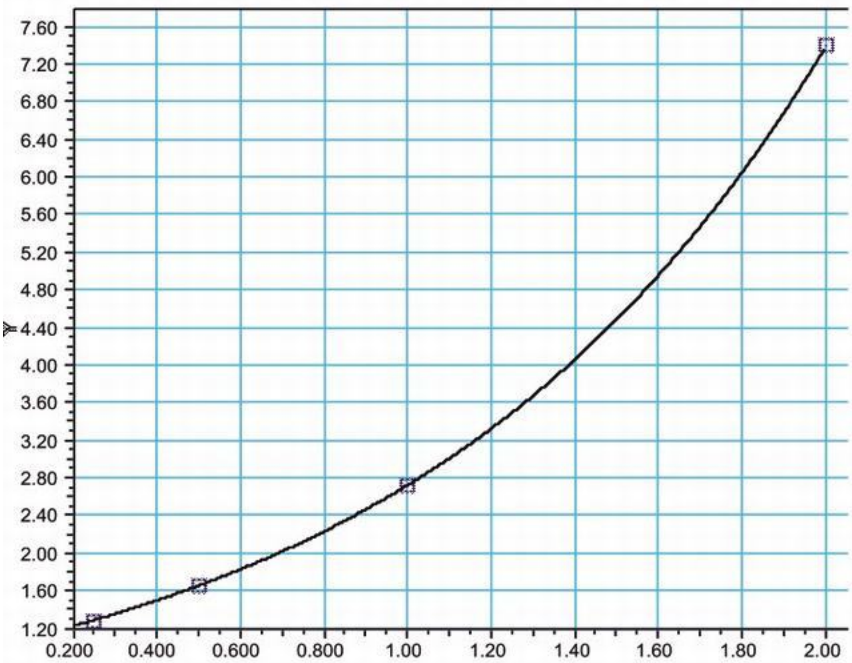
$$d = v_ot + \frac{1}{2}at^2$$



1.2.8 Exponential Relationship

Of the form Y = A * exp(BX). Characteristic of exponential growth or decay. Graph to left is exponential growth. The graph of exponential decay would look not unlike that of the inverse relationship. Characteristic of radioactive decay.

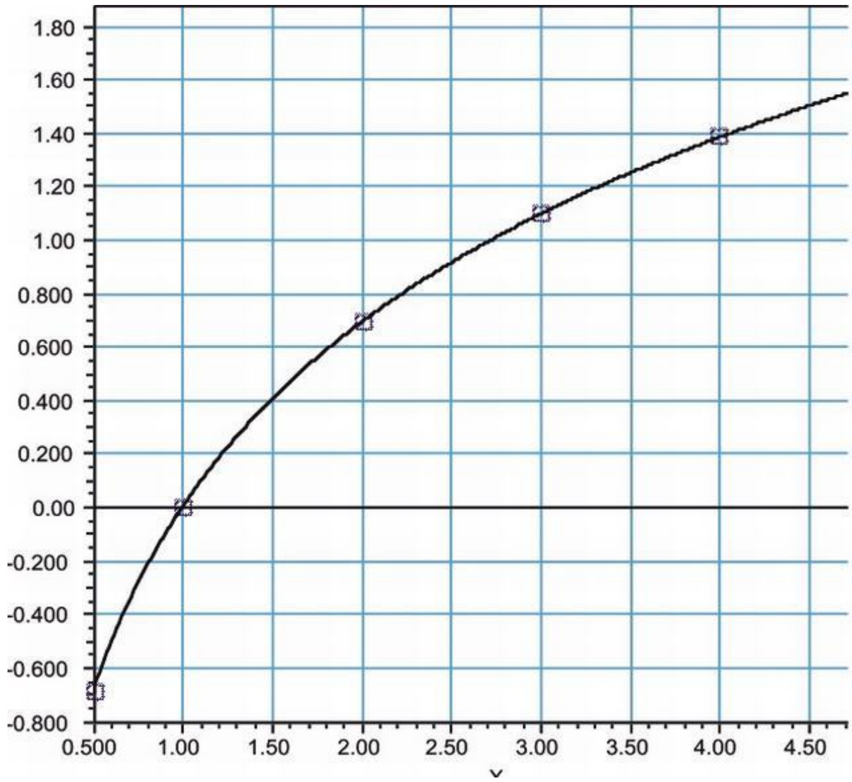
$N = Noe^{-\lambda t}$



1.2.9 Natural Log (LN) Relationship

Of the form Y = A ln(BX). Characteristic of entropy change during a free expansion:

$S_f - S_i = nRln\,(V_f/V_i)$


1.80
1.60
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Chapter 2

Mechanics

2.1 Kinematics

2.1.1 The Equations of motion and the origin of Graph Handling

2.1.1.1 The First Equation

The Equation $v = \dfrac{dx}{dt}$ in linear motion implies

i) The **Slope** of **Position-Time Graph** is **Instantaneous Velocity.**

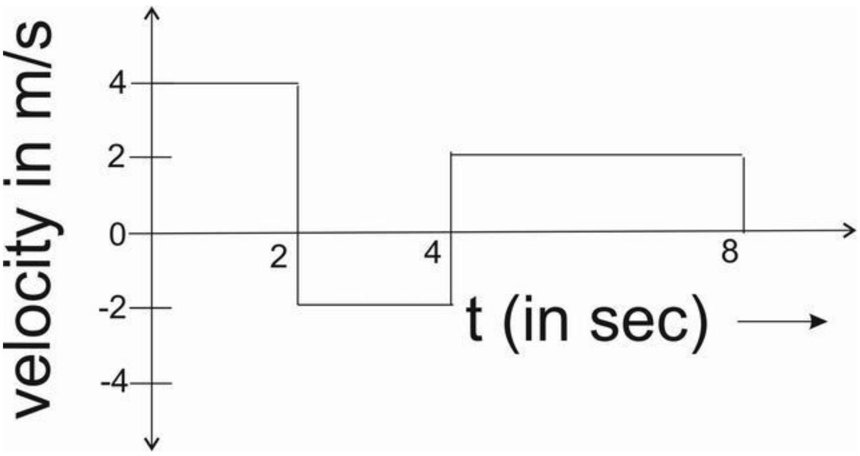
ii) The **Area** under the **Velocity-Time Graph** is **Change in Position**.

{ The second one requires the manipulation , $dx = vdt$ i.e. $\int dx = \int vdt$ }

The equations can be further manipulated to obtain the Speed Time Graph , where

speed = rate of change of distance wrt time

Example : A body is moving in a straight line as shown in velocity-time graph. The displacement and distance travelled by body in 8 second are respectively:



a) 12 m, 20 m

b) 20 m, 12 m

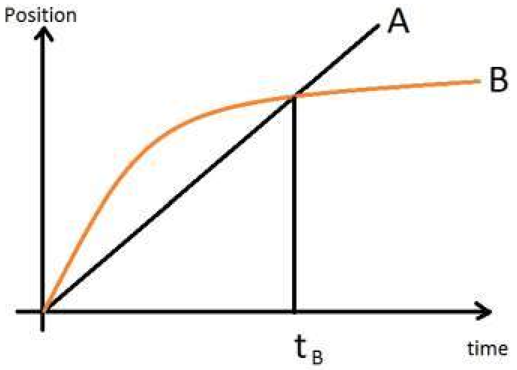
c) 12 m, 12 m

d) 20 m, 20 m

{ Hint: The displacement in a velocity-time graph is given by the area under the graph with proper signs. From 0s - to 2s , the area is 8m . From 2s - to 4s , the area is -4m . From 4s - to 8s , the area is 8m. Adding these 3 values , we get 8m + (-4m) + 8m = 12m. } The distance in a v-t graph is given by the absolute area under the graph. So, taking the absolute values of individual area divisions, we get 8m + 4m + 8m =20m

Answer: a) is the correct answer. }

Example : The graph shows position as a function of time for two trains running on parallel tracks. Which statement is true?



a) At time t_B both trains have the same velocity.

b) Both trains have the same velocity at some time after t_B .

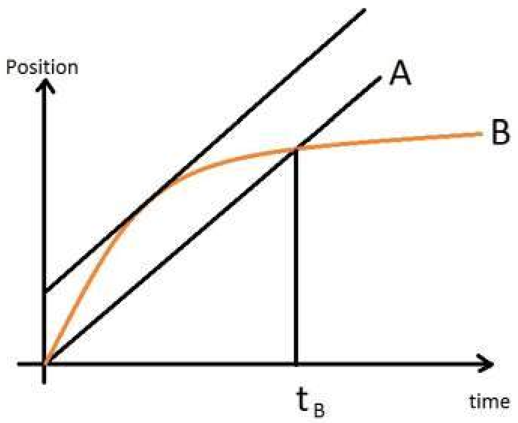
c) Both trains have the same velocity at some time before t_B .

d) Somewhere on the graph, both trains have the same acceleration.

{ Hint: Depending on the question requirements, we'll have to check all the assertions one by one.

a) In a position time graph, the slope gives velocity. It can be clearly seen that Graph B has a much lower slope than Graph A at time t_B. So, the assertion is wrong.

b,c) By drawing a line parallel to the line A which is a tangent to Graph B , it can be seen where the two graphs have same slope. It is clear that the graphs have same slope between 0 and t_B as noted from the figure. So, assertion b is wrong while c is correct.

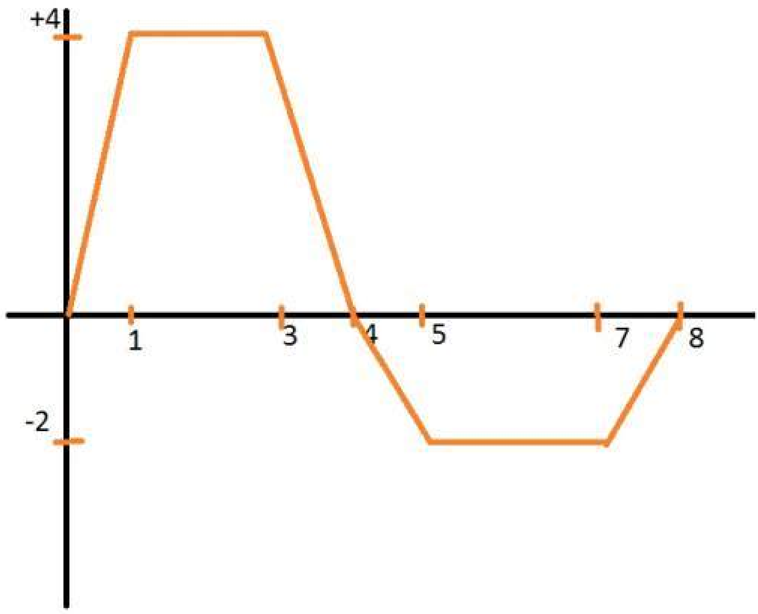


d) As the Graph A has a constant slope, so the acceleration of body A is zero. Whereas Graph B is constantly turning, so the slope can be assumed to be non-zero throughout. According to some revelations, however it is noted that the figure is not clear enough to show whether Graph B is straight after t_B or bending. In case it is assumed to be straight, then after t_B both trains will have same (zero) acceleration. Also at start both have large (infinite) accelration, in which case the ratio of the two large (infinite) values may be calculated if initial conditions are mentioned and is required.

At our level we would assume this assertion to be wrong, however making a note that the image should have been more clearly presented.

Answer: c) is the correct assertion. }

Example : The velocity-time graph of a particle in linear motion is as shown. Both v and t are in SI units. The displacement of the particle is



a) 6 m

b) 8 m

c) 16 m

d) 18 m

{ Hint : For displacement calculations, between 0 - to 4 , area of the positive trapesium = (4+2)X4 = 24

between 4 - to 8, area of negative trapesium = (2+4)X(-2) = -12.

So , the answer is +12 , which is not in the options.

So , the anser is None of these. }

2.1.1.2 The Second Equation

Proceeding similar to above, the equation $a = \frac{dv}{dt}$ implies

i) The **Slope** of **Velocity-Time Graph** is **Instantaneous Acceleration**.

ii) The **Area** under **Acceleration-Time Graph** is **Change in Velocity**.

{ The second one requires the manipulation , $dv = adt$ i.e. $\int dv = \int adt$ }

A few of the following examples illustrate it.

Example : A car starts from rest acquires a velocity v with uniform acceleration $2ms^{-2}$ then it comes to stop with uniform retardation $4ms^{-2}$. If the total time for which it remains in motion is 3 sec, the total distance travelled is:

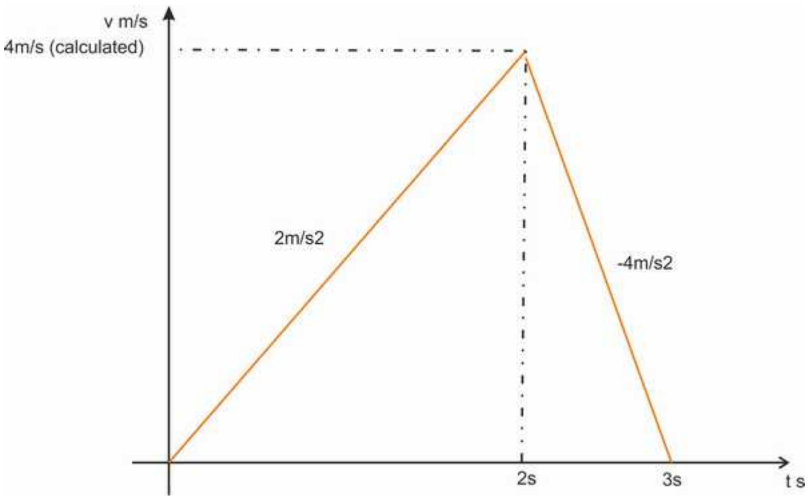
a) 2 m

b) 3 m

c) 4 m

d) 6 m

{Hint: For solving this problem, we draw the graph of the problem,



Accordning to graph, let the time when it reaches maximum velocity be T, and the maximum velocity be V.

⇒V = 2XT and also V = 4X(3-T)

Equating the equations,

2T = 12-4T = V

⇒6T = 12

⇒T = 2

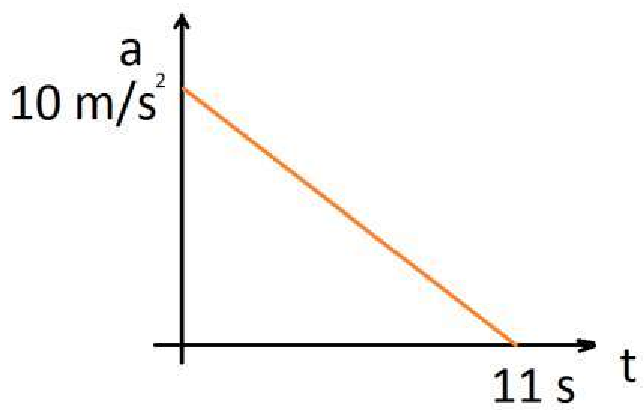
⇒ V = 2T = 4

Calculating the area under the graph using the calculated parameters, Area = 1/2 X 4 X 3 = 6m

So , area under the graph is 6m = displacement . Also, as all the area is on the positive side, so distance = 6m.

}

Example : A particle starts from rest. Its acceleration (a) vs time (t) is as shown in the Figure. The maximum speed of the particle will be



a) 110 m/s

b) 55 m/s

c) 550 m/s

d) 660 m/s

{ Hint : Writing the equation of the graph , we get $\frac{a}{10} + \frac{t}{11} = 1$

$\Rightarrow a = \frac{10}{11}(11 - t)$

Integrating, (we will assume initial velocity to be zero as the body starts from rest.)

$v = \frac{10}{11}(11t - \frac{1}{2}t^2)$

Substituting $t = 11s$

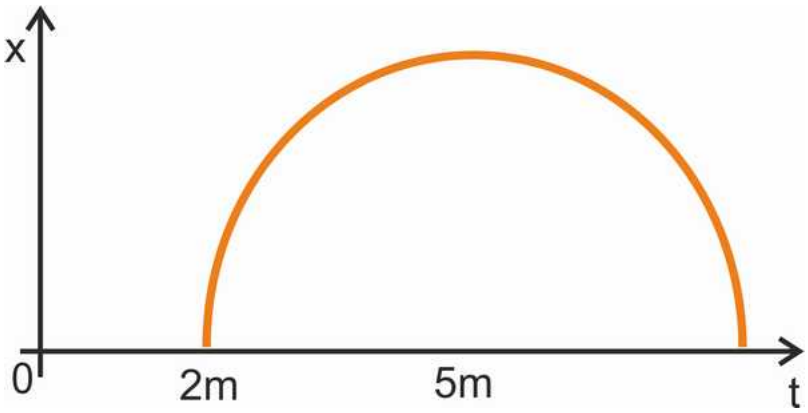
$v_{11s} = 55m/s$

Answer: b) is the correct answer }

2.1.1.3 The Average-Velocity / Instantaneous Velocity , Equal Case

We know , that (in a x-t graph) the slope of the Secant is the Average Velocity , whereas the slope of Tangent is the Instaneous Velocity. The point where these two lines coincide, is the point where Average Velocity is equal to Instantaneous Velocity.

Example : Position-time graph is shown which is a semicircle from t = 2 to t = 8 s. Find time t at which the instantaneous velocity is equal to average velocity over first t seconds,



a) 4.8 s

b) 3.2 s

c) 2.4 s

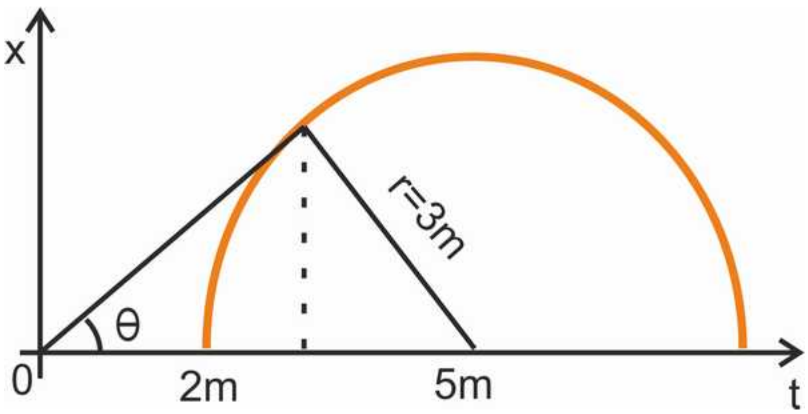
d) 5 s

{ Hint: The tangent from 0 to the circle is drawn. It's normal passes through the center of the circle. Time at this instant needs to be calculated.

If H=5 , R = 3 , Length of tangent = 4. (By Pythagoras.)

Angle which the tangent makes with the t axis is $\theta = sin^{-1}(3/5)$

So, the projection of tangent on t axis (i.e. the required time) = 4 cos $\theta = 4 \times \frac{4}{5} = 3.2$



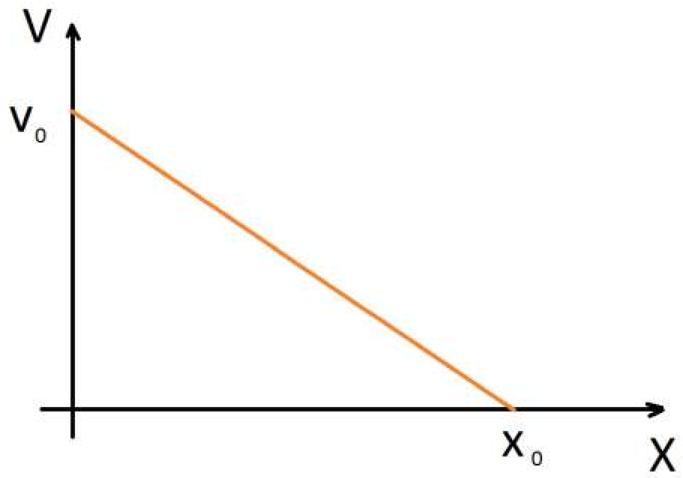
}

2.1.1.4 The Velocity-Displacement Case

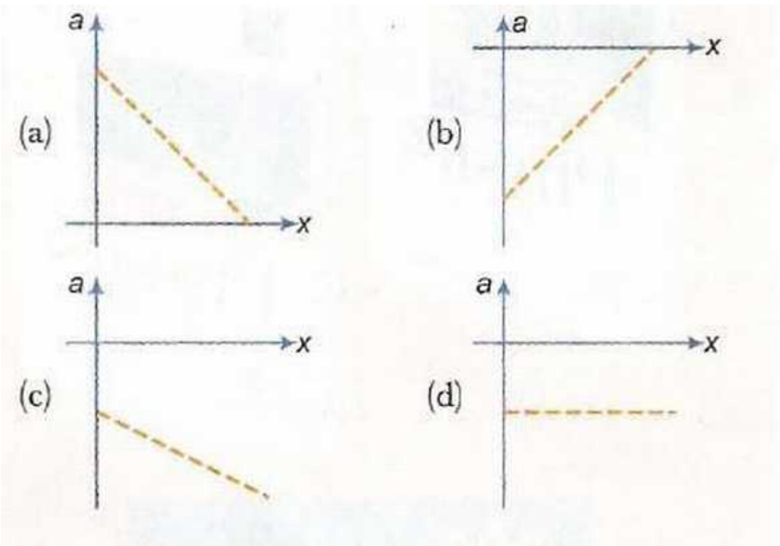
This can be handled in a similar way as Acceleration-Displacement case by integrating the respective equation. Here the problem is of v = f(x) type, which can be integrated by writing $\frac{dx}{dt} = f(x)$

i.e. dx = f(x)dt

Example : The velocity-displacement graph of a particle moving along a straight line is shown here.



The most suitable acceleration-displacement graph will be



{Hint: Using Co-Ordinate Geometry Result studied in +1 Mathematics, we get the equation of the graph

$\frac{v}{v_o} + \frac{x}{x_o} = 1$

We are supposed to find the a-x graph from this.

So, we rewrite this equation as $v = v_o(1 - \frac{x}{x_o})$

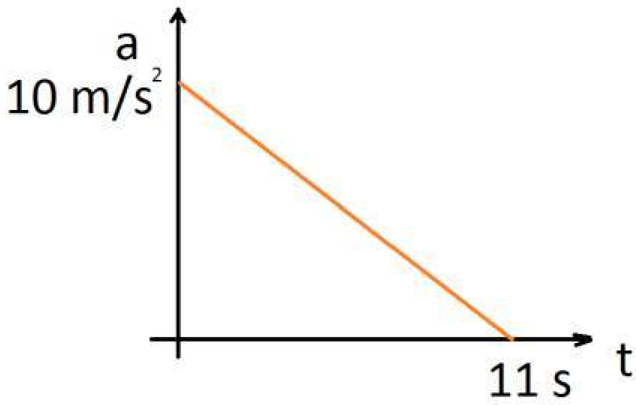
Differentiating, we get $a = -\frac{v_o}{x_o}v = -\frac{v_o^2}{x_o}(1 - \frac{x}{x_o})$

Hence b) is the requisite graph, the only graph with a +ve slope , a negative y intercept and a positive x intercept.

Answer: b) is the correct answer. }

2.1.2 Previous Years IIT Problems

Q1: A particle starts from rest. Its acceleration (a) versus time (t) is as shown in the gure. The maximum speed of the particle will be



a) 110 m /s

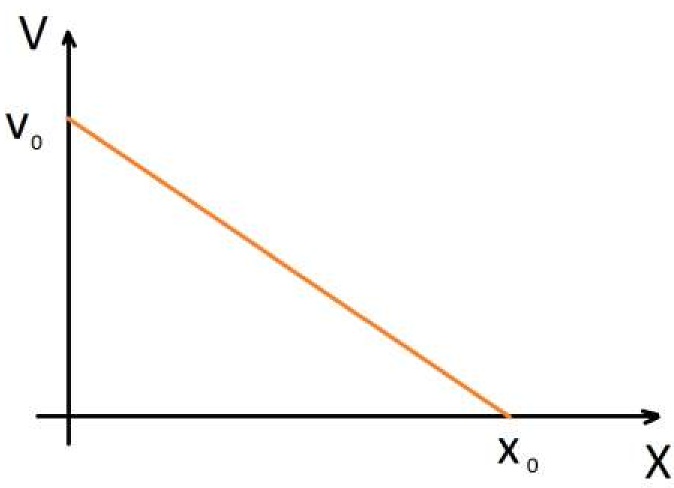
b) 55 m /s

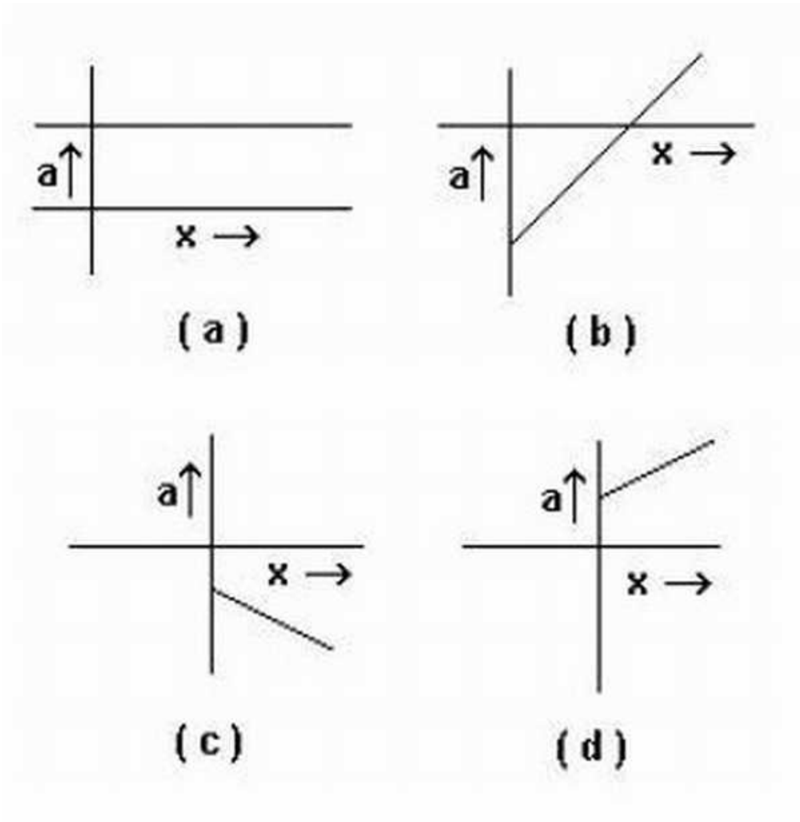
c) 550 m /s

d) 660 m /s

{ Hint : See In chapter examples for solution. }

Q2: If graph of velocity vs. distance is as shown, which of the following graphs correctly represents the variation of acceleration with displacement ?





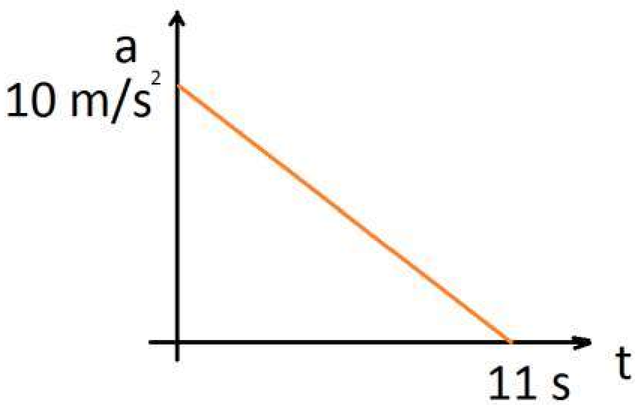
{ Hint : The graph of the question is a straight line with the equation, $\frac{x}{x_o} + \frac{v}{v_o} = 1$

This gives , $v = v_o(1 - x/x_o)$

So, diffrentiating it, we get

$a = -v_o/x_o$ which is a constant. Only in a) it is shown to be a constant.

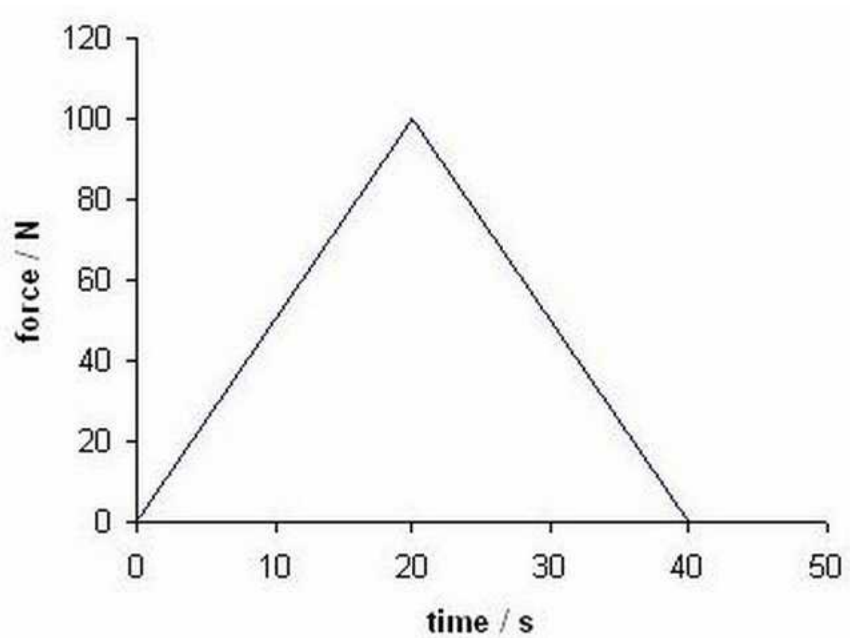
So, a) }



2.2 Laws of Motion

2.2.1 Abstract Introduction

2.2.1.1 Force – Time Graphs



The area under a force – time graph gives us the impulse of the force applied (and hence the change in momentum of the object). For this graph the impulse (the area under the graph) is 2000 kg ms^{-1}. n.d.

2.2.1.2 Change in Momentum or the "Impulse"

The change in the momentum of a system (or the impulse delivered by the net force) is given mathematically by the Momentum Principle,

$\triangle\overrightarrow{p} = \overrightarrow{F_{net}}\triangle t$ n.d.

In this form, the change in momentum is calculated over a "discrete" time step. That is, the calculation is done over a known or determined time interval. If the force is non-constant (i.e., depends on location or velocity), this calculation is not exact. In fact, in this case, the net force is the average net force over the time interval. So that a better definition is this:

$\triangle\overrightarrow{p} = \overrightarrow{F_{net,avg}}\triangle t$

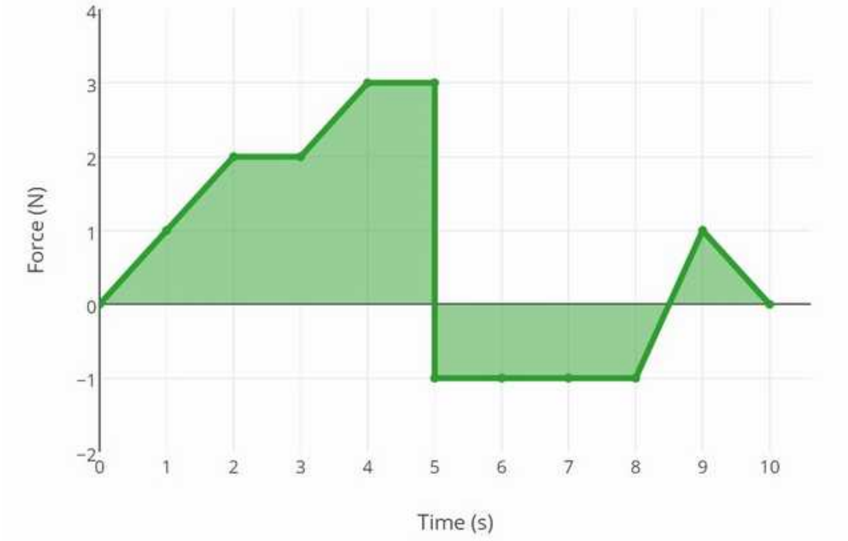
This definition works well for case where you might use iterative procedures to determine the change in momentum over small time intervals. If on the other hand, you can analytically integrate the force (e.g., it is or can be put into a form which is time dependent), then you can use the derivative form of the Momentum Principle,

$\Delta\overrightarrow{p} = \int_{t_i}^{t_f}\overrightarrow{F}_{net}dt$

In any event, either (or both) can be useful to think about graphs of force vs time.

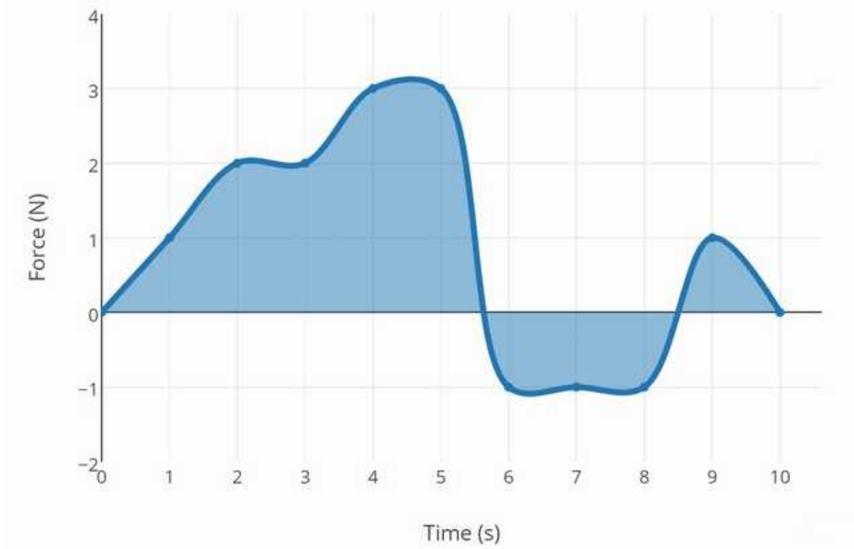
Force vs Time Graphs In some situations, it is easier to empirically measure force versus time graphs because the situations lend themselves more easily to these empirical measurements rather than what might be more complex physical theories. This is true in different engineering contexts (e.g., impact design and the flow of fluids). In these cases, you are interested in determining the change in momentum (and thus the velocity) of the system in question1).

Below is a force vs time graph where the "area under the curve" has been highlighted. In this example, we are only looking at the component of the net force in the xx-direction. Such graphs can be produced for each component of the net force, but let's say that for this system, there was a non-zero component of the net force only in the x-direction.



For the above figure, the momentum change over the complete time interval can be determined in a straightforward way due to the simple geometric shapes produced. Area above the zero line are positive momentum changes, and area below are negative. By adding up the “area under the curve” in this way, we obtain a momentum change of 7 Ns.

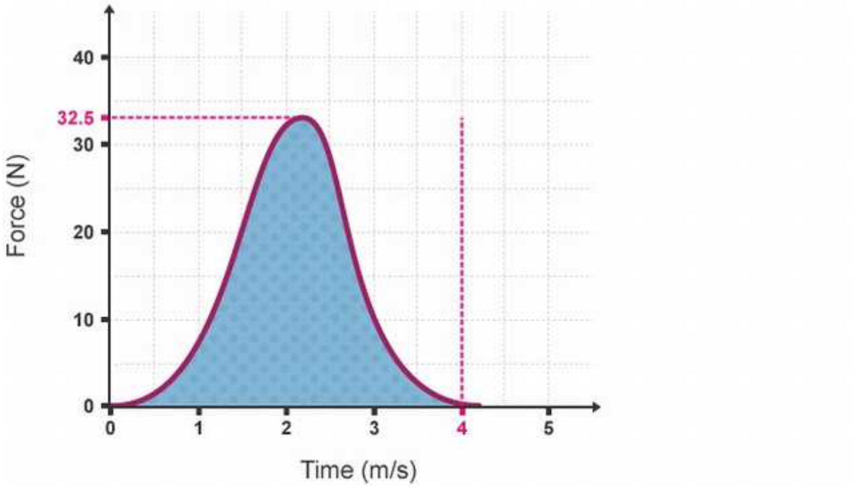
The figure below shows the force vs time graph for another system. In this case, the graph has a smooth form, which doesn't appear to be analytic. The “area under the curve” for this graph could be analyzed computationally, by taking small steps (i.e., Riemann Sum), and the change in momentum could be determined.



1) It is possible to determine the displacement of such systems as well. This can be done using velocity vs time graphs that are produced from the analysis of force vs time graphs.

2.2.1.3 Impulse graphs (A Case Study) n.d.

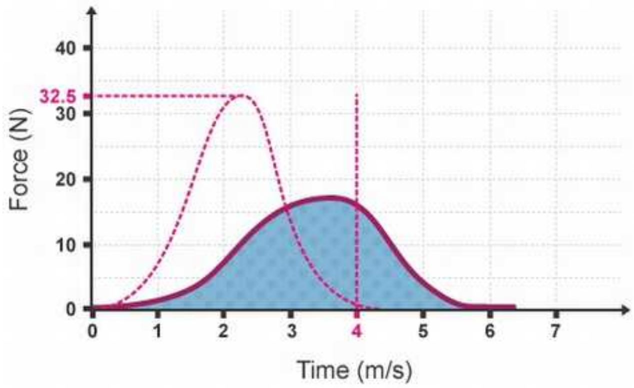
The force on the squash ball in the previous question is an average force and often the force changes during the collision. For this example the force–time graph could look like this.



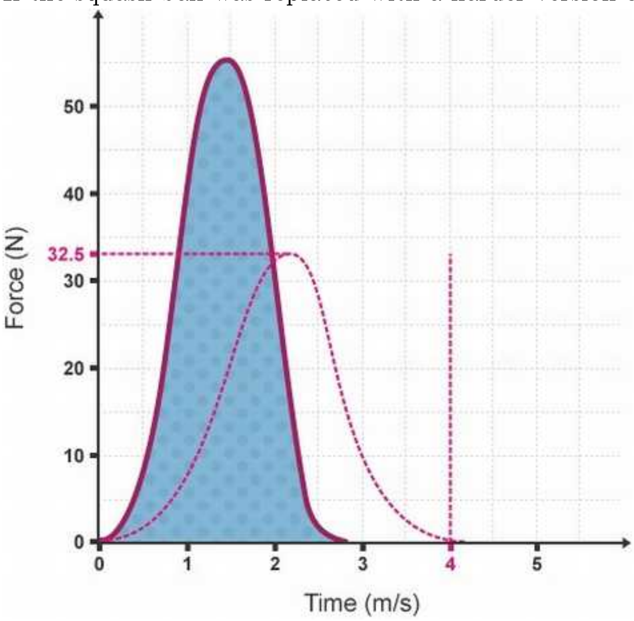
Notice the peak force is greater than the average force calculated.

The area under a force time graph is equal to the impulse. For any collision with a fixed change in momentum, if the time of contact can be increased, the peak force is reduced:

For example if the squash ball was replaced with a softer version of same mass the collision graph would look like this:



If the squash ball was replaced with a harder version of same mass the collision graph would look like this:



Question Modern cars are designed to crumple on impact in a collision. How does this help to protect the occupants from harm?

Answer The change in momentum (area under the force time graph) can't be changed at the time of the accident (mass is fixed and it is too late for the driver to slow down!) By increasing the time of collision the peak force is less and hopefully lets the occupants come to less harm as a result.

Question : How do I find "Velocity" from "Force vs. Time" graph?

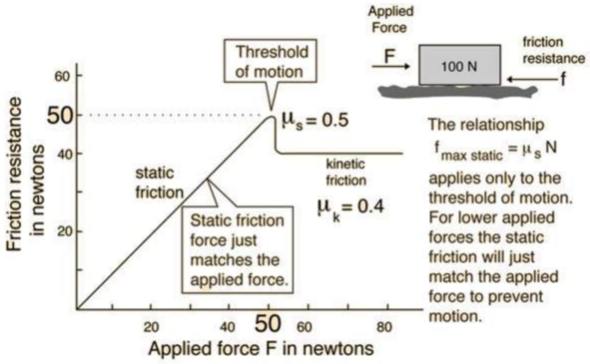
Solution : You need two additional pieces of information: the mass of the object and its initial velocity. Given those, the relation is:

$v(t) = v_o + \frac{1}{m}\int_0^t F(t)dt$

2.2.2 Friction

2.2.2.1 Static Friction 2017

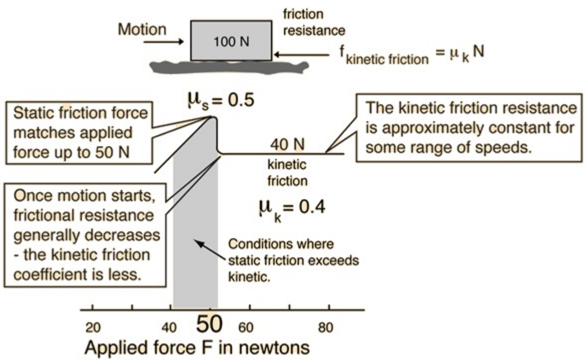
Static frictional forces from the interlocking of the irregularities of two surfaces will increase to prevent any relative motion up until some limit where motion occurs. It is that threshold of motion which is characterized by the coefficient of static friction. The coefficient of static friction is typically larger than the coefficient of kinetic friction.



In making a distinction between static and kinetic coefficients of friction, we are dealing with an aspect of "real world" common experience with a phenomenon which cannot be simply characterized. The difference between static and kinetic coefficients obtained in simple experiments like wooden blocks sliding on wooden inclines roughly follows the model depicted in the friction plot from which the illustration above is taken. This difference may arise from irregularities, surface contaminants, etc. which defy precise description. When such experiments are carried out with smooth metal blocks which are carefully cleaned, the difference between static and kinetic coefficients tends to disappear. When coefficients of friction are quoted for specific surface combinations are quoted, it is the kinetic coefficient which is generally quoted since it is the more reliable number.

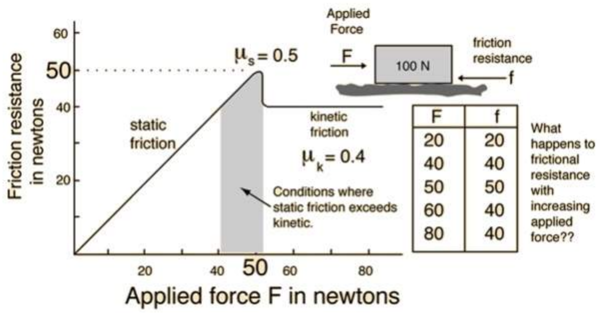
2.2.2.2 Kinetic Friction

When two surfaces are moving with respect to one another, the frictional resistance is almost constant over a wide range of low speeds, and in the standard model of friction the frictional force is described by the relationship below. The coefficient is typically less than the coefficient of static friction, reflecting the common experience that it is easier to keep something in motion across a horizontal surface than to start it in motion from rest.

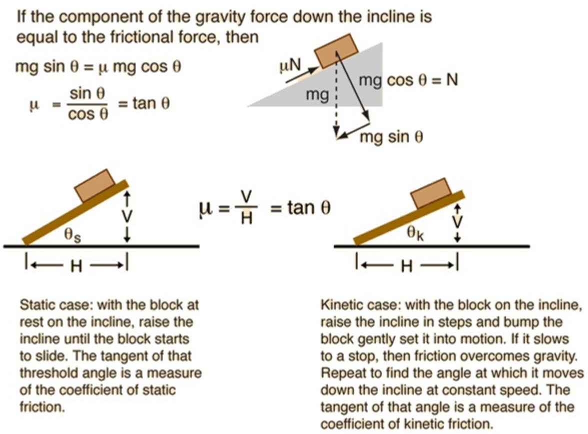


2.2.2.3 Friction Plot

Static friction resistance will match the applied force up until the threshold of motion. Then the kinetic frictional resistance stays about constant. This plot illustrates the standard model of friction.



The above plot, though representing a simplistic view of friction, agrees fairly well with the results of simple experiments with wooden blocks on wooden inclines. The experimental procedure described below equates the vector component of the weight down the incline to the coefficient of friction times the normal force produced by the weight on the incline.



s

Having taken a large number of students through this experiment, I can report that the coefficient of static friction obtained is almost always greater than the coefficient of kinetic friction. Typical results for the woods I have used are 0.4 for the static coefficient and 0.3 for the kinetic coefficient.

When carefully standardized surfaces are used to measure the friction coefficients, the difference between static and kinetic coefficients tends to disappear, indicating that the difference may have to do with irregular surfaces, impurities, or other factors which can be frustratingly non-reproducible. To quote a view counter to the above model of friction: "Many people believe that the friction to be overcome to get something started (static friction) exceeds the force required to keep it sliding (sliding friction), but with dry metals it is very hard to show any difference. The opinion probably arises from experiences where small bits of oil or lubricant are present, or where blocks, for example, are supported by springs or other flexible supports so that they appear to bind." R. P. Feynman, R. P. Leighton, and M. Sands, The Feynman Lectures on Physics, Vol. I, p. 12-5, Addison-Wesley, 1964.

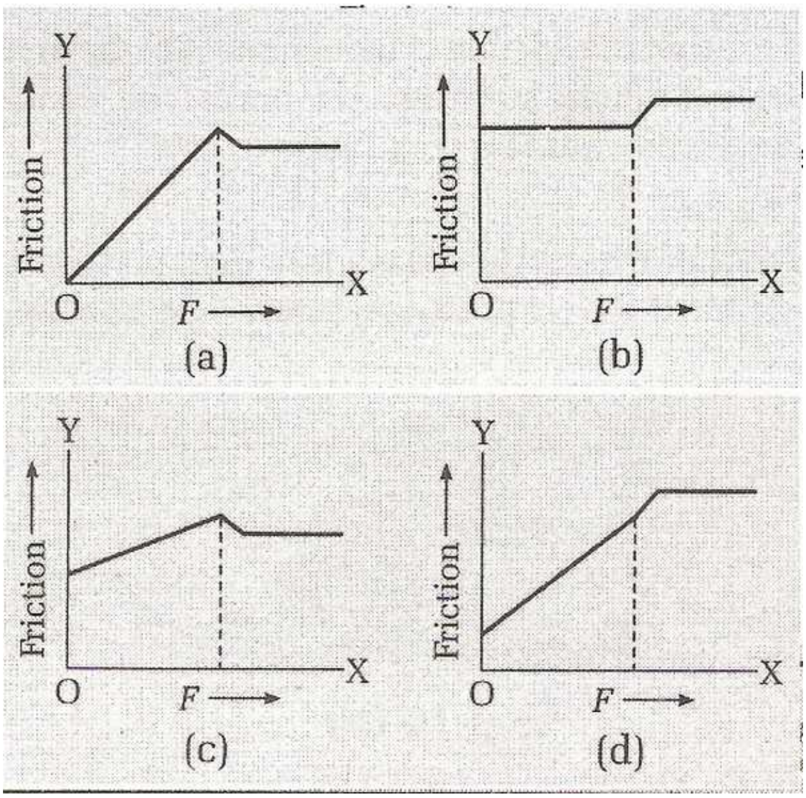
2.2.2.4 Rolling Friction

A rolling wheel requires a certain amount of friction so that the point of contact of the wheel with the surface will not slip. The amount of traction which can be obtained for an auto tire is determined by the coefficient of static friction between the tire and the road. If the wheel is locked and sliding, the force of friction is determined by the coefficient of kinetic friction and is usually significantly less.

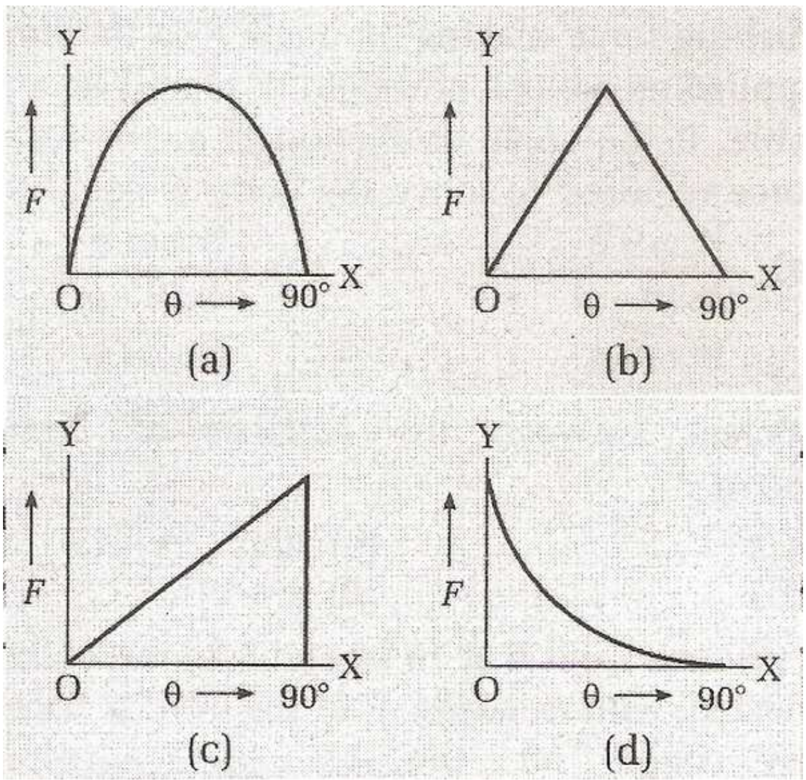
Assuming that a wheel is rolling without slipping, the surface friction does no work against the motion of the wheel and no energy is lost at that point. However, there is some loss of energy and some deceleration from friction for any real wheel, and this is sometimes referred to as rolling friction. It is partly friction at the axle and can be partly due to flexing of the wheel which will dissipate some energy. Figures of 0.02 to 0.06 have been reported as effective coefficients of rolling friction for automobile tires, compared to about 0.8 for the maximum static friction coefficient between the tire and the road.

2.2.2.5 Few problems related to Friction

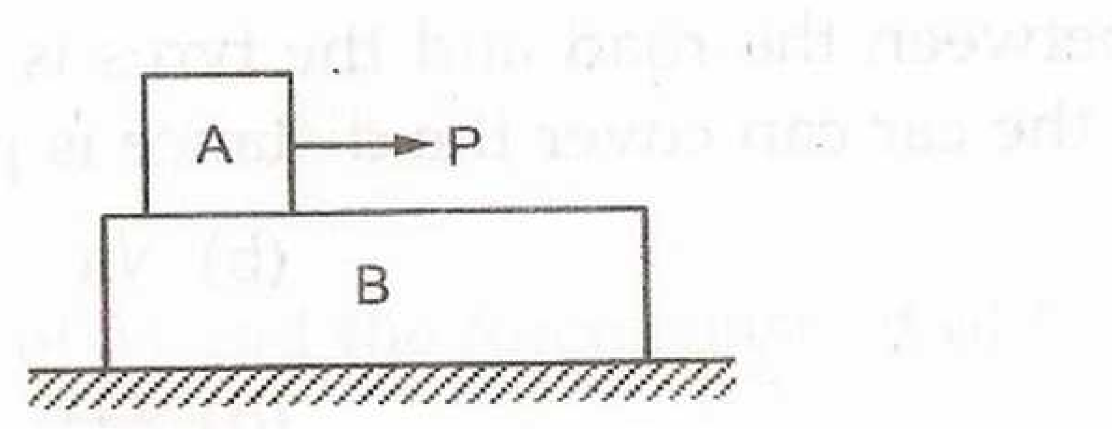
Example : A block on the horizontal table is acted upon by a force F. The graph of frictional force against F is



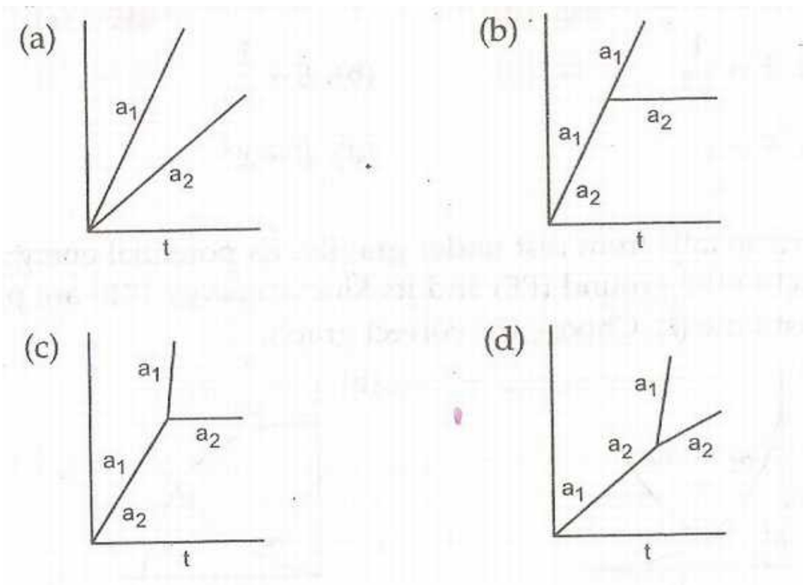
Example: A block rests on a rough plane whose inclination θ to the horizontal can be varied. Which of the following graphs indicates how the frictional force F between the block and plane varies as θ is increased ?



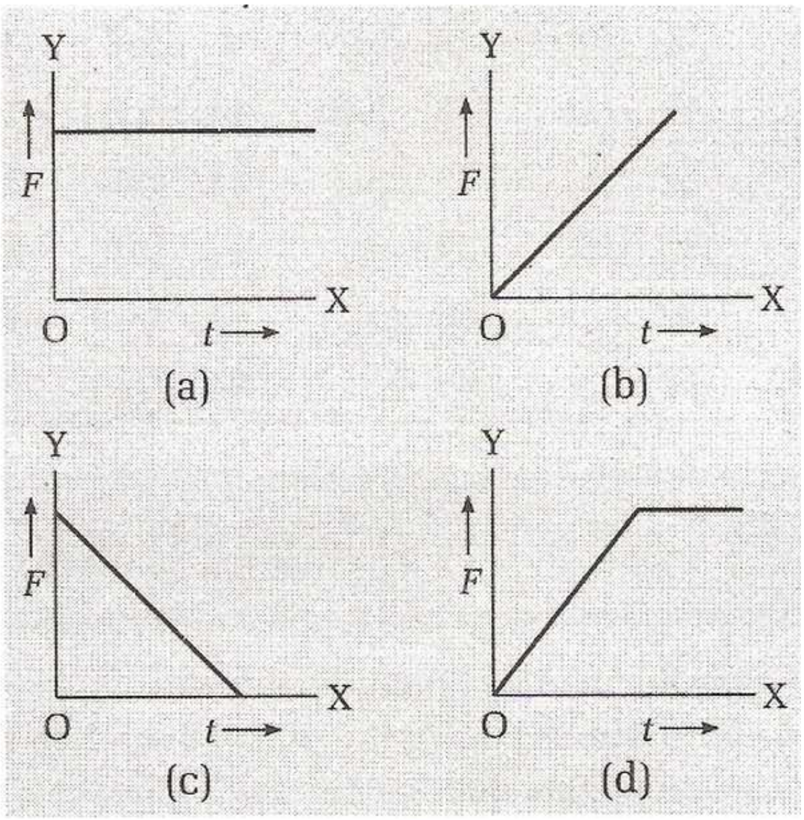
Example : Block A is placed on block B, whose mass is greater than that of A.



There is friction between the blocks, while the ground is smooth. A horizontal force P, increasing linearly with time, begins to act on A. The accelerations a_1 and a_2 of A and B respectively are plotted against time (t). Choose the correct graph.



Example : A body moves with uniform speed on a rough surface. If force F of dynamic friction is plotted with time t as shown in figure, the graph will be



2.2.3 Theory and Problems

2.2.3.1 Impulse as Force-time Graph

Example : For the graph shown above, assume that it shows a constant force of 25 N acting over a 10 s period of time. Determine the impulse.

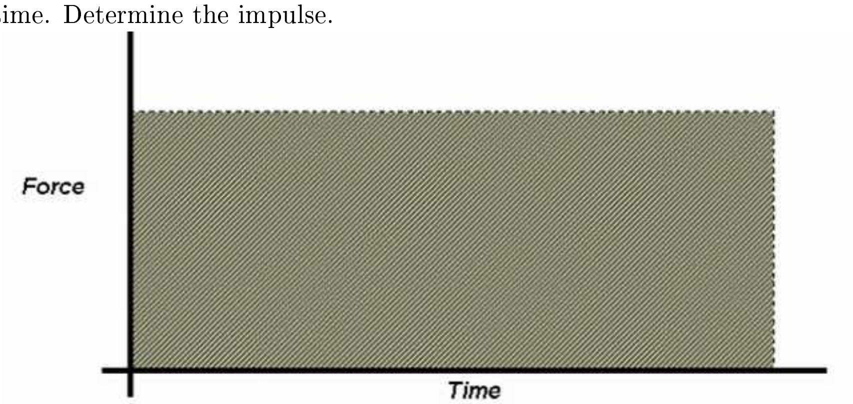


Illustration : Graph of Example (Force as a function of Time)

Solution : (Theory : So far we've implied some things about what is constant and what can change in the impulse formula F Δt = m Δv.

We look at situations where we expect the mass of the object will stay constant. • The velocity will change, and that's why we put a delta in front of it. • Time is changing (sort of) as we measure it over a period of time. • Force must be a constant. We assume that the force being exerted on the object was always the same, causing a constant acceleration. If we are looking at a simple impulse question (where the force is constant), we can figure out exactly what we can interpret from a graph. • Later this may help us to figure out a more complicated question, like if the force changes. The following graph is an example of one of those simple situations where the force remains constant during the entire time. • If we look at what the slope might represent, we get...

slope = rise / run

slope = F / $\triangle$t

Since nothing in the impulse formula can be rearranged to give us force over time, the slope doesn't mean anything to us in this situation.

If we look at the area under the line, we get something a bit better...

Area = lw = F$\triangle$t = $\triangle$p)

Since area under the line is equal to impulse...
Area = lw
Area = 25×10
Area = 2.5e2
p = 2.5e2Ns
If we really wanted to, we could have simply use $\Delta p = F\Delta t$ to figure out the impulse. We could do this in this situation because the force is constant. • If we need to do a question where the force is not constant, we can still use the area under the line to get the impulse, even though the formula $\Delta p = F\Delta t$ can not be used.

Example : I am in a car that is accelerating from rest at a red light. I want to calculate the impulse that is acting on the car during the first 5.78s. If I know that the force on the car steadily increases from 0 N to 3012 N over this time, determine the impulse. If the mass of the car is 1500 kg, also determine the final velocity of the car.

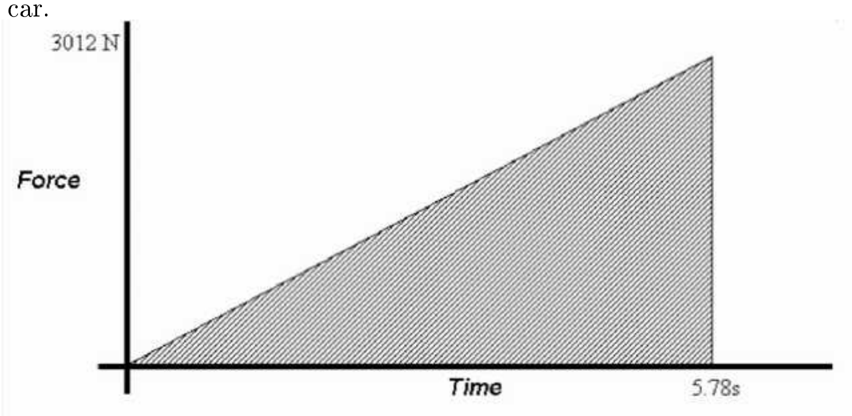


Illustration : Graph for Example (Force as a function of Time)

Solution : Let's start by graphing the information we were given. We will get a nice linear graph, since it said that the force steadily increases.

If we calculate the area under the graph (a triangle) we will know what the impulse is.
$A = \frac{1}{2}$ bh
$= \frac{1}{2}$ (5.78 s)(3012 N)
= 8704.68 A
= 8.70e3 kgm/s

To calculate the final velocity, we can use the value for the impulse we just got with the right hand side of the impulse formula. Remember that the initial velocity (sitting at the light) is zero...
Δ p=mΔv
Δ p=m(v_f−v_i)
Δ p=mv_f
v_f=Δ p m
v_f=8704.68/1500
v_f=5.80312
v_f=5.80m/s

The graph that we make does not have to be a pretty right angle triangle either. We can also do some crazy stuff with what we are looking for in the question, as the next example shows.

Example : This graph shows the result of applying 500 kgm/s of impulse to an object as it moved across the floor for 10.0 s. Determine the maximum force that was exerted.

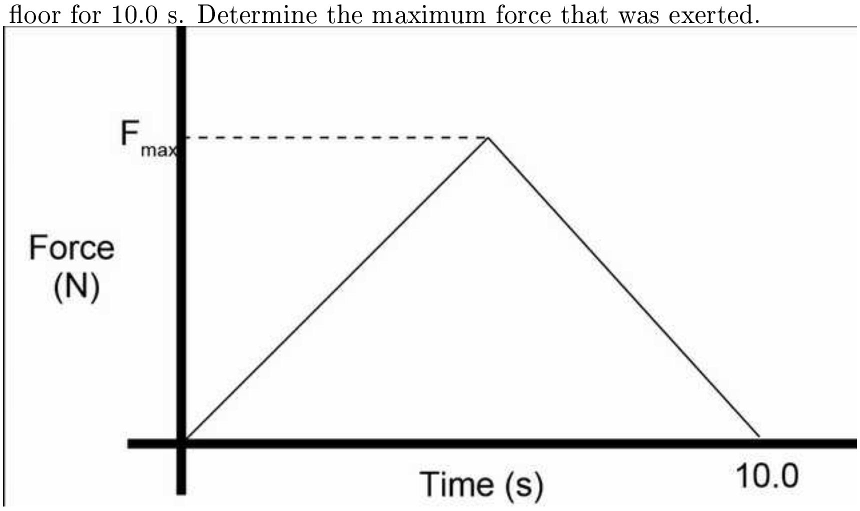


Illustration 3: Pushing object across the floor.

Solution : Even though it is not a right angle triangle, this graph still shows a triangle that we can use the regular area formula with. In this case, we already know the area (the impulse is 500 kgm/s) and we know the base (10.0 s). All we want is the height of the triangle, since that is the magnitude of the maximum force.

Area=bh/2

$\triangle$p=F$\triangle$t/2

F=2 $\triangle$p/ t = 2×500/10.0 F=100N

Even if the graph is a curved line, you can still at least estimate the area under the graph. • Although this will only be an approximate area, without getting into calculus it's as good as you'll get and as good as you need. ∘ On the graph shown below we have an s-curve that would be difficult to calculate the exact area of. ∘ Instead, we just look at the triangle drawn in red. For the little bit extra it has near the beginning, it misses a bit later on. These two parts should more or less make up for each other, so that the area of the triangle will be about the same as the area under the curve.

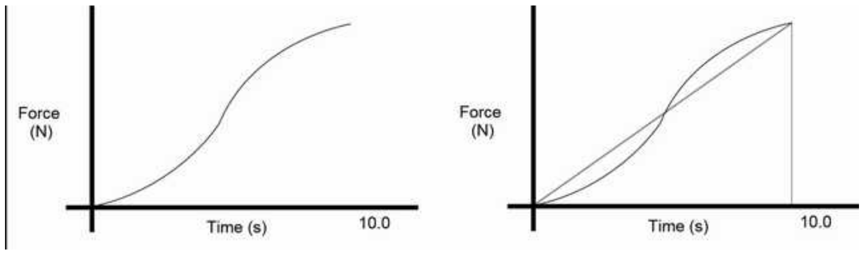
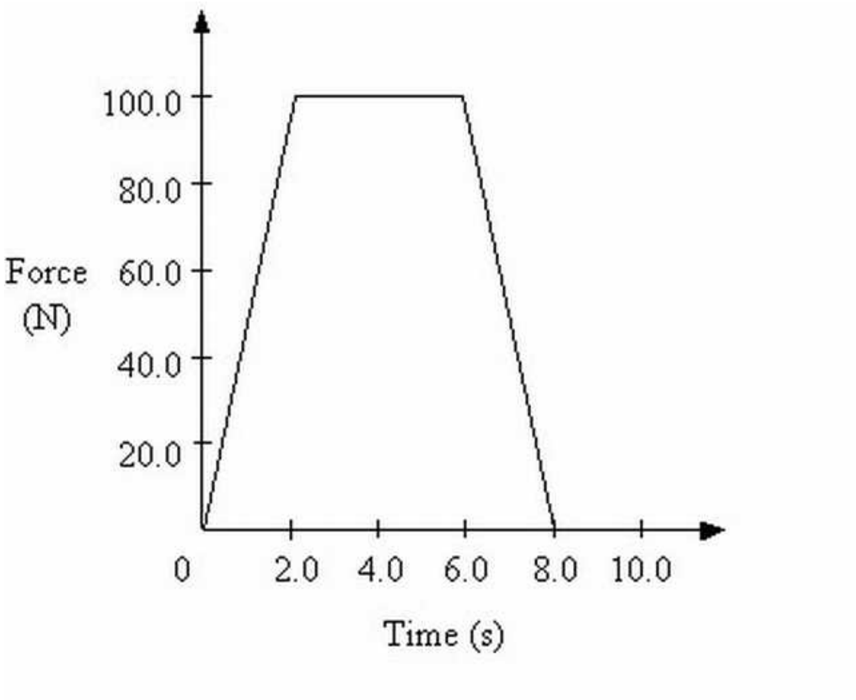


Illustration 4: Instead of trying to figure out the area of the curve exactly, we just use the area of the triangle as an approximation.

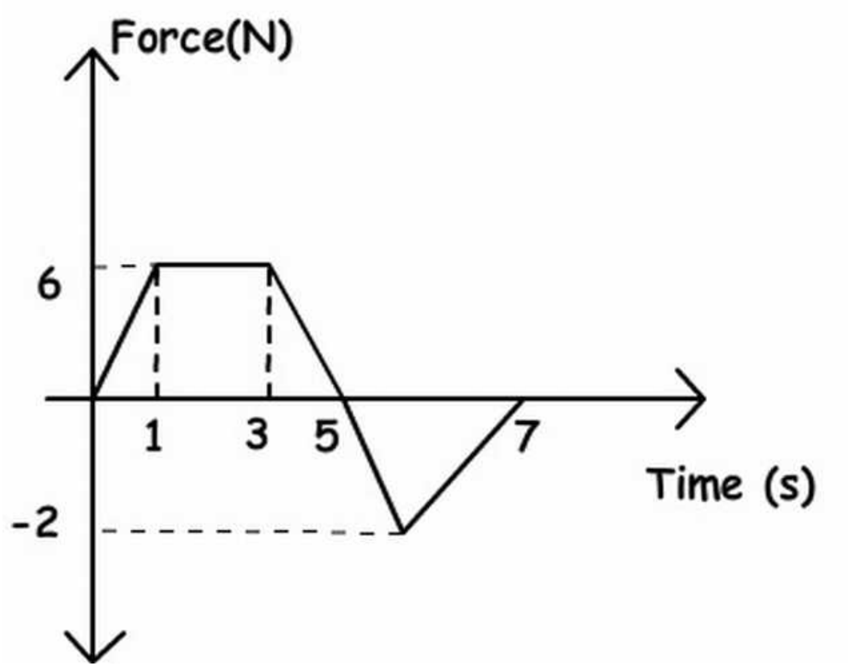
2.2.3.2 Force Time graph with respect to momentum



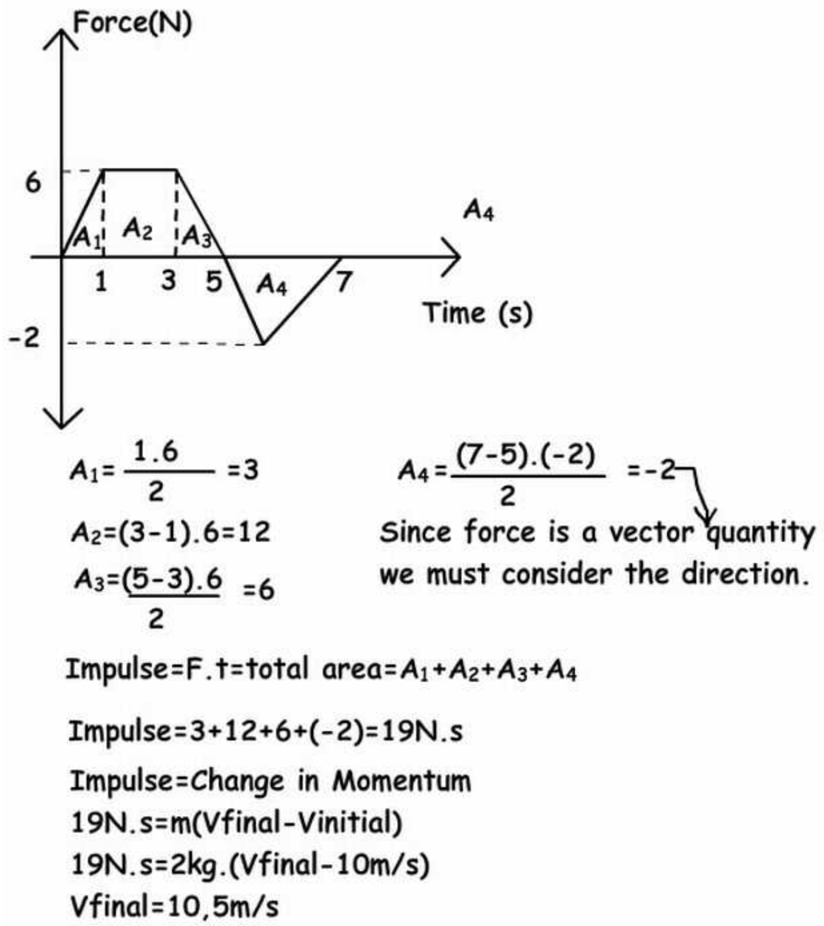
Example : If the mass of the object is 3.0 kg, what is its final velocity over the 8.0 s time period? n.d.

Solution : You need the area under the curve, and you do not need calculus, as that is a trapezoid. The area of a trapezoid is the average of the bases times the height, which is (4 + 8)seconds/2*100 N = 600 N*s. Set this to $mv - mv_o$, and assuming v_o is zero get v final = 200 m/s.

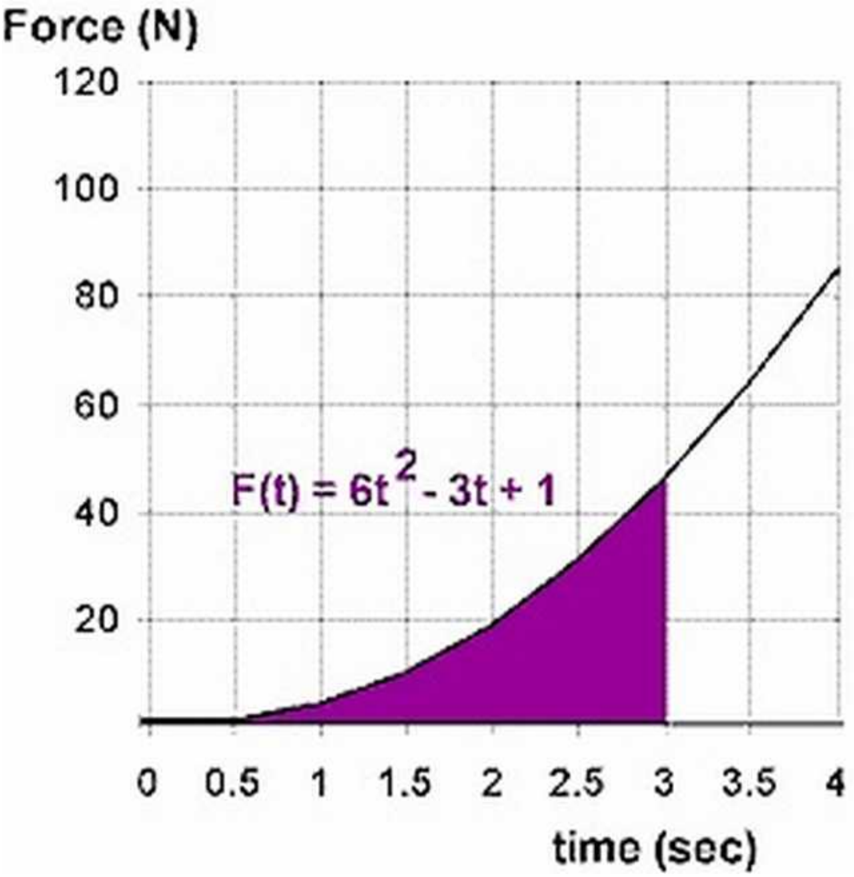
Example : The graph given below belongs to an object having mass 2kg and velocity 10m/s. It moves on a horizontal surface. If a force is applied to this object between (1-7) seconds find the velocity of the object at 7 seconds. n.d.



Solution : Area under the graph gives us impulse. First, we find the total impulse with the help of graph given above then total impulse gives us the momentum change. Finally, we find the final velocity of the object from the momentum change.



Example : Suppose a force,$F(t) = 6t^2 - 3t + 1$, acts on an 7-kg mass for three seconds.



a) What impulse will the 7-kg object receive in the first three seconds?

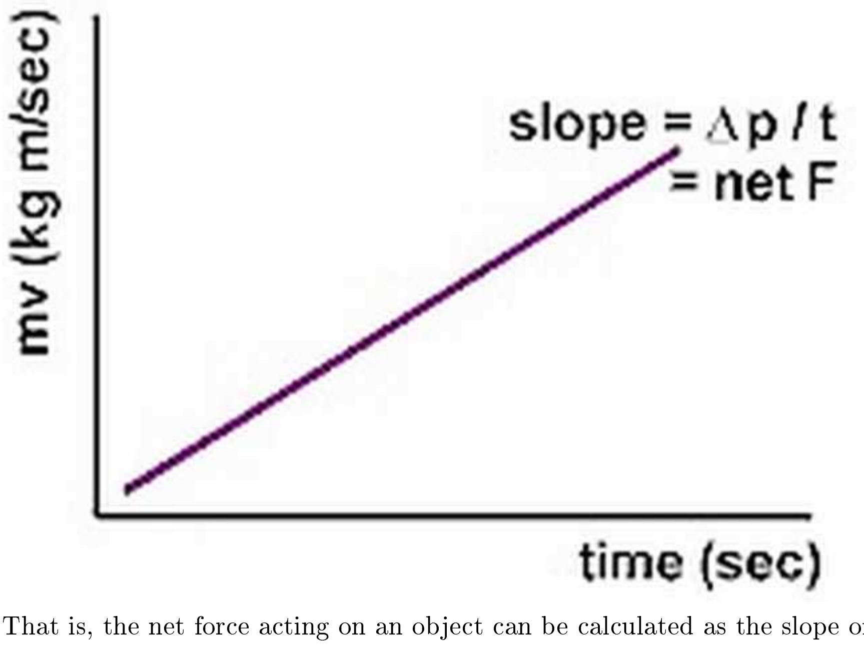
b) If the mass started from rest, what is its final velocity?

Solution : Force as the rate of change of momentum The impulse equation J = (net F)t = Δp where p = mv can be rearranged to state that the applied net force applied to an object equals the rate of change of the its momentum.

net F $= \dfrac{\triangle p}{\triangle t}$

net F $= \dfrac{m\triangle v}{\triangle t}$

net F = ma

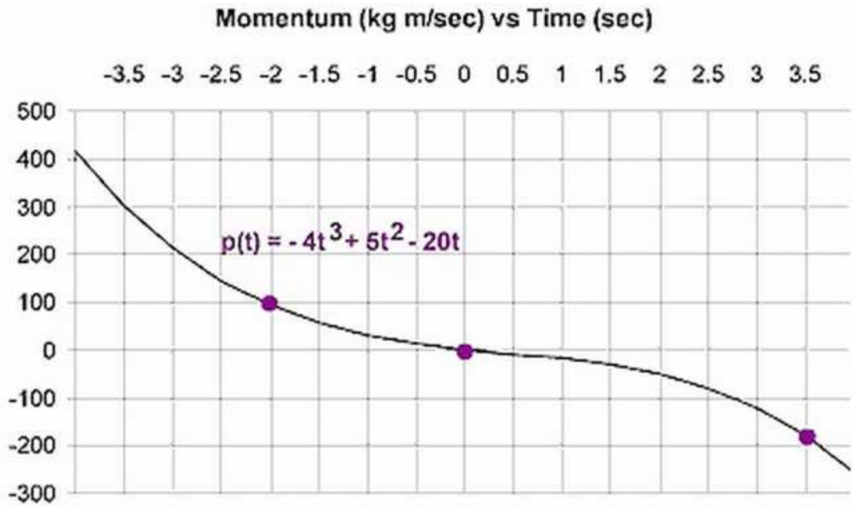


That is, the net force acting on an object can be calculated as the slope of a momentum vs time graph. In terms of the calculus, this result equates to taking the derivative.

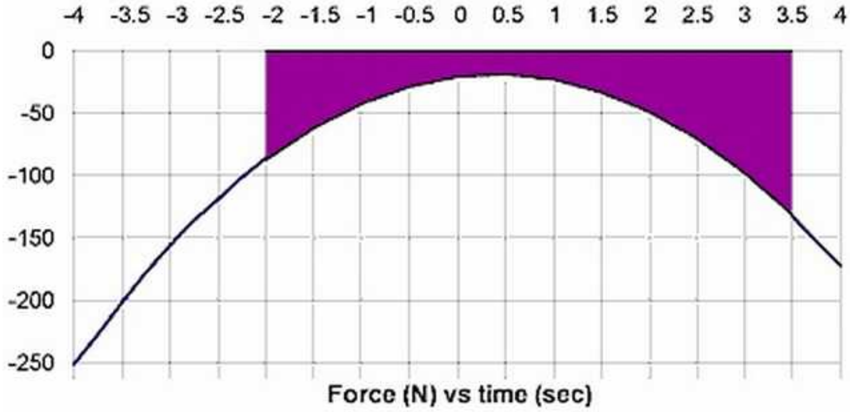
$\dfrac{dp(t)}{dt} = F(t)$

Notice that force must expressed as a function in terms of time, not displacement. Calculus will allow us to determine expressions for instantaneous, non-constant forces and thus is applicable to a wider range of situations. Let's work an example using this relationship.

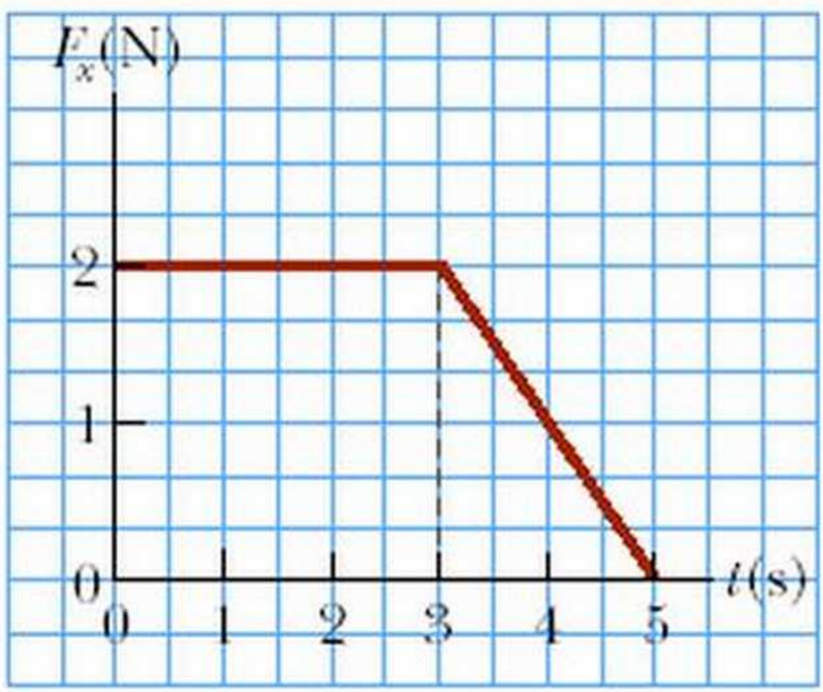
Using the graph provided below, determine the instantaneous force acting on the 7-kg mass at each of the specified times:



t = -2 seconds,
t = 0 seconds,
t = 3.5 seconds.



Example : The force shown in the force vs time graph below acts on a 1.7 kg object.



(a) Find the magnitude of the impulse of the force.
Ans : 8 kg m/s.
(b) Find the final velocity of the object if the object was initially at rest.
Ans : 2.76 m/s
(c) Find the final velocity of the object if the object was initially moving along the x axis with a velocity of -1.7 m/s.
Ans : " "
Note : The force shown in the force vs time graph below acts on a 1.7 kg object. Find the final velocity of the object if the object was initially at rest. Find the final velocity of the object if the object was initially moving along the x axis with a velocity of -1.7 m/s.

Example : Relating Momentum and Impulse

EXPLORATION – An impulsive bike ride Suki is riding her bicycle, in a straight line, along a flat road. Suki and her bike have a combined mass of 50 kg. At t = 0, Suki is traveling at 8.0 m/s. Suki coasts for 10 seconds, but when she realizes she is slowing down, she pedals for the next 20 seconds. Suki pedals so that the static friction force exerted on the bike by the road increases linearly with time from 0 to 40 N, in the direction Suki is traveling, over that 20-second period. Assume there is constant 10 N resistive force, from air resistance and other factors, acting on her and the bicycle the entire time. Step 1 - Sketch a diagram of the situation. The diagram is shown in Figure 6.2, along with the free-body diagram that applies for the first 10 s and the free-body diagram that applies for the 20second period while Suki is pedaling.

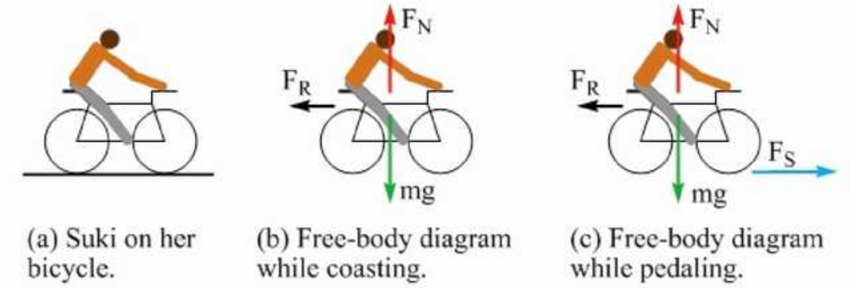


Figure : A diagram of (a) Suki on her bike, as well as free-body diagrams while she is (b) coasting and while she is (c) pedaling. Note that in free-body diagram (c), the static friction force $\vec{F_S}$ gradually increases because of the way Suki pedals.

Step 2 - Sketch a graph of the net force acting on Suki and her bicycle as a function of time. Take the positive direction to be the direction Suki is traveling. In the vertical direction, the normal force exactly balances the force of gravity, so we can focus on the horizontal forces. For the first 10 seconds, we have only the 10 N resistive force, which acts to oppose the motion and is thus in the negative direction. For the next 20 seconds, we have to account for the friction force that acts in the direction of motion and the resistive force. We can account for their combined effect by drawing a straight line that goes from –10 N at t = 10 s, to +30 N (40 N – 10N) at t = 30 s. The result is shown in Figure 6.3.

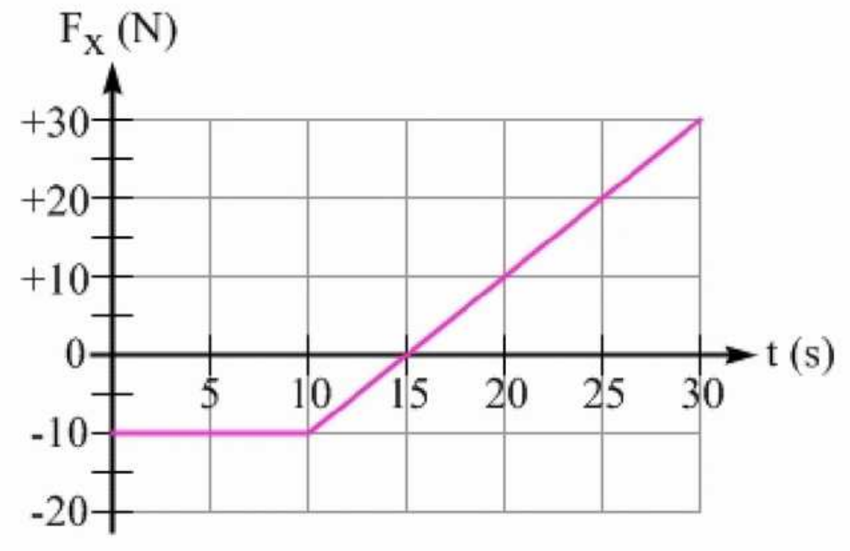


Figure 6.3: A graph of the net force acting on Suki and her bicycle as a function of time.

Step 3 - What is Suki's speed at t = 10 s? Let's apply Equation 6.3, which we can write as:

$\vec{F_{net}}\Delta t = \Delta(m\vec{v}) = m\Delta\vec{v} = m(\overrightarrow{v_{10s}} - \vec{v_i})$.

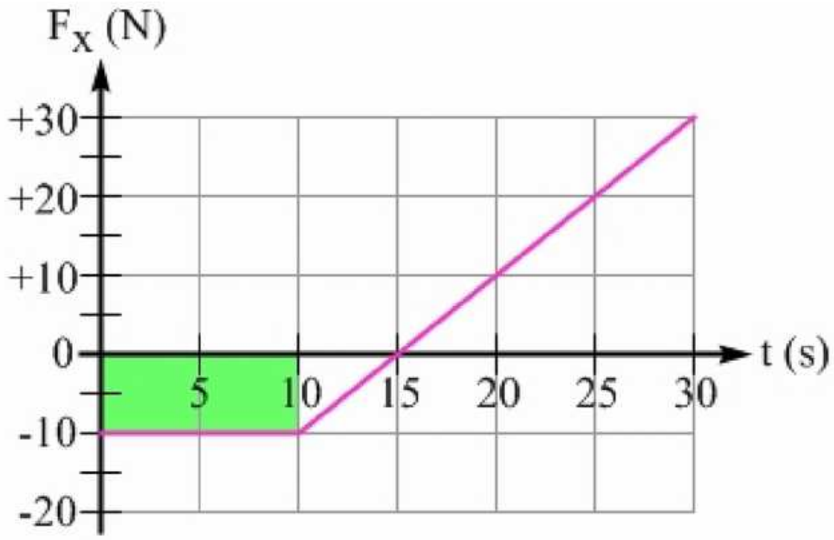
Solving for the velocity at t = 10 s gives:

$\overrightarrow{v_{10s}} = \overrightarrow{v_i} + \frac{\overrightarrow{F_{net}}\Delta t}{m} = +8.0m/s + \frac{(-10N)(10s)}{50kg} = +8.0m/s - 2.0m/s = +6.0m/s$

Thus, Suki's speed at t = 10 s is 6.0 m/s. We can also obtain this result from the forceversus-time graph, by recognizing that the impulse, $\overrightarrow{F_{net}}\Delta t$, represents the area under this graph over some time interval Δt . Let's find the area under the graph, over the first 10 seconds, shown highlighted in green in Figure 6.4. The area is negative, because the net force is negative over that time interval. The area under the graph is the impulse:

$\overrightarrow{F_{net}}\Delta t = -10N \times 10s = -100Ns = -100kg\ m/s$

Figure 6.4: The green rectangle represents the area under the graph for the first 10 s. The area is negative, because the force is negative. From Equation 6.3, we know the impulse is equal to the change in momentum.



Suki's initial momentum is

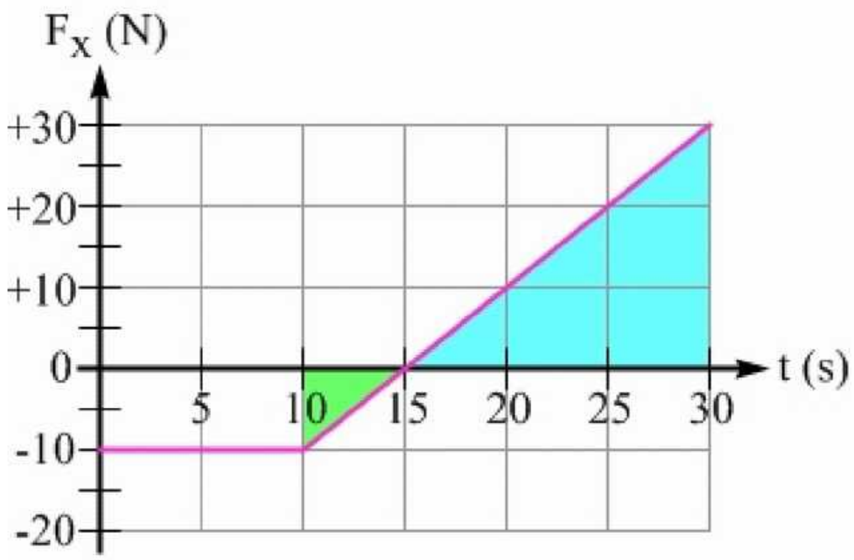
$m\overrightarrow{v_i} = 50kg \times 8.0m/s = +400kg\ m/s$. Her momentum at t = 10 s is therefore + 400kg m/s - 100kg m/s = + 300kg m/s . Dividing this by the mass to find the velocity at t = 10 s confirms what we found above:

$\overrightarrow{v}_{10s} = \frac{\overrightarrow{p_{10s}}}{m} = \frac{\overrightarrow{p_i} + \triangle\overrightarrow{p}}{m} = \frac{+400kgm/s - 100kgm/s}{50kg} = \frac{+300kgm/s}{50kg} = +6.0m/s$

Step 4 - What is Suki's speed at t = 30 s? Let's use the area under the force-versus-time graph, between t = 10 s and t = 30 s, to find Suki's change in momentum over that 20-second period. This area is highlighted in Figure 6.5, split into a negative area for the time between t = 10 s and t = 15 s, and a positive area between t = 15 s and t = 30 s. These regions are triangles, so we can use the equation for the area of a triangle, 0.5 × base × height . The area under the curve, between 10 s and 15 s, is 0.5 × (5.0s) ×(- 10 N) = - 25kg m/s . The area between 15 s and 30 s is 0.5 × (15s) × (30 N) = + 225kg m/s . The total area (total change in momentum) is +200 kg m/s.

Note that another approach is to multiply the average net force acting on Suki and the bicycle (+10 N) over this interval, by the time interval (20 s), for a +200 kg m/s change in momentum.

Figure 6.5: The shaded regions correspond to the area under the curve for the time interval from t = 10 s to t = 30 s.



In step 3, we determined that Suki's momentum at t = 10 s is +300 kg m/s. With the additional 200 kg m/s, the net momentum at t = 10 s is +500 kg m/s. Dividing by the 50 kg mass gives a velocity at t = 30 s of +10 m/s.

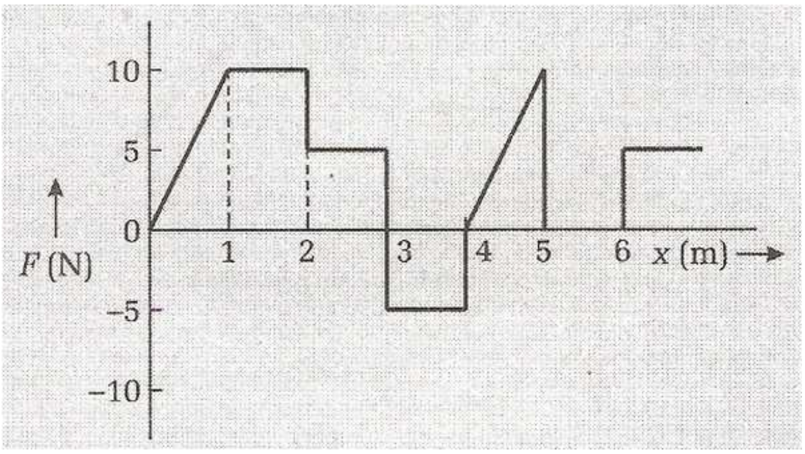
Key idea for the graphical interpretation of impulse: The area under the net force versus time graph for a particular time interval is equal to the change in momentum during that time interval.

2.2.4 Problems for Practice

2.2.4.1 General Problem Set

Single Answer Type

Example : The relationship between the force F and position x of a body is as shown in figure. The work done in displacing the body from x=1m to x=5m will be



a) 30 J

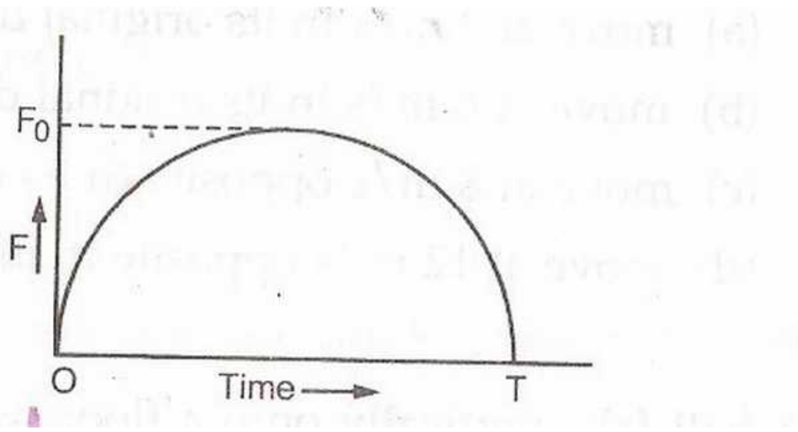
b) 15 J

c) 25 J

d) 20 J

{ Hint : Area under the graph from 1 to 5 taking signs , 15 J}

Example: A particle of mass m, initially at rest, is acted upon by a variable force F for a brief interval of time T. It begins to move with a velocity u after the force stops acting. F is shown in the graph as a function of time. The curve is a semicircle



a) $u = \frac{\pi F_0^2}{2m}$

b) $u = \frac{\pi T^2}{8m}$

c) $u = \frac{\pi F_0 T}{4m}$

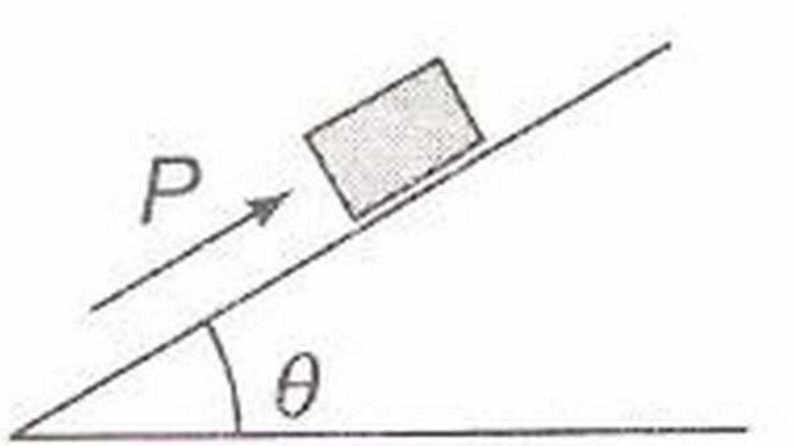
d) $u = \frac{F_0 T}{2m}$

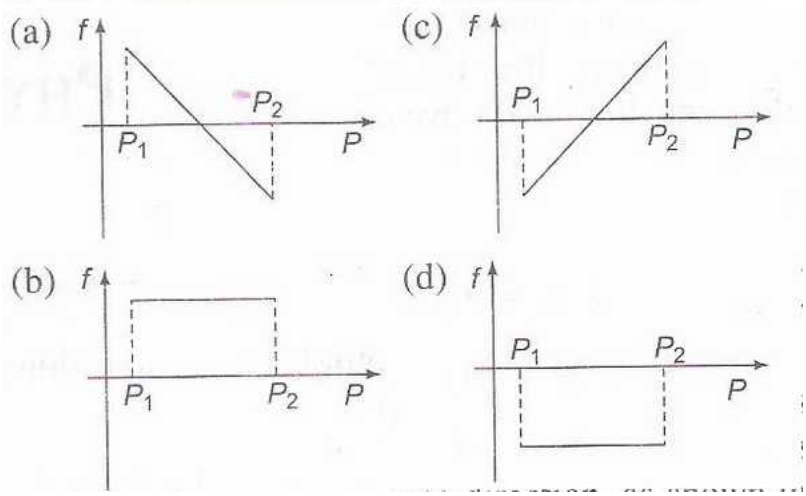
{ Hint : From impulse relation , $mv_f = \frac{\pi F_0^2}{2}$. So, a, b,c}

2.2.4.2 Previous Years IIT Problems

Single Answer

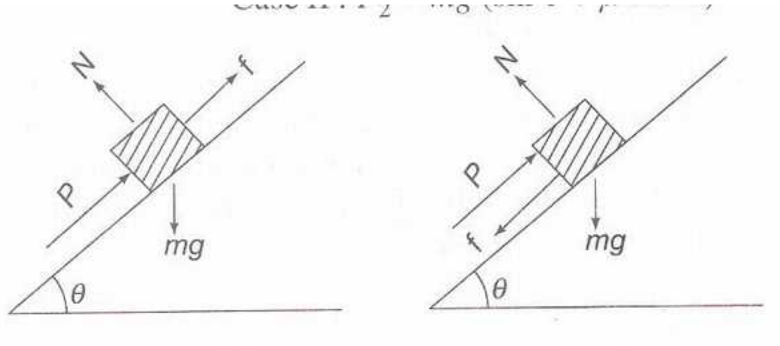
Example: A block of mass m is on an inclined plane of angle θ. The coefficient of friction between the block and the plane is $\mu$and $tan\theta > \mu$. The block is held stationary by applying a force P parallel to the plane. The direction of force pointing up the plane is taken to be positive. As P is varied from $P_1 = mg\,(sin\theta - \mu cos\theta)$ to $P_2 = mg\,(sin\theta + \mu cos\theta)$, the frictional force f versus P graph will look like





{ Solution: Case I : $P_1 = mg\,(sin\theta - \mu cos\theta)$

Case II: $P_2 = mg\,(sin\theta + \mu cos\theta)$



Case (a) Case (b)

In case a), frictional force f is positive and in case b), f is negative. When $P = mgsin\theta$,$f = \mu mgcos\theta = 0$

When $P < mgsin\theta$,$f = mgsin\theta - P$

When $P > mgsin\theta$,$f = P - mgsin\theta$

Thus f vaires linearly with P, is positive when $P = P_1$ and negative when $P = P_2$. So the correct option is a)

}

2.3 Energy Conservation

2.3.1 Abstract Introduction

2.3.1.1 KINETIC ENERGY

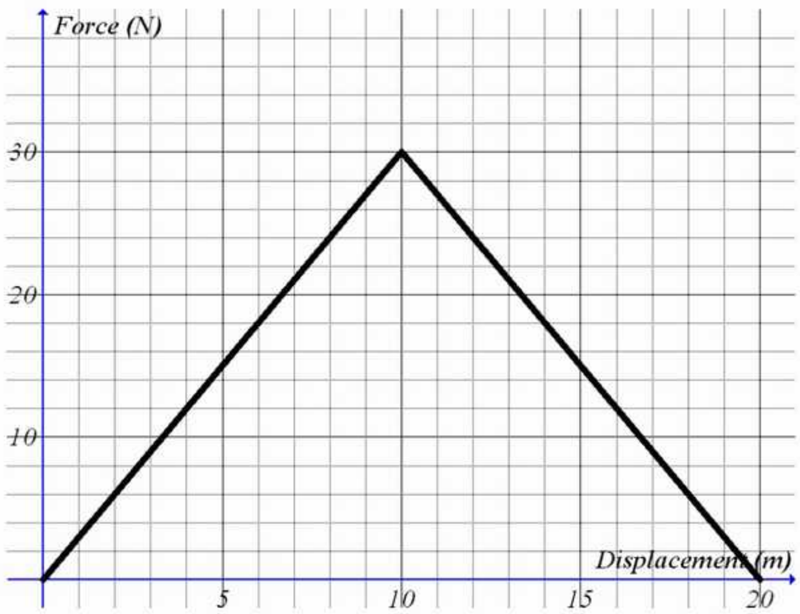
Objects have energy because of their motion; this energy is called kinetic energy. Kinetic energy of the objects having mass m and velocity v can be calculated with the formula given below;

$E_k = \frac{1}{2}mv^2$

As you see from the formula, kinetic energy of the objects is only affected by the mass and velocity of the objects. The unit of the E_k is again from the formula $kg.m^2/s^2$ or in general use joule.

2.3.1.2 Work Done by a Variable Force

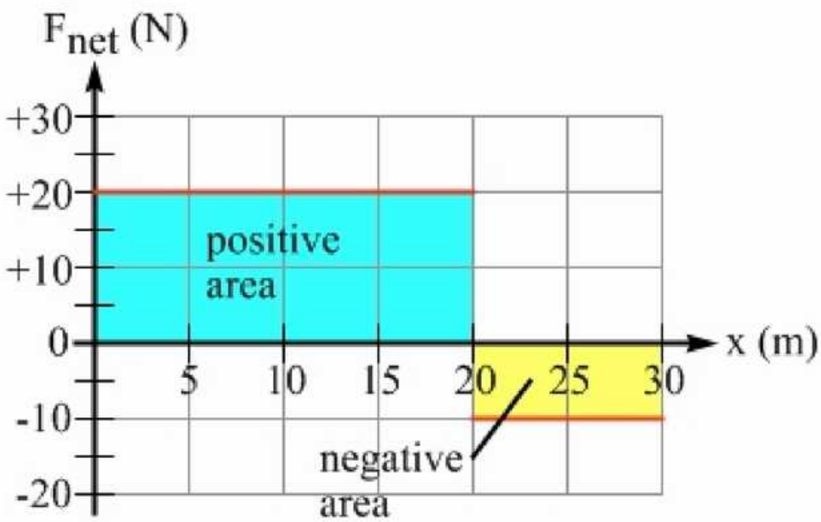
Graphically, the work done on an object or system is equal to the area under a Force vs. displacement graph:



The area under the graph from zero to 20 meters is 300 N m. Thus, the force represented by the graph does 300 J of work. This work is also a measure of the energy which was transferred while the force was being applied

2.3.1.3 The net force vs. position graph

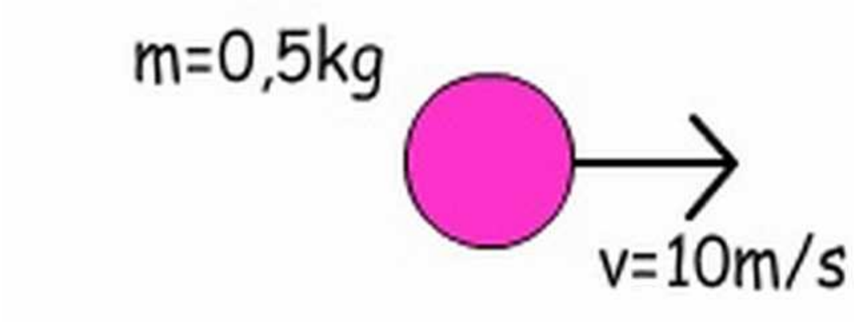
The area under the net force vs. position graph represents the change in kinetic energy (also known as the net work).



2.3.2 Theory and Problems

2.3.2.1 Force vs. Distance graph.

Examples : Find the kinetic energy of the ball having mass 0,5 kg and velocity 10m/s.

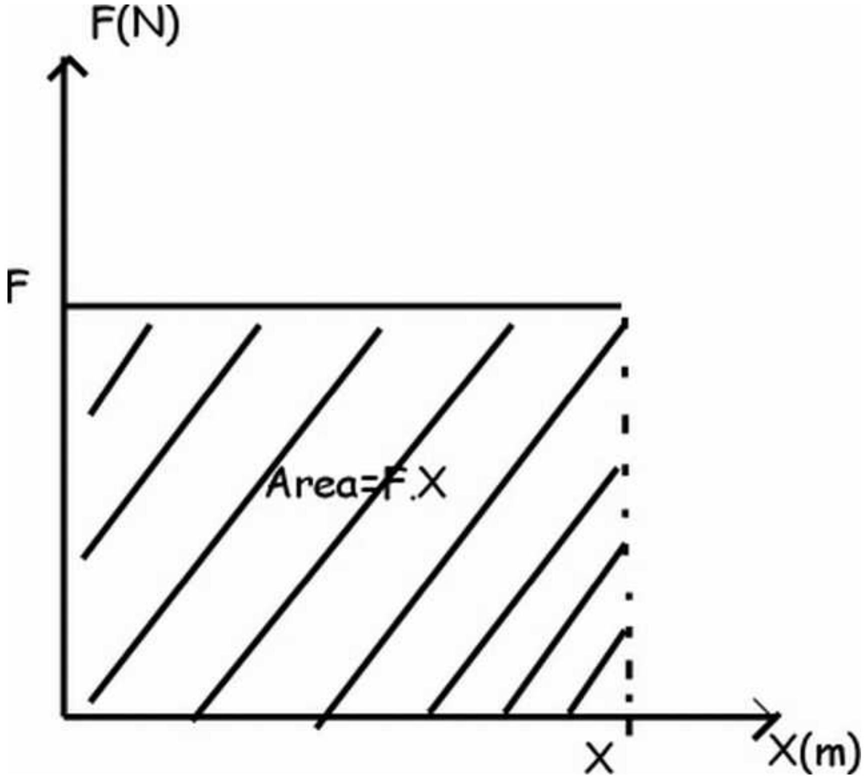


Ek=1/2mv^2

Ek=1/2.0, 5. (10) ^{2}

Ek=25joule

As in the case of Kinematics we can use graphs to show the relations of the concepts here. Look at the given graph of

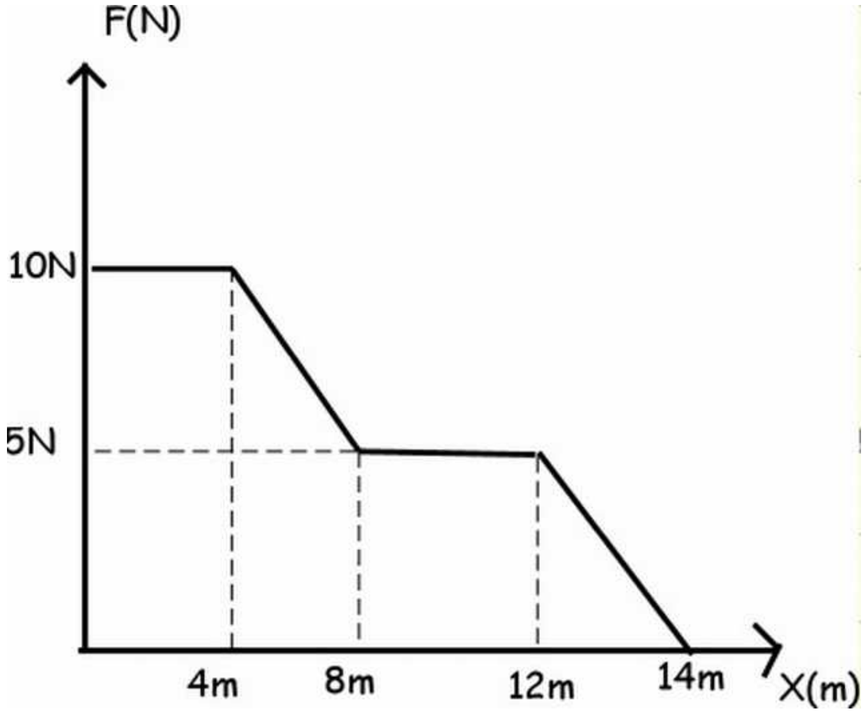


Area under the force vs. distance graph gives us work

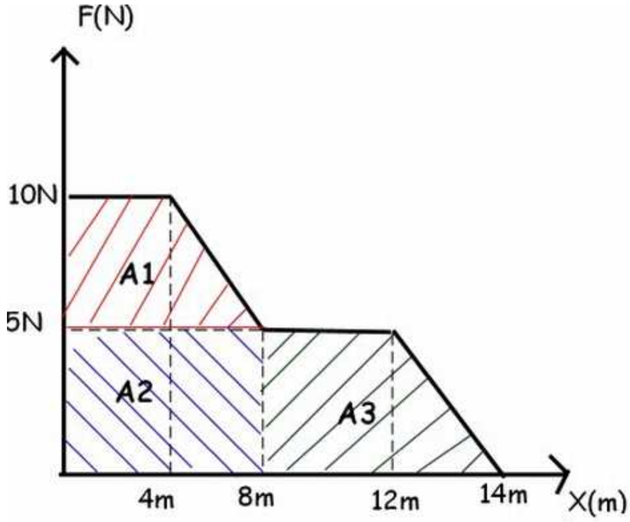
Work=Force. Distance=Area=F.X (distance)

We can find energy of the objects from their Force vs. Distance graph.

Example : Find the Kinetic Energy of the object at 14m from the given graph below.



We can find the total kinetic energy of the object after 14m from the graph; we use area under it to find energy.



$$A1 = \frac{(8+4).5}{2} = 30 \qquad A3 = \frac{(6+4).5}{2} = 25$$

$$A2 = 5.8 = 40$$

Total Area=A1+A2+A3

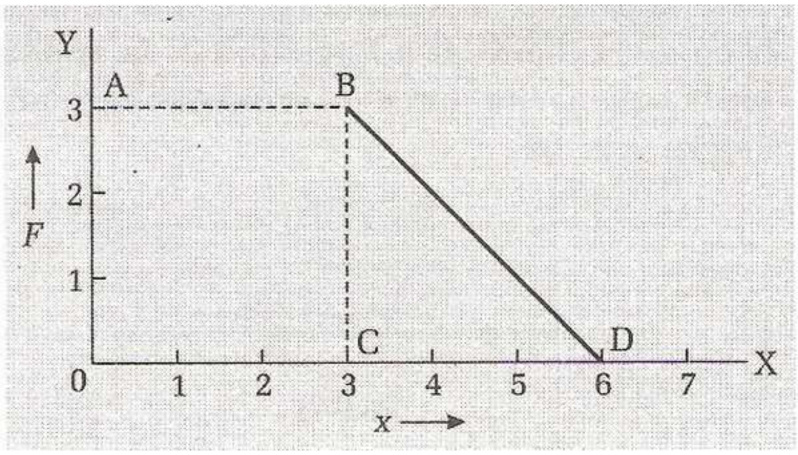
Total Area=30+40+25=95

Ek=Total Area=95joule

2.3.3 Practice Problems

2.3.3.1 General Problem Set

Example : A force F acting on an object varies with distance x as shown in Figure. The force is in newton (N) and the distance (x) in metre. The work done by the force in moving from x=0 to x=6m is



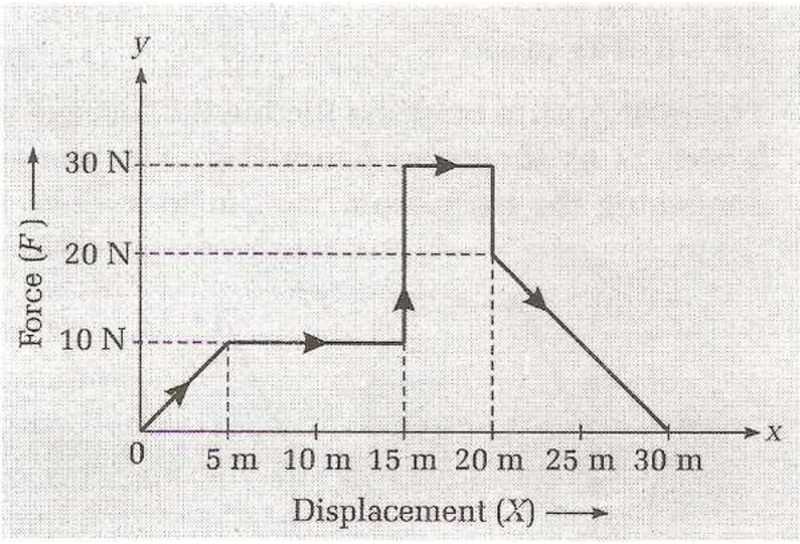
a) 4.5 J

b) 9.0 J

c) 14.5 J

d) 15 J

Example : Given below is a graph between a variable force (F) (along y-axis) and the displacement (X) (along x-axis) of a particle in one dimension. The work done by the force in the displacement interval between 0 m and 30 m is



a) 275 J

b) 375 J

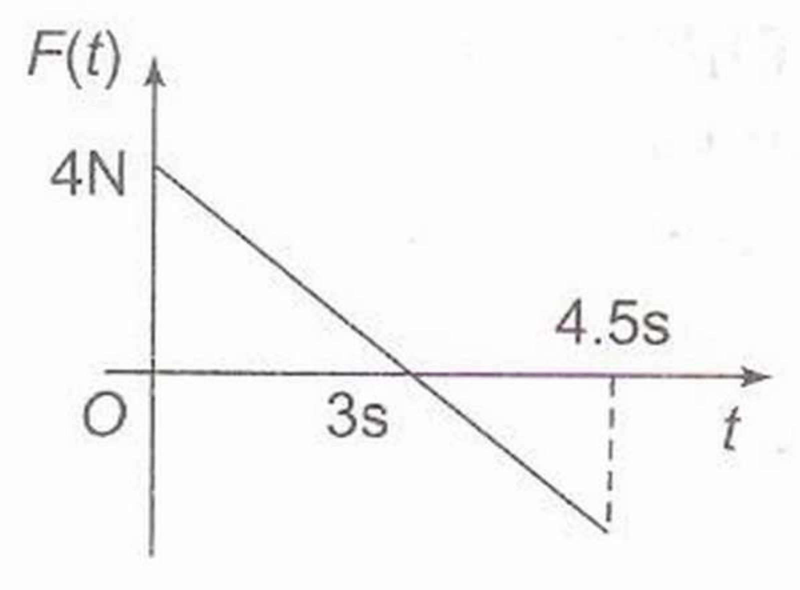
c) 400 J

d) 300 J

2.3.3.2 Previous Years IIT Problems

Single Answer

Example: A block of mass 2 kg is free to move along the x-axis. It is at rest and from t=0 onwards it is subjected to a time-dependent force F(t) in the x direction. The force F(t) varies with t as shown in the figure. The kinetic energy of the block after 4.5 seconds is



a) 4.50J

b) 7.50J

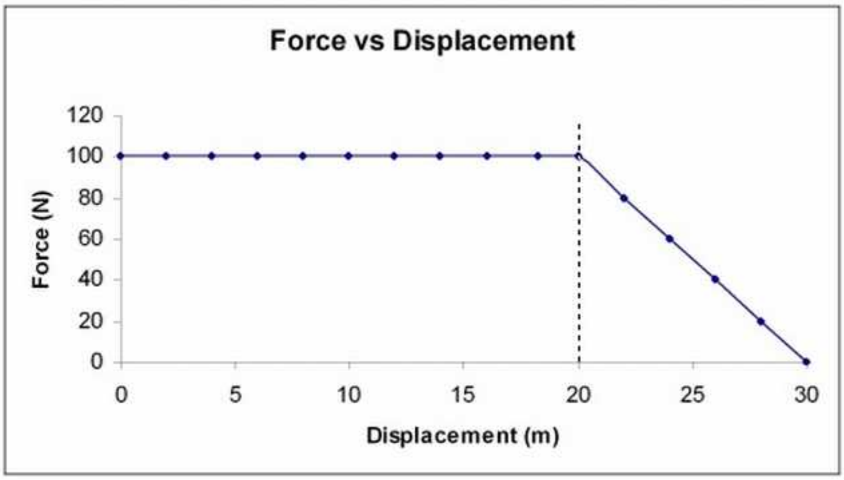
c) 5.06J

d) 14.06J

2.3.4 Review Questions I

Refer to the following information for the next thirteen questions. n.d.

A 5.0-kg mass is pushed along a straight line by a net force described in the graph below. The object is at rest at t = 0 and x = 0.



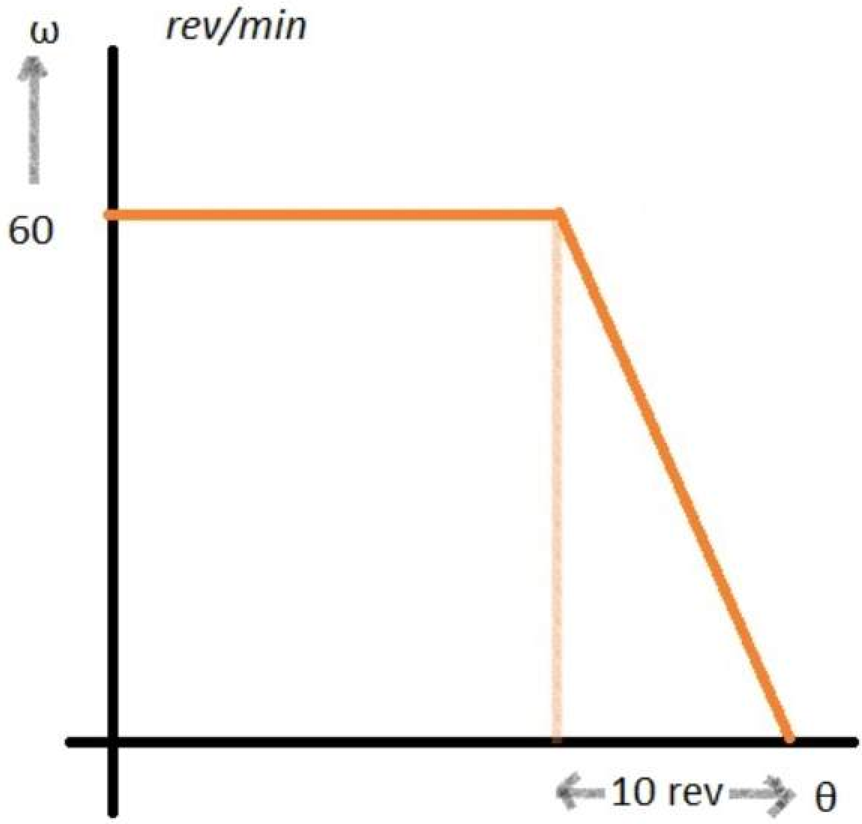
a) During which displacement interval was the object's acceleration uniform?
b) What acceleration did the object experience when x = 10 meters?
c) How much work was done on the object during the first 20 meters?
d) How much kinetic energy did the object gain during the first 20 meters?
e) What was the object's instantaneous velocity at x = 20 meters?
f) How much time was required to move it through the first 20 meters?
g) How much did the object's momentum change in the first 20 meters?
h) What was the object's instantaneous acceleration at x = 22 meters?
i) Why can't the kinematics equations for uniformly accelerated motion be used to calculate the object's instantaneous velocity at x = 30 meters? What method should be used?
j) How much work was done to move the object from 20 meters to 30 meters?
k) What was the object's instantaneous speed at x = 30 meters?
l) What was the total impulse delivered to the object from x = 0 to x = 30 meters?
m) What percent of the impulse was delivered in the last 10 meters?

2.4 Rotatory Motion

2.4.1 Problems for Practice

2.4.1.1 General Problem Set

Single Answer Type Example : The angular velocity of a rotating disc decreases linearly with angular displacement from 60 rev/min. to zero during 10 rev as shown. Determine the angular velocity of the disc 3 sec after it begins to slow down



(a) $\frac{20\pi}{10}$

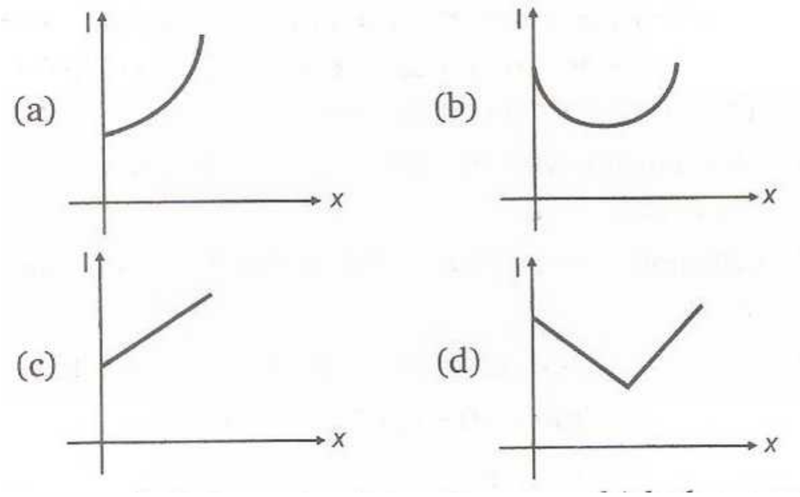
(b) $\frac{17\pi}{10}$

(c) $\frac{7\pi}{3}$

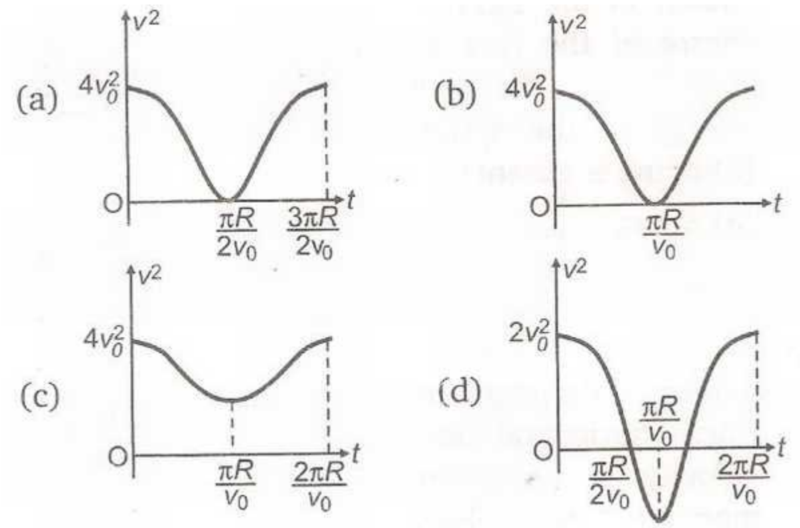
(d) $\frac{10\pi}{3}$

{ Hint: Answer C }

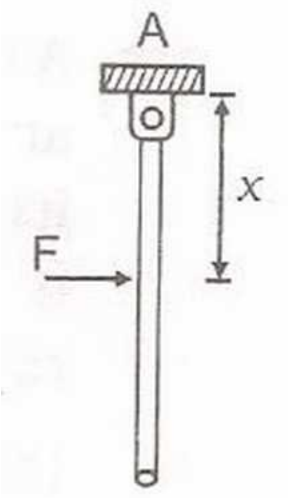
Example: Moment of Inertia I of a solid sphere about an axis parallel to a diameter and at a distance x from its centre of mass varies as



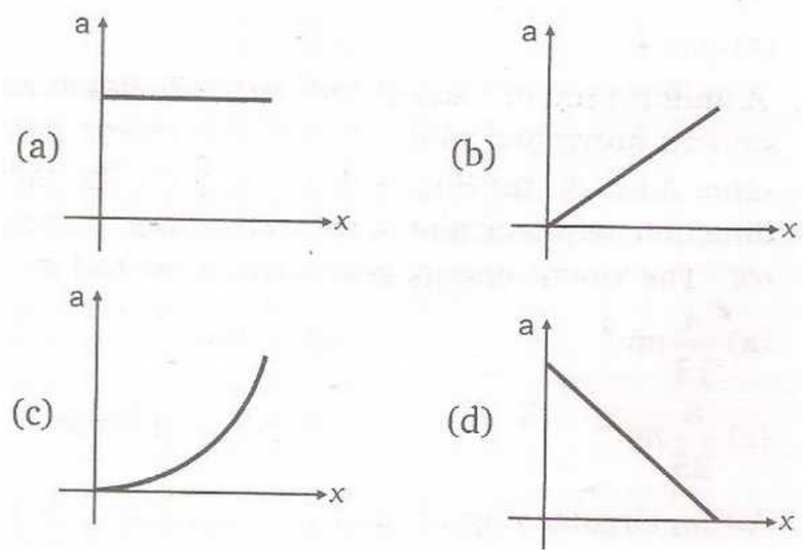
Example: A wheel is rolling without sliding on a horizontal surface. The centre of the wheel moves with a constant speed v_o . Consider a point P on the ri which is at the top at time t=0. The square of speed of point P is plotted against time t. The correct plot is (R is radius of the wheel)



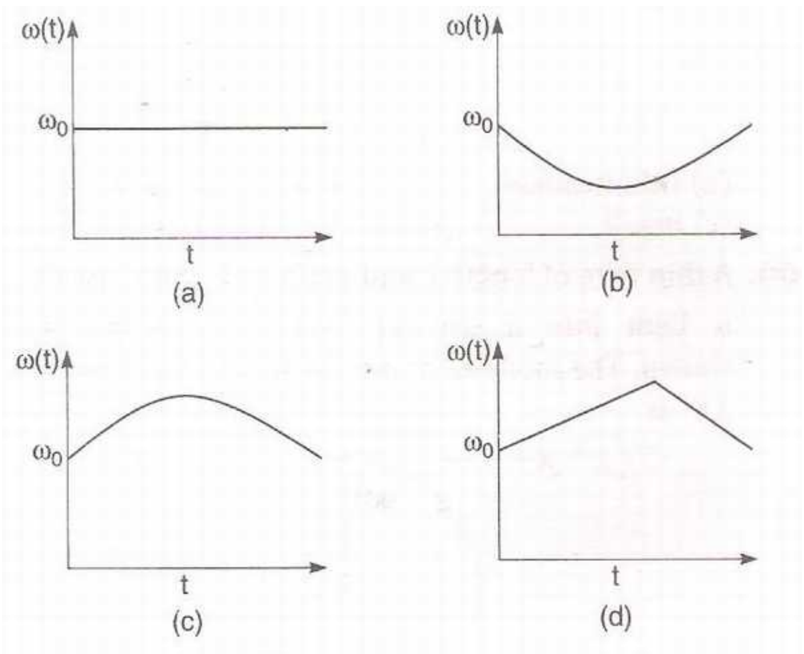
Example: A rod of mass m and length l is hinged at one of its end A as shown in figure.



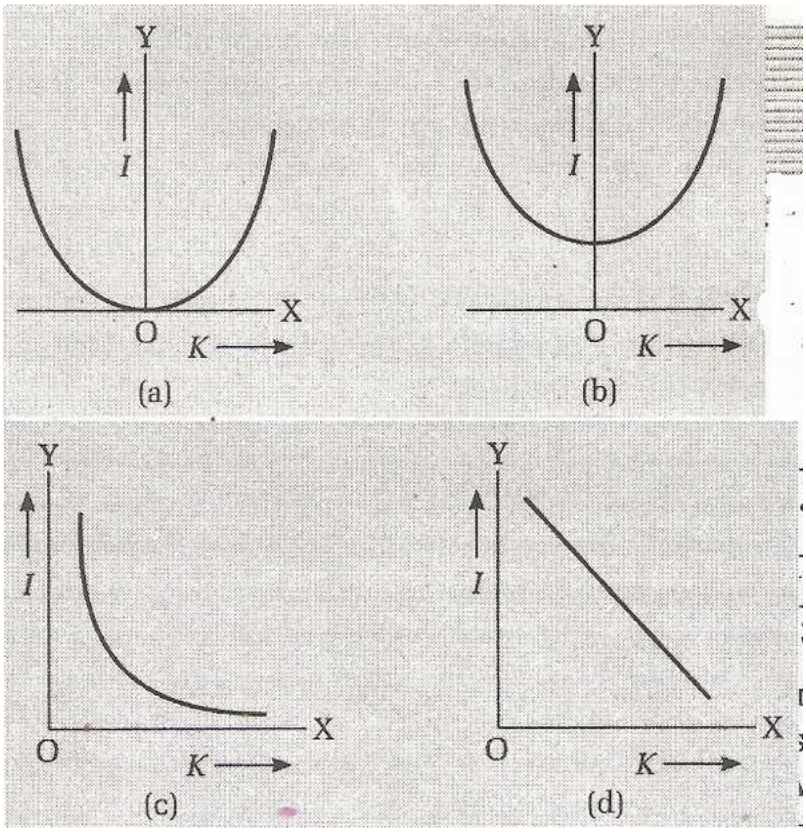
A force F is applied at a distance x from A. The acceleration of centre of mass (a) varies with x as



Example: A circular platform is free to rotate in a horizontal plane about a vertical axis passing through its centre. A tortoise is sitting at the edge of the platform. Now the platform is given an angular velocity ω_o. When the tortoise move along a chord of the platform with a constant velocity (with respect to the platform). The angular velocity of the platform $\omega(t)$ will vary with time t as

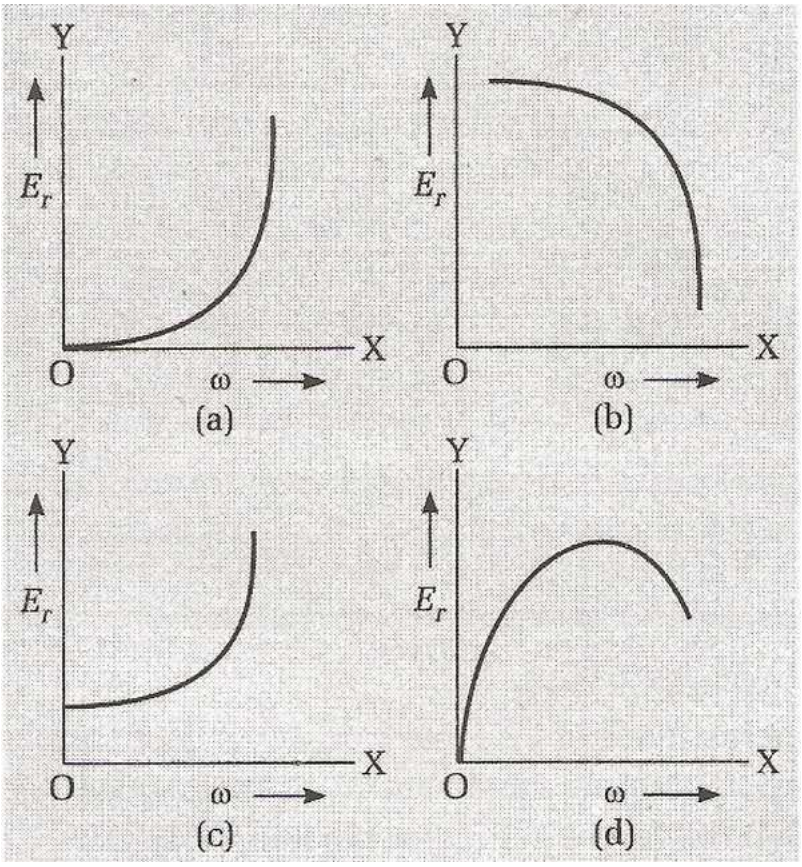


Example: The curve for the moment of inertia of a sphere of constant mass M versus its radius of gyration K will be



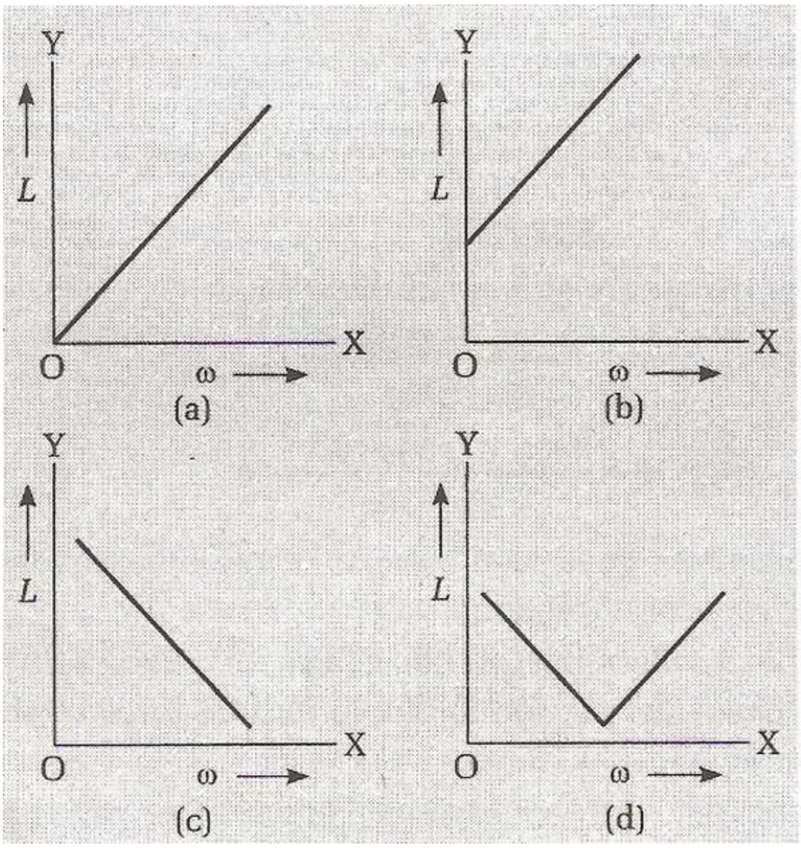
{ Hint : a) }

Example : The graph between rotational energy E_r and angular velocity ω is represented by which curve



{ Hint : a) }

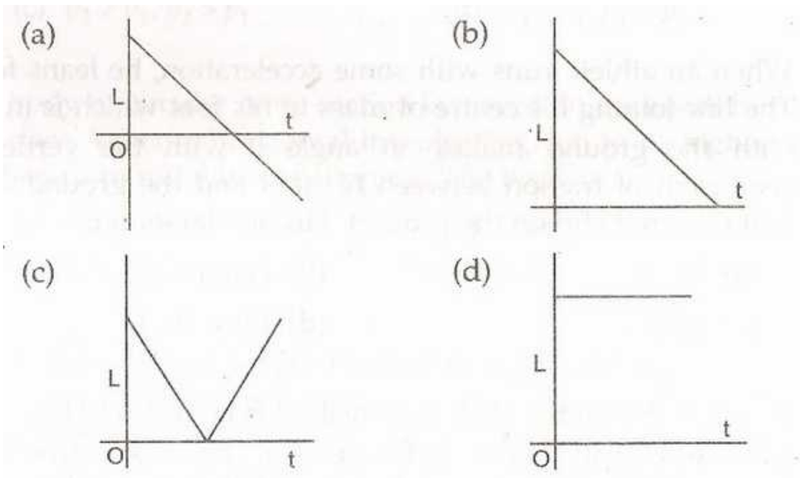
Example : The curve between angular momentum L and angular velocity ω will be



{ Hint : a) }

2.4.2 Angular Momentum Conservation

Example : A block slides on a rough horizontal ground from point A to point B. Point C is midway between A and B. The coefficient of friction between the block and the ground is constant. Its angular momentum L about C is plotted against time t. Which of the following curves is correct?



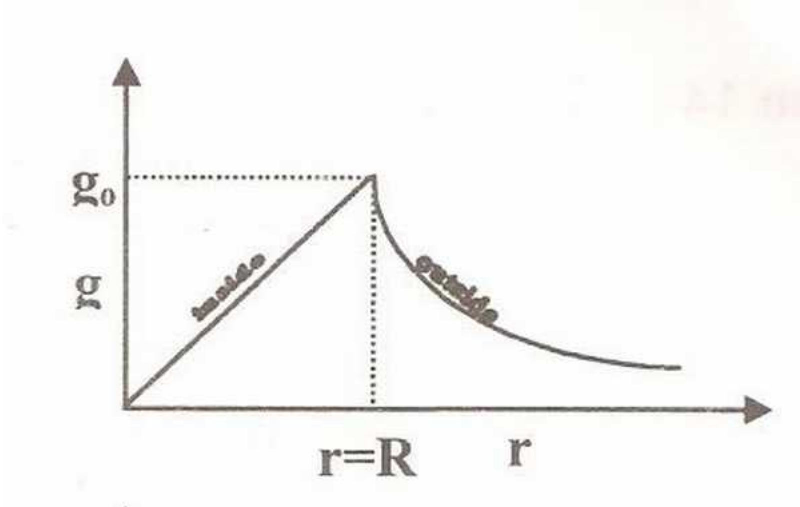
2.5 Gravitation

2.5.1 Basics

2.5.1.1 Variation of "g"

Outside the earth $g = g_o \left(\frac{R}{r}\right)^2$

where g_o is the acceleration due to gravity at the surface of earth.



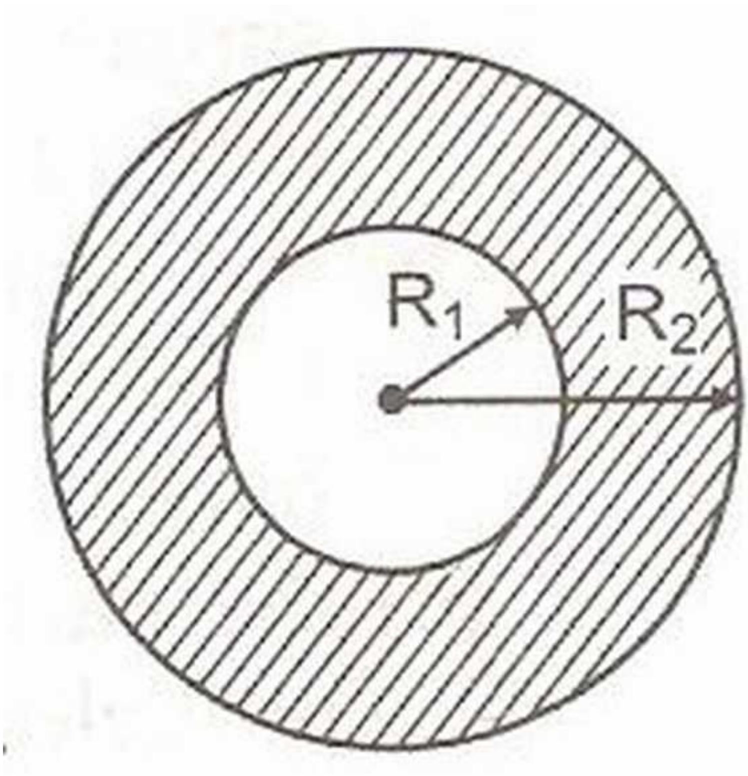
$g_o = \dfrac{GM}{R^2}$, R = radius of earth

2.5.2 Problems for Practice

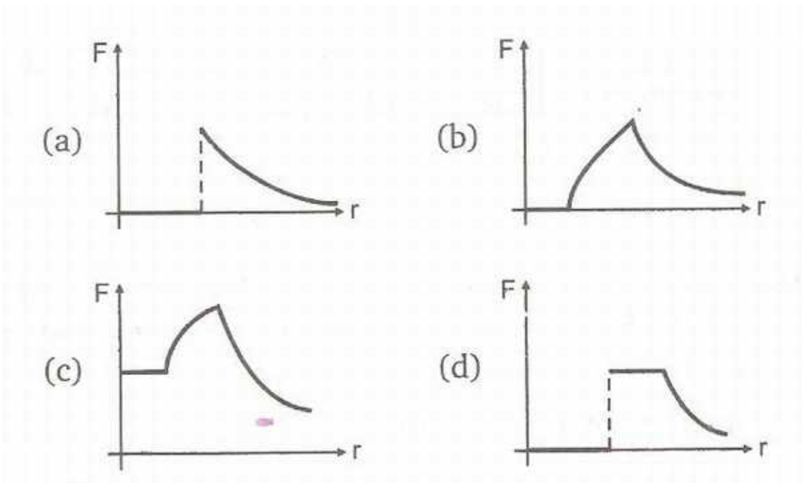
2.5.2.1 General Problem Set

Single Answer

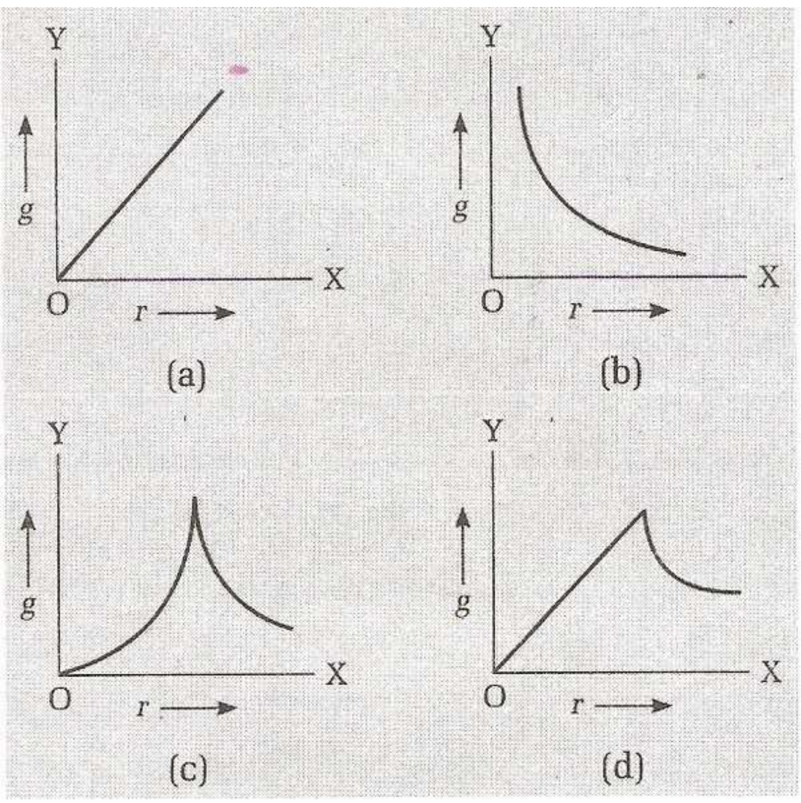
Example: A sphere of mass M and radius R_2 has a concentric cavity of radius R_1 as shown in figure.



The force F exerted by the sphere on a particle of mass m located at a distance r from the centre of sphere varies as $(0 \leq r \leq \infty)$

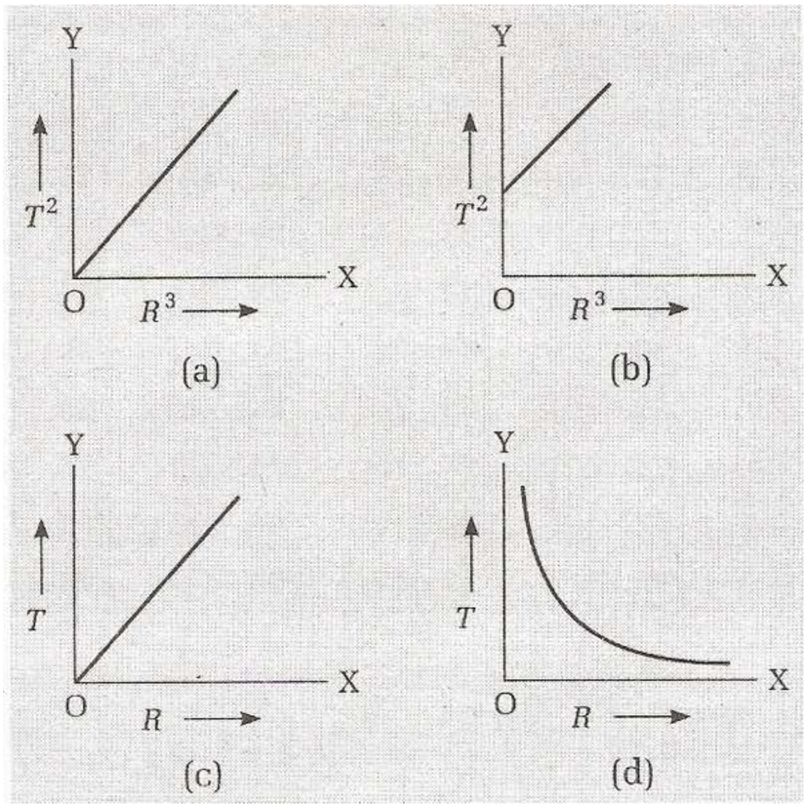


Example : The variation of acceleration due to gravity as one moves away from earth's centre is given by the graph



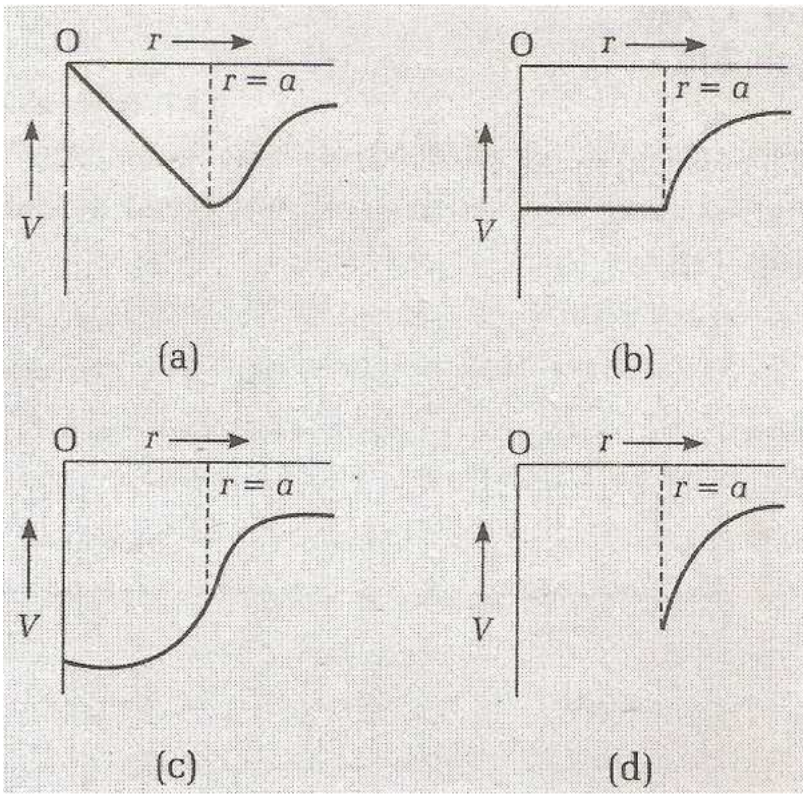
{ Hint: c) }

Example : Which of the following graphs represents the motion of a planet moving about the sun?



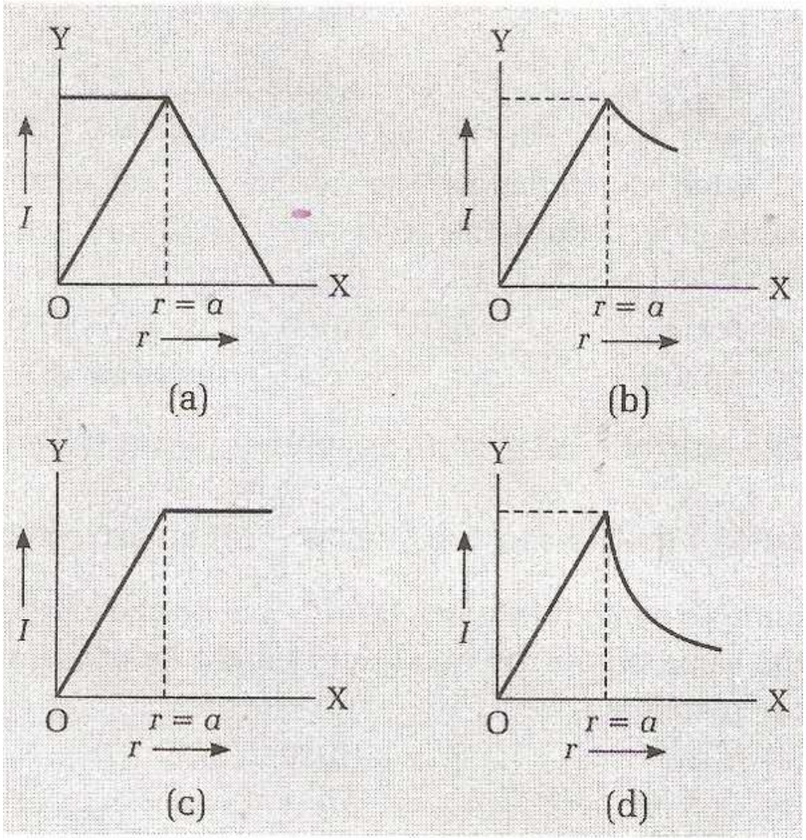
{ Hint : a) }

Example : P is a point at a distance r from the centre of a solid sphere of radius a. The gravitational potential at P is V. If V is plotted as a function of r, which is the correct curve?



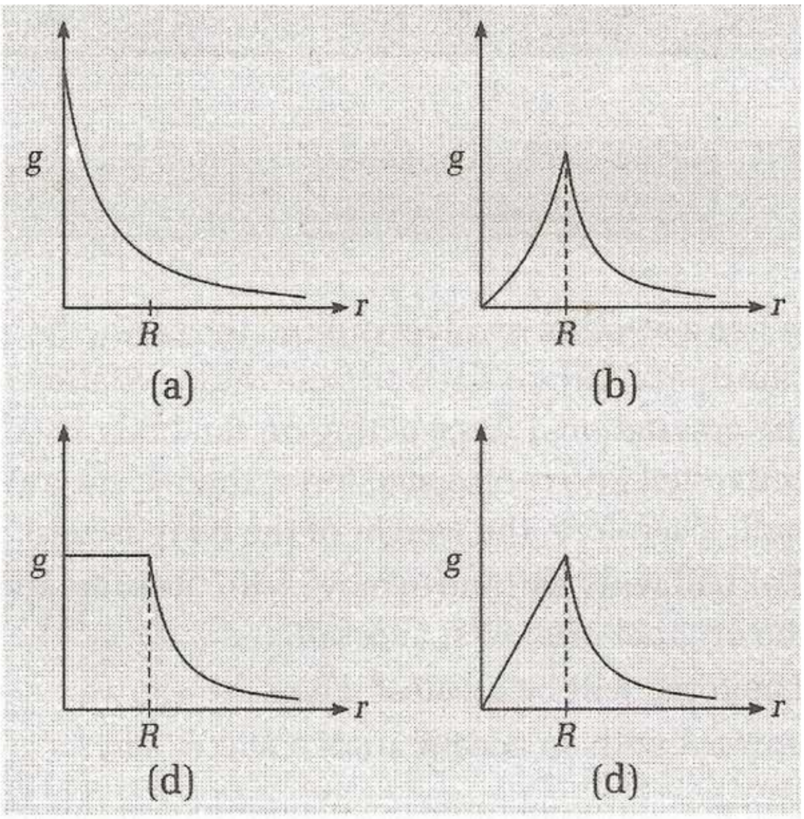
{ Hint : c) }

Example : Which of the graphs represents correctly the variation of intensity of gravitational field I with the distance r from the centre of a spherical shell of mass M and radius a?



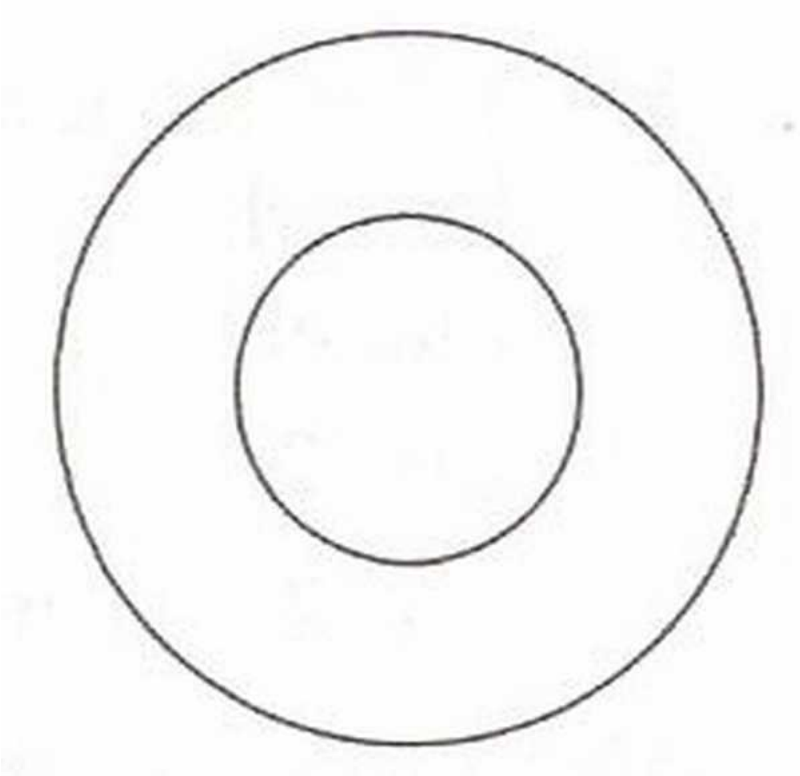
{ Hint : d) }

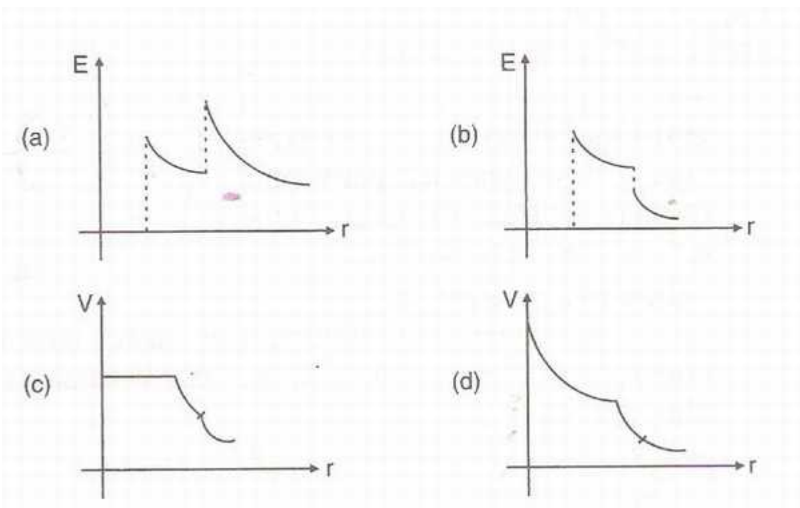
Example : The dependence of acceleration due to gravity g on the distance r from the center of the earth, assumed to be a sphere of radius R of uniform density is as shown in figure below



Multiple Answer

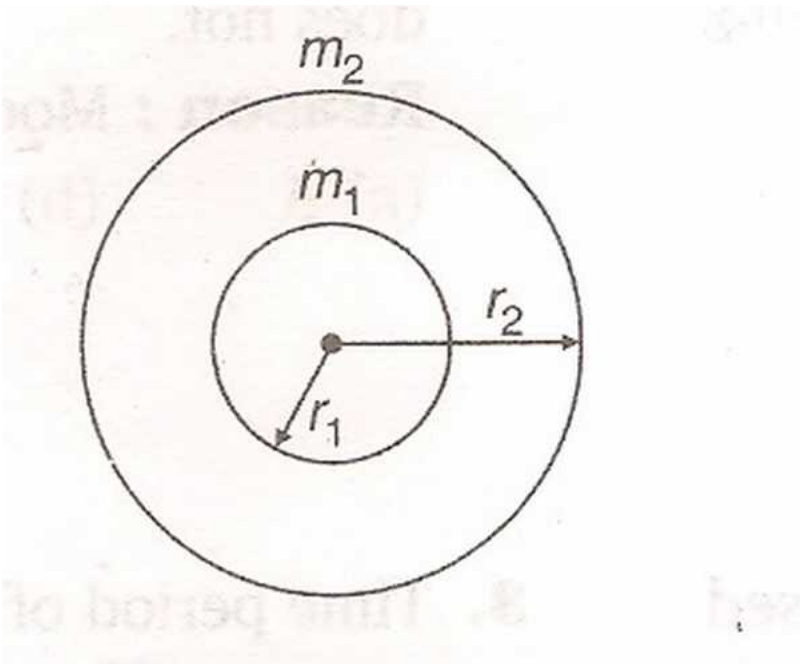
Example: Two concentric spherical shells are as shown in figure. The magnitude of gravitational potential (V) and field strength (E) vary with distance (r) from centre as



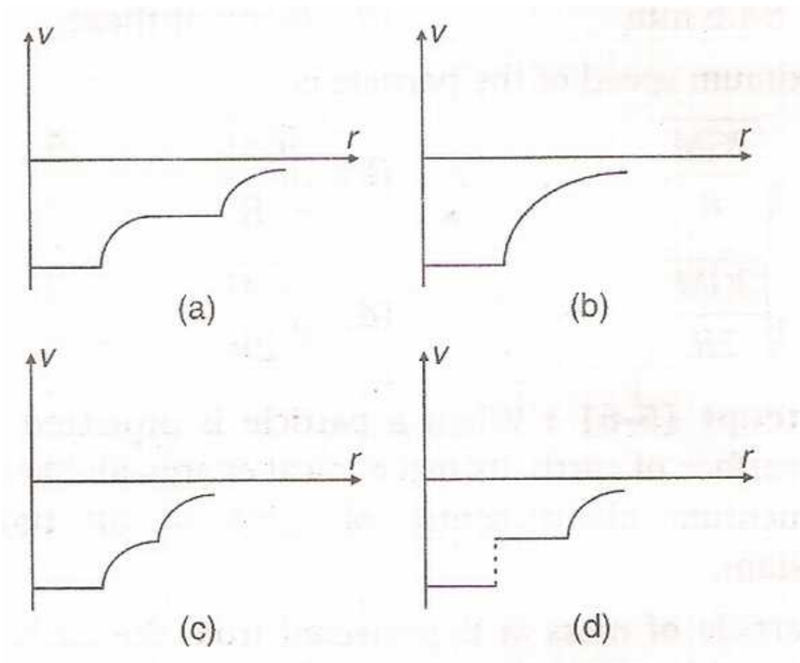


Comprehension Type

2.5.2.2 Concept 1 Gravitational potential inside a sphereical shell is constant and outside the shell it varies as $V \propto \frac{1}{r}$ (with negative sign). Here r is the distance from centre.

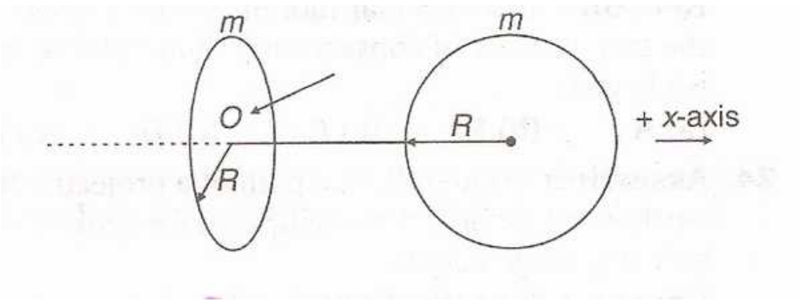


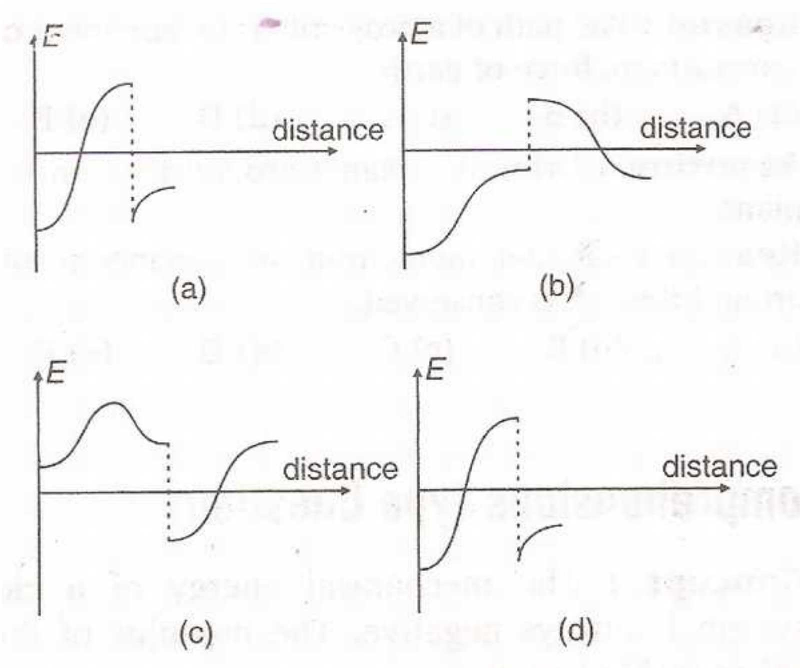
Example1: Two concentric spherical shells are as shown in figure. The V-r graph will be as



Statement : 1: Gravitational Field at distance x from the centre on the axis of a ring is given as
$E = \frac{Gmx}{(R^2 + x^2)^{3/2}}$
Here m is the mass of ring and R its radius.
2: Gravitational field at distance x($\geq R$) from centre of a solid sphere is given as,
$E = \frac{Gm}{x^2}$
Here m is the mass of solid sphere.

Example2: One ring of radius R and mass m and one solid sphere of same mass m and same radius R are placed with their centres on positive x-axis. We are moving from some finite distance on negative x-axis towards positive x-axis. Plane of the ring is perpendicular to x-axis. How will the net gravitational field vary with distance moved on x-axis. We move only up to surface of solid sphere. O is the origin,





Statement: Change in potential energy when a mass m is taken to a height h from the surface of earth is given by

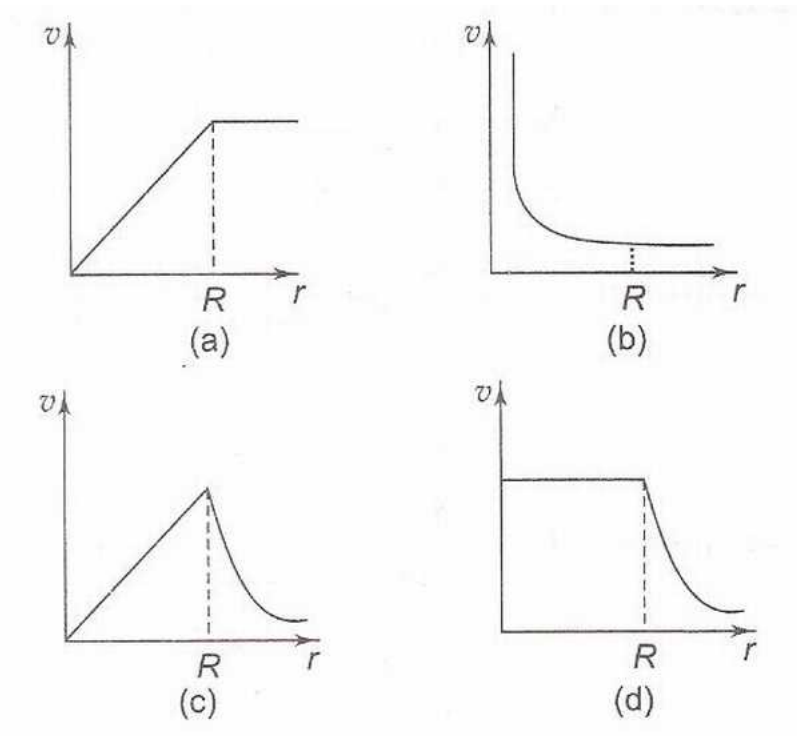
$$\triangle U = \frac{mgh}{1+h/R}$$

2.5.2.3 Previous Years IIT Problems

Single Answer A spherically symmetric gravitational system of particles has a mass density

$$\rho = \begin{cases} \rho_o & for\, r \leq R \\ 0 & for\, r > R \end{cases}$$

where ρ_o is a constant. A test mass can undergo circular motion under the influence of the gravitational field of particles. Its speed v as a function of distance r ($0 < r < \infty$) from the centre of the system is represented by



{Solution: If M is the total mass of the system of particles, the orbital speed of the test mass is

$v = \sqrt{\dfrac{GM}{r}}$

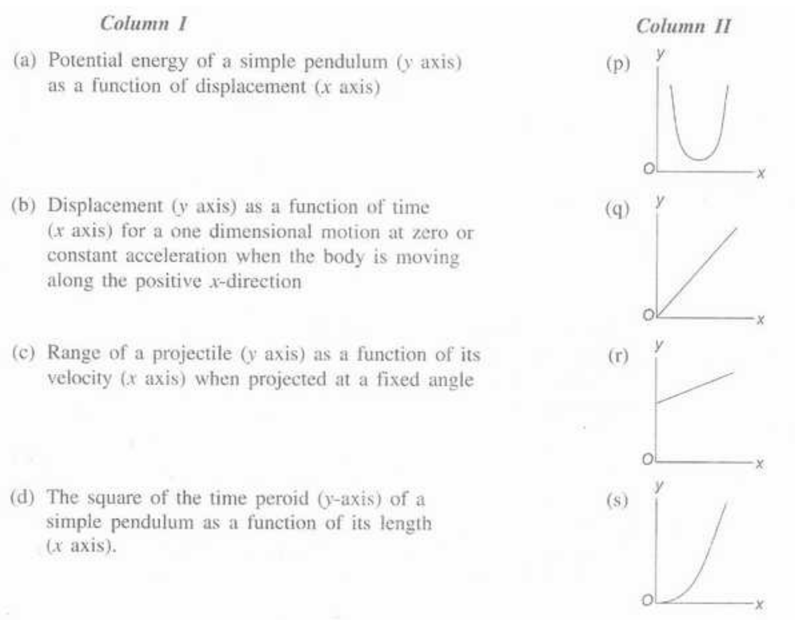
For $r \leq R$, $v = \sqrt{\dfrac{G \times \dfrac{4\pi}{3} r^3 \rho_o}{r}}$ which gives $v \propto r$

i.e. v increases linearly with r up to r=R. Hence choices b) and d) are wrong.

For r>R, the whole mass of the system is $M = \dfrac{4\pi}{3} R^3 \rho_o$, which is constant. Hence for r>R,

$v = \sqrt{\dfrac{GM}{r}}$

i.e, $v \propto \dfrac{1}{\sqrt{r}}$. Hence, the correct choice is c). }



Column I	Column II
(a) Potential energy of a simple pendulum (y axis) as a function of displacement (x axis)	(p)
(b) Displacement (y axis) as a function of time (x axis) for a one dimensional motion at zero or constant acceleration when the body is moving along the positive x-direction	(q)
(c) Range of a projectile (y axis) as a function of its velocity (x axis) when projected at a fixed angle	(r)
(d) The square of the time peroid (y-axis) of a simple pendulum as a function of its length (x axis).	(s)

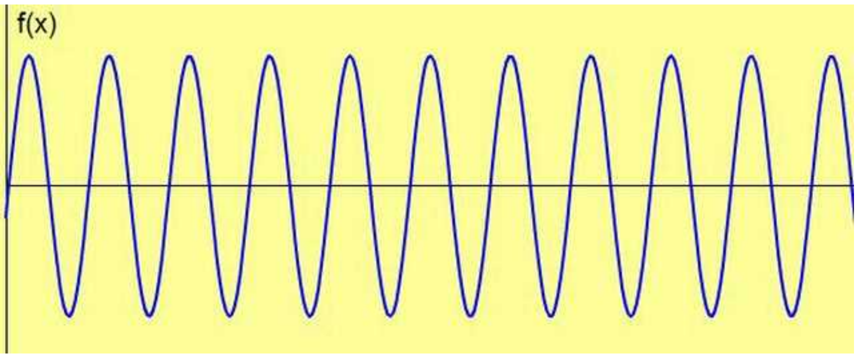
Matching

2.6 Periodic Motion

2.6.1 Abstract Introduction (SHM)

2.6.1.1 Position vs time

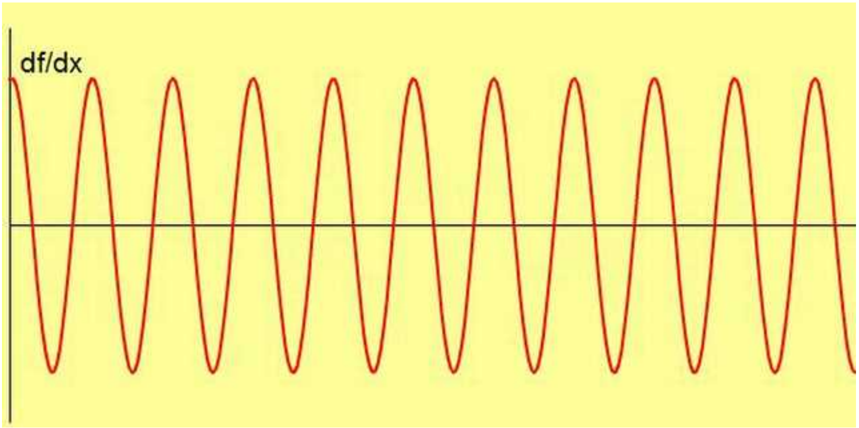
The graph of position verse time is a sine wave with a possible phase shift. The phase shift is how much the ahead or behind the position is on the sine wave. n.d.



Consider this graph, if the "clock" is started at 0.05 (where the mass is at it's maximum stretch) seconds then there would be a phase shift of 90 degrees (or we could replace the sine function for a cosine). If the "clock" is started at 0.1 seconds (where mass moves down instead of up; left instead of right) then the phase shift would be 180 degrees (a negative sine function).

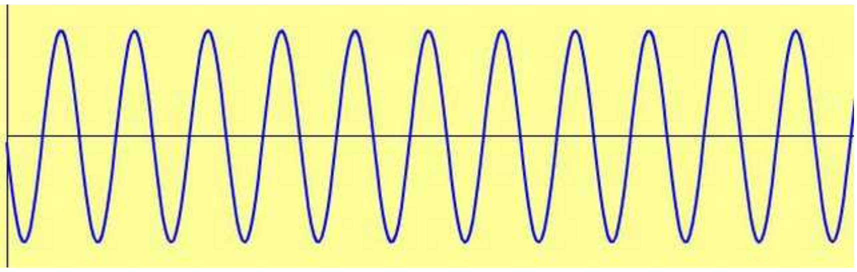
2.6.1.2 Velocity vs time

Consider the position verses time graph, at any point were the mass has reached the amplitude (maximum distance from from the equilibrium point) the speed of the mass at these point is zero. When the position is at zero then the speed is at a maximum (if you don't believe it, consider conservation of energy). This "shifts" the position graph by 90 degrees "creating" a cosine graph for velocity. The other way of thinking about is velocity is the change in position with respect to time, the change in a sine wave with respect to time is a cosine graph.



2.6.1.3 Acceleration vs time

The acceleration verse time graph is the easiest of the graphs to make. The simple harmonic motion is based on a relationship between position and acceleration; x = - Ka. So the graph of position and acceleration should look alike, except for the negative sign. In fact position and acceleration are the same shape just mirror copies of each other.

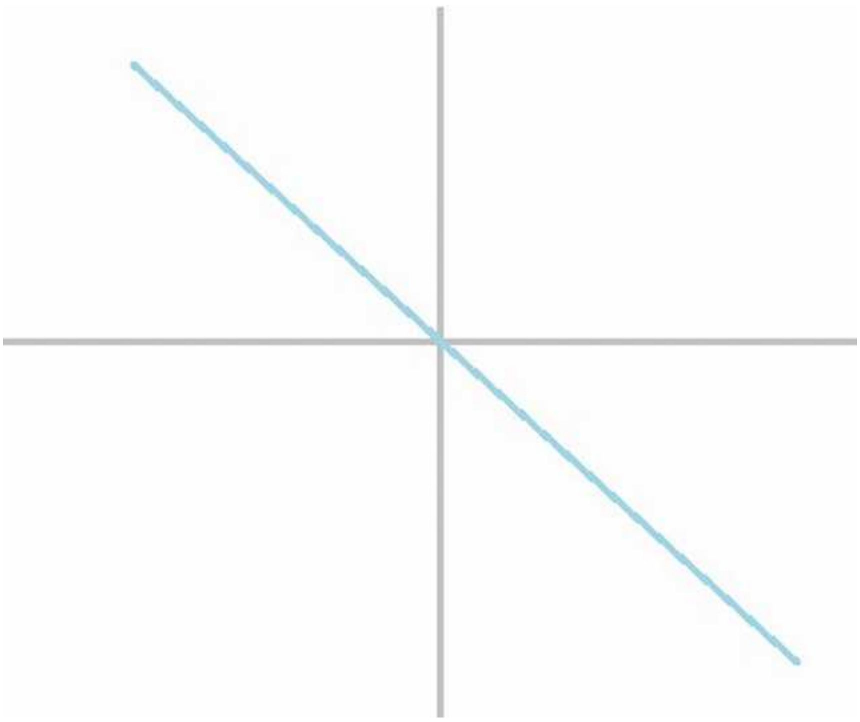


2.6.1.4 Peak Height

It is important to note that the shapes of each graph is simular, that each graph has the same frequency, period and wavelength, but they don't have the same amplitude, for common simple harmonic motion, the height the peak of the function (not to confused with the amplitude, amplitude refers to the height of the position graph alone, i'm talking about the height of the position, velocity and acceleration graphs) tend to get smaller and smaller starting with the position graph being the tallest and the acceleration being the shortest.

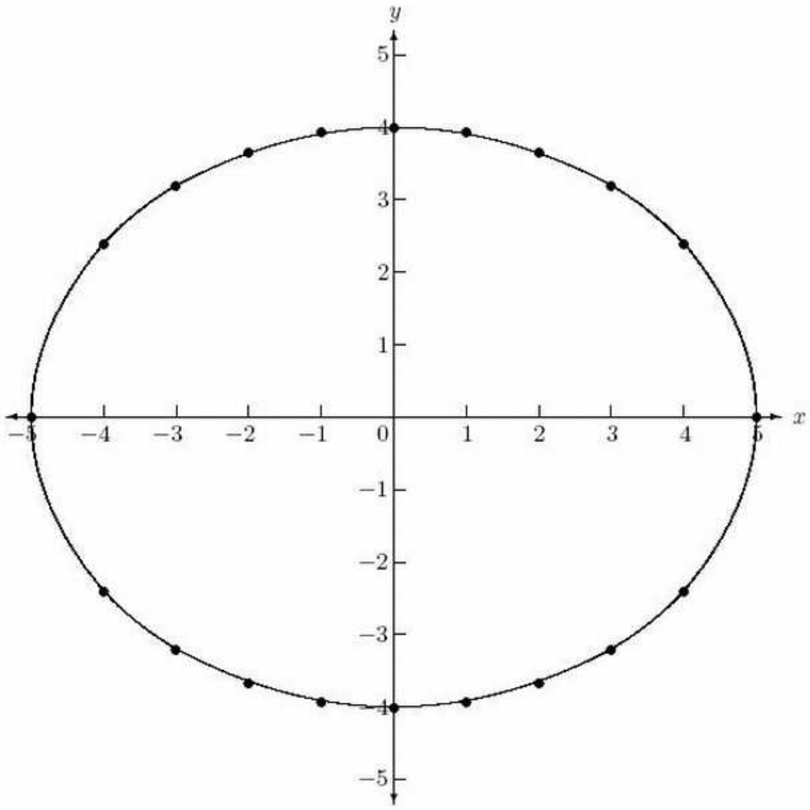
2.6.1.5 Graphing position, velocity, and acceleration with respect to each other

Graphing position to the other functions can be complicated and when tested on it, most student are unable to give the right answer. First consider this, for simple harmonic motion position and acceleration are proportional. x = -k a this is a linear relationship so the graph is a line, the slope is negative so the line is heading down.



2.6.1.6 Position and acceleration verses velocity

The position and acceleration verse velocity graph look entirely different. First off the straight line test fails when plotting position vs velocity or acceleration verse velocity. Take the point were x and a are zero, there are two possible answers for the point (the speed maybe at maximum) the object could be moving down or up at the point. That means the velocity can be a positive maximum or a negative maximum, two separate values. Looking a the points were the velocity is equal to zero, there are two possible answers, either at the top or at the bottom. If you continue to plot data points graph that is developed is a ellisipe

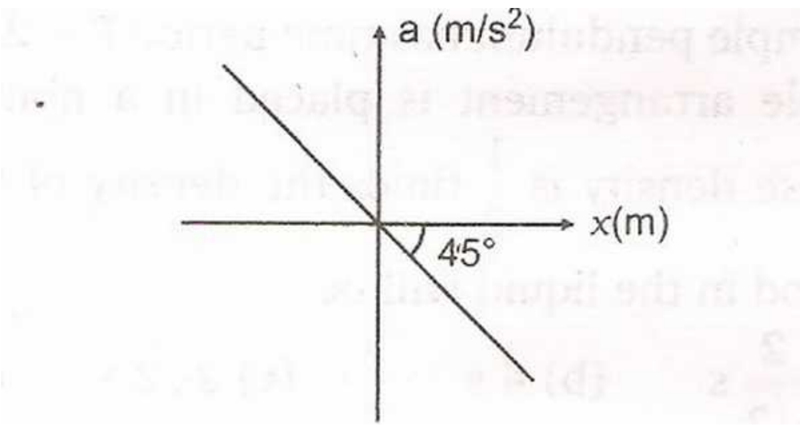


2.6.2 Problems

2.6.2.1 General Problem Set

Single Answer Type

Example 1: Aceleration-displacement graph of a particle executing SHM is as shown in given figure. The time period of its oscillation is (in sec)



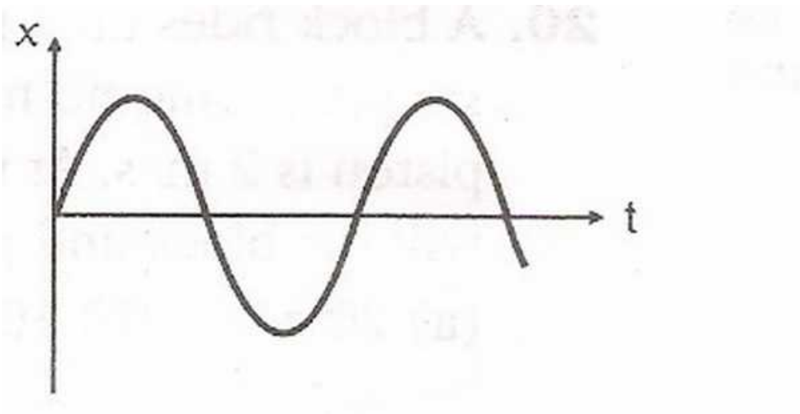
a) $\pi/2$

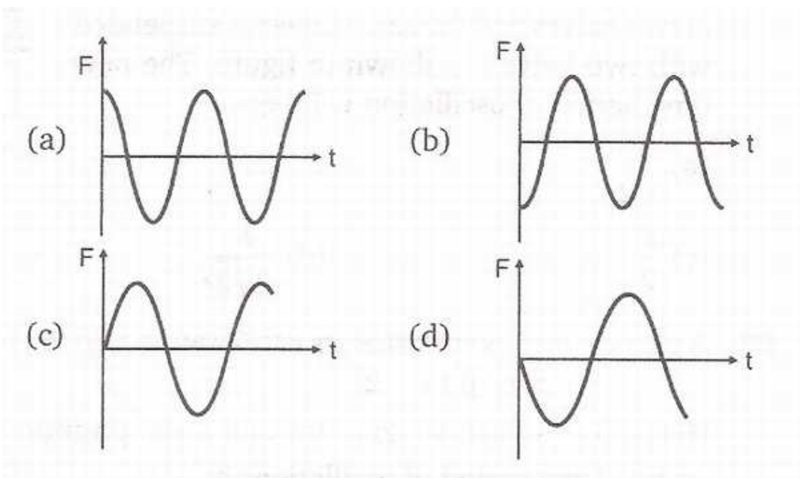
b) 2π

c) π

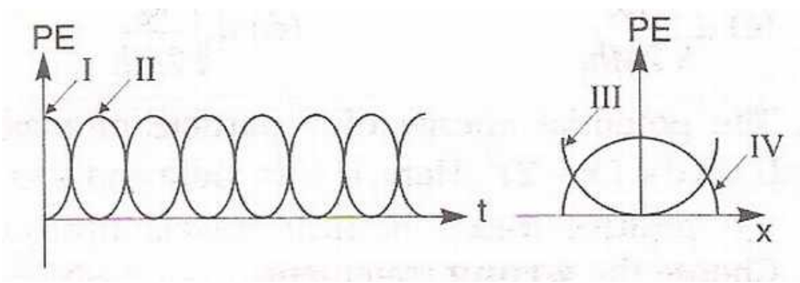
d) $\pi/4$

Example 2: Displacement-time graph of a particle executing SHM is as shown.





Example 3: For a particle executing SHM the displacement x is given by $x = A\cos\omega t$. Identify the graph which represents the variation of potential energy (PE) as a function of time t and displacement x



a) I, III

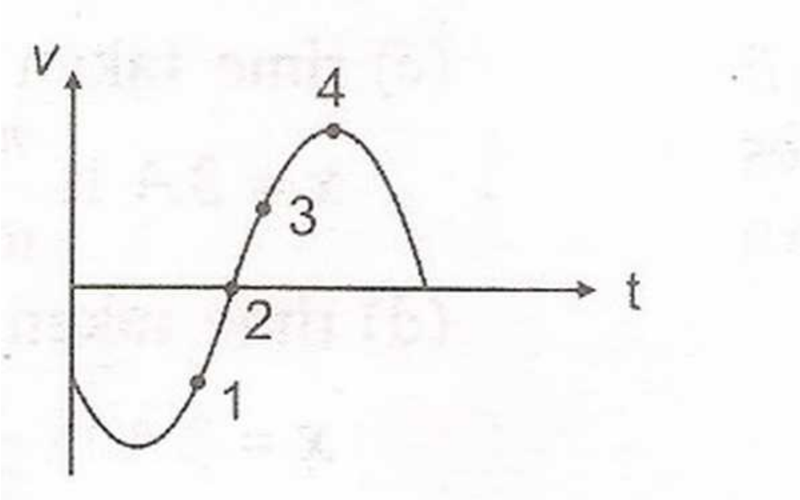
b) II, IV

c) II, III

d) I, IV

Multiple Answer Type

Example 1: Velocity-time graph of a particle executing SHM is shown in figure. Select the correct alternative(s).



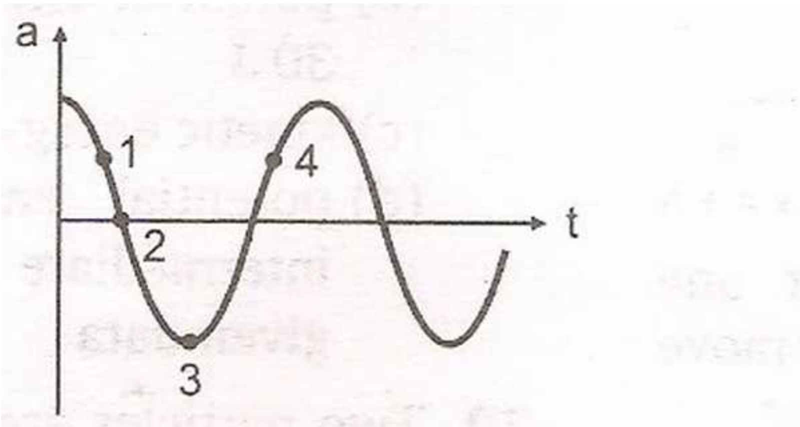
a) At position 1 displacement of particle may be positive or negative.

b) At position 2 displacement of particle is negative.

c) At position 3 acceleration of particle is positive.

d) At position 4 acceleration of particle is positive.

Example 2: Acceleration-time graph of a particle executing SHM is as shown in figure. Select the correct alternative(s).



a) Displacement of particle at 1 is negative.

b) Velocity of particle at 2 is positive.

c) Potential energy of particle at 3 is maximum.

d) Speed of particle at 3 is decreasing.

Matching Type Questions

Example 1: Velocity-time graph of a particle in SHM is as shown in figure. Match the following

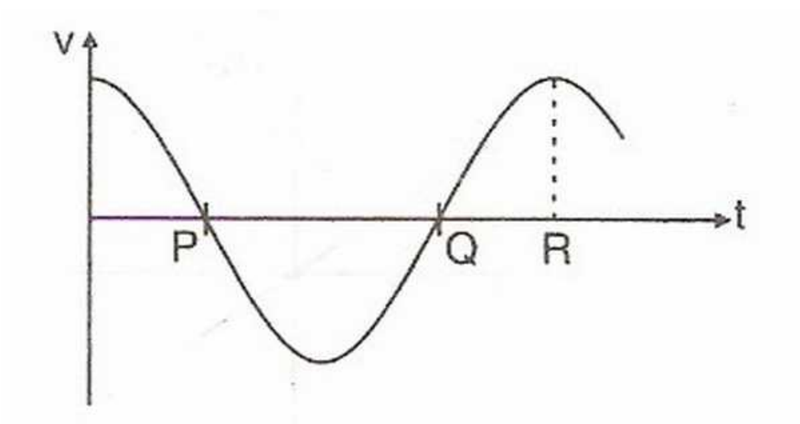
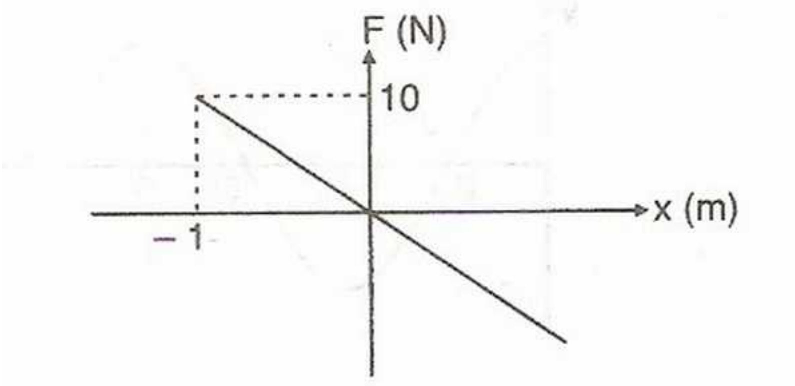


Table-1	Table-2
(A) At P	(P) Particle is at $x = -A$
(B) At Q	(Q) Acceleration of particle is maximum
(C) At R	(R) Displacement of particle is zero
	(S) Acceleration of particle is zero
	(T) None

Example 2: F-x and x-t graph of a particle in SHM are as shown in figure. Match the following



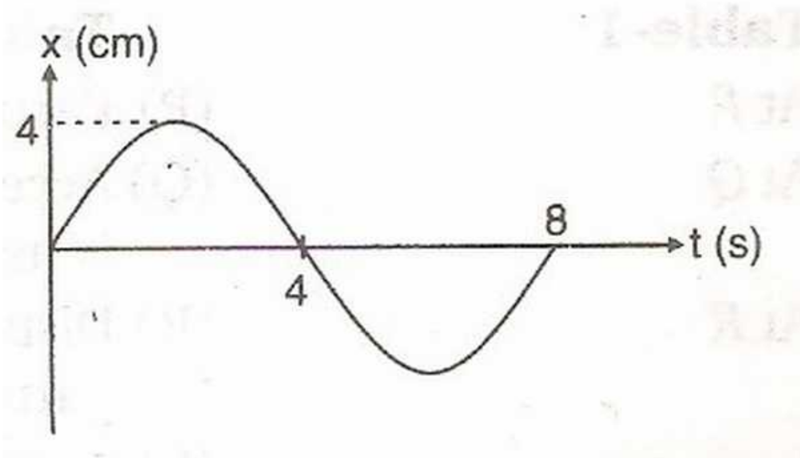
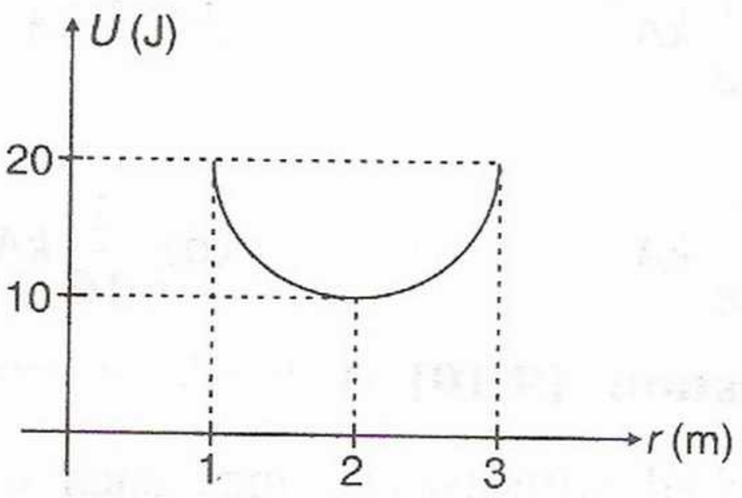


Table-1	Table-2
(A) Mass of the particle	(P) $\pi/2$ SI unit
(B) Maximum kinetic energy of particle	(Q) $(160/\pi^2)$ SI unit
(C) Angular frequency of particle	(R) (8.0×10^{-3}) SI unit
	(T) None

Comprehension Type Questions

Comprehension 1 Concept : In SHM, force $\left(F = -\frac{dU}{dr}\right)$ on the particle at mean position is zero. Potential energy at extreme position is maximum

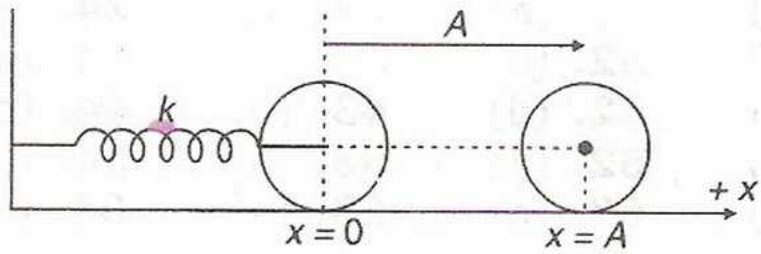
Example: U-r graph of a particle which can be under SHM is as shown in figure. What conclusion cannot be drawn from the graph?



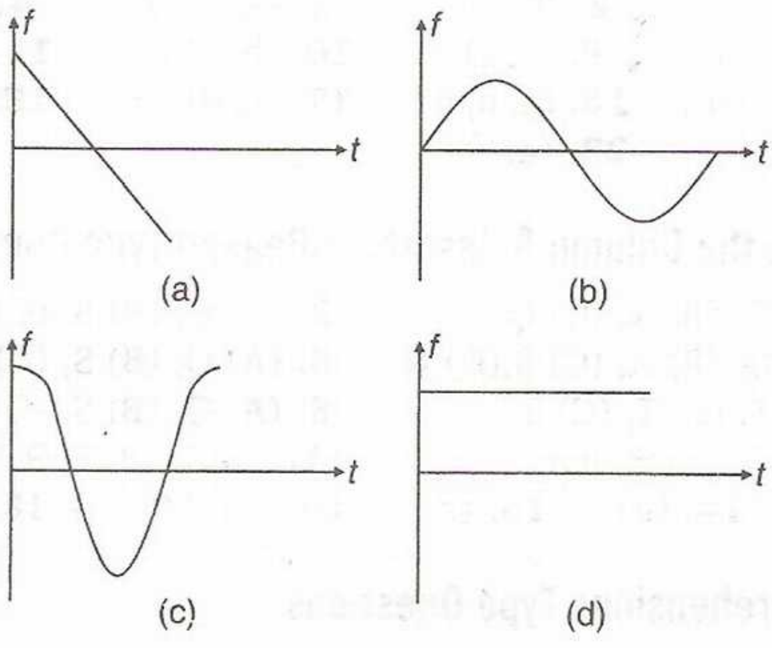
a) Mean position of the particle is at r=2m.
b) Potential energy of particle at mean position is 10 J.
c) Amplitude of oscillation is 1 m.
d) None of these.

Comprehension 2 Statement : In case of pure rolling $a = R\alpha$, where a is the linear acceleration and α the angular acceleration.

Question : A disc of mass m and radius R is attached with a spring of force constant k at its centre as shown in figure. At x=0, spring is unstretched. The disc is moved to x=A and then released. There is no slipping between disc and ground. Let f be the force of friction on the disc from the ground.



Example 1 : f versus t (time) graph will be as



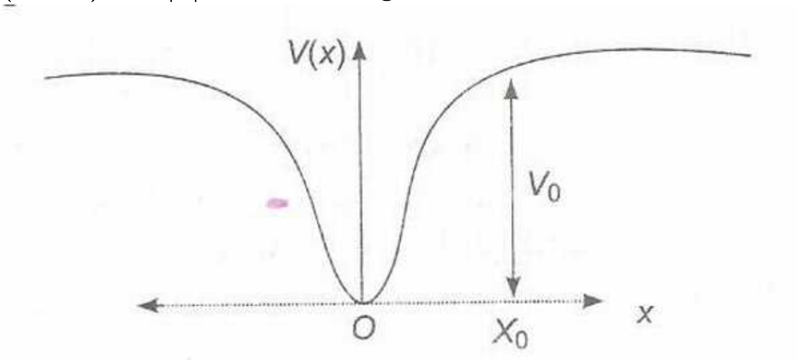
Example 2 : In the problem if k = 10 N/m, m= 2 kg, R = 1 m and A = 2 m. Find linear speed of the disc at mean position

a) $\sqrt{\frac{40}{3}}$ m/s
b) $\sqrt{20}$ m/s
c) $\sqrt{\frac{10}{3}}$m/s
d) $\sqrt{\frac{50}{3}}$ m/s

2.6.2.2 Previous Years IIT Problems

Paragraph

Paragraph 1: When a particle of mass m moves on the x-axis in a potential of the form $V(x) = kx^2$ it performs simple harmonic motion. The corresponding time period is proportional to $\sqrt{\frac{m}{k}}$, as can be seen easily using dimensional analysis. However, the motion of a particle can be periodic even when its potential energy increases on both sides of x=0 in a way different from kx^2 and its total energy is such that the particle does not escape to infinity. Consider a particle of mass m moving on the x-axis. Its potential energy is $V(x) = \alpha x^4$ ($\alpha > 0$) for |x| near the origin and becomes a constant equal to Vo for $|x| \geq X_0$ (see figure)



1. If the total energy of the particle is E, it will perform periodic motion only if

a) E<0
b) E>0
c) Vo>E>0
d) E>Vo

2. For periodic motion of small amplitude A, the time period T of this particle is proportional to

a) $A\sqrt{\frac{m}{\alpha}}$
b) $\frac{1}{A}\sqrt{\frac{m}{\alpha}}$
c) $A\sqrt{\frac{\alpha}{m}}$
d) $\frac{1}{A}\sqrt{\frac{\alpha}{m}}$

3. The acceleration of this particle for |x| > Xo is

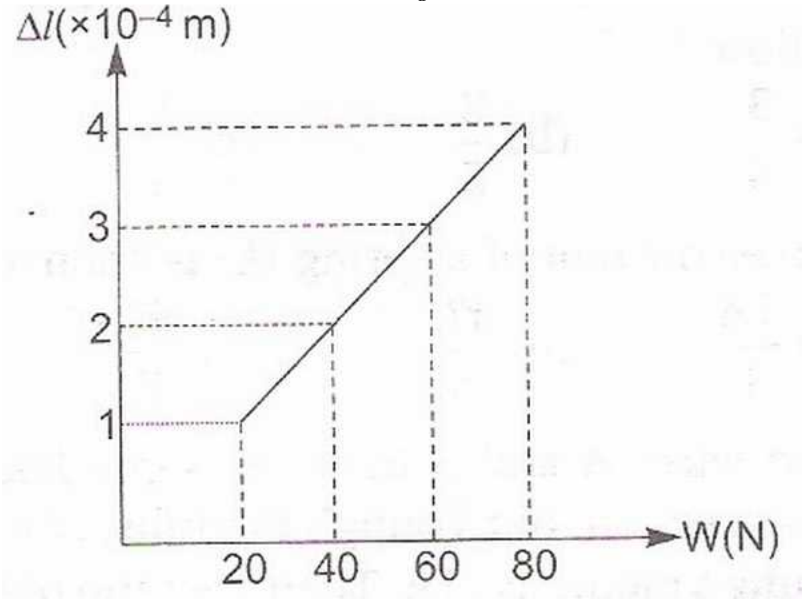
a) Proportional to Vo
b) Proportional to Vo/mXo
c) Proportional to $\sqrt{Vo/mXo}$
d) Zero

2.7 Statics

2.7.1 Modulii of Elasticity

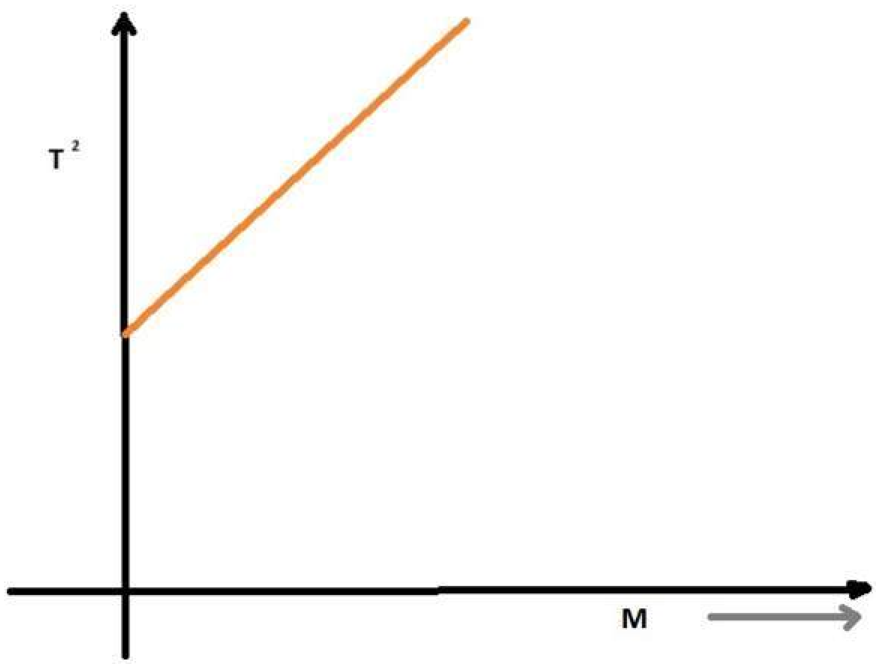
2.7.1.1 General Problem Set

Single Answer Type Example 1 : The graph shows the extension ($\triangle l$) of a wire of length 1.0m suspended from the top of a roof at one end and with a load W connected to the other end. If the cross-sectional area of the wire is $10^{-6}m^2$, calculate the Young's modulus of the material of the wire



a) 2×10^{11} N/m^2
b) 2×10^{10} N/m^2
c) 2×10^{12} N/m^2
d) 2×10^{13} N/m^2

Example : The graph shown was obtained from experimental measurements of the period of oscillation T for different masses M placed in the scale pan on the lower end of the spring balance. The most likely reason for the line not passing through the origin is that the



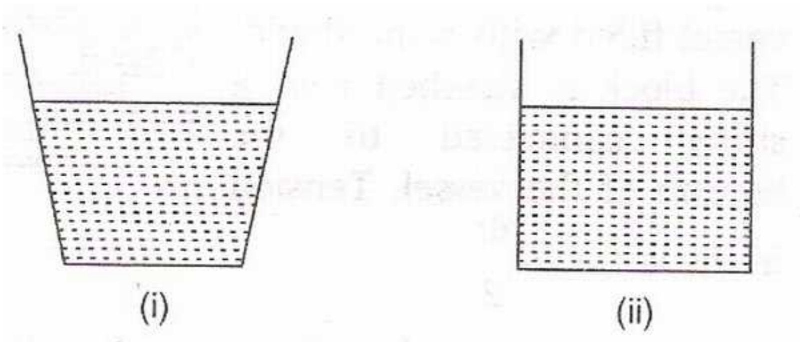
(a) Spring did not obey Hooke's law
(b) Amplitude of oscillation was too large
(c) Clock used needed regulating
(d) Mass of the pan was neglected

{ Hint: Answer D }

Comprehension Type

Comprehension 1 Concept : Free body diagram of liquid can be drawn in similar manner as we draw the free body diagram of a solid. The only difference is, what we call the normal reaction between solid-solid boundary, we here call it pressure X area in case of liquids. Both are prependicular to the surface.

Example : Equal amounts of liquid are filled in two vessels of different shapes as shown in figure. Let F_1be the force by the base on liquid in case (i) and F_2 in case (ii) Then



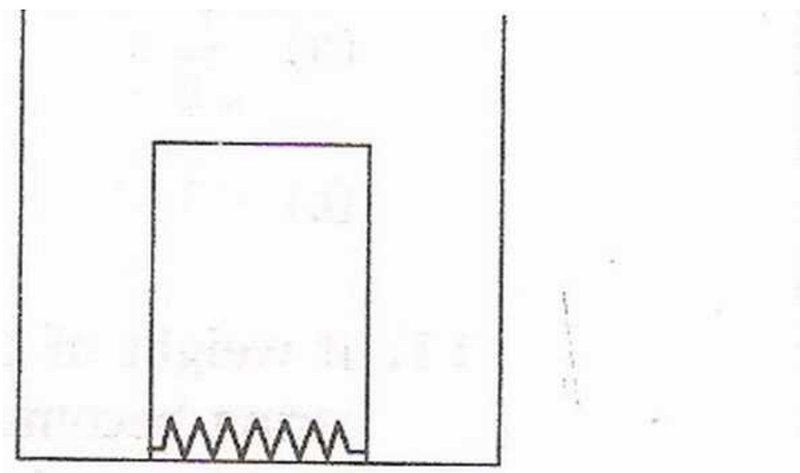
a) F_1 >F_2

b) F_1 <F_2

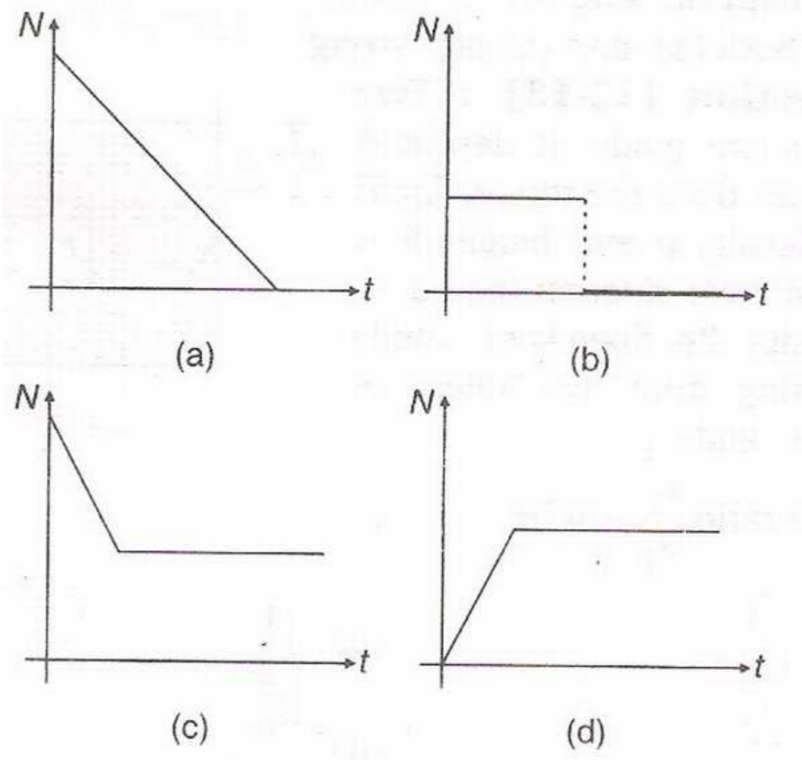
c) F_1 =F_2

d) Data insufficient

Question : A cube (side = 10 cm) of density 0.5 g/cm^3 is placed in a vessel of base area 20 cm X 20 cm. A liquid of density 1.0 g/cm^3 is gradually filled in the vessel at a constant rate Q = 50 cm^3/s.



Example : If we plot a graph between the normal reaction on cube by the vessel versus time. The graph will be like



Example: The cube will leave contact with the vessel after time t= .... s
a) 30
b) 40
c) 60
d) 20

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Chapter 3

Heat

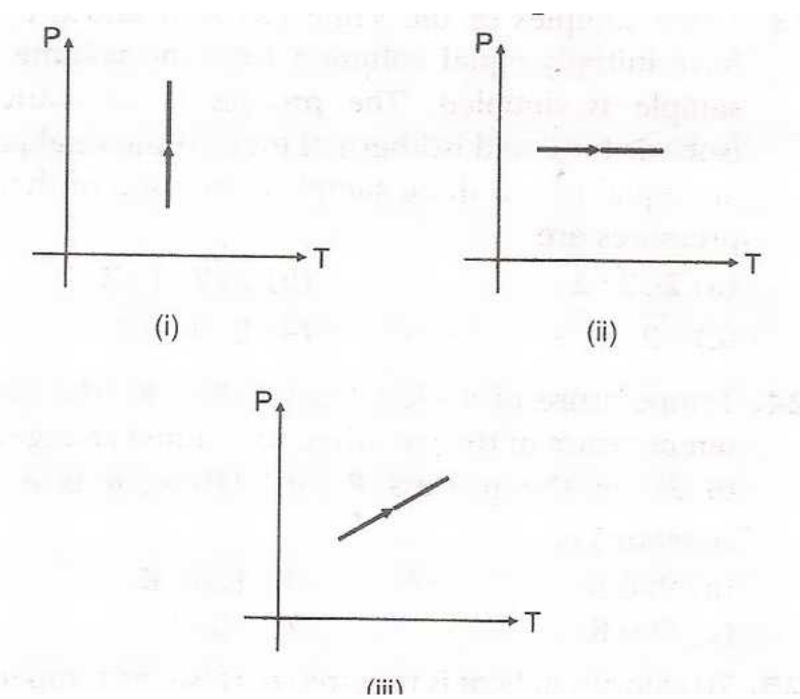
3.1 Thermodynamics

3.1.1 Practice Problems

3.1.1.1 General Problem Set

Single Answer Type

Example : Pressure versus temperature graphs of an ideal gas are as shown in figure. Choose the wrong statement



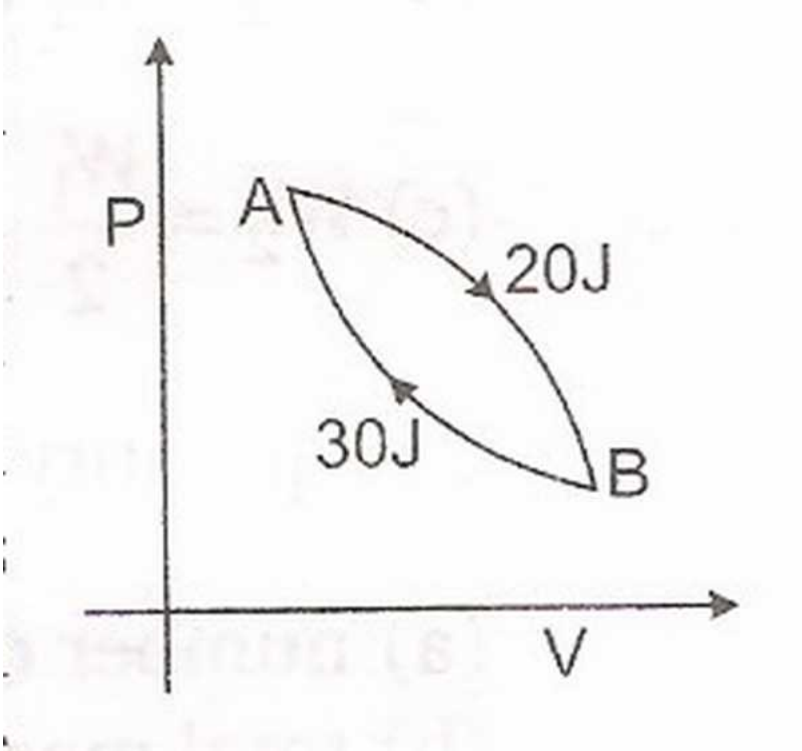
a) Density of gas is increasing in graph (i)

b) Density of gas is decreasing in graph (ii)

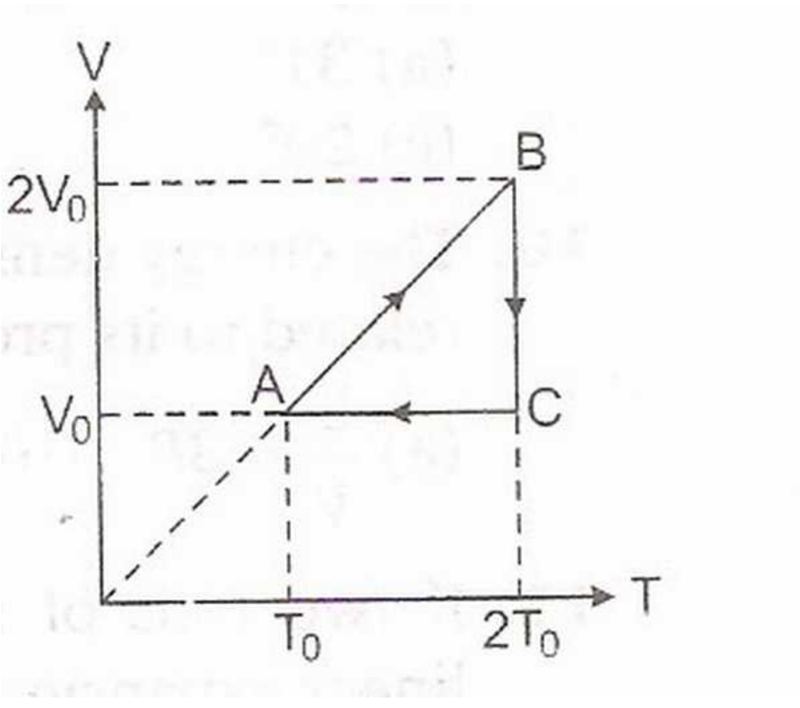
c) Density of gas is constant in graph (iii)

d) None of the above

Example : In a cyclic process shown in the figure an ideal gas is adiabatically taken from B to A, the work done on the gas during the process B->A is 30J, when the gas is taken from A->B the heat absorbed by the gas is 20 J. The change in internal energy of the gas in the process A-> B is



Example : An ideal monoatomic gas undergoes a cyclic process ABCA as shown in the figure. The ratio of heat absorbed during AB to the work done on the gas during BC is



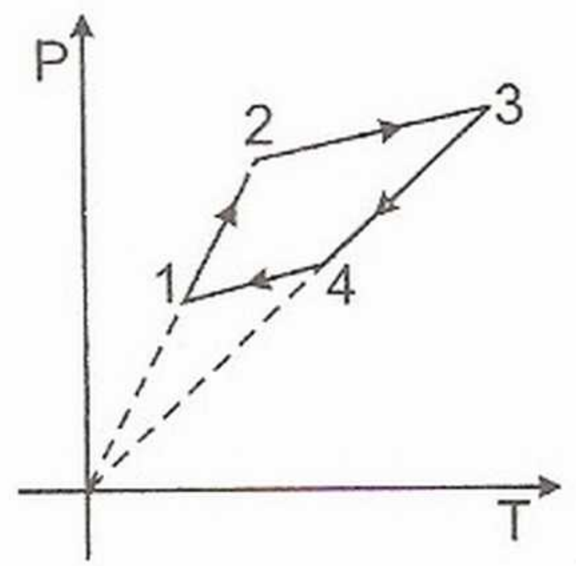
a) 5/2ln2

b) 5/3

c) 5/4ln2

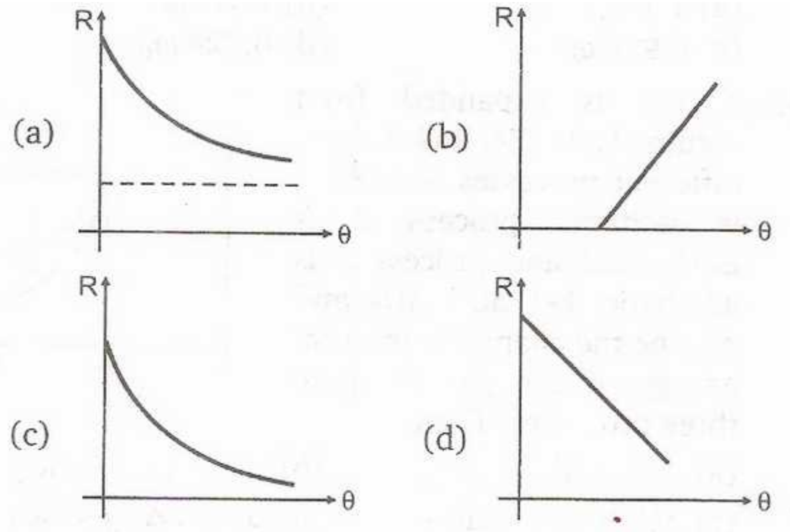
d) 5/6

Example : Three moles of an ideal monoatomic gas performs a cycle 1->2->3->4->1 as shown. The gas temperatures in different states are T1=400K, T2=800K, T3=2400K and T4=1200K. The work done by the gas during the cycle is (2-3 and 4-1 are isobaric)

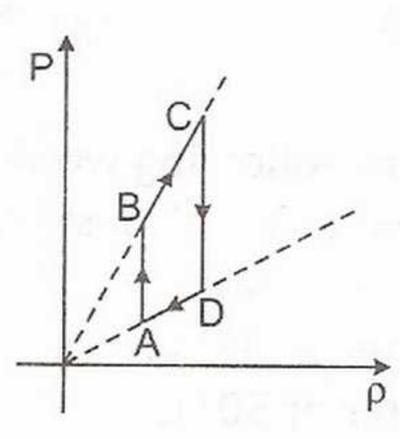


a) 1200 R
b) 3600 R
c) 2400 R
d) 2000 R

Example : Temperature of a body θ is slightly more than the temperature of the surrounding θ_o . Its rate of cooling (R) versus temperature of body (θ) is plotted, its shape would be



Example : Pressure versus density graph of an ideal gas is shown in figure



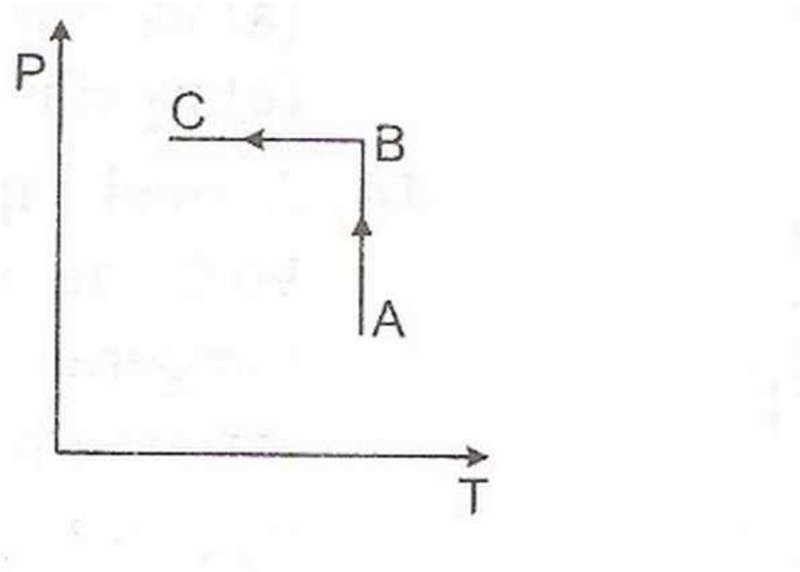
a) during the process AB work done by the gas is positive

b) during the process AB work done by the gas is negative

c) during the process BC internal energy of the gas is increasing

d) None of the above

Example : Ideal gas is taken through the process shown in the figure



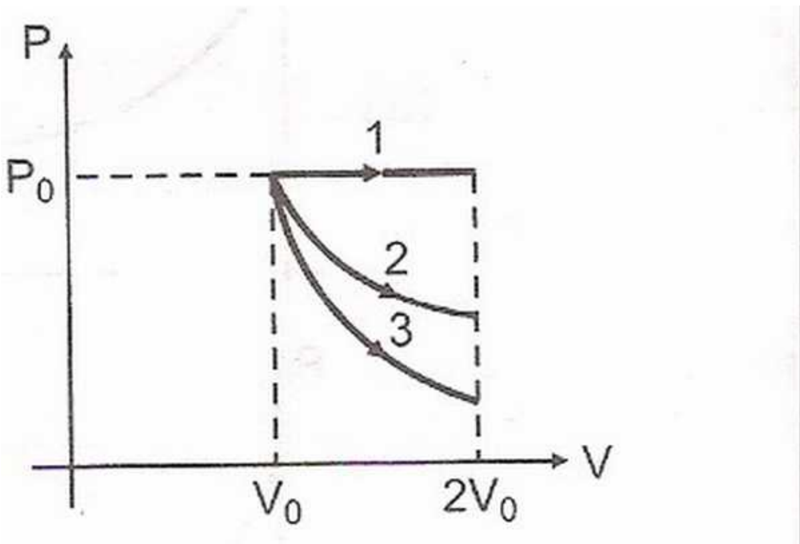
a) In process AB, work done by system is positive

b) In process AB, heat is rejected

c) In process AB, internal energy increases

d) In process AB internal energy decreases and in process BC, internal energy increases

Example : A gas is expanded from volume V_0 to $2V_0$ under three different processes. Process 1 is isobaric, process 2 is isothermal and process 3 is adiabatic. Let $\triangle U_1$,$\triangle U_2$ and $\triangle U_3$ be the change in internal energy of the gas in these three processes. Then



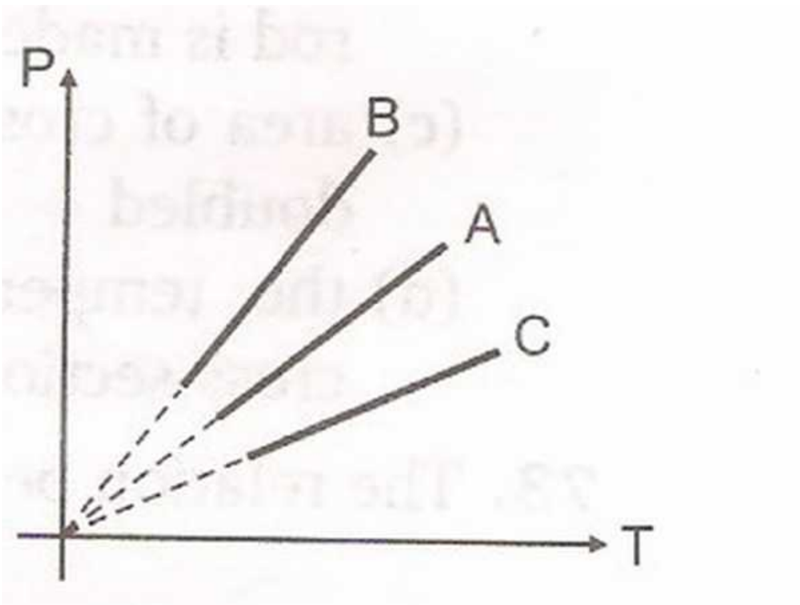
a) $\triangle U_1 > \triangle U_2 > \triangle U_3$

b) $\triangle U_1 < \triangle U_2 < \triangle U_3$

c) $\triangle U_2 < \triangle U_1 < \triangle U_3$

d) $\triangle U_2 < \triangle U_3 < \triangle U_1$

Example : Pressure versus temperature graph of an ideal gas at constant volume V is shown by the straight line A. Now mass of the gas is doubled and the volume is halved, then the corresponding pressure versus temperature graph will be shown by the line



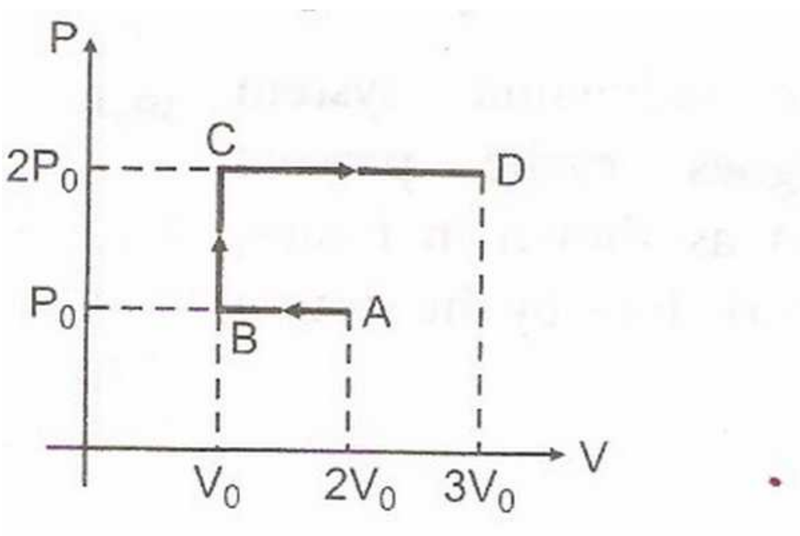
a) A

b) B

c) C

d) None of these

Example : P-V diagram of an ideal gas is as shown in figure. Work done by the gas in the process ABCD is



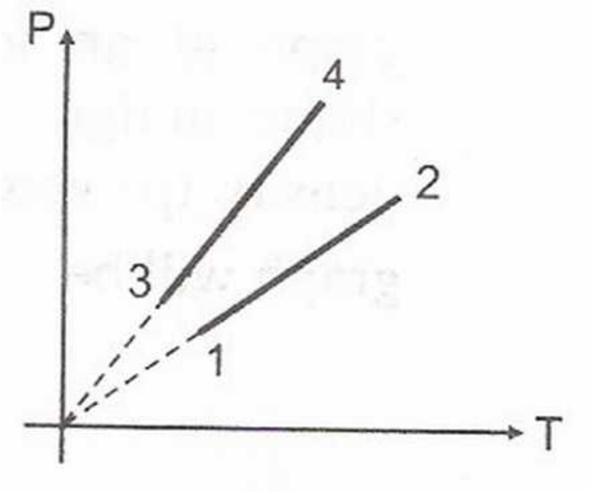
a) $4P_0V_0$

b) $2P_0V_0$

c) $3P_0V_0$

d) P_0V_0

Example : Pressure versus temperature graph of an ideal gas of equal number of moles of different volumes are plotted as shown in figure. Choose the correct alternatives.



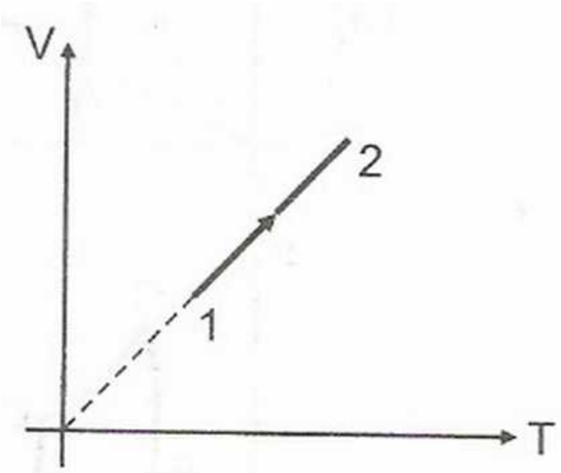
a) $V_1 = V_2$, $V_3 = V_4$ and $V_2 > V_3$

b) $V_1 = V_2$, $V_3 = V_4$ and $V_2 < V_3$

c) $V_1 = V_2 = V_3 = V_4$

d) $V_4 > V_3 > V_2 > V_1$

Example : Volume versus temperature graph of two moles of helium gas is as shown in figure. The ratio of heat absorbed and the work done by the gas in process 1-2 is



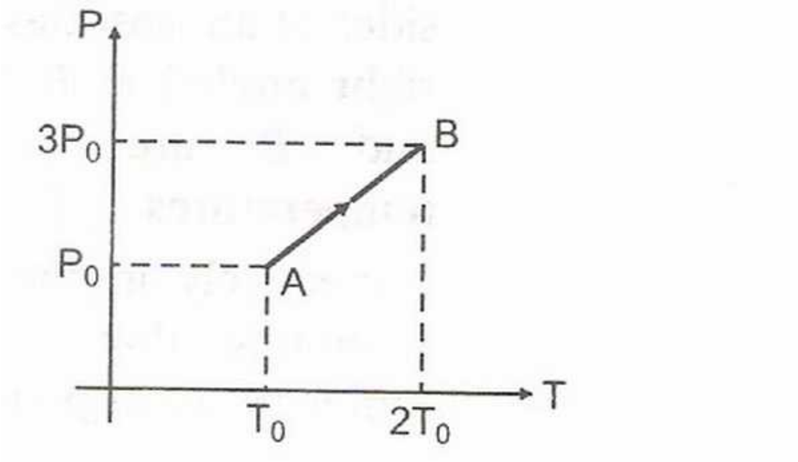
a) 3

b) 5/2

c) 5/3

d) 7/2

Example : Pressure versus temperature graph of an ideal gas is as shown in figure. Density of the gas at point A is ρ_o. Density at B will be



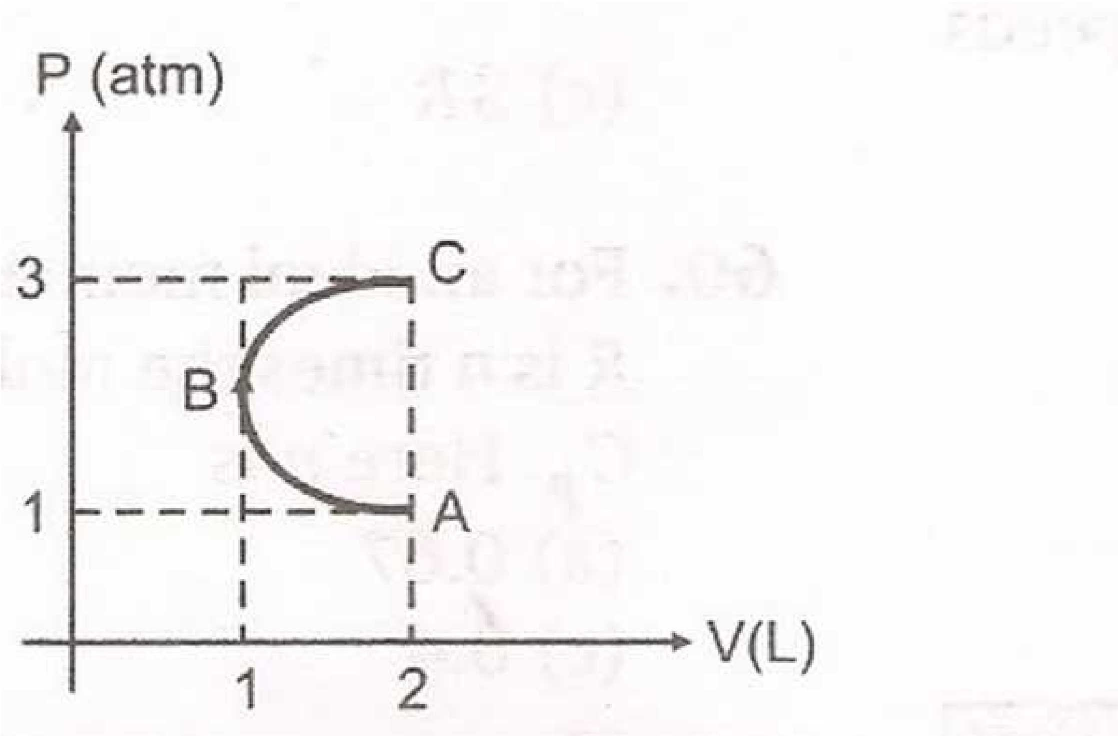
a) $\frac{3}{4}\rho_o$.

b) $\frac{3}{2}\rho_o$.

c) $\frac{4}{3}\rho_o$.

d) $2\rho_o$.

Example : In the P-V diagram shown in figure ABC is a semicircle. The work done in the process ABC is



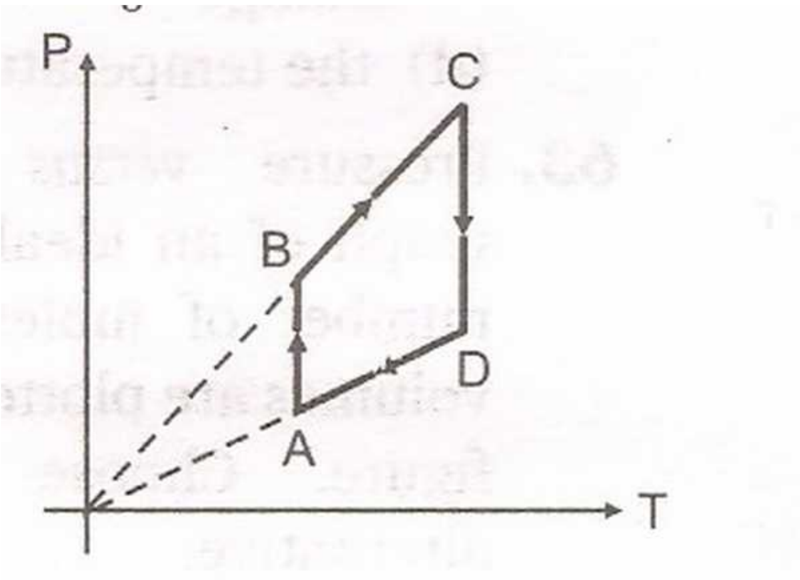
a) zero

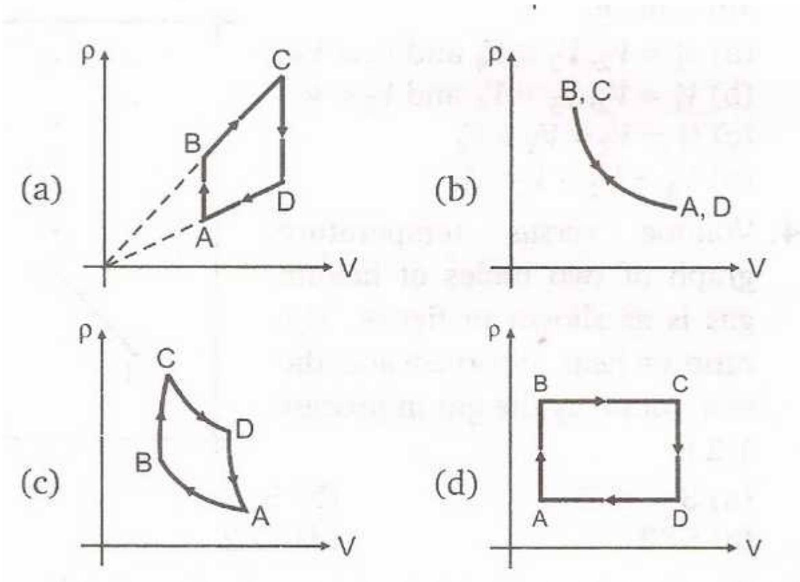
b) $\frac{\pi}{2}$ atm-L

c) -$\frac{\pi}{2}$ atm-L

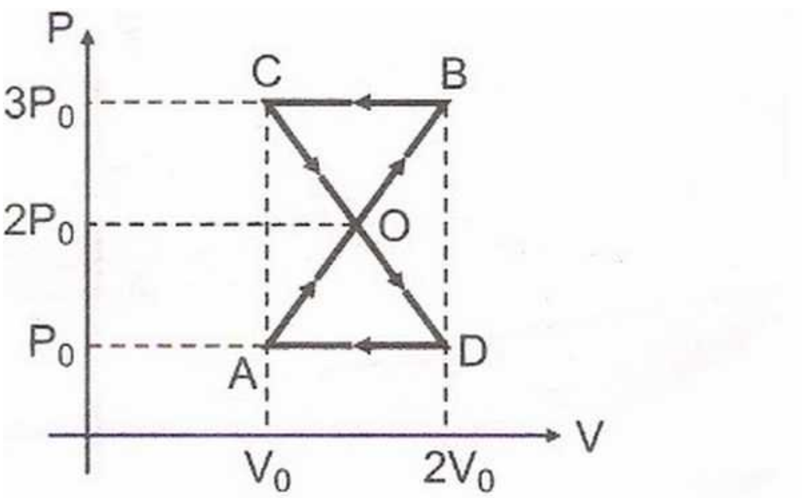
d) 4 atm-L

Example : Pressure versus temperature graph of an ideal gas is as shown in figure corresponding density (ρ) versus volume (V) graph will be





Example: A thermodynamic system undergoes cyclic process ABCDA as shown in figure.The work done by the system is



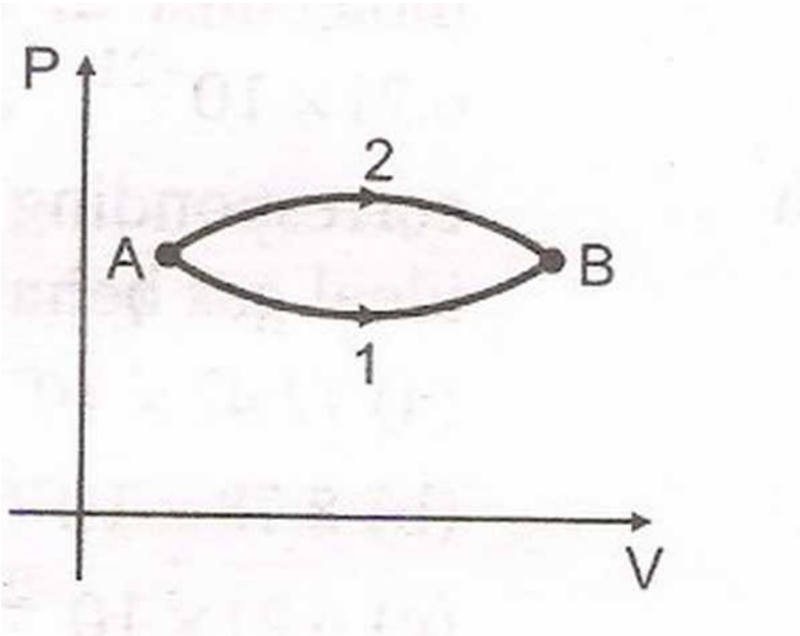
a) PoVo

b) 2PoVo

c) PoVo/2

d) zero

Example : The figure shows two paths for the change of state of a gas from A to B. The ratio of molar heat capacities in path 1 and path 2 is



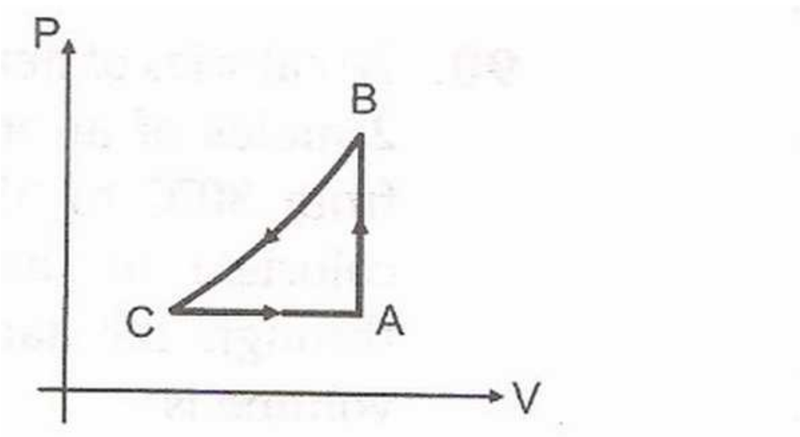
a) > 1

b) < 1

c) 1

d) Data insufficient

Example : A sample of an ideal gas is taken through a cycle as shown in figure. It absorbs 50 J of energy during the process AB, no heat during BC, rejects 70J during CA. 40 J of work is done on the gas during BC. Internal energy of gas at A is 1500 J, the internal energy at C would be



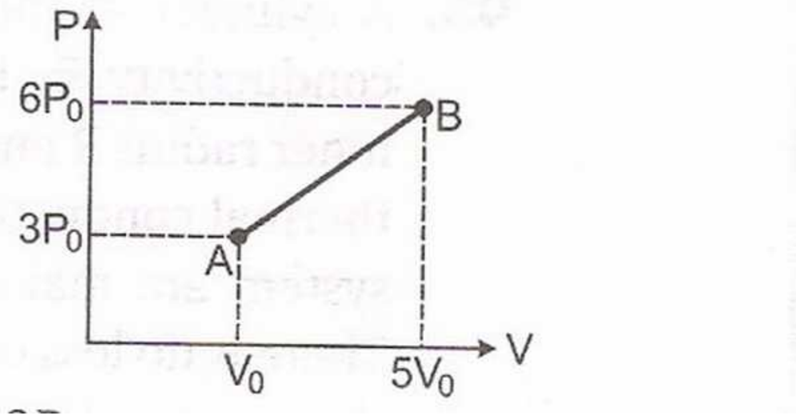
a) 1590 J

b) 1620 J

c) 1540 J

d) 1570 J

Example : One mole of a monoatomic ideal gas undergoes the process A -> B in the given P-V diagram. The specific heat for this process is



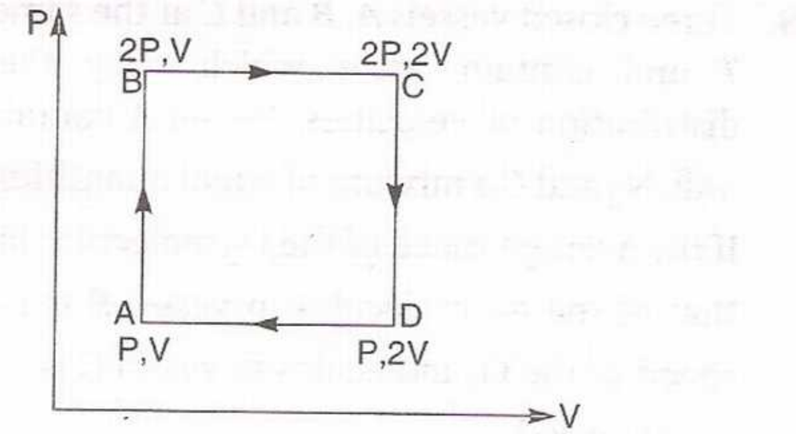
a) 3R/2

b) 13R/6

c) 5R/2

d) 2R

Example : An ideal monoatomic gas is taken round the cycle ABCDA as shown in the P-V diagram (see figure). The work done during the cycle is



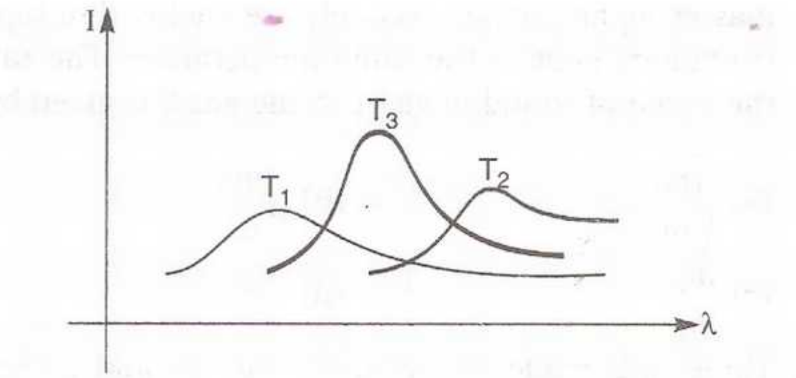
a) PV

b) 2 PV

c) PV/2

d) zero

Example : The plots of intensity versus wavelength for three black bodies at temperatures T_1 ,T_2 and T_3 respectively are as shown. Their temperatures are such that



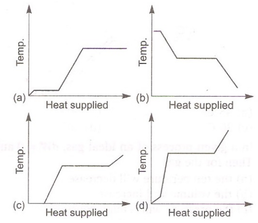
a) T_1 >T_2 > T_3

b) T_1 >T_3 > T_2

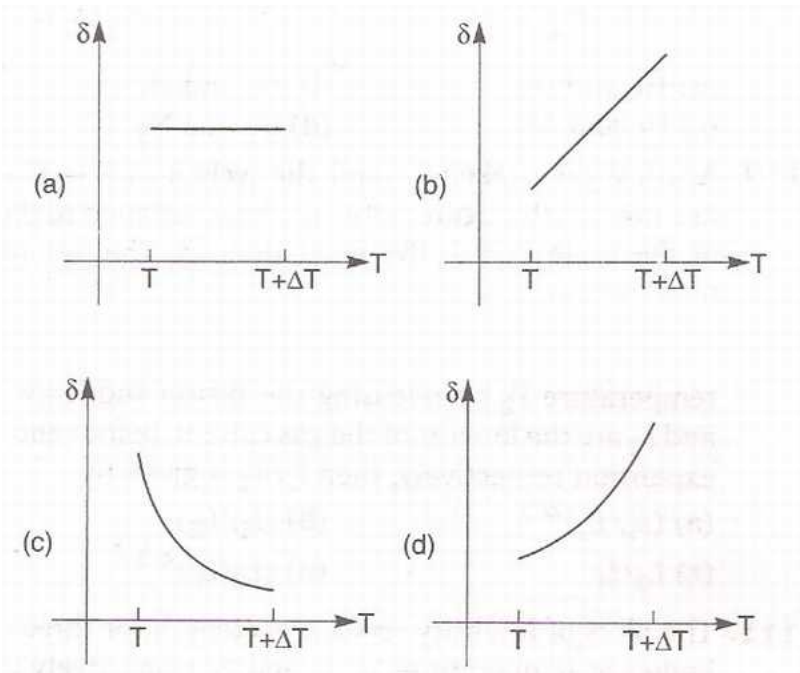
c) $T_2 > T_3 > T_1$

d) $T_3 > T_2 > T_1$

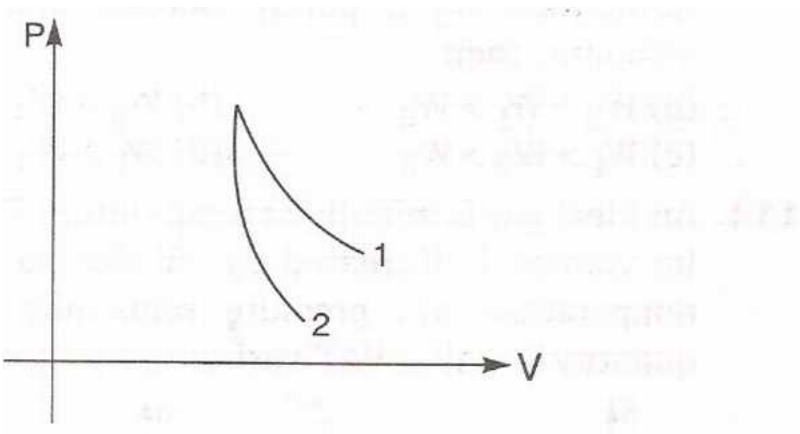
Example : A block of ice at -10^oC is slowly heated and converted to steam at 100^oC . Which of the following curvers represents the phenomenon?



Example : An ideal gas is initially at temperature T and volume V. Its volume is increased by $\triangle V$ due to an increase in temperature $\triangle T$, pressure remaining constant. The quantity $\delta = \dfrac{\triangle V}{V \triangle T}$ varies with temperature as



Example : P-V plots for two gases during adiabatic processes are shown in the figure. Plots 1 and 2 should correspond respectively to



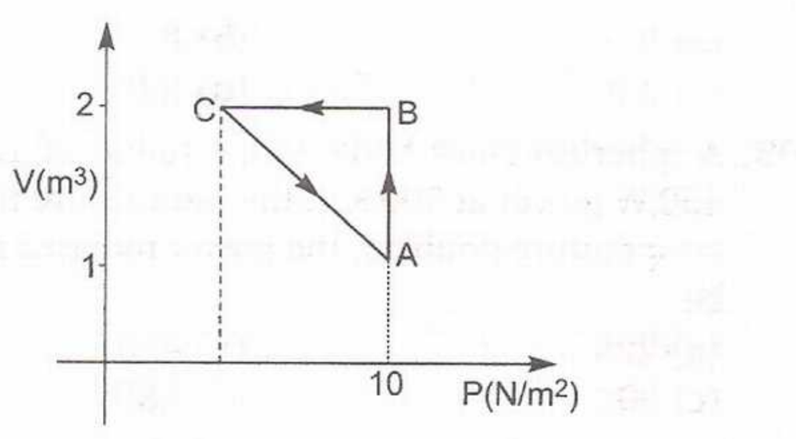
a) He and O_2

b) O_2 and He

c) He and Ar

d) O_2 and N_2

Example : An ideal gas is taken through the cycle A->B->C->A as shown in the figure. If the net heat supplied to the gas in the cycle is 5 J, the work done by the gas in the process C->A is



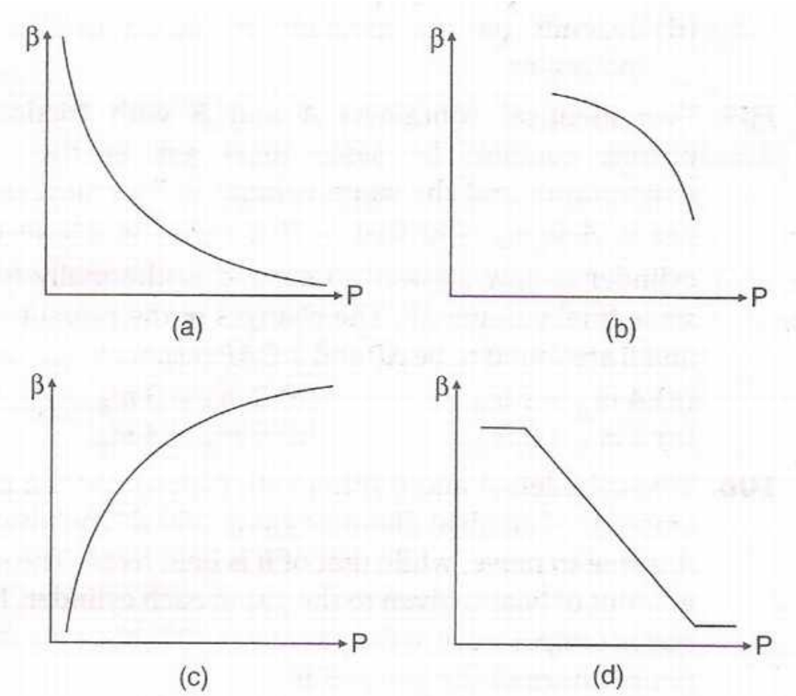
a) -5 J

b) -10 J

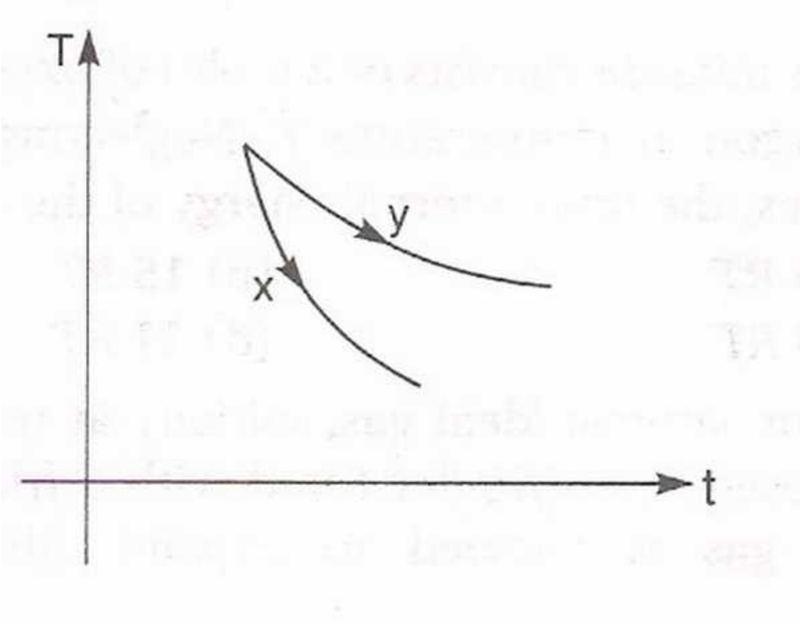
c) -15 J

d) -20 J

Example : Which of the following graphs correctly represents the variation of $\beta = -\frac{dV/dP}{V}$ with P for an ideal gas at constant temperature?



Example : The graph, shown in the diagram, represents the variation of temperature (T) of the bodies, x and y having same surface area, with time (t) due to the emission of radiation. Find the correct relation between the emissivity and absorptivity of the two bodies



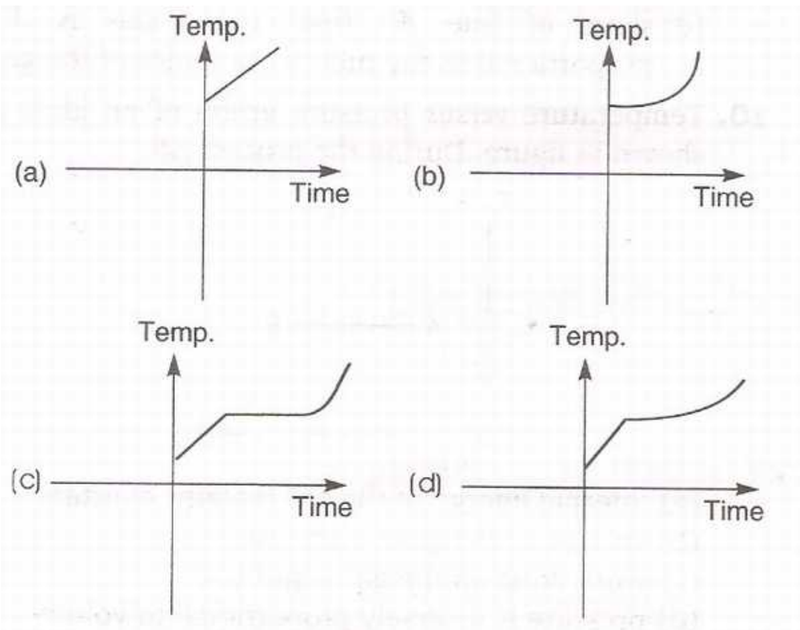
a) $E_x > E_y$ and $a_x < a_y$

b) $E_x < E_y$ and $a_x > a_y$

c) $E_x > E_y$ and $a_x > a_y$

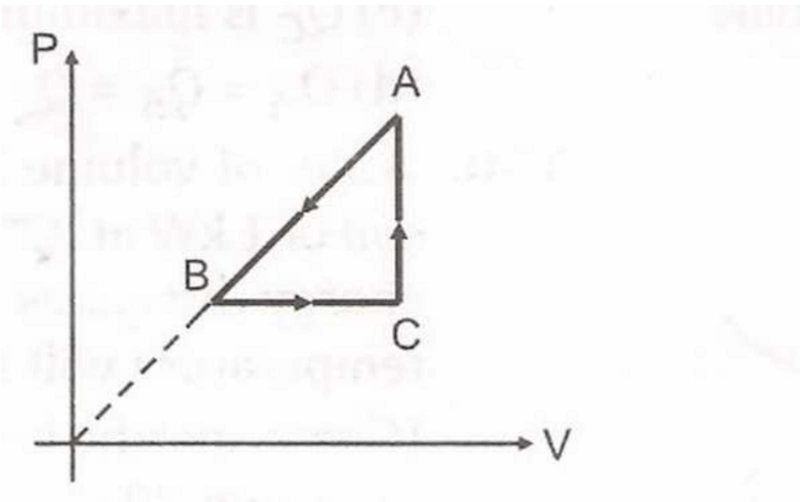
d) $E_x < E_y$ and $a_x < a_y$

Example : Liquid oxygen at 50 K is heated to 300K at constant pressure of 1 atm. The rate of heating is constant. Which of the following graphs represent the variation of temperature with time?



Multiple Answer Type

Example : P-V diagram of a cyclic process ABCA is as shown in figure. Choose the correct statement (s)



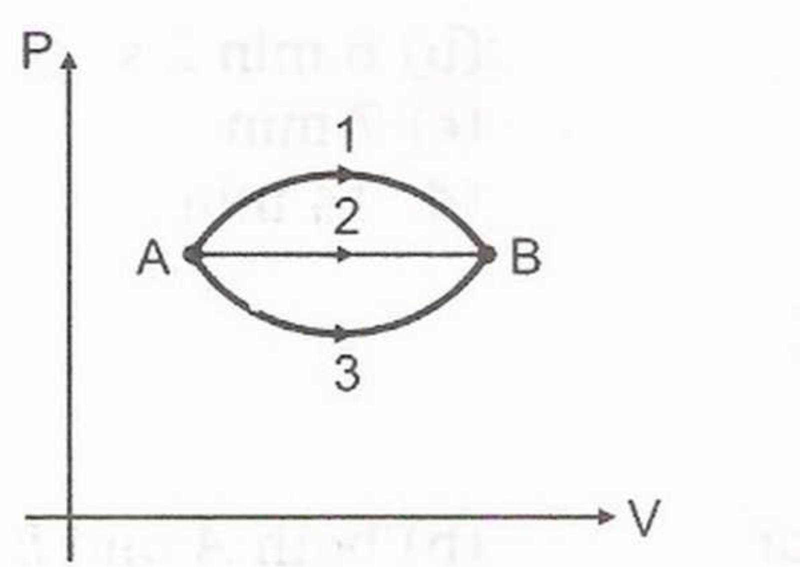
a) $\triangle Q_{A\to B}$ = negative

b) $\triangle U_{B\to C}$ = positive

c) $\triangle U_{C\to A}$ = negative

d) $\triangle W_{CAB}$ = negative

Example : A gas undergoes the change in its state from position A to position B via three different paths as shown in figure.



Select the correct alternative (s).

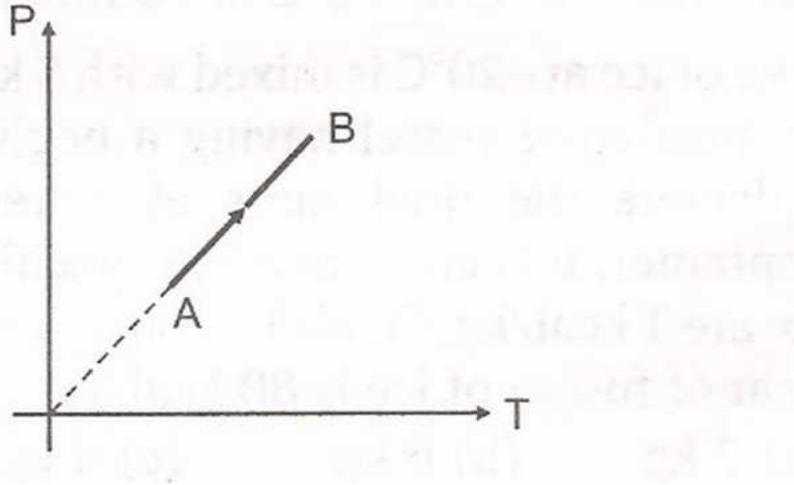
a) Change in internal energy in all the three paths is equal.

b) In all the three paths heat is absorbed by the gas.

c) Heat absorbed / released by the gas is maximum in path 1

d) Temperature of the gas first increases and then decreases in path 1

Example : During the process A-B of an ideal gas



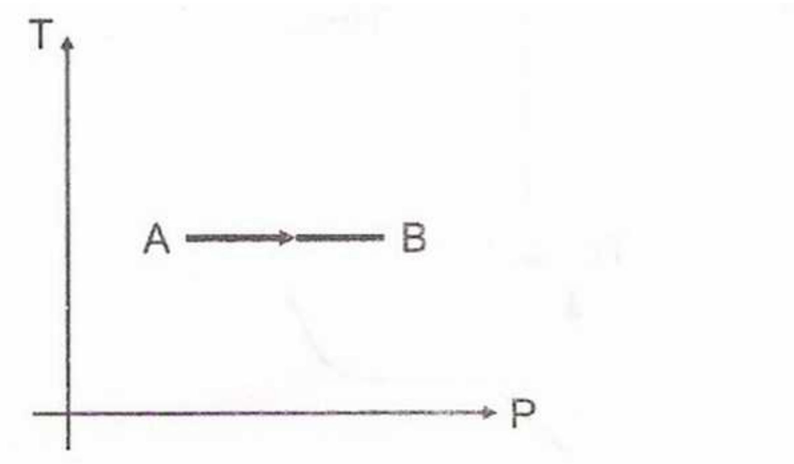
a) work done on the gas is zero

b) density of the gas is constant

c) slope of line AB from the T-axis is inversely proportional to the number of moles of the gas

d) slope of line AB from the T-axis is directly proportional to the number of moles of the gas

Example : Temperature versus pressure graph of an ideal gas is shown in figure. During the process AB



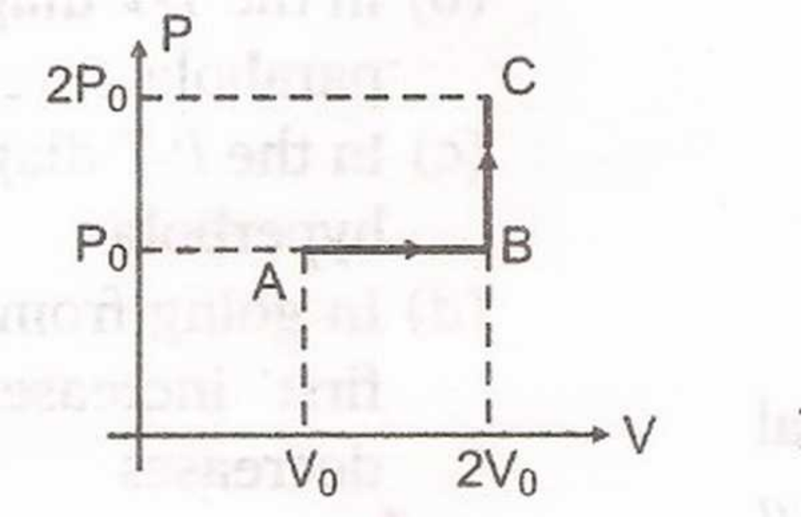
a) internal energy of the gas remains constant

b) volue of the gas is increased

c) work done on the gas is positive

d) pressure is inversely proportional to volume

Example : One mole of an ideal monochromatic gas is taken from A to C along the path ABC. The temperature of the gas at A is T_o. For the process ABC



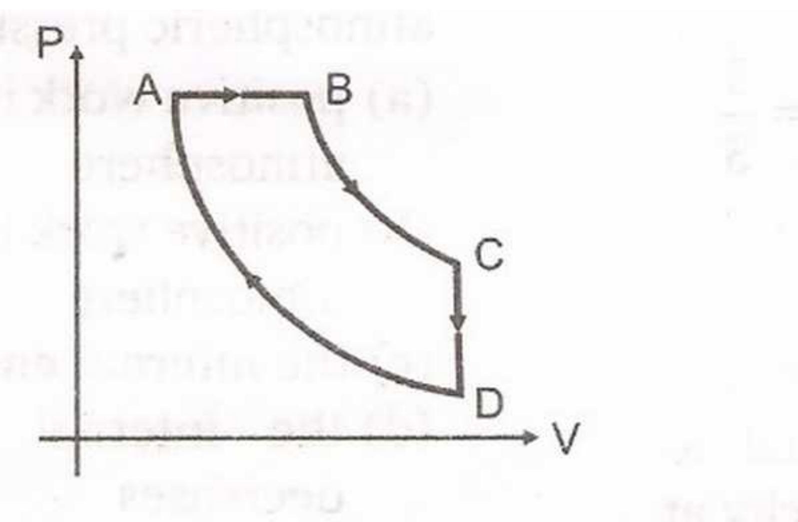
a) work done by the gas is RT_o

b) change in internal energy of the gas is $\frac{11}{2}RT_o$

c) heat absorbed by the gase is $\frac{11}{2}RT_o$

d) heat absorbed by the gas is $\frac{13}{2}RT_o$

Example : n moles of a monoatomic gas undergo a cyclic process ABCDA as shown in figure. Process AB is isobaric, BC is adiabatic, CD is isochoric and DA is isothermal. The maximum and minimum temperature in the cycle are $4T_o$ and T_o respectively. Then



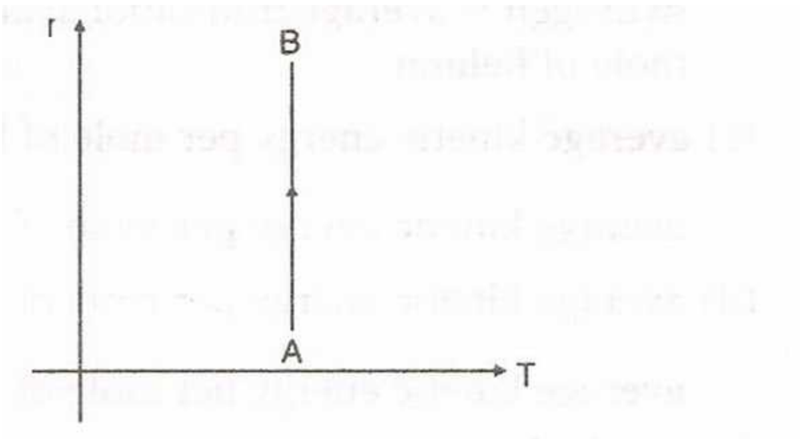
a) $T_B > T_C > T_D$

b) heat is released by the gas in the process CD

c) heat is supplied to the gas in the process AB

d) total heat supplied to the gas is 2nRT_o ln (2)

Example : The density (ρ) of an ideal gas varies with temperature T as shown in figure. Then



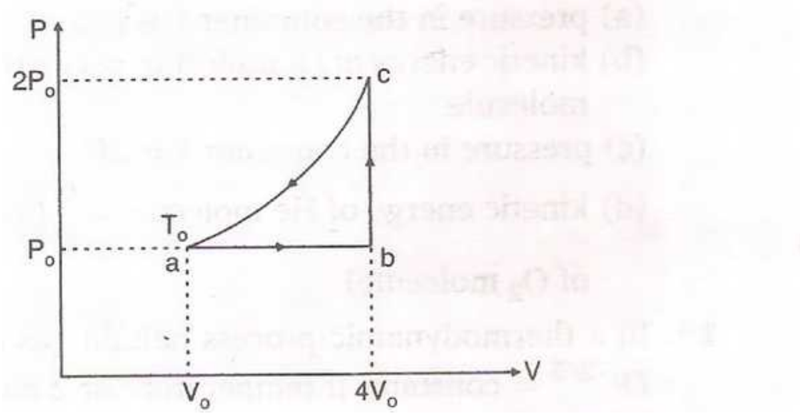
a) the product of P & V at A is equal to the product of P & V at B

b) pressure at B is greater than the pressure at A

c) work done by the gas during the process AB is negative

d) the change in internal energy from A to B is zero

Example : One mole of an ideal monoatomic gas (initial temperature T_o) is made to go through the cycle abca shown in the figure. If U denotes the internal energy, then choose the correct alternatives



a) $U_c - U_a = 10.5RT_o$

b) $U_b - U_a = 4.5RT_o$

c) $U_c > U_b > U_a$

d) $U_c - U_b = 6RT_o$

Matching Type

Matrix Match 1 In the $\rho - T$ graph shown in figure, match the following

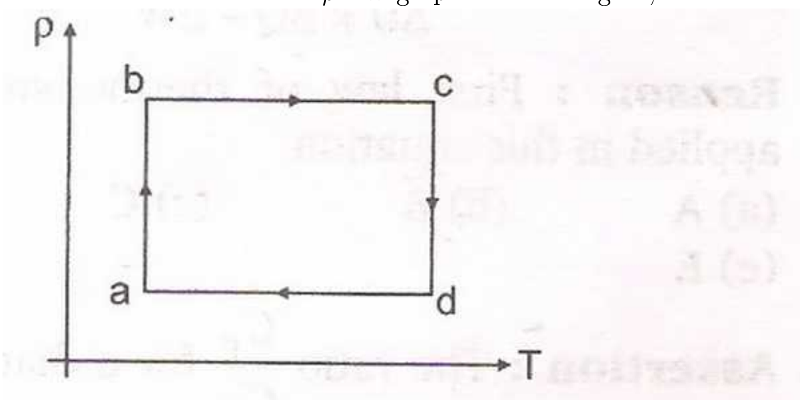


Table-1	Table-2
(A) Process *a*-*b*	(P) Isochoric process
(B) Process *b*-*c*	(Q) $\Delta U = 0$
(C) Process *c*-*d*	(R) P increasing
(D) Process *d*-*a*	(S) P decreasing

Matrix Match 2 In the V-T graph shown in figure match the following

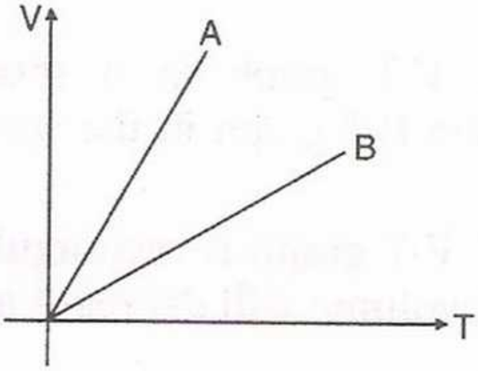
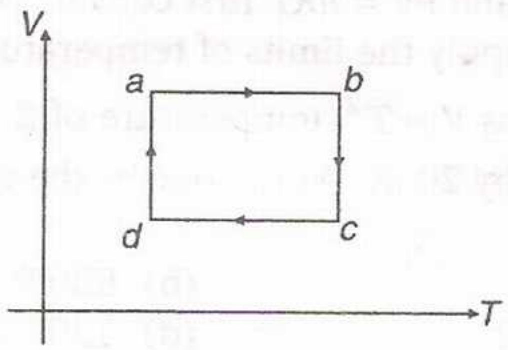


Table-1	Table-2
(A) Gas A and Gas B are ...	(P) monoatomic, diatomic
(B) P_A/P_B is	(Q) diatomic, monoatomic
(C) n_A/n_B is	(R) > 1
	(S) < 1
	(T) Cannot say any thing

Comprehension Type

Comprehension 1

Statement : V-T graph of an ideal gas is as shown in figure.



Question : Work done by the gas in complete cyclic process abcd is

a) zero

b) positive

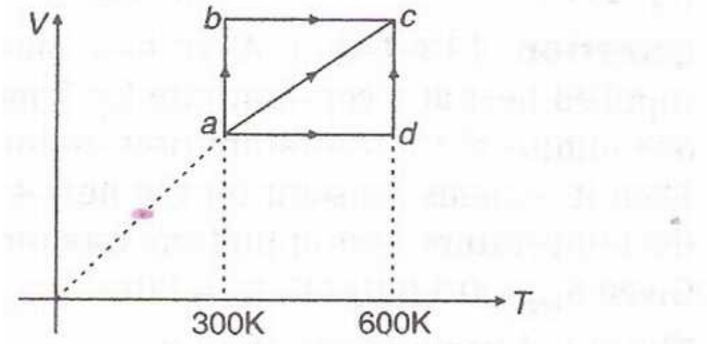
c) negative

d) Data is insufficient

Question : Heat is supplied to the gas in process (es)

a) da, ab and bc
b) da and ab only
c) da only
d) ab and bc only

Comprehension 2 Two moles of a monoatomic gas are taken from a to c, via three paths abc, ac and adc.



Question : Work done by the gas in process ac is

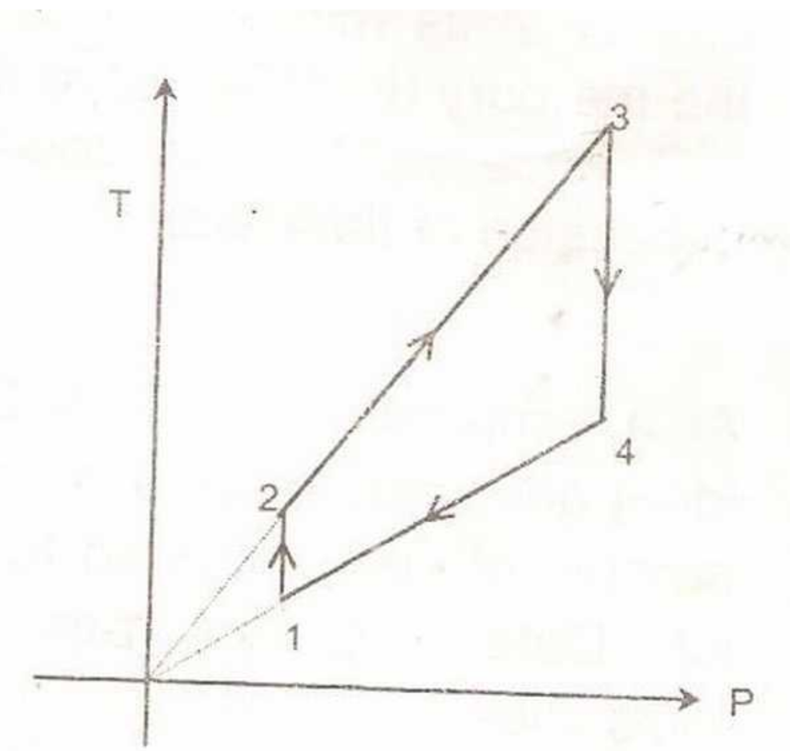
a) 1000 R
b) 900 R
c) 600 R
d) 1500 R

Question : If work done by the gas in abc is W_1, in ac work done is W_2 and in adc work done is W_3, then

a) $W_2 > W_3 > W_1$
b) $W_1 > W_2 > W_3$
c) $W_2 > W_1 > W_3$
d) $W_3 > W_2 > W_1$

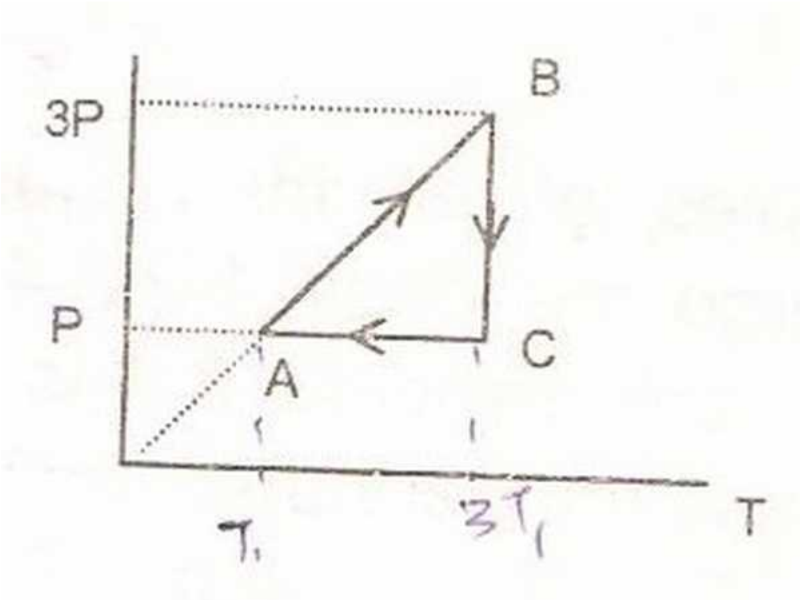
Subjective Problems

Example: The moles of an ideal monoatomic gas undergoes a cyclic process as shown in the figure.

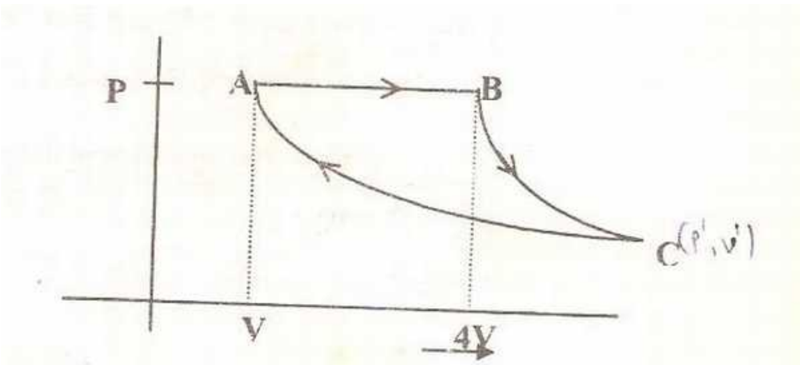


The temperatures in different states are $6T_1 = 3T_2 = 2T_4 = T_3 = 1800$K . Determine the work done by the gas during the cycle.

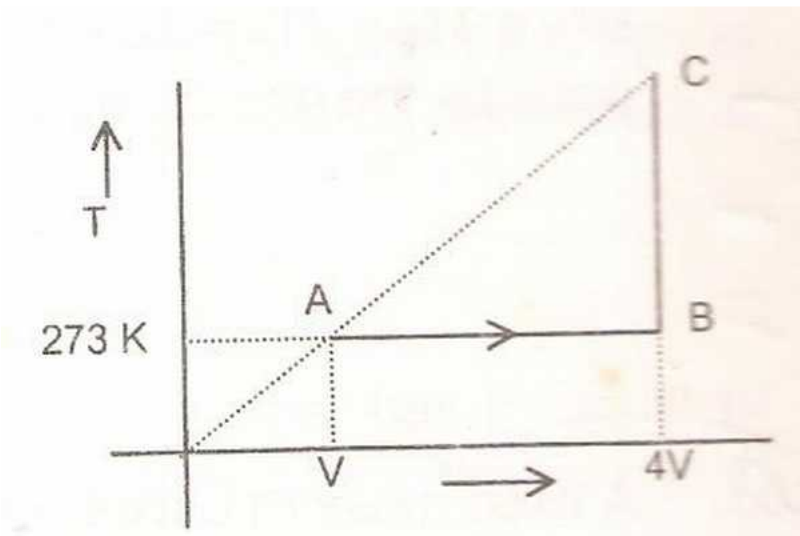
Example: A fixed mass of oxygen gas performs a cycle ABCA as shown. Find efficiency of the process.



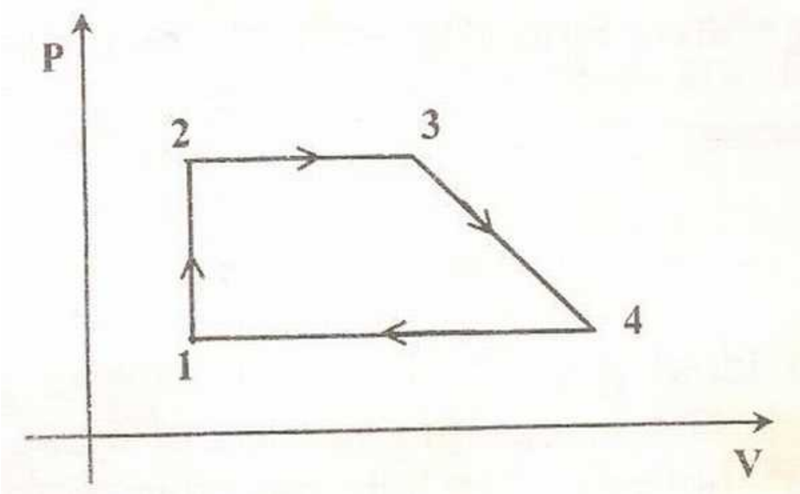
Example: A fixed mass of gas is taken through a process $A \to B \to C \to A$. Here $A \to B$ is isobaric.$B \to C$ is adiabatic and $C \to A$ is isothermal. Find efficiency of process.(Take $\gamma = 1.5$)



Example: At a temperature of $T_o = 273^oK$, two moles of an ideal gas undergoes a process as shown. The total amount of heat imparted to the gas equals $Q = 27.7kJ$. Determine the ratio of molar specific heat capacities.



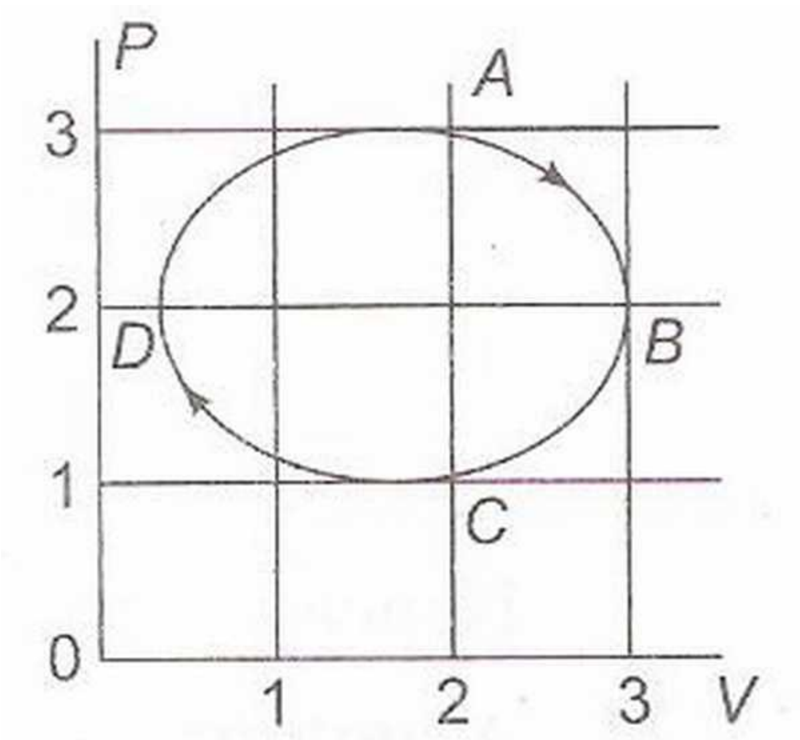
Example: n moles of an ideal gas is made to undergo the cycle 1-2-3-4-1 as shown in the figure. Process 3-4 is a straight line. The gas temperatures in states 1,2 and 3 are T_1, $T_2$and T_3 respectively. Temperature at 3 and 4 are equal. Determine the work done by the ga during the cycle.



3.1.1.2 Previous Years IIT Problems

Mutliple Answer

Example: The figure shows the P-V plot of an ideal gas taken through a cycle ABCDA. The part ABC is a semi-circle and CDA is half of an ellipse. Then,



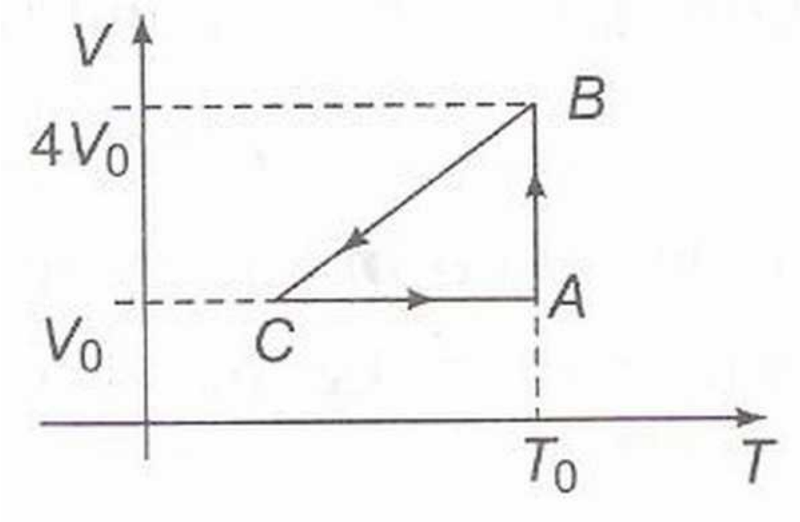
a) the process during the path A->B is isothermal.

b) heat flows out of the gas during the path B->C->D

c) work done during the path A->B->C is zero.

d) positive work is done by the gas in the cycle ABCDA

Example: One mole of an ideal gas in initial state A undergoes a cyclic process ABCA, as shown in the figure. Its pressure at A is P_o . Choose the correct option(s) from the following



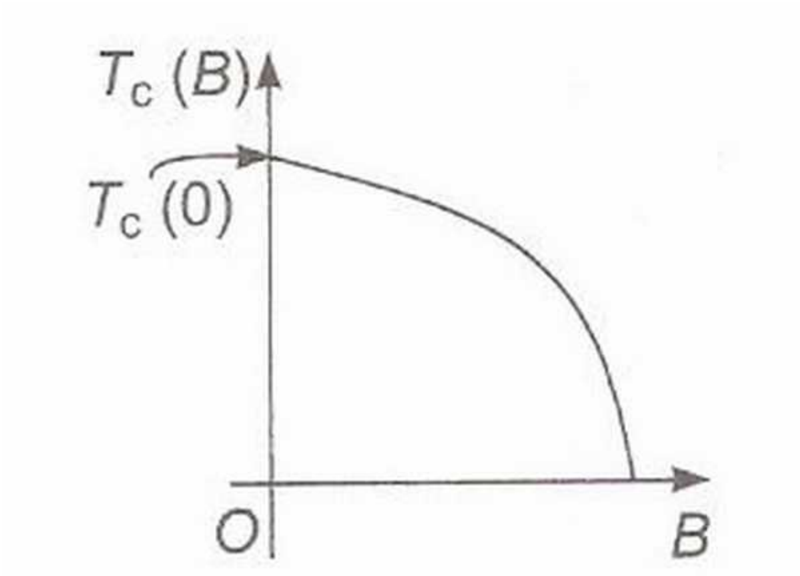
a) Internal energies at A and B are the same

b) Work done by the gas in process AB is $P_o V_o$ ln 4

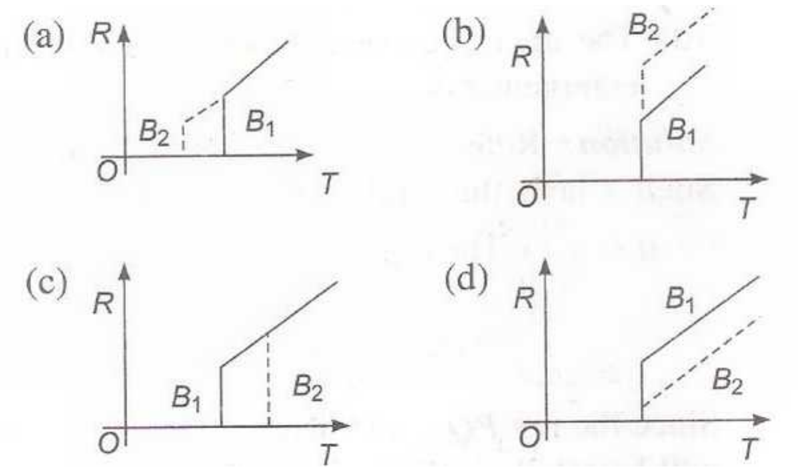
c) Presssure at C is $\frac{P_0}{4}$.

d) Temperature at C is $\frac{T_0}{4}$.

Paragraph Paragraph 1: Electrical resistance of certain materials, known as superconductors, changes abruptly from a nonzero value to zero as their temperature is lowered below a critical temperature Tc(0). An intersting property of superconductors is that their critical temperatre becomes smaler than Tc(0) if they are placed in a magnetic field, i.e., the critical temperature Tc(B) is a function of the magnetic field strength B. The dependence of Tc(B) on B is shown in the figure.



1: In the graphs below, the resistance R of a superconductor is shown as a function of its temperature T for two different magnetic fields B1 (solid line) and B2 (dashed line). If B2 is larger than B1 which of the following graphs shows the correct variation of R with T in these fields?



2: A superconductor has Tc(0) = 100K. When a magnetic field of 7.5 Tesla is applied, its Tc decreases to 75 K. For this material one can definitely say that when

a) B = 5 Tesla, Tc(B) = 80 K

b) B = 5 Tesla, 75 K < Tc(B) < 100K

c) B = 10 Tesla, 75K < Tc < 100K

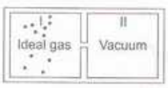
d) B = 10 Tesla, Tc = 70 K

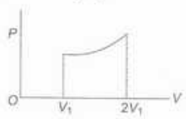
Matching

Example:

Column I contains a list of processes involving expansion of an ideal gas. Match this with Column II describing the thermodynamic change during this process.

Column I	*Column II*
(a) An insulated containar has two chambers separated by a valve. Chamber I contains an ideal gas and the chamber II has vacuum. The valve is opened.	(p) The temperature of the gas decreases
(b) An ideal monoatomic gas expands to twice its original volume such that its pressure $P \propto \frac{1}{V^2}$, where V is the volume of the gas.	(q) The temperature of the gas increases or remains constant
(c) An ideal monoatomic gas expands to twice its original volume such that its pressure $P \propto \frac{1}{V^{4/3}}$, where V is its volume	(r) The gas loses heat
(d) An ideal monoatamic gas expands such that its pressure P and volumn V follow the behaviour shown in the graph.	(s) The gas gains heat





Chapter 4

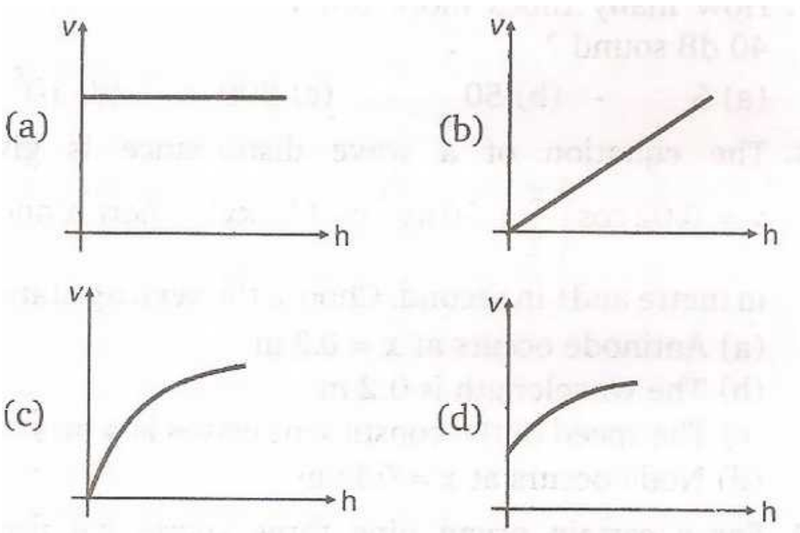
Waves

4.1 Mechanical Waves

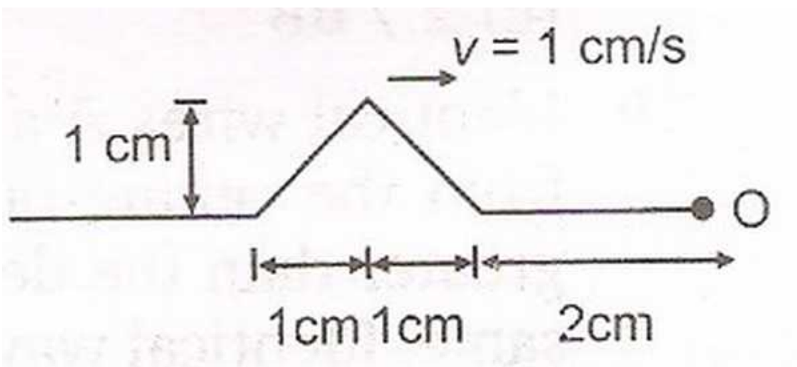
4.1.1 General Problem Set

4.1.1.1 Single Answer Questions

Example 1: A uniform rope having mass m hangs vertically from a rigid support. A transverse wave pulse is produced at the lower end. The speed ν of wave pulse varies with height h from the lower end as



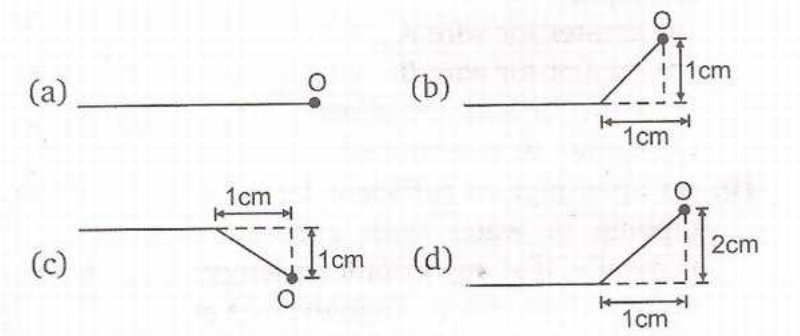
Example 2: A wave pulse on a string has the dimension shown in figure. The wave speed is v=1 cm/s . If point O is a free end.



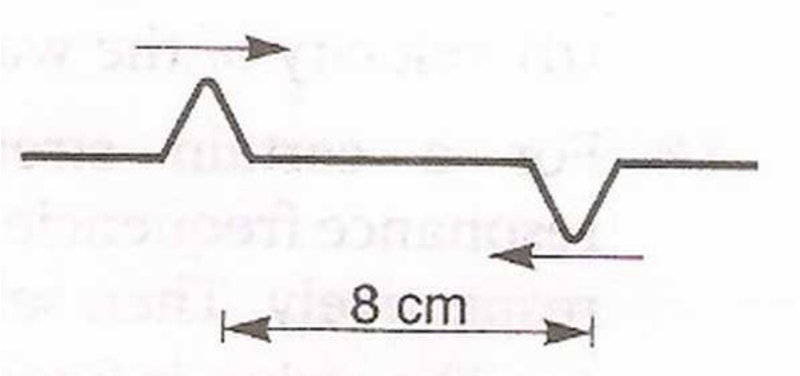
(i) The shape of wave at time t=3s is

(ii) The shape of the wave at time t=3s if O is a fixed end will be

{ Both answers from the image below }



Example 3 : Two pulses in a stretched string, whose centres are initially 8 cm apart, are moving towards each other as shown in the figure. The speed of each pulse is 2 cm/s. After 2s the total energy of the pulses will be



a) zero

b) purely kinetic

c) purely potential

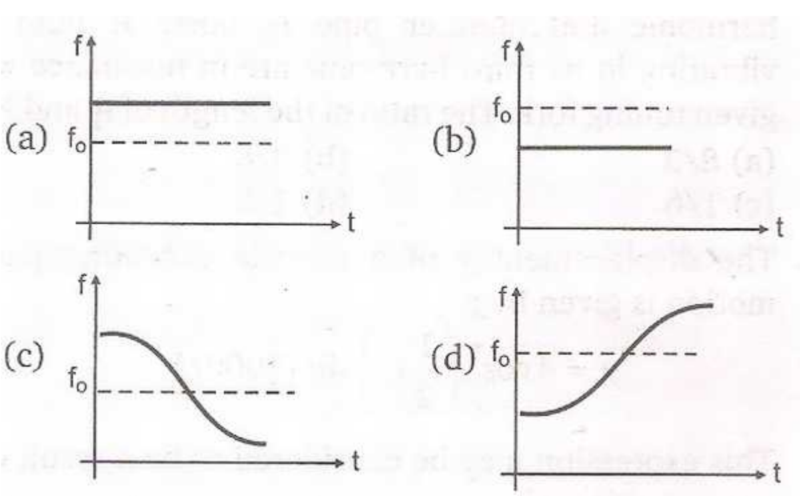
d) partly kinetic and partly potential

4.2 Sound

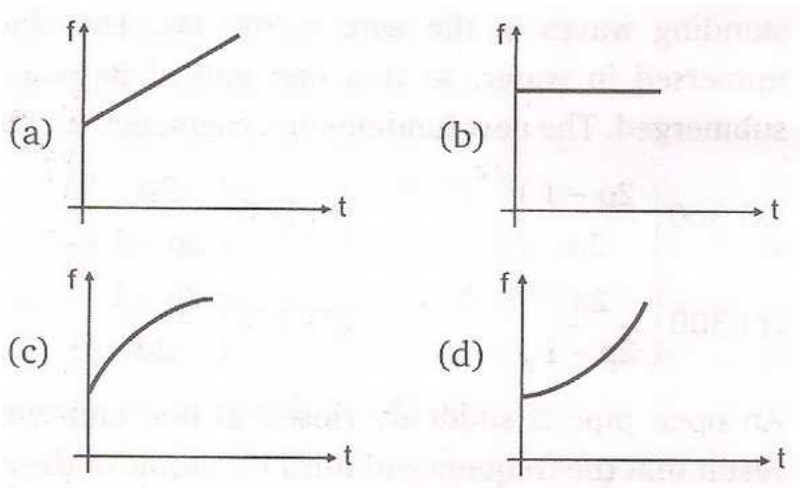
4.2.0.1 General Problem Set

Single Answer Type

Example 1 : Source and observer both start moving simultaneously from origin one along x-axis and the other along y-axis with speed of source = 2 (speed of observer). The graph between the apparent frequency observed by observer (f) and time (t) would be



Example 2: An observer starts moving with uniform accelration a towards a stationary sound source of frequency f_o. As the observe approaches the source, the apparent frequency f heard by the observer varies with time t as



Multiple Answer Type

Example 1 : A stationary observer receives a sound of frequency $f_o = 2000$ Hz. Source is moving with constant velocity on a road at some non-zero perpendicular distance from observer. The apparent frequency f varies with time as shown in figure. Speed of sound = 300 m/s. Choose the correct alternative(s).

a) Speed of source is 66.7 m/s

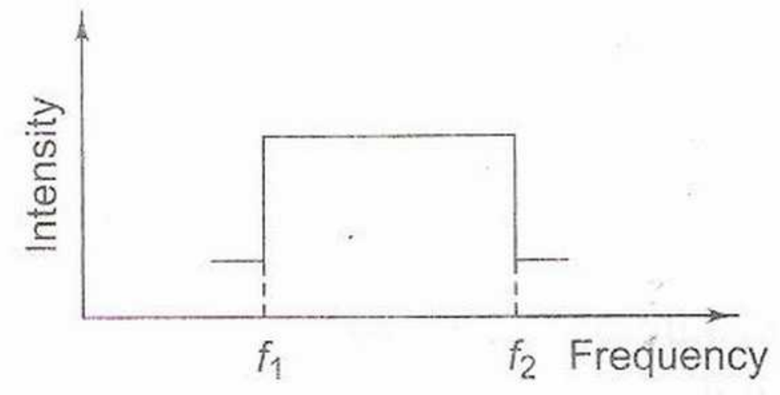
b) fm shown in figure cannot be greater than 2500 Hz

c) Speed of source is 33.33 m/s

d) f_m shown in figure cannot be greater than 2250 Hz

4.2.0.2 Previous Years IIT Problems

Passage Two trains A and B are moving with speed 20 m/s and 30 m/s respectively in the same direction on the same straight track, with B ahead of A. The engines are at the front ends. The engine of train A blows a long whistle. Assume that the sound of the whistle is composed of components varying in frequency from $f_1 = 800Hz$ to $f_2 = 1120Hz$, as shown in the figure. The spread in the frequency (highest frequency-lowest frequency) is thus 320Hz. The speed of sound in still air is 340 m/s.



1: The speed of sound of the whistle is

a) 340 m/s for passengers in A and 310 m/s for passenger in B

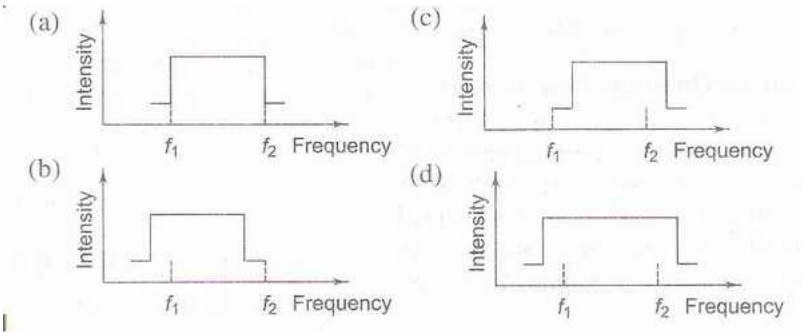
b) 360 m/s for passengers in A and 310 m/s for passenger in B

c) 310 m/s for passengers in A and 360 m/s for passenger in B

d) 340 m/s for passengers in both the trains

{ Solution: The speed of sound depends only on the modulus of elasticity and the density of the medium in which it travels. The speed of sound does not depend on the speed of the source of sound or of the observer. Hence the correct option is d) }

2: The distribution of the sound intensity of the whistle as observed by the passengers in train A is best represented by



{ Solution: For train A, there is no relative motion between the source and the passengers. Hence the frequency of sound heard by passengers in train A will be the same as the frequency of sound emitted by the whistle. Therefore, the correct choice is a). }

3: The spread of frequency as observed by the passenger in train B is

a) 310 Hz

b) 330 Hz

c) 350 Hz

d) 290 Hz

{ Solution: The apparent frequency of sound as heard by passengers in train B is given by

$$f' = f_o \left(\frac{v - u_B}{v - u_A} \right)$$

where f_o = actual frequency, v = speed of sound, u_B= speed of train B and u_A = speed of train A.

$$f'(\text{For } f_o = 800 \text{ Hz}) = 800 \times \left(\frac{340 - 30}{340 - 20} \right) = 775 \text{ Hz}$$

$$f'(\text{For } f_o = 1120 \text{ Hz}) = 1120 \times \left(\frac{340 - 30}{340 - 20} \right) = 1085 \text{ Hz}$$

$\therefore$ Spread of frequency = 1085 -775 =310 Hz Hence the correct choice is a) }

Chapter 5

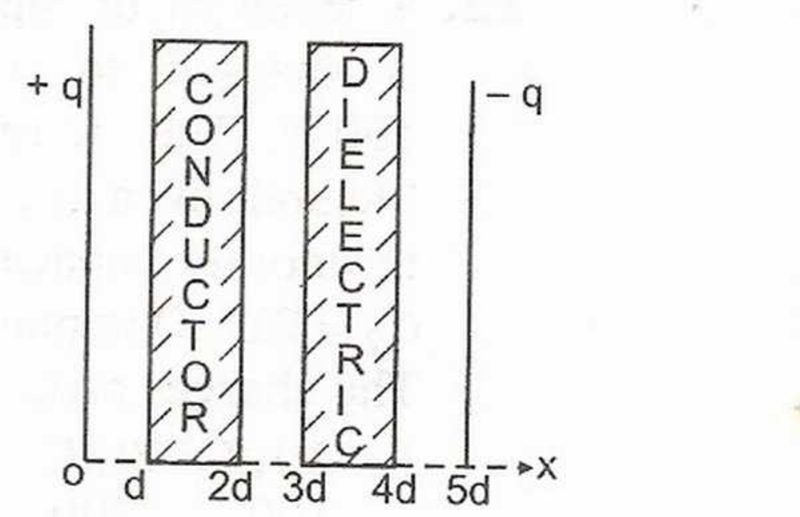
Electromagnetism

5.1 Electrostatics

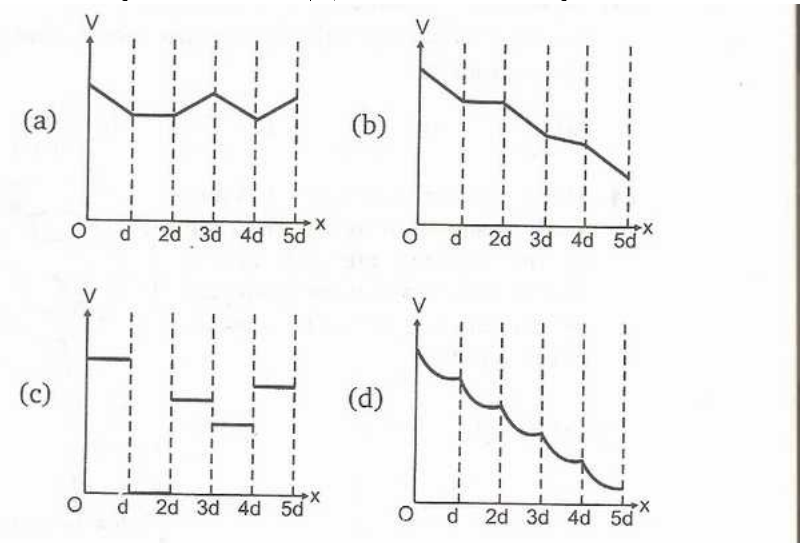
5.1.1 Problems for Practice

5.1.1.1 General Problem Set

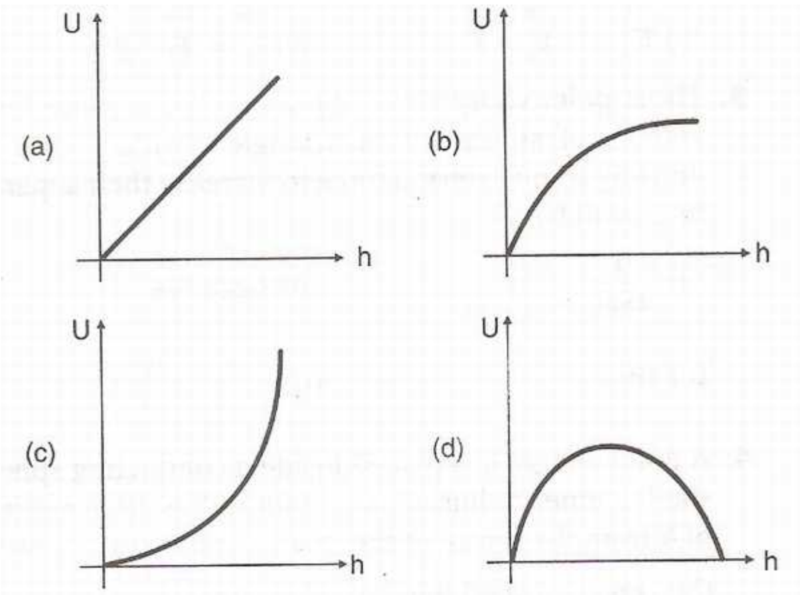
Single Answer Type Example : The distance between plates of a parallel plate capacitor is 5d. The positively charged plate is at x=0 and negatively charged plate is at x=5d.



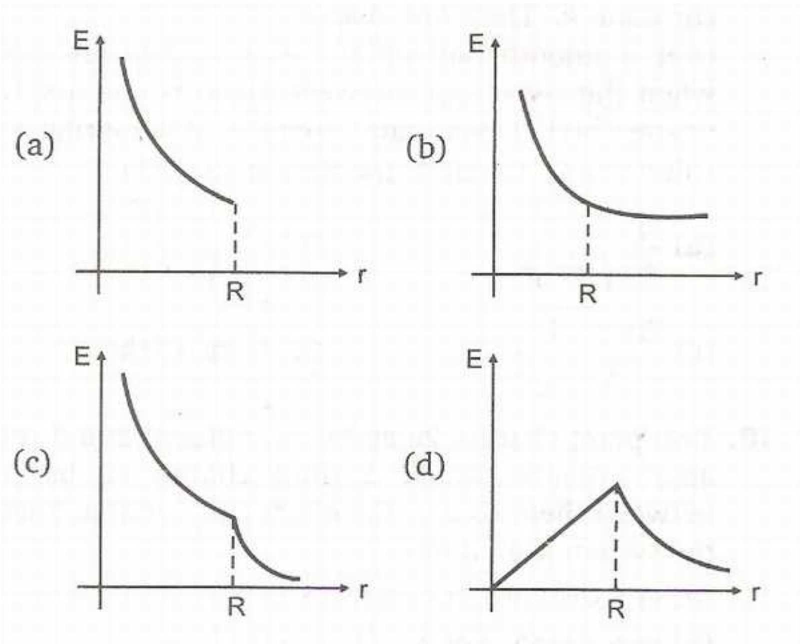
Two slabs one of conductor and the other of a dielectric of same thickness d are inserted between the plates as shown in figure. Potential (V) versus distance x graph will be



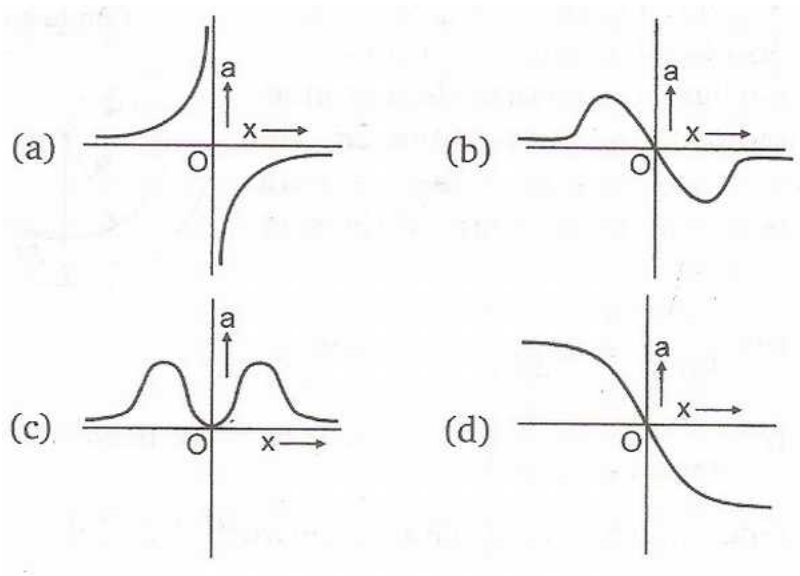
Example : A particle of mass m and charge q is projected vertically upwards. A uniform electric field $\vec{E}$ is acted vertically downwards. The most appropriate graph between potential energy U (gravitational plus electrostatic) and height h (<< radius of earth) is (assume U to be zero on surface of earth)



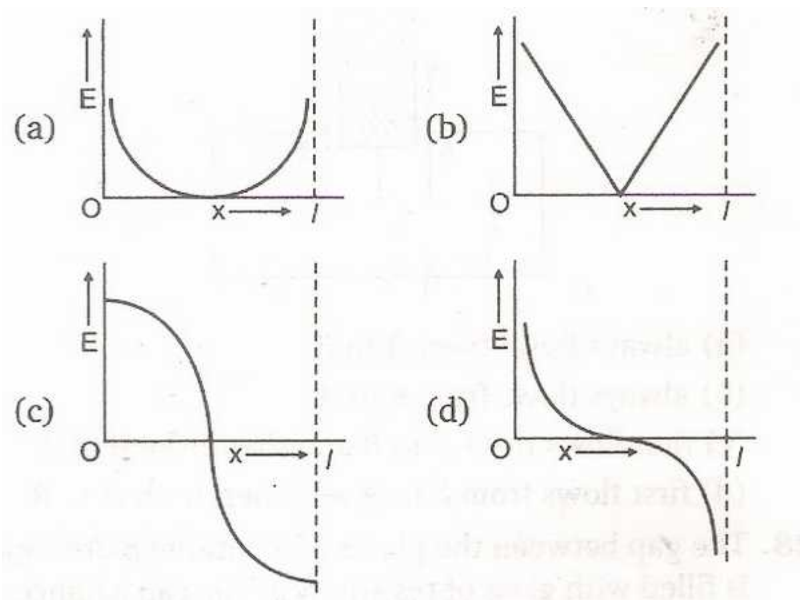
Example : A conducting shell of radius R carries charge -Q. A point charge +Q is placed at the centre. The electric field E varies with distance r (from the centre of the shell) as



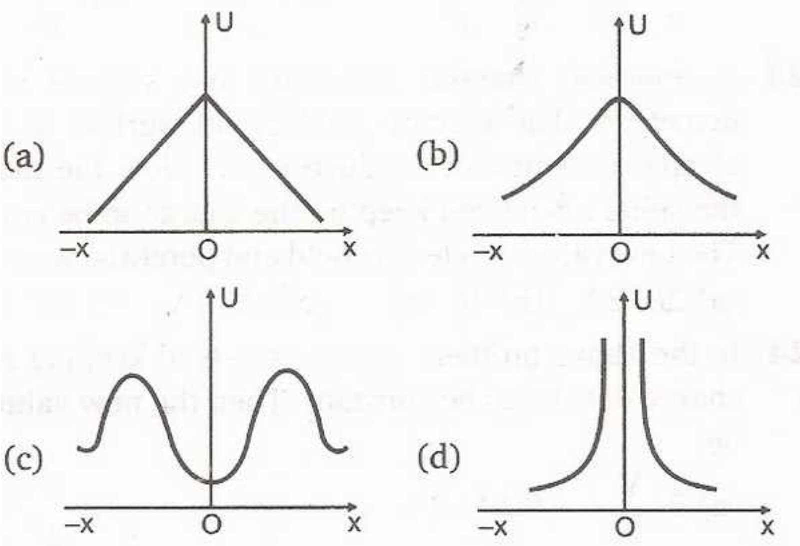
Example : Two identical positive charges are fixed on the y-axis, at equal distances from the origin O. A particle with a negative charge starts on the negative x-axis at a large distance from O, moves along the x-axis, passed through O and moves far away from O. Its acceleration a is taken as positive along its direction of motion. The particle's acceleration a is plotted against its x-coordinate. Which of the following best represents the plot ?



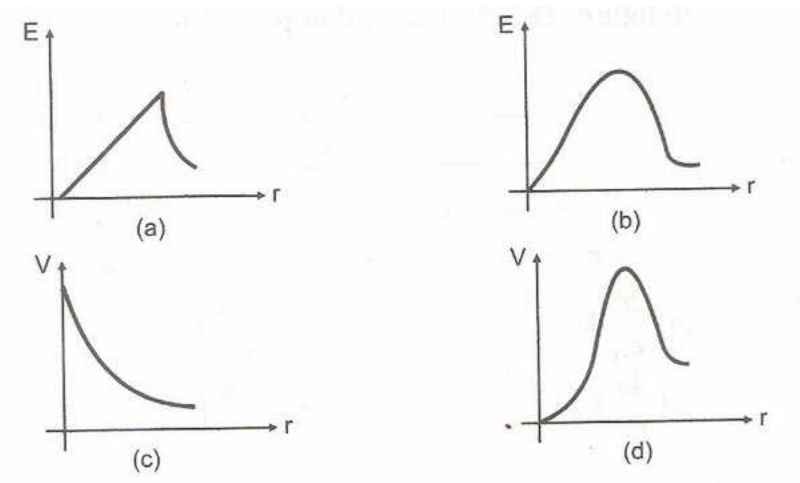
Example: Two identical point charges are placed at a separation of l. P is a point on the line joining the charges, at a distance x from any one charge. The field at P is E. E is plotted against x for values of x from close to zero to slightly less than l. Which of the following best represents the resulting curve?



Example : Four equal charges of magnitude q each are placed at four corners of a square with its centre at origin and lying in y-z plane. A fifth charge +Q is moved along x-axis. The electrostatic potential energy (U) varies on x-axis as



Example : A circular ring carries a uniformly distributed positive charge. The electric field (E) and potential (V) varies with distance (r) from the centre of the ring along its axis as



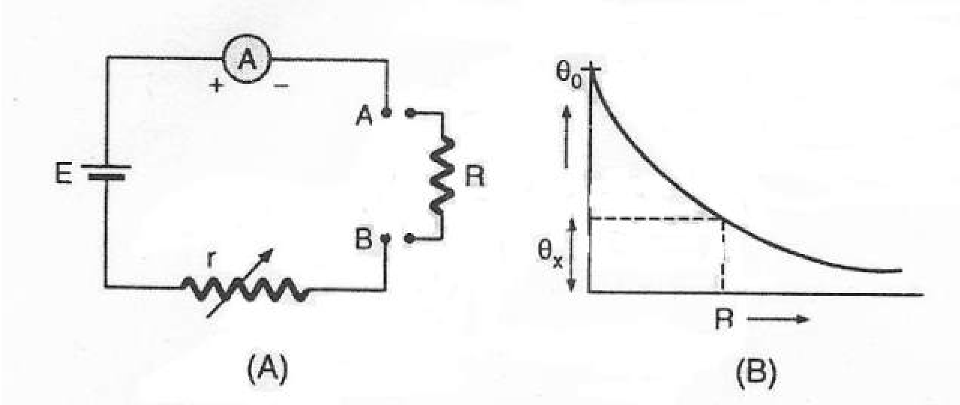
5.2 Current Electricity

5.2.1 Basics

5.2.1.1 Theory

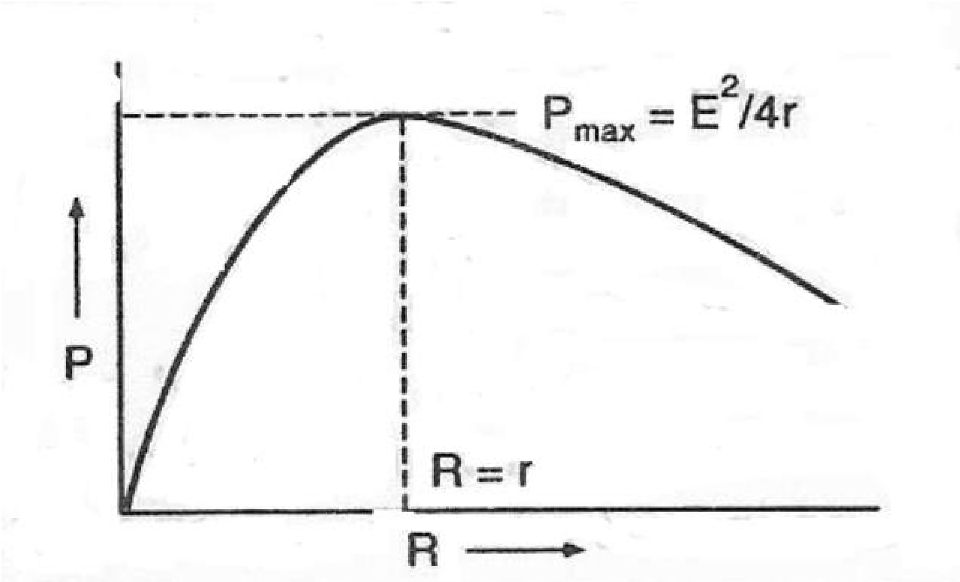
Ohm-meter It is an instrument designed to measure resistance. It contains an Ammeter, a Battery and a Rheostat as shown in Figure. The terminals A and B are first short circuited and the Rheostat is adjusted to show full deflection on Ammeter. The full scale deflection corresponds to zero external resistance.

Now , connecting a resistance box between points A and B , ammeter deflection θ is noted for different values of R and a graph is plotted between θ and R. The graph is called Calibration Curve and is shown in Figure. Now the resistance box is removed and an unknown resistance is connected to the circuit. The deflection is noted down and from the calibration curve, the value of R is found out.



Power Transfer to a load The power transfer to the load by the cell will be $P = I^2R = \dfrac{E^2R}{(R+r)^2}$

From the equation, it is clear that Power would be zero , if R=0 or ∞ and gives the minima.



$\dfrac{dP}{dR} = 0$ i.e. $\dfrac{d}{dR}\left[\dfrac{E^2R}{(R+r)^2}\right] = 0$ will give any other local maxima / local minima

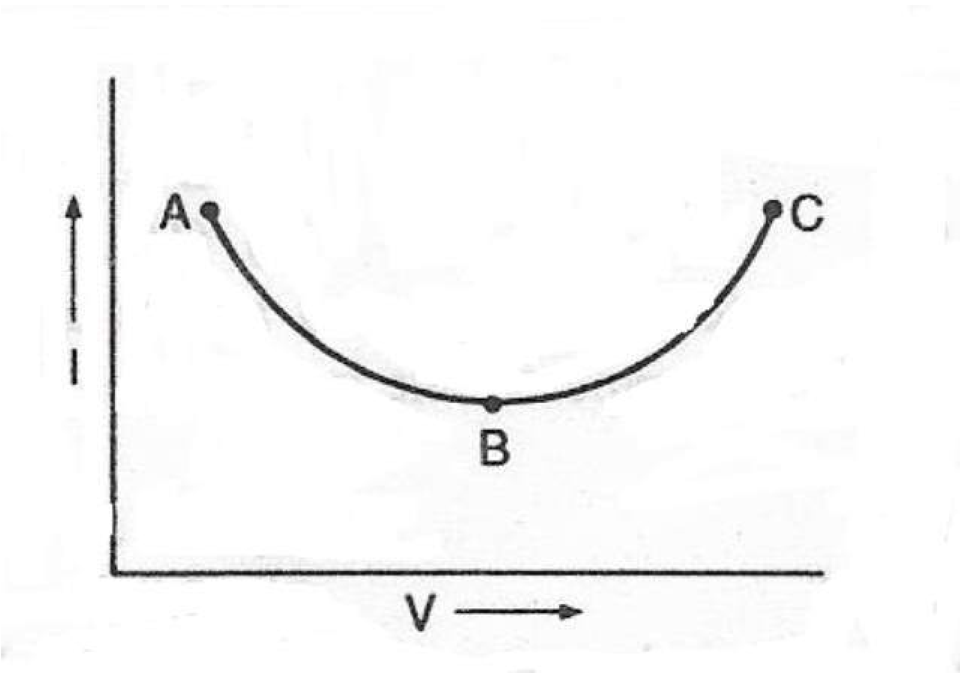
It is zero at R=r which gives a maxima

i.e. power transfer to the load by a cell is maximum when R=r and $P_{max} = \dfrac{E^2}{4r}$

5.2.1.2 Problems

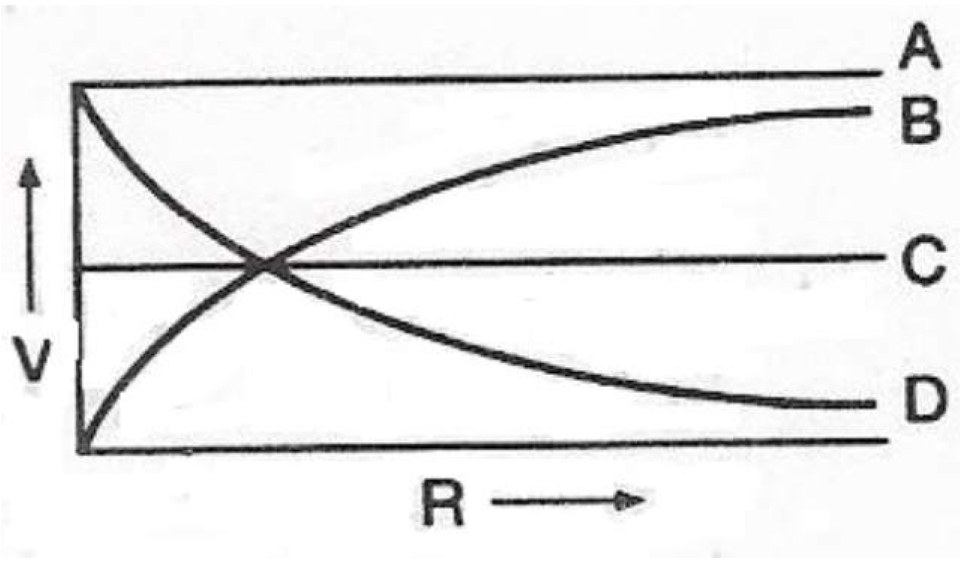
Objective Type Questions

Example: Resistance as shown in Figure is negative at



a) A
b) B
c) C
d) None of these
{Hint: The resistance is given by V/I and NOT dV/dI . V and I are shown positive , so R (their ratio) is also +ve.
d) is the correct answer. }

Example: A cell of EMF E having an Internal Resistance r is connected to an External Resistance R. The potential difference V across the resistance R varies with R as the curve :



a) A

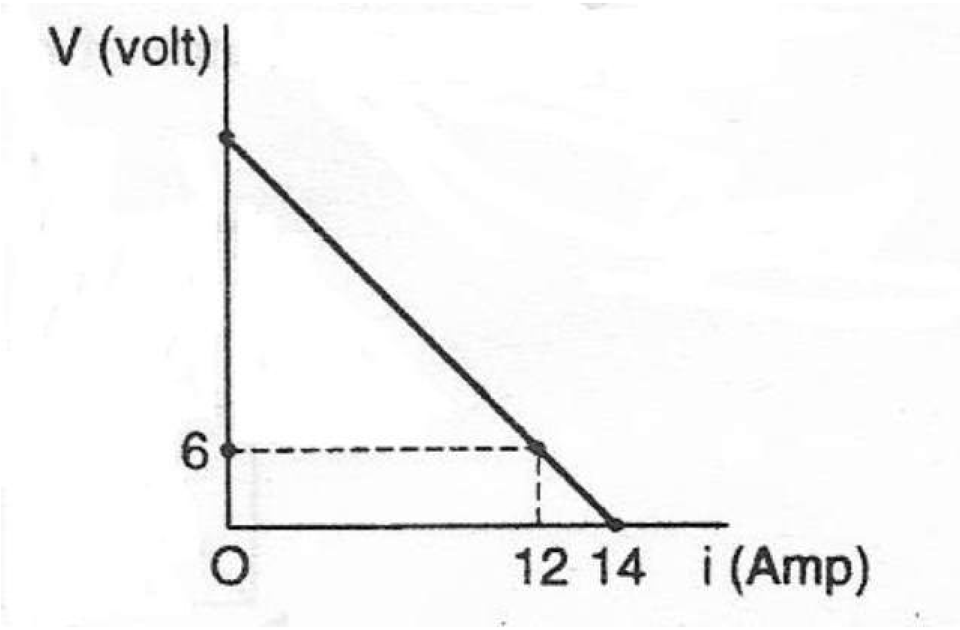
b) B

c) C

d) D

{ Hint: V=ER/(r+R), So, B is the required curve as of y=xc'/(x+c") }

Example: 10 cells , each of EMF E and internal resistance r are connected in series to a variable external resistance. Figure shows the variation of terminal potential difference with the current drawn from the combination. EMF of each cell is :



a) 1.6 V

b) 3.6 V

c) 1.4 V

d) 4.2 V

{Hint : The equaiton of the line is V/42 + i/14 =1

Also, V = 10ER/10r+R

So, 10ER/42(10r+R) + i/14=1

10ER+3i(10r+R)=42(10r+R)}

As R is Variable, setting R as infinity

10E+3i=42

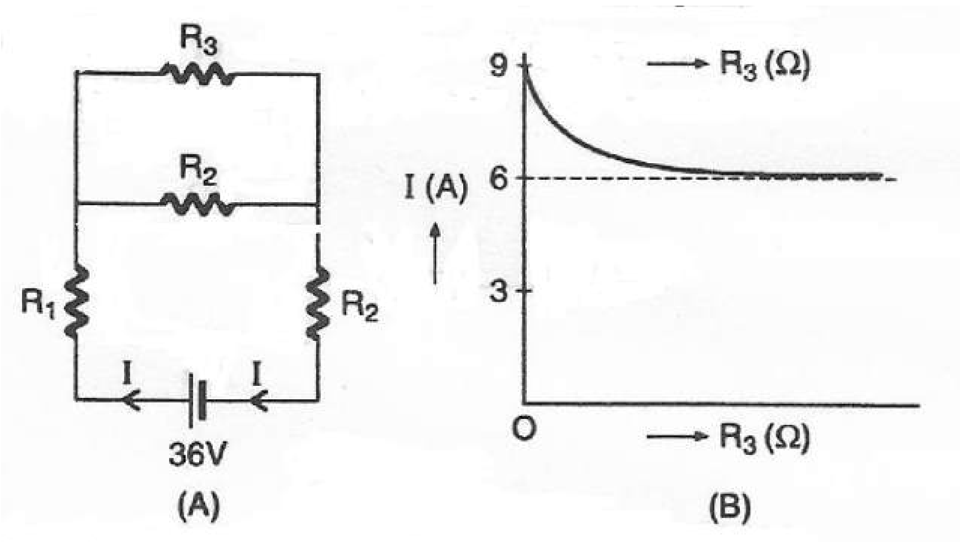
E = (42 - 3i) / 10 = 4.2 - 0.3 i

Also at R infinity i would be zero as it is a series connected circuit

So , E = 4.2 V

d) is the correct answer }

Example: In the circuit shown in figure, R3 is a variable resistance. As the value of R3 is changed, current I through the cell varies as shown. Obviously, the variation is asymptotic, i.e. I -> 6A as R_3->∞ . Resistance R_1 and R_2 are respectively, :



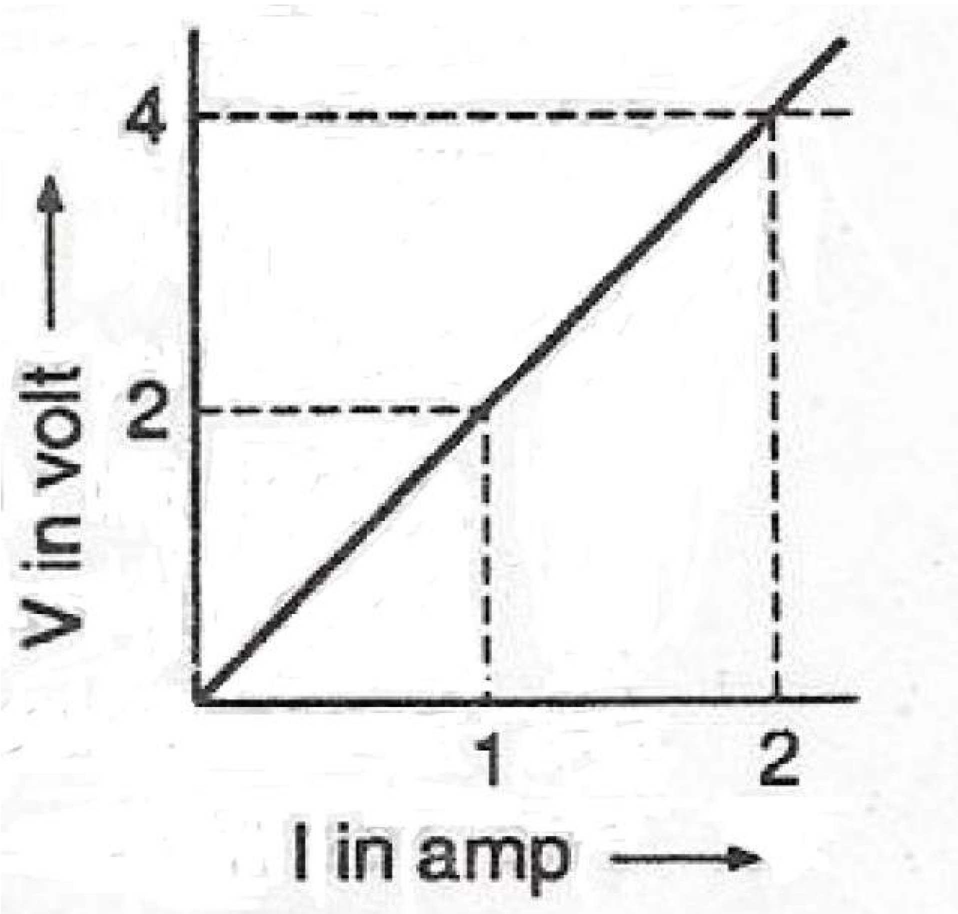
a) 4Ω, 2Ω

b) 2Ω, 4Ω

c) 2Ω,2Ω

d) 1Ω,4Ω

Example: The variation of current with potential difference is as shown in Figure. The resistance of the conductor is :



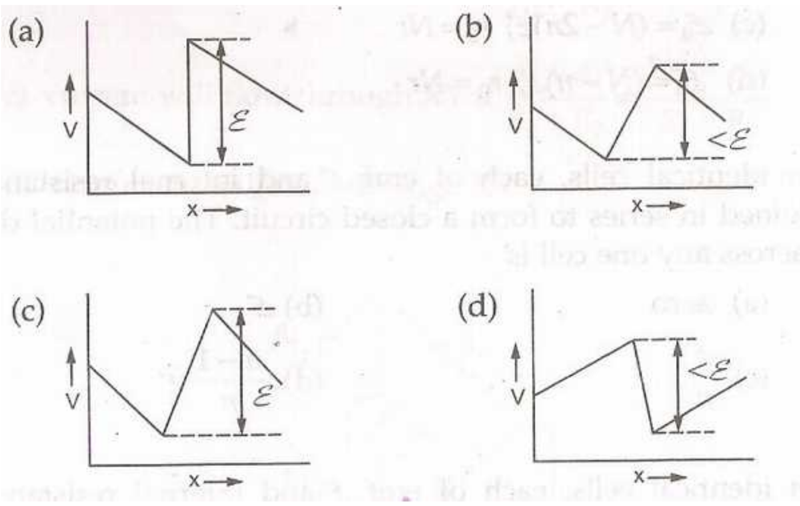
a) 1Ω

b) 2Ω

c) 3Ω

d) 4Ω

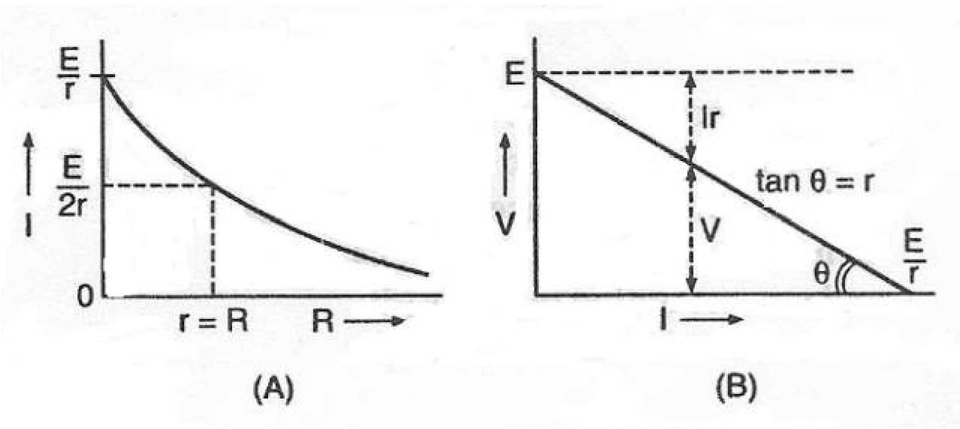
Example : The two ends of a uniform conductor are joined to a cell of emf E and some internal resistance. Starting from the midpoint P of the conductor, we move in the direction of the current and return to P. The potential V at every point on the path is plotted against the distance covered (x). Which of the following best represents the resulting curve?



Subjective

Example: Draw a) I vs R b) V vs I , characteristics for a cell.

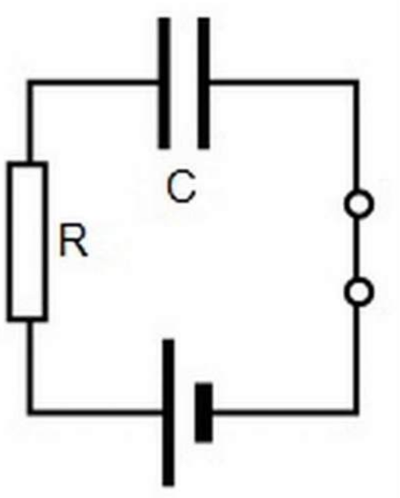
{ Hint: For a cell, as I = E/(R+r) and V = E - Ir ,



}

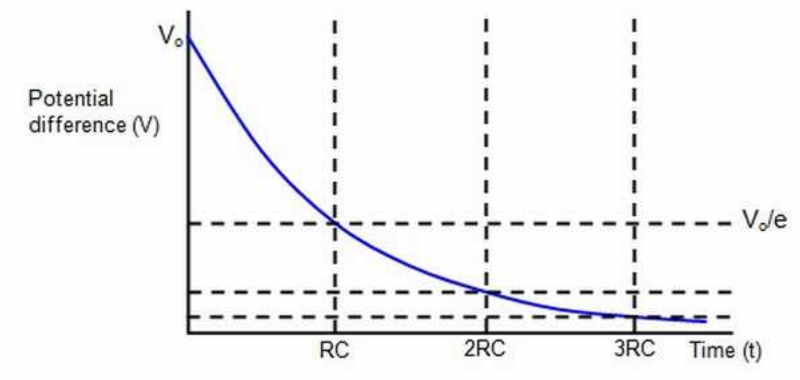
5.2.2 Capacitors

Mathematical treatment of charging and discharging a capacitor n.d.



5.2.2.1 Discharge of a capacitor

The area under the current-time discharge graph gives the charge held by the capacitor. The gradient of the charge-time graph gives the current flowing from the capacitor at that moment.



In Figure let the charge on a capacitor of capacitance C at any instant be q, and let V be the potential difference across it at that instant.

The current (I) in the discharge at that instant is therefore: I = - dq/dt

But V = IR and q = CV so dq/dt = d(CV)/dt = C dV/dt Therefore we have V = -CR dV/dt Rearranging and integrating gives:

Capacitor discharge (voltage decay): $V = V_o e{-}^{(t/RC)}$

where Vo is the initial voltage applied to the capacitor. A graph of this exponential discharge is shown below in Figure

Since Q = CV the equation for the charge (Q) on the capacitor after a time t is therefore:

Capacitor discharge (charge decay): $Q = Q_o e^{-(t/RC)}$

V = $V_o e^{-(t/RC)}$ and also $I = I_o e^{-(t/RC)}$ $Q = Q_o e^{-(t/RC)}$

You should realise that the term RC governs the rate at which the charge on the capacitor decays.

When t = RC, V = Vo/e = 0.37 Vo and the product RC is known as the time constant for the circuit. The bigger the value of RC the slower the rate at which the capacitor discharges.

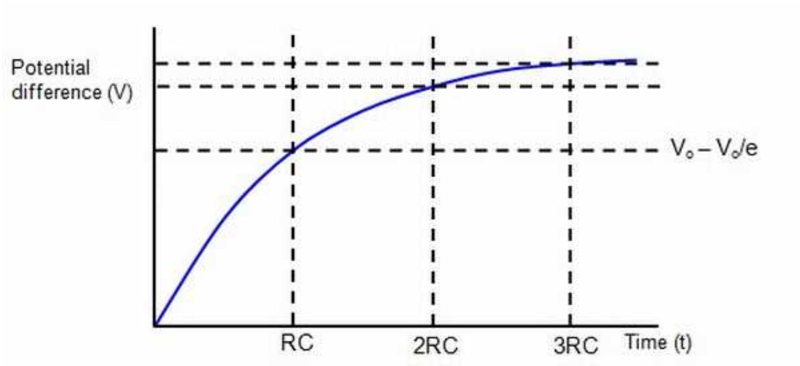
The value of C can be found from this discharge curve if R is known.

5.2.2.2 Charging a capacitor

When a capacitor (C) is being charged through a resistance (R) to a final potential Vo the equation giving the voltage (V) across the capacitor at any time t is given by:

Capacitor charging (potential difference): $V = V_o[1 - e^{-(t/RC)}]$

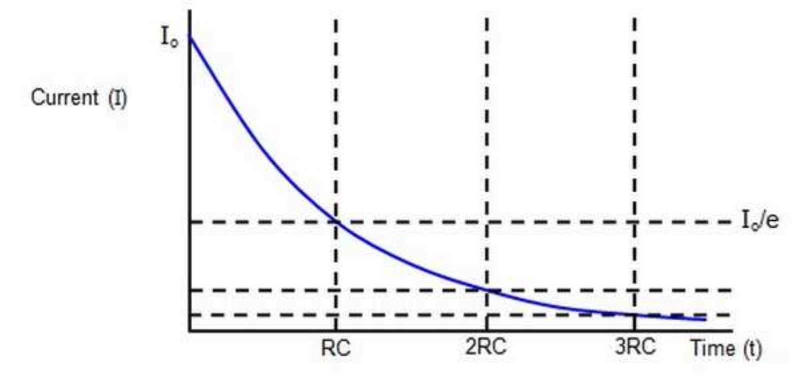
and the variation of potential with time is shown in Figure



As the capacitor charges the charging current decreases since the potential across the resistance decreases as the potential across the capacitor increases.

Figure shows how both the potential difference across the capacitor and the charge on the plates vary with time during charging.

The charging current would be given by the gradient of the curve in Figure at any time and the graph of charging current against time is shown in next Figure.



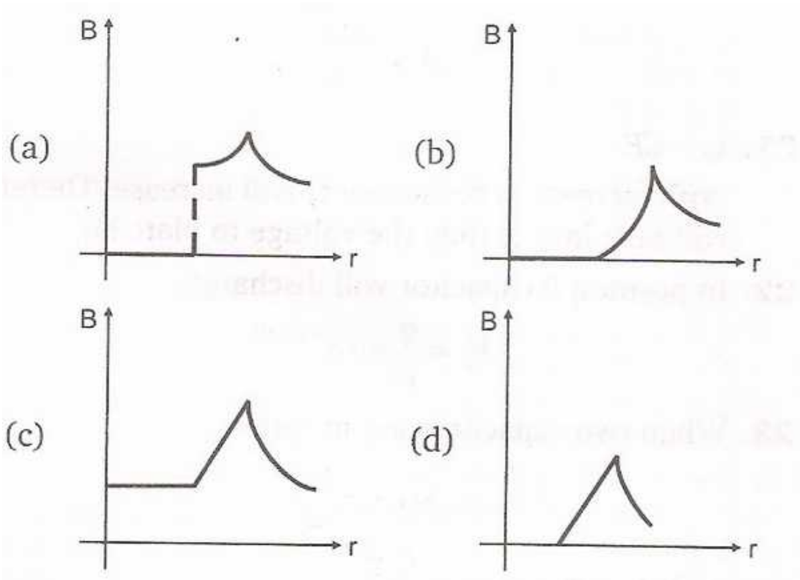
The area below the current-time curve in both charg ing and discharging represents the total charge held by the capacitor.

5.3 Magnetic Field

5.3.1 Problems for Practice

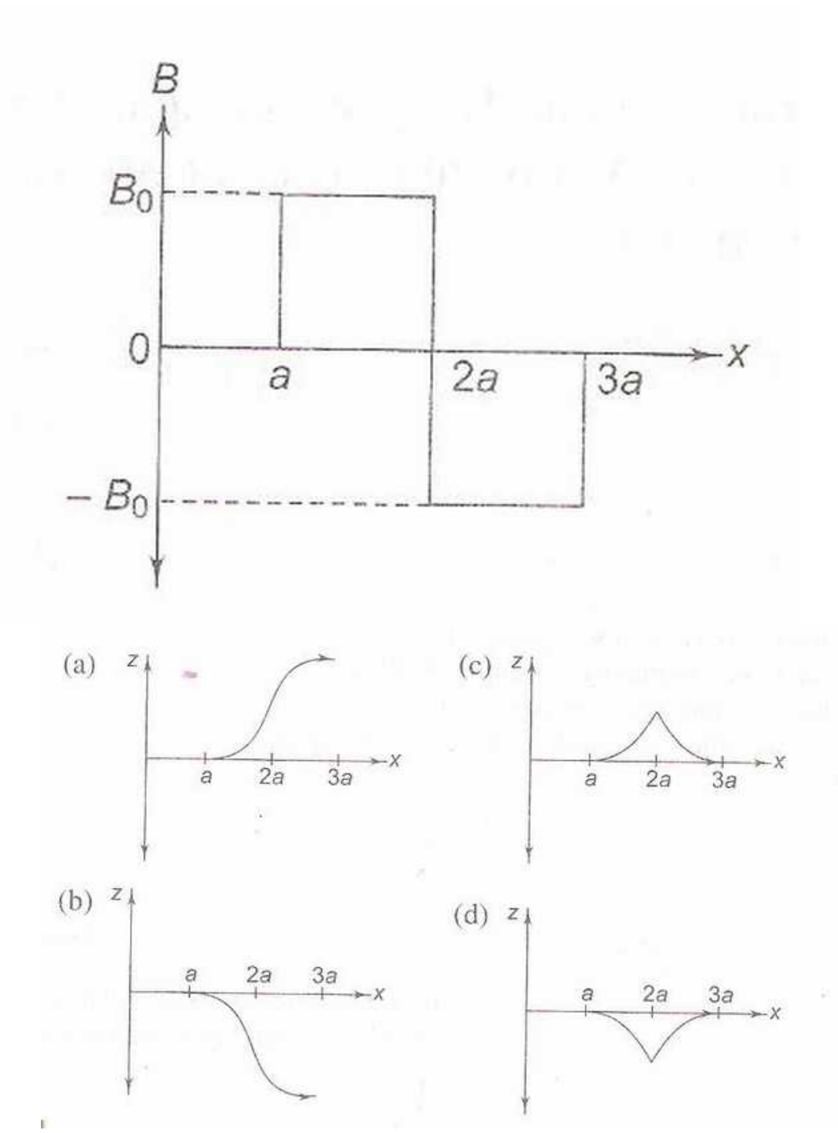
5.3.1.1 General Problem Set

Example : A current i is uniformly distributed over the cross section of a long hollow cylindrical wire of inner radius R_1 and outrer radus R_2. Magnetic field B varies with distance r from the axis of the cylinder as



5.3.1.2 IIT Previous Years Problems

Example: A magnetic field $\overrightarrow{B} = B_o\hat{j}$ exists in the region a<x<2a and $\overrightarrow{B} = -B_o\hat{j}$ in the region $2a < x < 3a$ where B_o is a positive constant. A positive point charge moving with a velocity $\vec{v} = v_o\hat{i}$, where v_o is a positive constant , enters the magnetic field at x=a. The trajectory of the charge in this region can be like



Solution: Force experienced by the charge q is given by

$\overrightarrow{F} = q\left(\overrightarrow{v} \times \overrightarrow{B}\right)$

In the region from x=a to x=2a, the force is $\overrightarrow{F_1} = q\left(v_o\hat{i} \times B_o\hat{j}\right) = qv_oB_o\hat{k}$ directed along the positive z-axis.

In the region from x=a to x=2a to x=3a, the force is $\overrightarrow{F_2} = q\left(v_o\hat{i} \times (-B_o)\,\hat{j}\right) = -qv_oB_o\hat{k}$ directed along the negative z-axis.

Since force $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$are perpendicular to velocity $\overrightarrow{v}$, the correct trajectory is as shown in option a).

Bibliography

(N.d.). URL: http://www.schoolphysics.co.uk/age16-19/Optics/Refraction/text/Lenses_graphs/index.html.

Chapter 6

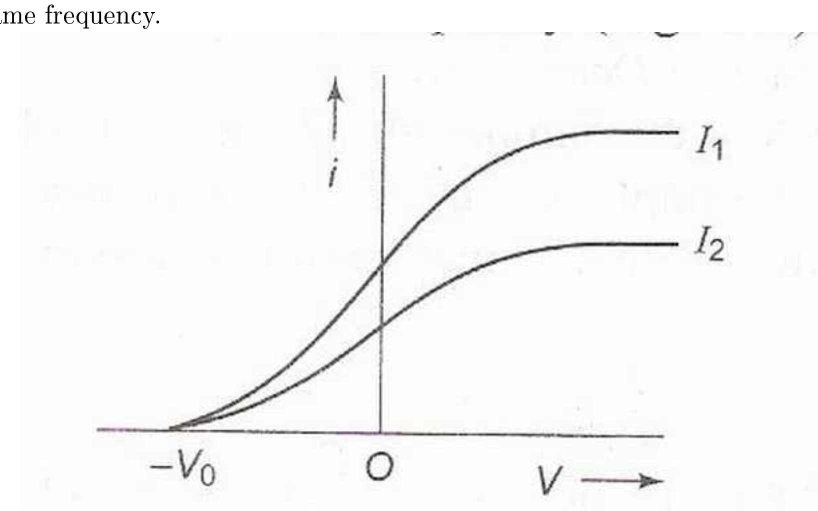
Matter

6.1 Lattice

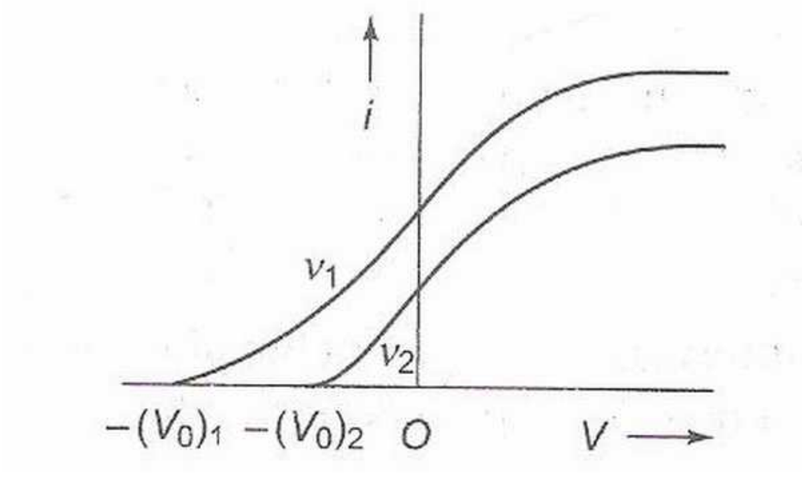
6.1.1 Experiments

Photoelectric Effect

Graphs of Photoelectric Current vs Voltage For radiation of different Intensities ($I_1 > I_2$) but the same frequency.



For radiation of different frequencies ($\nu_1 > \nu_2$) but of the same intensity.

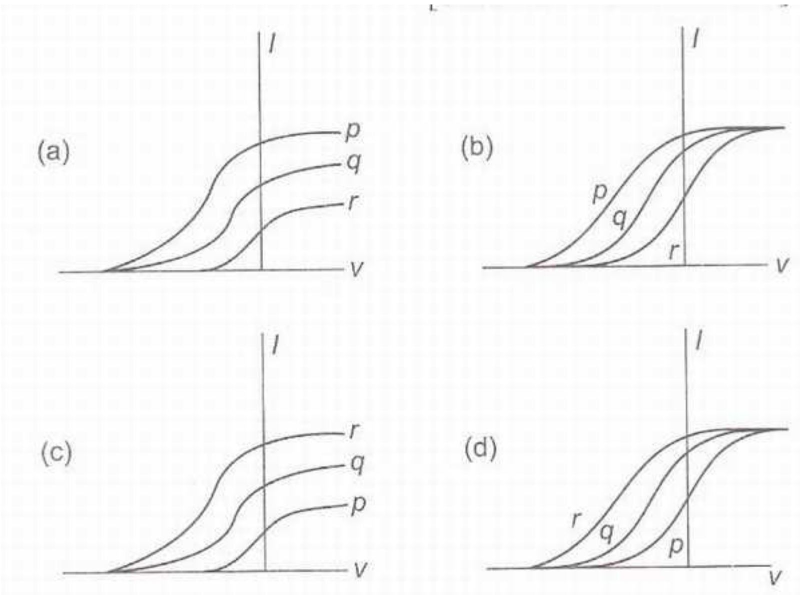


6.1.2 Problems

6.1.2.1 Previous Years IIT Problems

Single Answer Questions

Example: Photoelectric effect experiments are performed using three different metal plates p,q and r having work functions $\phi_p = 2.0eV$. $\phi_q = 2.5eV$. $\phi_r = 3.0eV$ respectively. A light beam containing wavelengths of 550nm, 450nm and 350nm with equal intensitites illuminates each of the plates. The correct I-V graph for the experiment is [Take hc=1240 eV nm]



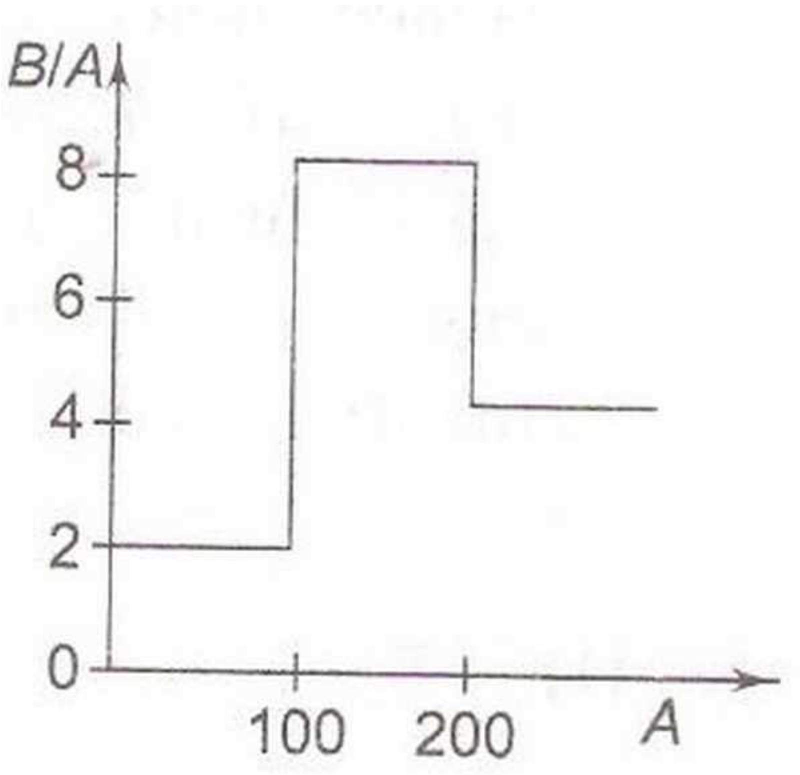
6.2 Nucleus

6.2.1 Problems

6.2.1.1 Previous Years IIT Problems

Multiple Answer

Example: Assume that the nuclear binding energy per nucleon (B/A) versus mass number is as shown in the figure. Use this plot to choose the correct choice (s) given below.



a) Fusion of two nuclei with mass numbers lying in the range 1<A<50 will release energy
b) Fusion of two nuclei with mass numbers lying in the range of 51<A<100 will release energy

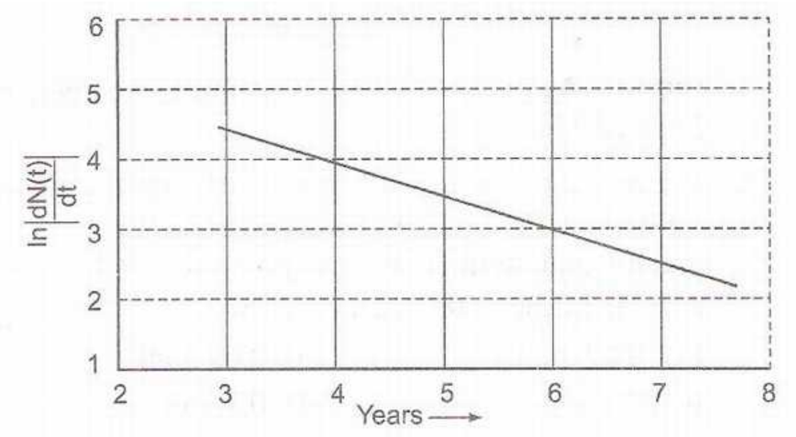
c) Fission of a nucleus lying in the mass number range of 100<A<200 will release energy when broken into equal fragments

d) Fission of a nucleus lying in the mass number range of 200<A<260 will release energy when broken into equal fragments

{ Solution: Energy is released if the total binding energy of the products is greater than the total binding energy of the reactants. This is not possible in choices a) and c). The correct choices are b) and d). }

Integer Type

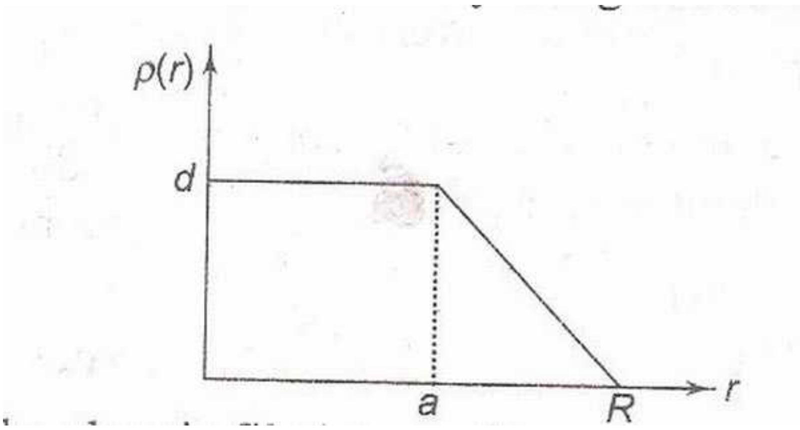
Question : To determine the half life of radioactive element, a student plots a praph of $ln\left|\frac{dN(t)}{dt}\right|$ vs t. Here $\frac{dN(t)}{dt}$ is the rate of radioactive decay at time t. If the number of radioactive nuclei of this element decreases by a factor of p after 4.16 years, find the value of p.



{ Answer: 8 }

Paragraph

Question: The nuclear charge (Ze) is non-uniformly distributed within a nucleus of radius R. The charge density (ρ) (charge per unit volume) is dependent only on the radial distance r from centre of the nucleus as shown in figure. The electric field is only along the radial direction.



1. The electric field at r=R is

a) independent of a

b) directly poportional to a

c) directly proportial to a^2

d) inversely proportional to a

{ Solution: The charge q=Ze can be assumed to be concetrated at the centre of the nucleus. The electric field at r=R is

$E = \frac{q}{4\pi\varepsilon_o R^2} = \frac{Ze}{4\pi\varepsilon_o R^2}$

which is a constant. Hence the correct choice is a) . }

2. For a=0, the value of d (maximum value of ρ as shown in the figure) is

a) $\dfrac{3Ze}{4\pi R^3}$

b) $\dfrac{3Ze}{\pi R^3}$

c) $\dfrac{4Ze}{3\pi R^3}$

d) $\dfrac{Ze}{3\pi R^3}$

{ Solution: Total charge is

$q = \int_0^R 4\pi r^2 \left(d - \dfrac{d}{R} r \right) dr$

$= 4\pi \left[d \int_0^R r^2 dr - \dfrac{d}{R} \int_0^R r^3 dr \right]$

$= \dfrac{\pi d R^3}{3}$

Now that $d = \dfrac{3Ze}{\pi R^3}$

So, the correct choice is b) }

3. The electric field within the nucleus is generally observed to be linearly dependent on r. This implies

a) a=0

b) a=R/2

c) a=R

d) a=2R/3

{ Solution: For spherical charge distribution, the electric field is linearly dependent on r if the charge density ρ is uniform, i.e. a=R. Hence the correct choice is c). }

Chapter 7

Optics

7.1 Ray Optics

7.1.1 Theory

7.1.1.1 Graphs for convex and concave lenses

(Real is positive sign convention) n.d.

Figure shows a graph where the reciprocal of the image distance is plotted against the reciprocal of the object distance. The graph is a straight line that intercepts both axes at 1/f where f is the focal length of the lens. Since the object is real this graph is for a convex lens.

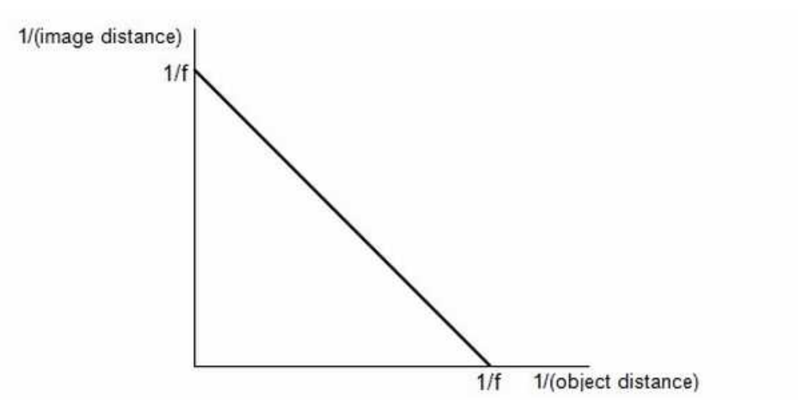
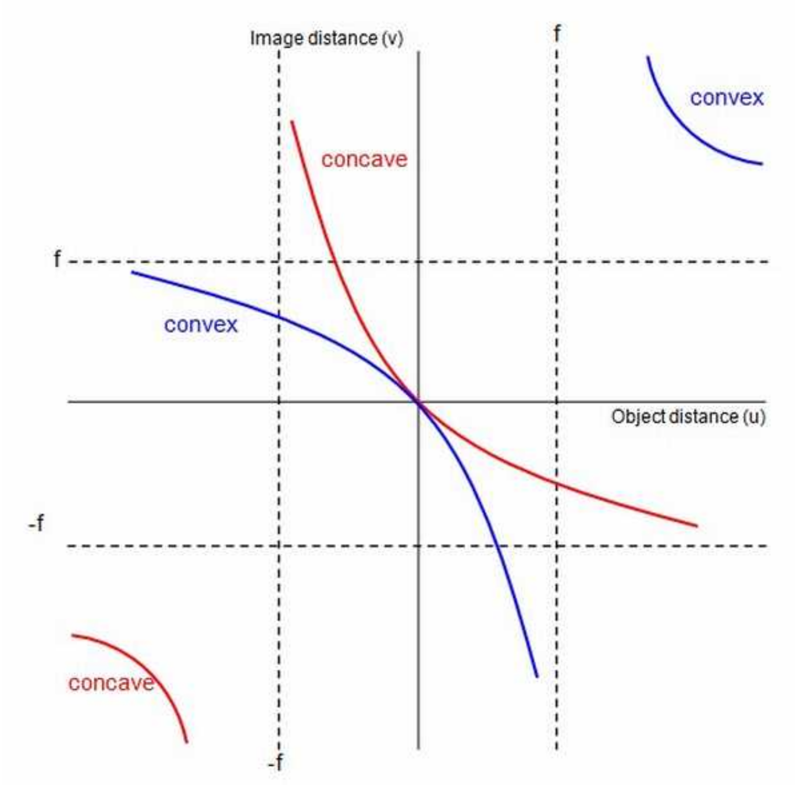
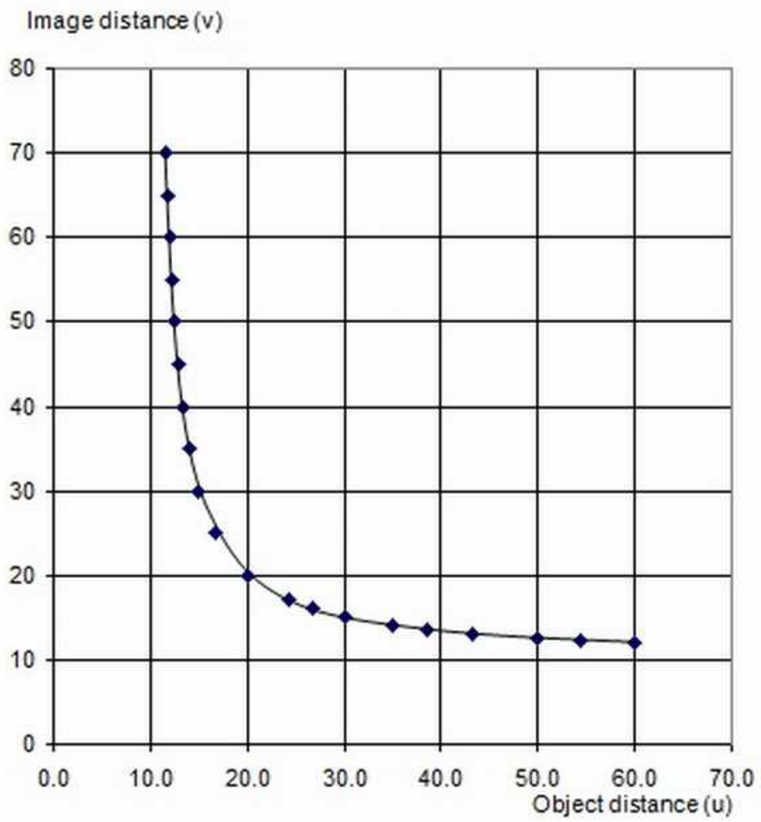


Figure shows a graph of the object distance plotted against the image distance for both convex and concave lenses. Both real and virtual objects and images are shown.



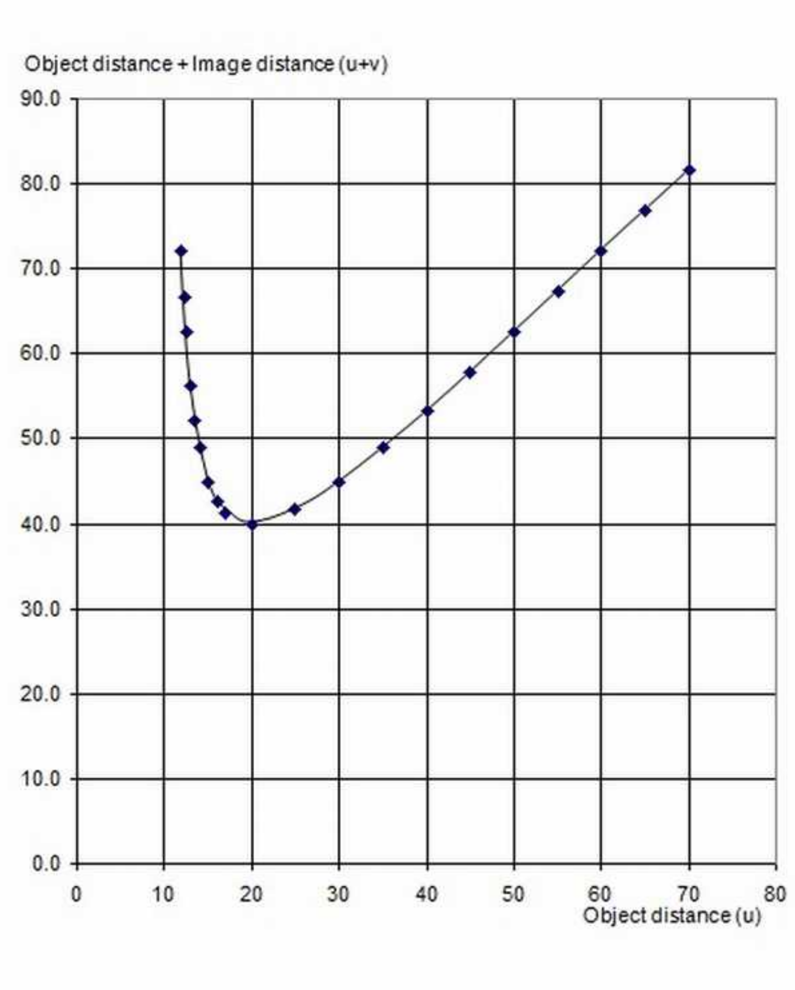
Enlarged view of an object distance (u) against image distance (v) graph The focal length of the lens is 10 cm.

The graph is completely symmetrical so that when u= 2f ,v also equals 2f.



Minimum distance The next graph shows the distance between the object and image (u+v) plotted against the object distance (u) (it could equally well have been v).

The minimum value for (u+v) is 4f when u=v=2f. This means that no image can be formed with a convex lens of focal length 10 cm if the object and the screen are closer than 40 cm.

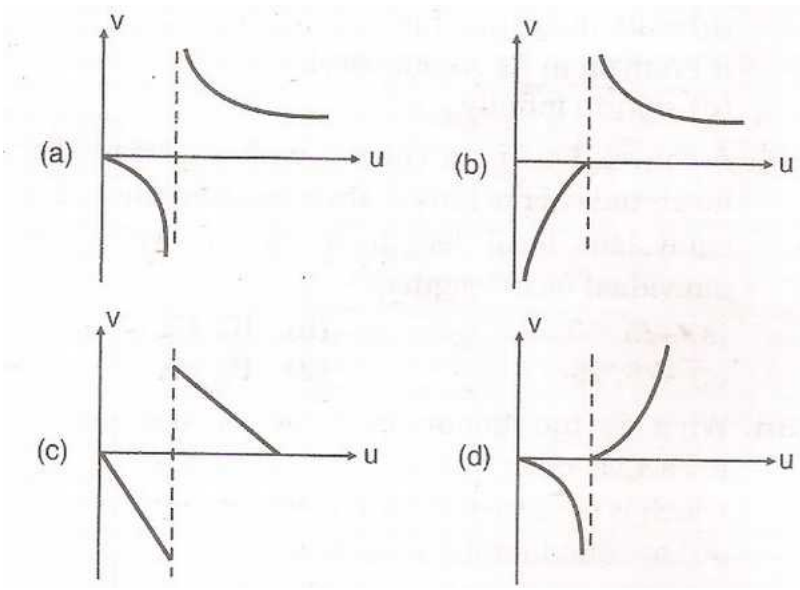


7.1.2 Problems for Practice

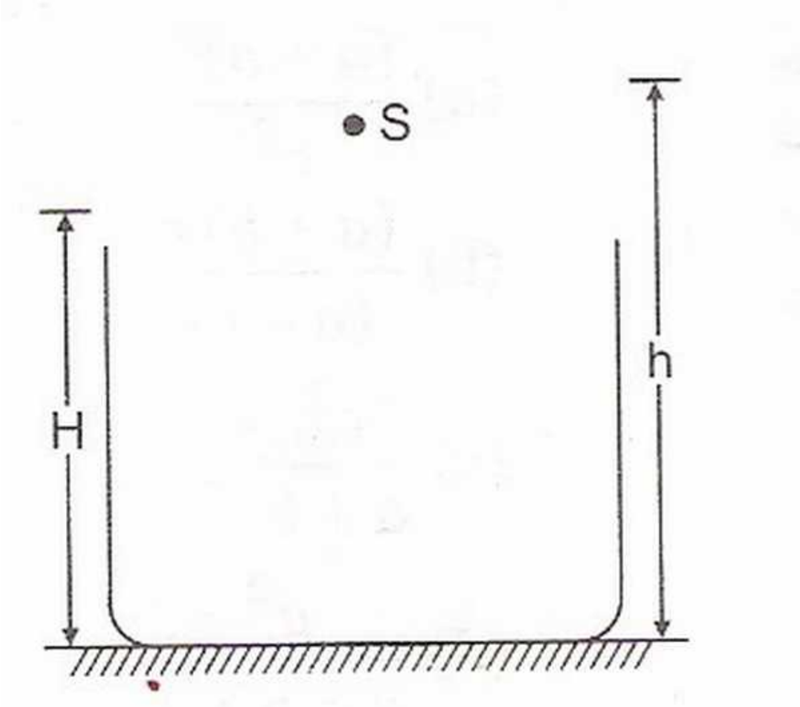
7.1.2.1 General Problem Set

Single Answer Type

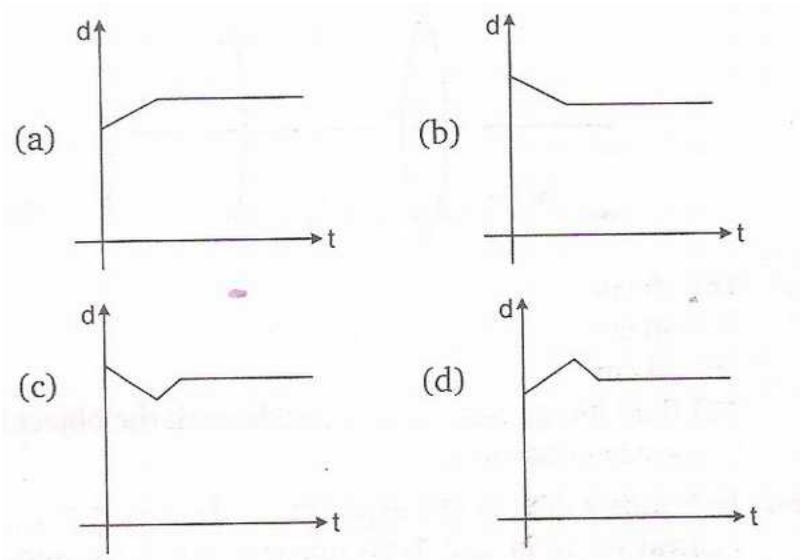
Example : As the position of an object (u) reflected from a concave mirror is varied, the position of the image (v) also varies. By letting the u change from 0 to $+\infty$ the graph between v versus u will be



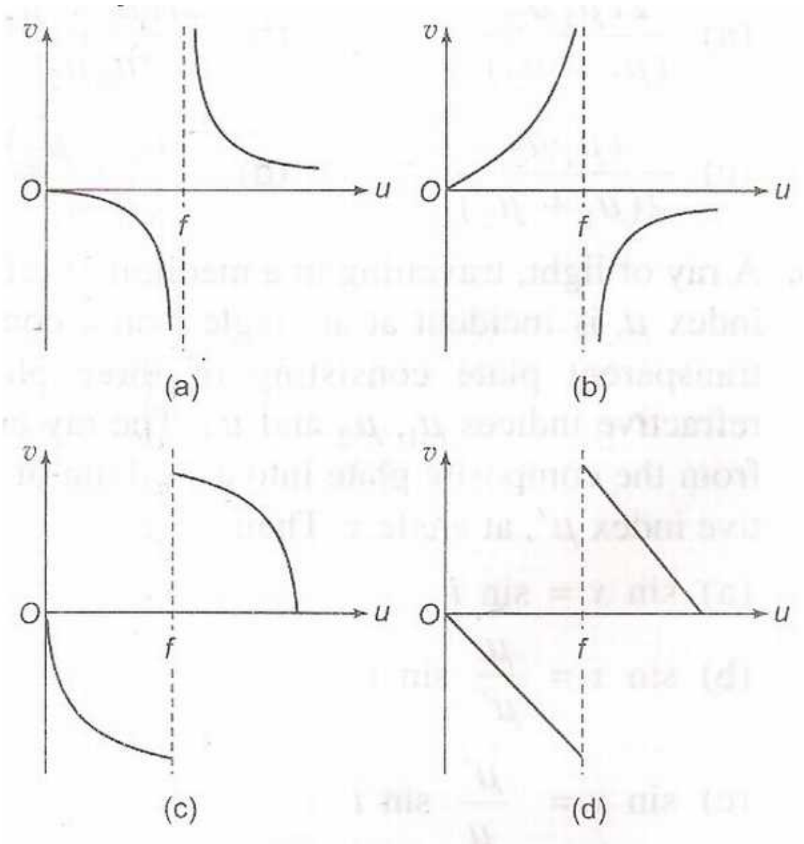
Example : A point source S is placed at a height h from the bottom of a vessel of height H ($<$ h). The vessel is polished at the base. Water is gradually filled in the vessel at a constant rate $\alpha m^3/s$.



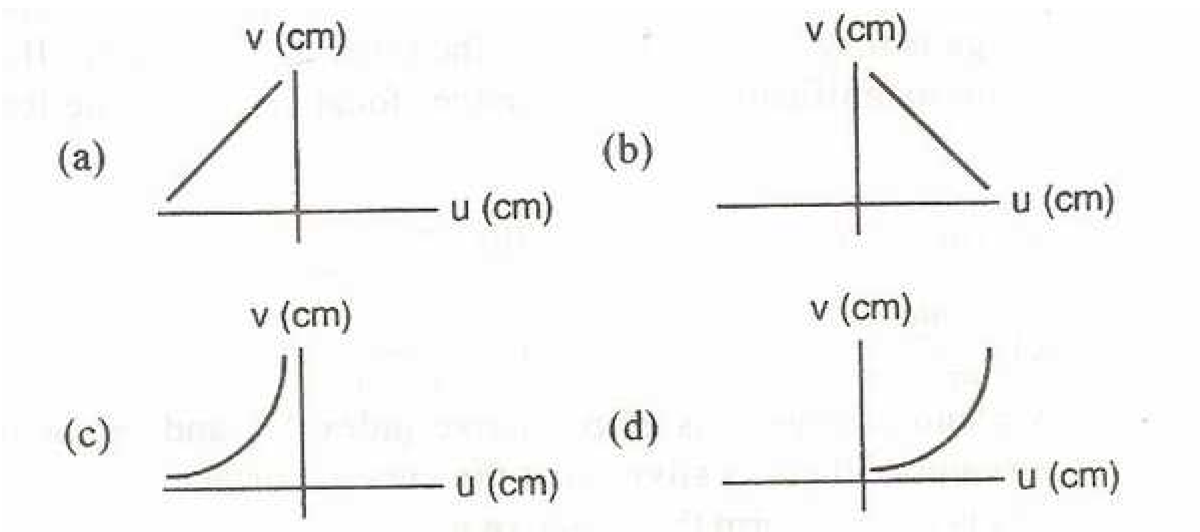
The distance d of image of the source after reflection from mirror from the bottom of the vessel varies with time t as



Multiple Answer Type Example : The image distance (v) is plotted against the object distance (u) for a concave mirror of focal length f. Which of the graphs shown in Figure represents the variation of v versus u as u is varied from zero to infinity?

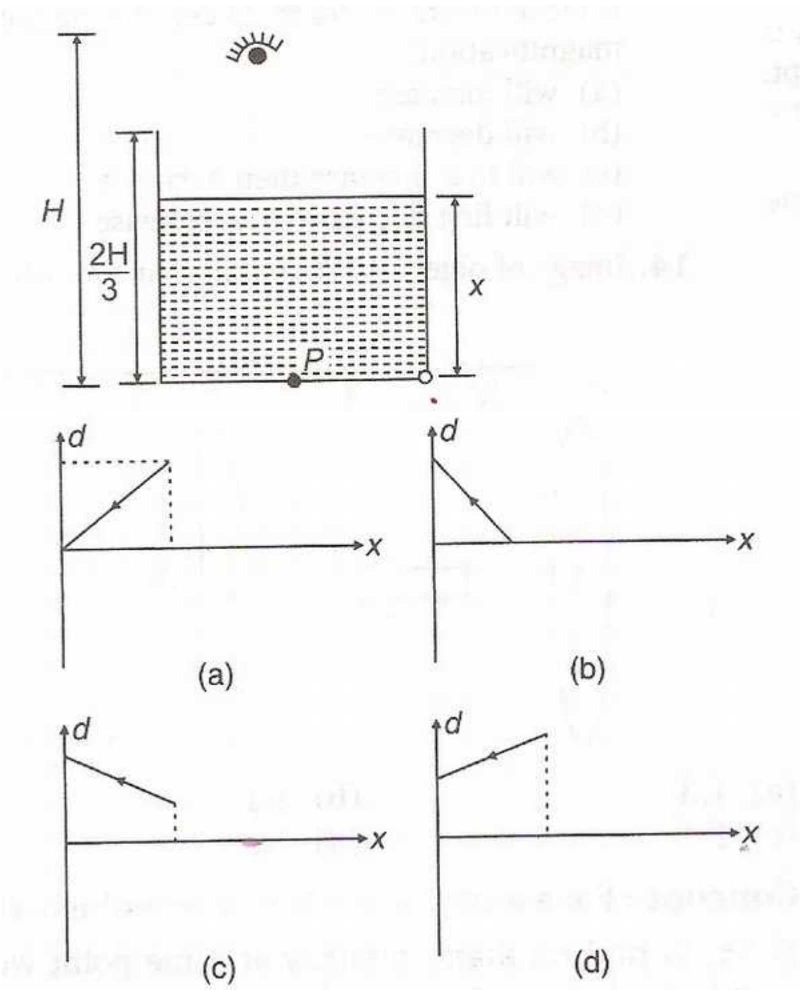


Example : A student measures the focal length of a convex lens by putting an object pin at a distance U from the lens and measuring the distance v of the image pin. Graph between u and v plotted by the student should look like :



Comprehension Type

Comprehension 1 Liquid is filled in a vessel of height 2H/3. At the bottom of the vessel there is a spot P and a hole from which liquid is coming out. Let d be the distance of image of P from an eye at height H from bottom at an instant when level of liquid in the vessel is x. If we plot a graph between d and x it will be like


H
2H/3
x
P
d
x
(a)
(b)
(c)
(d)

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(N.d.). URL: http://www.schoolphysics.co.uk/age16-19/Optics/Refraction/text/Lenses_graphs/index.html.

Chapter 8

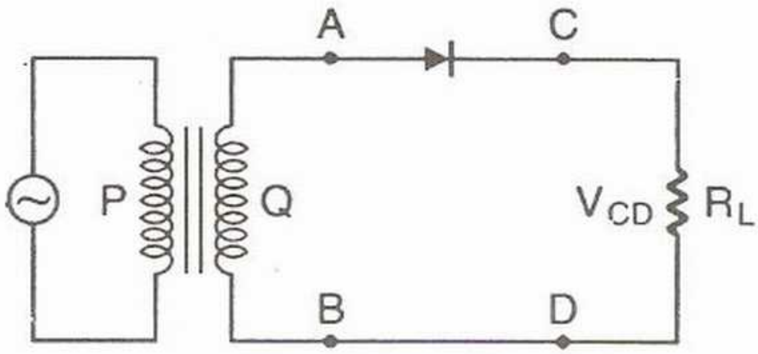
Microelectronics

8.1 Semiconductors

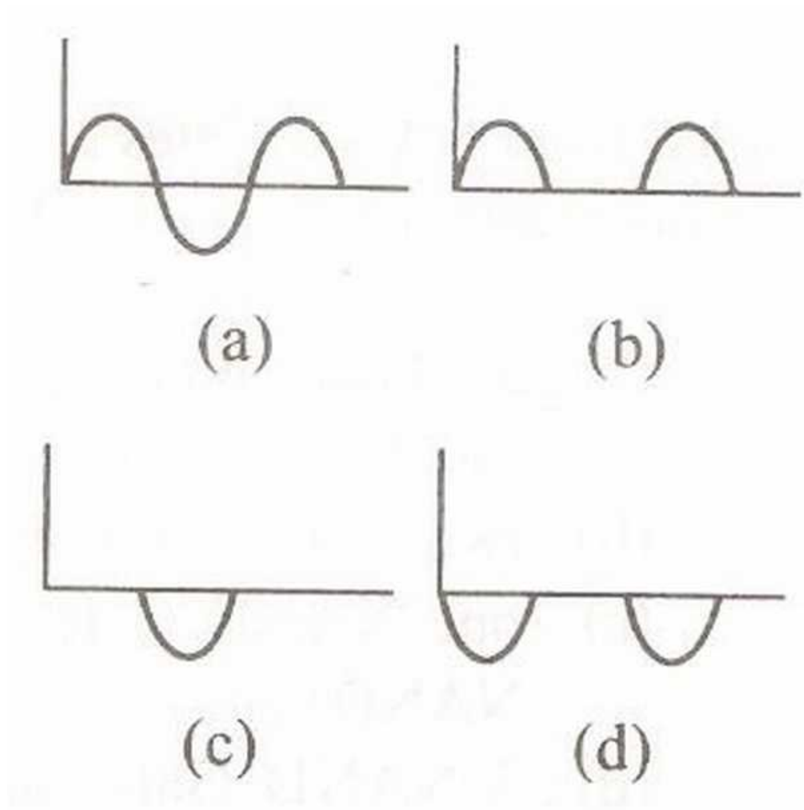
8.1.1 Diode

8.2 Rectifier

Example: In the half-wave rectifier circuit shown:

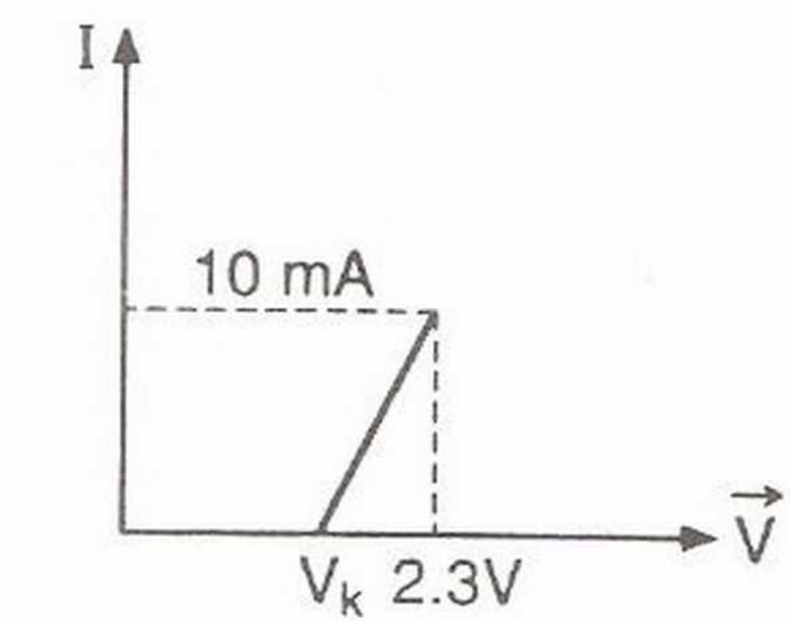


Which one of the following wave forms is true for V_{CD} , the



8.2.0.1 Junction Diode

Example: The reistance of a germanium junction diode, whose V-I is shown in figure is: ($V_k = 0.3V$)



a) 5 $k\Omega$

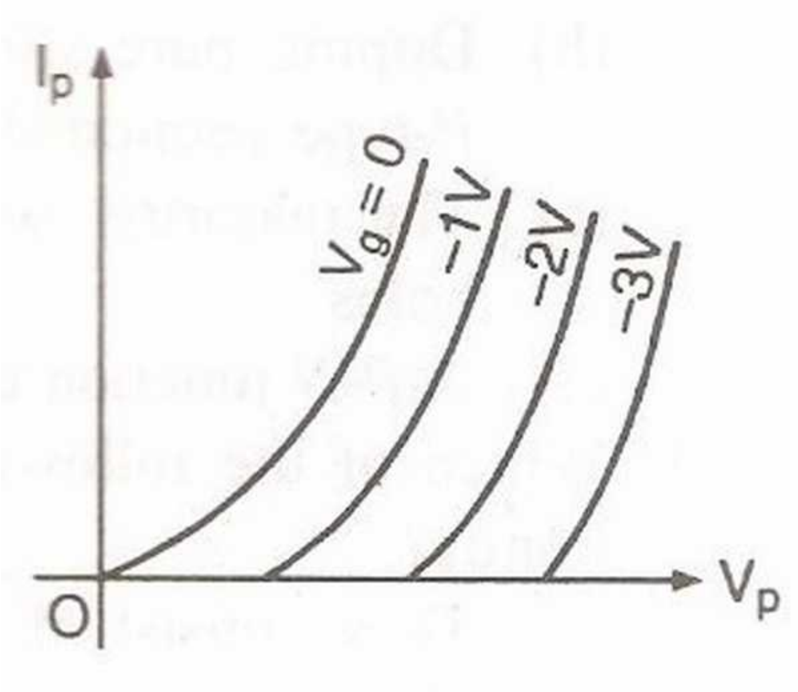
b) 0.2 $k\Omega$

c) 2.3 $k\Omega$

d) $\left(\frac{10}{2.3}\right) k\Omega$

8.2.1 Triode

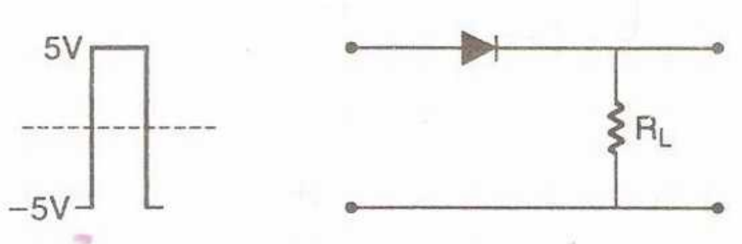
Example: The characteristic of triode shown in Figure. is known as :



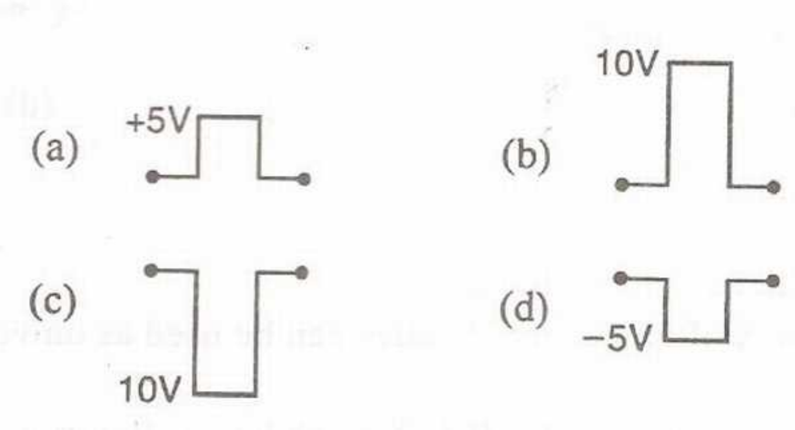
a) mutual characteristic
b) transfer characteristic
c) static plate characteristic
d) voltage transfer characteristic

8.2.2 p-n Junction

If in a p-n junction diode, a square input signal of 10 V is applied as shown



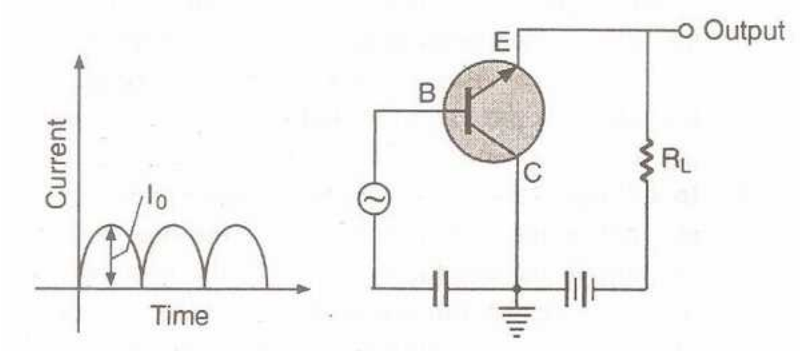
Then the output signal across R_L will be:



{Answer: a) }

8.2.3 Transistors

Example: The output current versus time curve of a rectifier is shown in figure. The average value of the output current in this case is:

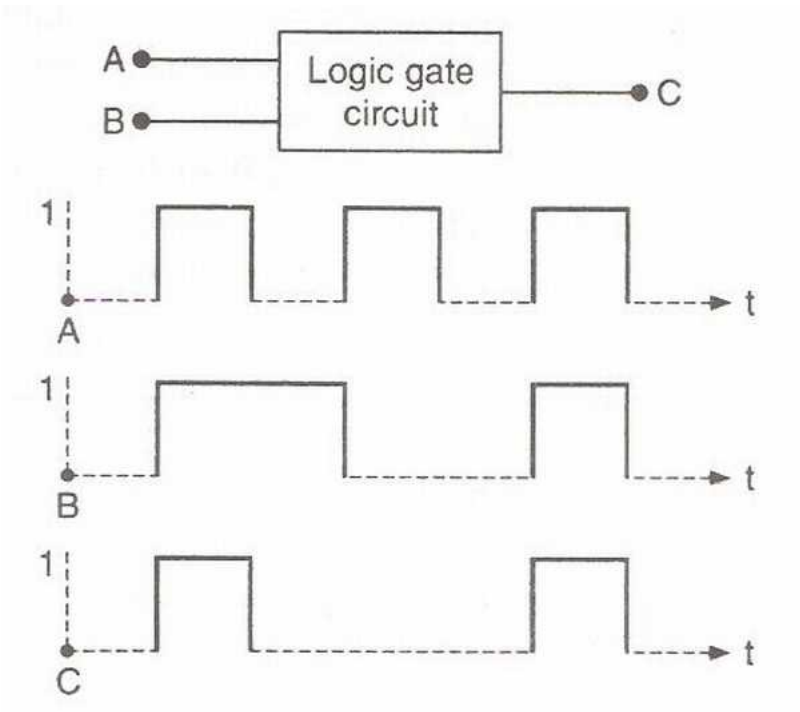


a) 0
b) $\frac{I_0}{2}$
c) $\frac{2I_0}{\pi}$
d) I_0

8.3 Logic Gates

8.3.1 Problems

Example: The folowing figure shows a logic gate circuit with two inputs A and B and the output C. The voltage wave forms of A, B and C are as shown below:



The logic circuit gate is
a) AND gate
b) NAND gate
c) NOR gate
d) OR gate
{ Answer: a) }

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