

The Light of Physics - First Edition

Manas Kalia and Rajat Kalia

14 April, 2018

764, 41 A, Chandigarh 160036

Contents

I Introduction	9
1 Physics in a nutshell	11
2 Something more Concrete (than merely panchtantra stories)	15
3 A brief overview of various disciplines connected to Physics	17
3.0.1 Medical Science	17
3.0.2 Electrical Engineering and Electrical / Communication engineering	17
3.0.3 Mechanical Engineering	17
3.0.4 Civil Engineering	17
II Mathematical Tools	19
4 Trigonometry	21
5 Calculus	23
5.1 Derivatives	23
III A Concise Course in Graphs	25
6 Introduction to Graphs	27
6.1 Theory	27
6.1.1 Introduction 2016	27
6.1.2 Elements of a good Graph	27
6.1.3 Graphical Representation of Uncertainties	29
6.1.4 Curve Fitting	29
6.1.5 Uncertainty in a Slope	30
6.1.6 Graphical Analysis of Data	30
6.1.7 EXERCISES	32
6.2 Common Graph Forms in Physics	32
6.2.1 Linear Relationship	32
6.2.2 Inverse Relationship	33
6.2.3 Inverse-Square Relationship	33
6.2.4 Double-Inverse Relationship	34
6.2.5 Power Relationship	35
6.2.6 Power Relationship 2	35
6.2.7 Polynomial of Second Degree	35
6.2.8 Exponential Relationship	36
6.2.9 Natural Log (LN) Relationship	36
7 Mechanics	41
7.1 Kinematics	41
7.1.1 The Equations of motion and the origin of Graph Handling	41
7.1.1.1 The First Equation	41
7.1.1.2 The Second Equation	43
7.1.1.3 The Average-Velocity / Instantaneous Velocity , Equal Case	45
7.1.1.4 The Velocity-Displacement Case	46
7.1.2 Previous Years IIT Problems	47

7.2	Laws of Motion	49
7.2.1	Abstract Introduction	49
7.2.1.1	Force – Time Graphs	49
7.2.1.2	Change in Momentum or the "Impulse"	49
7.2.1.3	Impulse graphs (A Case Study) n.d.	50
7.2.2	Friction	52
7.2.2.1	Static Friction 2017	52
7.2.2.2	Kinetic Friction	52
7.2.2.3	Friction Plot	52
7.2.2.4	Rolling Friction	53
7.2.2.5	Few problems related to Friction	53
7.2.3	Theory and Problems	56
7.2.3.1	Impulse as Force-time Graph	56
7.2.3.2	Force Time graph with respect to momentum	58
7.2.4	Problems for Practice	64
7.2.4.1	General Problem Set	64
7.2.4.2	Previous Years IIT Problems	65
7.3	Energy Conservation	66
7.3.1	Abstract Introduction	66
7.3.1.1	KINETIC ENERGY	66
7.3.1.2	Work Done by a Variable Force	66
7.3.1.3	The net force vs. position graph	66
7.3.2	Theory and Problems	67
7.3.2.1	Force vs. Distance graph.	67
7.3.3	Practice Problems	68
7.3.3.1	General Problem Set	68
7.3.3.2	Previous Years IIT Problems	69
7.3.4	Review Questions I	70
7.4	Rotatory Motion	71
7.4.1	Problems for Practice	71
7.4.1.1	General Problem Set	71
7.4.2	Angular Momentum Conservation	75
7.5	Gravitation	75
7.5.1	Basics	75
7.5.1.1	Variation of "g"	75
7.5.2	Problems for Practice	76
7.5.2.1	General Problem Set	76
7.5.2.2	Concept 1 Gravitational potential inside a spherical shell is constant and outside the shell it varies as $V \propto \frac{1}{r}$ (with negative sign). Here r is the distance from centre.	81
7.5.2.3	Previous Years IIT Problems	82
7.6	Periodic Motion	84
7.6.1	Abstract Introduction (SHM)	84
7.6.1.1	Position vs time	84
7.6.1.2	Velocity vs time	84
7.6.1.3	Acceleration vs time	84
7.6.1.4	Peak Height	85
7.6.1.5	Graphing position, velocity, and acceleration with respect to each other	85
7.6.1.6	Position and acceleration verses velocity	85
7.6.2	Problems	86
7.6.2.1	General Problem Set	86
7.6.2.2	Previous Years IIT Problems	91
7.7	Statics	92
7.7.1	Modulii of Elasticity	92
7.7.1.1	General Problem Set	92

8 Heat	97
8.1 Thermodynamics	97
8.1.1 Practice Problems	97
8.1.1.1 General Problem Set	97
8.1.1.2 Previous Years IIT Problems	117
9 Waves	121
9.1 Mechanical Waves	121
9.1.1 General Problem Set	121
9.1.1.1 Single Answer Questions	121
9.2 Sound	122
9.2.0.1 General Problem Set	122
9.2.0.2 Previous Years IIT Problems	123
10 Electromagnetism	125
10.1 Electrostatics	125
10.1.1 Problems for Practice	125
10.1.1.1 General Problem Set	125
10.2 Current Electricity	128
10.2.1 Basics	128
10.2.1.1 Theory	128
10.2.1.2 Problems	129
10.2.2 Capacitors	133
10.2.2.1 Discharge of a capacitor	134
10.2.2.2 Charging a capacitor	134
10.3 Magnetic Field	135
10.3.1 Problems for Practice	135
10.3.1.1 General Problem Set	135
10.3.1.2 IIT Previous Years Problems	135
11 Matter	139
11.1 Lattice	139
11.1.1 Experiments	139
11.1.2 Problems	139
11.1.2.1 Previous Years IIT Problems	139
11.2 Nucleus	140
11.2.1 Problems	140
11.2.1.1 Previous Years IIT Problems	140
12 Optics	143
12.1 Ray Optics	143
12.1.1 Theory	143
12.1.1.1 Graphs for convex and concave lenses	143
12.1.2 Problems for Practice	145
12.1.2.1 General Problem Set	145
13 Microelectronics	151
13.1 Semiconductors	151
13.1.1 Diode	151
13.2 Rectifier	151
13.2.0.1 Junction Diode	152
13.2.1 Triode	152
13.2.2 p-n Junction	153
13.2.3 Transistors	153
13.3 Logic Gates	154
13.3.1 Problems	154

IV Mechanics	155
14 Kinematics	159
14.1 Derivation of Newton's Equations of Motion from basic forms and graphs	159
14.1.1 Newton's first equation of motion	159
14.1.1.1 Basic Derivation (without use of calculus or graphs)	159
14.1.1.2 Derivation from v-t graph(Scalar form)	159
14.1.1.3 Calculus Derivation(to be used in our class)	160
14.1.2 Newton's second equation of motion	160
14.1.2.1 Basic Derivation (without use of calculus or graphs)	160
14.1.2.2 Derivation from v-t graph(Scalar form)	160
14.1.2.3 Calculus Derivation(to be used in our class)	161
14.1.3 Newton's third equation of motion	161
14.1.3.1 Basic Derivation (without use of calculus or graphs)	161
14.1.3.2 Derivation from v-t graph(Scalar form)	161
14.1.3.3 Derivation(to be used in our class)	162
14.2 Relative Velocity	162
14.2.1 Crossing the River problems (Theory)	162
14.3 Motion in 2D	165
14.3.1 2D Projectile Motion	165
14.3.1.1 Projection at an angle to the Horizontal	165
14.3.1.2 Horizontal Projection (Corollary)	166
14.3.1.3 Projection on Inclined Plane	166
15 Laws of Motion	169
15.1 Problem solving techniques	169
15.1.1 Free Body Diagrams	169
15.1.2 The Constraint equation	169

Preface

This book is based on existing knowledge on Physics already available with various Physics Institutions and now being presented in Author's own words. It is really fortunate that lots of good reading material is already available in the market so that a physics learner can quench his or her appetite. However , we would like to go a step further and present the India's native way of learning physics presented by one of World's topers in his academics and also an enticing journalist in his work field, the Author Mr. Rajat Kalia . He further wants his son Manas Kalia to take this book forward during his lifetime and be a part of his father's contributions to the world education system. Rajat wants his son Manas Kalia to be a doctor from AIIMS , though at the time of this writing , Manas is only 7 years old. So this much ahead in future is being planned all due to God's grace . So this book would have nearly two generations of our family mostly devoted to it and both having top of the world class education themselves, Rajat an IITian and Manas if he is able to join AIIMS then he too would be the righteous and just candidate for writing this book further.

Rajat spent part of his career at very small coaching institutes and software companies with very small infrastructure compared to what IITians usually work in ,in Chandigarh as during the very initial phase of his career , while working in Gameloft at their Hyderabad development centre he met with a very serious accident. Fortune 500 MNC's and bon bon America IITian dream completely shattered. Though although a topper , he managed to pull himself back out of the mess but then again the failed marriage was part of his before 30 years life. Still the son Manas Kalia was very intelligent since very early childhood and definitely deserved an already good work from his father which he could take further during his lifetime. Further Rajat started suffering from a mental disease due to the Military beating he got in Hyderabad accident , the disease Reverse Schizophrenia. This disease coupled with meditation he learned from Guru Sham Giri (real name Surender Giri) gave him a perfect vision for seeing the physics happenings with his own eyes. Later Rajat himself figured out that the American dream wasn't exactly all he was looking for and India too had everything needfull . Being a pampered child since childhood did the work .

Rajat has written some AOE2:TC 1.0 and 1.0e maps with one map Attila@Gmetro_v3 getting very popular. His as a player career was totally dwindling and he could not be in world's 250 ranks till one day one 4v4 with Chris having Nilgiri's BSF and Aravali's Arjun Gaur in his team and Rajat's team having Cuitlauac and Kumaon's cultural secretary Harry (with handle creator) , the game Rajat's team won and he had a 15 minutes castle and an Elephant Flush. He however mistook Chris for Voodoo as he wasn't playing in his usual form or probably 1.0 Fast speed was bit new settings for him or maybe India's whether too bad for him . He was probably staying in some IIT hostel too it is believed and probably came for intern. People said he was real L_Clan_Chris later. After that Rajat's teams won most matches they played and remained in amongst Game's top players in India. Later other smaller groups too established including the BSK and Indus etc but were wiped out with time as the game's new mod didn't have much people playing it . One guy Sasi Sekhar died too probably owing to his due to the game hallucinations. He was from BITS Pilani and doing in Indus clan during the initial phase of it's formation and died in America. He was nearly Rajat's age . Moreover Rajat's father claims that Rajat's disease has a lot to do with the game too and it's the game which is the real cause of the mental disease. However , Rajat later played World at Arms and Castle Siege (AOE) and was a fierce player and managed his groups well. Rajat also played in 7 Hardest game and won using Koreans Civilization . Rajat's friend and game critic Rahul Aggarwal claims that Hardest didn't play well in that game, while Rajat says that 3 of the scouts of the computers were killed in the initial 7 or 8 minutes and this leads to the AI performing really bad in 1.0 settings. Moreover Rajat was playing aggressive and one of the hardest resigned in 20 minutes due to tower attack, and only one or two of them could reach imperial. The part which usual fans like the most is that Rajat had nearly full population of Elite War Wagons (200 then) by the end of the game. The movie "3 Idiots" shows the name of IIT as Imperial college of Engineering as most of the people in IIT's during those times played Age of Empires (especially the people whom Aamir represents , the common engineers) and play in Castle age and don't like Imperial age wars. AIIMS is a similar way in Medical and has even lesser seats than IIT and Manas would also be my kind i think. The name change of IIT to Imperial college of Engineering simply let the movie connect to other smaller good colleges where AOE was the face of a magna , and good for the movie's earnings and also a kind of joke on IITians , that they do this game and all stuff once they reach IIT and spoil their career. Not the same with the story and the presentation of the

movie and also the theme song “Saari umra ham” which only represents IIT and nothing else. Other games Rajat had a faction HN_Clan like in World at Arms and Castle Siege and they did good too. Earlier in olden times wikipedia pages of HN Clan and India Cup were created and they too added to the publicity. The Indian Gamers Organization owned by Rajat and having website with the same name had massive response . Article series were written on AOE India orkut page and some initial 15 or 16 articles are still there on the web. Rajat wrote a total of around 250 articles in the Newbie FAQ thread. Abis Ottoman a pune guy with this handle wrote well in Rajat’s favour some interesting prose and was our good friend till he all of a sudden vanished and Rajat too met with accident and that’s how our team ended . Beyonder (Vishesh Dahinwal) was a good guide. Niraj Patel (BSF) too played and guided well till one day he switched sides in favour of Nilgiri people who were playing puny hostel politics in an Inter India competition and then Akshay Kumar from Oracle , he too later created an Anti group on Orkut. Rajat won Tryst 2005/2006 tourneys and was amongst the winners in War of Million , these being the India’s top AOE tourneys. He had a massive fan following and critic and anti groups too in large number as the supporters. In Computer Games he was the only phenomenon India ever had even to this date. Mostly because no single game could ever get as popular as AOE in India and Rajat was a popular software worker with his software tobu one of the everydoor name before he began his AOE career, the new advent of Internet post to 2000 Y2K fear and hype and he has contributed to a great extent in promoting India’s games industry and it’s not considered that bad a taboo as it was in those days though Rajat is all set to receive his 5 pending electric shocks for Reverse Schizophrenia treatment but has contributed in a great deal to pave way for future generations in safety while his own son probably would have some psychiatric problem probably due to excessive exposure to games and computers too.

In Software too Rajat has done extremely well . Rajat completed his software learning during the very beginning of his IIT phase and being one of the pioneers introducing Computers to other likers post to the 2001 entry in IIT. During that phase Games had not at all got introduced in IIT and Rajat and friends were all learning and exploring. Rajat got to admin a partially corrupted software Tobi which got out of control due to some accident with it’s server and none of the creator’s or caretakers had any clue on what had gone wrong . Rajat was the hot shot software guy and was approached for dead tobi’s revival and it ran for nearly 2 years till 2004-05 and became the face of IIT to the outside visitors . A parallel Kumon e-governance site was also created by Rajat’s juniors under his guidance during that period. Then one Startup guy got Rajat and Party for creating some mobile phone messenger back in 2004 , we see those concepts now in practicality now as in 2018 while those were budding concepts then and they were amongst the core introducers. Hacking too he did considerable work on but were appropriately punished by the IIT authorities. Then Rajat was creating one inventory control system for Indian Air Force , BRD 13 . He could not complete the project due to the ongoing hacking case on him during that period in IIT but atleast IAF got the concept and would have probably got it completed as IITians were specially hired during that period and Rajat’s concept was really pathbreaking in those times(2006) and later now in 2018 we see even very small companies using inventory control using that concept. Gameloft job was a dream come true for Rajat , job in Game industry and that too in a top Game company. Rajat worked further too on many projects like the varnishlog filtering for Xataka and Graphs Engine of Intellectual Property Rights in GreyB and all have been known as good and pathbreaking concepts which can absorb future generations to work on them and even in the intervening 12-15 years large portions of software industry have worked on them and they were Rajat’s concepts initially and it would look like a sort of wonder to you. Wherever Rajat has gone for working in software , he has introduced new concepts and it’s strange that we have got most of the software population working on those concepts till much later ages.

Part I

Introduction

Chapter 1

Physics in a nutshell

Now I won't start with the statement "Physics is everywhere". It's an old statement. I would instead start with let's say , what's a blackhole or a quasar or a pulsar.

Now you are in +1/+2 so mostly you know about gravitation , current , pressure etc. So, in a blackhole, let's see at the centre first, one possibility is that there's extreme pressure, temperature gravitation and current. Second possibility is that above the centre the forces balance out, like in case of sun, it's being said that even though above it's extremely hot with plasma and nuclear fusion reactions going on , in the inside it is not so and there are people out there trying to scientifically prove it. There are sci-fi movies like the "The Core" which say that earth's centre is also the same as sun and there is no hot core and it's a primitive civilization living inside the earth, however in case of earth, these claims have been widely rejected, maybe possible in case of somewhere else in the universe's earth, i mean some other earth.

So, what is a blackhole. Is it something to be inquisitive of. Is it something to be afraid of?? Or is it the solution to all our problems. And why am i interested in blackhole and how can i help in learning about it.

{Gandhiji's 3 monkeys : Don't see bad, don't speak bad , don't chat bad. Now every mental patient can be classified in one of these three types, the ones who use their vision to see bad things etc. and in return lose both their eyes in old age as a punishment and then there are other two types of monkeys}

So , due to my disease which is a form of Paranoid Schizophrenia , i can look into matter (where i am guided to see it) . I can use it to peek into some girl's bathroom or else i can see and show a blackhole's structure. Don't worry I am not a dangerous person who can spy on you, I take medicine now. These things which i am showing here are what I saw before taking medicine and now i have faint memory of them. Gandhiji claimed that every person in the world is one of these three types of monkeys and has one of these special gifts which connects them to God and after 35 years of age everybody has old age protection from god's side.

So what exactly the use of these blackholes , quasars and pulsar's etc.

They can create heavier elements than our normal 118 elements . But then why don't we find these elements created by blackholes anywhere.

Now I will tell what I have seen , then later I'll tell the fantasy. Blackholes are the excretion points of the universe where they consume worn out planets and stars and exit them out of this universe by feeding them to the other end of the opening of other universes about to be created . Now a question arises, if this much mass was teleported out of the universe, where does the universe balance it's mass from. The answer is again what I can see , the universe boundaries have a shell like covering made of faint white and black salt which creates life and matter. from there the material enters , from blackholes it exits. But then we know of the neutron burst which created the universe? Don't blackholes consume all mass and all planets and stars and grow big to become the neutron star? and then it bursts again to create the universe again . The answer is , yes this is one alternate mechanism. I told our universe's mechanism and you are telling the one connected to yours.

Million dollar question? What happens to US when these blackholes consume everything.

And there is a 2 Rupee answer (provided free of cost or covered in the price of this book). We have already moved to alternate universes and blackholes consume only completely dead planets and solar systems. So when the universe is completely dead , then the neutron star is formed.

Another irritation in brain. What is a universe according to hindu religion? It's the body of bhagwan in spiritual domain. Like we have a body, now we are on the outside , we can't take care or look inside , so we are the devil who is greater being than god. Under our skin , there is a force always looking inwards and providing all the necessary things to all cells who are inside and taking care of them . Now we see these cells on operation or a cut in physical domain , but if we close our eyes and look inside we can see a similar universe to ours from a position outside it , near our head , our eyes. So universe is nothing but a person's body, inwardly controlled by God and not that person, however that person is the one who provides food and takes work out of the body. Interestingly what we see outside is also another universe with all the people and we are a small being in this universe while that inner universe is so cool we are even bigger than it. So what are we? we are at the

boundary. Why we see both differently is because , like we say that the universe has all the planets and solar system and so frightening things and inside our body there are the cells the tissues and the brain , heart etc frightening things. Why do we see them different, This is because we are seeing the outside planets and solar systems etc in the spiritual domain , similarly in our body we can look in spiritual domain by closing our eyes and looking inwards. While the cells and tissues and organs etc are in Physical domain . Our universe also has it's physical domain similar to us and we tried to visualize the blackhole as an excretory organ. There is another domain, the mechanical domain one of the three domains which i have seen and it has all the medicine reservoirs in our body and we simply refill them or change the medicine already present in our bodies in these reservoirs. There can be other domains in auditory and speech sections, I myself haven't seen them how they work, but i hope there would be atleast 3 domains there too in each method.

The fantasy

But some people say a blackhole is extremely dangerous, tell some horror stories or other intersting stories.

Fruits and vegetables

Now there is an intersting story going, and I have seen it with my eyes as well. 10000 years punishment in a blackhole. It occurs when someone becomes a vegetable or a fruit. Now everyday the birth begins , there is very faint light inside the blackhole. We can move slightly in the room , the blade starts and we are cut completely and then we die. Then we wake up and again have a fresh body and it's again cut and then we are left to die, it's repeated 84 lakh times in a total time span of 10000 years. This punishment starts when 84 lakh vegetables or fruits are consumed by one person. Interstingly a chicken on the other hand has relatively much more freedom than this case . So, a chicken consuming person can be in much better state than this person, however traditional hindu religion says not to consume non-veg. Possibly the removal of shell of the vegetable , and offering the first piece to god etc are possible escape routes from this punishment, while Guru Nanak Dev ji says "sayanpan lakh howen , ik na chale naal ", meaning , that in front of almighty no cleverness works. (**Don't worry, this doesn't happen ever. I'll tell later why.)**

Howcome?

The universe is only a small part of the world without much significance. Now if the complete logic of everything fails, then say a billion universe group fails, in that case the error recovery runs. I have seen this happeing in my vision (Live) , I know the pain too as it happened to me too and also saw the difference in magnitude of pain in blackhole, quasar, pulsar etc. Blackhole is amongst the easiest as an enemy punishes there. In a pulsar , you keep thinking all the time that it's your father who is running it. All are extremely painfull.

Joke???

Well there are people out there who think that if everything fails it reestablishes in no time but in fact it reestablishes in 10000 years.

What's the best part.

The best part is that I am a mad person. Mostly my sayings are not meant to be believed and what's the use also to be getting scared without much reason. Thinking about this punishment can give us extreme courage in very dangerous situations as any situation can be very very better than this situation.

Other kind of blackhole

There is a very funny blackhole created in the name of one person named Krishan (not to be confused with Lord Krishna). In this blackhole there are in infinite number of machine buffalows with plastic casings as skin. This person krishan has to milk them to survive and he lives in a room at the center of that blackhole.

Any other kind

One normal blackhole similar to a mother's womb had a person like me at it's centre and as that person is big in size, the currents appear small , the pressure appears not much etc. But then that blackhole starts to end and persons dig into it to live there and in the centre that person wakes up as a Black monkey. When he woke up the last time there was ola shower in chandigarh, we call it mullets or what???

Other interesting visions

There are visions of Durga with 1 tiger and 999 tigresses.

Another is Durga with 8 billion tigers and tigresses

Another is Kali standing on a Damped wave with infinite tigers and tigresses.

Now these three visions, the scenario is seen at the beginning and starts, later what happens is very interesting.

In one tiger and 999 tigresses, the tiger is able to save durga and takes her away. He is Man (the inner heart) and durga is probably saved in this scenario.

In 2nd 8 billion tigers and tigresses, all of the tigers and tigresses are born out of durga's womb with an explosion and she gets red clothing due to the blood which came out along with the birth and due to the explosion it spreads on her body. In this scenario , durga is later killed by the tigers while some tigers try to save her too and fight for righteousness. There is an invisible tiger too in this case.

In the Kali one , she looks like priyanka chopra.

Then there is a 13 member family , where one boy is born at the 7th place and he has a sister. He kills the sister with her own nail and she later becomes goddess and creates the world with one tiger . Now in that case none of the 13 family members experienced any physical pain and the brother who had killed the girl and tiger later gets left alone and with one word Aum he sets into meditation and he is one person in that world and takes all births of humans and animals etc all alone. Now there was a very massive operation which took place when that guy killed the girl and the tiger and implanted them in his body. It was a perfect operation. That guy is dil and there is a floor also where he meditated and that was also dil. (It's the heart in hindi). Had that person sinned or not is not known but he is the normal living thing we see ourselves and around us. Moreover this girl looks a bit clever in the vision and probably knew or was pretending that she was not getting harmed in the operation (atleast that's what appears from the facial expressions which I saw.). Aah I remember, before finally setting into meditation, he bent his body into shapes thinking that the tiger's body shape was different than his and he bent his body into 8400000 shapes too(Or probably 8.4 , i.e. 8 or 9 shapes, or best just 8 shapes for ease.). So, for example if he bent his body like a mosquito , the mosquito he made would be as big as a human. The girl's name was Suman Radaa (similar sounding to modern day Radha) , boy's name was Rahul .

The bible's begining of the world

In one of the most popular version's of bible , it is said that the earth was already present in the beginning, it had matter and water , these two components and probably no air . Then the holy spirit (probably god) roamed near it in space and with one command all the living beings were created.

Now I want to tell what happened in the delta time that command was being spoken, we know that before it no living being was there and after it all the living beings came to existence. Now lets see what happened in that small time. Now before that time the matter looks completely calm and dark and water was also calm and only spilt when the holy spirit passed by (sort of primitive ocean waves) . The matter was clear in it's mind that it is the strongest and it can't be beaten, while the water looks extremely happy and enjoyed even more when holy spirit came near and sort of played with it. In the place of contact of water and matter, there was some sort of contact which was not a matter of conflict between matter and water till the holy spirit came to picture. Now when the command was spoken, water was the first to get frightened and it immediately realised that it's not a single entity and composed of smaller particles and they tried to come near each other in a state of fear created by the command and disturbances were set in water. As the intensity of shrill of the shriek kept on increasing, the disturbances further increased and vapour was created. Matter was still like a single entity and it was feeling like a pain in head (it's body) due to the sudden noise and wanted to reply back but didn't have a tongue or face for that matter. It started withering near the contact , not due to water causing itching but because the noise was so painful to it. Later we know that now earth has a hot core and an atmosphere and probably formed in the beginning itself . All the living things were created after the peak of the delta was reached (similar to birth case) and slightly after the peak noise . All living beings were created, now where did the holy spirit go, it was also a human in the plane where lots of other humans were standing and he brought out a mic from his pocket and made the first announcement and explained the key things to everyone just born. Now he was completely mad while everyone else just born was partially mad. It's not known what happened to him afterwards but then we know of the Adam and eve story and see that he was still present or maybe some of his descendants or he himself. Then probably Christ was the last in their lineage and now there further cults like illuminati claiming that maravengians in france is a group of people who have christ's descendants and lineage, and many movies like Da Vinci Code showing the legend.

Now here , from physics point of view , what's important is that matter is present, water is separate from matter and then all the charge is concentrated in one supernatural being , the holy spirit . Now every human

has a soul which is small chage with shiny black colour or blue color matter in the interior. So we see that Physics mostly is dealing in the Spiritual domain of the world and not physical or mechanical domain as much.

Criticism Now this is a partially mad man's view , i.e. me . I can se souls and male souls have mostly black condensed liquid in the interior , probably a mixture of matter and water for completeness.

However , drug addicts on the other hand think that this cant be the case with them. They say that their soul is completely white , they say that matter and liquid combination can be created as a grass mixture too and can be fed inside the charge outer shell of the soul. Now interestingly I have myself seen souls which are dense white and are very big in size (3 to 4 cm dia compared to usual 2mm dia(meter)) . They look like a spherical egg. There are other smaller completely white souls too (with 2.5 cm dia kindaa) , now i don't know whose souls these are in all but some people have really peculiar souls. It's possible that I am consuming excessive amount of coca cola coke and it's visible in my soul. In other people's case it can be some other substance they are consuming heavily .

Chapter 2

Something more Concrete (than merely panchtantra stories)

Now, we know of Hydrogen. What's it like ? does it smell good or bad ? (Well actually hydrogen has a punchy smell, you can buy uncle chips and it is packed in Nitrogen getting punchy odour due to the chips and the masala) We know it can be used as fuel for state of the art combustion cells, and is seen as a modern alternative to petroleum. We know fusion bombs (also some are hydrogen bombs) work on this principle , that a nuclear reaction hydrogen atoms combine to form Helium and produce lots of energy. Hitler's Archeotype which was a balloon like aeroplane having hydrogen caught fire and all people inside died (America was an enemy though but also the technology was flawed too). Modern cars and space shuttles might benefit from it.

Now hydrogen done. Do we discuss all elements or few of them as we are not a chemistry course.

Hmmm. Hydrogen has importance to physics. Then which other elements . Let's see . Ummm... let have a brief look at all.

Helium, it's the product of a fusion reaction, and an inert gas. Used by scuba divers when mixed with oxygen in some particular ratio, it has upper hand to nitrogen due to non formation of compounds with blood under large pressure.

Lithium, used in some logic gates and transistors (probably ic chips too), first alkali , light in weight atoms.

Berillium, well benadryl cough medicine tastes like it. lots reactive.

Boron, a nuclear product and having nuclear active isotopes too. Having interesting reactions in chemistry salt analysis.

Carbon, Now this deserves some space. Your complete organic chemistry is compounds of carbon. Human body has mostly organic compounds as it's living. Then we gift diamonds , they are pure carbon. What else, we have the carbon contact points in the dc generator motor etc. We have charcoal , a type of slightly impure carbon used as the bed in chemistry salt analysis. We have lots of electrical equipments using carbon parts , batteries and all, even cells and high voltage ones too have large percentage of carbon present. Hmmm. Is that all, we see that all things which are black are not carbon. So we couldn't find as many uses of carbon as it appeared , a blackhole is not made of carbon , it can have probably very little percentage of it. But the human organs are all carbon based organic compounds made . We see that Carbon has ample applications in Organic chemistry and biology but not as much in Physics but still it can have say few useful applications we can say. Mostly coz we talk electricity , we need metals, we talk machines we need metals, we talk say civil engineering , we have asphalt there , a carbon compound and oil's last residue but otherwise civil engineering has dams and large constructions and carbon can have small applications somewhere or the other in it but not a massively important material . Actually why i am speaking a bit anti is coz since childhood i have been told that it's extremely important material and now that i am trying to write a paragraph , i find not so many uses to be able to write a good paragraph. But the atomic number 6 and mass number 12 can say something. But then these are human relation talk and not physics talk. Let's stay in physics domain only and the judgement is , it's another normal element like the rest of the elements, not too useless , not too useful.

Nitrogen, It's odourless . Air is odourless and has large percentage of Nitrogen. It is used by plants and nitrogenous manures are considered extremely good for crops. Nitrogen fixing bacteria fix them for plants. And I have already said we have nitrogen packaging of food too. In a sci-fi movie terminator 2 , the enemy gets nearly killed by the accident with a liquid nitrogen tank and few minutes later gets completely killed too due to some other reason.

Oxygen, the massively important material for breathing widely portrayed in sci-fi that people in mars and the rebellion there are getting lesser of it. Mostly kept and extracted for artificial respiration systems. Oxides are a trouble as they are to be broken back to get the pure element and the pure element sometimes comes in contact with air and forms oxide again. No special use as far as physics is concerned. If we spell it as Oak-season

then it refers to the dirtiest enemy in the Mythology movies. And they keep on attacking in their season. Now clean air is devtas according to some people(according to me too as i have seen them) and bhagwan is foul air (clean air which has gathered some foul and is smelling bad now), now what is oxygen it remains to be questioned as it's a part of both clean and foul air .

Fluorine, I have my toothpaste which has fluorine, close up is the name of the paste. I don't have very good reviews for my teeth, but that's mostly due to lesser hardwork from my side . The toothpaste is amongst the best or I would say, the best toothpaste.

Neon, earlier neon lamps were made for light bulbs. Then we had neon gas leakage in large hadron collider due to which id didn't start on the recommended date. These are the only two instances i heard of neon. Inert gas.

Sodium, now Natrium (Na) . Kept in kerosene so that it doesn't catch fire in air. In water if it's kept it burns beautifully . A component of caustic soda and baking powder. 23 it's atomic mass and 11 it's atomic number. Sodna means a very bad term in punjabi sexual terminology. It's intersting that the english terminology has chosen the name sodium and not Natrium completely ignoring the punjabi meaning. As far as physics is concerned , in present times we don't see much use except compounds of it can be usefull. Direct Sodium we don't use much except state of art projects i think.

Magnesium, magnisium ribbon burning in air and MgO_2 ash collection experiments we read in 8th class. It's intersting that the ash is not MgO but with two oxygen atoms even though both O and Mg are divalent.

Aluminium, aha , 13 atomic number and 27 mass number. We used to have aluminium wires in electrical transmission earlier and now we have copper wires instead. Aluminium sheets are used widely in cutting industry. Aluminium parts used in machinery. Aluminium has good ductility , malleability etc.

Silicon, 14 atomic number. Used in most semiconductor devices. Widely available in earth's crust as silicon dioxide. Artificial chests for ladies in modelling careers. In physics, semiconductors is a vast topic and extends to electronics enginering apart from connecting to various other desciplines. In a computer the Processor and Motherboard are both silicon based.

Phosphorus, 15 atomic number. In this particular subastance , we can say we know one natural application. We see bodies burning in shamshan ghat in night due to the phosphorus in bones catching fire. This makes shamshan ghats really haunted places as they create misconception of ghosts behind these burning bones.

Sulphur, 16 atomic number. Sulphur dioxide has fart like pungent smell. Sulphur is supposed to be present in alien clothes which sci-fi movies show have smell. Then sulphur is a component of tubeless tyres etc. Also present in some chemicals. Match stick's burning masala's component etc are few of it's uses.

Chlorine, 17 atomic number but less reactive than fluorine in group 17 elements. Common Salt is abundant in nature and can be used to create chlorine using electrolysis. Detective television series show that it could be used in killings back in 1940's and then it used to be to be very hard to trap them due to weak medical technology for ascertianing the cause of death, however these days it's not used for killing as medical science has grown strong.

Argon, 18 atomic number, (not to be confused with Aragorn of LOTR, he was not inert and was linked to many women and few elvies too) . Inert gas.

Potassium, also known as Kalium, 19 Atomic number, My wife was 19 years old when she married me, She was duggal before marriage and became Kalia after marriage with me , then later she left me and married someone else.

Chapter 3

A brief overview of various disciplines connected to Physics

3.0.1 Medical Science

There is a special discipline which produces “Physicians” . Now these special doctors check the body part movement especially after a broken bone at joint area or some muscle or ligaments breakage in joint areas. In modern days however Physician is a term generally used for a normal doctor too however it is a special discipline too, we are interested particularly not in this field , we want to show how knowledge of physics can be useful in medical science.

The machinery has joints and rotations sometimes , so knowledge of physics can be helpful, It's in general an introduction to engineering as engineering is related to medical science always.

3.0.2 Electrical Engineering and Electrical / Communication engineering

The topics of these streams are briefly and in a general sense introduced in Physics courses before a full fledged course is discussed in the actual discipline. The circuit laws, the basic communication, some machinery are already studied in Physics these days.

3.0.3 Mechanical Engineering

Mechanics is a topic especially devoted to creating the base for mechanical engineering. The ideal state studied in intermediate can be easily extended to real state in senior classes.

3.0.4 Civil Engineering

The hydrostatics for dams , the cantilevers and moduli of elasticity for strengths of materials can be easily extended to all of civil engineering.

Part II

Mathematical Tools

Chapter 4

Trigonometry

Chapter 5

Calculus

5.1 Derivatives

Rule - I : The derivative of a constant is Zero.

Examples : The derivative of a constant is always zero, e.g. Derivatives of all constants like 5, π , e^3 , $\ln 10$ are all zero.

Rule - II : Derivative of x^n is nx^{n-1}

Derivative of x is 1

Derivative of x^3 is $3x^2$

Derivative of $x^{3/\sqrt{2}}$ is $\frac{3}{\sqrt{2}}x^{3/\sqrt{2}-1}$

Derivative of $\frac{1}{x}$ is $-\frac{1}{x^2}$

Derivative of $\frac{1}{x^e}$ is $-ex^{-e-1}$

Part III

A Concise Course in Graphs

Chapter 6

Introduction to Graphs

6.1 Theory

6.1.1 Introduction 2016

A graph is an accurate pictorial representation of data. The accuracy of data in physics requires that graphs be made on good quality graph paper. Nearly all graphs in physics are smooth line graphs; broken line (connect the dots) graphs and bar graphs are seldom appropriate.

The style and format of a graph will depend upon its intended purpose. Three types are common in physics

1. PICTORIAL GRAPHS. These are the kind found in mathematics and physics textbooks. Their purpose is to simply and clearly illustrate a mathematical relation. No attempt is made to show data points or errors on such a graph.

2. DISPLAY GRAPHS. These present the data from an experiment. They are found in laboratory reports, research journals, and sometimes in textbooks. They show the data points as well as a smooth line representing the mathematical relation.

3. COMPUTATIONAL GRAPHS. These are drawn for the purpose of extracting a numerical result from the data. An example is the calculation of the slope of a straight line graph, or its intercepts.

6.1.2 Elements of a good Graph

Certain informational and stylistic features are required in all graphs:

1. The graph must have a descriptive title or caption, clearly stating what the graph illustrates.
2. Data points are plotted as small dots with a sharp pencil, or as pinpricks. Some method should be used to emphasize the location of the points, for example, a neat circle drawn around each point.

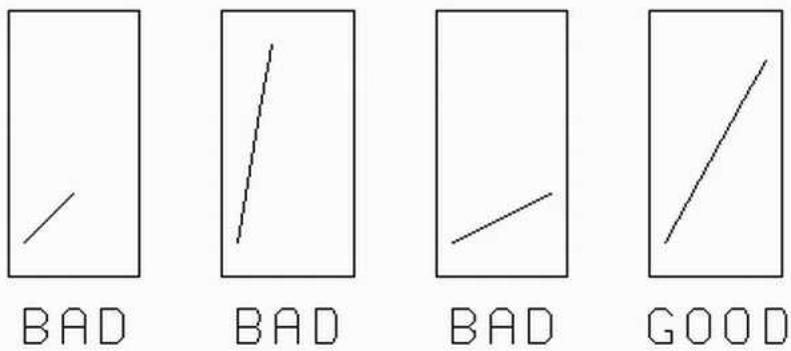


Figure :Good and bad choices of geometric slope of a graph.

3. Curves drawn through the points should be smooth (use ¹ French curves if your hand is not steady). The curve should stand out clearly.

4. Choose scales that are convenient to plot and easy to read.

¹A French curve is a template usually made from metal, wood or plastic composed of many different curves. It is used in manual drafting to draw smooth curves of varying radii. The shapes are segments of the Euler spiral or clothoid curve. The curve is placed on the drawing material, and a pencil, knife or other implement is traced around its curves to produce the desired result.

Modern computer-aided design (CAD) systems use vector-based graphics to achieve a precise radius, so no template is required. Digital computers can also be used to generate a set of coordinates that accurately describe an arbitrary curve, and the points can be connected with line segments to approximate the curve with a high degree of accuracy. Some computer-graphics systems make use of Bézier curves, which allow a curve to be bent in real time on a display screen to follow a set of coordinates, much in the way a French curve would be placed on a set of three or four points on paper.

5. Choose scales such that the graph occupies most of the page. The two scales need not have the same size units. Also, the scales need not begin at zero.
6. Indicate the name, letter symbol and units of each variable plotted on each axis.
7. All text (title, labels, etc.) should be printed.

PHYSICAL SLOPE AND GEOMETRIC SLOPE

Slope. When textbooks refer to the "slope" of a plotted graph line we mean the "physical slope"

$$\text{physical slope} = \frac{\Delta y}{\Delta x}$$

where Δy and Δx are expressed in the physical units of the x and y axes. This slope has physical significance in describing the physical data.

Geometric slope. A line which makes a 45° angle with an axis will not necessarily have a physical slope of size 1. Some authors introduce the term "geometric slope" to describe the tilt of the line on the page. This is a ratio of lengths of the legs of the triangle, without reference to the units plotted on the axes.

There is seldom (probably never) any need to calculate the geometric slope of a line on a graph. The idea is only useful when describing the appearance of the graph on the page. One rule of graph construction states that the graph should occupy most of the page. For square graph paper this suggests a geometric slope of 45° . See Fig for examples of good and bad choices of geometric slope.

THE APPEARANCE OF THE GRAPH

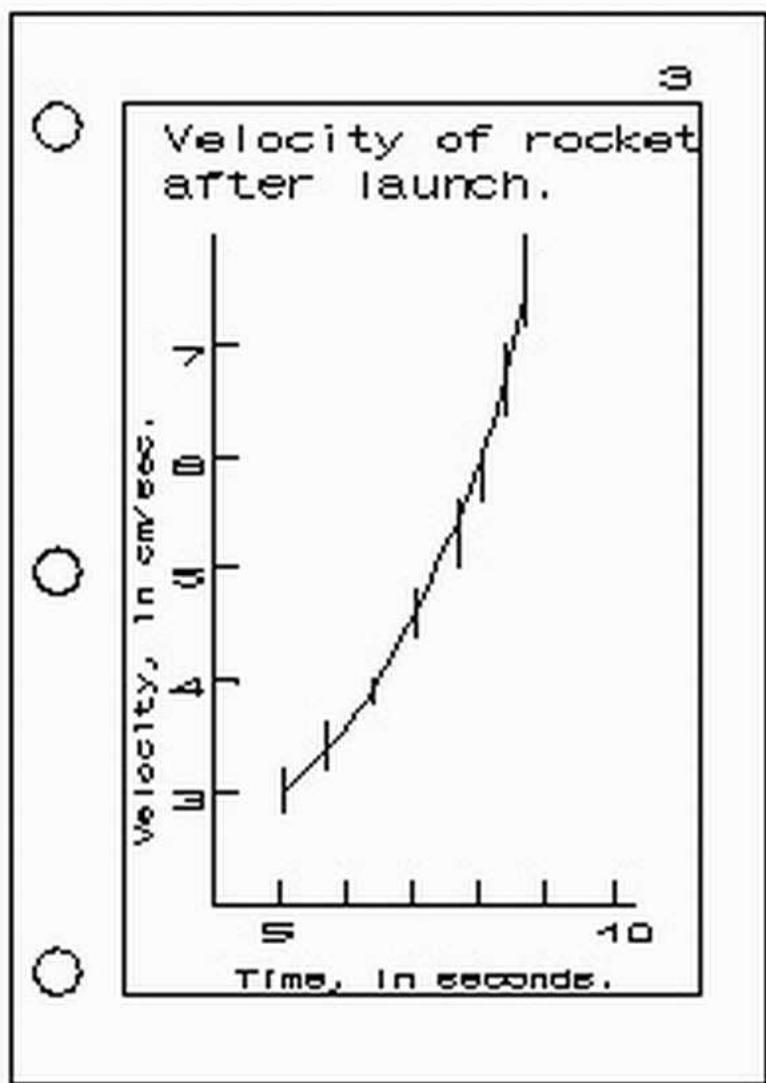


Figure : Elements of a graph.

Use quality graph paper, size 8.5 by 11 inches only.

The left margin is largest, for binding or stapling.

Nothing should be in the white margins except a page number. Axes, lettering and labeling should all be within the printed grid area. [The grid lines serve as guide lines for neat, uniform printed lettering.]

The title must be descriptive.

Both axes are labeled with the full name of the quantity (not merely its symbol), and its units.

The plotted points and curve should occupy most (more than half) of the area of the graph paper.

Sometimes a small sketch of the experimental situation may be included, located where it will not confuse the interpretation of the graph. In the same manner an equation, or short explanatory comment, may be included.

6.1.3 Graphical Representation of Uncertainties

Display graphs and computational graphs should clearly show the size of the experimental uncertainties (errors) in each plotted point. There are several conventional ways to do this, the commonest being the use of error bars illustrated below:

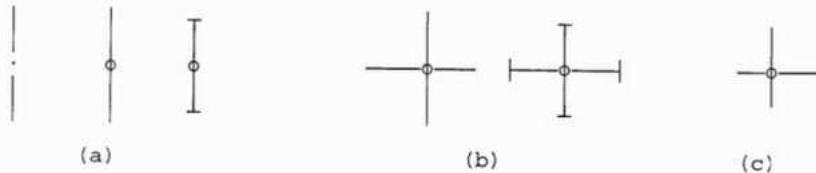


Figure : Various styles of error bars.

The plotted point is represented as a dot, and the range of uncertainty is shown by the extent of the bars on either side. The types shown in (a) are suitable where the error is entirely in one variable, or where the errors in both variables have been lumped together. The types shown in (b) are preferred where it is necessary to show the error in each variable explicitly.

When the uncertainties have a symmetric distribution about the mean, the error bars extend equally on either side of the points. If the data distributions are not symmetric, the plotted points will not be centrally located in the range of uncertainty and the error bars might look like those in Figure. part (c).

Error bars may not be necessary when the data points are so numerous that their scatter is clearly shows the uncertainty. In these cases error bars would clutter the graph making it difficult to interpret. Another situation where error bars are inappropriate is when the scale of the graph is such that the bars would be very small. In this case, it may be possible to indicate the uncertainty by the size of the circle or rectangle surrounding each point.

6.1.4 Curve Fitting

The curve drawn through plotted data need not pass exactly through every data point. But usually the curve should pass within the uncertainty range of each point, that is, within the error bars, if the bars represent limits of error.

One principle of curve fitting is also a fundamental rule of science itself:

Assume the simplest relation consistent with the data. We are not justified in assuming a more complex relation than can be demonstrated by the data. If a curve were drawn with detail smaller than the data uncertainty, that detail would be only a guess.

This rule of simplicity may also be expressed mathematically. The mathematical relations encountered in physics may often be represented by power series such as

$$y = A + Bx + Cx^2 + Dx^3 + Ex^4 + \dots$$

where A, B, C, ... are constants.

For very "wiggly" curves, many terms of this equation, including high powers of x, might be required to express the equation of the relation. The simplest relations are those which contain the smallest powers of x. The simplest relations of all are

$$y = a \text{ or } y = a + bx$$

which describe straight lines. Many relations in physics are, fortunately, of this form. Others only include the x^2 term, describing a parabolic curve. Note that double valued curves, sometimes encountered in physics, cannot be represented by Equation.

When sizable amounts of data are taken, standard mathematical methods are available which generate the equation of the simplest curve which statistically "best fits" the data.

The student may wonder how one can be certain that the curve fitted to the data is the "correct" curve. The answer is that relations are never known with certainty. The uncertainty of available data always limits the certainty of the results. Someday someone may obtain more accurate data and be able to show that the old relations are slightly incorrect, and provide us with better ones. As data improves, so does our understanding of relations—this is the way of scientific progress. But we never should claim to know a relation better than the data allows.

6.1.5 Uncertainty in a Slope

One use of a computational graph is to determine the slope of a straight line. This is illustrated in Figure. Eight data points are shown with error bars on each. If these bars represent maximum error, any line drawn to represent this data should pass within all bars.

If the error bars represent error estimates smaller than the maximum (average deviation, standard deviation, etc.), then the fitted curve need not pass within all of the error bars, just most of them.

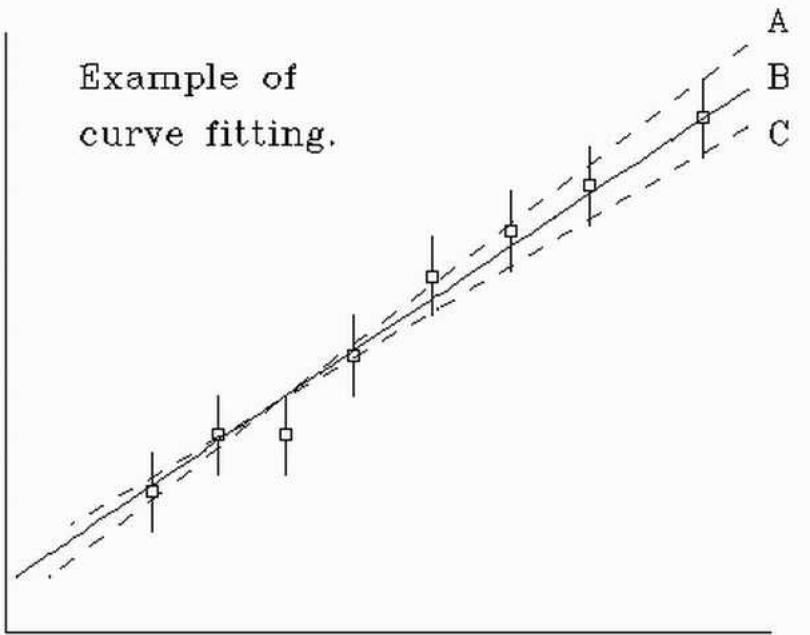


Figure : Fitting a curve.

Even a simple "manual" curve fit with a ruler can reveal the uncertainties in the slope resulting from uncertainties in the data. Figure. illustrates this process.

The dotted lines A and C fall within the error bars, and represent the maximum and minimum slope one could justify from this data. The "best" value of slope might be that of solid line B.

The third point from the left seems to limit the slope the most, and would appear to be "suspect." But one ought not to "throw it out" without better reason, based on further investigation.

6.1.6 Graphical Analysis of Data

Graphs can be a valuable tool for determining or verifying functional relations between variables. Many special types of graph paper are available for handling the most frequently encountered relations. You are probably already familiar with linear graph paper and polar coordinate paper.

You may have purchased a packet of graph paper for this course. It includes samples of graph papers you will use in this course, and a few other types. As you read the material below, examine the corresponding papers from your packet.

LINEAR RELATIONS are those which satisfy the equation

$$y = mx + b$$

where the variables are x and y , and m and b are constants. When y is plotted against x on ordinary Cartesian (linear) graph paper, the points fall on a straight line with slope m and a y -intercept b , as shown in Fig. 1.5.

The slope of an experimental relation is often physically significant. It is obtained by choosing two well-separated points on the line (x_1, y_1) and (x_2, y_2) . From Eq. 7-3:

$$y_1 = mx_1 + b$$

$$\text{and } y_2 = mx_2 + b$$

Subtract the first from the second.

$$(y_2 - y_1) = m(x_2 - x_1).$$

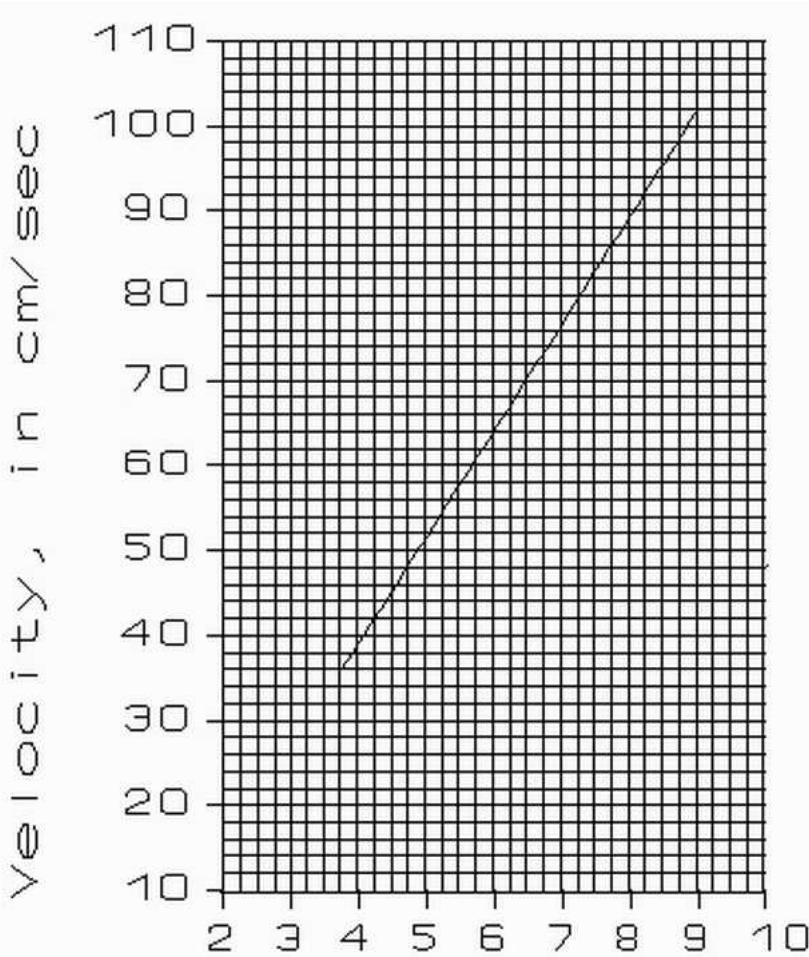


Figure : Measuring a slope on linear graph paper.

Therefore,

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$

The slope of a straight line is a ratio of the "lengths" of two legs of a right triangle constructed with the legs parallel to the graph axes and with the graph line along the hypotenuse. Figure. illustrates this, the slope being $\Delta y/\Delta x$.

So far this discussion has been strictly mathematical. Now let's consider a fairly realistic physical example. Figure. shows the curve from measurements of the velocity of a moving body as a function of time.

If we use letters v for velocity and t for time, we'd expect this curve to be described by the relation:

$$v = v_o + at$$

Here the constant a (acceleration) is the slope of the line, while v_o plays the role of b in Equation. These two constants are physically significant, and we wish to find their values from the graph.

We choose two points on the line at $t = 4.25$ and 8.75 , with corresponding values of velocity: 42 cm/s and 98 cm/s . Mark these points on Figure. , to confirm these values. The slope of the line is therefore:

$$a = \frac{(98 \text{ cm/s} - 42 \text{ cm/s})}{8.75 \text{ s} - 4.25 \text{ s}} = \frac{56}{4.5} \text{ cm/s}^2 = 56/4.5 \text{ cm/s}^2 = 12.44 \text{ cm/s}^2$$

When calculating this ratio do not use ruler-measured lengths. The lengths are expressed in the units marked on the graph axes. The calculated slope is therefore independent of the particular choice of units, of the way you choose to label the graph scale divisions, and is also independent of the size of the graph paper.

INTERCEPTS: The values of the intercepts are often physically significant. They can be simply read from the graph—if the $x = 0$ and $y = 0$ axes happen to be within the graph's boundaries. In the equation $y = mx + b$, the y intercept is b.

The v intercept of Figure. is the value of v when $t = 0$. It has the same units and dimensions as y. If, as in this case, the v intercept does not lie within the area of the graph, it may be calculated using the slope and one value taken from a point on the fitted line. Take the point $v = 98 \text{ cm/s}$ when $t = 8.75 \text{ sec}$.

$$v = v_o + at, \text{ in our case, } v = v_o + 12.44 t$$

so,

$$v = v_o - 12.44 t = 98 - 12.44(8.75) = -10.89 \text{ cm/s}$$

A check of the graph, Figure. , shows that this looks reasonable.

STRAIGHTENING A CURVE. When it is possible to convert an experimental relation to a straight line graph it is usually useful to do so. Look for such opportunities. For example, when studying gases at constant temperature we find that

$$PV = C$$

where P is pressure, V is the gas volume and C is constant. The graph of P vs. V is a branch of an hyperbola. But if we graph P vs. $1/V$, or V vs. $1/P$, the data would fall on a straight line.

$$P = \left(\frac{1}{V}\right)C$$

One reason for doing this is that it is easier to fit the experimental data with a ruler-drawn straight line, than to draw the best hyperbola on a PV graph. Another advantage is that the P vs. $1/V$ graph has a slope

$$C = \frac{\Delta P}{\Delta \left(\frac{1}{V}\right)}$$

Therefore the constant C is easily determined from the straight line. This constant was not evident, nor was it easy to determine from the PV graph!

Inexpensive electronic calculators make it so easy to manipulate data that there is no good excuse to pass up an opportunity to "linearize" experimental graphs.

6.1.7 EXERCISES.

In each case state how you could plot (x,y) data on linear paper to obtain a straight line graph. What quantity in the equation is determinable from the slope of the straight line? What quantity is determinable from an intercept?

$$(1.1) x(y + 1) = 3$$

$$(1.2) 1/x + 1/y = 5$$

$$(1.3) y = Ae^{-x}$$

$$(1.4) y = \sqrt{A - x}$$

$$(1.5) y^2 + x^2 = 7$$

6.2 Common Graph Forms in Physics

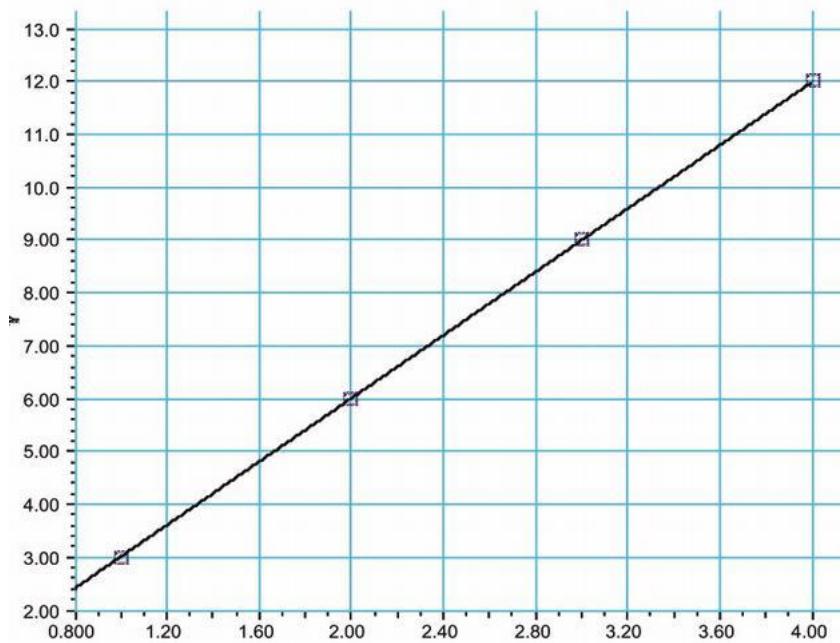
Working with graphs – interpreting, creating, and employing – is an essential skill in the sciences, and especially in physics where relationships need to be derived. As an introductory physics student you should be familiar with the typical forms of graphs that appear in physics. Below are a number of typical physical relationships exhibited graphically using standard X-Y coordinates (e.g., no logarithmic, power, trigonometric, or inverse plots, etc.). Study the forms of the graphs carefully, and be prepared to use the program Graphical Analysis to formulate relationships between variables by using appropriate curve-fitting strategies. Note that all non-linear forms of graphs can be made to appear linear by "linearizing" the data. Linearization consists of such things as plotting X versus Y^2 or X versus $1/Y$ or Y versus $\log(X)$, etc. Note: While a 5th order polynomial might give you a better fit to the data, it might not represent the simplest model.

6.2.1 Linear Relationship

What happens if you get a graph of data that looks like this? How does one relate the X variable to the Y variable? It's simple, $Y = A + BX$ where B is the slope of the line and A is the Y-intercept. This is characteristic of Newton's second law of motion and of Charles' law:

$$F = ma$$

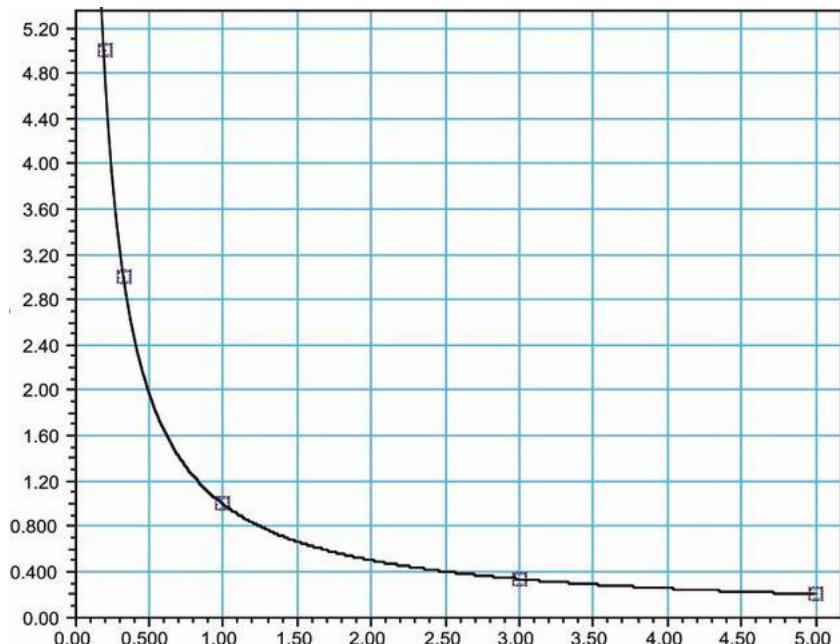
$$P/T = \text{const.}$$



6.2.2 Inverse Relationship

This might be a graph of the pressure and temperature for a changing volume constant temperature gas. How would you find this relationship short of using a computer package? The answer is to simplify the plot by manipulating the data. Plot the Y variable versus the inverse of the X variable. The graph becomes a straight line. The resulting formula will be $Y = A/X$ or $XY = A$. This is typical of Boyle's law:

$$PV = \text{const}$$

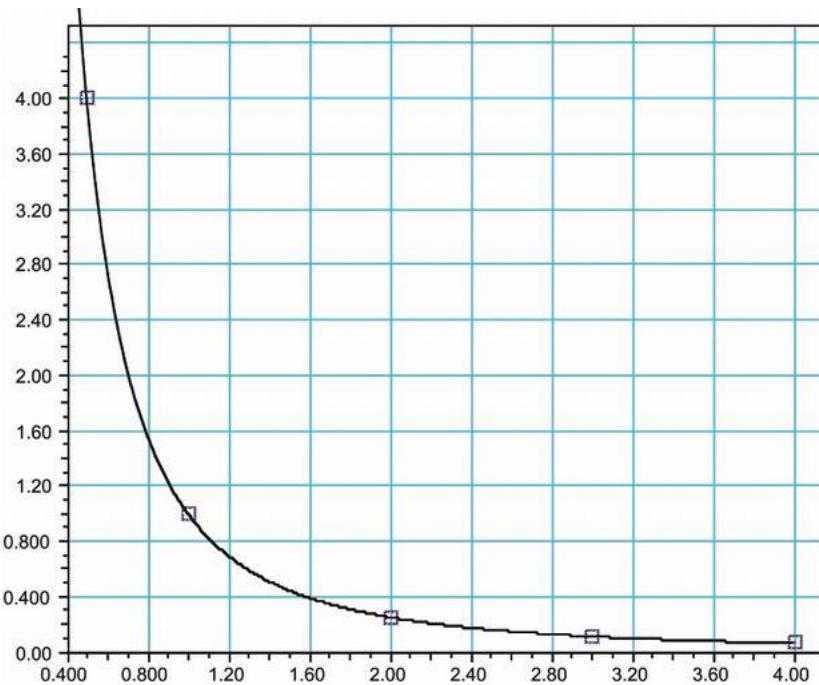


6.2.3 Inverse-Square Relationship

Of the form $Y = A/X^2$. Characteristic of Newton's law of universal gravitation, and the electrostatic force law:

$$F = \frac{Gm_1m_2}{r^2}$$

$$F = \frac{kq_1q_2}{r^2}$$



In the latter two examples above there are only subtle differences in form. Many common graph forms in physics appear quite similar. Only by looking at the "RMSE" (root mean square error provided in Graphical Analysis) can one conclude whether one fit is better than another. The better fit is the one with the smaller RMSE. See below for more examples of common graph forms in physics.

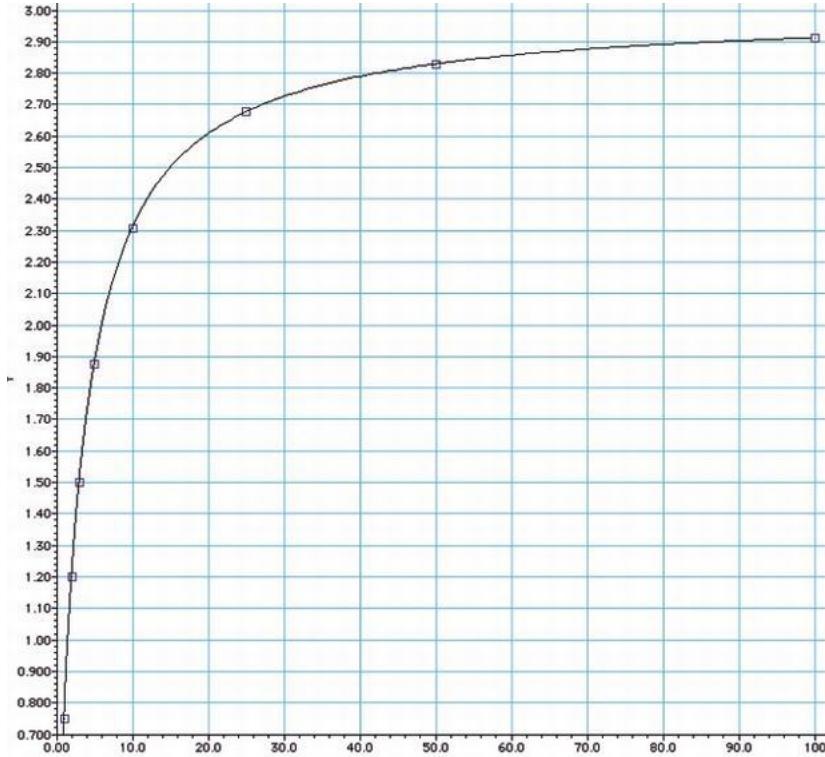
6.2.4 Double-Inverse Relationship

Of the form $1/Y = 1/X + 1/A$. Most readily identified by the presence of an asymptotic boundary ($y = A$) within the graph. This form is characteristic of the thin lens and parallel resistance formulas.

$$\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}$$

and

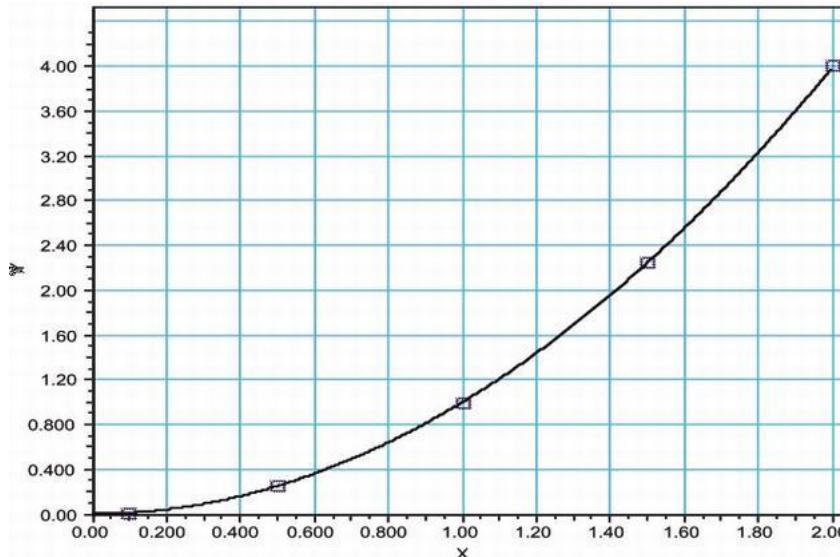
$$\frac{1}{R_t} = \frac{1}{R_1} + \frac{1}{R_2}$$



6.2.5 Power Relationship

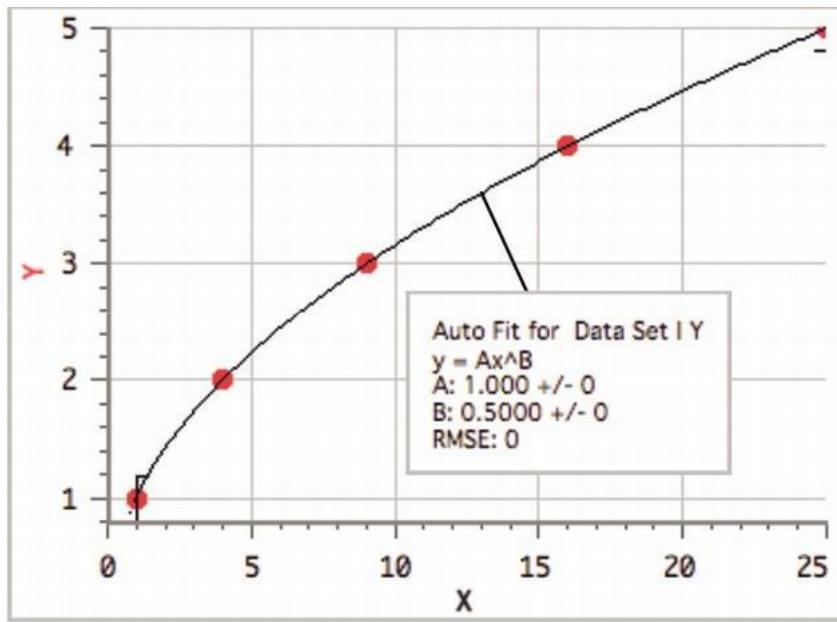
Top opening parabola. Of the form $Y = AX^2$. Typical of the distance-time relationship:

$$d = \frac{1}{2}at^2$$



6.2.6 Power Relationship 2

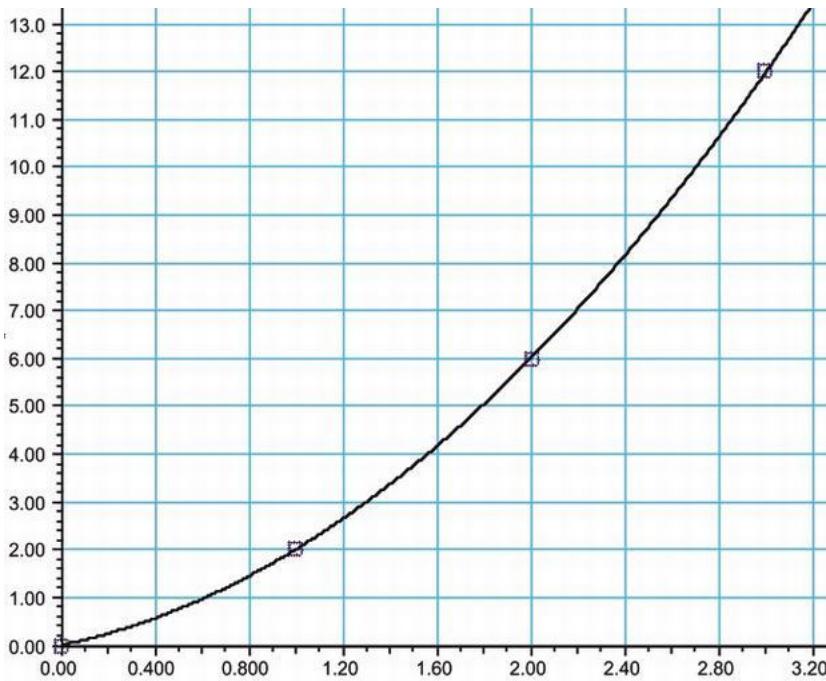
Side opening parabola. Of the form $Y^2 = A_1X$ or $Y = A_2X^{1/2}$. Typical of the simple pendulum relationship: $P^2 = k_1l$ or $P = k_2\sqrt{l}$



6.2.7 Polynomial of Second Degree

Of the form $Y = AX + BX^2$. Typical of the kinematics equation:

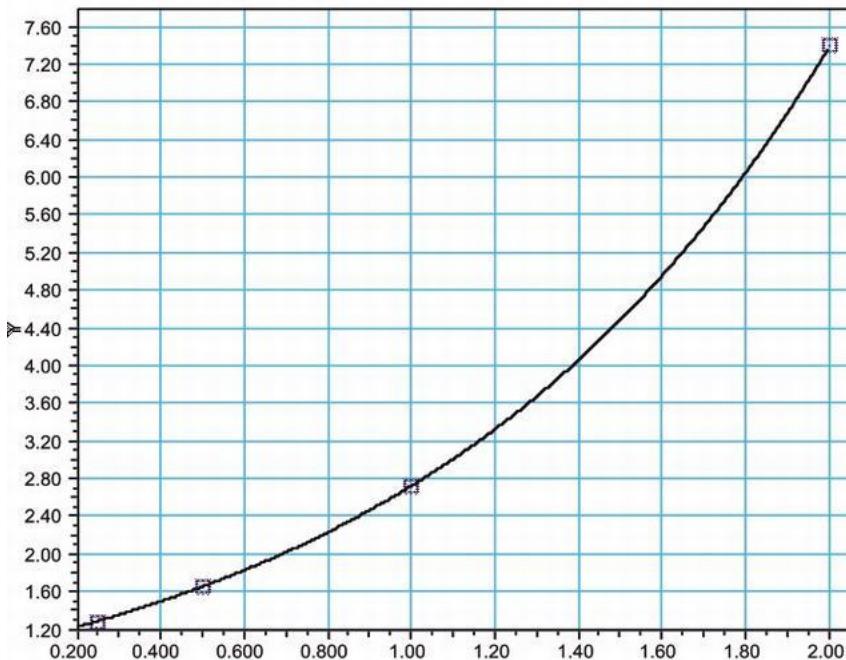
$$d = v_o t + \frac{1}{2}at^2$$



6.2.8 Exponential Relationship

Of the form $Y = A * \exp(BX)$. Characteristic of exponential growth or decay. Graph to left is exponential growth. The graph of exponential decay would look not unlike that of the inverse relationship. Characteristic of radioactive decay.

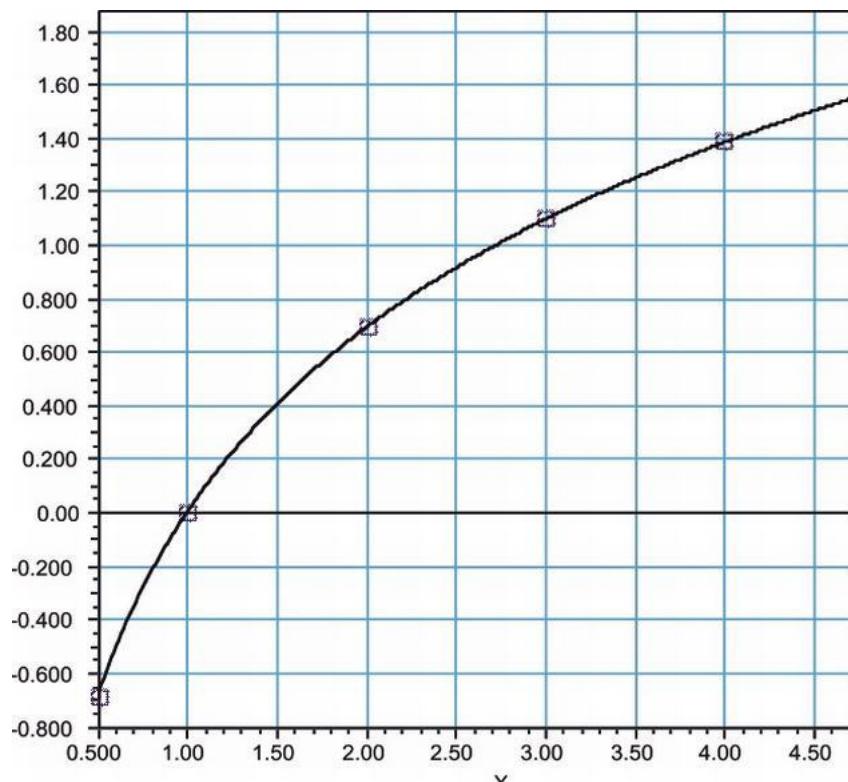
$$N = Noe^{-\lambda t}$$



6.2.9 Natural Log (LN) Relationship

Of the form $Y = A \ln(BX)$. Characteristic of entropy change during a free expansion:

$$S_f - S_i = nR \ln(V_f/V_i)$$



Bibliography

(2016). URL: <https://www.lhup.edu/~dsimanek/scenario/errorman/graphs.htm>.

Chapter 7

Mechanics

7.1 Kinematics

7.1.1 The Equations of motion and the origin of Graph Handling

7.1.1.1 The First Equation

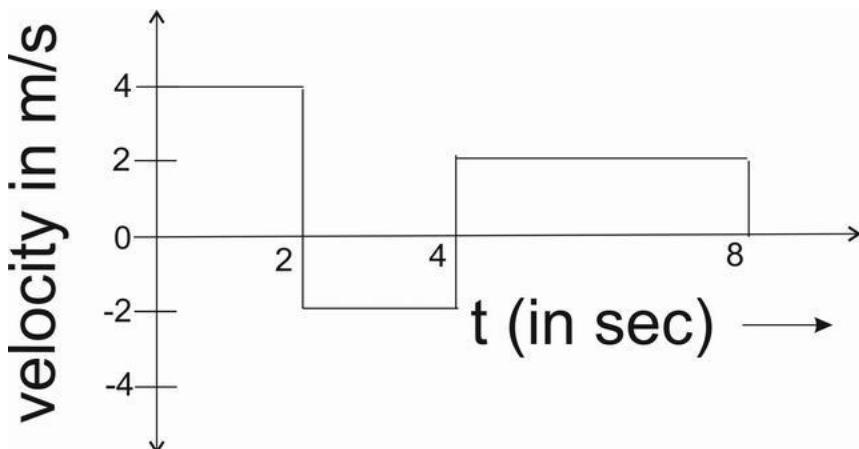
The Equation $v = \frac{dx}{dt}$ in linear motion implies

- i) The **Slope of Position-Time Graph** is **Instantaneous Velocity**.
- ii) The **Area under the Velocity-Time Graph** is **Change in Position**.
- { The second one requires the manipulation , $dx = vdt$ i.e. $\int dx = \int vdt$ }

The equations can be further manipulated to obtain the Speed Time Graph , where

speed = rate of change of distance wrt time

Example : A body is moving in a straight line as shown in velocity-time graph. The displacement and distance travelled by body in 8 second are respectively:

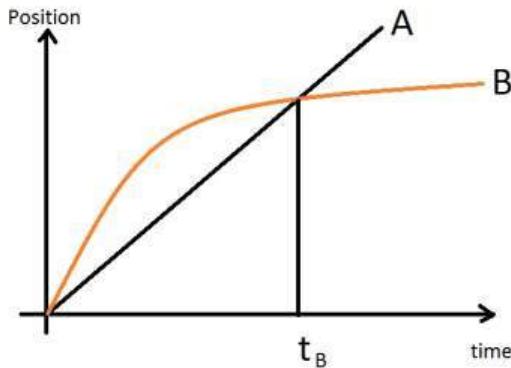


- a) 12 m, 20 m
- b) 20 m, 12 m
- c) 12 m, 12 m
- d) 20 m, 20 m

{ Hint: The displacement in a velocity-time graph is given by the area under the graph with proper signs. From 0s - to 2s , the area is 8m . From 2s - to 4s , the area is -4m . From 4s - to 8s , the area is 8m. Adding these 3 values , we get $8m + (-4m) + 8m = 12m$. } The distance in a v-t graph is given by the absolute area under the graph. So, taking the absolute values of individual area divisions, we get $8m + 4m + 8m = 20m$

Answer: a) is the correct answer. }

Example : The graph shows position as a function of time for two trains running on parallel tracks. Which statement is true?

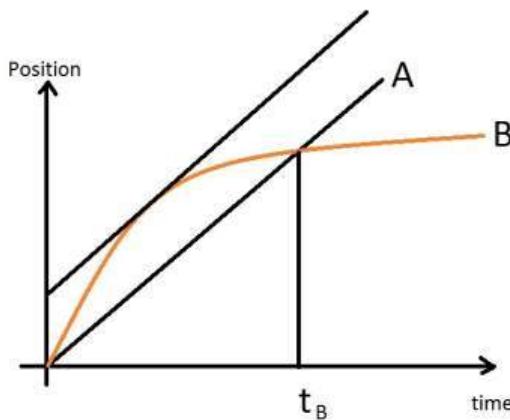


- a) At time t_B both trains have the same velocity.
- b) Both trains have the same velocity at some time after t_B .
- c) Both trains have the same velocity at some time before t_B .
- d) Somewhere on the graph, both trains have the same acceleration.

{ Hint: Depending on the question requirements, we'll have to check all the assertions one by one.

a) In a position time graph, the slope gives velocity. It can be clearly seen that Graph B has a much lower slope than Graph A at time t_B . So, the assertion is wrong.

b,c) By drawing a line parallel to the line A which is a tangent to Graph B, it can be seen where the two graphs have same slope. It is clear that the graphs have same slope between 0 and t_B as noted from the figure. So, assertion b is wrong while c is correct.

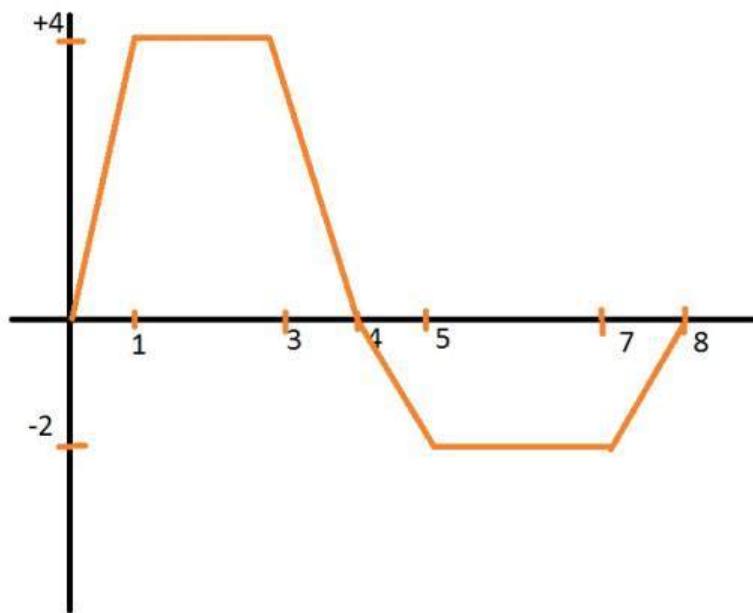


- d) As the Graph A has a constant slope, so the acceleration of body A is zero. Whereas Graph B is constantly turning, so the slope can be assumed to be non-zero throughout. According to some revelations, however it is noted that the figure is not clear enough to show whether Graph B is straight after t_B or bending. In case it is assumed to be straight, then after t_B both trains will have same (zero) acceleration. Also at start both have large (infinite) acceleration, in which case the ratio of the two large (infinite) values may be calculated if initial conditions are mentioned and is required.

At our level we would assume this assertion to be wrong, however making a note that the image should have been more clearly presented.

Answer: c) is the correct assertion. }

Example : The velocity-time graph of a particle in linear motion is as shown. Both v and t are in SI units. The displacement of the particle is



- a) 6 m
- b) 8 m
- c) 16 m
- d) 18 m

{ Hint : For displacement calculations, between 0 - to 4 , area of the positive trapezium = $(4+2) \times 4 = 24$

between 4 - to 8, area of negative trapezium = $(2+4) \times (-2) = -12$.

So , the answer is +12 , which is not in the options.

So , the answer is None of these. }

7.1.1.2 The Second Equation

Proceeding similar to above, the equation $a = \frac{dv}{dt}$ implies

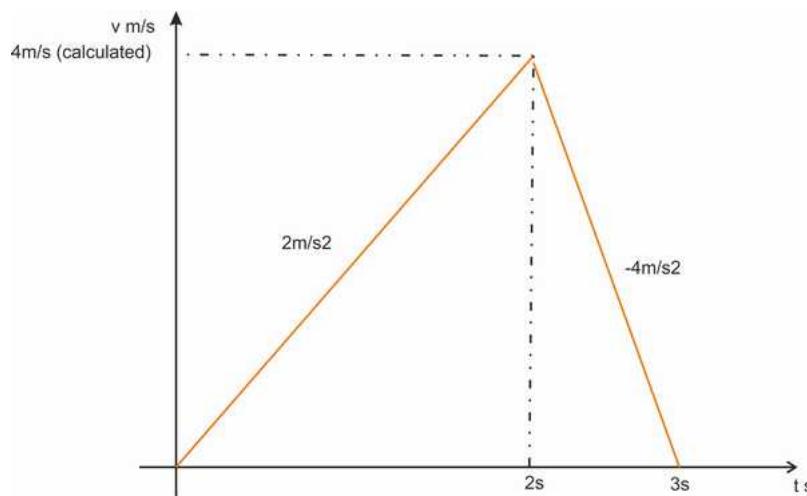
- i) The Slope of Velocity-Time Graph is Instantaneous Acceleration.
 - ii) The Area under Acceleration-Time Graph is Change in Velocity.
- { The second one requires the manipulation , $dv = adt$ i.e. $\int dv = \int adt$ }

A few of the following examples illustrate it.

Example : A car starts from rest acquires a velocity v with uniform acceleration $2ms^{-2}$ then it comes to stop with uniform retardation $4ms^{-2}$. If the total time for which it remains in motion is 3 sec, the total distance travelled is:

- a) 2 m
- b) 3 m
- c) 4 m
- d) 6 m

{Hint: For solving this problem, we draw the graph of the problem,



According to graph, let the time when it reaches maximum velocity be T , and the maximum velocity be V .

$$\Rightarrow V = 2XT \text{ and also } V = 4X(3-T)$$

Equating the equations,

$$2T = 12 - 4T = V$$

$$\Rightarrow 6T = 12$$

$$\Rightarrow T = 2$$

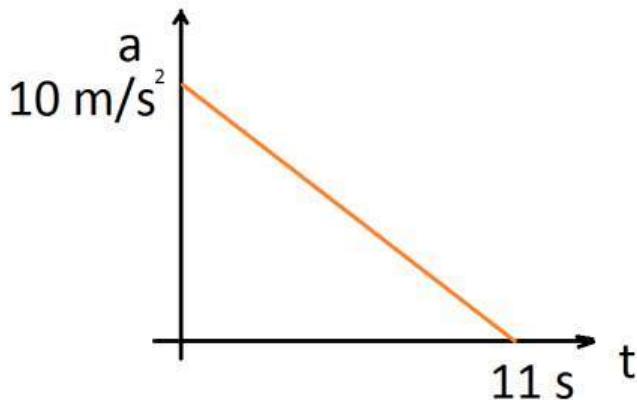
$$\Rightarrow V = 2T = 4$$

Calculating the area under the graph using the calculated parameters, Area = $1/2 \times 4 \times 3 = 6\text{m}$

So, area under the graph is 6m = displacement. Also, as all the area is on the positive side, so distance = 6m .

}

Example : A particle starts from rest. Its acceleration (a) vs time (t) is as shown in the Figure. The maximum speed of the particle will be



- a) 110 m/s
- b) 55 m/s
- c) 550 m/s
- d) 660 m/s

{ Hint : Writing the equation of the graph , we get $\frac{a}{10} + \frac{t}{11} = 1$

$$\Rightarrow a = \frac{10}{11}(11 - t)$$

Integrating, (we will assume initial velocity to be zero as the body starts from rest.)

$$v = \frac{10}{11}(11t - \frac{1}{2}t^2)$$

Substituting $t = 11s$

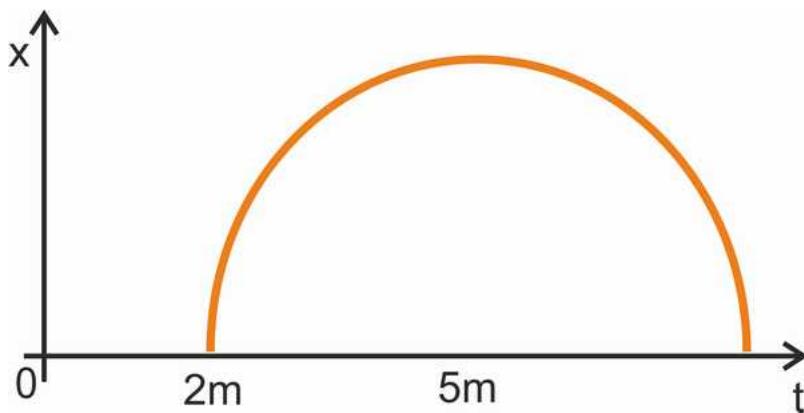
$$v_{11s} = 55m/s$$

Answer: b) is the correct answer }

7.1.1.3 The Average-Velocity / Instantaneous Velocity , Equal Case

We know , that (in a x-t graph) the slope of the Secant is the Average Velocity , whereas the slope of Tangent is the Instantaneous Velocity. The point where these two lines coincide, is the point where Average Velocity is equal to Instantaneous Velocity.

Example : Position-time graph is shown which is a semicircle from $t = 2$ to $t = 8$ s. Find time t at which the instantaneous velocity is equal to average velocity over first t seconds,



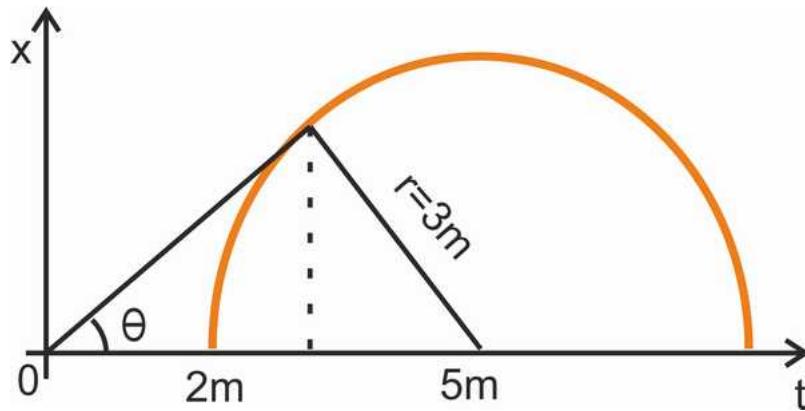
- a) 4.8 s
- b) 3.2 s
- c) 2.4 s
- d) 5 s

{ Hint: The tangent from 0 to the circle is drawn. It's normal passes through the center of the circle. Time at this instant needs to be calculated.

If $H=5$, $R = 3$, Length of tangent = 4. (By Pythagoras.)

Angle which the tangent makes with the t axis is $\theta = \sin^{-1}(3/5)$

So, the projection of tangent on t axis (i.e. the required time) = $4 \cos \theta = 4 \times \frac{4}{5} = 3.2$

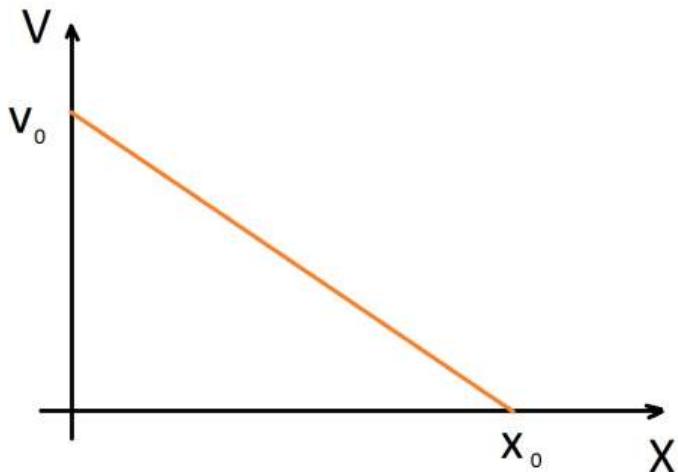


}

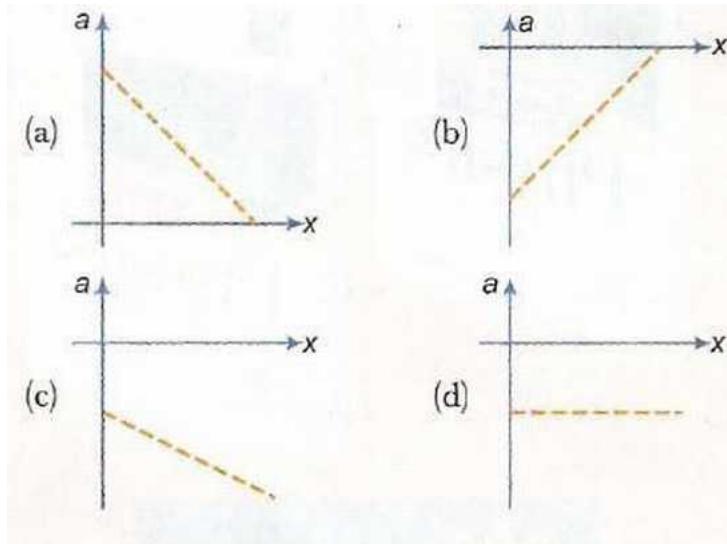
7.1.1.4 The Velocity-Displacement Case

This can be handled in a similar way as Acceleration-Displacement case by integrating the respective equation. Here the problem is of $v = f(x)$ type, which can be integrated by writing $\frac{dx}{dt} = f(x)$
i.e. $dx = f(x)dt$

Example : The velocity-displacement graph of a particle moving along a straight line is shown here.



The most suitable acceleration-displacement graph will be



{Hint: Using Co-Ordinate Geometry Result studied in +1 Mathematics, we get the equation of the graph

$$\frac{v}{v_o} + \frac{x}{x_o} = 1$$

We are supposed to find the a - x graph from this.

So, we rewrite this equation as $v = v_o(1 - \frac{x}{x_o})$

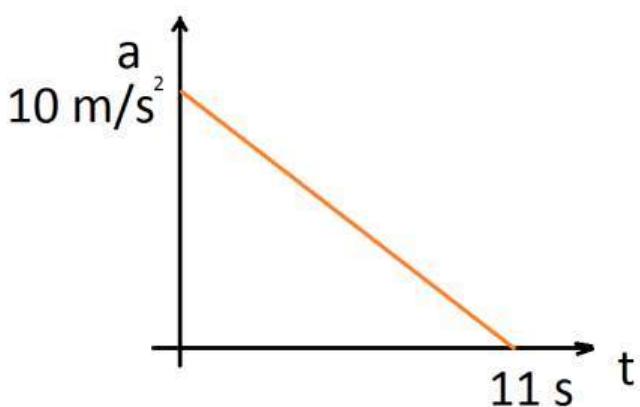
$$\text{Differentiating, we get } a = -\frac{v_o}{x_o}v = -\frac{v_o^2}{x_o}(1 - \frac{x}{x_o})$$

Hence b) is the requisite graph, the only graph with a +ve slope, a negative y intercept and a positive x intercept.

Answer: b) is the correct answer. }

7.1.2 Previous Years IIT Problems

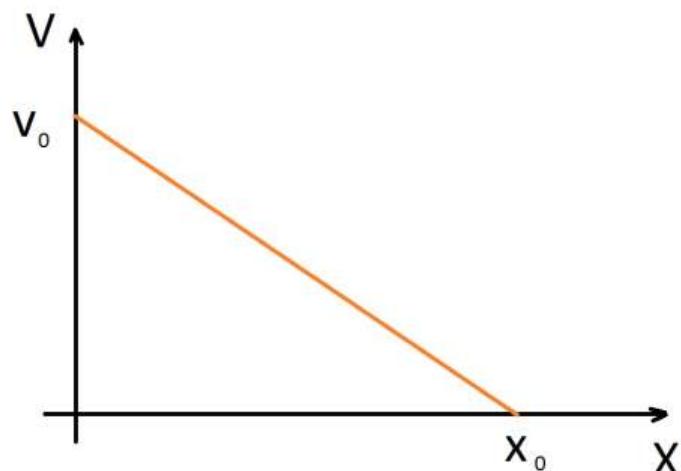
Q1: A particle starts from rest. Its acceleration (a) versus time (t) is as shown in the figure. The maximum speed of the particle will be

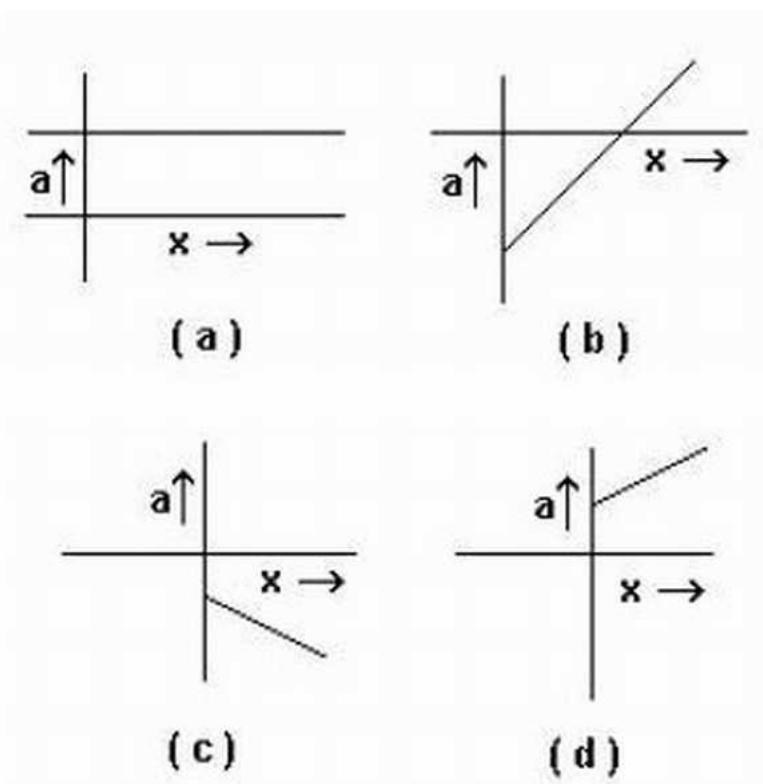


- a) 110 m /s
- b) 55 m /s
- c) 550 m /s
- d) 660 m /s

{ Hint : See In chapter examples for solution. }

Q2: If graph of velocity vs. distance is as shown, which of the following graphs correctly represents the variation of acceleration with displacement ?





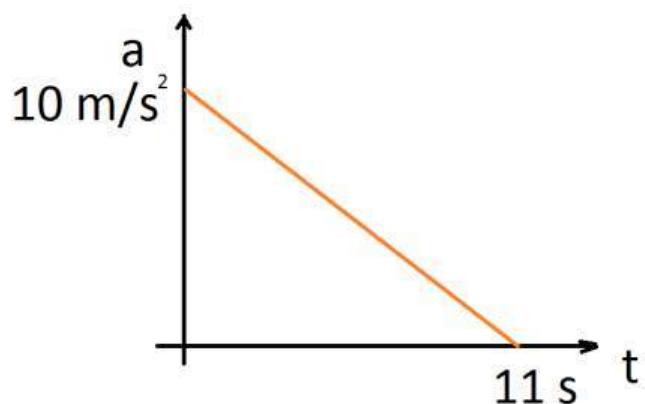
{ Hint : The graph of the question is a straight line with the equation, $\frac{x}{x_o} + \frac{v}{v_o} = 1$

This gives , $v = v_o(1 - x/x_o)$

So, differentiating it, we get

$a = -v_o/x_o$ which is a constant. Only in a) it is shown to be a constant.

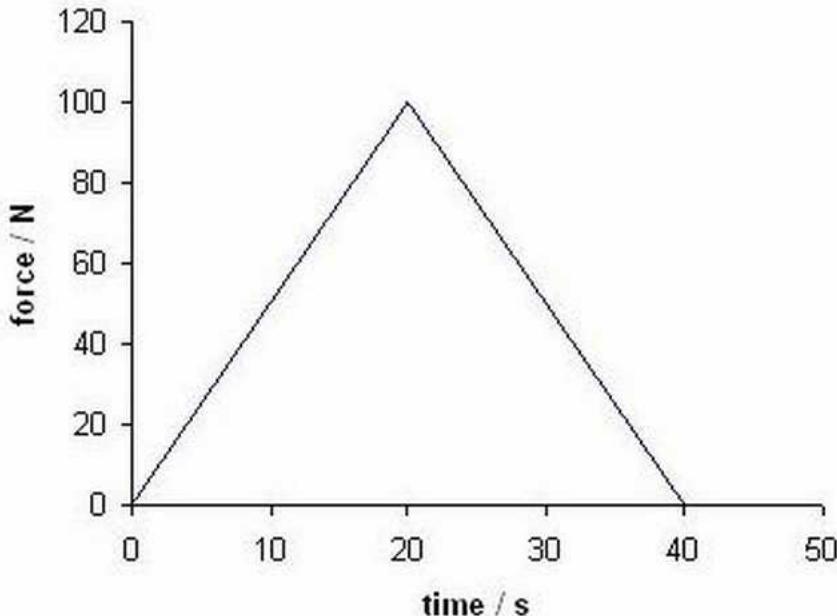
So, a) }



7.2 Laws of Motion

7.2.1 Abstract Introduction

7.2.1.1 Force – Time Graphs



The area under a force – time graph gives us the impulse of the force applied (and hence the change in momentum of the object). For this graph the impulse (the area under the graph) is 2000 kg ms^{-1} . n.d.

7.2.1.2 Change in Momentum or the "Impulse"

The change in the momentum of a system (or the impulse delivered by the net force) is given mathematically by the Momentum Principle,

$$\Delta \vec{p} = \overrightarrow{F_{net}} \Delta t \text{ n.d.}$$

In this form, the change in momentum is calculated over a “discrete” time step. That is, the calculation is done over a known or determined time interval. If the force is non-constant (i.e., depends on location or velocity), this calculation is not exact. In fact, in this case, the net force is the average net force over the time interval. So that a better definition is this:

$$\Delta \vec{p} = \overrightarrow{F_{net,avg}} \Delta t$$

This definition works well for cases where you might use iterative procedures to determine the change in momentum over small time intervals. If on the other hand, you can analytically integrate the force (e.g., it is or can be put into a form which is time dependent), then you can use the derivative form of the Momentum Principle,

$$\Delta \vec{p} = \int_{t_i}^{t_f} \overrightarrow{F_{net}} dt$$

In any event, either (or both) can be useful to think about graphs of force vs time.

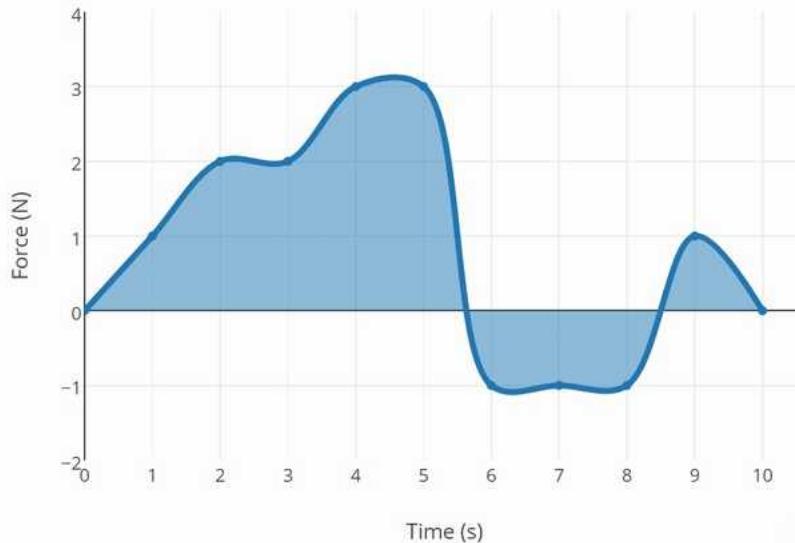
Force vs Time Graphs In some situations, it is easier to empirically measure force versus time graphs because the situations lend themselves more easily to these empirical measurements rather than what might be more complex physical theories. This is true in different engineering contexts (e.g., impact design and the flow of fluids). In these cases, you are interested in determining the change in momentum (and thus the velocity) of the system in question1).

Below is a force vs time graph where the “area under the curve” has been highlighted. In this example, we are only looking at the component of the net force in the xx-direction. Such graphs can be produced for each component of the net force, but let’s say that for this system, there was a non-zero component of the net force only in the x-direction.



For the above figure, the momentum change over the complete time interval can be determined in a straightforward way due to the simple geometric shapes produced. Area above the zero line are positive momentum changes, and area below are negative. By adding up the “area under the curve” in this way, we obtain a momentum change of 7 Ns.

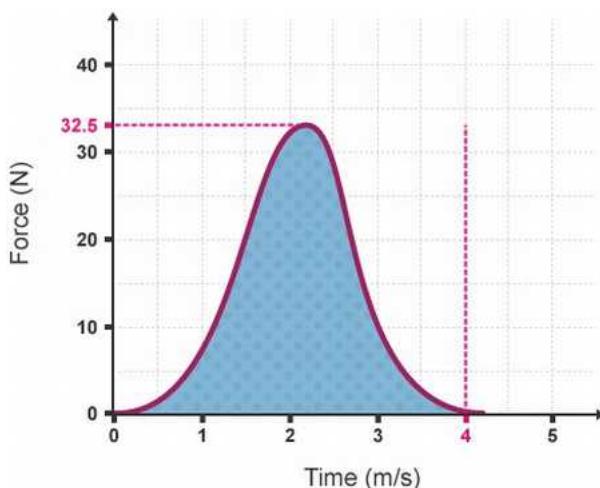
The figure below shows the force vs time graph for another system. In this case, the graph has a smooth form, which doesn’t appear to be analytic. The “area under the curve” for this graph could be analyzed computationally, by taking small steps (i.e., Riemann Sum), and the change in momentum could be determined.



- 1) It is possible to determine the displacement of such systems as well. This can be done using velocity vs time graphs that are produced from the analysis of force vs time graphs.

7.2.1.3 Impulse graphs (A Case Study) n.d.

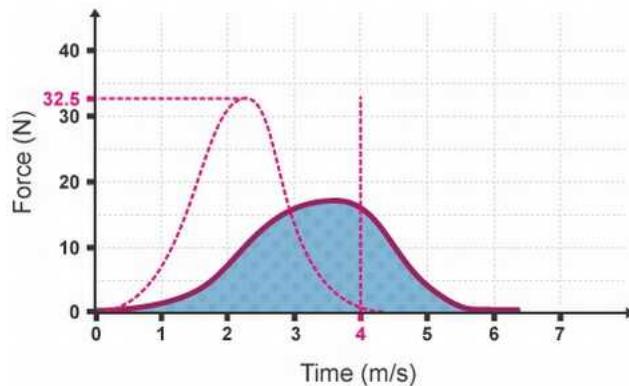
The force on the squash ball in the previous question is an average force and often the force changes during the collision. For this example the force-time graph could look like this.



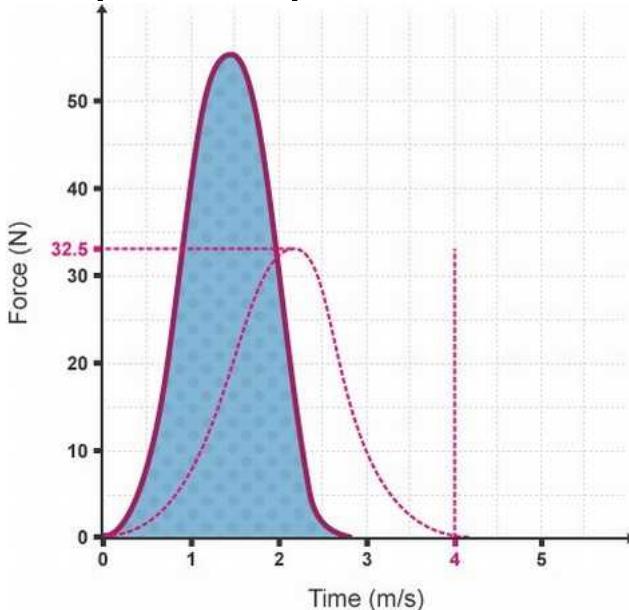
Notice the peak force is greater than the average force calculated.

The area under a force time graph is equal to the impulse. For any collision with a fixed change in momentum, if the time of contact can be increased, the peak force is reduced:

For example if the squash ball was replaced with a softer version of same mass the collision graph would look like this:



If the squash ball was replaced with a harder version of same mass the collision graph would look like this:



Question Modern cars are designed to crumple on impact in a collision. How does this help to protect the occupants from harm?

Answer The change in momentum (area under the force time graph) can't be changed at the time of the accident (mass is fixed and it is too late for the driver to slow down!) By increasing the time of collision the peak force is less and hopefully lets the occupants come to less harm as a result.

Question : How do I find "Velocity" from "Force vs. Time" graph?

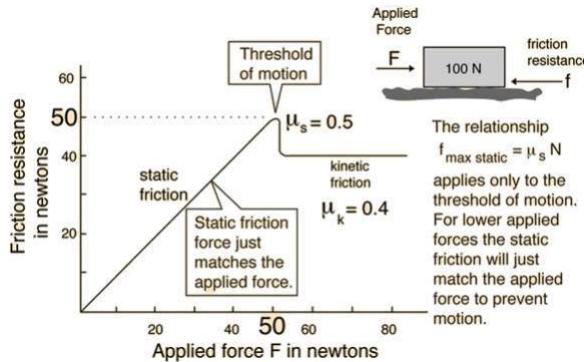
Solution : You need two additional pieces of information: the mass of the object and its initial velocity. Given those, the relation is:

$$v(t) = v_o + \frac{1}{m} \int_0^t F(t) dt$$

7.2.2 Friction

7.2.2.1 Static Friction 2017

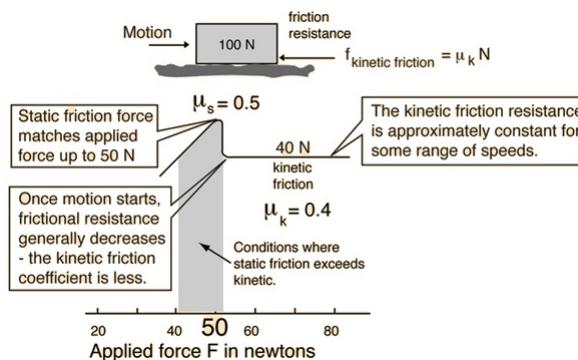
Static frictional forces from the interlocking of the irregularities of two surfaces will increase to prevent any relative motion up until some limit where motion occurs. It is that threshold of motion which is characterized by the coefficient of static friction. The coefficient of static friction is typically larger than the coefficient of kinetic friction.



In making a distinction between static and kinetic coefficients of friction, we are dealing with an aspect of "real world" common experience with a phenomenon which cannot be simply characterized. The difference between static and kinetic coefficients obtained in simple experiments like wooden blocks sliding on wooden inclines roughly follows the model depicted in the friction plot from which the illustration above is taken. This difference may arise from irregularities, surface contaminants, etc. which defy precise description. When such experiments are carried out with smooth metal blocks which are carefully cleaned, the difference between static and kinetic coefficients tends to disappear. When coefficients of friction are quoted for specific surface combinations are quoted, it is the kinetic coefficient which is generally quoted since it is the more reliable number.

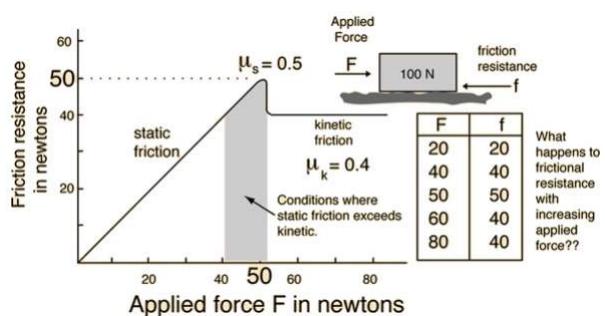
7.2.2.2 Kinetic Friction

When two surfaces are moving with respect to one another, the frictional resistance is almost constant over a wide range of low speeds, and in the standard model of friction the frictional force is described by the relationship below. The coefficient is typically less than the coefficient of static friction, reflecting the common experience that it is easier to keep something in motion across a horizontal surface than to start it in motion from rest.

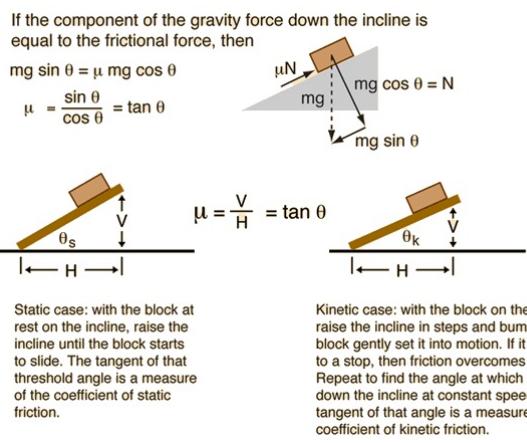


7.2.2.3 Friction Plot

Static friction resistance will match the applied force up until the threshold of motion. Then the kinetic frictional resistance stays about constant. This plot illustrates the standard model of friction.



The above plot, though representing a simplistic view of friction, agrees fairly well with the results of simple experiments with wooden blocks on wooden inclines. The experimental procedure described below equates the vector component of the weight down the incline to the coefficient of friction times the normal force produced by the weight on the incline.



Having taken a large number of students through this experiment, I can report that the coefficient of static friction obtained is almost always greater than the coefficient of kinetic friction. Typical results for the woods I have used are 0.4 for the static coefficient and 0.3 for the kinetic coefficient.

When carefully standardized surfaces are used to measure the friction coefficients, the difference between static and kinetic coefficients tends to disappear, indicating that the difference may have to do with irregular surfaces, impurities, or other factors which can be frustratingly non-reproducible. To quote a view counter to the above model of friction: "Many people believe that the friction to be overcome to get something started (static friction) exceeds the force required to keep it sliding (sliding friction), but with dry metals it is very hard to show any difference. The opinion probably arises from experiences where small bits of oil or lubricant are present, or where blocks, for example, are supported by springs or other flexible supports so that they appear to bind." R. P. Feynman, R. P. Leighton, and M. Sands, *The Feynman Lectures on Physics*, Vol. I, p. 12-5, Addison-Wesley, 1964.

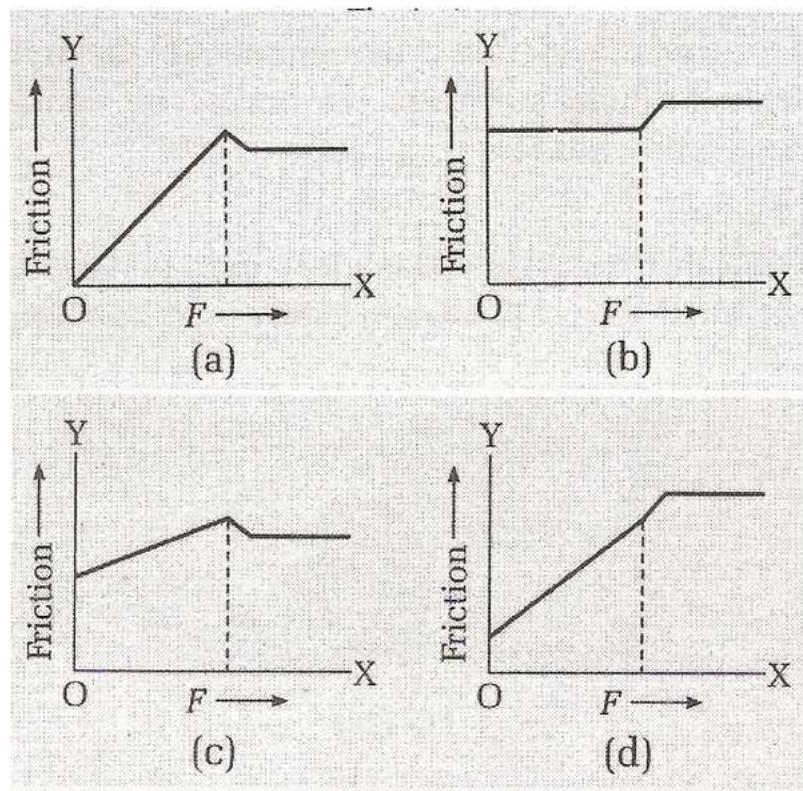
7.2.2.4 Rolling Friction

A rolling wheel requires a certain amount of friction so that the point of contact of the wheel with the surface will not slip. The amount of traction which can be obtained for an auto tire is determined by the coefficient of static friction between the tire and the road. If the wheel is locked and sliding, the force of friction is determined by the coefficient of kinetic friction and is usually significantly less.

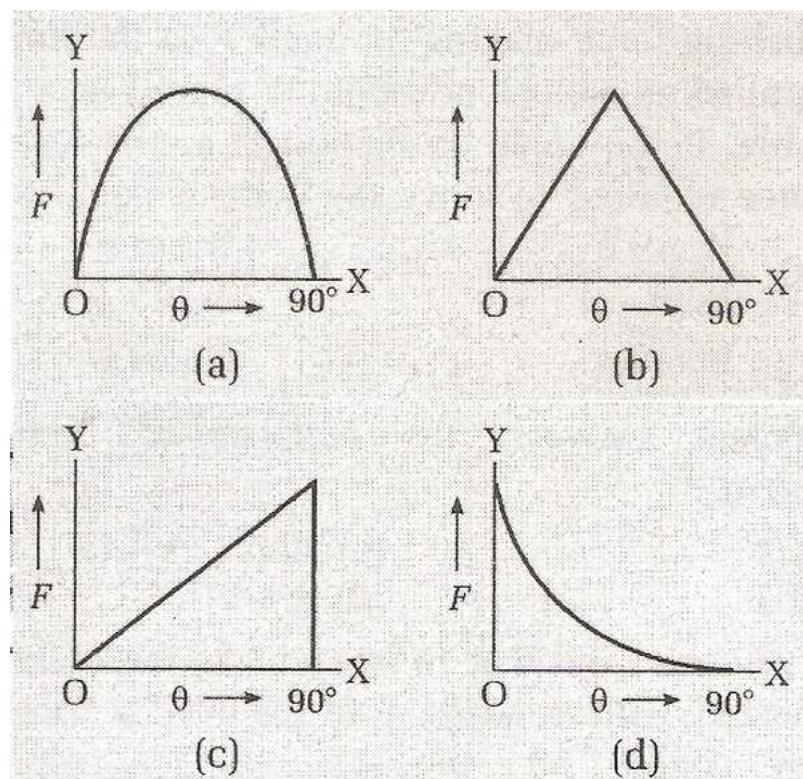
Assuming that a wheel is rolling without slipping, the surface friction does no work against the motion of the wheel and no energy is lost at that point. However, there is some loss of energy and some deceleration from friction for any real wheel, and this is sometimes referred to as rolling friction. It is partly friction at the axle and can be partly due to flexing of the wheel which will dissipate some energy. Figures of 0.02 to 0.06 have been reported as effective coefficients of rolling friction for automobile tires, compared to about 0.8 for the maximum static friction coefficient between the tire and the road.

7.2.2.5 Few problems related to Friction

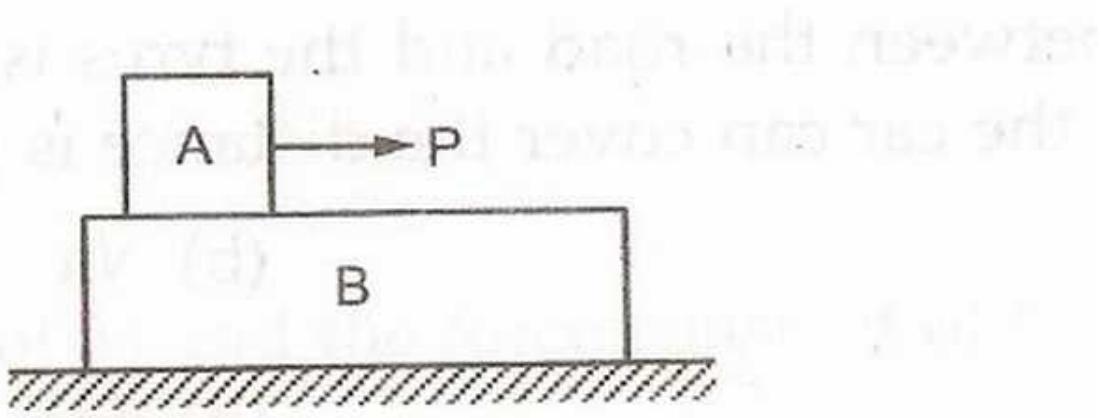
Example : A block on the horizontal table is acted upon by a force F . The graph of frictional force against F is



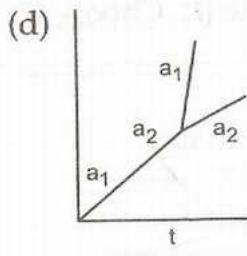
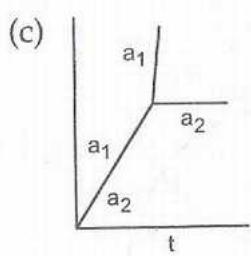
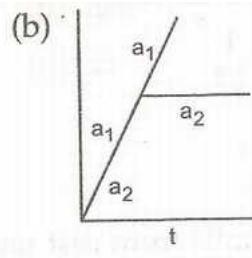
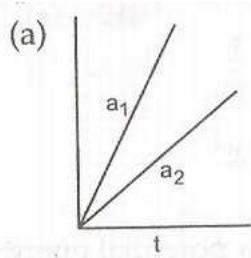
Example: A block rests on a rough plane whose inclination θ to the horizontal can be varied. Which of the following graphs indicates how the frictional force F between the block and plane varies as θ is increased?



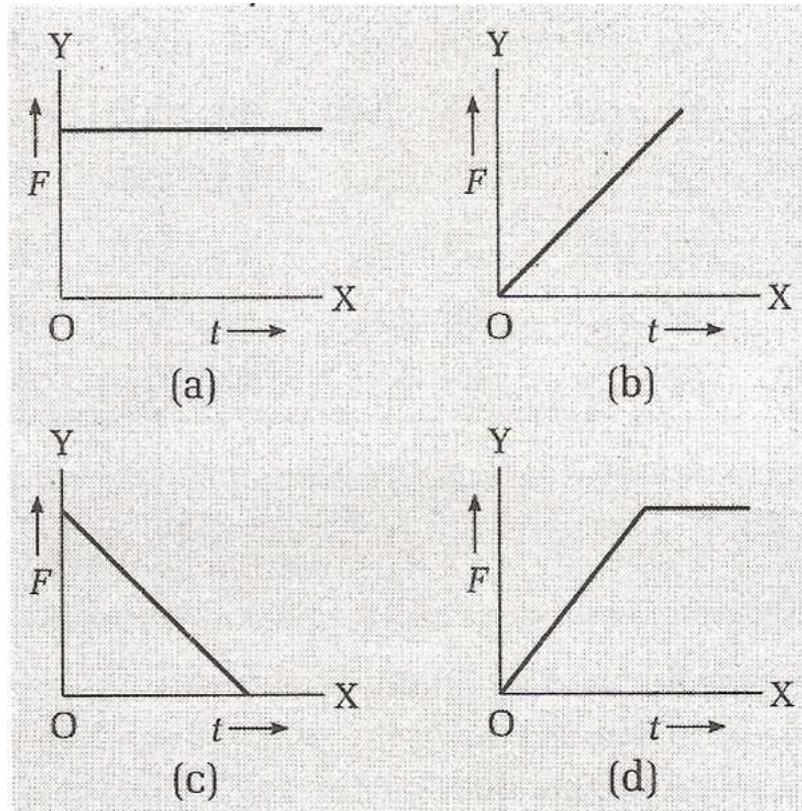
Example : Block A is placed on block B, whose mass is greater than that of A.



There is friction between the blocks, while the ground is smooth. A horizontal force P , increasing linearly with time, begins to act on A. The accelerations a_1 and a_2 of A and B respectively are plotted against time (t). Choose the correct graph.



Example : A body moves with uniform speed on a rough surface. If force F of dynamic friction is plotted with time t as shown in figure, the graph will be



7.2.3 Theory and Problems

7.2.3.1 Impulse as Force-time Graph

Example : For the graph shown above, assume that it shows a constant force of 25 N acting over a 10 s period of time. Determine the impulse.

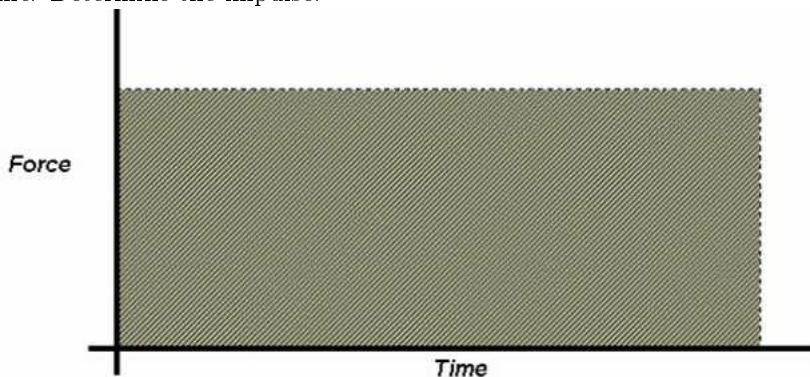


Illustration : Graph of Example (Force as a function of Time)

Solution : (Theory : So far we've implied some things about what is constant and what can change in the impulse formula $F \Delta t = m \Delta v$.

We look at situations where we expect the mass of the object will stay constant. • The velocity will change, and that's why we put a delta in front of it. • Time is changing (sort of) as we measure it over a period of time. • Force must be a constant. We assume that the force being exerted on the object was always the same, causing a constant acceleration. If we are looking at a simple impulse question (where the force is constant), we can figure out exactly what we can interpret from a graph. • Later this may help us to figure out a more complicated question, like if the force changes. The following graph is an example of one of those simple situations where the force remains constant during the entire time. • If we look at what the slope might represent, we get...

$$\text{slope} = \text{rise} / \text{run}$$

$$\text{slope} = F / \Delta t$$

Since nothing in the impulse formula can be rearranged to give us force over time, the slope doesn't mean anything to us in this situation.

If we look at the area under the line, we get something a bit better...

$$\text{Area} = lw = F \Delta t = \Delta p$$

Since area under the line is equal to impulse...

$$\text{Area} = lw$$

$$\text{Area} = 25 \times 10$$

$$\text{Area} = 2.5e2$$

$$p = 2.5e2 \text{Ns}$$

If we really wanted to, we could have simply used $\Delta p = F\Delta t$ to figure out the impulse. We could do this in this situation because the force is constant. • If we need to do a question where the force is not constant, we can still use the area under the line to get the impulse, even though the formula $\Delta p = F\Delta t$ can not be used.

Example : I am in a car that is accelerating from rest at a red light. I want to calculate the impulse that is acting on the car during the first 5.78s. If I know that the force on the car steadily increases from 0 N to 3012 N over this time, determine the impulse. If the mass of the car is 1500 kg, also determine the final velocity of the car.

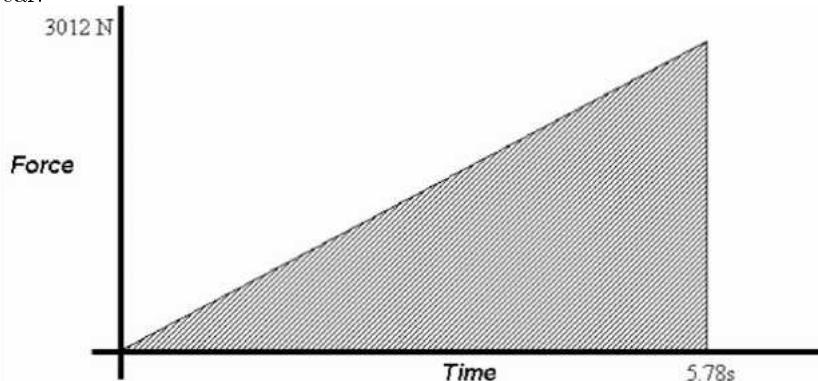


Illustration : Graph for Example (Force as a function of Time)

Solution : Let's start by graphing the information we were given. We will get a nice linear graph, since it said that the force steadily increases.

If we calculate the area under the graph (a triangle) we will know what the impulse is.

$$A = \frac{1}{2} bh$$

$$= \frac{1}{2} (5.78 \text{ s})(3012 \text{ N})$$

$$= 8704.68 \text{ A}$$

$$= 8.70e3 \text{ kgm/s}$$

To calculate the final velocity, we can use the value for the impulse we just got with the right hand side of the impulse formula. Remember that the initial velocity (sitting at the light) is zero...

$$\Delta p = m\Delta v$$

$$\Delta p = m(v_f - v_i)$$

$$\Delta p = mv_f$$

$$v_f = \Delta p / m$$

$$v_f = 8704.68 / 1500$$

$$v_f = 5.80312$$

$$v_f = 5.80 \text{ m/s}$$

The graph that we make does not have to be a pretty right angle triangle either. We can also do some crazy stuff with what we are looking for in the question, as the next example shows.

Example : This graph shows the result of applying 500 kgm/s of impulse to an object as it moved across the floor for 10.0 s. Determine the maximum force that was exerted.

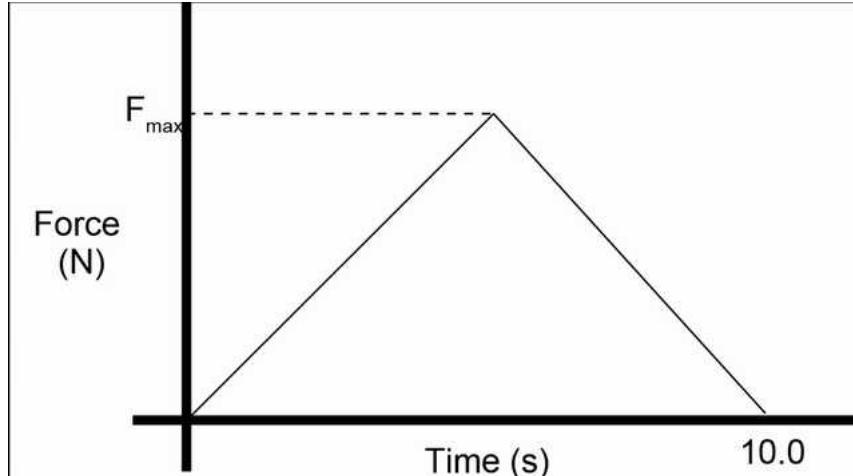


Illustration 3: Pushing object across the floor.

Solution : Even though it is not a right angle triangle, this graph still shows a triangle that we can use the regular area formula with. In this case, we already know the area (the impulse is 500 kgm/s) and we know the base (10.0 s). All we want is the height of the triangle, since that is the magnitude of the maximum force.

$$\text{Area} = \frac{bh}{2}$$

$$\Delta p = F \Delta t / 2$$

$$F = 2 \Delta p / t = 2 \times 500 / 10.0 \text{ N} = 100 \text{ N}$$

Even if the graph is a curved line, you can still at least estimate the area under the graph. • Although this will only be an approximate area, without getting into calculus it's as good as you'll get and as good as you need. ◦ On the graph shown below we have an s-curve that would be difficult to calculate the exact area of. ◦ Instead, we just look at the triangle drawn in red. For the little bit extra it has near the beginning, it misses a bit later on. These two parts should more or less make up for each other, so that the area of the triangle will be about the same as the area under the curve.

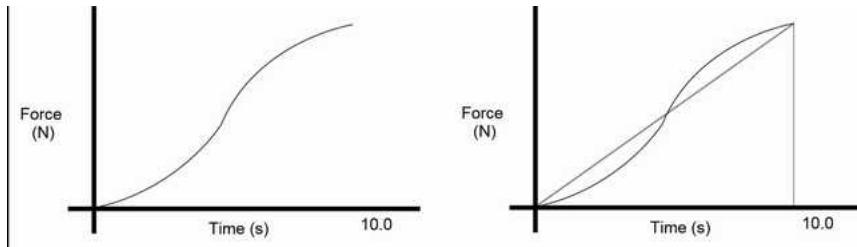
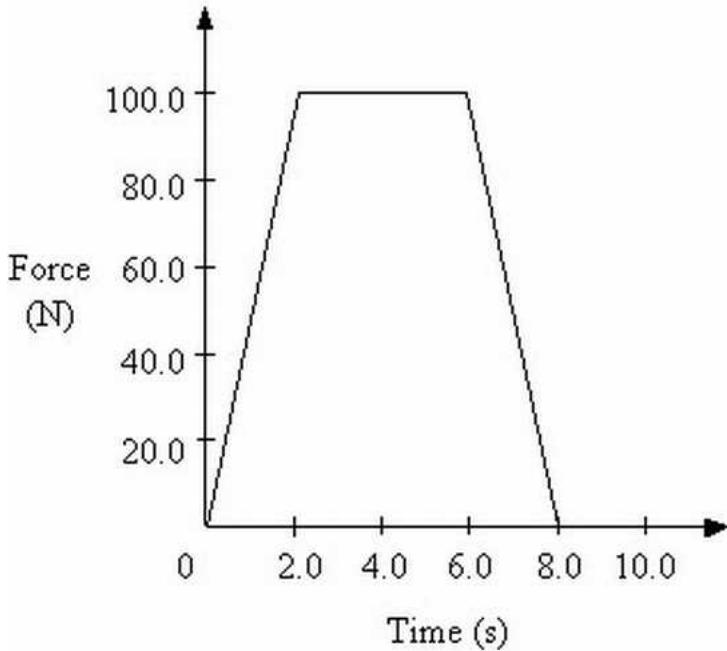


Illustration 4: Instead of trying to figure out the area of the curve exactly, we just use the area of the triangle as an approximation.

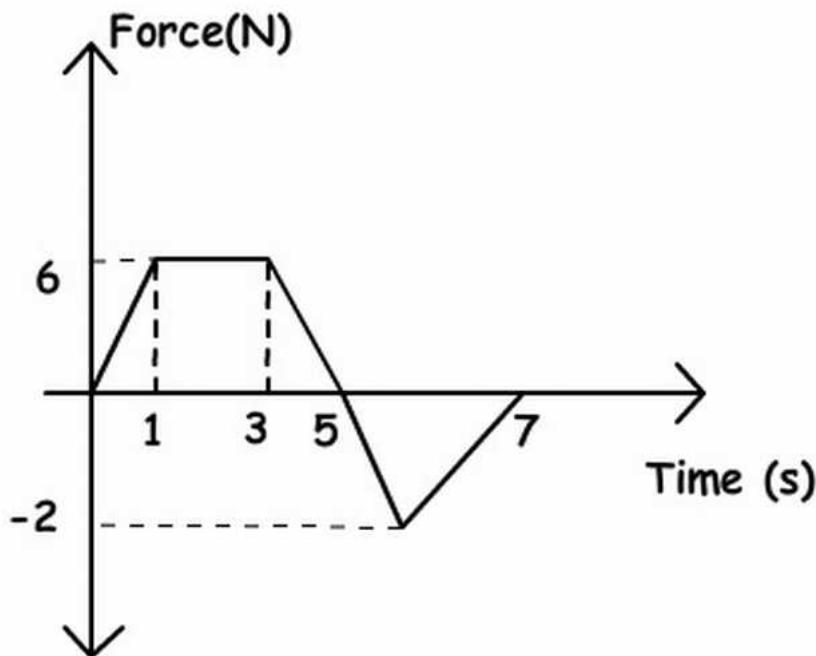
7.2.3.2 Force Time graph with respect to momentum



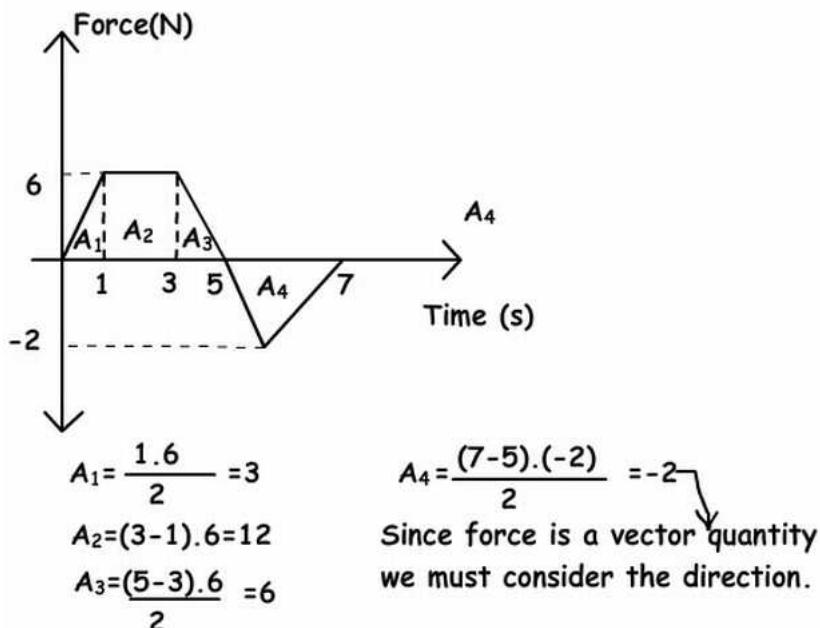
Example : If the mass of the object is 3.0 kg, what is its final velocity over the 8.0 s time period? n.d.

Solution : You need the area under the curve, and you do not need calculus, as that is a trapezoid. The area of a trapezoid is the average of the bases times the height, which is $(4 + 8)\text{seconds}/2 * 100 \text{ N} = 600 \text{ N*s}$. Set this to $mv - mv_o$, and assuming v_o is zero get $v_{\text{final}} = 200 \text{ m/s}$.

Example : The graph given below belongs to an object having mass 2kg and velocity 10m/s. It moves on a horizontal surface. If a force is applied to this object between (1-7) seconds find the velocity of the object at 7 seconds. n.d.



Solution : Area under the graph gives us impulse. First, we find the total impulse with the help of graph given above then total impulse gives us the momentum change. Finally, we find the final velocity of the object from the momentum change.



$$\text{Impulse} = F \cdot t = \text{total area} = A_1 + A_2 + A_3 + A_4$$

$$\text{Impulse} = 3 + 12 + 6 + (-2) = 19 \text{ N.s}$$

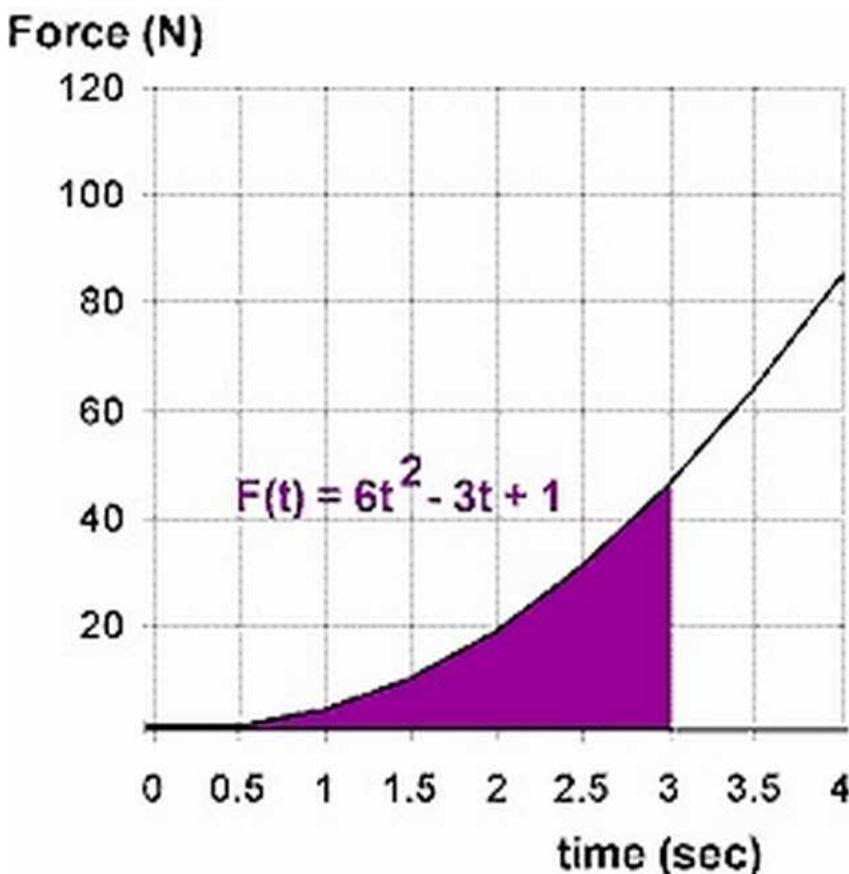
$$\text{Impulse} = \text{Change in Momentum}$$

$$19 \text{ N.s} = m(V_{\text{final}} - V_{\text{initial}})$$

$$19 \text{ N.s} = 2 \text{ kg} \cdot (V_{\text{final}} - 10 \text{ m/s})$$

$$V_{\text{final}} = 10.5 \text{ m/s}$$

Example : Suppose a force, $F(t) = 6t^2 - 3t + 1$, acts on an 7-kg mass for three seconds.



a) What impulse will the 7-kg object receive in the first three seconds?

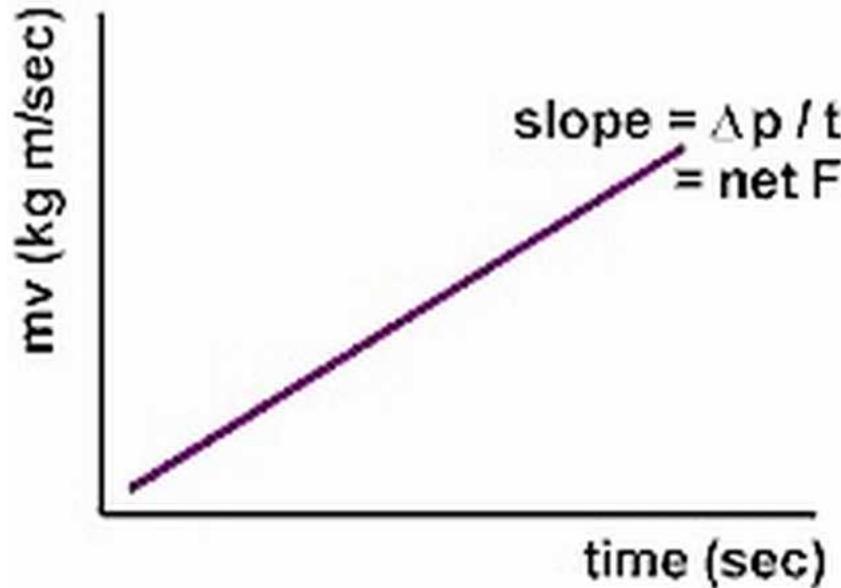
b) If the mass started from rest, what is its final velocity?

Solution : Force as the rate of change of momentum The impulse equation $J = (\text{net } F)t = \Delta p$ where $p = mv$ can be rearranged to state that the applied net force applied to an object equals the rate of change of the its momentum.

$$\text{net } F = \frac{\Delta p}{\Delta t}$$

$$\text{net } F = \frac{m\Delta v}{\Delta t}$$

$$\text{net } F = ma$$



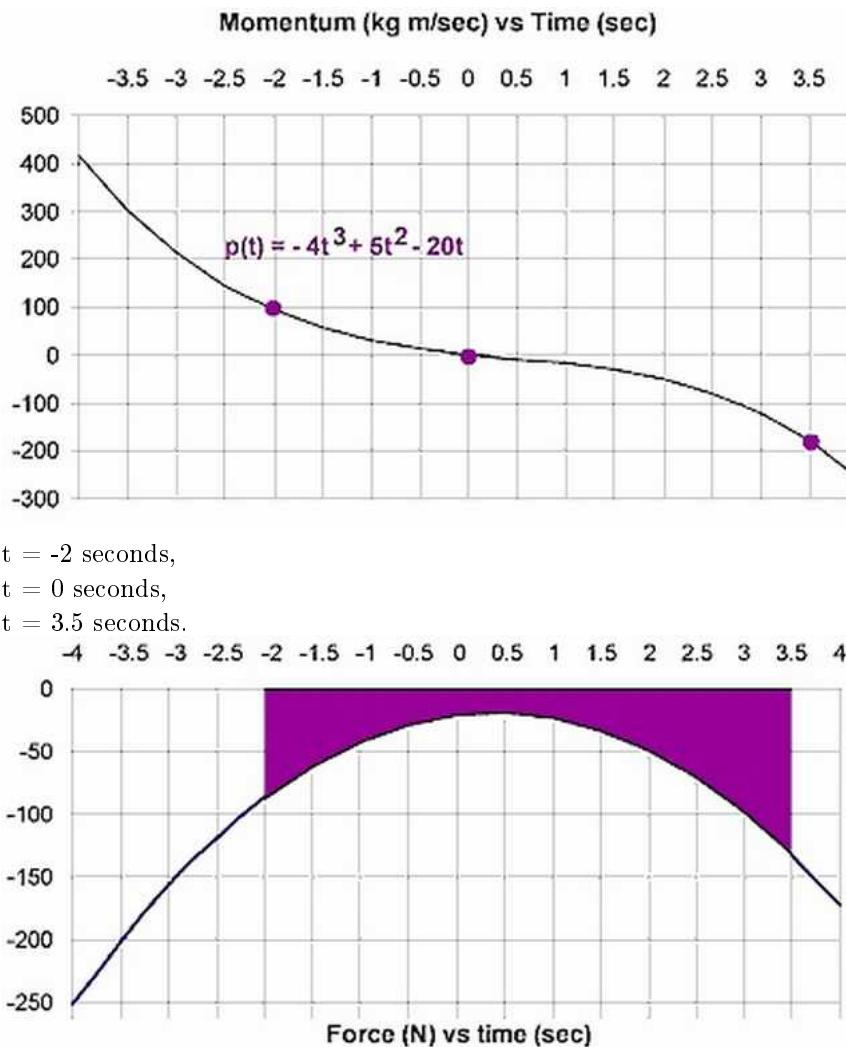
That is, the net force acting on an object can be calculated as the slope of a momentum vs time graph. In terms of the calculus, this result equates to taking the derivative.

$$\frac{dp(t)}{dt} = F(t)$$

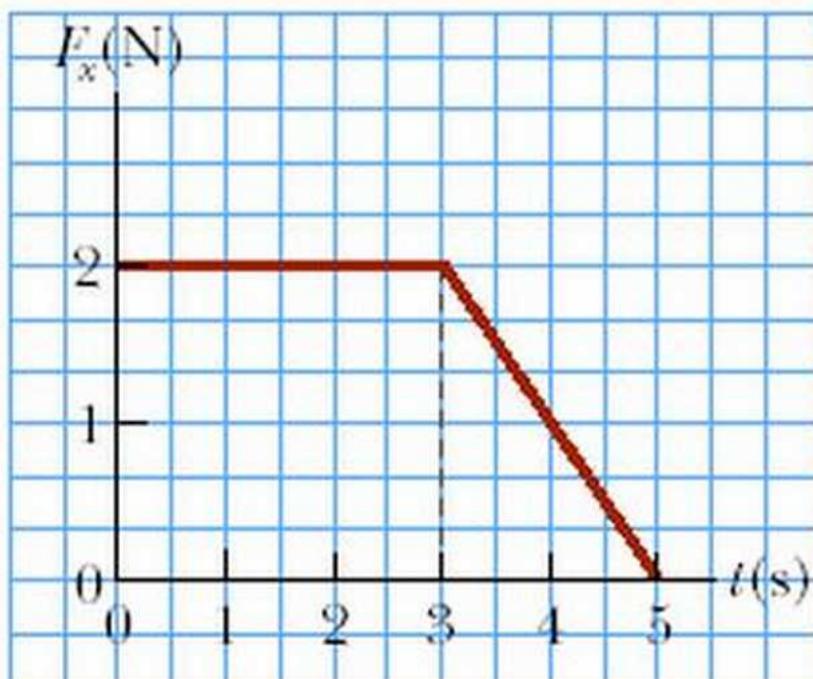
Notice that force must expressed as a function in terms of time, not displacement. Calculus will allow

us to determine expressions for instantaneous, non-constant forces and thus is applicable to a wider range of situations. Let's work an example using this relationship.

Using the graph provided below, determine the instantaneous force acting on the 7-kg mass at each of the specified times:



Example : The force shown in the force vs time graph below acts on a 1.7 kg object.



(a) Find the magnitude of the impulse of the force.

Ans : 8 kg m/s.

(b) Find the final velocity of the object if the object was initially at rest.

Ans : 2.76 m/s

(c) Find the final velocity of the object if the object was initially moving along the x axis with a velocity of -1.7 m/s.

Ans : “ ”

Note : The force shown in the force vs time graph below acts on a 1.7 kg object. Find the final velocity of the object if the object was initially at rest. Find the final velocity of the object if the object was initially moving along the x axis with a velocity of -1.7 m/s.

Example : Relating Momentum and Impulse

EXPLORATION – An impulsive bike ride Suki is riding her bicycle, in a straight line, along a flat road. Suki and her bike have a combined mass of 50 kg. At $t = 0$, Suki is traveling at 8.0 m/s. Suki coasts for 10 seconds, but when she realizes she is slowing down, she pedals for the next 20 seconds. Suki pedals so that the static friction force exerted on the bike by the road increases linearly with time from 0 to 40 N, in the direction Suki is traveling, over that 20-second period. Assume there is constant 10 N resistive force, from air resistance and other factors, acting on her and the bicycle the entire time. Step 1 - Sketch a diagram of the situation. The diagram is shown in Figure 6.2, along with the free-body diagram that applies for the first 10 s and the free-body diagram that applies for the 20second period while Suki is pedaling.

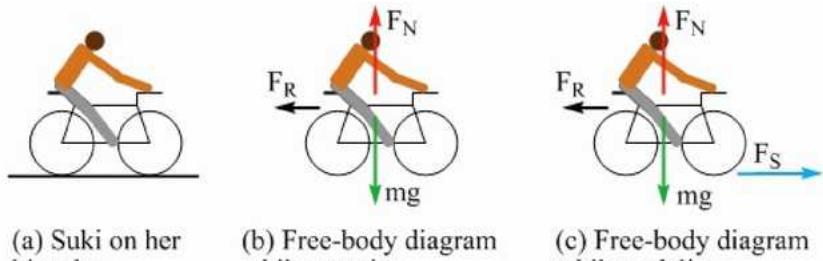


Figure : A diagram of (a) Suki on her bike, as well as free-body diagrams while she is (b) coasting and while she is (c) pedaling. Note that in free-body diagram (c), the static friction force \vec{F}_S gradually increases because of the way Suki pedals.

Step 2 - Sketch a graph of the net force acting on Suki and her bicycle as a function of time. Take the positive direction to be the direction Suki is traveling. In the vertical direction, the normal force exactly balances the force of gravity, so we can focus on the horizontal forces. For the first 10 seconds, we have only the 10 N resistive force, which acts to oppose the motion and is thus in the negative direction. For the next 20 seconds, we have to account for the friction force that acts in the direction of motion and the resistive force. We can account for their combined effect by drawing a straight line that goes from -10 N at $t = 10$ s, to +30 N (40 N - 10N) at $t = 30$ s. The result is shown in Figure 6.3.

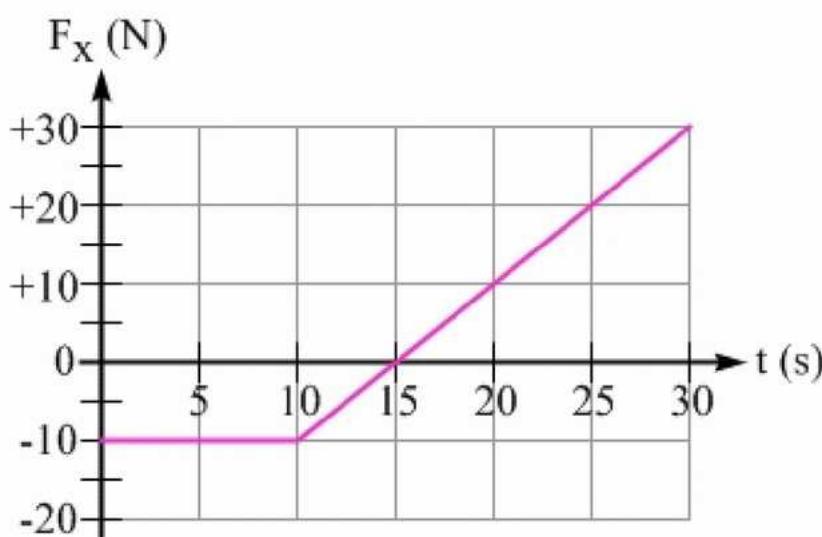


Figure 6.3: A graph of the net force acting on Suki and her bicycle as a function of time.

Step 3 - What is Suki's speed at $t = 10$ s? Let's apply Equation 6.3, which we can write as:

$$\vec{F}_{net} \Delta t = \Delta(m\vec{v}) = m\Delta\vec{v} = m(\vec{v}_{10s} - \vec{v}_i).$$

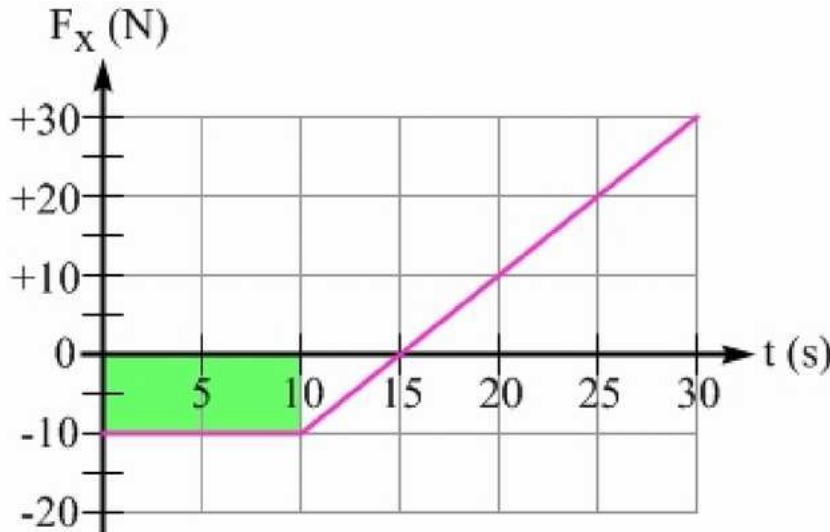
Solving for the velocity at $t = 10$ s gives:

$$\vec{v}_{10s} = \vec{v}_i + \frac{\vec{F}_{net}\Delta t}{m} = +8.0\text{ m/s} + \frac{(-10\text{ N})(10\text{ s})}{50\text{ kg}} = +8.0\text{ m/s} - 2.0\text{ m/s} = +6.0\text{ m/s}$$

Thus, Suki's speed at $t = 10\text{ s}$ is 6.0 m/s . We can also obtain this result from the force-versus-time graph, by recognizing that the impulse, $\vec{F}_{net}\Delta t$, represents the area under this graph over some time interval Δt . Let's find the area under the graph, over the first 10 seconds, shown highlighted in green in Figure 6.4. The area is negative, because the net force is negative over that time interval. The area under the graph is the impulse:

$$\vec{F}_{net}\Delta t = -10\text{ N} \times 10\text{ s} = -100\text{ N s} = -100\text{ kg m/s}$$

Figure 6.4: The green rectangle represents the area under the graph for the first 10 s. The area is negative, because the force is negative. From Equation 6.3, we know the impulse is equal to the change in momentum.



Suki's initial momentum is

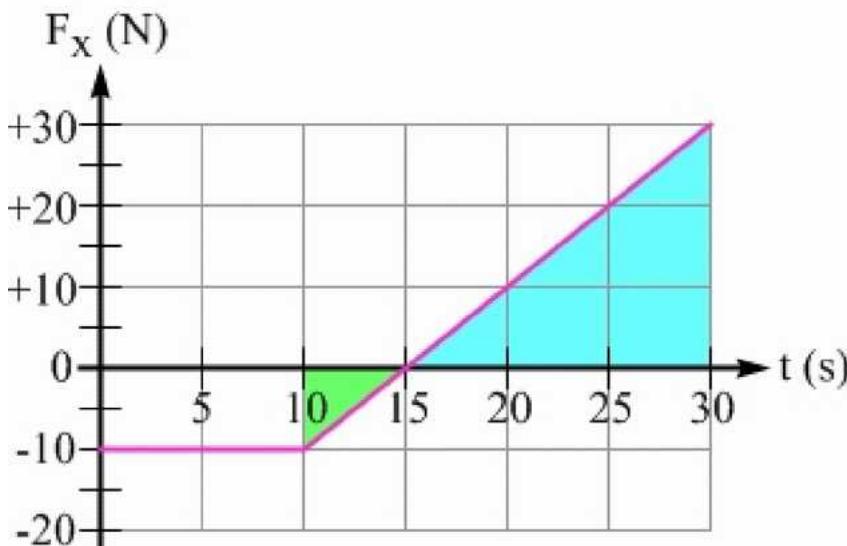
$m\vec{v}_i = 50\text{ kg} \times 8.0\text{ m/s} = +400\text{ kg m/s}$. Her momentum at $t = 10\text{ s}$ is therefore $+400\text{ kg m/s} - 100\text{ kg m/s} = +300\text{ kg m/s}$. Dividing this by the mass to find the velocity at $t = 10\text{ s}$ confirms what we found above:

$$\vec{v}_{10s} = \frac{\vec{p}_{10s}}{m} = \frac{\vec{p}_i + \Delta \vec{p}}{m} = \frac{+400\text{ kg m/s} - 100\text{ kg m/s}}{50\text{ kg}} = \frac{+300\text{ kg m/s}}{50\text{ kg}} = +6.0\text{ m/s}$$

Step 4 - What is Suki's speed at $t = 30\text{ s}$? Let's use the area under the force-versus-time graph, between $t = 10\text{ s}$ and $t = 30\text{ s}$, to find Suki's change in momentum over that 20-second period. This area is highlighted in Figure 6.5, split into a negative area for the time between $t = 10\text{ s}$ and $t = 15\text{ s}$, and a positive area between $t = 15\text{ s}$ and $t = 30\text{ s}$. These regions are triangles, so we can use the equation for the area of a triangle, $0.5 \times \text{base} \times \text{height}$. The area under the curve, between 10 s and 15 s, is $0.5 \times (5.0\text{ s}) \times (-10\text{ N}) = -25\text{ kg m/s}$. The area between 15 s and 30 s is $0.5 \times (15\text{ s}) \times (30\text{ N}) = +225\text{ kg m/s}$. The total area (total change in momentum) is $+200\text{ kg m/s}$.

Note that another approach is to multiply the average net force acting on Suki and the bicycle ($+10\text{ N}$) over this interval, by the time interval (20 s), for a $+200\text{ kg m/s}$ change in momentum.

Figure 6.5: The shaded regions correspond to the area under the curve for the time interval from $t = 10\text{ s}$ to $t = 30\text{ s}$.



In step 3, we determined that Suki's momentum at $t = 10$ s is $+300 \text{ kg m/s}$. With the additional 200 kg m/s, the net momentum at $t = 10$ s is $+500 \text{ kg m/s}$. Dividing by the 50 kg mass gives a velocity at $t = 30$ s of $+10 \text{ m/s}$.

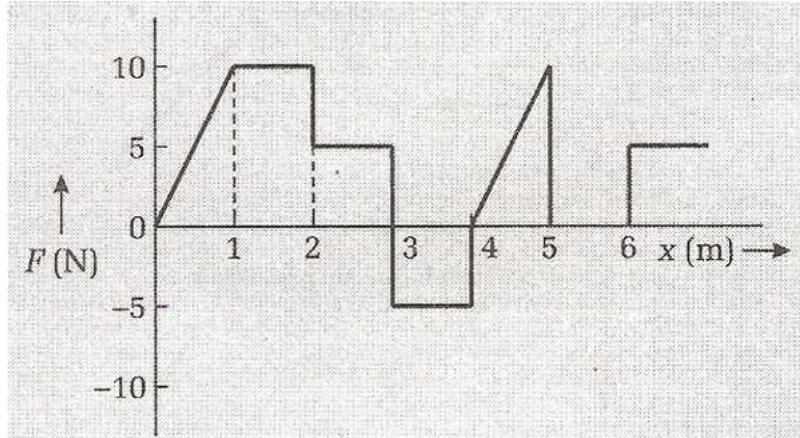
Key idea for the graphical interpretation of impulse: The area under the net force versus time graph for a particular time interval is equal to the change in momentum during that time interval.

7.2.4 Problems for Practice

7.2.4.1 General Problem Set

Single Answer Type

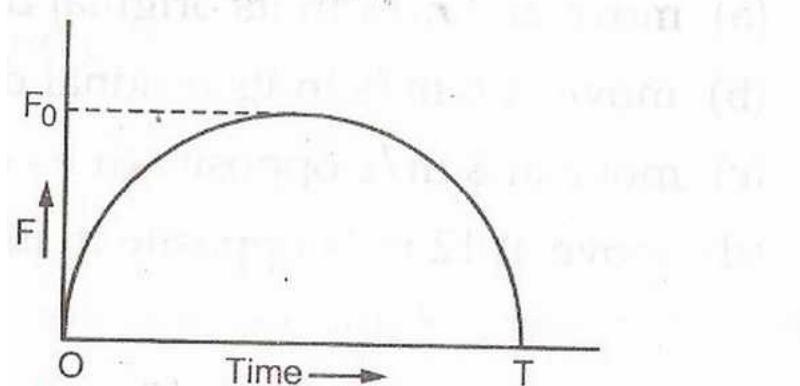
Example : The relationship between the force F and position x of a body is as shown in figure. The work done in displacing the body from $x=1\text{m}$ to $x=5\text{m}$ will be



- a) 30 J
- b) 15 J
- c) 25 J
- d) 20 J

{ Hint : Area under the graph from 1 to 5 taking signs , 15 J }

Example: A particle of mass m, initially at rest, is acted upon by a variable force F for a brief interval of time T. It begins to move with a velocity u after the force stops acting. F is shown in the graph as a function of time. The curve is a semicircle



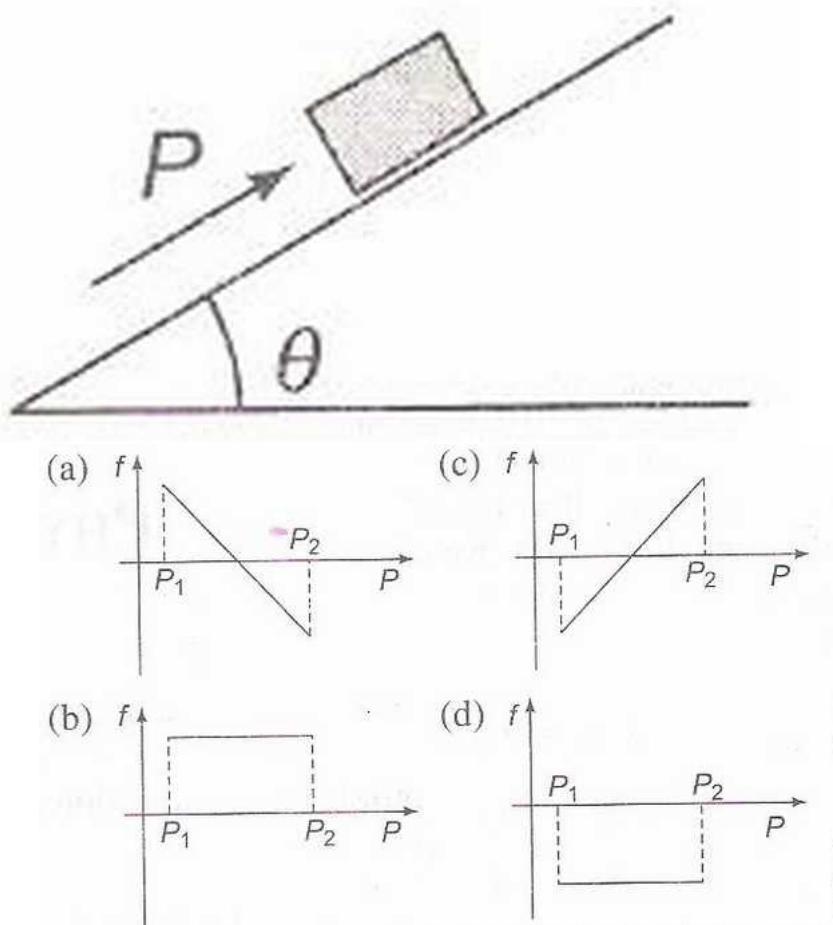
- a) $u = \frac{\pi F_0^2}{2m}$
- b) $u = \frac{\pi T^2}{8m}$
- c) $u = \frac{\pi F_0 T}{4m}$
- d) $u = \frac{F_0 T}{2m}$

{ Hint : From impulse relation , $mv_f = \frac{\pi F_0^2}{2}$. So, a, b,c }

7.2.4.2 Previous Years IIT Problems

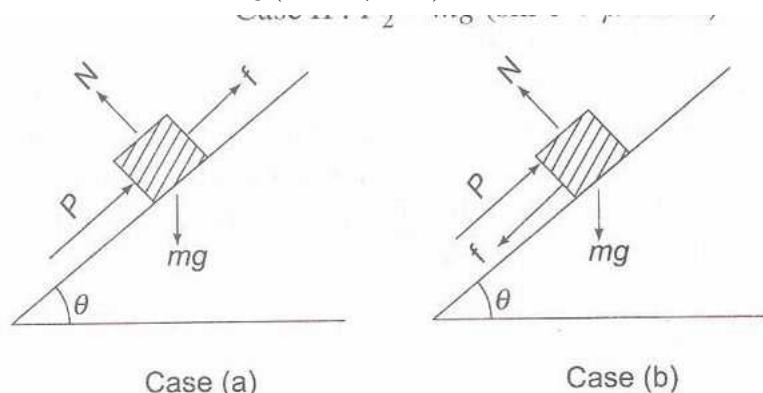
Single Answer

Example: A block of mass m is on an inclined plane of angle θ . The coefficient of friction between the block and the plane is μ and $\tan\theta > \mu$. The block is held stationary by applying a force P parallel to the plane. The direction of force pointing up the plane is taken to be positive. As P is varied from $P_1 = mg(\sin\theta - \mu\cos\theta)$ to $P_2 = mg(\sin\theta + \mu\cos\theta)$, the frictional force f versus P graph will look like



{ Solution: Case I : $P_1 = mg(\sin\theta - \mu\cos\theta)$

Case II: $P_2 = mg(\sin\theta + \mu\cos\theta)$



Case (a)

Case (b)

In case a), frictional force f is positive and in case b), f is negative. When $P = mgsin\theta$, $f = \mu mgcos\theta = 0$

When $P < mgsin\theta$, $f = mgsin\theta - P$

When $P > mgsin\theta$, $f = P - mgsin\theta$

Thus f varies linearly with P , is positive when $P = P_1$ and negative when $P = P_2$. So the correct option is a)

}

7.3 Energy Conservation

7.3.1 Abstract Introduction

7.3.1.1 KINETIC ENERGY

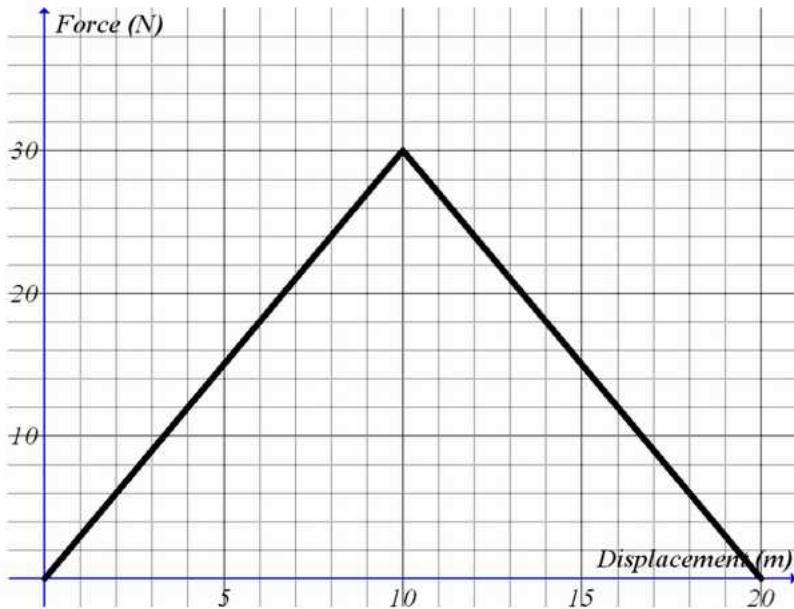
Objects have energy because of their motion; this energy is called kinetic energy. Kinetic energy of the objects having mass m and velocity v can be calculated with the formula given below;

$$E_k = \frac{1}{2}mv^2$$

As you see from the formula, kinetic energy of the objects is only affected by the mass and velocity of the objects. The unit of the E_k is again from the formula $\text{kg}\cdot\text{m}^2/\text{s}^2$ or in general use joule.

7.3.1.2 Work Done by a Variable Force

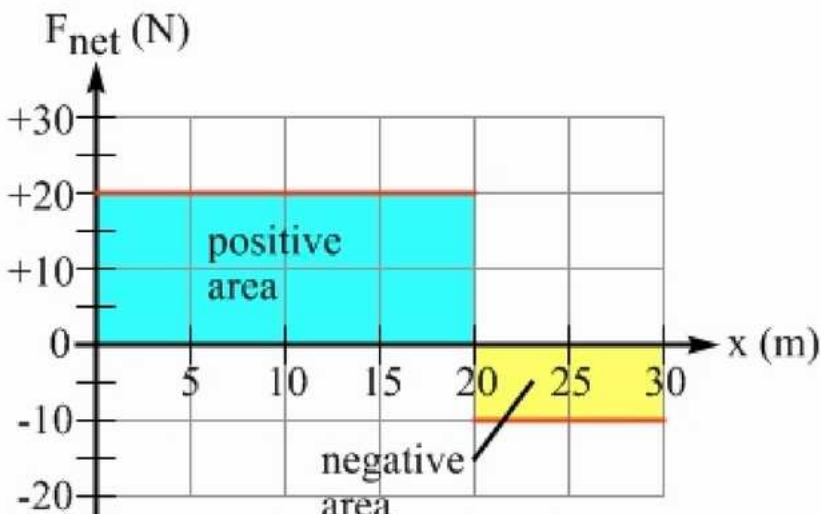
Graphically, the work done on an object or system is equal to the area under a Force vs. displacement graph:



The area under the graph from zero to 20 meters is 300 N m. Thus, the force represented by the graph does 300 J of work. This work is also a measure of the energy which was transferred while the force was being applied

7.3.1.3 The net force vs. position graph

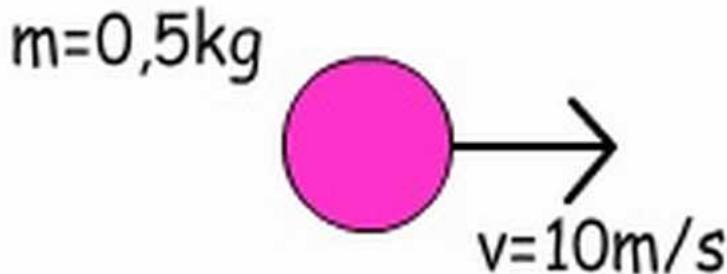
The area under the net force vs. position graph represents the change in kinetic energy (also known as the net work).



7.3.2 Theory and Problems

7.3.2.1 Force vs. Distance graph.

Examples : Find the kinetic energy of the ball having mass 0,5 kg and velocity 10m/s.

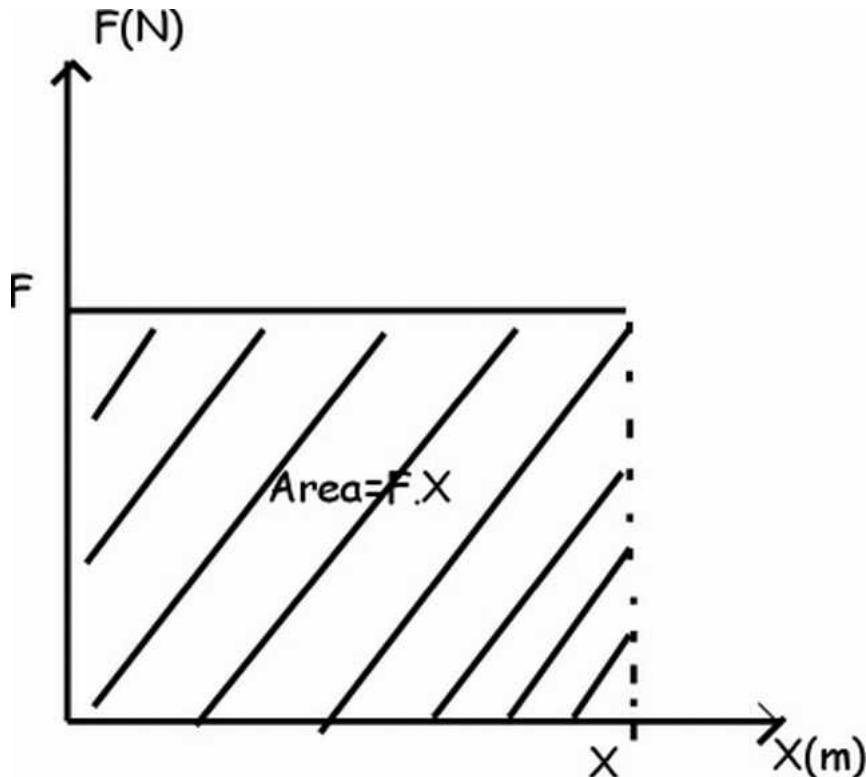


$$E_k = \frac{1}{2}mv^2$$

$$E_k = \frac{1}{2} \cdot 0,5 \cdot (10)^2$$

$$E_k = 25 \text{ joule}$$

As in the case of Kinematics we can use graphs to show the relations of the concepts here. Look at the given graph of

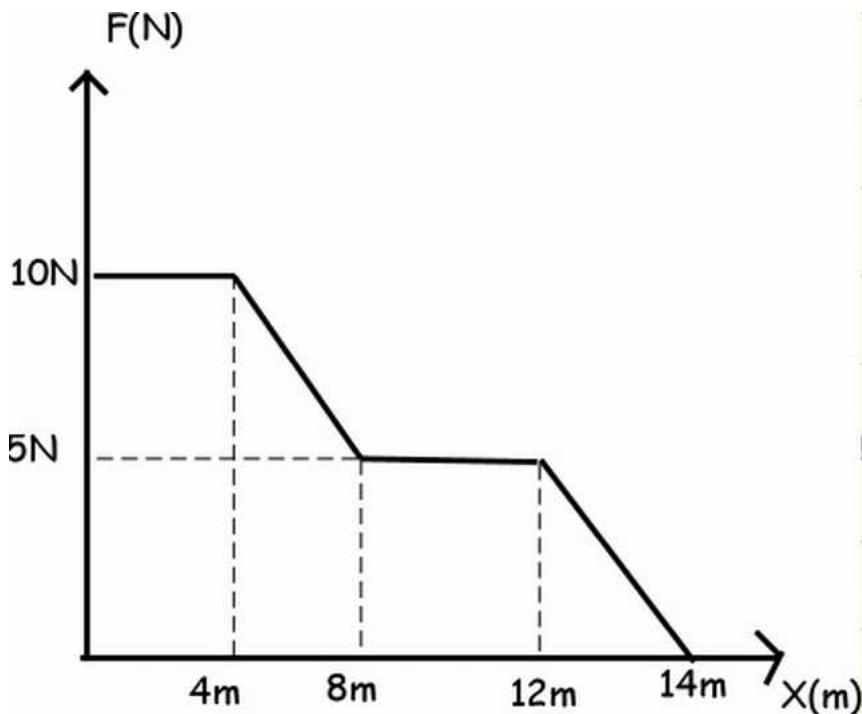


Area under the force vs. distance graph gives us work

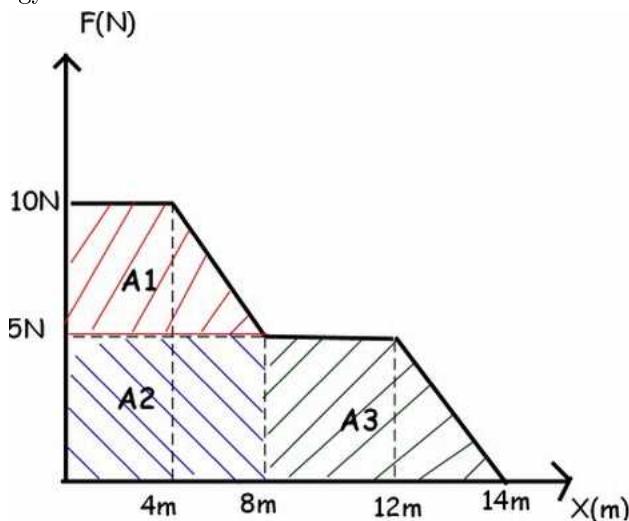
$$\text{Work} = \text{Force} \cdot \text{Distance} = \text{Area} = F \cdot X$$

We can find energy of the objects from their Force vs. Distance graph.

Example : Find the Kinetic Energy of the object at 14m from the given graph below.



We can find the total kinetic energy of the object after 14m from the graph; we use area under it to find energy.



$$A_1 = \frac{(8+4) \cdot 5}{2} = 30 \quad A_3 = \frac{(6+4) \cdot 5}{2} = 25$$

$$A_2 = 5 \cdot 8 = 40$$

$$\text{Total Area} = A_1 + A_2 + A_3$$

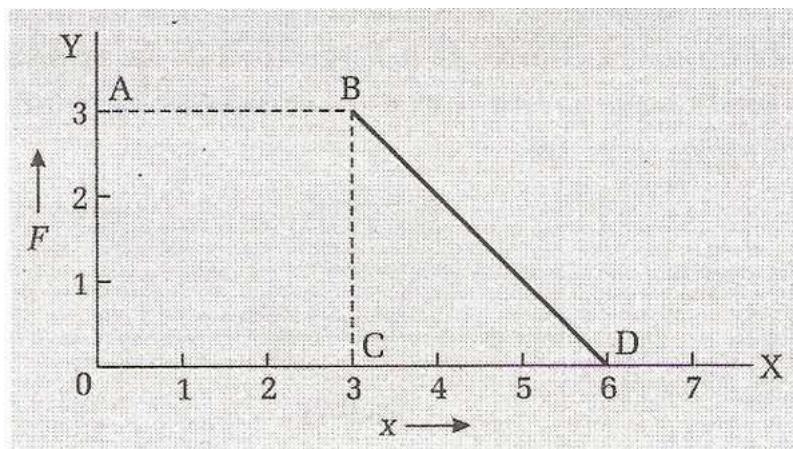
$$\text{Total Area} = 30 + 40 + 25 = 95$$

$$E_k = \text{Total Area} = 95 \text{ joule}$$

7.3.3 Practice Problems

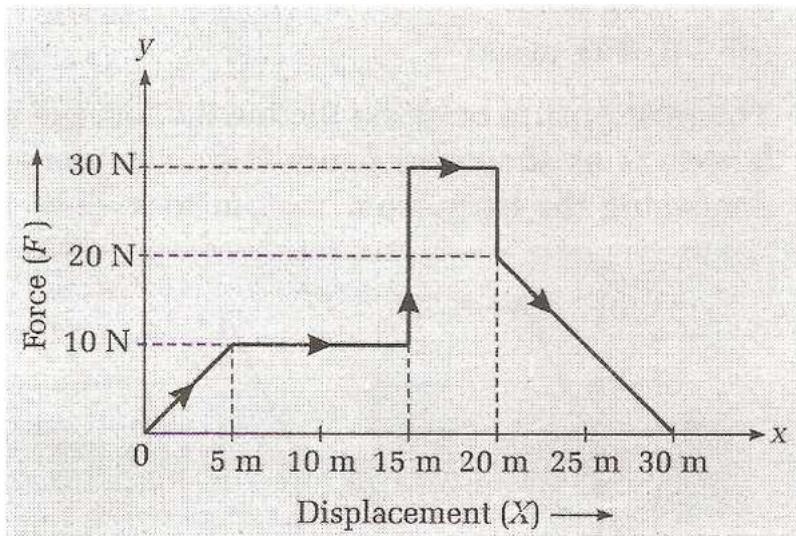
7.3.3.1 General Problem Set

Example : A force F acting on an object varies with distance x as shown in Figure. The force is in newton (N) and the distance (x) in metre. The work done by the force in moving from x=0 to x=6m is



- a) 4.5 J
- b) 9.0 J
- c) 14.5 J
- d) 15 J

Example : Given below is a graph between a variable force (F) (along y-axis) and the displacement (X) (along x-axis) of a particle in one dimension. The work done by the force in the displacement interval between 0 m and 30 m is

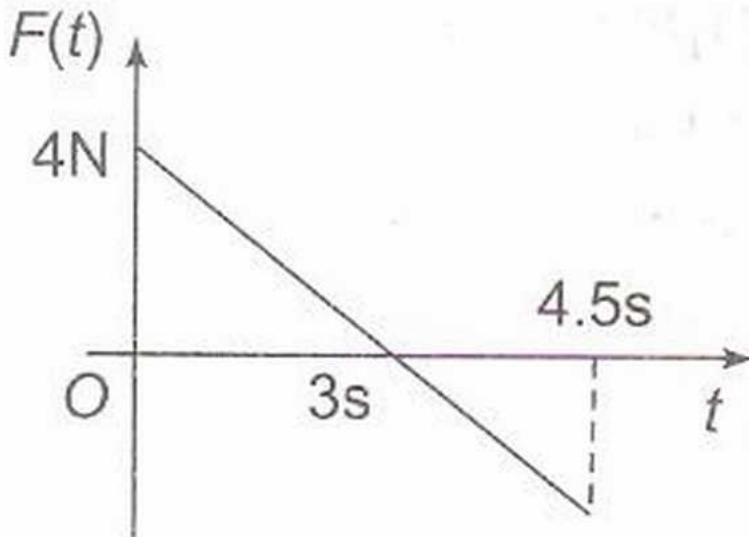


- a) 275 J
- b) 375 J
- c) 400 J
- d) 300 J

7.3.3.2 Previous Years IIT Problems

Single Answer

Example: A block of mass 2 kg is free to move along the x-axis. It is at rest and from $t=0$ onwards it is subjected to a time-dependent force $F(t)$ in the x direction. The force $F(t)$ varies with t as shown in the figure. The kinetic energy of the block after 4.5 seconds is

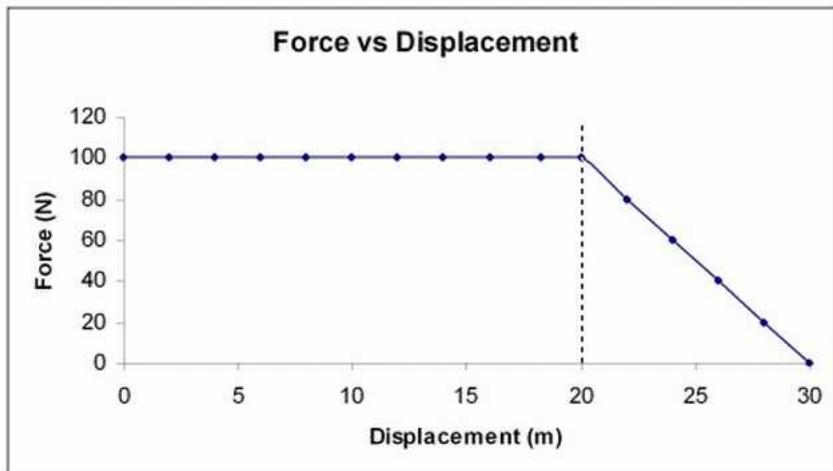


- a) 4.50J
- b) 7.50J
- c) 5.06J
- d) 14.06J

7.3.4 Review Questions I

Refer to the following information for the next thirteen questions. n.d.

A 5.0-kg mass is pushed along a straight line by a net force described in the graph below. The object is at rest at $t = 0$ and $x = 0$.



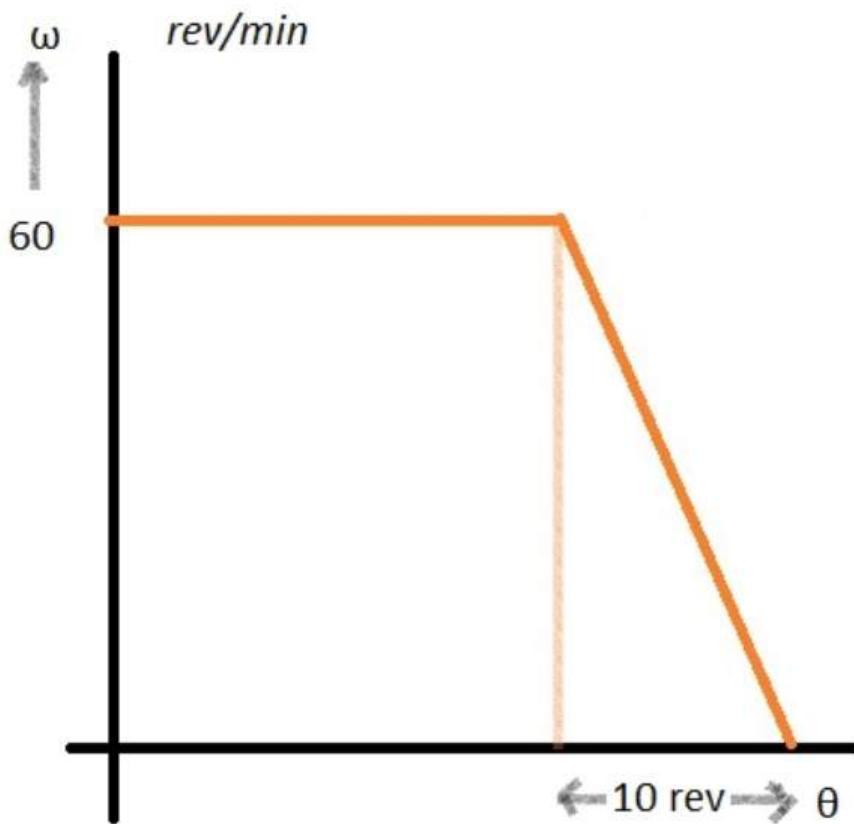
- a) During which displacement interval was the object's acceleration uniform?
- b) What acceleration did the object experience when $x = 10$ meters?
- c) How much work was done on the object during the first 20 meters?
- d) How much kinetic energy did the object gain during the first 20 meters?
- e) What was the object's instantaneous velocity at $x = 20$ meters?
- f) How much time was required to move it through the first 20 meters?
- g) How much did the object's momentum change in the first 20 meters?
- h) What was the object's instantaneous acceleration at $x = 22$ meters?
- i) Why can't the kinematics equations for uniformly accelerated motion be used to calculate the object's instantaneous velocity at $x = 30$ meters? What method should be used?
- j) How much work was done to move the object from 20 meters to 30 meters?
- k) What was the object's instantaneous speed at $x = 30$ meters?
- l) What was the total impulse delivered to the object from $x = 0$ to $x = 30$ meters?
- m) What percent of the impulse was delivered in the last 10 meters?

7.4 Rotatory Motion

7.4.1 Problems for Practice

7.4.1.1 General Problem Set

Single Answer Type Example : The angular velocity of a rotating disc decreases linearly with angular displacement from 60 rev/min. to zero during 10 rev as shown. Determine the angular velocity of the disc 3 sec after it begins to slow down



(a) $\frac{20\pi}{10}$

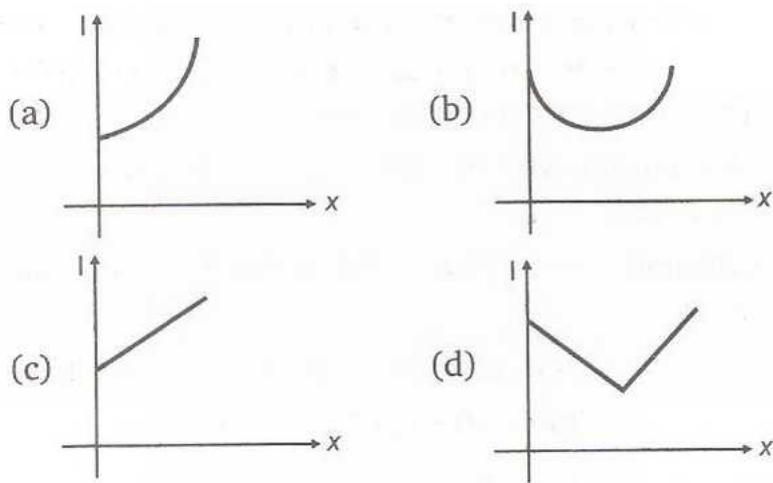
(b) $\frac{17\pi}{10}$

(c) $\frac{7\pi}{3}$

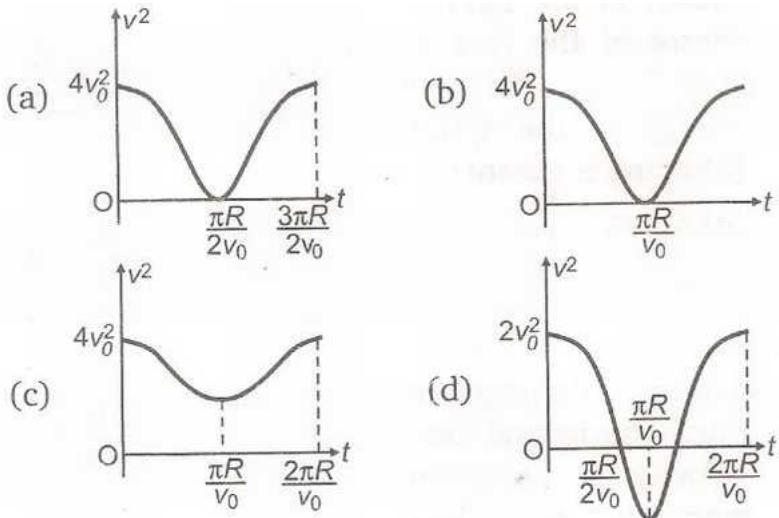
(d) $\frac{10\pi}{3}$

{ Hint: Answer C }

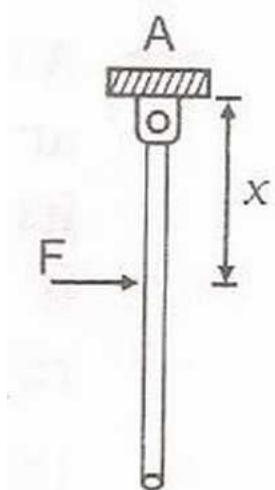
Example: Moment of Inertia I of a solid sphere about an axis parallel to a diameter and at a distance x from its centre of mass varies as



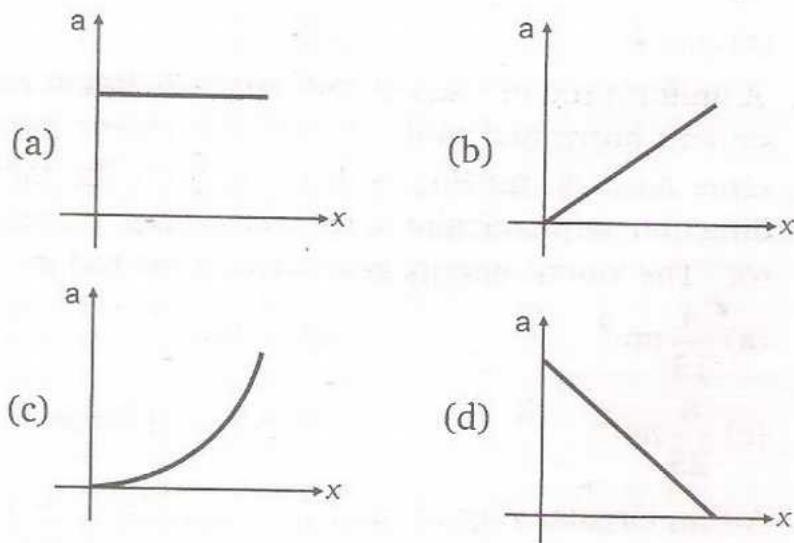
Example: A wheel is rolling without sliding on a horizontal surface. The centre of the wheel moves with a constant speed v_0 . Consider a point P on the rim which is at the top at time $t=0$. The square of speed of point P is plotted against time t. The correct plot is (R is radius of the wheel)



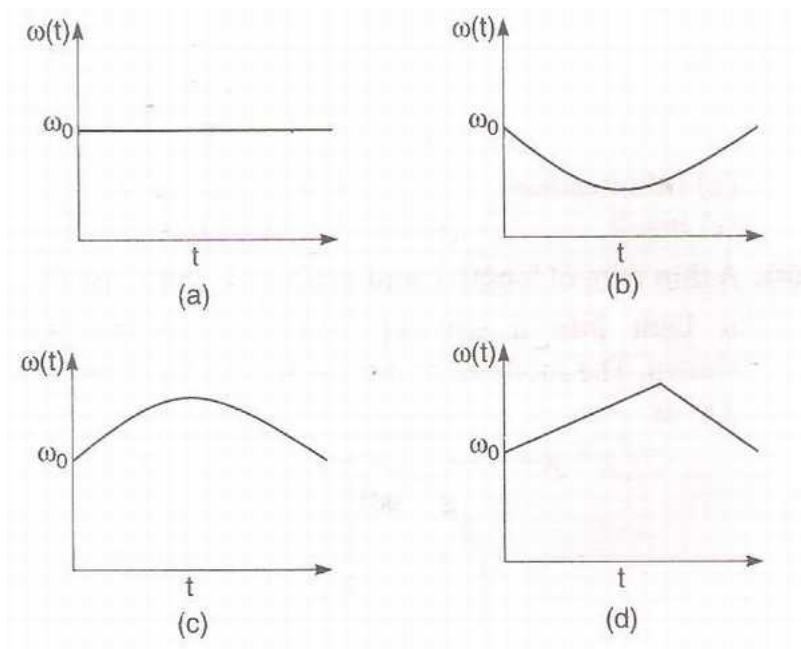
Example: A rod of mass m and length l is hinged at one of its end A as shown in figure.



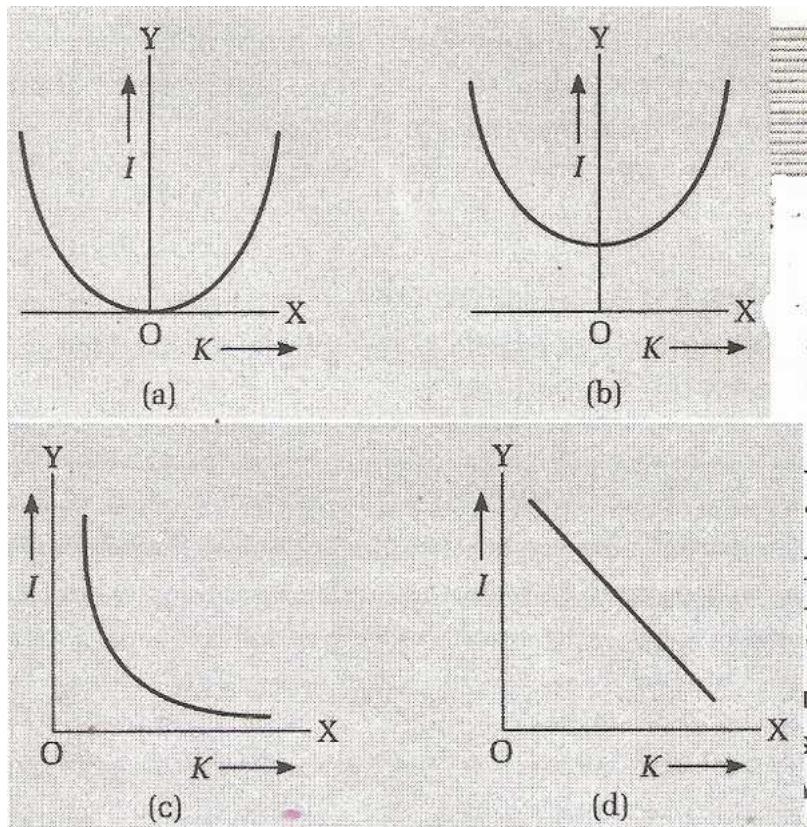
A force F is applied at a distance x from A. The acceleration of centre of mass (a) varies with x as



Example: A circular platform is free to rotate in a horizontal plane about a vertical axis passing through its centre. A tortoise is sitting at the edge of the platform. Now the platform is given an angular velocity ω_0 . When the tortoise moves along a chord of the platform with a constant velocity (with respect to the platform). The angular velocity of the platform $\omega(t)$ will vary with time t as

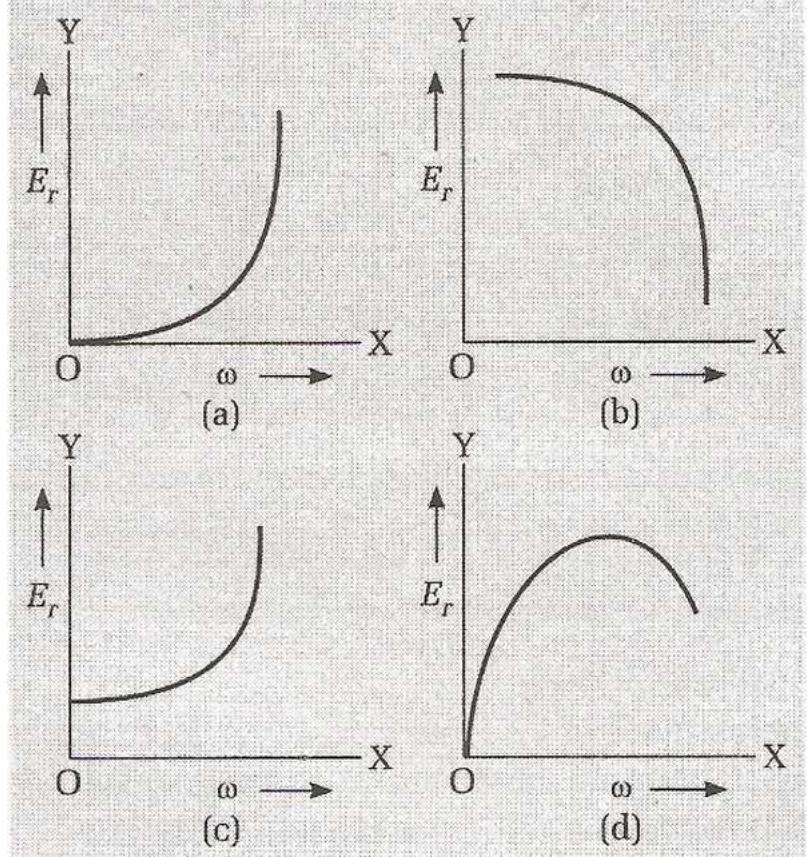


Example: The curve for the moment of inertia of a sphere of constant mass M versus its radius of gyration K will be



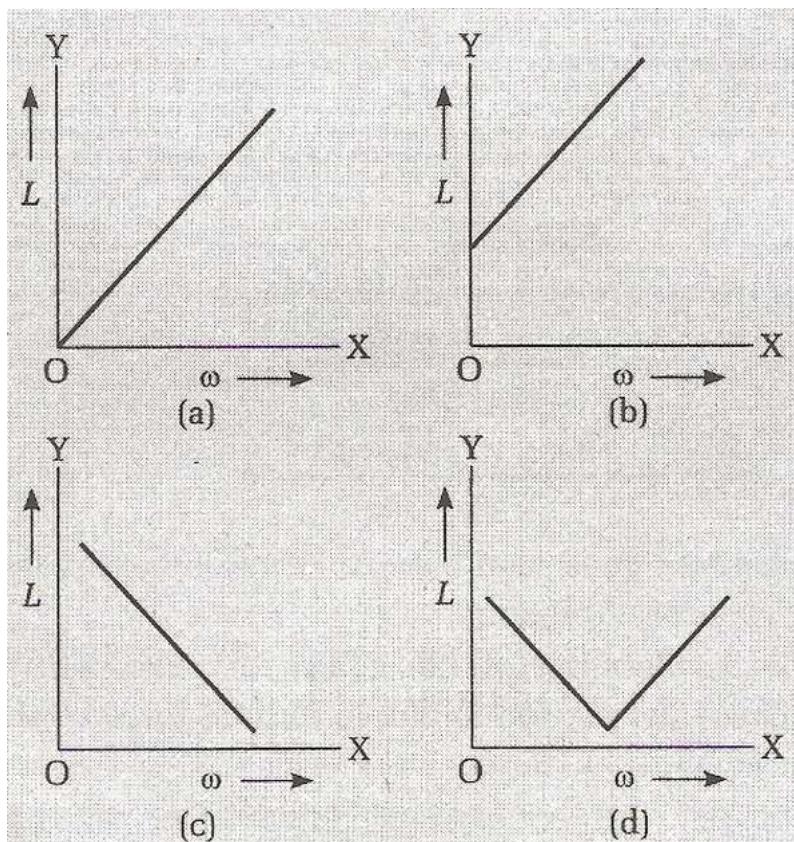
{ Hint : a) }

Example : The graph between rotational energy E_r and angular velocity ω is represented by which curve



{ Hint : a) }

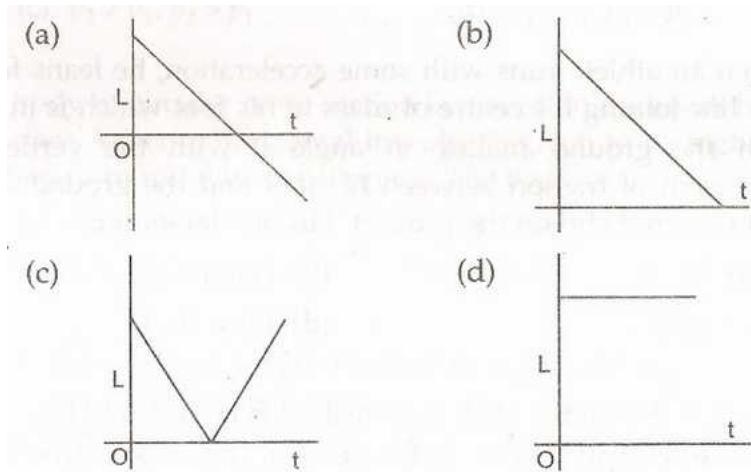
Example : The curve between angular momentum L and angular velocity ω will be



{ Hint : a) }

7.4.2 Angular Momentum Conservation

Example : A block slides on a rough horizontal ground from point A to point B. Point C is midway between A and B. The coefficient of friction between the block and the ground is constant. Its angular momentum L about C is plotted against time t. Which of the following curves is correct?



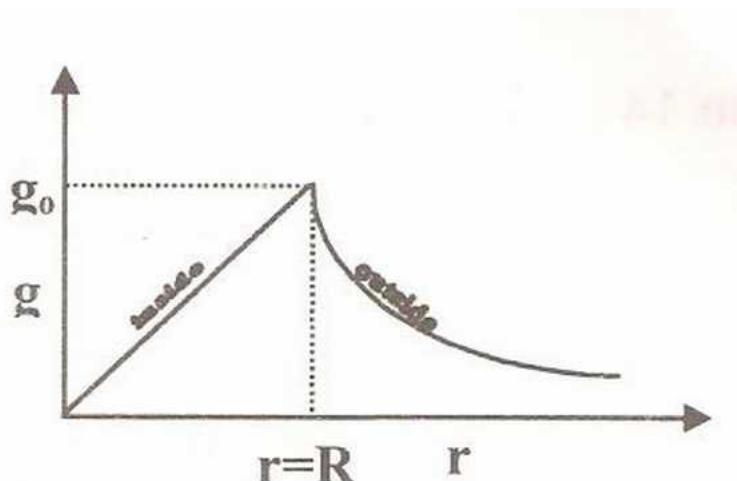
7.5 Gravitation

7.5.1 Basics

7.5.1.1 Variation of "g"

$$\text{Outside the earth } g = g_o \left(\frac{R}{r} \right)^2$$

where g_o is the acceleration due to gravity at the surface of earth.



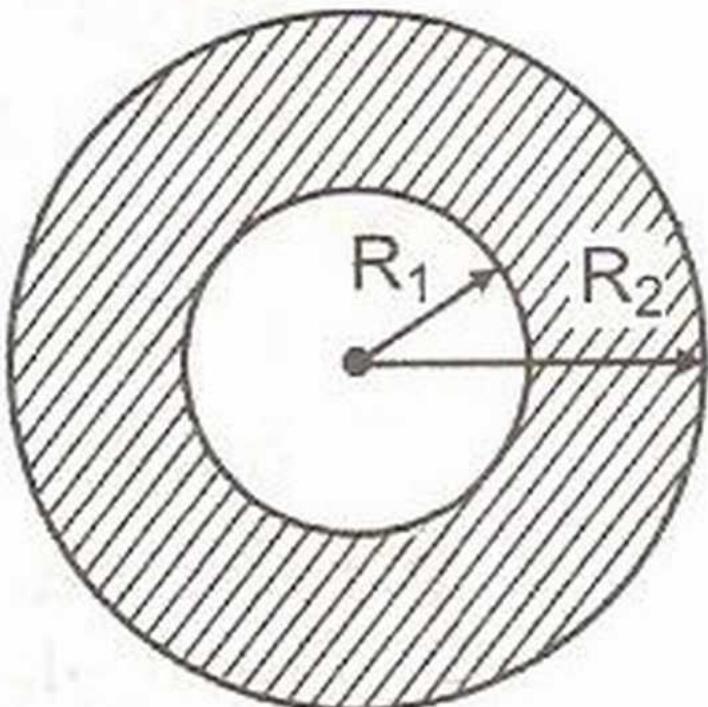
$$g_o = \frac{GM}{R^2}, R = \text{radius of earth}$$

7.5.2 Problems for Practice

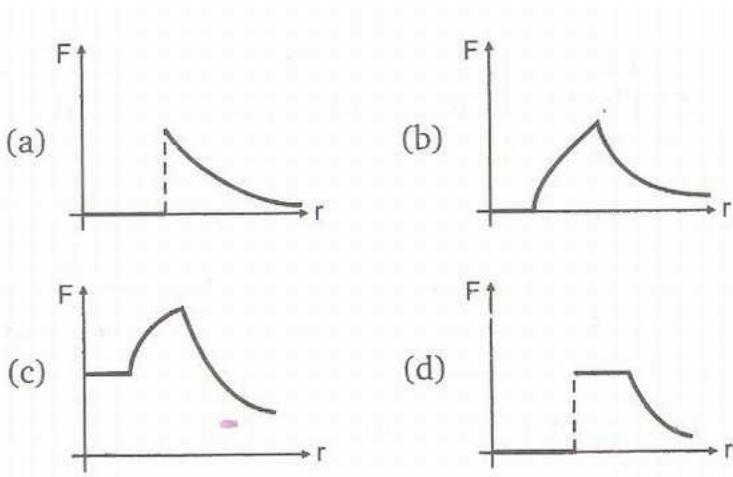
7.5.2.1 General Problem Set

Single Answer

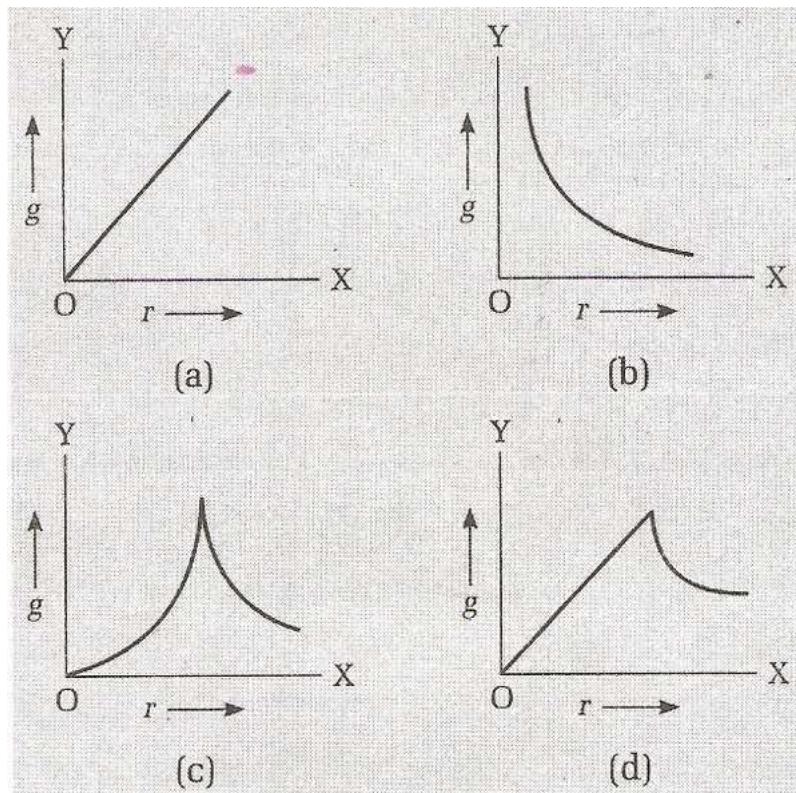
Example: A sphere of mass M and radius R_2 has a concentric cavity of radius R_1 as shown in figure.



The force F exerted by the sphere on a particle of mass m located at a distance r from the centre of sphere varies as ($0 \leq r \leq \infty$)

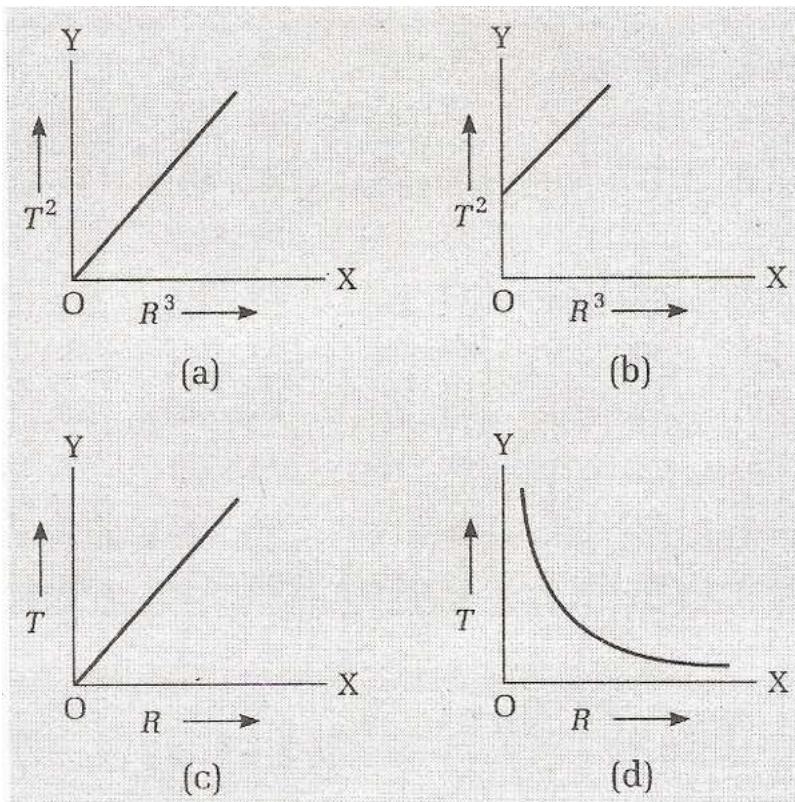


Example : The variation of acceleration due to gravity as one moves away from earth's centre is given by the graph



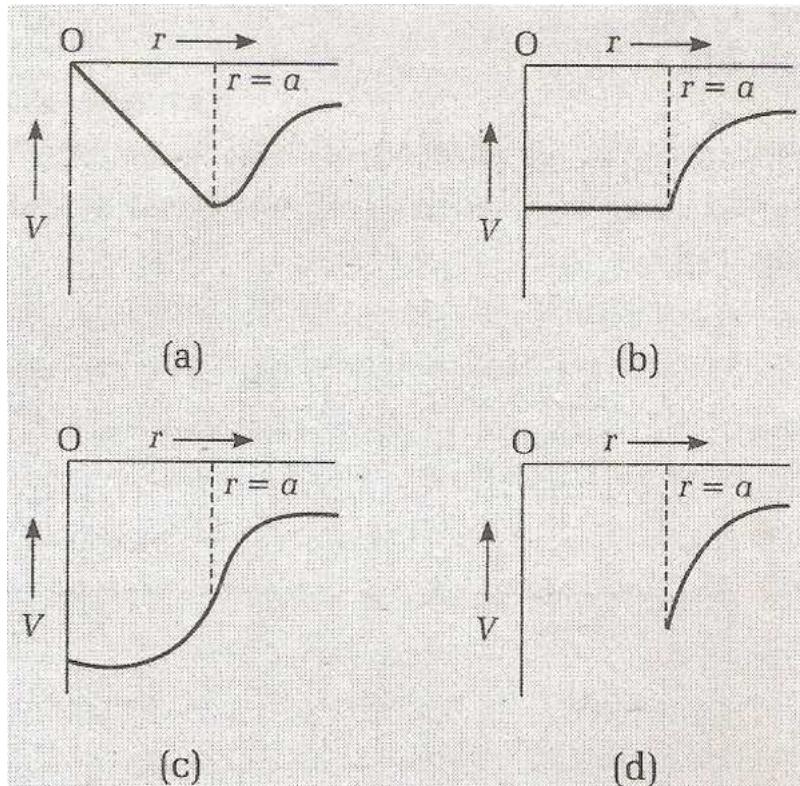
{ Hint: c) }

Example : Which of the following graphs represents the motion of a planet moving about the sun?



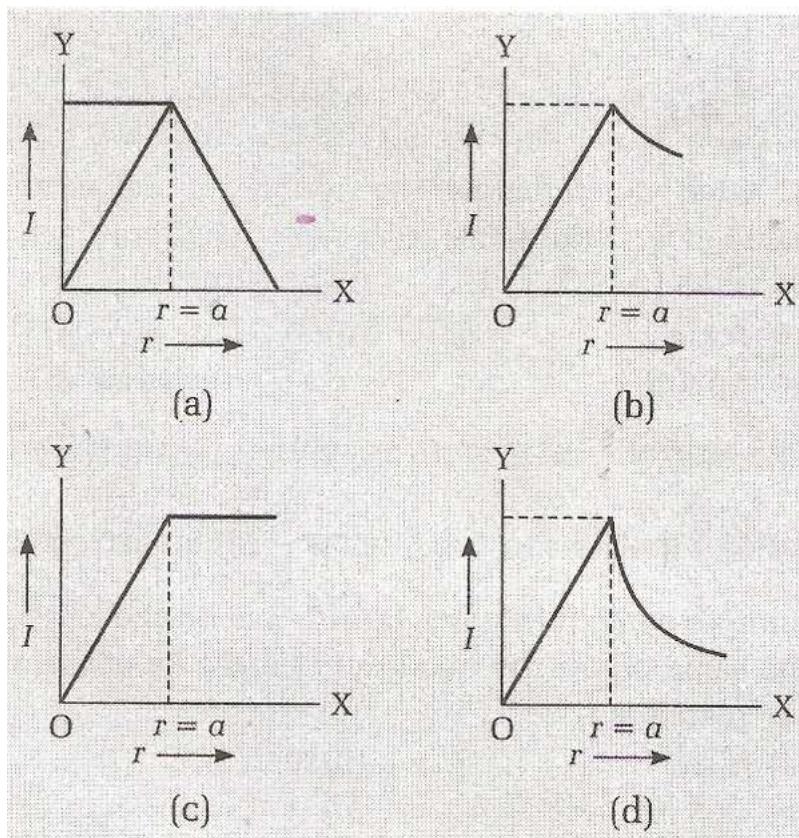
{ Hint : a) }

Example : P is a point at a distance r from the centre of a solid sphere of radius a . The gravitational potential at P is V . If V is plotted as a function of r , which is the correct curve?



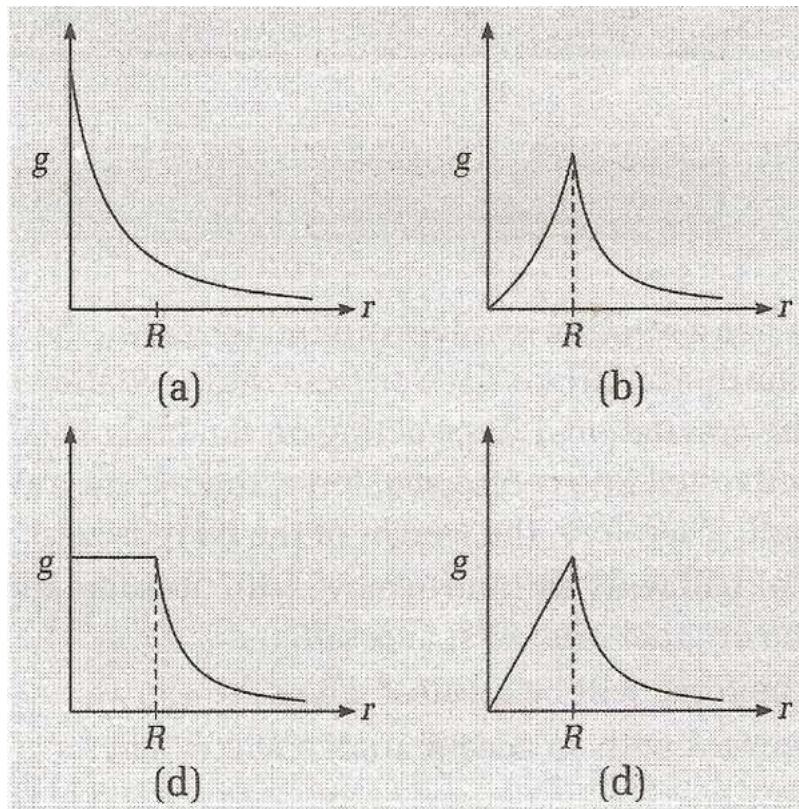
{ Hint : c) }

Example : Which of the graphs represents correctly the variation of intensity of gravitational field I with the distance r from the centre of a spherical shell of mass M and radius a ?



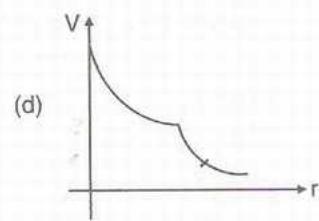
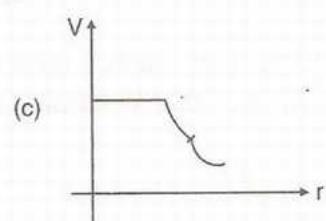
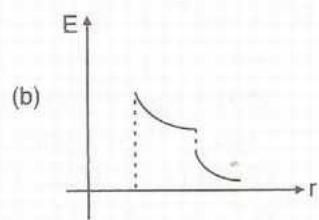
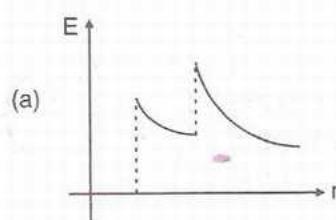
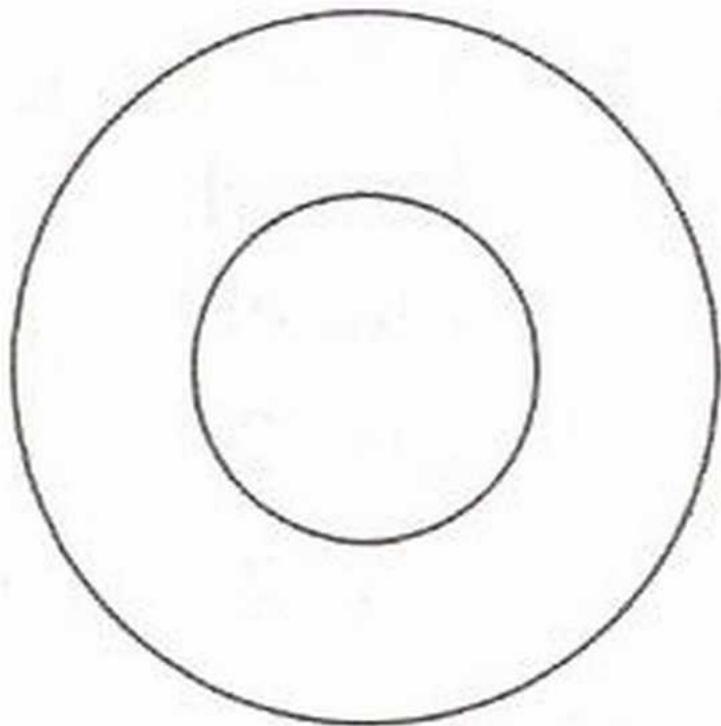
{ Hint : d) }

Example : The dependence of acceleration due to gravity g on the distance r from the center of the earth, assumed to be a sphere of radius R of uniform density is as shown in figure below

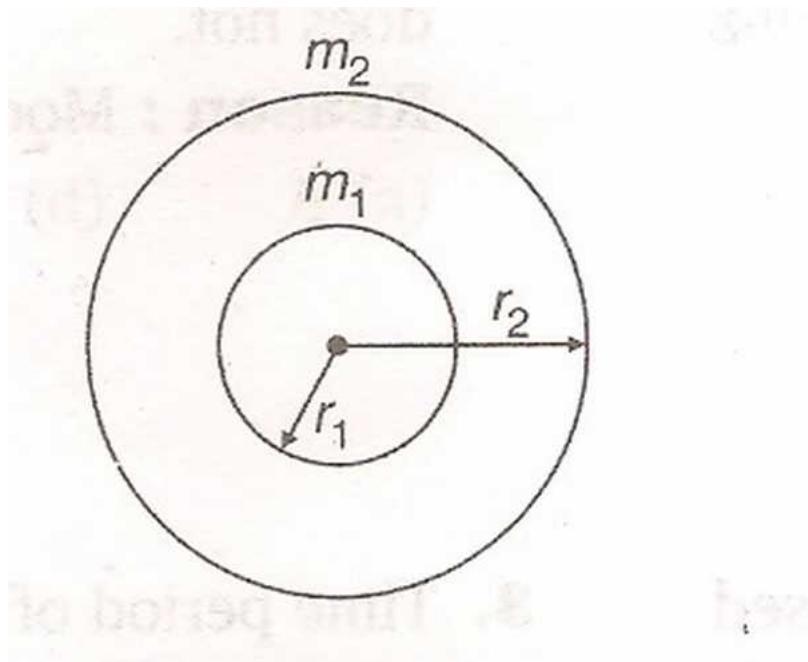


Multiple Answer

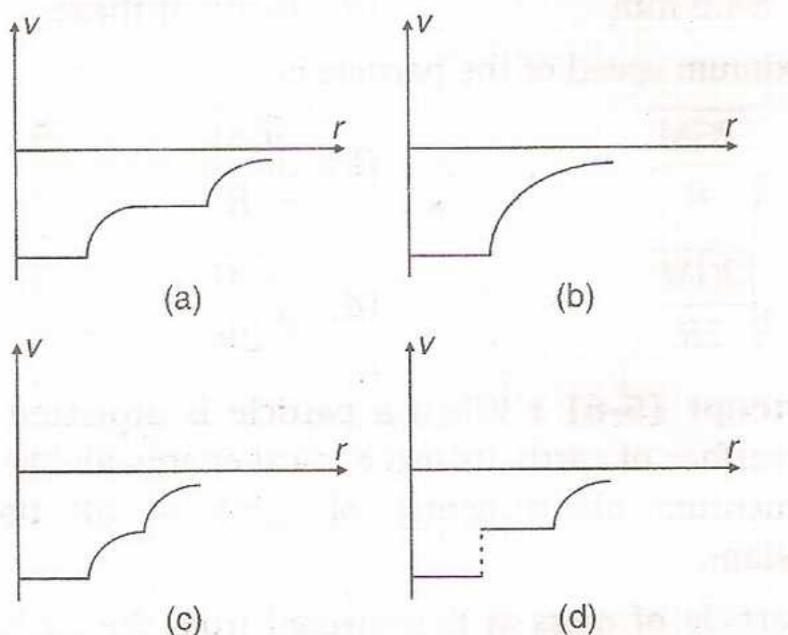
Example: Two concentric spherical shells are as shown in figure. The magnitude of gravitational potential (V) and field strength (E) vary with distance (r) from centre as



7.5.2.2 Concept 1 Gravitational potential inside a spherical shell is constant and outside the shell it varies as $V \propto \frac{1}{r}$ (with negative sign). Here r is the distance from centre.



Example1: Two concentric spherical shells are as shown in figure. The V-r graph will be as



Statement : 1: Gravitational Field at distance x from the centre on the axis of a ring is given as

$$E = \frac{Gmx}{(R^2 + x^2)^{3/2}}$$

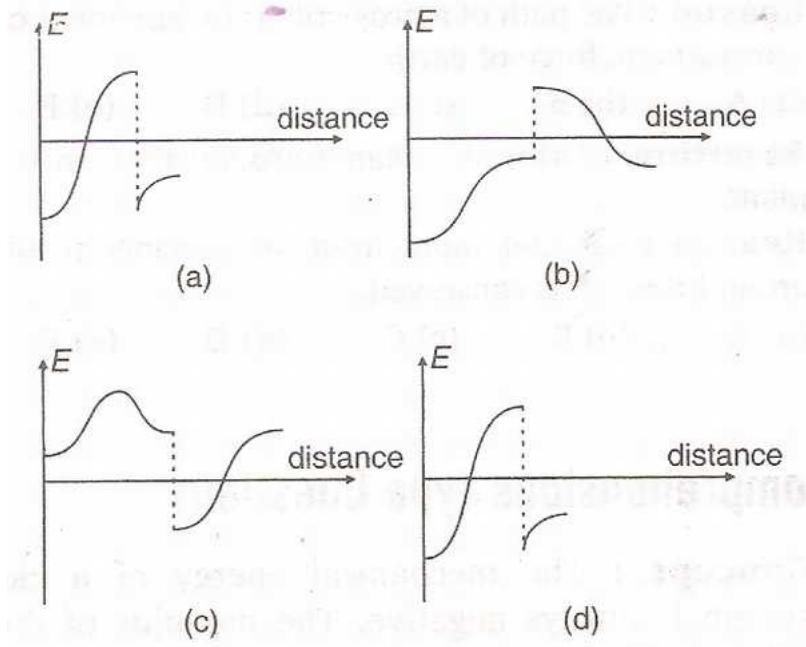
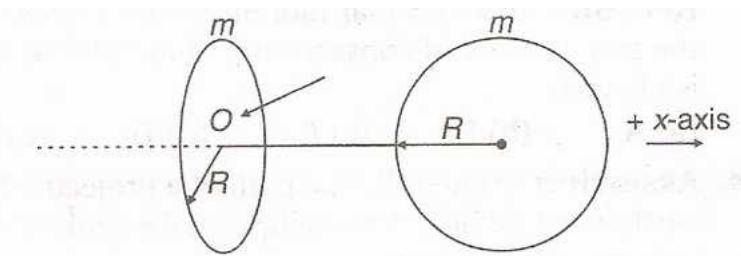
Here m is the mass of ring and R its radius.

2: Gravitational field at distance $x (\geq R)$ from centre of a solid sphere is given as,

$$E = \frac{Gm}{x^2}$$

Here m is the mass of solid sphere.

Example2: One ring of radius R and mass m and one solid sphere of same mass m and same radius R are placed with their centres on positive x-axis. We are moving from some finite distance on negative x-axis towards positive x-axis. Plane of the ring is perpendicular to x-axis. How will the net gravitational field vary with distance moved on x-axis. We move only up to surface of solid sphere. O is the origin,



Statement: Change in potential energy when a mass m is taken to a height h from the surface of earth is given by

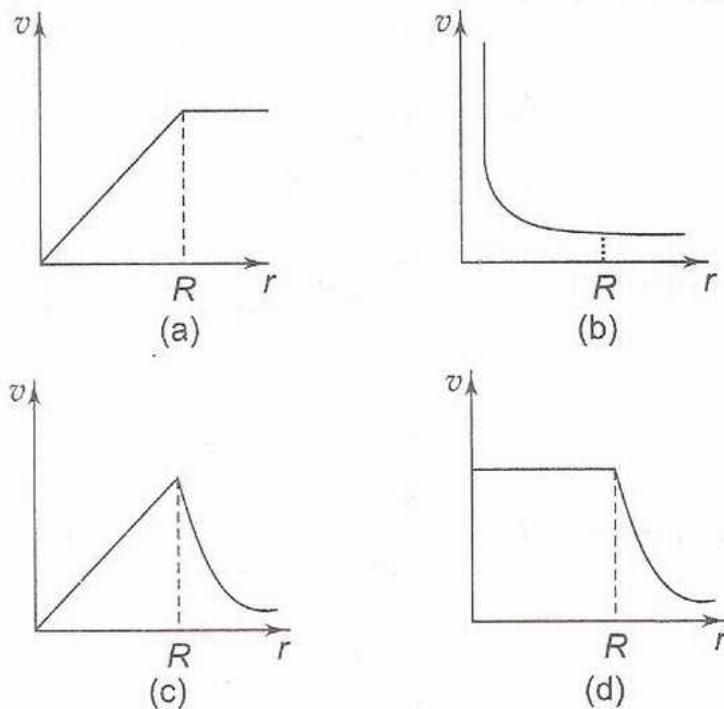
$$\Delta U = \frac{mgh}{1 + h/R}$$

7.5.2.3 Previous Years IIT Problems

Single Answer A spherically symmetric gravitational system of particles has a mass density

$$\rho = \begin{cases} \rho_o & \text{for } r \leq R \\ 0 & \text{for } r > R \end{cases}$$

where ρ_o is a constant. A test mass can undergo circular motion under the influence of the gravitational field of particles. Its speed v as a function of distance r ($0 < r < \infty$) from the centre of the system is represented by



{Solution: If M is the total mass of the system of particles, the orbital speed of the test mass is

$$v = \sqrt{\frac{GM}{r}}$$

$$\text{For } r \leq R, v = \sqrt{\frac{G \times \frac{4\pi}{3} r^3 \rho_o}{r}} \text{ which gives } v \propto r$$

i.e. v increases linearly with r up to $r=R$. Hence choices b) and d) are wrong.

For $r > R$, the whole mass of the system is $M = \frac{4\pi}{3} R^3 \rho_o$, which is constant. Hence for $r > R$,

$$v = \sqrt{\frac{GM}{r}}$$

i.e., $v \propto \frac{1}{\sqrt{r}}$. Hence, the correct choice is c). }

Column I

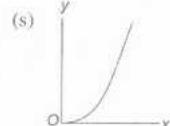
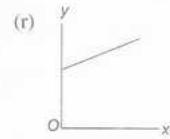
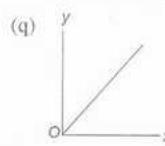
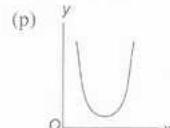
- (a) Potential energy of a simple pendulum (y axis) as a function of displacement (x axis)

- (b) Displacement (y axis) as a function of time (x axis) for a one dimensional motion at zero or constant acceleration when the body is moving along the positive x -direction

- (c) Range of a projectile (y axis) as a function of its velocity (x axis) when projected at a fixed angle

- (d) The square of the time period (y-axis) of a simple pendulum as a function of its length (x axis).

Column II

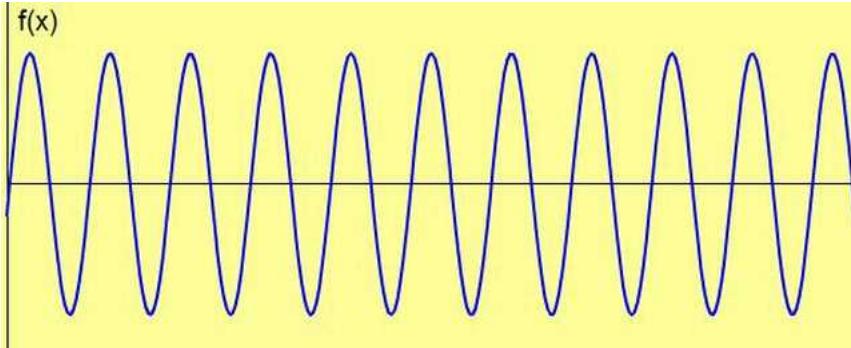


7.6 Periodic Motion

7.6.1 Abstract Introduction (SHM)

7.6.1.1 Position vs time

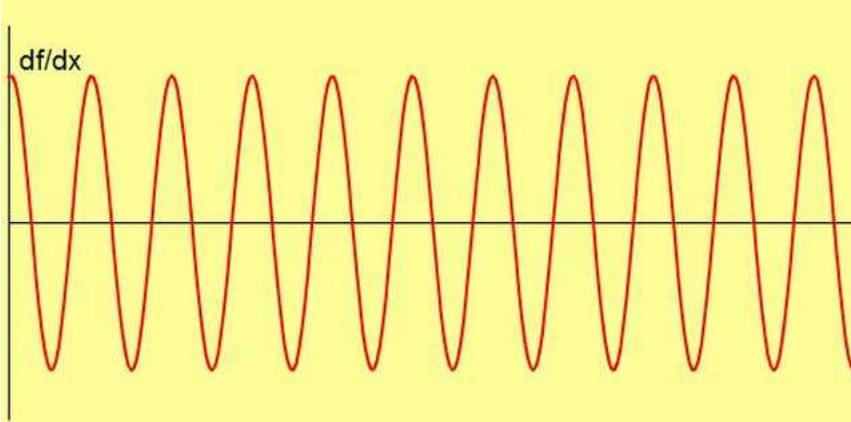
The graph of position verse time is a sine wave with a possible phase shift. The phase shift is how much the ahead or behind the position is on the sine wave. n.d.



Consider this graph, if the "clock" is started at 0.05 (where the mass is at it's maximum stretch) seconds then there would be a phase shift of 90 degrees (or we could replace the sine function for a cosine). If the "clock" is started at 0.1 seconds (where mass moves down instead of up; left instead of right) then the phase shift would be 180 degrees (a negative sine function).

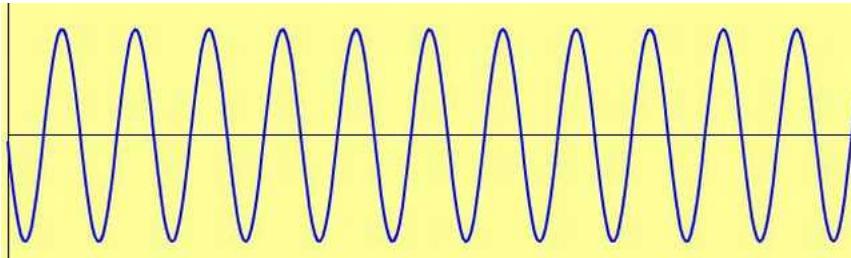
7.6.1.2 Velocity vs time

Consider the position verses time graph, at any point were the mass has reached the amplitude (maximum distance from from the equilibrium point) the speed of the mass at these point is zero. When the position is at zero then the speed is at a maximum (if you don't believe it, consider conservation of energy). This "shifts" the position graph by 90 degrees "creating" a cosine graph for velocity. The other way of thinking about is velocity is the change in position with respect to time, the change in a sine wave with respect to time is a cosine graph.



7.6.1.3 Acceleration vs time

The acceleration verse time graph is the easiest of the graphs to make. The simple harmonic motion is based on a relationship between position and acceleration; $x = -Ka$. So the graph of position and acceleration should look alike, except for the negative sign. In fact position and acceleration are the same shape just mirror copies of each other.

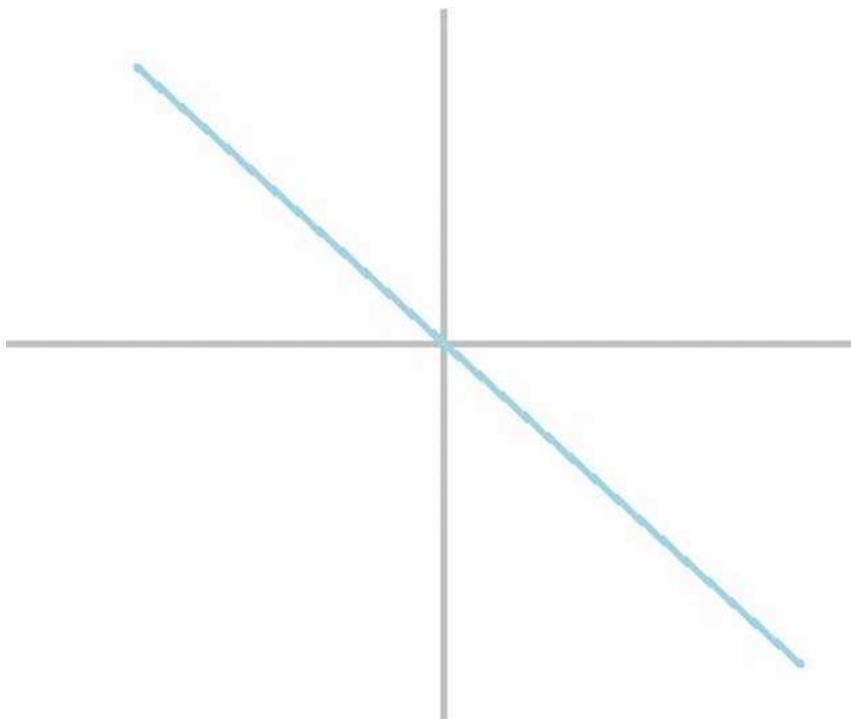


7.6.1.4 Peak Height

It is important to note that the shapes of each graph is similar, that each graph has the same frequency, period and wavelength, but they don't have the same amplitude, for common simple harmonic motion, the height the peak of the function (not to confused with the amplitude, amplitude refers to the height of the position graph alone, i'm talking about the height of the position, velocity and acceleration graphs) tend to get smaller and smaller starting with the position graph being the tallest and the acceleration being the shortest.

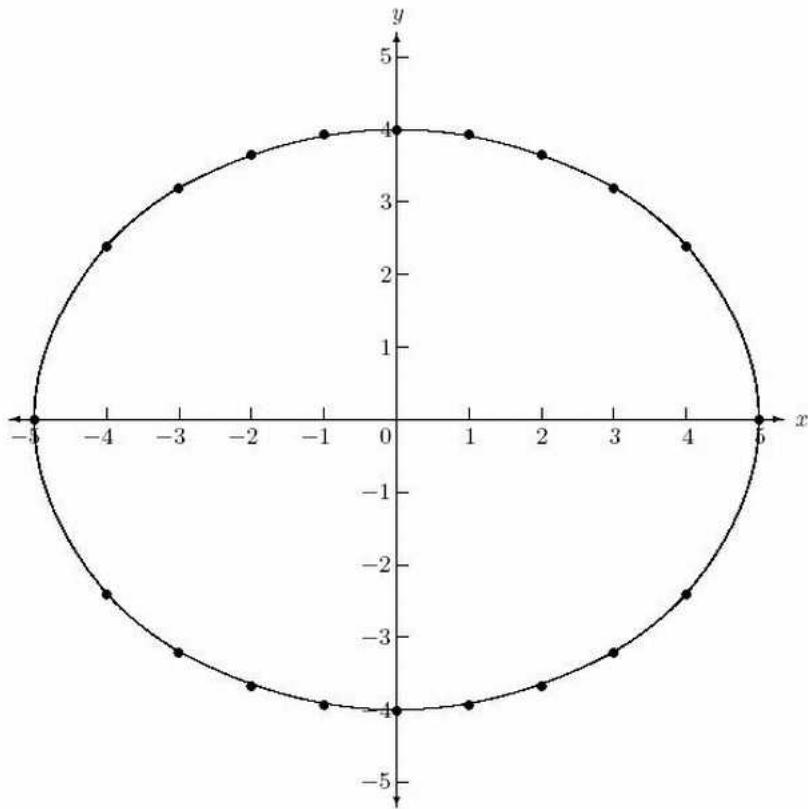
7.6.1.5 Graphing position, velocity, and acceleration with respect to each other

Graphing position to the other functions can be complicated and when tested on it, most student are unable to give the right answer. First consider this, for simple harmonic motion position and acceleration are proportional. $x = -k a$ this is a linear relationship so the graph is a line, the slope is negative so the line is heading down.



7.6.1.6 Position and acceleration verses velocity

The position and acceleration verse velocity graph look entirely different. First off the straight line test fails when plotting position vs velocity or acceleration verse velocity. Take the point were x and a are zero, there are two possible answers for the point (the speed maybe at maximum) the object could be moving down or up at the point. That means the velocity can be a positive maximum or a negative maximum, two separate values. Looking at the points were the velocity is equal to zero, there are two possible answers, either at the top or at the bottom. If you continue to plot data points graph that is developed is a ellisipe

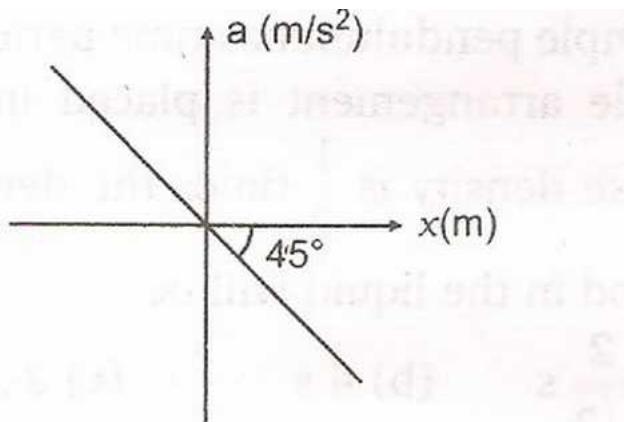


7.6.2 Problems

7.6.2.1 General Problem Set

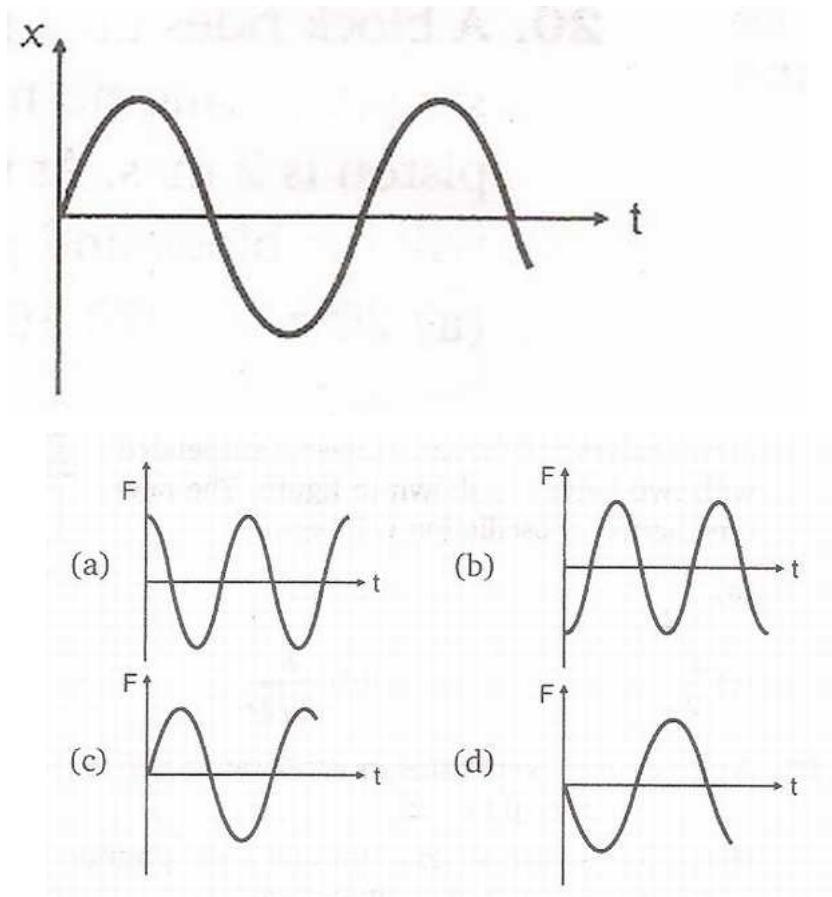
Single Answer Type

Example 1: Acceleration-displacement graph of a particle executing SHM is as shown in given figure. The time period of its oscillation is (in sec)

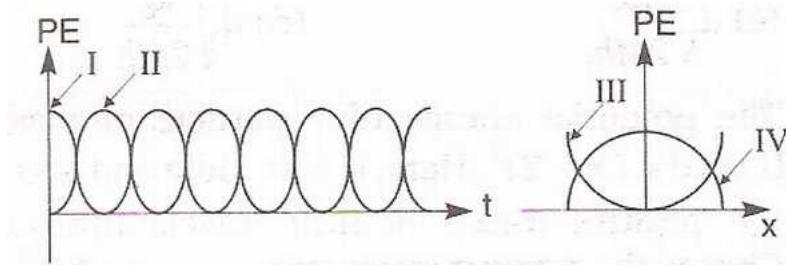


- a) $\pi/2$
- b) 2π
- c) π
- d) $\pi/4$

Example 2: Displacement-time graph of a particle executing SHM is as shown.



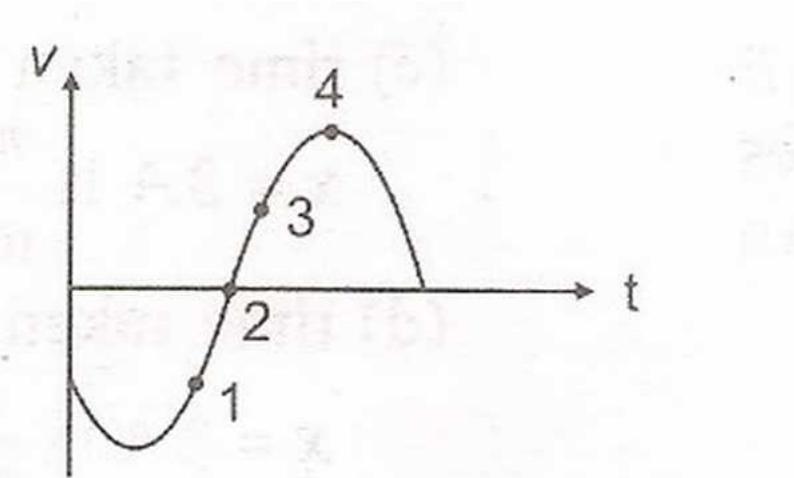
Example 3: For a particle executing SHM the displacement x is given by $x = A \cos \omega t$. Identify the graph which represents the variation of potential energy (PE) as a function of time t and displacement x



- a) I, III
- b) II, IV
- c) II, III
- d) I, IV

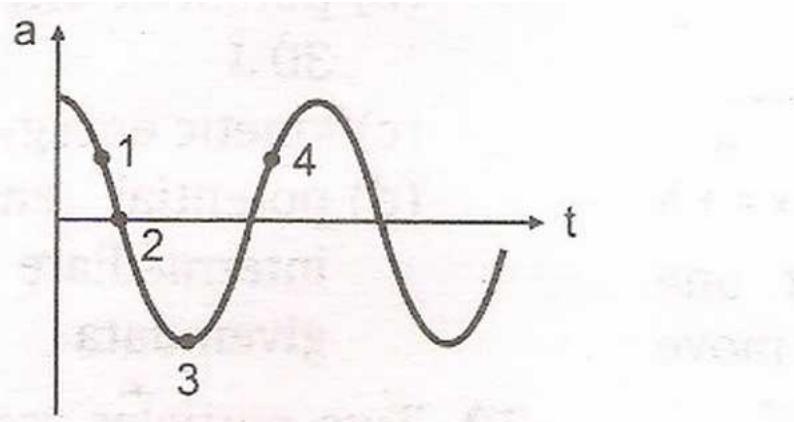
Multiple Answer Type

Example 1: Velocity-time graph of a particle executing SHM is shown in figure. Select the correct alternative(s).



- a) At position 1 displacement of particle may be positive or negative.
- b) At position 2 displacement of particle is negative.
- c) At position 3 acceleration of particle is positive.
- d) At position 4 acceleration of particle is positive.

Example 2: Acceleration-time graph of a particle executing SHM is as shown in figure. Select the correct alternative(s).



- a) Displacement of particle at 1 is negative.
- b) Velocity of particle at 2 is positive.
- c) Potential energy of particle at 3 is maximum.
- d) Speed of particle at 3 is decreasing.

Matching Type Questions

Example 1: Velocity-time graph of a particle in SHM is as shown in figure. Match the following

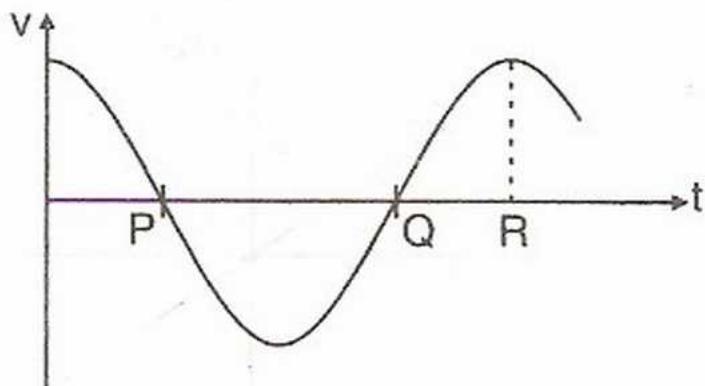


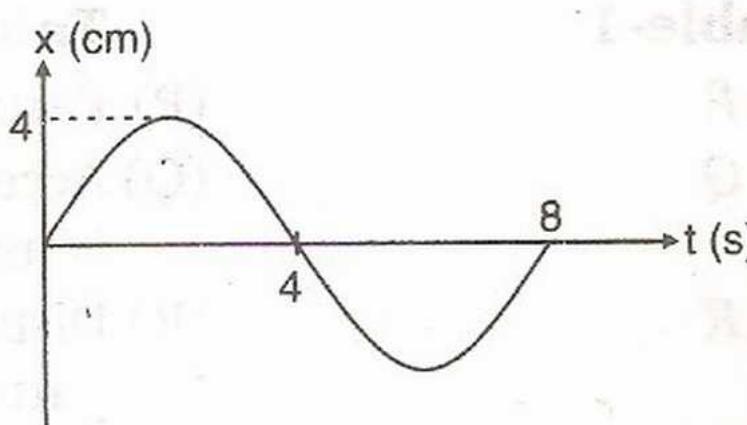
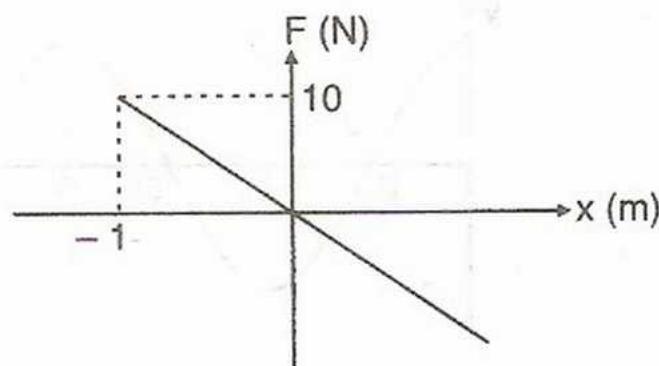
Table-1

- (A) At P
 (B) At Q
 (C) At R

Table-2

- (P) Particle is at $x = -A$
 (Q) Acceleration of particle is maximum
 (R) Displacement of particle is zero
 (S) Acceleration of particle is zero
 (T) None

Example 2: F-x and x-t graph of a particle in SHM are as shown in figure. Match the following

**Table-1**

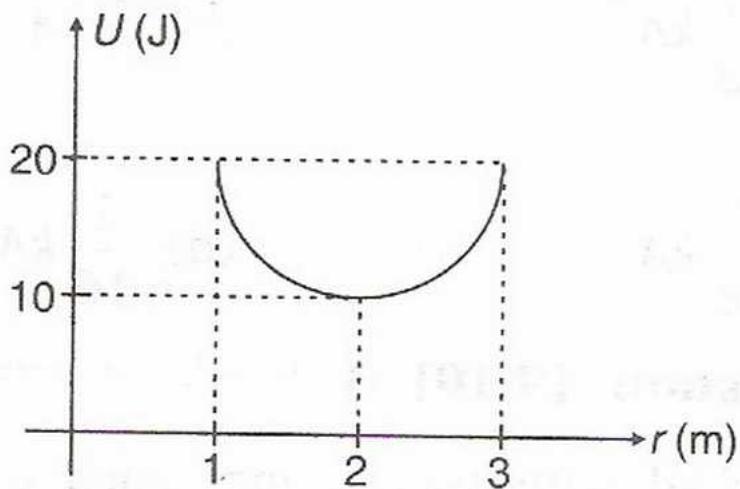
- (A) Mass of the particle (P) $\pi/2$ SI unit
 (B) Maximum kinetic energy of particle (Q) $(160/\pi^2)$ SI unit
 (C) Angular frequency of particle (R) (8.0×10^{-3}) SI unit
 (T) None

Table-2

Comprehension Type Questions

Comprehension 1 Concept : In SHM, force $\left(F = -\frac{dU}{dr}\right)$ on the particle at mean position is zero. Potential energy at extreme position is maximum

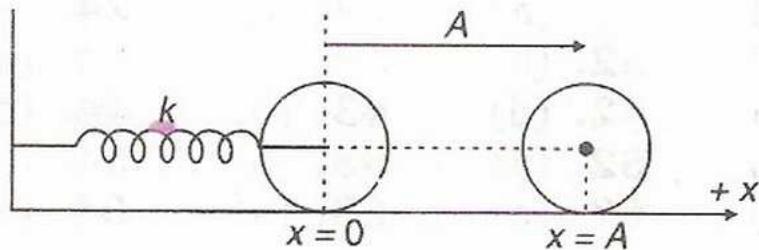
Example: U-r graph of a particle which can be under SHM is as shown in figure. What conclusion cannot be drawn from the graph?



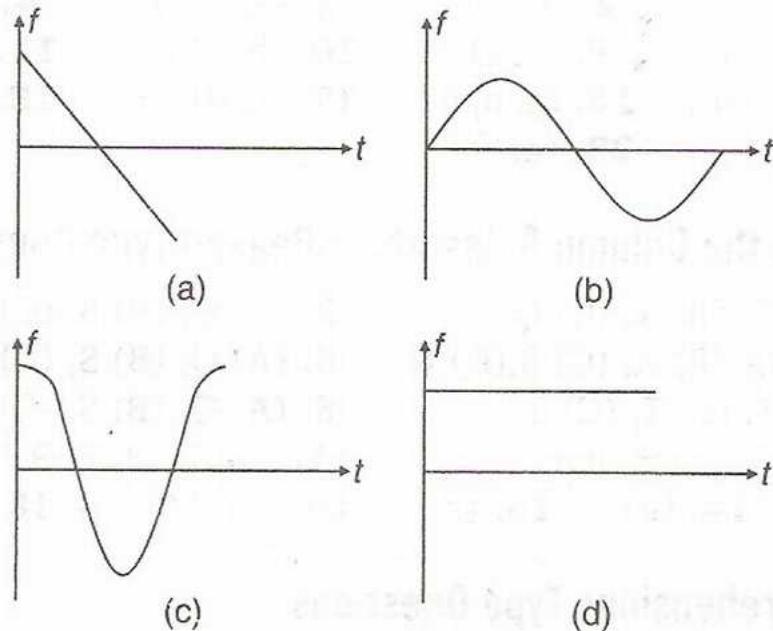
- a) Mean position of the particle is at $r=2\text{ m}$.
- b) Potential energy of particle at mean position is 10 J.
- c) Amplitude of oscillation is 1 m.
- d) None of these.

Comprehension 2 Statement : In case of pure rolling $a = R\alpha$, where a is the linear acceleration and α the angular acceleration.

Question : A disc of mass m and radius R is attached with a spring of force constant k at its centre as shown in figure. At $x=0$, spring is unstretched. The disc is moved to $x=A$ and then released. There is no slipping between disc and ground. Let f be the force of friction on the disc from the ground.



Example 1 : f versus t (time) graph will be as



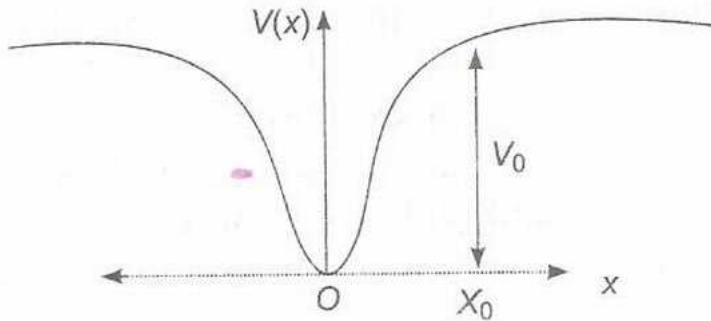
Example 2 : In the problem if $k = 10 \text{ N/m}$, $m = 2 \text{ kg}$, $R = 1 \text{ m}$ and $A = 2 \text{ m}$. Find linear speed of the disc at mean position

- a) $\sqrt{\frac{40}{3}}$ m/s
- b) $\sqrt{20}$ m/s
- c) $\sqrt{\frac{10}{3}}$ m/s
- d) $\sqrt{\frac{50}{3}}$ m/s

7.6.2.2 Previous Years IIT Problems

Paragraph

Paragraph 1: When a particle of mass m moves on the x -axis in a potential of the form $V(x) = kx^2$ it performs simple harmonic motion. The corresponding time period is proportional to $\sqrt{\frac{m}{k}}$, as can be seen easily using dimensional analysis. However, the motion of a particle can be periodic even when its potential energy increases on both sides of $x=0$ in a way different from kx^2 and its total energy is such that the particle does not escape to infinity. Consider a particle of mass m moving on the x -axis. Its potential energy is $V(x) = \alpha x^4$ ($\alpha > 0$) for $|x|$ near the origin and becomes a constant equal to V_0 for $|x| \geq X_0$ (see figure)



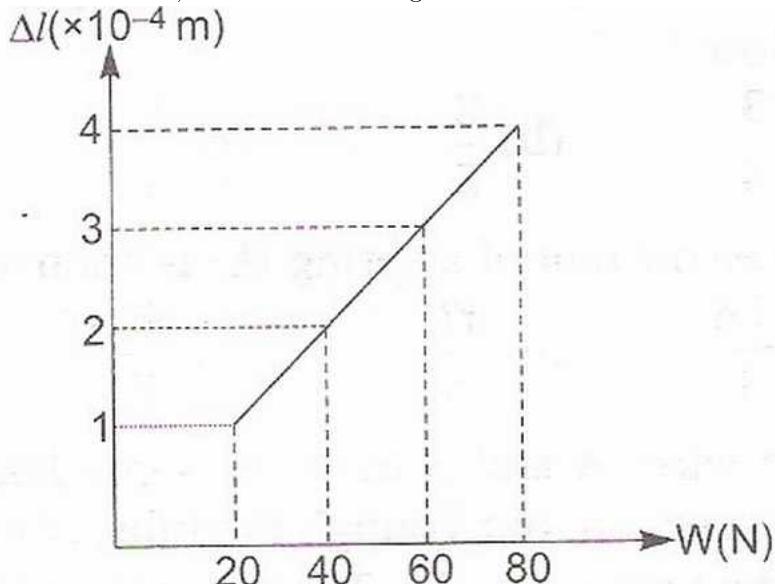
1. If the total energy of the particle is E , it will perform periodic motion only if
 - a) $E < 0$
 - b) $E > 0$
 - c) $V_0 > E > 0$
 - d) $E > V_0$
2. For periodic motion of small amplitude A , the time period T of this particle is proportional to
 - a) $A \sqrt{\frac{m}{\alpha}}$
 - b) $\frac{1}{A} \sqrt{\frac{m}{\alpha}}$
 - c) $A \sqrt{\frac{\alpha}{m}}$
 - d) $\frac{1}{A} \sqrt{\frac{\alpha}{m}}$
3. The acceleration of this particle for $|x| > X_0$ is
 - a) Proportional to V_0
 - b) Proportional to V_0/mX_0
 - c) Proportional to $\sqrt{V_0/mX_0}$
 - d) Zero

7.7 Statics

7.7.1 Modulii of Elasticity

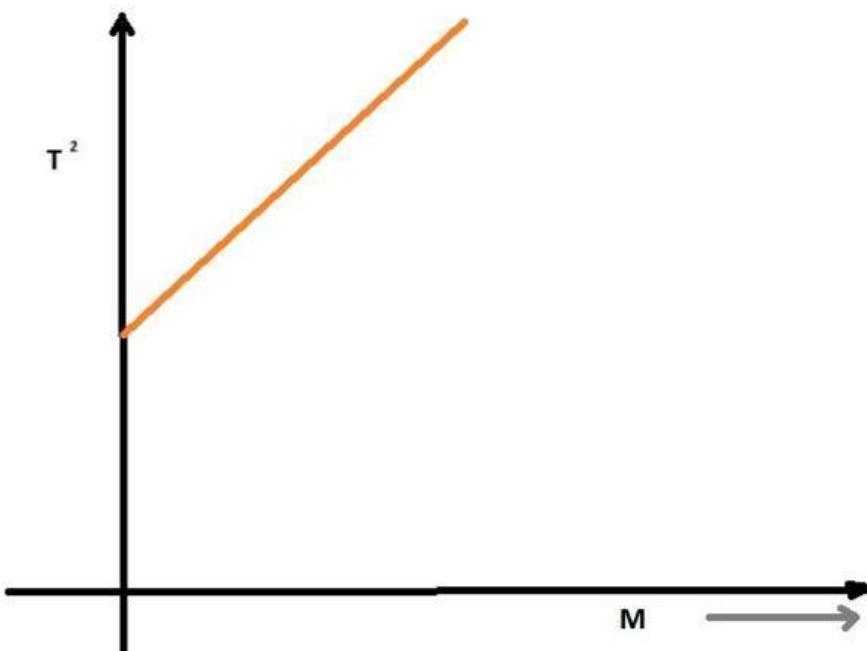
7.7.1.1 General Problem Set

Single Answer Type Example 1 : The graph shows the extension (Δl) of a wire of length 1.0m suspended from the top of a roof at one end and with a load W connected to the other end. If the cross-sectional area of the wire is $10^{-6} m^2$, calculate the Young's modulus of the material of the wire



- a) $2 \times 10^{11} N/m^2$
- b) $2 \times 10^{10} N/m^2$
- c) $2 \times 10^{12} N/m^2$
- d) $2 \times 10^{13} N/m^2$

Example : The graph shown was obtained from experimental measurements of the period of oscillation T for different masses M placed in the scale pan on the lower end of the spring balance. The most likely reason for the line not passing through the origin is that the



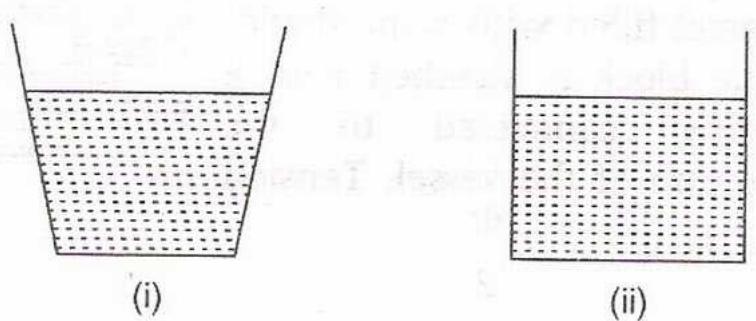
- (a) Spring did not obey Hooke's law
- (b) Amplitude of oscillation was too large
- (c) Clock used needed regulating
- (d) Mass of the pan was neglected

{ Hint: Answer D }

Comprehension Type

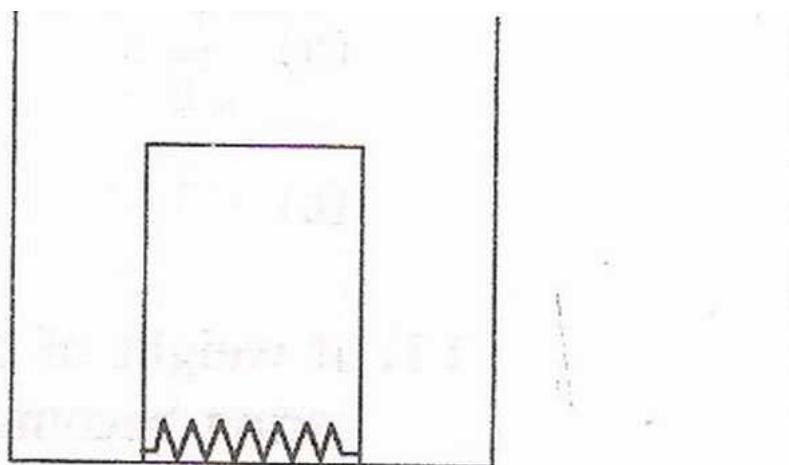
Comprehension 1 Concept : Free body diagram of liquid can be drawn in similar manner as we draw the free body diagram of a solid. The only difference is, what we call the normal reaction between solid-solid boundary, we here call it pressure X area in case of liquids. Both are perpendicular to the surface.

Example : Equal amounts of liquid are filled in two vessels of different shapes as shown in figure. Let F_1 be the force by the base on liquid in case (i) and F_2 in case (ii) Then

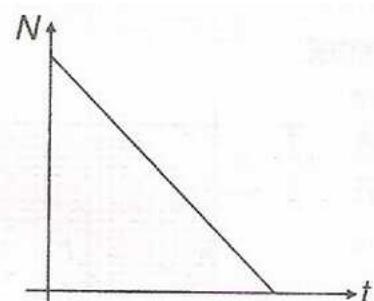


- a) $F_1 > F_2$
- b) $F_1 < F_2$
- c) $F_1 = F_2$
- d) Data insufficient

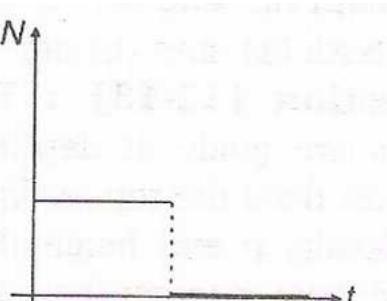
Question : A cube (side = 10 cm) of density 0.5 g/cm^3 is placed in a vessel of base area $20 \text{ cm} \times 20 \text{ cm}$. A liquid of density 1.0 g/cm^3 is gradually filled in the vessel at a constant rate $Q = 50 \text{ cm}^3/\text{s}$.



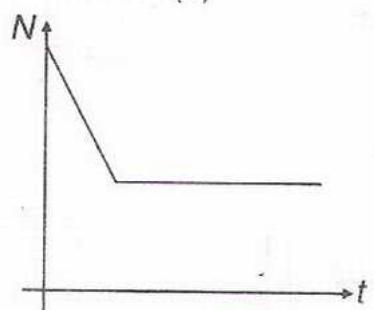
Example : If we plot a graph between the normal reaction on cube by the vessel versus time. The graph will be like



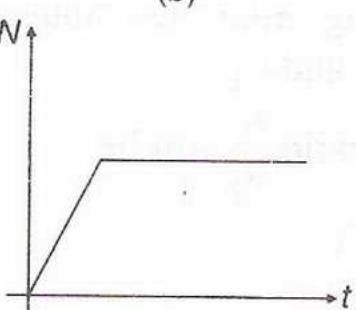
(a)



(b)



(c)



(d)

Example: The cube will leave contact with the vessel after time $t = \dots\dots\dots$ s

- a) 30
- b) 40
- c) 60
- d) 20

Bibliography

- (2017). URL: <http://hyperphysics.phy-astr.gsu.edu/hbase/frict2.html>.
- (N.d.). URL: <http://physicsnet.co.uk/a-level-physics-as-a2/further-mechanics/momentum-concepts/>.
- (N.d.). URL: http://p3server.pa.msu.edu/coursewiki/doku.php?id=183_notes:impulsegraphs.
- (N.d.). URL: <http://www.bbc.co.uk/education/guides/z9499j6/revision/5>.
- (N.d.). URL: Reference%20<https://www.physicsforums.com/threads/force-time-graph-with-respect-to-momentum.185149/>.
- (N.d.). URL: <http://www.physicstutorials.org/home/impulse-momentum/impulse>.
- (N.d.). URL: <http://dev.physicslab.org/DocumentPrint.aspx?doctype=5&filename=WorkEnergy-ForceDisplacementGraphs.xml>.
- (N.d.). URL: <http://ipodphysics.com/harmonic-graphing-position-velocity-acceleration.php>.

Chapter 8

Heat

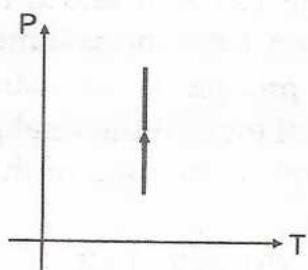
8.1 Thermodynamics

8.1.1 Practice Problems

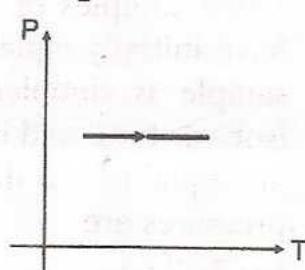
8.1.1.1 General Problem Set

Single Answer Type

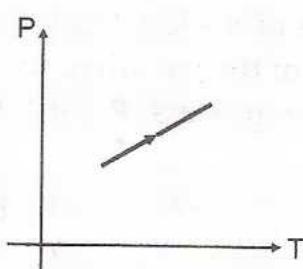
Example : Pressure versus temperature graphs of an ideal gas are as shown in figure. Choose the wrong statement



(i)



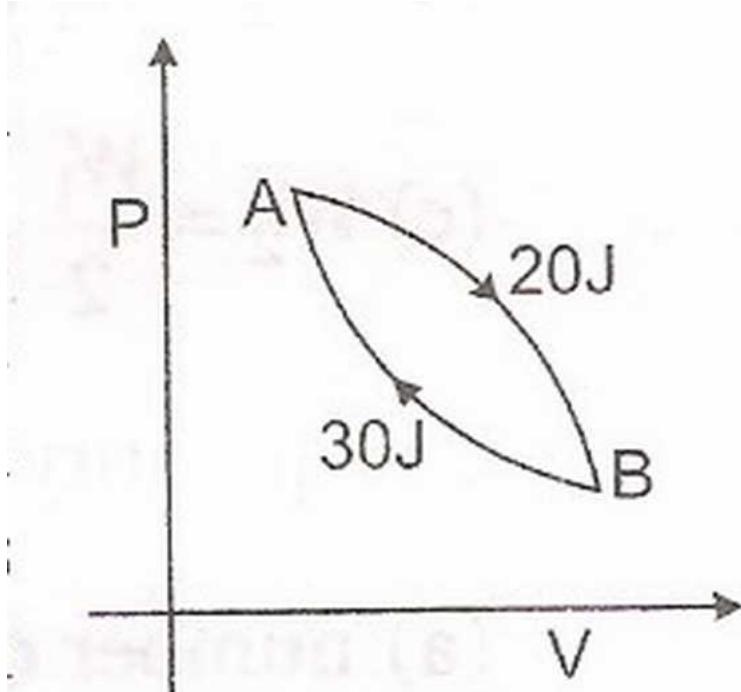
(ii)



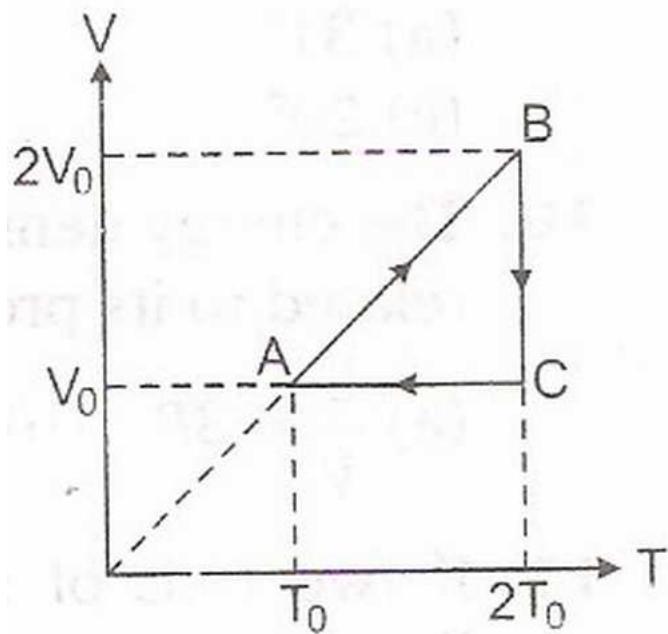
(iii)

- a) Density of gas is increasing in graph (i)
- b) Density of gas is decreasing in graph (ii)
- c) Density of gas is constant in graph (iii)
- d) None of the above

Example : In a cyclic process shown in the figure an ideal gas is adiabatically taken from B to A, the work done on the gas during the process B->A is 30J, when the gas is taken from A->B the heat absorbed by the gas is 20 J. The change in internal energy of the gas in the process A-> B is

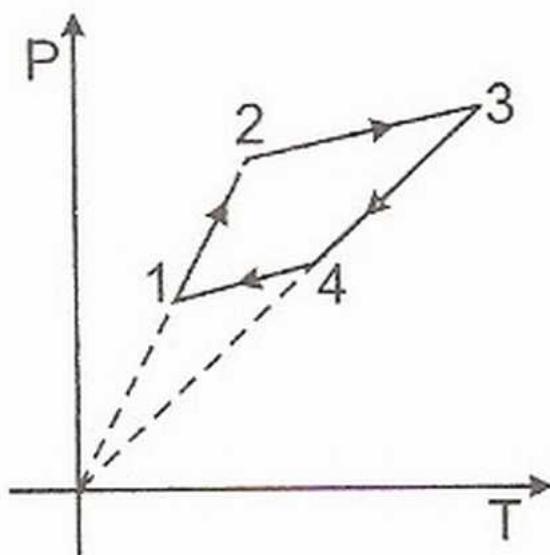


Example : An ideal monoatomic gas undergoes a cyclic process ABCA as shown in the figure. The ratio of heat absorbed during AB to the work done on the gas during BC is



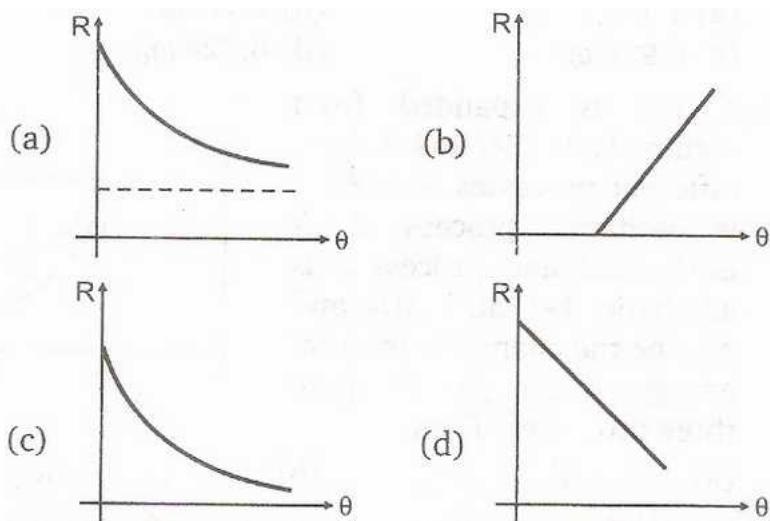
- a) $5/2\ln 2$
- b) $5/3$
- c) $5/4\ln 2$
- d) $5/6$

Example : Three moles of an ideal monoatomic gas performs a cycle 1->2->3->4->1 as shown. The gas temperatures in different states are $T_1=400K$, $T_2=800K$, $T_3=2400K$ and $T_4=1200K$. The work done by the gas during the cycle is (2-3 and 4-1 are isobaric)

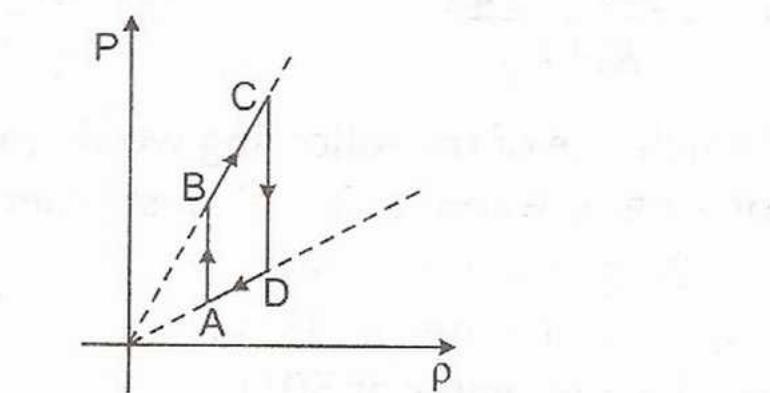


- a) 1200 R
- b) 3600 R
- c) 2400 R
- d) 2000 R

Example : Temperature of a body θ is slightly more than the temperature of the surrounding θ_o . Its rate of cooling (R) versus temperature of body (θ) is plotted, its shape would be

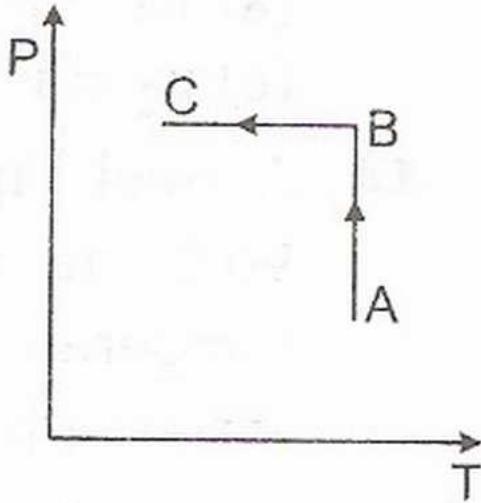


Example : Pressure versus density graph of an ideal gas is shown in figure



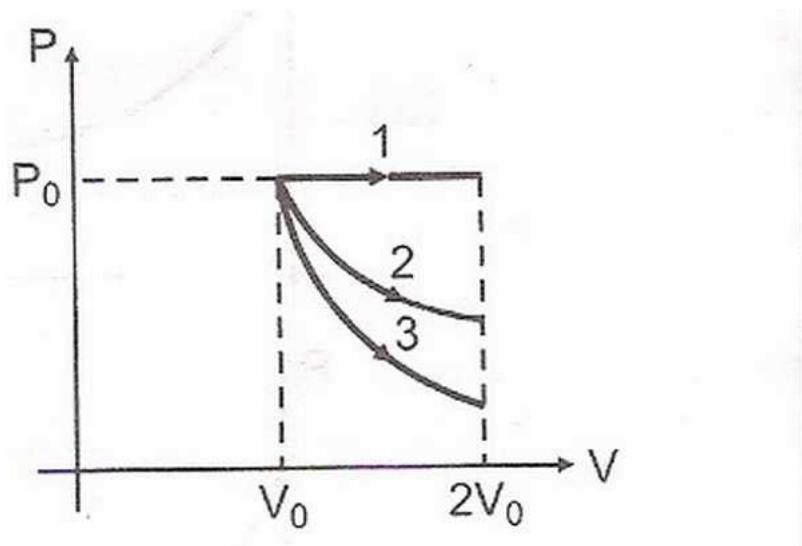
- a) during the process AB work done by the gas is positive
- b) during the process AB work done by the gas is negative
- c) during the process BC internal energy of the gas is increasing
- d) None of the above

Example : Ideal gas is taken through the process shown in the figure



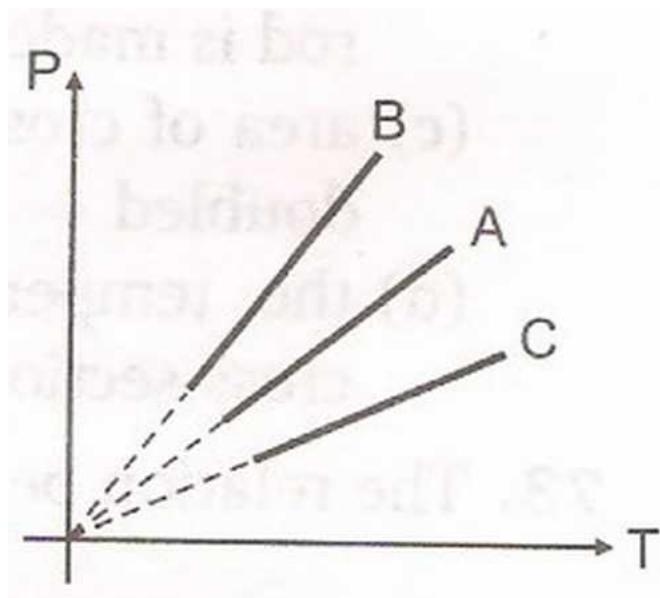
- a) In process AB, work done by system is positive
- b) In process AB, heat is rejected
- c) In process AB, internal energy increases
- d) In process AB internal energy decreases and in process BC, internal energy increases

Example : A gas is expanded from volume V_0 to $2V_0$ under three different processes. Process 1 is isobaric, process 2 is isothermal and process 3 is adiabatic. Let $\Delta U_1, \Delta U_2$ and ΔU_3 be the change in internal energy of the gas in these three processes. Then



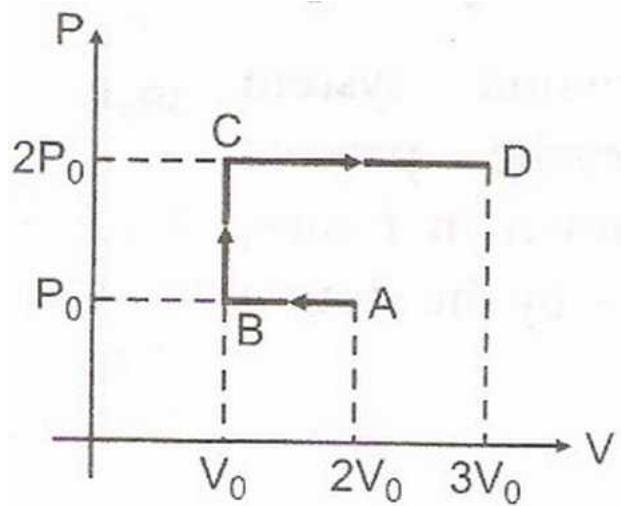
- a) $\Delta U_1 > \Delta U_2 > \Delta U_3$
- b) $\Delta U_1 < \Delta U_2 < \Delta U_3$
- c) $\Delta U_2 < \Delta U_1 < \Delta U_3$
- d) $\Delta U_2 < \Delta U_3 < \Delta U_1$

Example : Pressure versus temperature graph of an ideal gas at constant volume V is shown by the straight line A. Now mass of the gas is doubled and the volume is halved, then the corresponding pressure versus temperature graph will be shown by the line



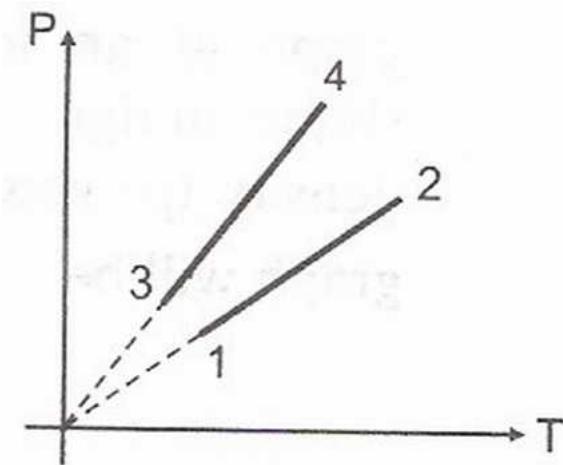
- a) A
- b) B
- c) C
- d) None of these

Example : P-V diagram of an ideal gas is as shown in figure. Work done by the gas in the process ABCD is



- a) $4P_0V_0$
- b) $2P_0V_0$
- c) $3P_0V_0$
- d) P_0V_0

Example : Pressure versus temperature graph of an ideal gas of equal number of moles of different volumes are plotted as shown in figure. Choose the correct alternatives.



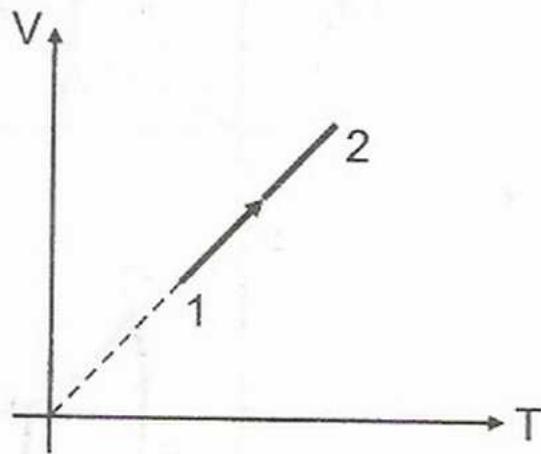
a) $V_1 = V_2, V_3 = V_4$ and $V_2 > V_3$

b) $V_1 = V_2, V_3 = V_4$ and $V_2 < V_3$

c) $V_1 = V_2 = V_3 = V_4$

d) $V_4 > V_3 > V_2 > V_1$

Example : Volume versus temperature graph of two moles of helium gas is as shown in figure. The ratio of heat absorbed and the work done by the gas in process 1-2 is



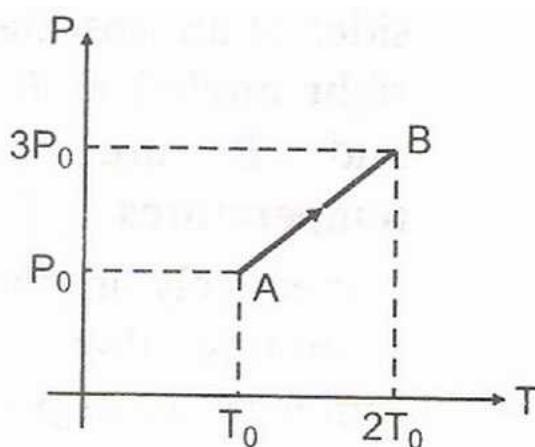
a) 3

b) $5/2$

c) $5/3$

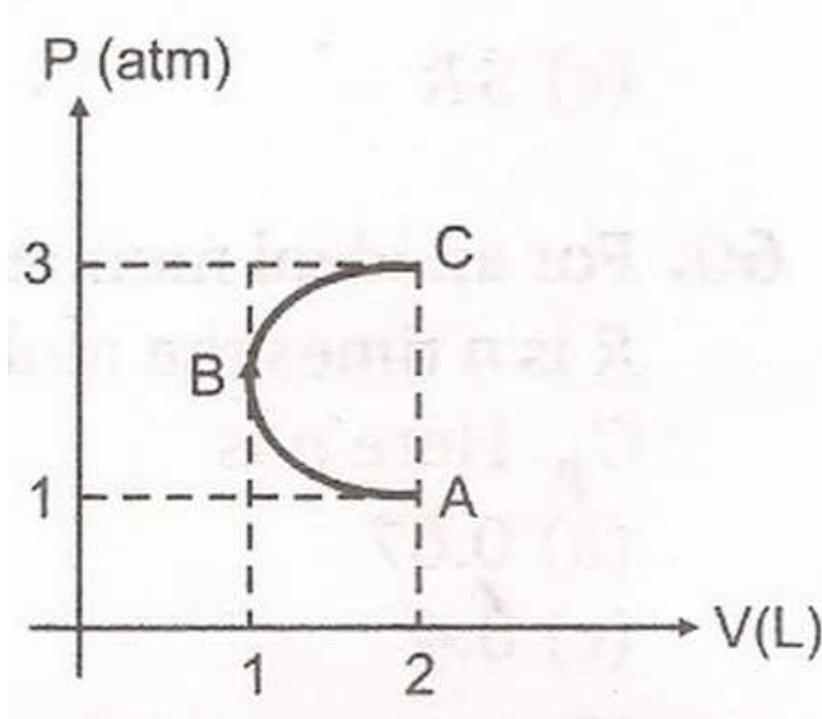
d) $7/2$

Example : Pressure versus temperature graph of an ideal gas is as shown in figure. Density of the gas at point A is ρ_o . Density at B will be



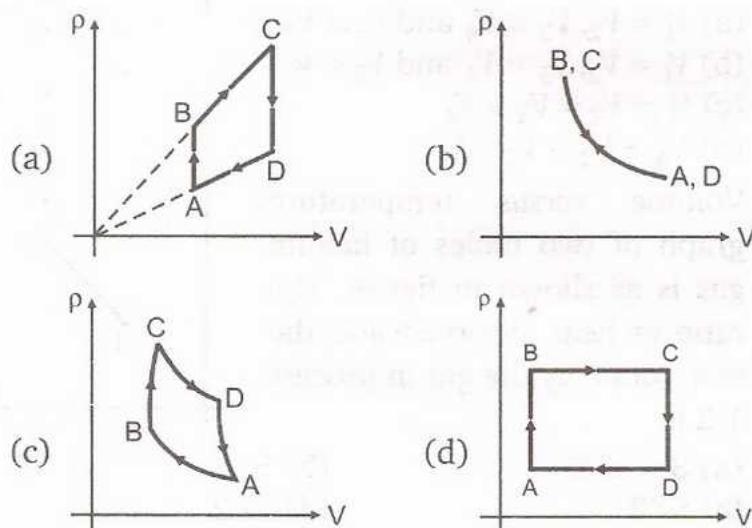
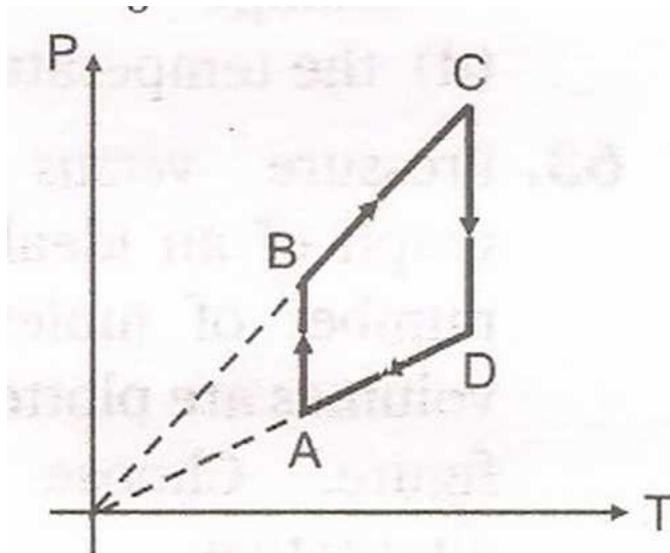
- a) $\frac{3}{4}\rho_o$.
- b) $\frac{3}{2}\rho_o$.
- c) $\frac{4}{3}\rho_o$.
- d) $2\rho_o$.

Example : In the P-V diagram shown in figure ABC is a semicircle. The work done in the process ABC is

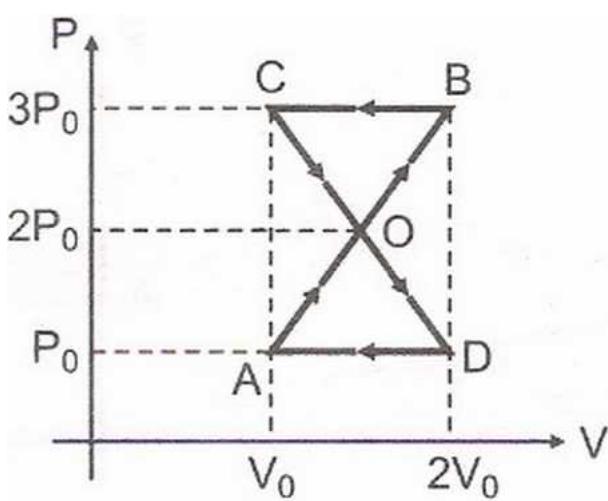


- a) zero
- b) $\frac{\pi}{2}$ atm-L
- c) $-\frac{\pi}{2}$ atm-L
- d) 4 atm-L

Example : Pressure versus temperature graph of an ideal gas is as shown in figure corresponding density (ρ) versus volume (V) graph will be

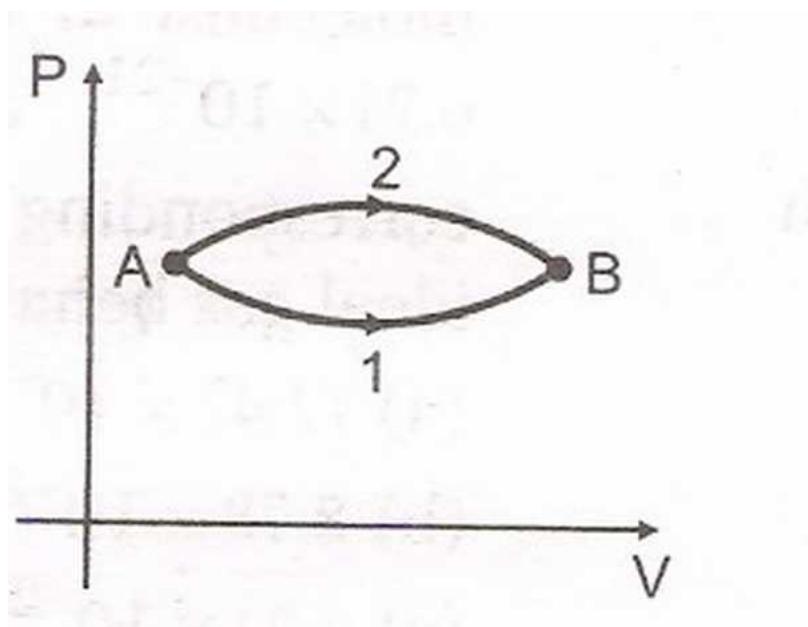


Example: A thermodynamic system undergoes cyclic process ABCDA as shown in figure. The work done by the system is



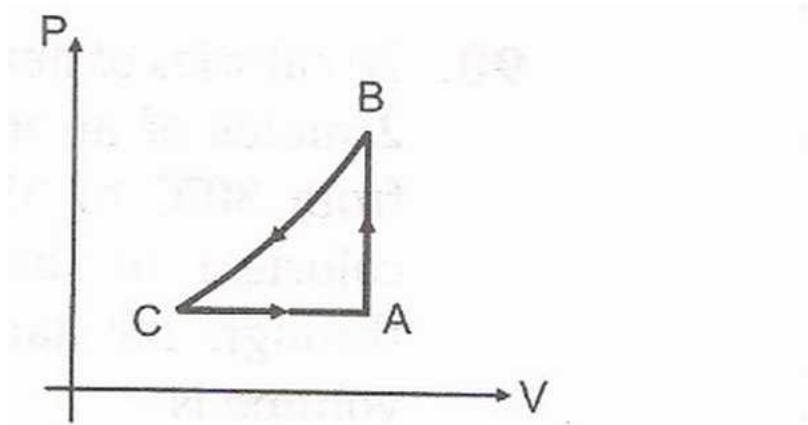
- a) $P_0 V_0$
- b) $2P_0 V_0$
- c) $P_0 V_0 / 2$
- d) zero

Example : The figure shows two paths for the change of state of a gas from A to B. The ratio of molar heat capacities in path 1 and path 2 is



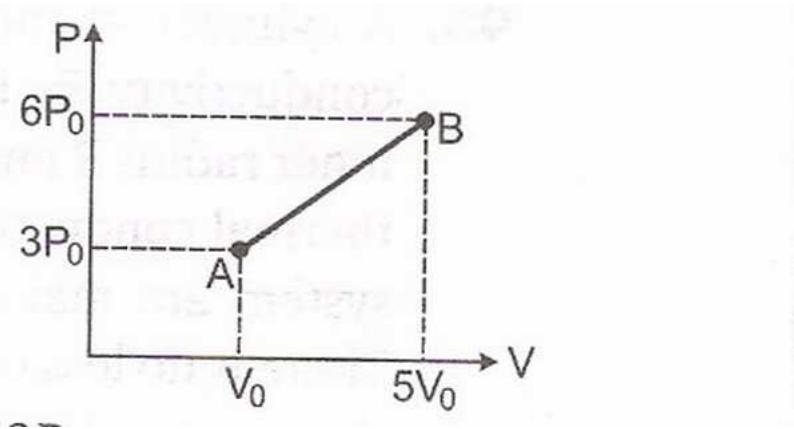
- a) > 1
- b) < 1
- c) 1
- d) Data insufficient

Example : A sample of an ideal gas is taken through a cycle as shown in figure. It absorbs 50 J of energy during the process AB, no heat during BC, rejects 70J during CA. 40 J of work is done on the gas during BC. Internal energy of gas at A is 1500 J, the internal energy at C would be



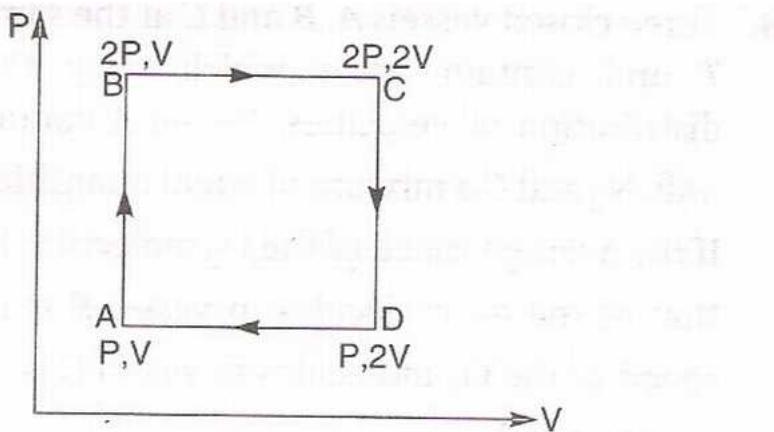
- a) 1590 J
- b) 1620 J
- c) 1540 J
- d) 1570 J

Example : One mole of a monoatomic ideal gas undergoes the process A \rightarrow B in the given P-V diagram. The specific heat for this process is



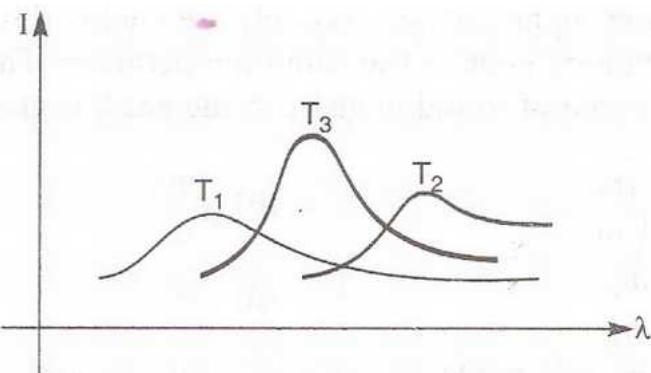
- a) $3R/2$
- b) $13R/6$
- c) $5R/2$
- d) $2R$

Example : An ideal monoatomic gas is taken round the cycle ABCDA as shown in the P-V diagram (see figure). The work done during the cycle is



- a) PV
- b) $2PV$
- c) $PV/2$
- d) zero

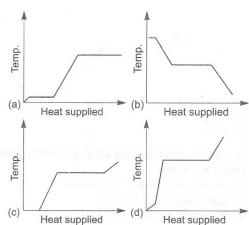
Example : The plots of intensity versus wavelength for three black bodies at temperatures T_1 , T_2 and T_3 respectively are as shown. Their temperatures are such that



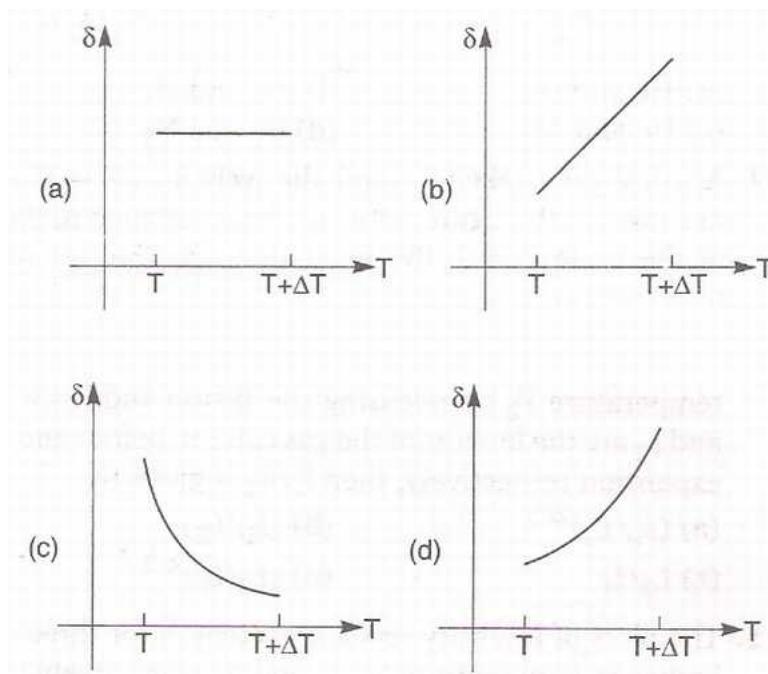
- a) $T_1 > T_2 > T_3$
- b) $T_1 > T_3 > T_2$

- c) $T_2 > T_3 > T_1$
d) $T_3 > T_2 > T_1$

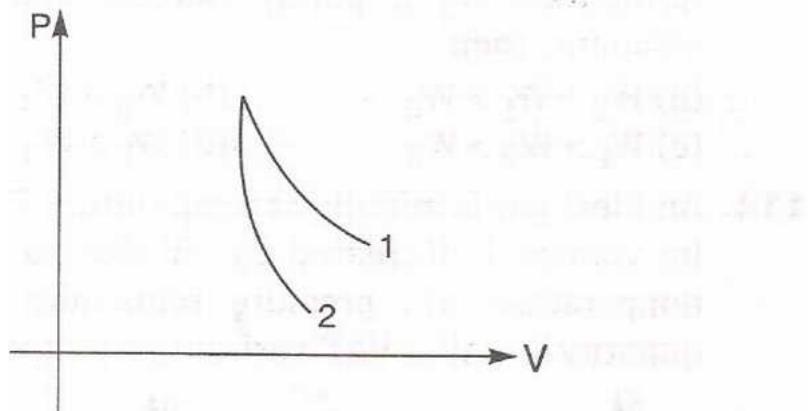
Example : A block of ice at -10°C is slowly heated and converted to steam at 100°C . Which of the following curves represents the phenomenon?



Example : An ideal gas is initially at temperature T and volume V . Its volume is increased by ΔV due to an increase in temperature ΔT , pressure remaining constant. The quantity $\delta = \frac{\Delta V}{V\Delta T}$ varies with temperature as

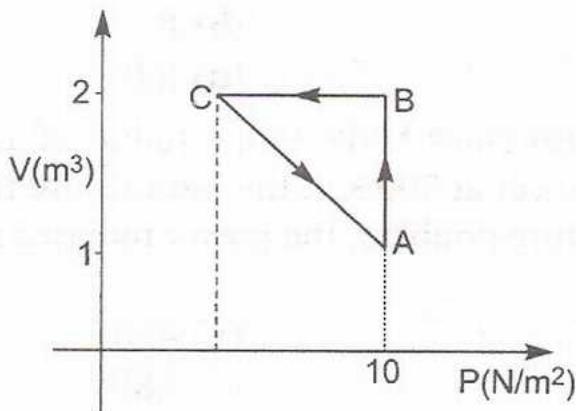


Example : P-V plots for two gases during adiabatic processes are shown in the figure. Plots 1 and 2 should correspond respectively to



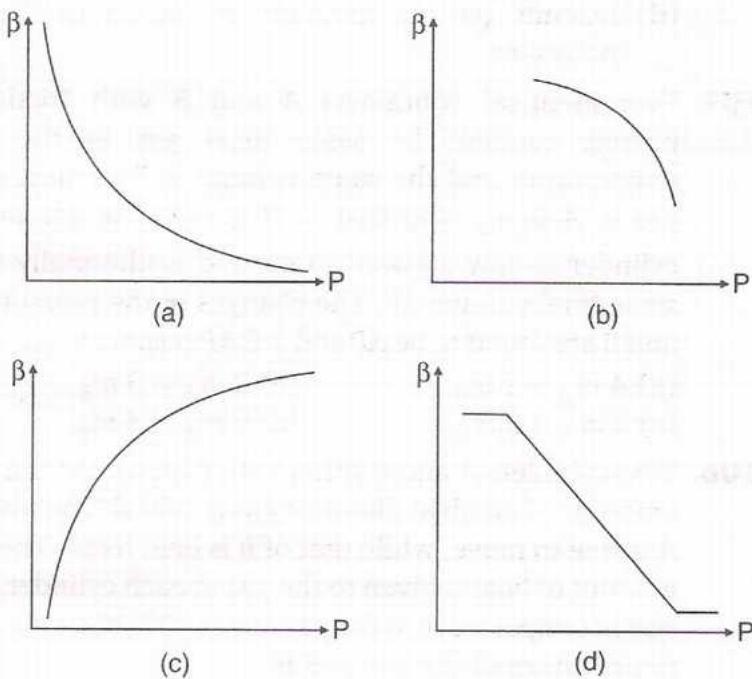
- a) He and O_2
b) O_2 and He
c) He and Ar
d) O_2 and N_2

Example : An ideal gas is taken through the cycle A->B->C->A as shown in the figure. If the net heat supplied to the gas in the cycle is 5 J, the work done by the gas in the process C->A is

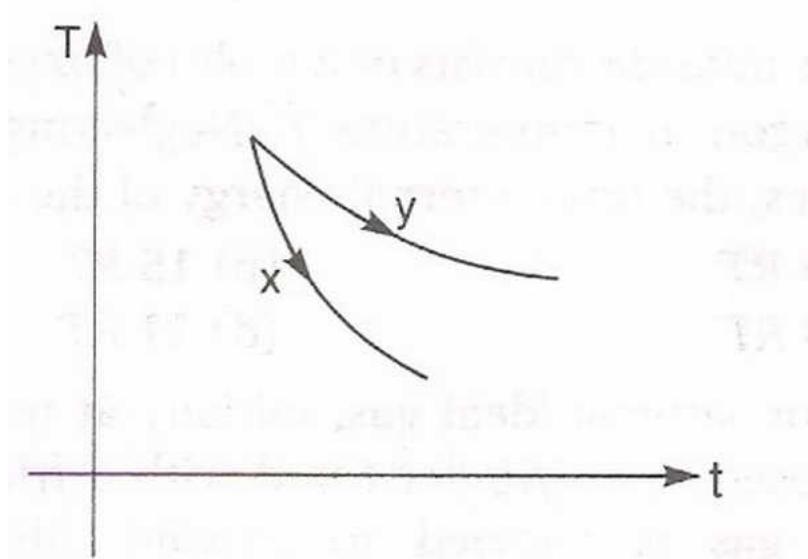


- a) -5 J
- b) -10 J
- c) -15 J
- d) -20 J

Example : Which of the following graphs correctly represents the variation of $\beta = -\frac{dV/dP}{V}$ with P for an ideal gas at constant temperature?

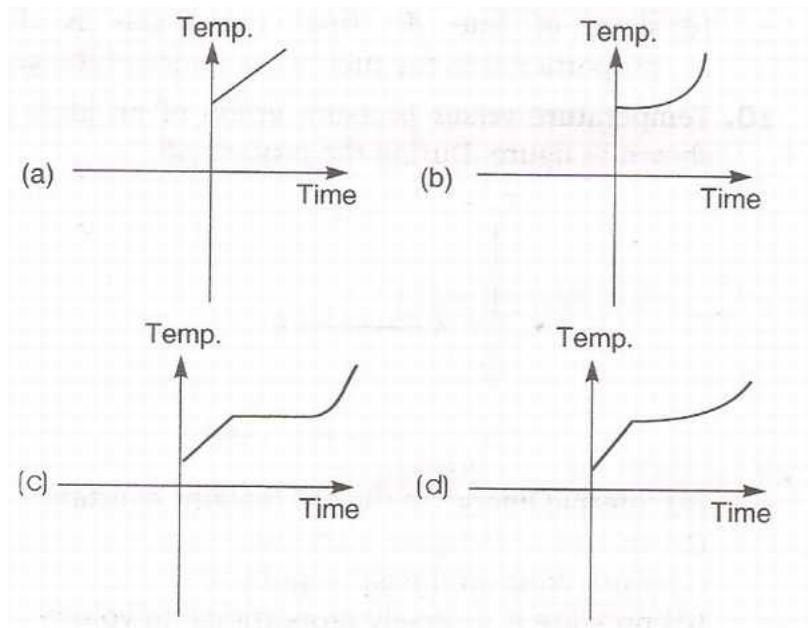


Example : The graph, shown in the diagram, represents the variation of temperature (T) of the bodies, x and y having same surface area, with time (t) due to the emission of radiation. Find the correct relation between the emissivity and absorptivity of the two bodies



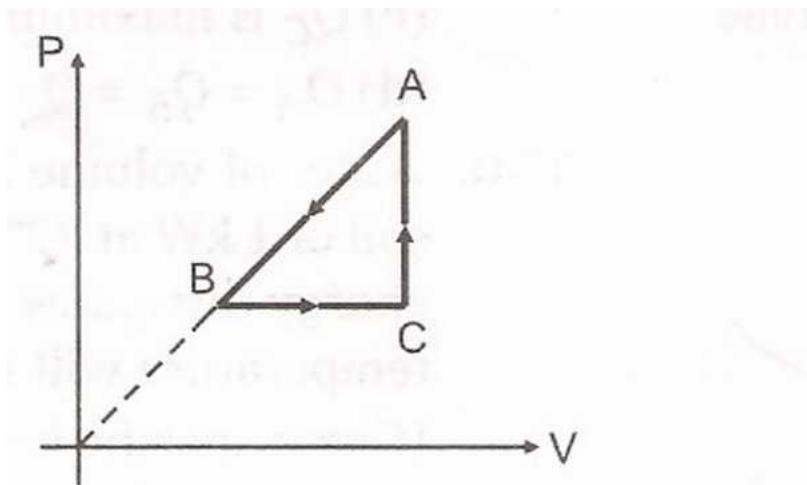
- a) $E_x > E_y$ and $a_x < a_y$
- b) $E_x < E_y$ and $a_x > a_y$
- c) $E_x > E_y$ and $a_x > a_y$
- d) $E_x < E_y$ and $a_x < a_y$

Example : Liquid oxygen at 50 K is heated to 300K at constant pressure of 1 atm. The rate of heating is constant. Which of the following graphs represent the variation of temperature with time?



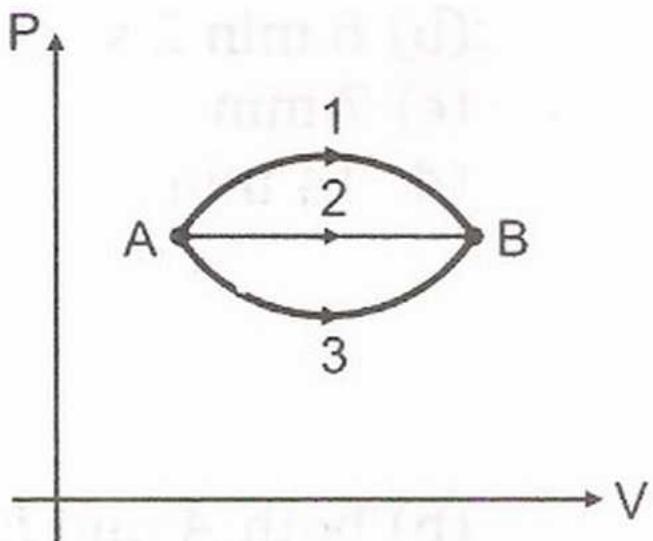
Multiple Answer Type

Example : P-V diagram of a cyclic process ABCA is as shown in figure. Choose the correct statement (s)



- a) $\Delta Q_{A \rightarrow B}$ = negative
- b) $\Delta U_{B \rightarrow C}$ = positive
- c) $\Delta U_{C \rightarrow A}$ = negative
- d) ΔW_{CAB} = negative

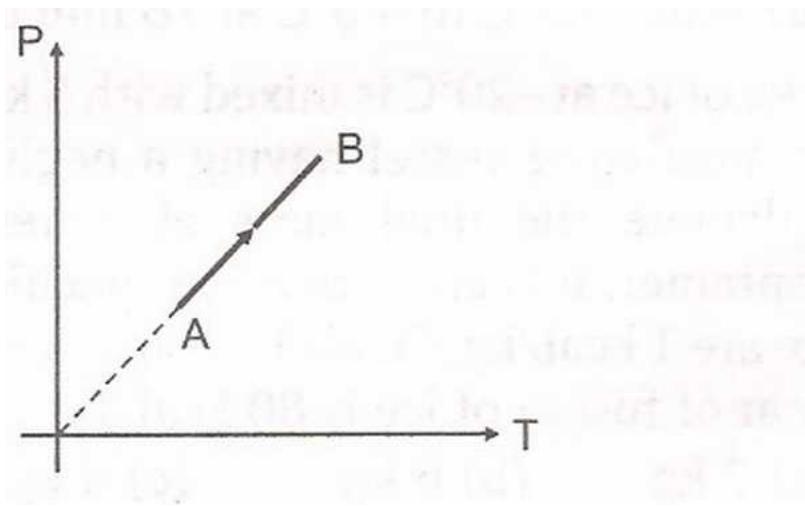
Example : A gas undergoes the change in its state from position A to position B via three different paths as shown in figure.



Select the correct alternative (s).

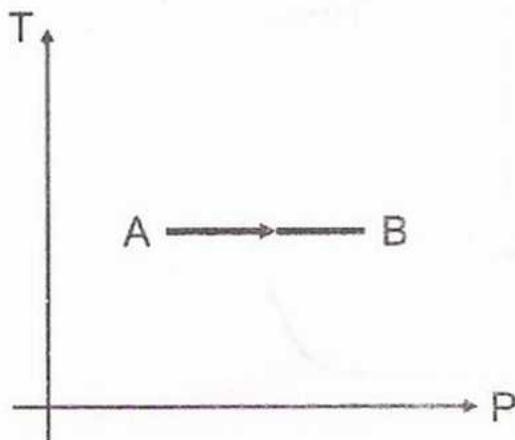
- a) Change in internal energy in all the three paths is equal.
- b) In all the three paths heat is absorbed by the gas.
- c) Heat absorbed / released by the gas is maximum in path 1
- d) Temperature of the gas first increases and then decreases in path 1

Example : During the process A-B of an ideal gas



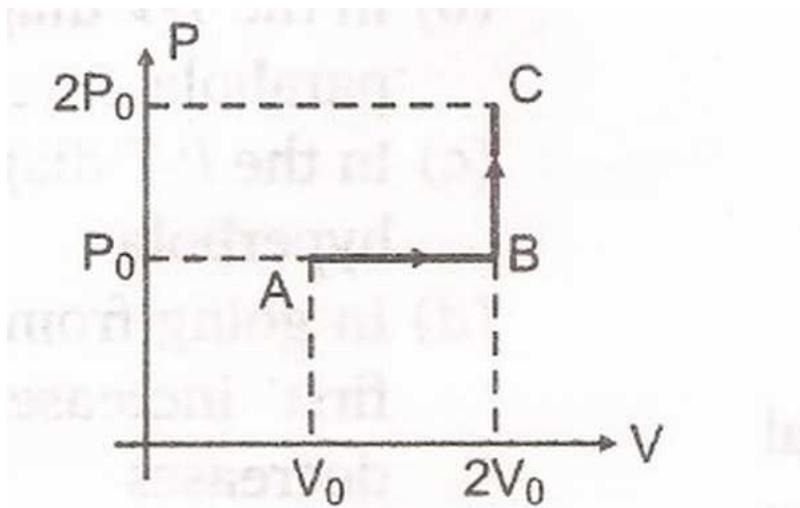
- a) work done on the gas is zero
- b) density of the gas is constant
- c) slope of line AB from the T-axis is inversely proportional to the number of moles of the gas
- d) slope of line AB from the T-axis is directly proportional to the number of moles of the gas

Example : Temperature versus pressure graph of an ideal gas is shown in figure. During the process AB



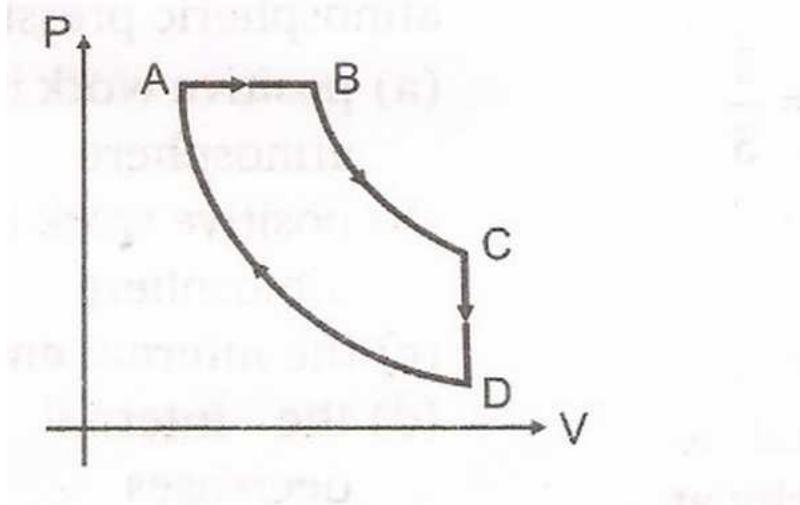
- a) internal energy of the gas remains constant
- b) value of the gas is increased
- c) work done on the gas is positive
- d) pressure is inversely proportional to volume

Example : One mole of an ideal monochromatic gas is taken from A to C along the path ABC. The temperature of the gas at A is T_o . For the process ABC



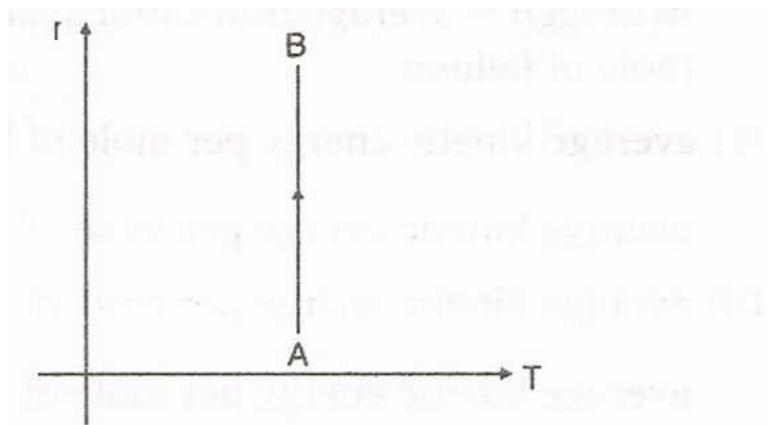
- a) work done by the gas is RT_o
- b) change in internal energy of the gas is $\frac{11}{2}RT_o$
- c) heat absorbed by the gas is $\frac{11}{2}RT_o$
- d) heat absorbed by the gas is $\frac{13}{2}RT_o$

Example : n moles of a monoatomic gas undergo a cyclic process ABCDA as shown in figure. Process AB is isobaric, BC is adiabatic, CD is isochoric and DA is isothermal. The maximum and minimum temperature in the cycle are $4T_o$ and T_o respectively. Then



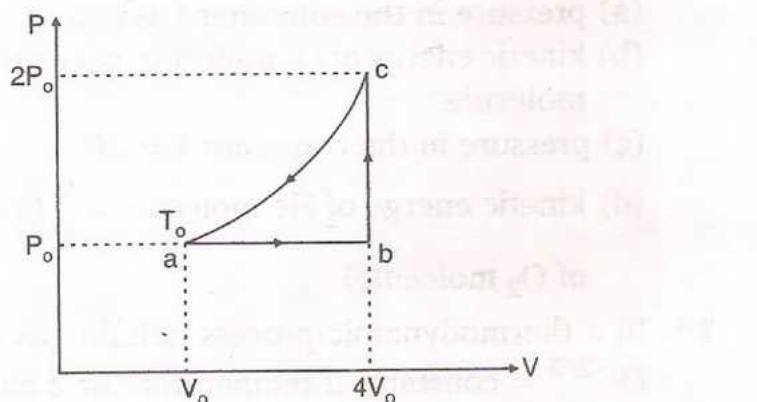
- a) $T_B > T_C > T_D$
- b) heat is released by the gas in the process CD
- c) heat is supplied to the gas in the process AB
- d) total heat supplied to the gas is $2nRT_o \ln(2)$

Example : The density (ρ) of an ideal gas varies with temperature T as shown in figure. Then



- a) the product of P & V at A is equal to the product of P & V at B
- b) pressure at B is greater than the pressure at A
- c) work done by the gas during the process AB is negative
- d) the change in internal energy from A to B is zero

Example : One mole of an ideal monoatomic gas (initial temperature T_o) is made to go through the cycle abca shown in the figure. If U denotes the internal energy, then choose the correct alternatives



- a) $U_c - U_a = 10.5RT_o$
- b) $U_b - U_a = 4.5RT_o$
- c) $U_c > U_b > U_a$
- d) $U_c - U_b = 6RT_o$

Matching Type

Matrix Match 1 In the $\rho - T$ graph shown in figure, match the following

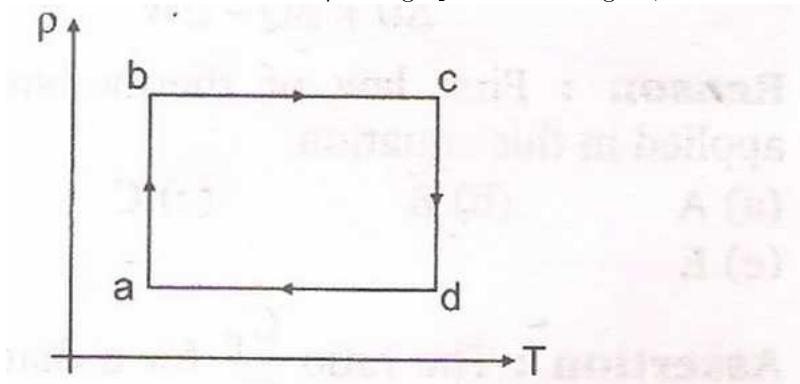


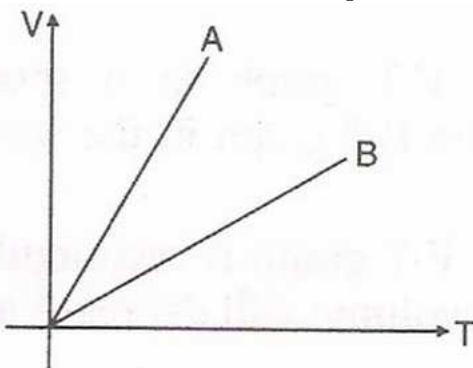
Table-1

- (A) Process $a-b$
 (B) Process $b-c$
 (C) Process $c-d$
 (D) Process $d-a$

Table-2

- (P) Isochoric process
 (Q) $\Delta U = 0$
 (R) P increasing
 (S) P decreasing

Matrix Match 2 In the V-T graph shown in figure match the following

**Table-1**

- (A) Gas A and Gas B are ...
 (B) P_A/P_B is
 (C) n_A/n_B is

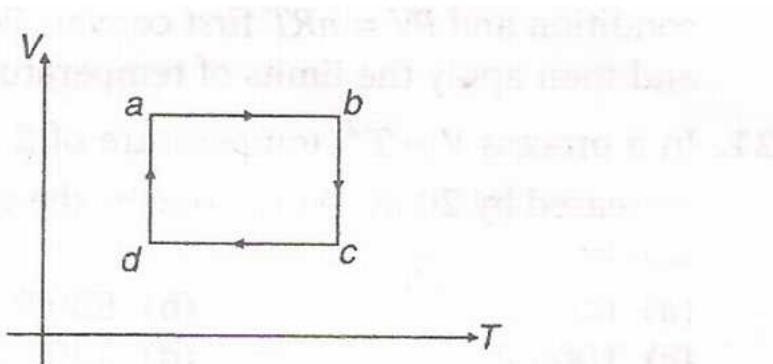
Table-2

- (P) monoatomic,
 diatomic
 (Q) diatomic,
 monoatomic
 (R) > 1
 (S) < 1
 (T) Cannot say any thing

Comprehension Type

Comprehension 1

Statement : V-T graph of an ideal gas is as shown in figure.



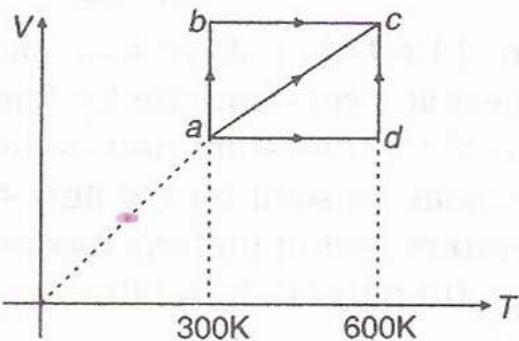
Question : Work done by the gas in complete cyclic process abcd is

- a) zero
- b) positive
- c) negative
- d) Data is insufficient

Question : Heat is supplied to the gas in process (es)

- a) da, ab and bc
- b) da and ab only
- c) da only
- d) ab and bc only

Comprehension 2 Two moles of a monoatomic gas are taken from a to c, via three paths abc, ac and adc.



Question : Work done by the gas in process ac is

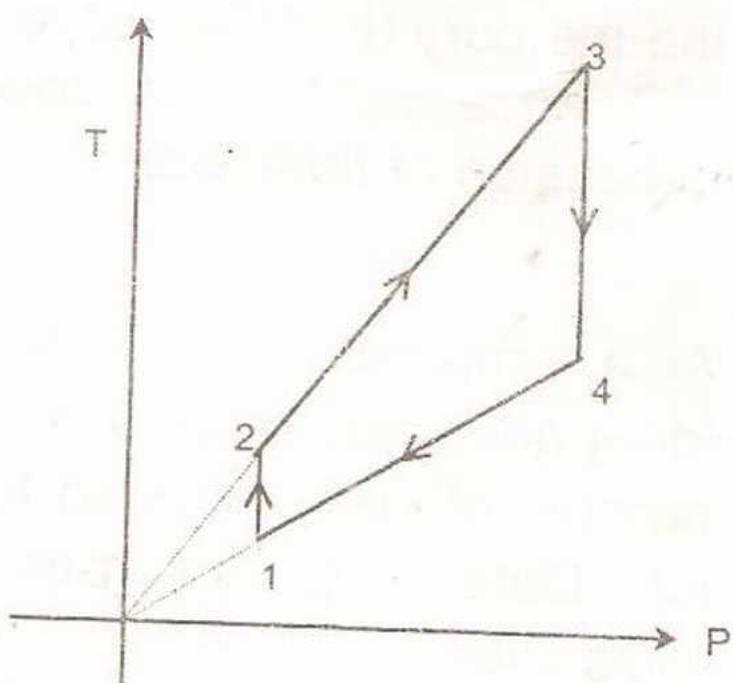
- a) 1000 R
- b) 900 R
- c) 600 R
- d) 1500 R

Question : If work done by the gas in abc is W_1 , in ac work done is W_2 and in adc work done is W_3 , then

- a) $W_2 > W_3 > W_1$
- b) $W_1 > W_2 > W_3$
- c) $W_2 > W_1 > W_3$
- d) $W_3 > W_2 > W_1$

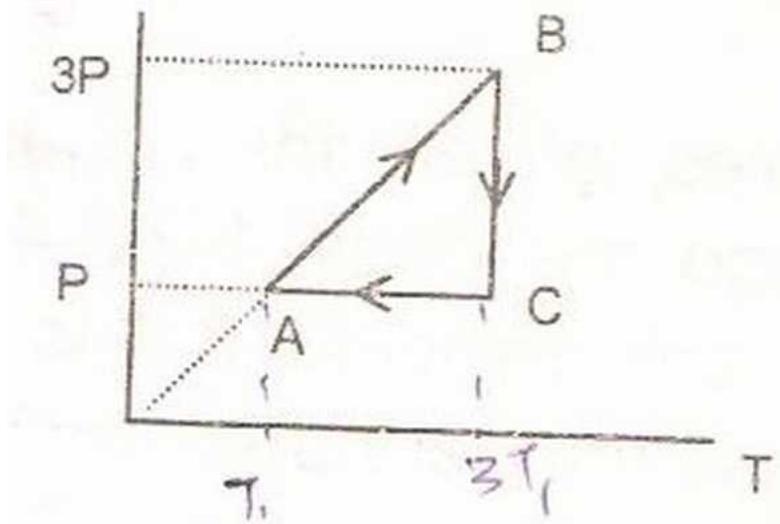
Subjective Problems

Example: The moles of an ideal monoatomic gas undergoes a cyclic process as shown in the figure.

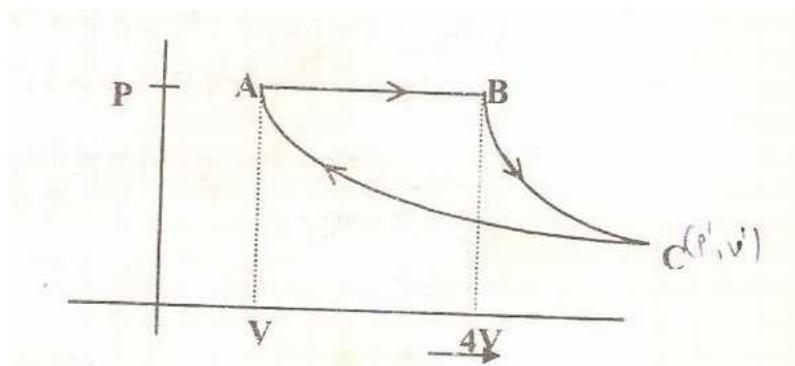


The temperatures in different states are $6T_1 = 3T_2 = 2T_4 = T_3 = 1800\text{K}$. Determine the work done by the gas during the cycle.

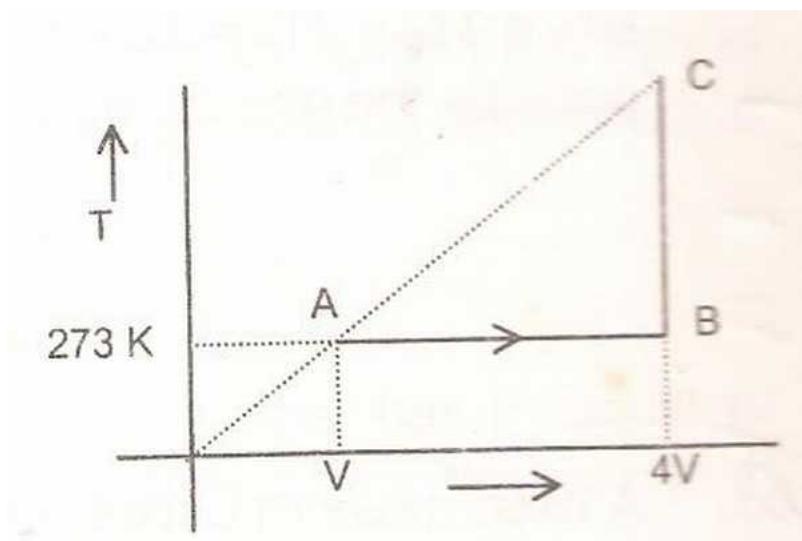
Example: A fixed mass of oxygen gas performs a cycle ABCA as shown. Find efficiency of the process.



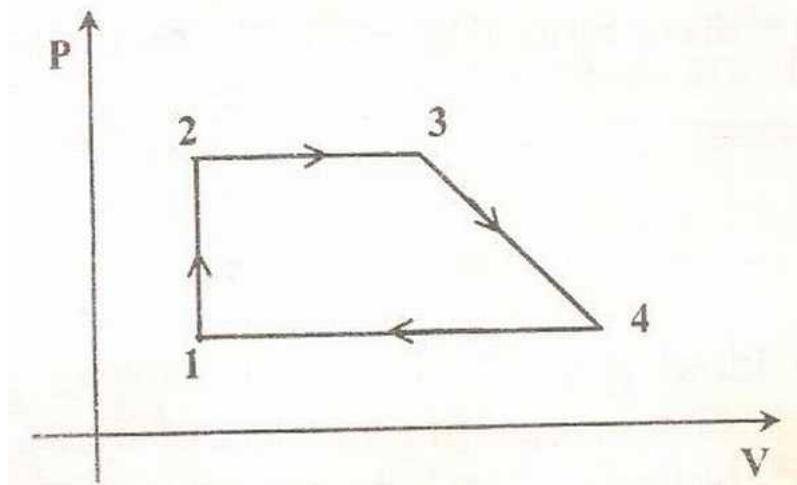
Example: A fixed mass of gas is taken through a process $A \rightarrow B \rightarrow C \rightarrow A$. Here $A \rightarrow B$ is isobaric, $B \rightarrow C$ is adiabatic and $C \rightarrow A$ is isothermal. Find efficiency of process. (Take $\gamma = 1.5$)



Example: At a temperature of $T_o = 273^\circ\text{K}$, two moles of an ideal gas undergoes a process as shown. The total amount of heat imparted to the gas equals $Q = 27.7\text{ kJ}$. Determine the ratio of molar specific heat capacities.



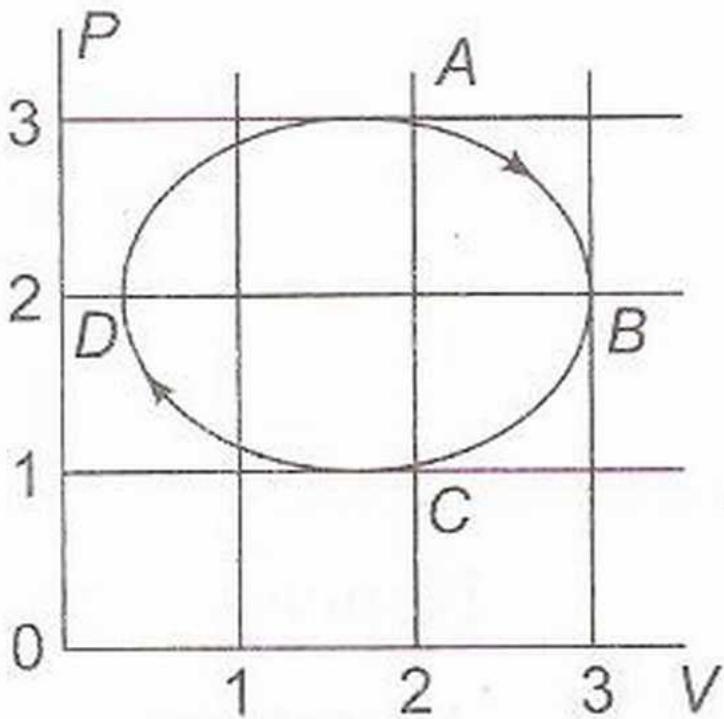
Example: n moles of an ideal gas undergoes the cycle 1-2-3-4-1 as shown in the figure. Process 3-4 is a straight line. The gas temperatures in states 1, 2 and 3 are T_1 , T_2 and T_3 respectively. Temperature at 3 and 4 are equal. Determine the work done by the gas during the cycle.



8.1.1.2 Previous Years IIT Problems

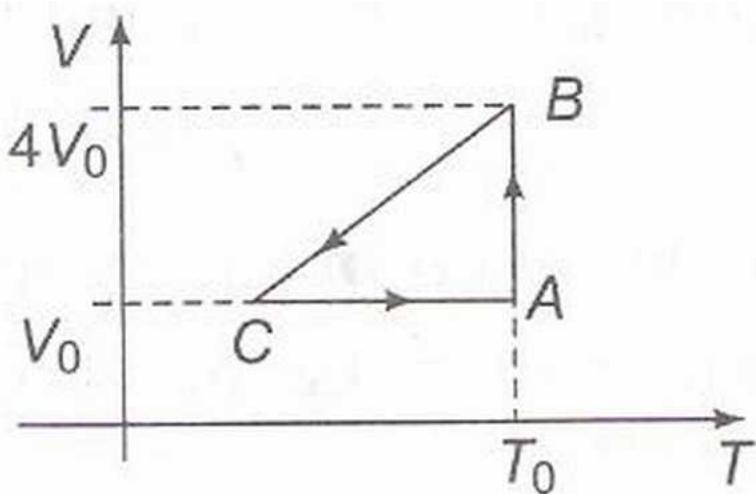
Mutliple Answer

Example: The figure shows the P-V plot of an ideal gas taken through a cycle ABCDA. The part ABC is a semi-circle and CDA is half of an ellipse. Then,



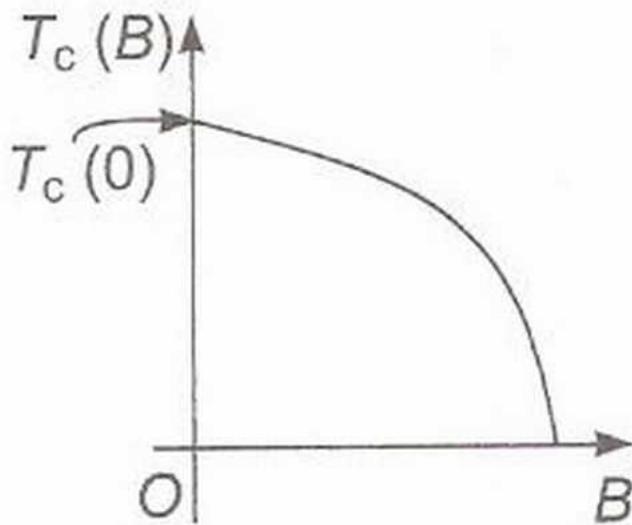
- a) the process during the path A->B is isothermal.
- b) heat flows out of the gas during the path B->C->D
- c) work done during the path A->B->C is zero.
- d) positive work is done by the gas in the cycle ABCDA

Example: One mole of an ideal gas in initial state A undergoes a cyclic process ABCA, as shown in the figure. Its pressure at A is P_o . Choose the correct option(s) from the following

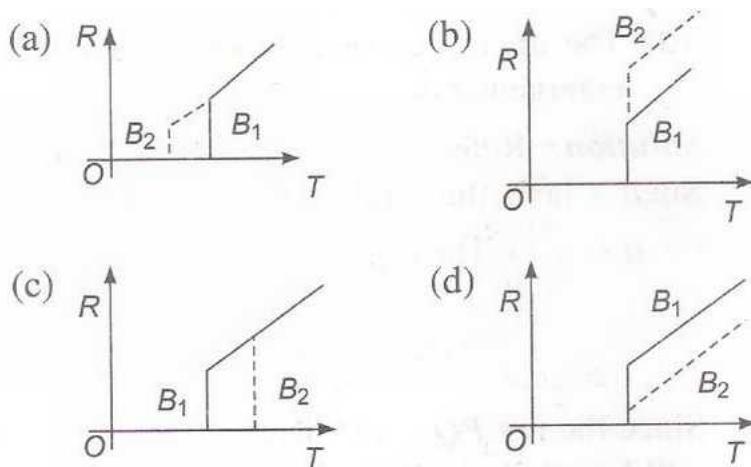


- a) Internal energies at A and B are the same
- b) Work done by the gas in process AB is $P_o V_o \ln 4$
- c) Pressure at C is $\frac{P_0}{4}$.
- d) Temperature at C is $\frac{T_0}{4}$.

Paragraph Paragraph 1: Electrical resistance of certain materials, known as superconductors, changes abruptly from a nonzero value to zero as their temperature is lowered below a critical temperature $T_c(0)$. An interesting property of superconductors is that their critical temperature becomes smaller than $T_c(0)$ if they are placed in a magnetic field, i.e., the critical temperature $T_c(B)$ is a function of the magnetic field strength B . The dependence of $T_c(B)$ on B is shown in the figure.



- 1: In the graphs below, the resistance R of a superconductor is shown as a function of its temperature T for two different magnetic fields B1 (solid line) and B2 (dashed line). If B2 is larger than B1 which of the following graphs shows the correct variation of R with T in these fields?



2: A superconductor has $T_c(0) = 100\text{K}$. When a magnetic field of 7.5 Tesla is applied, its T_c decreases to 75 K. For this material one can definitely say that when

- a) $B = 5$ Tesla, $T_c(B) = 80$ K
- b) $B = 5$ Tesla, $75 \text{ K} < T_c(B) < 100\text{K}$
- c) $B = 10$ Tesla, $75\text{K} < T_c < 100\text{K}$
- d) $B = 10$ Tesla, $T_c = 70$ K

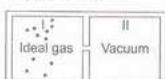
Matching

Example:

Column I contains a list of processes involving expansion of an ideal gas. Match this with Column II describing the thermodynamic change during this process.

Column I

- (a) An insulated container has two chambers separated by a valve. Chamber I contains an ideal gas and the chamber II has vacuum. The valve is opened.



Column II

- (p) The temperature of the gas decreases

- (b) An ideal monoatomic gas expands to twice its original volume such that its pressure $P \propto \frac{1}{V^2}$, where V is the volume of the gas.
(c) An ideal monoatomic gas expands to twice its original volume such that its pressure $P \propto \frac{1}{V^{4/3}}$, where V is its volume.
(d) An ideal monoatomic gas expands such that its pressure P and volume V follow the behaviour shown in the graph.

- (q) The temperature of the gas increases or remains constant

- (r) The gas loses heat

- (s) The gas gains heat



Chapter 9

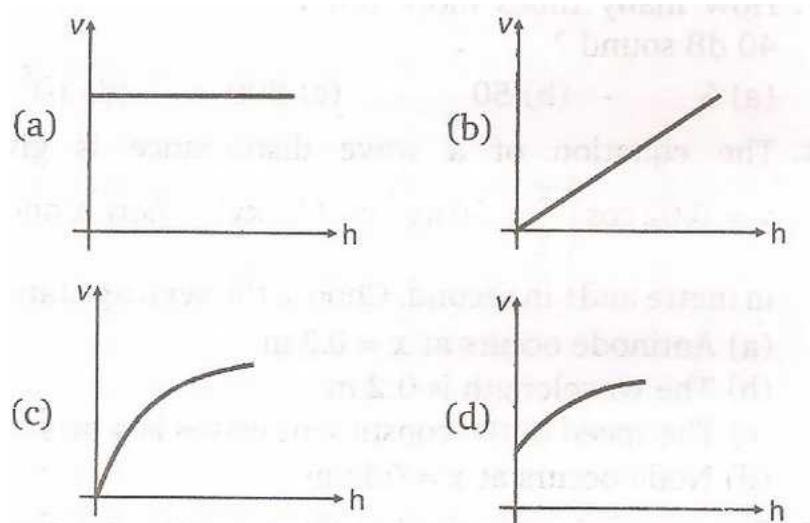
Waves

9.1 Mechanical Waves

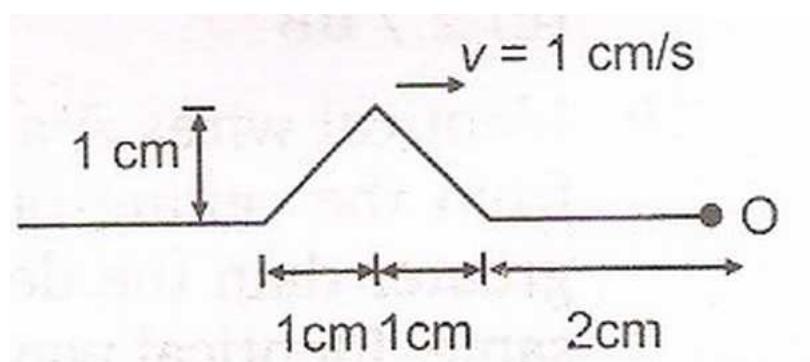
9.1.1 General Problem Set

9.1.1.1 Single Answer Questions

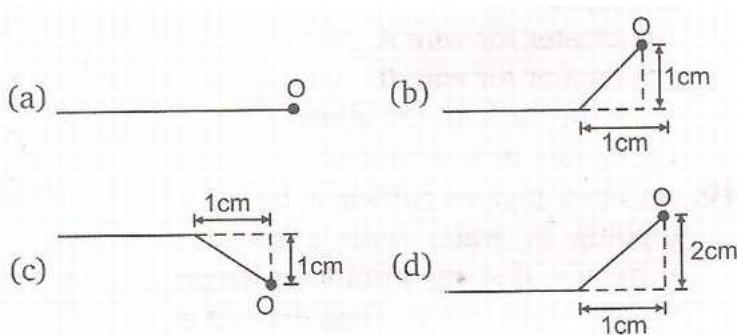
Example 1: A uniform rope having mass m hangs vertically from a rigid support. A transverse wave pulse is produced at the lower end. The speed v of wave pulse varies with height h from the lower end as



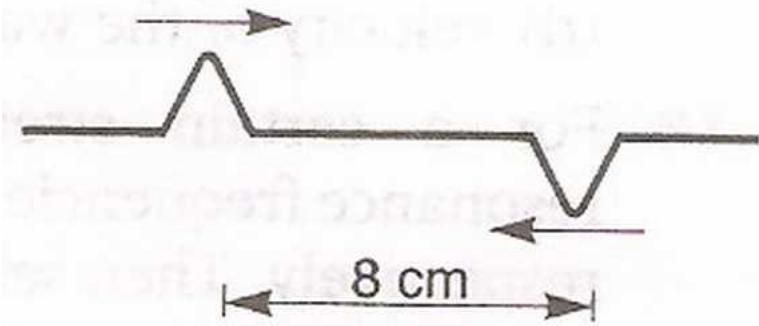
Example 2: A wave pulse on a string has the dimension shown in figure. The wave speed is $v=1 \text{ cm/s}$. If point O is a free end.



- The shape of wave at time $t=3\text{s}$ is
- The shape of the wave at time $t=3\text{s}$ if O is a fixed end will be
{ Both answers from the image below }



Example 3 : Two pulses in a stretched string, whose centres are initially 8 cm apart, are moving towards each other as shown in the figure. The speed of each pulse is 2 cm/s. After 2 s the total energy of the pulses will be



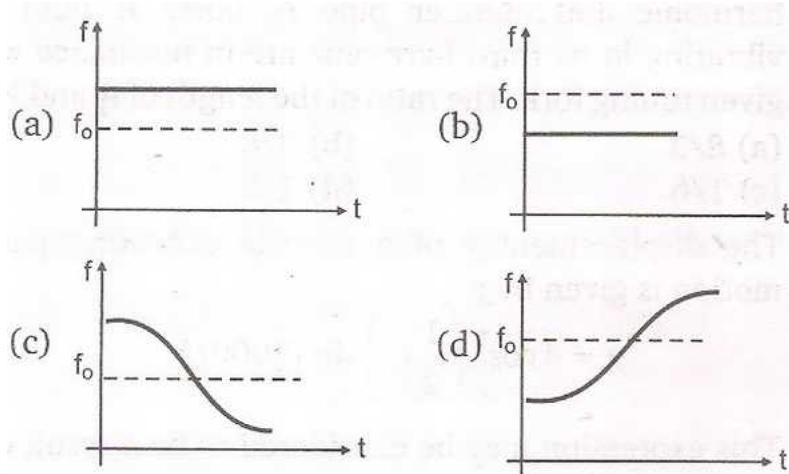
- a) zero
- b) purely kinetic
- c) purely potential
- d) partly kinetic and partly potential

9.2 Sound

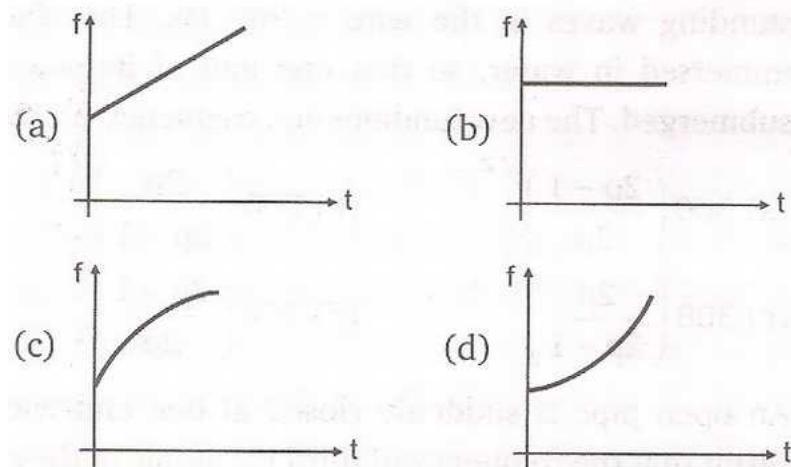
9.2.0.1 General Problem Set

Single Answer Type

Example 1 : Source and observer both start moving simultaneously from origin one along x-axis and the other along y-axis with speed of source = 2 (speed of observer). The graph between the apparent frequency observed by observer (f) and time (t) would be



Example 2: An observer starts moving with uniform acceleration towards a stationary sound source of frequency f_o . As the observer approaches the source, the apparent frequency f heard by the observer varies with time t as



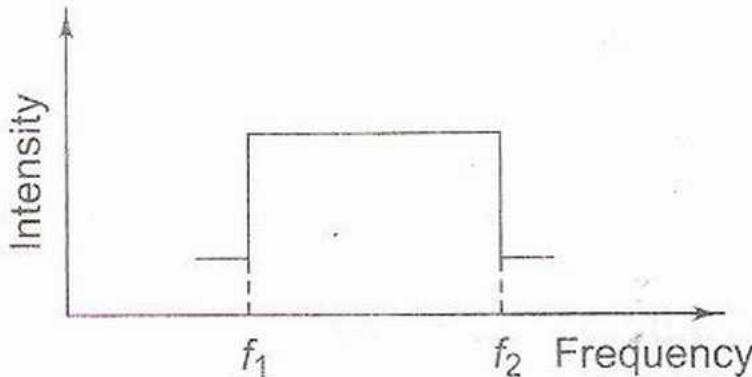
Multiple Answer Type

Example 1 : A stationary observer receives a sound of frequency $f_o = 2000$ Hz. Source is moving with constant velocity on a road at some non-zero perpendicular distance from observer. The apparent frequency f varies with time as shown in figure. Speed of sound = 300 m/s. Choose the correct alternative(s).

- a) Speed of source is 66.7 m/s
- b) f_m shown in figure cannot be greater than 2500 Hz
- c) Speed of source is 33.33 m/s
- d) f_m shown in figure cannot be greater than 2250 Hz

9.2.0.2 Previous Years IIT Problems

Passage Two trains A and B are moving with speed 20 m/s and 30 m/s respectively in the same direction on the same straight track, with B ahead of A. The engines are at the front ends. The engine of train A blows a long whistle. Assume that the sound of the whistle is composed of components varying in frequency from $f_1 = 800\text{Hz}$ to $f_2 = 1120\text{Hz}$, as shown in the figure. The spread in the frequency (highest frequency-lowest frequency) is thus 320Hz. The speed of sound in still air is 340 m/s.

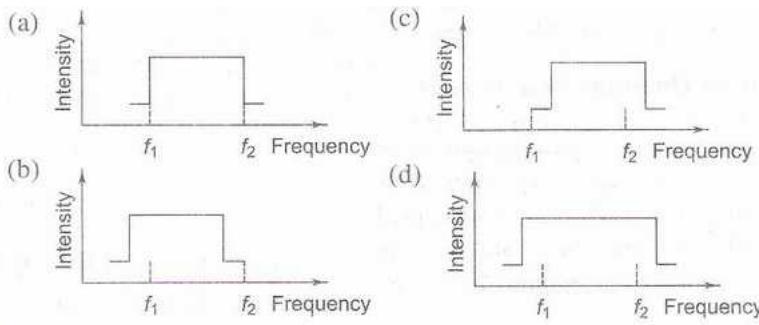


1: The speed of sound of the whistle is

- a) 340 m/s for passengers in A and 310 m/s for passenger in B
- b) 360 m/s for passengers in A and 310 m/s for passenger in B
- c) 310 m/s for passengers in A and 360 m/s for passenger in B
- d) 340 m/s for passengers in both the trains

{ Solution: The speed of sound depends only on the modulus of elasticity and the density of the medium in which it travels. The speed of sound does not depend on the speed of the source of sound or of the observer. Hence the correct option is d) }

- 2:** The distribution of the sound intensity of the whistle as observed by the passengers in train A is best represented by



{ Solution: For train A, there is no relative motion between the source and the passengers. Hence the frequency of sound heard by passengers in train A will be the same as the frequency of sound emitted by the whistle. Therefore, the correct choice is a). }

- 3:** The spread of frequency as observed by the passenger in train B is

- a) 310 Hz
- b) 330 Hz
- c) 350 Hz
- d) 290 Hz

{ Solution: The apparent frequency of sound as heard by passengers in train B is given by

$$f' = f_o \left(\frac{v - u_B}{v - u_A} \right)$$

where f_o = actual frequency, v = speed of sound, u_B = speed of train B and u_A = speed of train A.

$$f' (\text{For } f_o = 800 \text{ Hz}) = 800 \times \left(\frac{340 - 30}{340 - 20} \right) = 775 \text{ Hz}$$

$$f' (\text{For } f_o = 1120 \text{ Hz}) = 1120 \times \left(\frac{340 - 30}{340 - 20} \right) = 1085 \text{ Hz}$$

∴ Spread of frequency = $1085 - 775 = 310 \text{ Hz}$ Hence the correct choice is a) }

Chapter 10

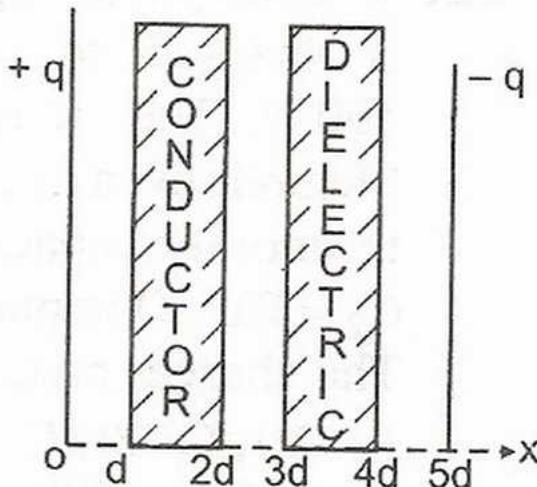
Electromagnetism

10.1 Electrostatics

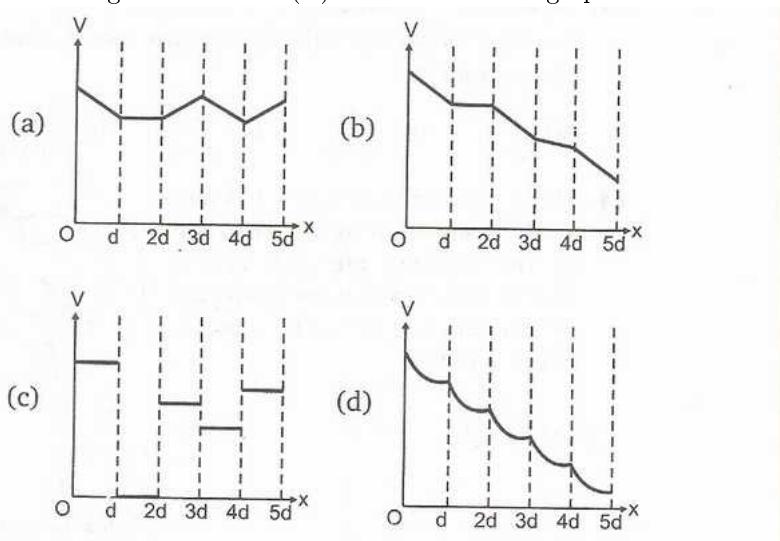
10.1.1 Problems for Practice

10.1.1.1 General Problem Set

Single Answer Type Example : The distance between plates of a parallel plate capacitor is $5d$. The positively charged plate is at $x=0$ and negatively charged plate is at $x=5d$.

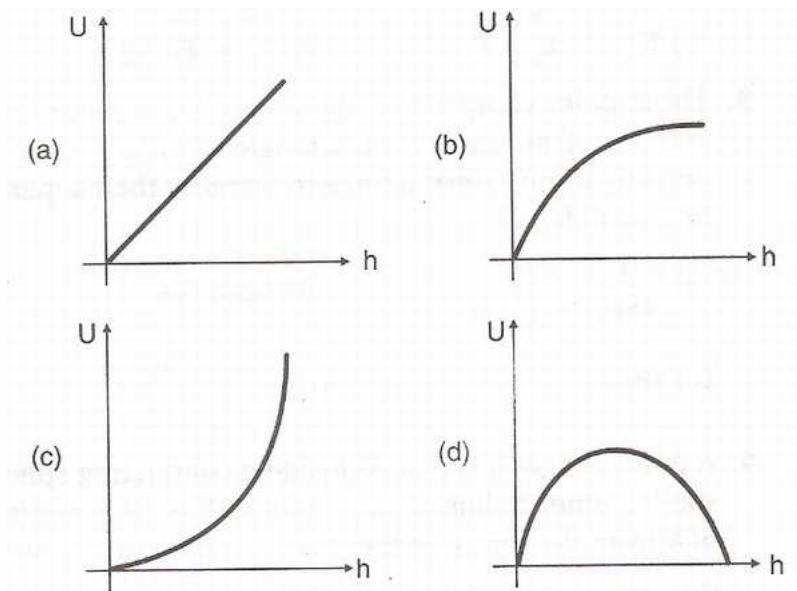


Two slabs one of conductor and the other of a dielectric of same thickness d are inserted between the plates as shown in figure. Potential (V) versus distance x graph will be

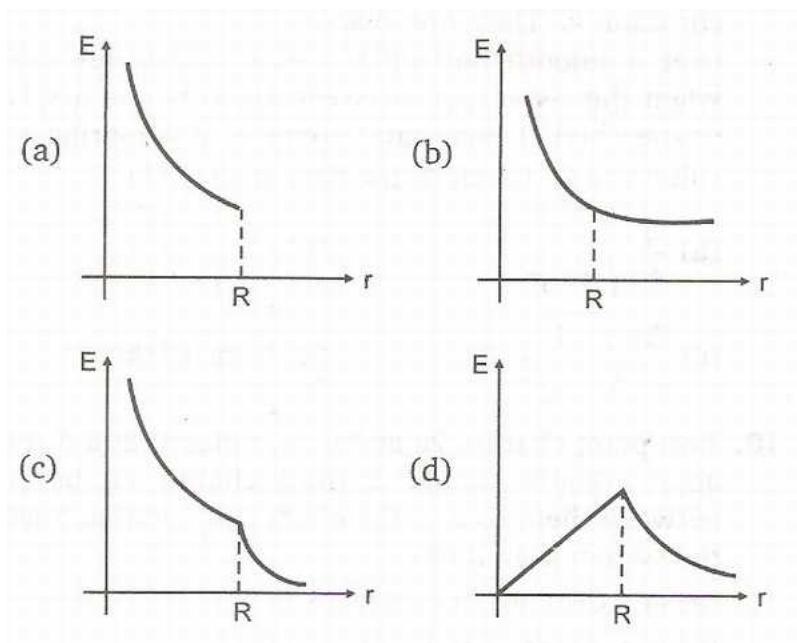


Example : A particle of mass m and charge q is projected vertically upwards. A uniform electric field \vec{E} is acted vertically downwards. The most appropriate graph between potential energy U (gravitational plus

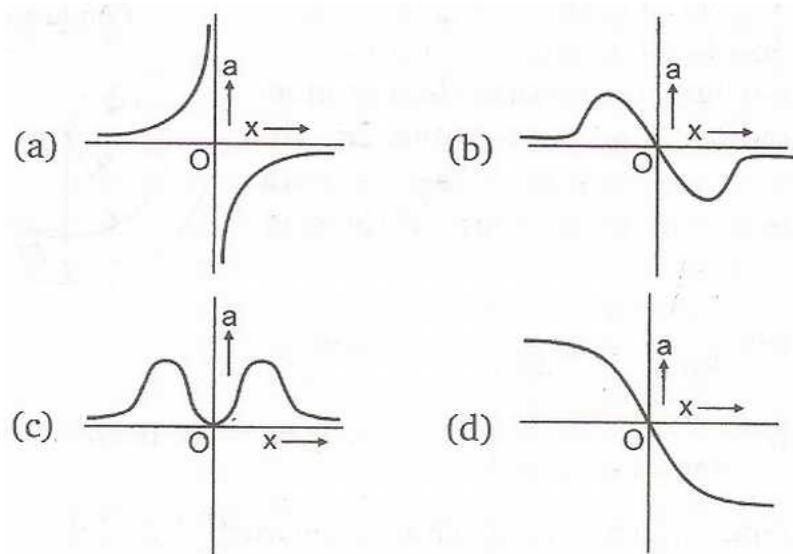
electrostatic) and height h (\ll radius of earth) is (assume U to be zero on surface of earth)



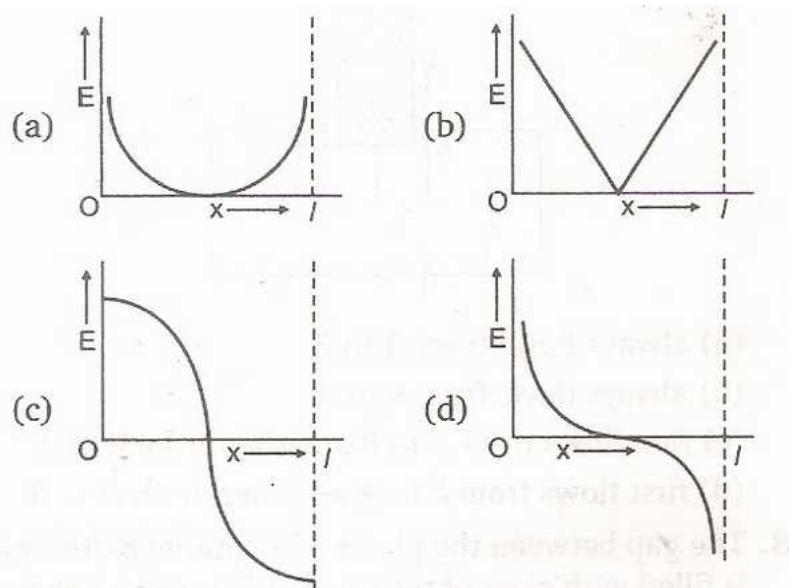
Example : A conducting shell of radius R carries charge $-Q$. A point charge $+Q$ is placed at the centre. The electric field E varies with distance r (from the centre of the shell) as



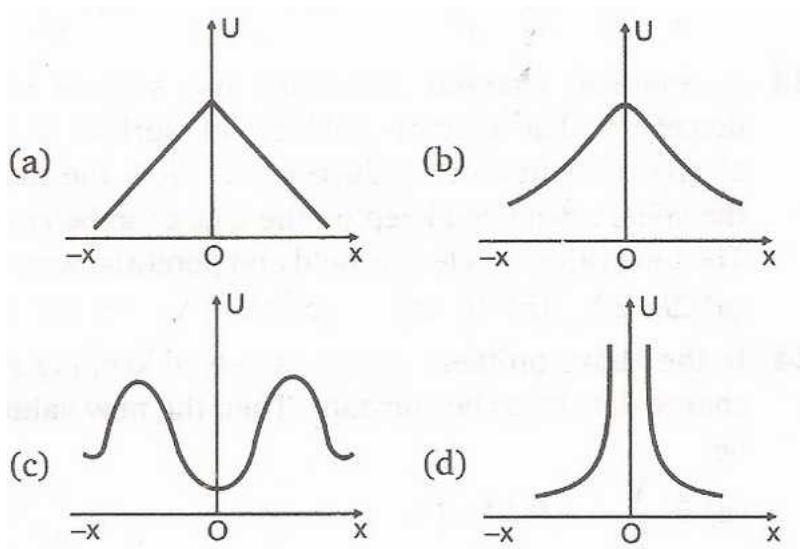
Example : Two identical positive charges are fixed on the y -axis, at equal distances from the origin O . A particle with a negative charge starts on the negative x -axis at a large distance from O , moves along the x -axis, passed through O and moves far away from O . Its acceleration a is taken as positive along its direction of motion. The particle's acceleration a is plotted against its x -coordinate. Which of the following best represents the plot ?



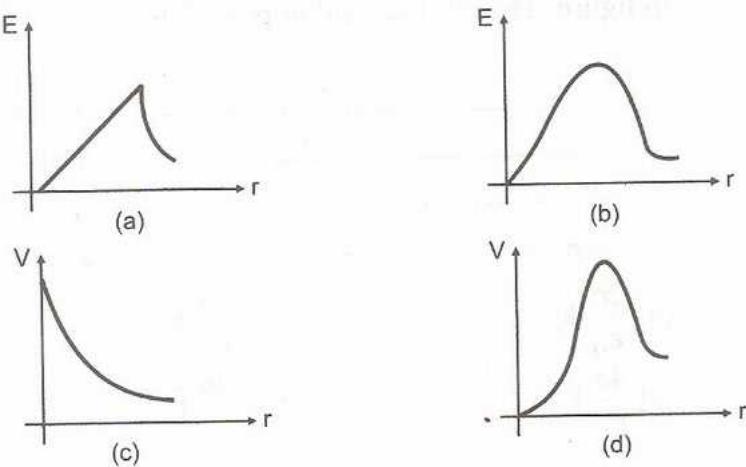
Example: Two identical point charges are placed at a separation of l . P is a point on the line joining the charges, at a distance x from any one charge. The field at P is E . E is plotted against x for values of x from close to zero to slightly less than l . Which of the following best represents the resulting curve?



Example : Four equal charges of magnitude q each are placed at four corners of a square with its centre at origin and lying in y - z plane. A fifth charge $+Q$ is moved along x -axis. The electrostatic potential energy (U) varies on x -axis as



Example : A circular ring carries a uniformly distributed positive charge. The electric field (E) and potential (V) varies with distance (r) from the centre of the ring along its axis as



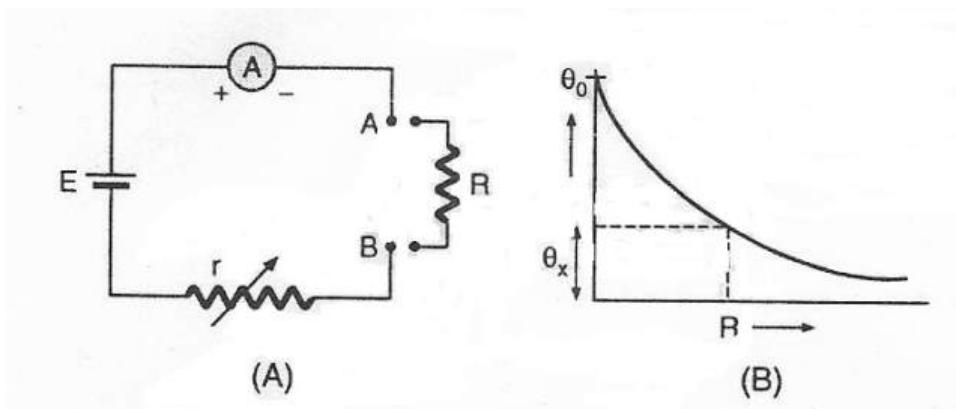
10.2 Current Electricity

10.2.1 Basics

10.2.1.1 Theory

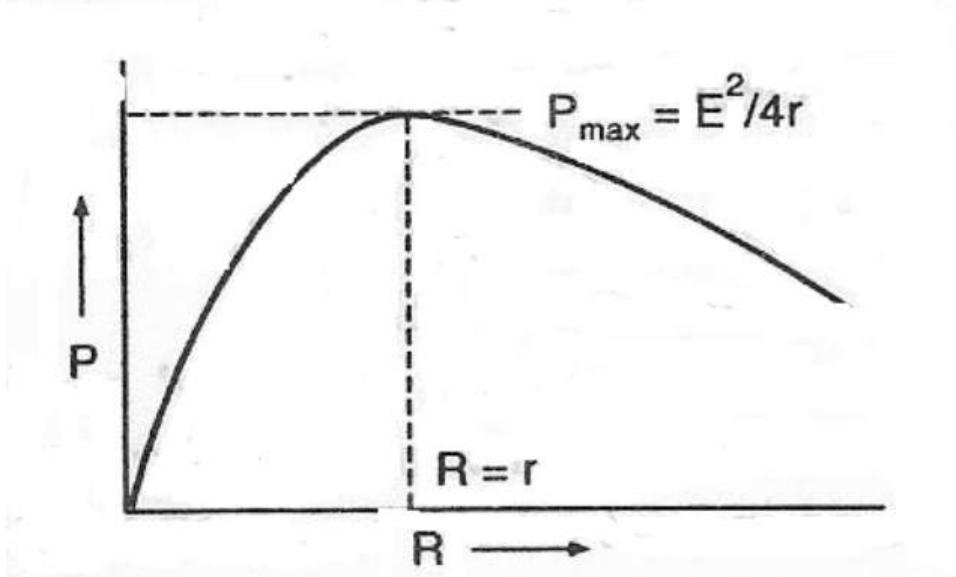
Ohm-meter It is an instrument designed to measure resistance. It contains an Ammeter, a Battery and a Rheostat as shown in Figure. The terminals A and B are first short circuited and the Rheostat is adjusted to show full deflection on Ammeter. The full scale deflection corresponds to zero external resistance.

Now, connecting a resistance box between points A and B, ammeter deflection θ is noted for different values of R and a graph is plotted between θ and R. The graph is called Calibration Curve and is shown in Figure. Now the resistance box is removed and an unknown resistance is connected to the circuit. The deflection is noted down and from the calibration curve, the value of R is found out.



Power Transfer to a load The power transfer to the load by the cell will be $P = I^2R = \frac{E^2R}{(R+r)^2}$

From the equation, it is clear that Power would be zero, if $R=0$ or ∞ and gives the minima.



$$\frac{dP}{dR} = 0 \text{ i.e. } \frac{d}{dR} \left[\frac{E^2R}{(R+r)^2} \right] = 0 \text{ will give any other local maxima / local minima}$$

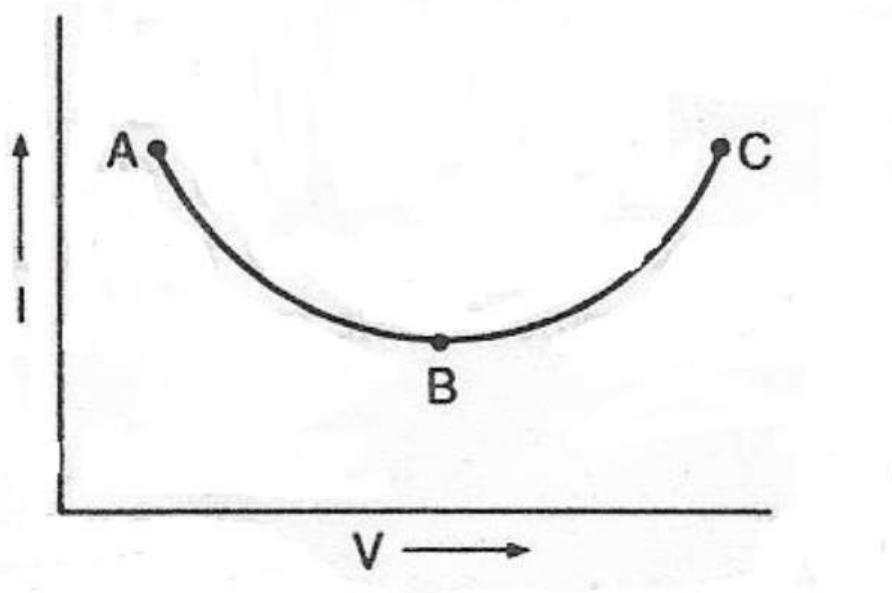
It is zero at $R=r$ which gives a maxima

i.e. power transfer to the load by a cell is maximum when $R=r$ and $P_{max} = \frac{E^2}{4r}$

10.2.1.2 Problems

Objective Type Questions

Example: Resistance as shown in Figure is negative at

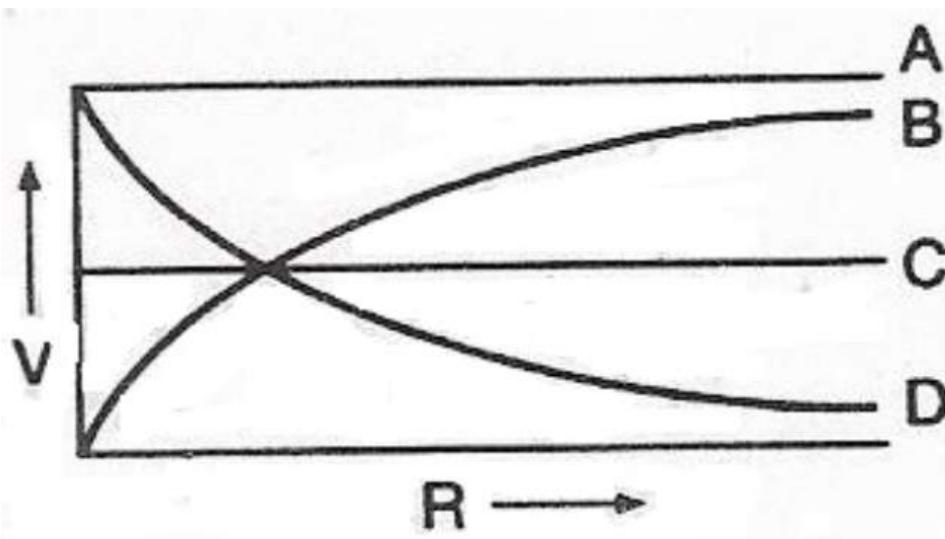


- a) A
- b) B
- c) C
- d) None of these

{Hint: The resistance is given by V/I and NOT dV/dI . V and I are shown positive, so R (their ratio) is also +ve.

d) is the correct answer. }

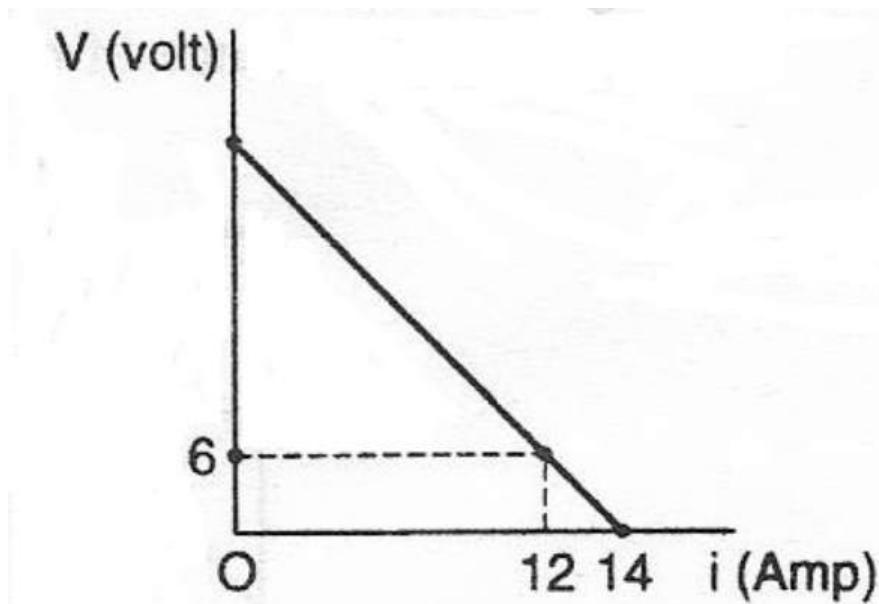
Example: A cell of EMF E having an Internal Resistance r is connected to an External Resistance R . The potential difference V across the resistance R varies with R as the curve :



- a) A
- b) B
- c) C
- d) D

{ Hint: $V=ER/(r+R)$, So, B is the required curve as of $y=xc'/(x+c')$ }

Example: 10 cells, each of EMF E and internal resistance r are connected in series to a variable external resistance. Figure shows the variation of terminal potential difference with the current drawn from the combination. EMF of each cell is :



- a) 1.6 V
- b) 3.6 V
- c) 1.4 V
- d) 4.2 V

{Hint : The equation of the line is $V/42 + i/14 = 1$

$$\text{Also, } V = 10E/10r+R$$

$$\text{So, } 10E/42(10r+R) + i/14 = 1$$

$$10E + 3i(10r+R) = 42(10r+R)$$

As R is Variable, setting R as infinity

$$10E + 3i = 42$$

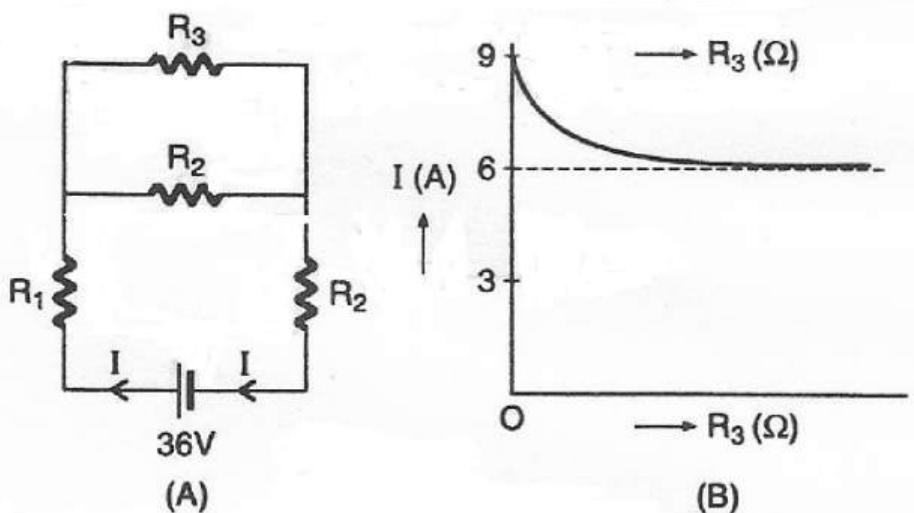
$$E = (42 - 3i) / 10 = 4.2 - 0.3i$$

Also at R infinity i would be zero as it is a series connected circuit

$$\text{So, } E = 4.2 \text{ V}$$

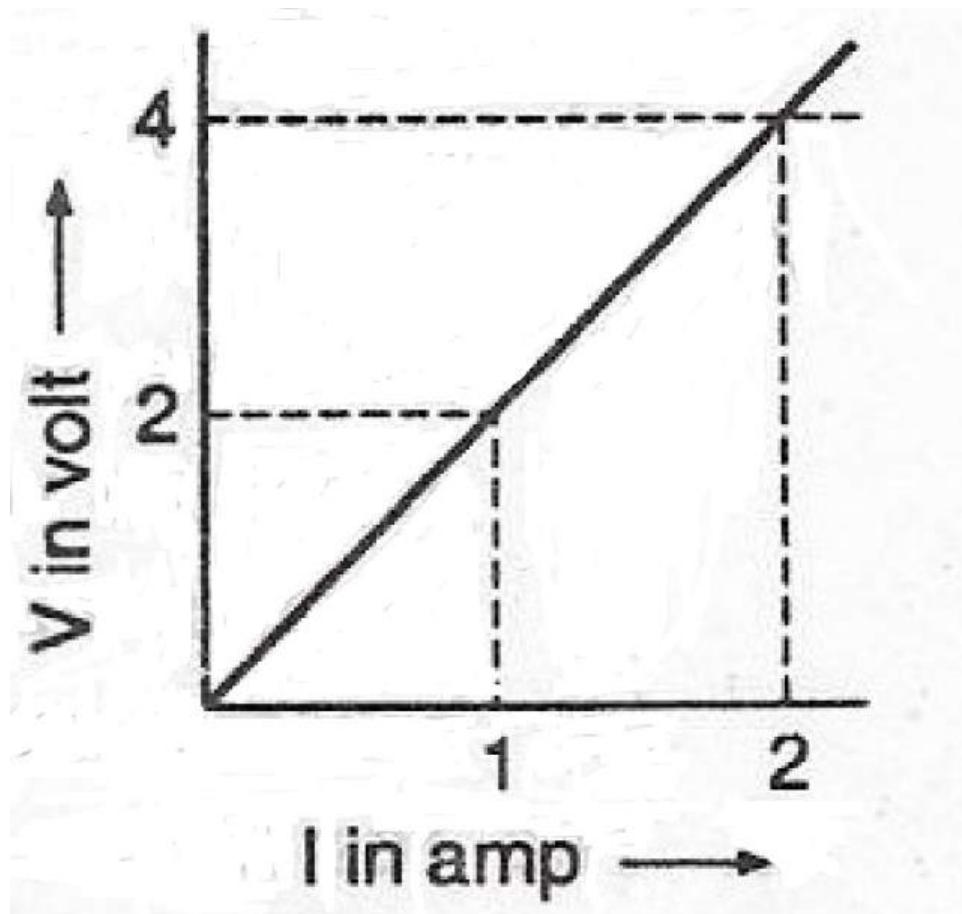
d) is the correct answer }

Example: In the circuit shown in figure, R_3 is a variable resistance. As the value of R_3 is changed, current I through the cell varies as shown. Obviously, the variation is asymptotic, i.e. $I \rightarrow 6$ A as $R_3 \rightarrow \infty$. Resistance R_1 and R_2 are respectively :



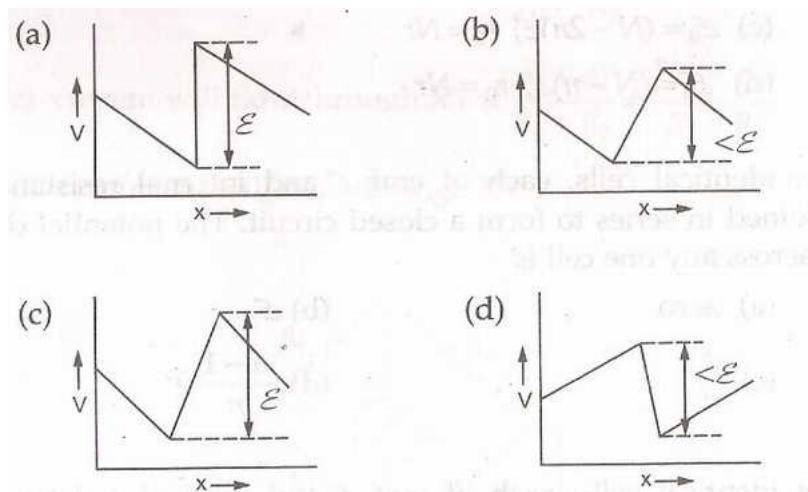
- a) $4\Omega, 2\Omega$
- b) $2\Omega, 4\Omega$
- c) $2\Omega, 2\Omega$
- d) $1\Omega, 4\Omega$

Example: The variation of current with potential difference is as shown in Figure. The resistance of the conductor is :



- a) 1Ω
- b) 2Ω
- c) 3Ω
- d) 4Ω

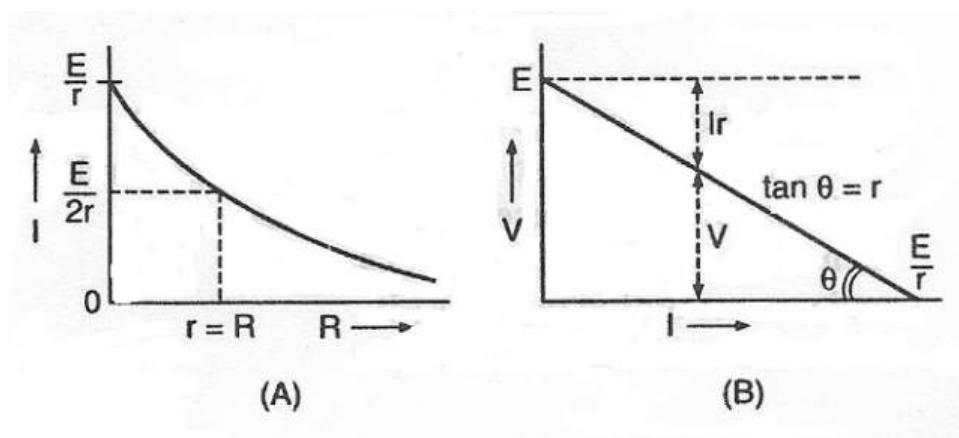
Example : The two ends of a uniform conductor are joined to a cell of emf E and some internal resistance. Starting from the midpoint P of the conductor, we move in the direction of the current and return to P. The potential V at every point on the path is plotted against the distance covered (x). Which of the following best represents the resulting curve?



Subjective

Example: Draw a) I vs R b) V vs I , characteristics for a cell.

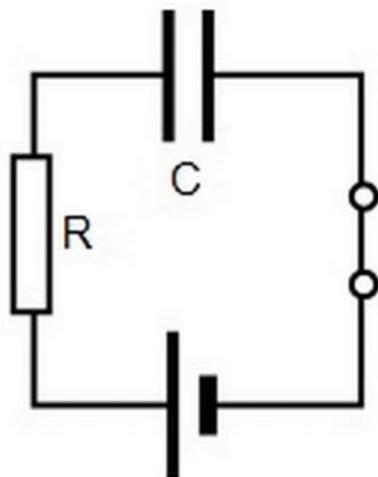
{ Hint: For a cell, as $I = E/(R+r)$ and $V = E - Ir$,



}

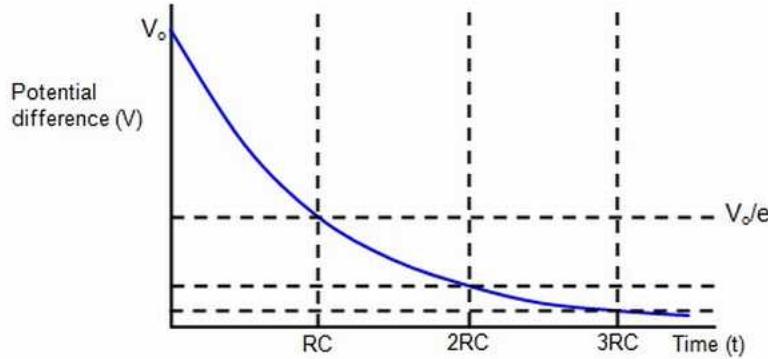
10.2.2 Capacitors

Mathematical treatment of charging and discharging a capacitor n.d.



10.2.2.1 Discharge of a capacitor

The area under the current-time discharge graph gives the charge held by the capacitor. The gradient of the charge-time graph gives the current flowing from the capacitor at that moment.



In Figure let the charge on a capacitor of capacitance C at any instant be q , and let V be the potential across it at that instant.

The current (I) in the discharge at that instant is therefore: $I = -dq/dt$

But $V = IR$ and $q = CV$ so $dq/dt = d(CV)/dt = C dV/dt$. Therefore we have $V = -CR dV/dt$. Rearranging and integrating gives:

Capacitor discharge (voltage decay): $V = V_0 e^{-(t/RC)}$

where V_0 is the initial voltage applied to the capacitor. A graph of this exponential discharge is shown below in Figure

Since $Q = CV$ the equation for the charge (Q) on the capacitor after a time t is therefore:

Capacitor discharge (charge decay): $Q = Q_0 e^{-(t/RC)}$

$V = V_0 e^{-(t/RC)}$ and also $I = I_0 e^{-(t/RC)}$ $Q = Q_0 e^{-(t/RC)}$

You should realise that the term RC governs the rate at which the charge on the capacitor decays.

When $t = RC$, $V = V_0/e = 0.37 V_0$ and the product RC is known as the time constant for the circuit. The bigger the value of RC the slower the rate at which the capacitor discharges.

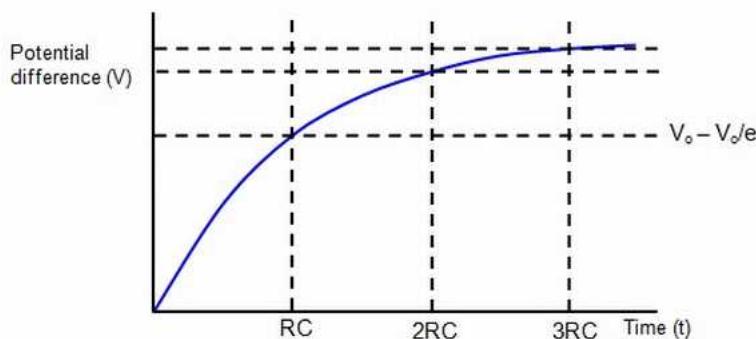
The value of C can be found from this discharge curve if R is known.

10.2.2.2 Charging a capacitor

When a capacitor (C) is being charged through a resistance (R) to a final potential V_0 the equation giving the voltage (V) across the capacitor at any time t is given by:

Capacitor charging (potential difference): $V = V_0 [1 - e^{-(t/RC)}]$

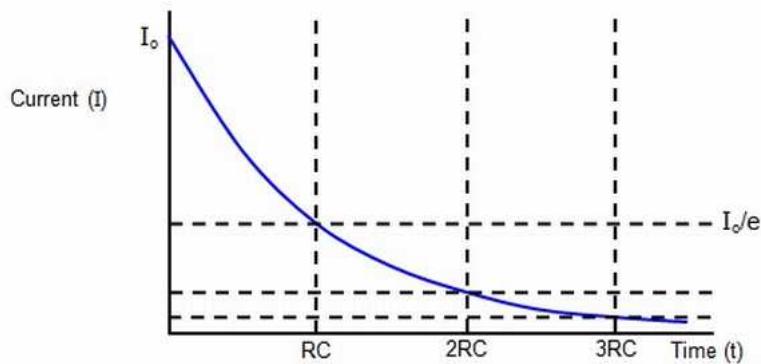
and the variation of potential with time is shown in Figure



As the capacitor charges the charging current decreases since the potential across the resistance decreases as the potential across the capacitor increases.

Figure shows how both the potential difference across the capacitor and the charge on the plates vary with time during charging.

The charging current would be given by the gradient of the curve in Figure at any time and the graph of charging current against time is shown in next Figure.



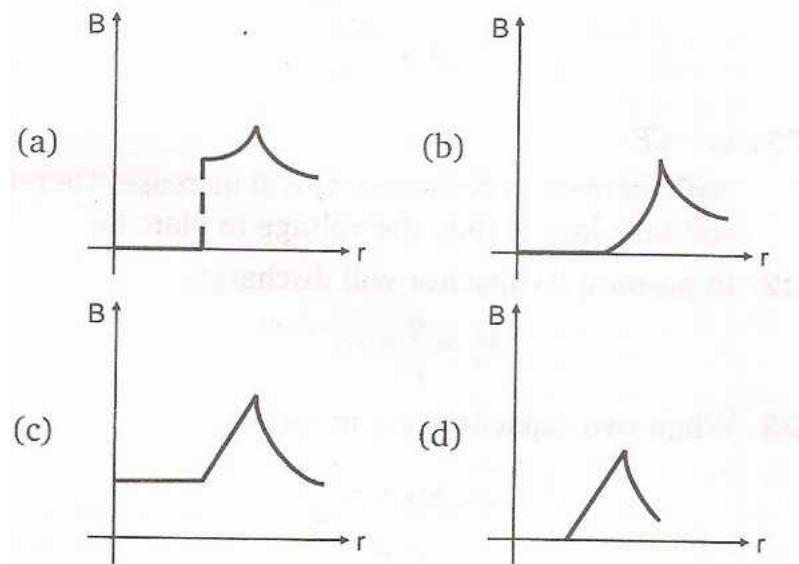
The area below the current-time curve in both charging and discharging represents the total charge held by the capacitor.

10.3 Magnetic Field

10.3.1 Problems for Practice

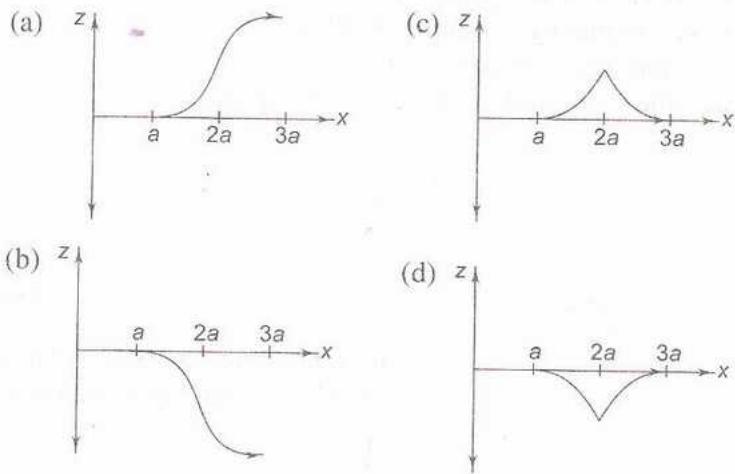
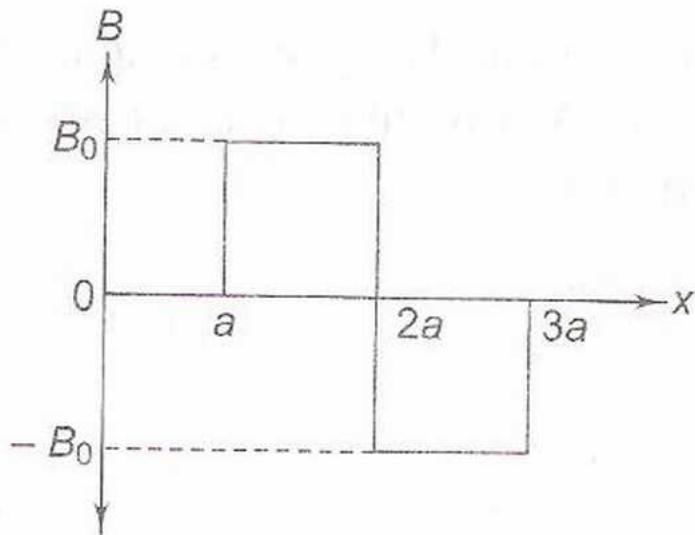
10.3.1.1 General Problem Set

Example : A current i is uniformly distributed over the cross section of a long hollow cylindrical wire of inner radius R_1 and outer radius R_2 . Magnetic field B varies with distance r from the axis of the cylinder as



10.3.1.2 IIT Previous Years Problems

Example: A magnetic field $\vec{B} = B_o \hat{j}$ exists in the region $a < x < 2a$ and $\vec{B} = -B_o \hat{j}$ in the region $2a < x < 3a$ where B_o is a positive constant. A positive point charge moving with a velocity $\vec{v} = v_o \hat{i}$, where v_o is a positive constant, enters the magnetic field at $x=a$. The trajectory of the charge in this region can be like



Solution: Force experienced by the charge q is given by

$$\vec{F} = q(\vec{v} \times \vec{B})$$

In the region from $x=a$ to $x=2a$, the force is $\vec{F}_1 = q(v_o \hat{i} \times B_o \hat{j}) = qv_o B_o \hat{k}$ directed along the positive z -axis.

In the region from $x=a$ to $x=2a$ to $x=3a$, the force is

$$\vec{F}_2 = q(v_o \hat{i} \times (-B_o) \hat{j}) = -qv_o B_o \hat{k}$$

directed along the negative z -axis.

Since force \vec{F}_1 and \vec{F}_2 are perpendicular to velocity \vec{v} , the correct trajectory is as shown in option a).

Bibliography

(N.d.). URL: http://www.schoolphysics.co.uk/age16-19/Optics/Refraction/text/Lenses_graphs/index.html.

Chapter 11

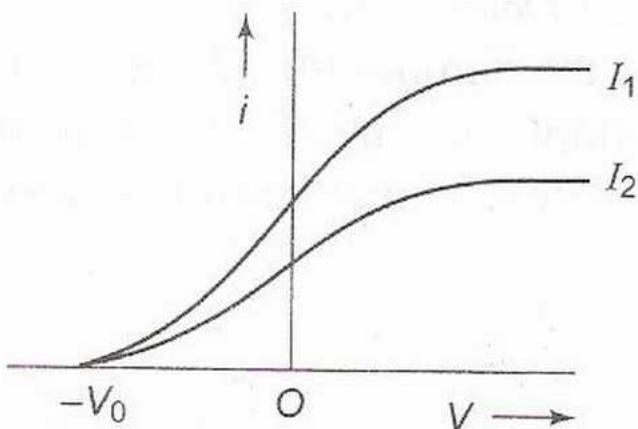
Matter

11.1 Lattice

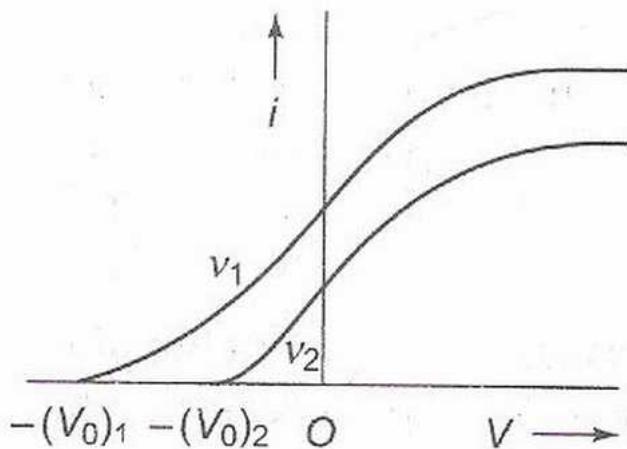
11.1.1 Experiments

Photoelectric Effect

Graphs of Photoelectric Current vs Voltage For radiation of different Intensities ($I_1 > I_2$) but the same frequency.



For radiation of different frequencies ($\nu_1 > \nu_2$) but of the same intensity.

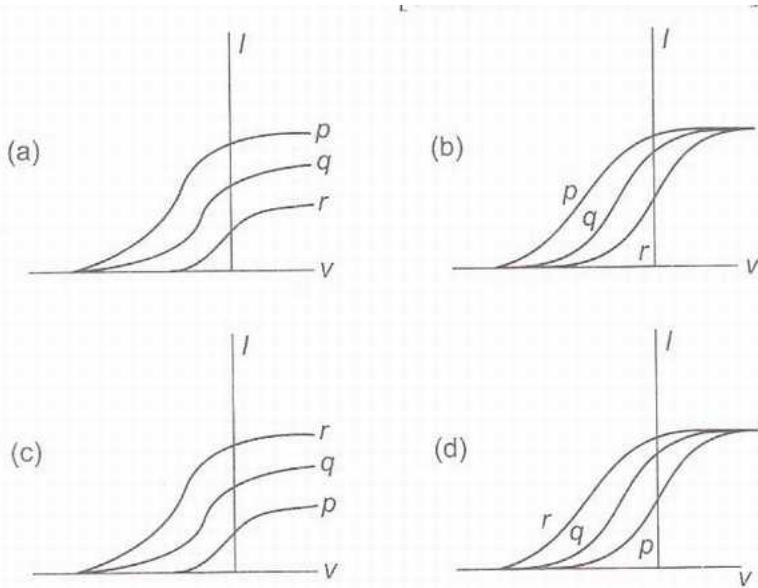


11.1.2 Problems

11.1.2.1 Previous Years IIT Problems

Single Answer Questions

Example: Photoelectric effect experiments are performed using three different metal plates p,q and r having work functions $\phi_p = 2.0\text{eV}$. $\phi_q = 2.5\text{eV}$. $\phi_r = 3.0\text{eV}$ respectively. A light beam containing wavelengths of 550nm, 450nm and 350nm with equal intensities illuminates each of the plates. The correct I-V graph for the experiment is [Take $hc=1240 \text{ eV nm}$]



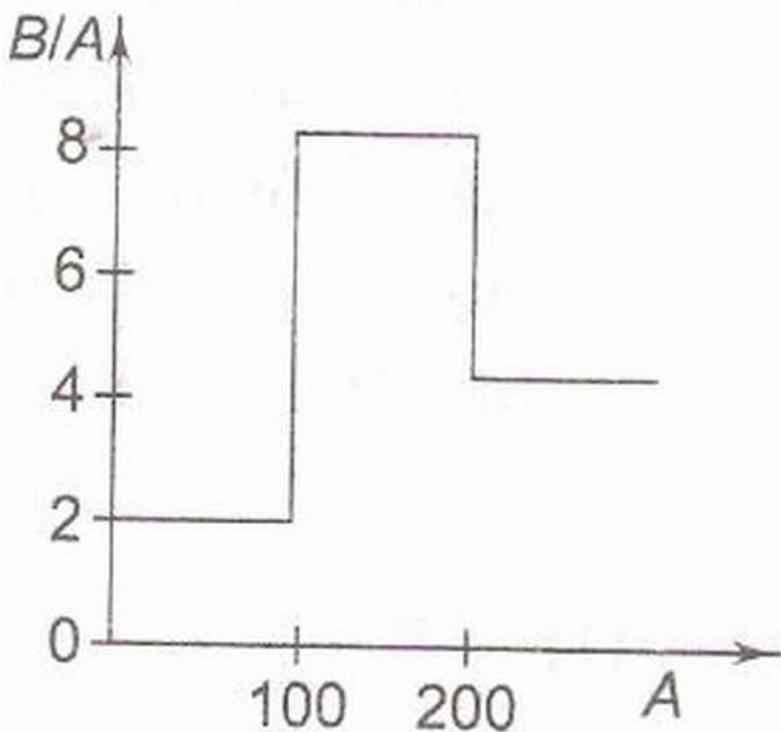
11.2 Nucleus

11.2.1 Problems

11.2.1.1 Previous Years IIT Problems

Multiple Answer

Example: Assume that the nuclear binding energy per nucleon (B/A) versus mass number is as shown in the figure. Use this plot to choose the correct choice (s) given below.

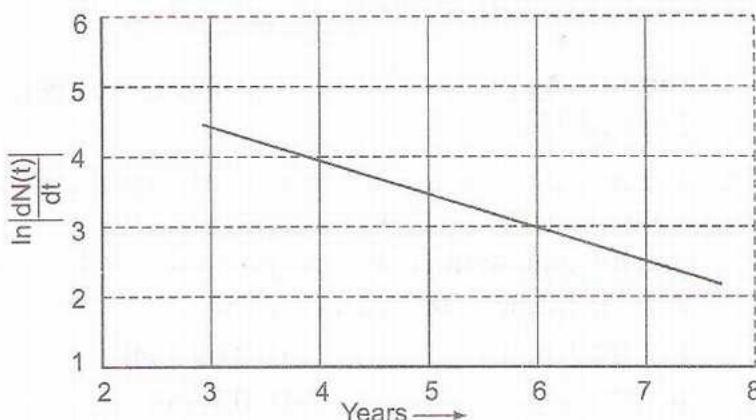


- a) Fusion of two nuclei with mass numbers lying in the range $1 < A < 50$ will release energy
- b) Fusion of two nuclei with mass numbers lying in the range of $51 < A < 100$ will release energy

- c) Fission of a nucleus lying in the mass number range of $100 < A < 200$ will release energy when broken into equal fragments
d) Fission of a nucleus lying in the mass number range of $200 < A < 260$ will release energy when broken into equal fragments
{ Solution: Energy is released if the total binding energy of the products is greater than the total binding energy of the reactants. This is not possible in choices a) and c). The correct choices are b) and d). }

Integer Type

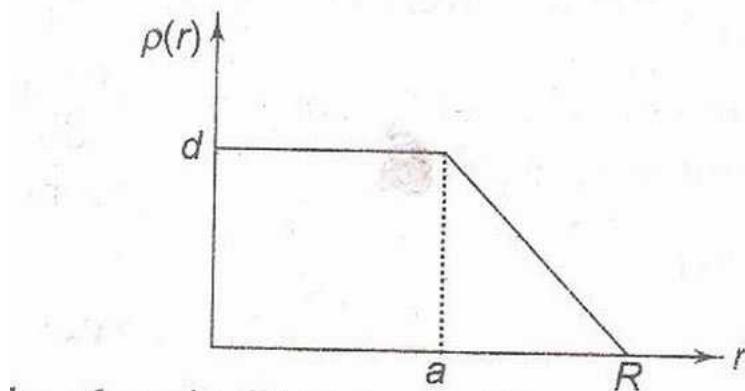
Question : To determine the half life of radioactive element, a student plots a graph of $\ln \left| \frac{dN(t)}{dt} \right|$ vs t. Here $\frac{dN(t)}{dt}$ is the rate of radioactive decay at time t. If the number of radioactive nuclei of this element decreases by a factor of p after 4.16 years, find the value of p.



{ Answer: 8 }

Paragraph

Question: The nuclear charge (Ze) is non-uniformly distributed within a nucleus of radius R. The charge density (ρ) (charge per unit volume) is dependent only on the radial distance r from centre of the nucleus as shown in figure. The electric field is only along the radial direction.



1. The electric field at $r=R$ is

- a) independent of a
- b) directly proportional to a
- c) directly proportional to a^2
- d) inversely proportional to a

{ Solution: The charge $q=Ze$ can be assumed to be concentrated at the centre of the nucleus. The electric field at $r=R$ is

$$E = \frac{q}{4\pi\epsilon_0 R^2} = \frac{Ze}{4\pi\epsilon_0 R^2}$$

which is a constant. Hence the correct choice is a). }

2. For $a=0$, the value of d (maximum value of ρ as shown in the figure) is

a) $\frac{3Ze}{4\pi R^3}$

b) $\frac{3Ze}{\pi R^3}$

c) $\frac{4Ze}{3\pi R^3}$

d) $\frac{Ze}{3\pi R^3}$

{ Solution: Total charge is }

$$\begin{aligned} q &= \int_0^R 4\pi r^2 \left(d - \frac{d}{R}r \right) dr \\ &= 4\pi \left[d \int_0^R r^2 dr - \frac{d}{R} \int_0^R r^3 dr \right] \\ &= \frac{\pi d R^3}{3} \end{aligned}$$

Now that $d = \frac{3Ze}{\pi R^3}$

So, the correct choice is b) }

3. The electric field within the nucleus is generally observed to be linearly dependent on r . This implies

a) $a=0$

b) $a=R/2$

c) $a=R$

d) $a=2R/3$

{ Solution: For spherical charge distribution, the electric field is linearly dependent on r if the charge density ρ is uniform, i.e. $a=R$. Hence the correct choice is c). }

Chapter 12

Optics

12.1 Ray Optics

12.1.1 Theory

12.1.1.1 Graphs for convex and concave lenses

(Real is positive sign convention) n.d.

Figure shows a graph where the reciprocal of the image distance is plotted against the reciprocal of the object distance. The graph is a straight line that intercepts both axes at $1/f$ where f is the focal length of the lens. Since the object is real this graph is for a convex lens.

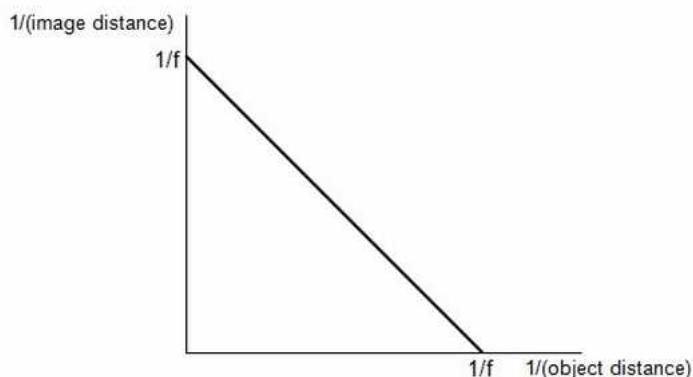
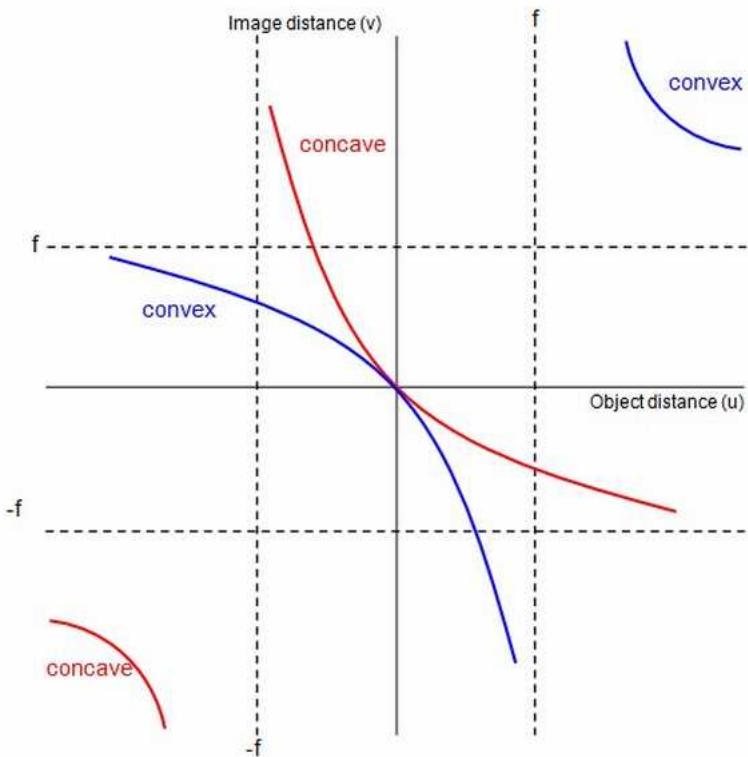
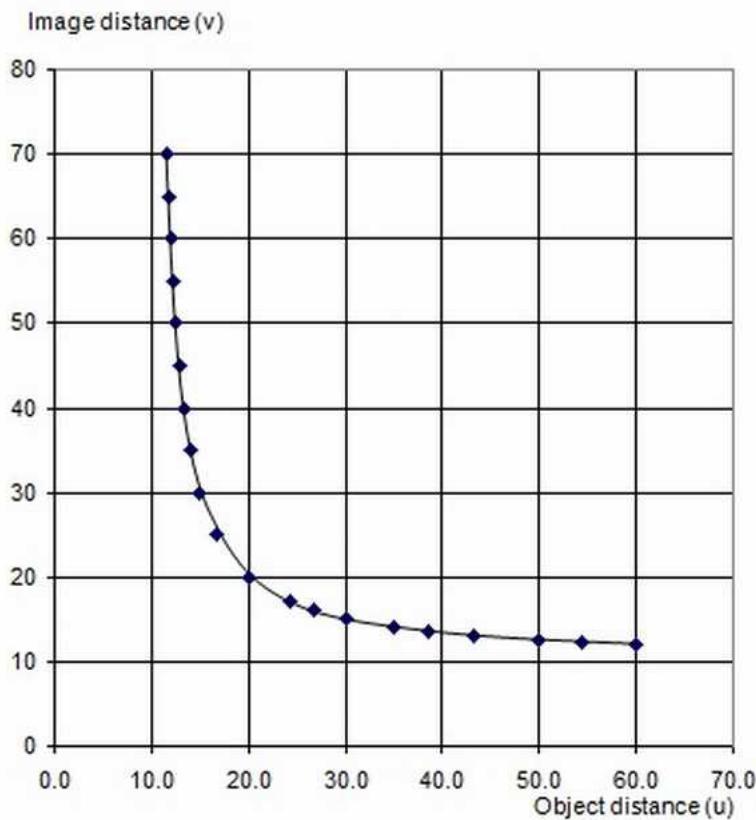


Figure shows a graph of the object distance plotted against the image distance for both convex and concave lenses. Both real and virtual objects and images are shown.



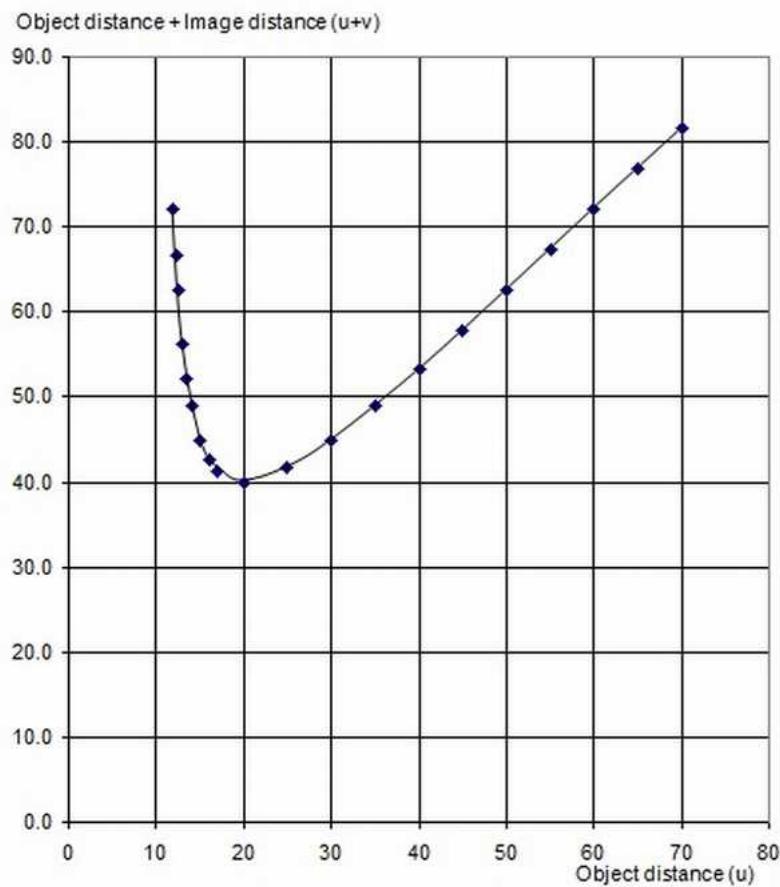
Enlarged view of an object distance (u) against image distance (v) graph The focal length of the lens is 10 cm.

The graph is completely symmetrical so that when $u = 2f$, v also equals $2f$.



Minimum distance The next graph shows the distance between the object and image ($u+v$) plotted against the object distance (u) (it could equally well have been v).

The minimum value for $(u+v)$ is $4f$ when $u=v=2f$. This means that no image can be formed with a convex lens of focal length 10 cm if the object and the screen are closer than 40 cm.

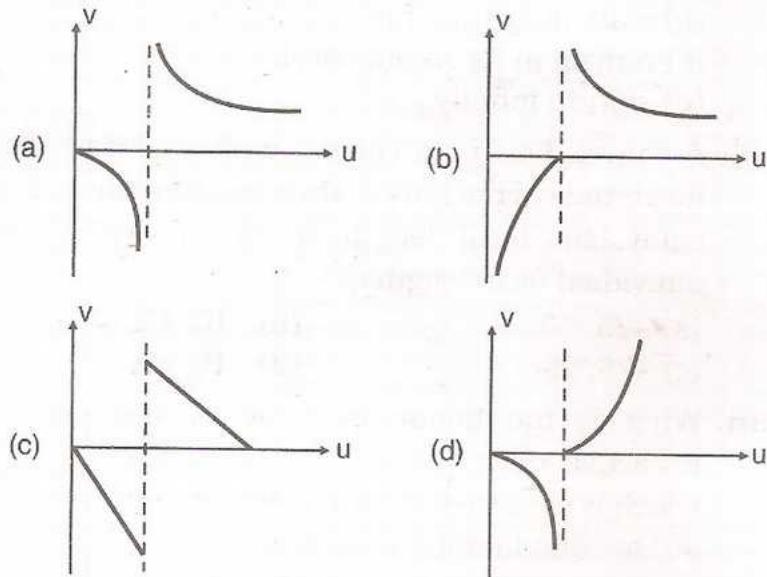


12.1.2 Problems for Practice

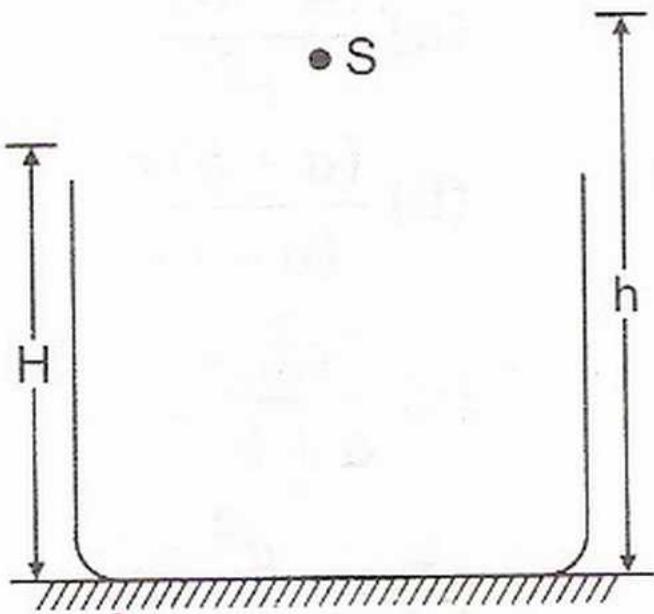
12.1.2.1 General Problem Set

Single Answer Type

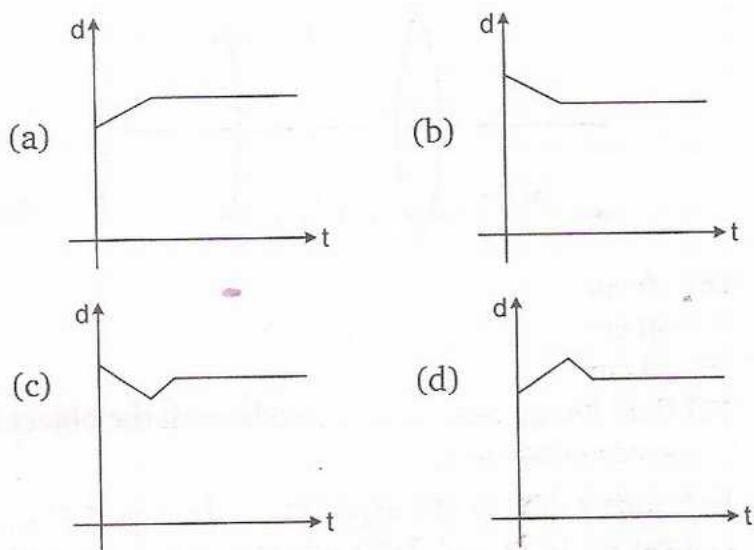
Example : As the position of an object (u) reflected from a concave mirror is varied, the position of the image (v) also varies. By letting the u change from 0 to $+\infty$ the graph between v versus u will be



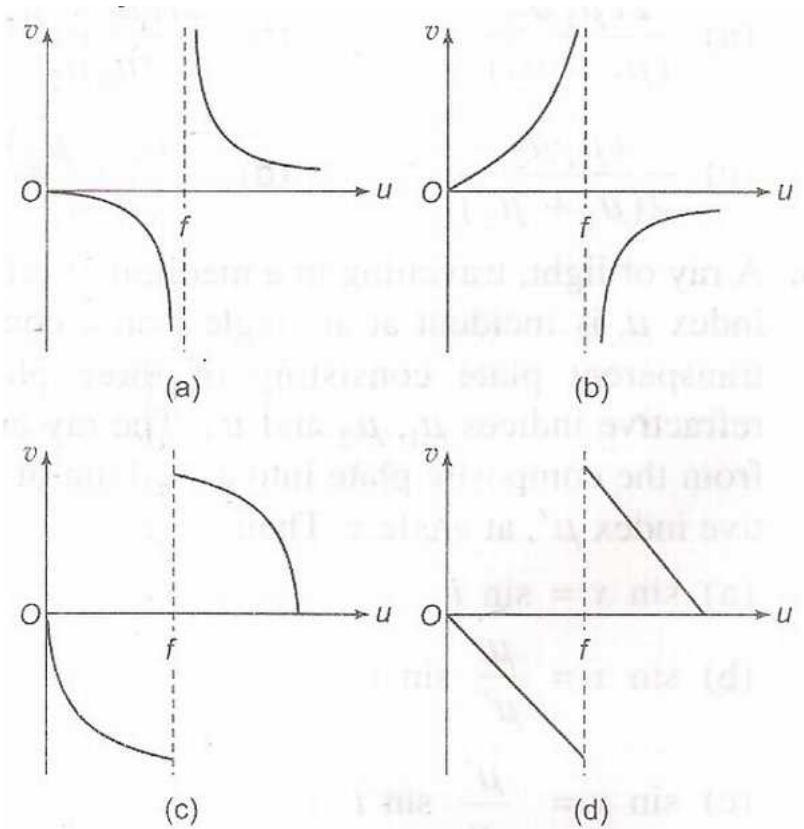
Example : A point source S is placed at a height h from the bottom of a vessel of height H ($< h$). The vessel is polished at the base. Water is gradually filled in the vessel at a constant rate $\alpha m^3/s$.



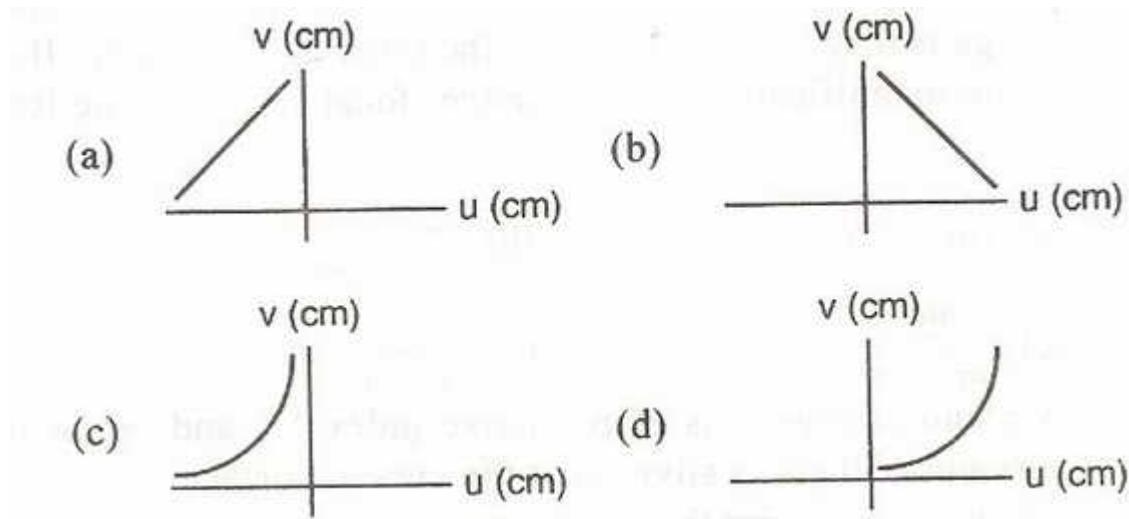
The distance d of image of the source after reflection from mirror from the bottom of the vessel varies with time t as



Multiple Answer Type Example : The image distance (v) is plotted against the object distance (u) for a concave mirror of focal length f . Which of the graphs shown in Figure represents the variation of v versus u as u is varied from zero to infinity?

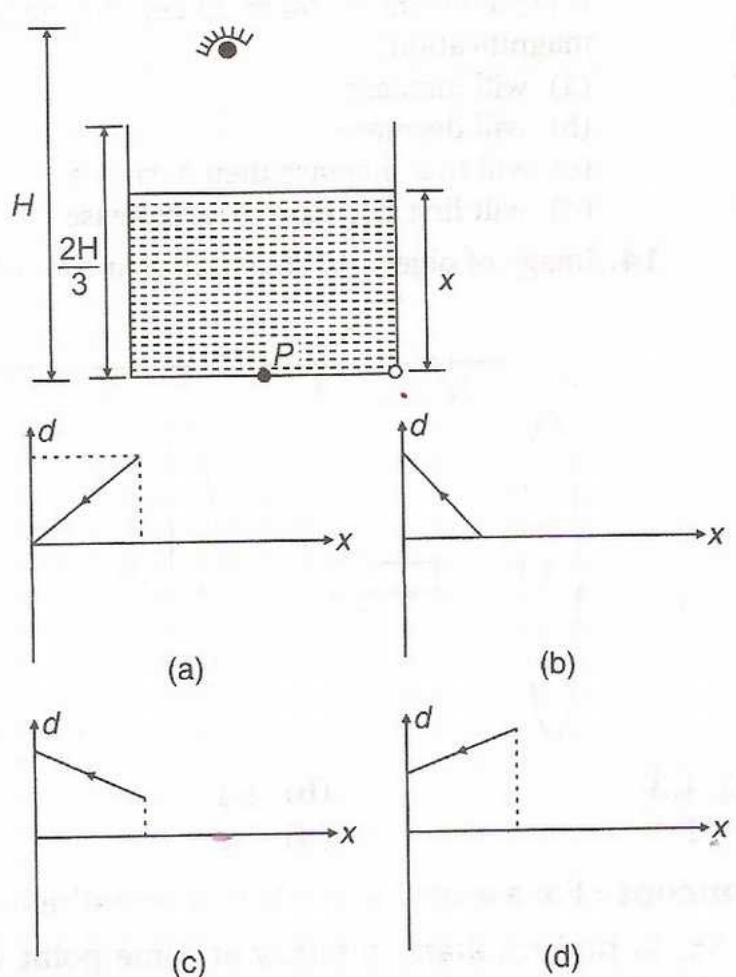


Example : A student measures the focal length of a convex lens by putting an object pin at a distance U from the lens and measuring the distance v of the image pin. Graph between u and v plotted by the student should look like :



Comprehension Type

Comprehension 1 Liquid is filled in a vessel of height $2H/3$. At the bottom of the vessel there is a spot P and a hole from which liquid is coming out. Let d be the distance of image of P from an eye at height H from bottom at an instant when level of liquid in the vessel is x . If we plot a graph between d and x it will be like



Bibliography

(N.d.). URL: http://www.schoolphysics.co.uk/age16-19/Optics/Refraction/text/Lenses_graphs/index.html.

Chapter 13

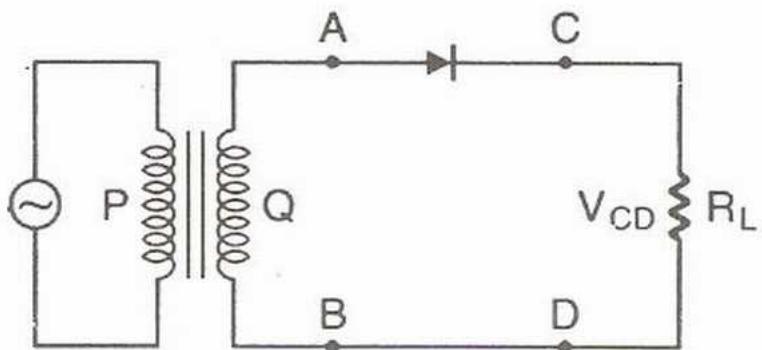
Microelectronics

13.1 Semiconductors

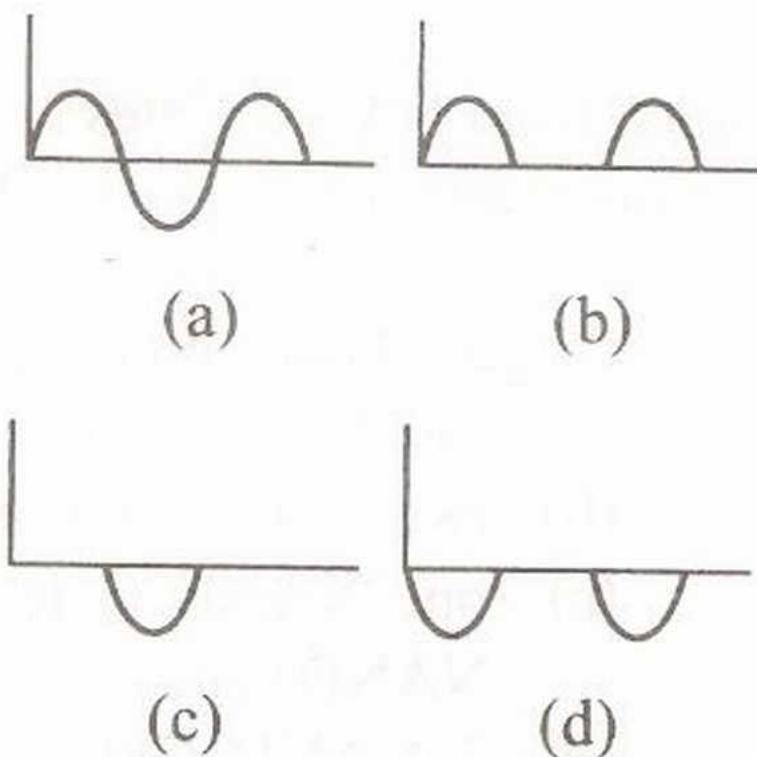
13.1.1 Diode

13.2 Rectifier

Example: In the half-wave rectifier circuit shown:

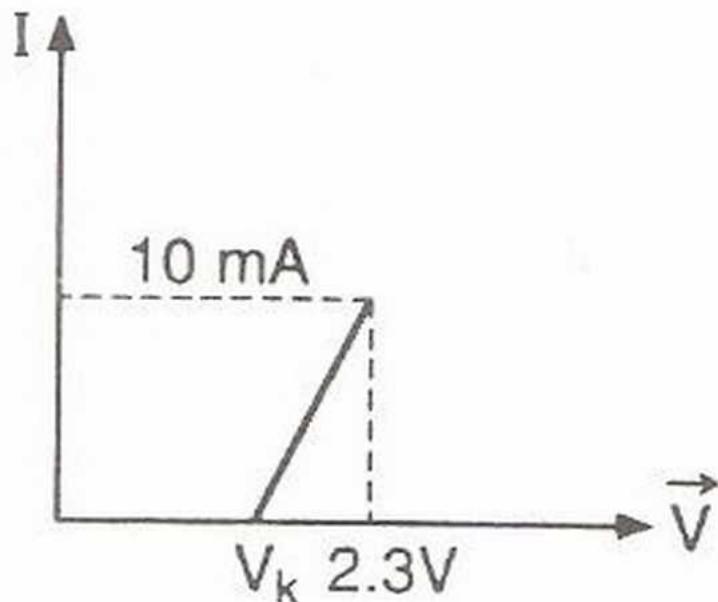


Which one of the following wave forms is true for V_{CD} , the



13.2.0.1 Junction Diode

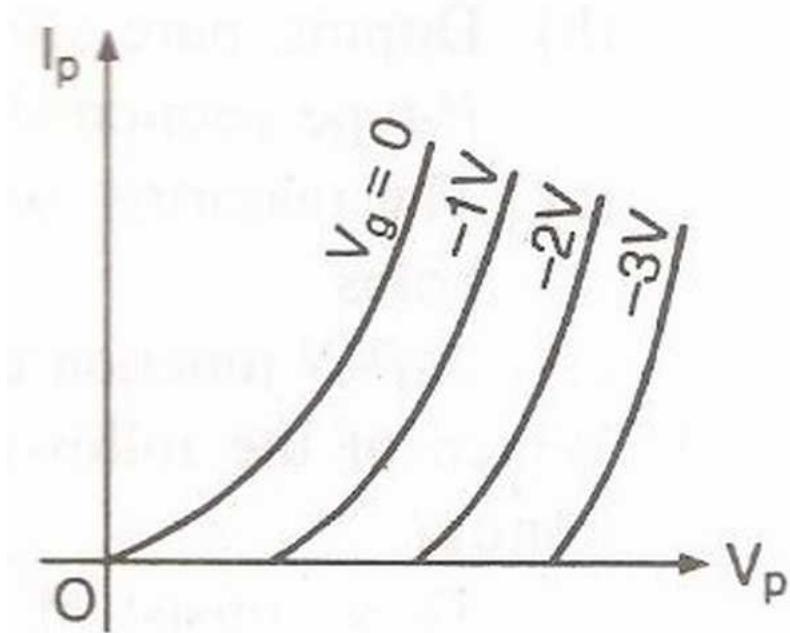
Example: The resistance of a germanium junction diode, whose V-I is shown in figure is: ($V_k = 0.3V$)



- a) $5 \text{ k}\Omega$
- b) $0.2 \text{ k}\Omega$
- c) $2.3 \text{ k}\Omega$
- d) $\left(\frac{10}{2.3}\right) \text{k}\Omega$

13.2.1 Triode

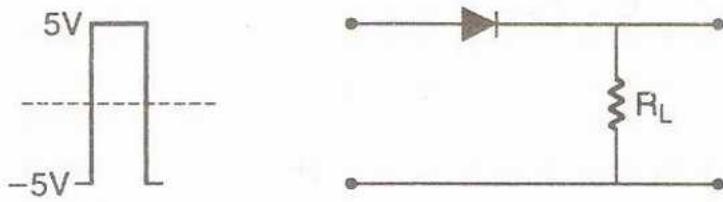
Example: The characteristic of triode shown in Figure. is known as :



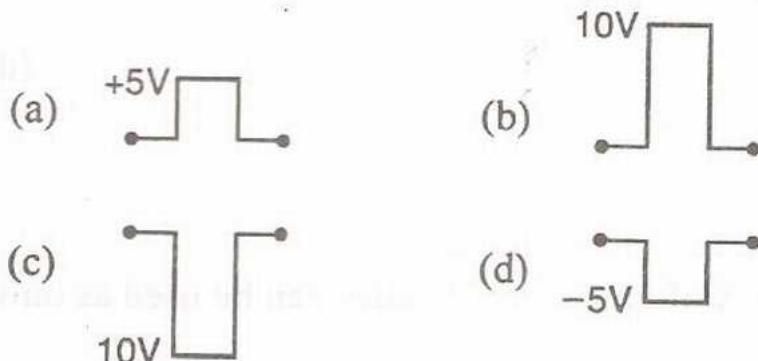
- a) mutual characteristic
- b) transfer characteristic
- c) static plate characteristic
- d) voltage transfer characteristic

13.2.2 p-n Junction

If in a p-n junction diode, a square input signal of 10 V is applied as shown



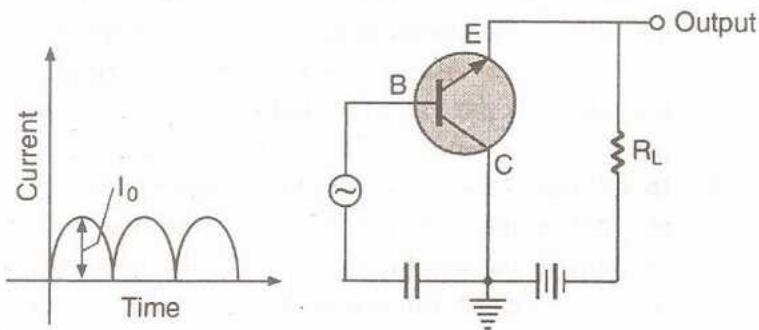
Then the output signal across R_L will be:



{Answer: a) }

13.2.3 Transistors

Example: The output current versus time curve of a rectifier is shown in figure. The average value of the output current in this case is:

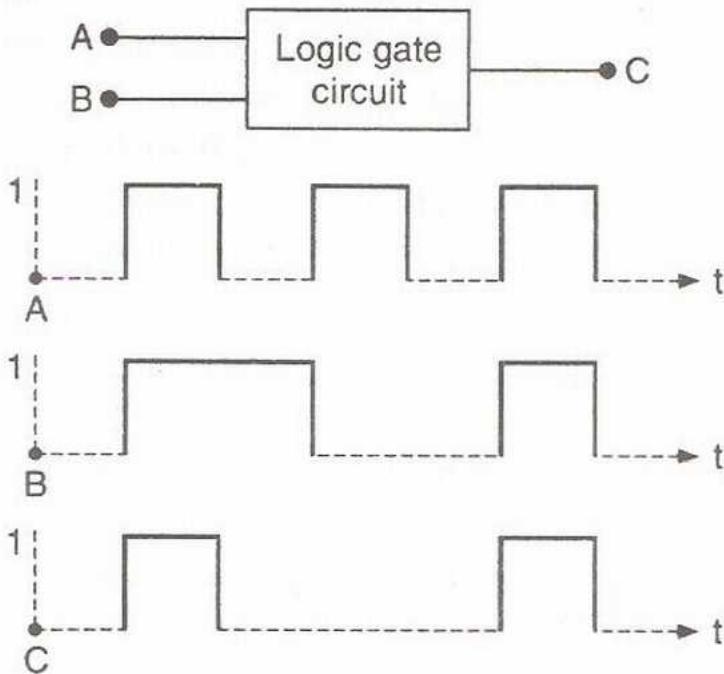


- a) 0
- b) $\frac{I_0}{2}$
- c) $\frac{2I_0}{\pi}$
- d) I_0

13.3 Logic Gates

13.3.1 Problems

Example: The following figure shows a logic gate circuit with two inputs A and B and the output C. The voltage wave forms of A, B and C are as shown below:



The logic circuit gate is

- a) AND gate
 - b) NAND gate
 - c) NOR gate
 - d) OR gate
- { Answer: a) }

Part IV

Mechanics

Mechanics is basically introduction to Mechanical Engineering , that's true but for intermediate level , we mostly present the ideal case only and not the practical cases. So , for example we can assume that a block moving with an acceleration can keep on moving like that for as long time as we want, another particle moving with constant velocity moving like that , now this thing might not look very cool to a proper mechanical engineer say from Delhi College of engineering like my brother Manish Kalia who goes on further to steer in college designed racing cars and airplane designs. However to an IITian mechanical engineer like the Fiitjee head , he goes on further to make new questions and theory working on the ideal case . So, there is a difference, if one is interested in designing real mechanics in India, he should choose DCE and if one wants to work on ideal theory one should choose ME in IIT. What are the other options? You can very well join MIT if you can clear the entrance procedure or some other good college. Now as far as india is concerned DCE is the best in Mechanical and IIT is the in general best college. MIT on the other hand is the world's best college (and the oldest as well). Now manish is into teaching Mathematics for indian college entrance and one of his students Keshav went to MIT and other girl Ravi to UC and there are plenty people in other world over good universities.

So Mechanics which we would study won't have car design constraints, it won't have the windscreen mechanisms it won't have too many examples of tyre motion or other practical situation things. We would be dealing with problems mostly in the ideal case which would appear impractical to someone working on real cases since childhood. Now for the MIT case, it is a research university i would say and people there are really smart and immediately recognized worldover but they are also real humans and have real life problems too. UC i don't know, those people are also doing good i think. Then there is the Stanford where i studied in previous birth , it appears very good to me .

Chapter 14

Kinematics

14.1 Derivation of Newton's Equations of Motion from basic forms and graphs

We will use the basic assumptions like $\vec{v} = \frac{d\vec{r}}{dt}$ (First Equation) and $\vec{a} = \frac{d\vec{v}}{dt}$ (2nd Equation) and the v-t graph to derive the following Newton's equations of motion for constant acceleration case(i.e. in Newton's equations we assume that \vec{a} is constant while the basic first and second equation , we can use everywhere[non-uniform acceleration cases too]).

1. $\vec{v} = \vec{v}_o + \vec{a}t$ (Newton's first equation of motion)
2. $\vec{r} = \vec{r}_o + \vec{v}_o t + \frac{1}{2}\vec{a}t^2$ (Newton's second equation of motion)
3. $\vec{v} \cdot \vec{v} = \vec{v}_o \cdot \vec{v}_o + 2\vec{a} \cdot (\vec{r} - \vec{r}_o)$ (Newton's third equation of motion)

14.1.1 Newton's first equation of motion

The equation is $\vec{v} = \vec{v}_o + \vec{a}t$

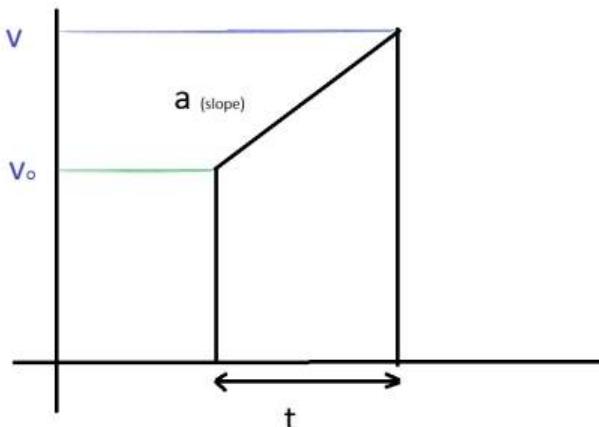
We'll first rewrite a 9th class derivation without the use of first equation.

14.1.1.1 Basic Derivation (without use of calculus or graphs)

We define $\vec{a} = \frac{\vec{v} - \vec{v}_o}{t}$

Cross multiplying, $\vec{a}t = \vec{v} - \vec{v}_o$
 $\Rightarrow \vec{v} = \vec{v}_o + \vec{a}t$

14.1.1.2 Derivation from v-t graph(Scalar form)



We define the slope of v-t graph as a, so it gives $a = \frac{v - v_o}{t}$

Manipulating the form of this equation, we get $v = v_o + at$, which is newton's first equation of motion in scalar form.

14.1.1.3 Calculus Derivation(to be used in our class)

By the 2nd equation General Definition, we have

$$\begin{aligned}\vec{a} &= \frac{\vec{v}}{dt} \\ \Rightarrow d\vec{v} &= \vec{a} dt \\ \Rightarrow \int_{\vec{v}_o}^{\vec{v}} d\vec{v} &= \int_0^t \vec{a} dt \\ \Rightarrow [\vec{v}]_{\vec{v}_o}^{\vec{v}} &= \vec{a} [t]_0^t \\ \Rightarrow \vec{v} - \vec{v}_o &= \vec{a} t \\ \Rightarrow \vec{v} &= \vec{v}_o + \vec{a} t \quad (\text{Derived})\end{aligned}$$

14.1.2 Newton's second equation of motion

The equation is $\vec{r} = \vec{r}_o + \vec{v}_o t + \frac{1}{2} \vec{a} t^2$

We'll first rewrite a 9th class derivation without the use of first equation.

14.1.2.1 Basic Derivation (without use of calculus or graphs)

We'll be using newton's first equation of motion and the definition of average velocity to derive it.

$\vec{v} = \frac{\vec{v} + \vec{v}_o}{2}$ i.e. Average velocity vector is the average of initial velocity and final velocity vectors.

Also, $\vec{s} = \vec{v} t$

$$\Rightarrow \vec{s} = \frac{\vec{v} + \vec{v}_o}{2} t$$

Substituting, $\vec{v} = \vec{v}_o + \vec{a} t$, i.e. the newton's first equation of motion. We get

$$\vec{s} = \frac{\vec{v}_o + \vec{a} t + \vec{v}_o}{2} t$$

Now, $\vec{s} = \vec{r} - \vec{r}_o$, displacement vector is the difference of final position vector and initial position vector.

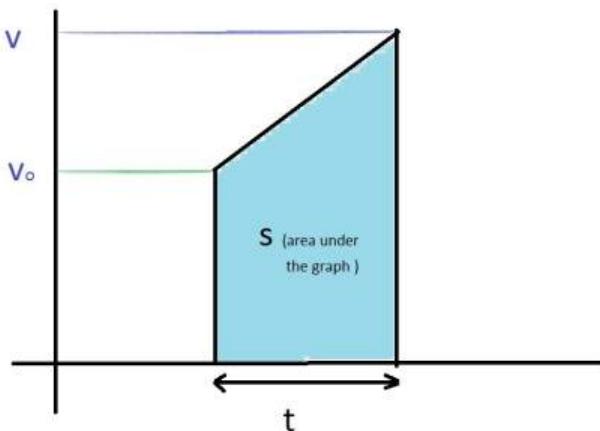
Manipulating, we get

$$\vec{r} - \vec{r}_o = \frac{2\vec{v}_o + \vec{a} t}{2} t$$

OR

$$\vec{r} = \vec{r}_o + \vec{v}_o t + \frac{1}{2} \vec{a} t^2$$

14.1.2.2 Derivation from v-t graph(Scalar form)



Area under the v-t graph is displacement(s)

Area of a trapezium is sum of parallel sides X distance between them

$$\Rightarrow s = \frac{v + v_o}{2} t$$

Substituting, the previously derived newton's first equation of motion in scalar form

$$\Rightarrow s = \frac{v_o + at + v_o}{2} t$$

OR

$$s = v_o t + \frac{1}{2} at^2$$

14.1.2.3 Calculus Derivation(to be used in our class)

We have, $\vec{v} = \frac{\vec{dr}}{dt}$, by first equation general definition

Substituting, newton's first equation of motion

We get

$$\begin{aligned}\vec{v}_o + \vec{a}t &= \frac{\vec{dr}}{dt} \\ \Rightarrow \int_{\vec{r}_o}^{\vec{r}} d\vec{r} &= \int_0^t (\vec{v}_o + \vec{a}t) dt \\ \Rightarrow [\vec{r}]_{\vec{r}_o}^{\vec{r}} &= \left[\vec{v}_o t + \frac{1}{2} \vec{a} t^2 \right]_0^t \\ \Rightarrow \vec{r} - \vec{r}_o &= \vec{v}_o t + \frac{1}{2} \vec{a} t^2 \\ \Rightarrow \vec{r} &= \vec{r}_o + \vec{v}_o t + \frac{1}{2} \vec{a} t^2 \quad (\text{Derived})\end{aligned}$$

14.1.3 Newton's third equation of motion

The equation is $\vec{v} \cdot \vec{v} = \vec{v}_o \cdot \vec{v}_o + 2 \vec{a} \cdot (\vec{r} - \vec{r}_o)$

We'll first rewrite a basic derivation without the use of first equation. It will require dot product.

14.1.3.1 Basic Derivation (without use of calculus or graphs)

We have,

$$\begin{aligned}\vec{v} - \vec{v}_o &= \vec{a}t, \text{ by newton's first equation of motion} \\ \frac{\vec{v} + \vec{v}_o}{2} t &= \vec{s}\end{aligned}$$

Taking dot product

$$(\vec{v} - \vec{v}_o) \cdot \left(\frac{\vec{v} + \vec{v}_o}{2} t \right) = \vec{a}t \cdot \vec{s}$$

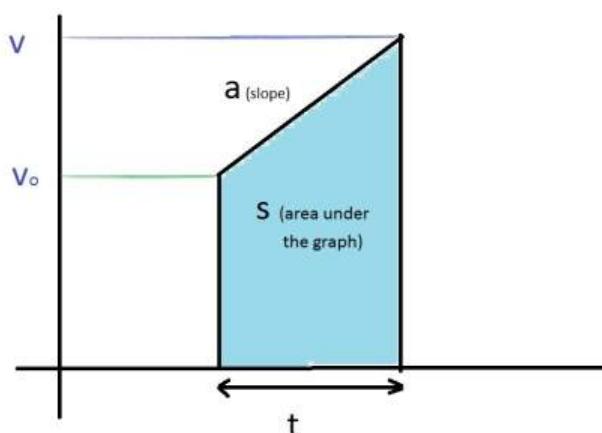
Cancelling t , transferring 2 to right hand side in numerator and opening the brackets, we get

$$\vec{v} \cdot \vec{v} - \vec{v}_o \cdot \vec{v}_o = 2 \vec{a} \cdot \vec{s}$$

Manipulating, substituting the value of displacement vector, we get

$$\vec{v} \cdot \vec{v} = \vec{v}_o \cdot \vec{v}_o + 2 \vec{a} \cdot (\vec{r} - \vec{r}_o)$$

14.1.3.2 Derivation from v-t graph(Scalar form)



In deriving the newton's third equation by graph, two values are taken from the graph

$$a = \frac{v - v_o}{t} \text{ and } s = \frac{v + v_o}{2} t$$

Multiplying both the equations,

$$as = \frac{v^2 - v_o^2}{2}$$

OR

$$v^2 - v_o^2 = 2as$$

14.1.3.3 Derivation(to be used in our class)

We have, $\vec{v} = \frac{\vec{dr}}{dt}$ (First Equation) and $\vec{a} = \frac{\vec{dv}}{dt}$ (2nd Equation) as the basic General Definition

Reversing the sides of the second equation and taking dot product, we get

$$\vec{v} \cdot \vec{dv} = \vec{a} \cdot \vec{dr}$$

Integrating,

$$\int_{\vec{v}_o}^{\vec{v}} \vec{v} \cdot \vec{dv} = \int_{\vec{r}_o}^{\vec{r}} \vec{a} \cdot \vec{dr}$$

$\Rightarrow \left[\frac{\vec{v} \cdot \vec{v}}{2} \right]_{\vec{v}_o}^{\vec{v}} = \vec{a} \cdot [\vec{r}]_{\vec{r}_o}^{\vec{r}}$ (This type of Integral in dot product, we'll study at bachelor's level, here we can prove it using vector's components)

Substituting the values of limits,

$$\frac{\vec{v} \cdot \vec{v} - \vec{v}_o \cdot \vec{v}_o}{2} = \vec{a} \cdot \vec{s}$$

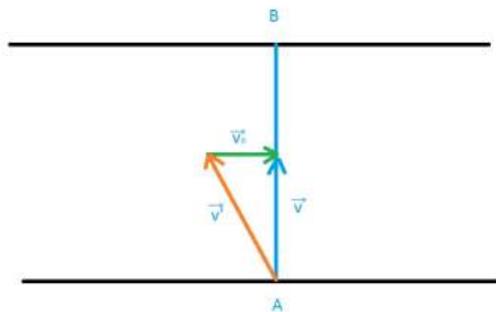
$$\Rightarrow \vec{v} \cdot \vec{v} - \vec{v}_o \cdot \vec{v}_o = 2 \vec{a} \cdot \vec{s} \quad \text{——— (Derived)}$$

14.2 Relative Velocity

14.2.1 Crossing the River problems (Theory)

Theory Problem 1 : Two swimmers leave point A on one bank of the river to reach point B lying right across on the other bank. One of them crosses the river along the straight line AB while the other swims at right angles to the stream and then walks the distance that he has been carried away by the stream to get to point B. What was the velocity u of his walking if both swimmers reached the destination simultaneously? The stream velocity $v_o = 2.0$ km/hour and the velocity v' of each swimmer with respect to water equals 2.5 km per hour.

Solution : Case I Swimmer swims with final velocity along AB, this case is also called the **Shortest Path** case.

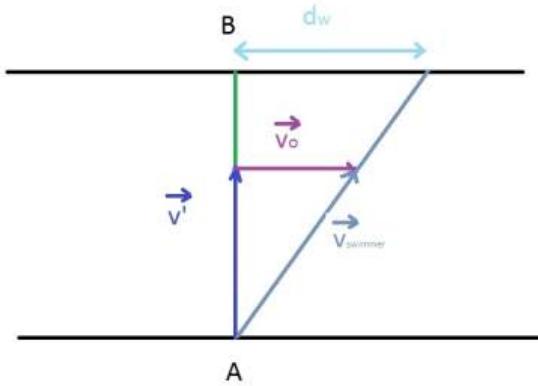


We see that $\vec{v}' + \vec{v}_o = \vec{v}$ and they form a right triangle

$$\Rightarrow v = \sqrt{v'^2 + v_o^2}$$

$$\text{time to reach point B} = \frac{AB}{\sqrt{v'^2 + v_o^2}}$$

Case II : Swimmer swims at right angle to the stream, this would require the "**Shortest time in crossing the river**" but the problem requires that he would have to walk to get to B.



We see that $\vec{v'} + \vec{v}_o = \vec{v}$ and they form a right triangle, though this time with \vec{v} as the hypotenuse. It should be noted however that this time v' , v_o and v vectors are different from Case I while the magnitudes of v' and v_o are the same. Our problem Case II is independent in this regard from case I and we can choose the same names without loss of generality

Proceeding to solve the question

$$\frac{AB}{v'} = \frac{d_w}{v_o} \text{ (By similarity of triangles, from figure)}$$

$$\Rightarrow d_w = \frac{v_o}{v'} \cdot AB$$

Time t_1 to cross the river and reach the point (say C)

$$t_1 = \frac{AB}{v'}$$

Time t_2 to cross the distance d_w back to B

$$t_2 = \frac{d_w}{u}$$

$$\text{Total time} = t_1 + t_2 = \frac{AB}{v'} + \frac{d_w}{u} = \frac{AB}{v'} + \frac{\frac{v_o}{v'} \cdot AB}{u}$$

Proceeding to solve the problem by comparing both the cases,

$$\frac{AB}{\sqrt{v'^2 - v_o^2}} = \frac{AB}{v'} + \frac{\frac{v_o}{v'} \cdot AB}{u}$$

Solving further, we get the required form, first cancelling AB and cross multiplying v'

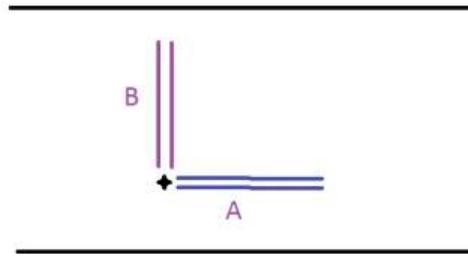
$$\frac{v'}{\sqrt{v'^2 - v_o^2}} - 1 = \frac{v_o}{u}$$

$$\Rightarrow u = \frac{\frac{v_o}{v'}}{\frac{v'}{\sqrt{v'^2 - v_o^2}} - 1}$$

Calculating the value, $u = 3 \text{ km/hr}$

Theory Problem 2 : Two boats, A and B, move away from a buoy anchored at the middle of a river along the mutually perpendicular straight lines: the boat A along the river, and the boat B across the river. Having moved off an equal distance from the buoy the boats returned. Find the ratio of times of motion of boats τ_A/τ_B if the velocity of each boat with respect to water is $\eta = 1.2$ times greater than the stream velocity.

Solution:



Let the stream velocity be v_o , then the boat velocity with respect to water is ηv_o . Also let us assume that the equal distance be d .

Case A : Final velocity of boat A in forward journey (in the stream direction)

$$v = \eta v_o + v_o = (\eta + 1)v_o$$

$$\text{Time to reach the destination} = t_1 = \frac{d}{(\eta + 1)v_o} \dots\dots\dots(1)$$

Final velocity of boat A in the backward journey (opposite to the stream direction)

$$v = \eta v_o - v_o = (\eta - 1)v_o$$

$$\text{Time to reach back to the buoy} = t_2 = \frac{d}{(\eta - 1)v_o} \dots\dots\dots(2)$$

$$\tau_A = t_1 + t_2 = \frac{d}{(\eta + 1)v_o} + \frac{d}{(\eta - 1)v_o} = \frac{2\eta}{\eta^2 - 1} \frac{d}{v_o}$$

Case B : Here we proceed according to the shortest path case in Theory Problem 1.

In Forward Journey (Upwards in the figure)

$$t_1 = \frac{d}{\sqrt{\eta^2 - 1} v_o}$$

In Backwards Journey (Downwards in the figure)

$$t_2 = \frac{d}{\sqrt{\eta^2 - 1} v_o} = t_1$$

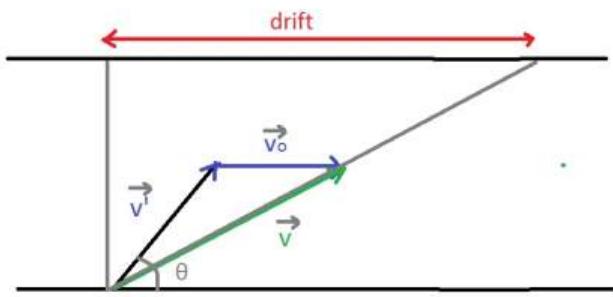
$$\tau_B = t_1 + t_2 = 2t_1 = \frac{2}{\sqrt{\eta^2 - 1}} \frac{d}{v_o}$$

$$\frac{\tau_A}{\tau_B} = \frac{\frac{2\eta}{\eta^2 - 1} \frac{d}{v_o}}{\frac{2}{\sqrt{\eta^2 - 1}} \frac{d}{v_o}} = \frac{\eta}{\sqrt{\eta^2 - 1}}$$

Substituting the value, we get $6/\sqrt{11}$ as the ratio. = 1.81 (approx.)

Theory Problem 3 : A boat moves relative to water with a velocity which is $n = 2.0$ times less than the river flow velocity. At what angle to the stream direction must the boat move to minimize drifting?

Solution : Let the river flow velocity = v_o . and velocity of boat relative to water would be then $v' = v_o/n$



Across component(along y axis) of v is $v' \sin \theta$

Drift component(along x axis) of v is $v' \cos \vartheta + v_o$

$$\text{drift} = \frac{AB}{v' \sin \theta} (v' \cos \vartheta + v_o) = AB \left(\cot \theta + \frac{v_o}{v'} \operatorname{cosec} \theta \right) = AB (\cot \theta + n \operatorname{cosec} \theta)$$

Now we have to minimize drift,

$$\text{At minima, } \frac{d}{d\theta} \text{drift} = 0$$

$$\Rightarrow \frac{d}{d\theta} AB (\cot \theta + n \operatorname{cosec} \theta) = 0$$

$$\Rightarrow AB (\operatorname{cosec}^2 \theta + n \operatorname{cosec} \theta \cot \theta) = 0$$

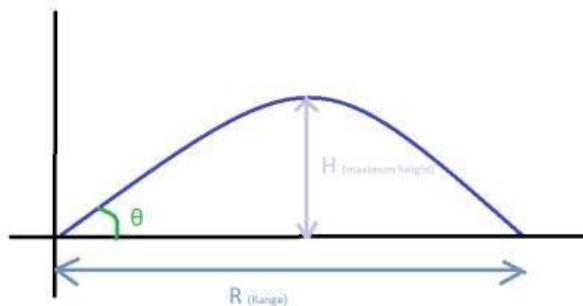
$$\Rightarrow \cot \theta = -\frac{1}{n}$$

The only solution in $(0, \pi)$ is 120° .

14.3 Motion in 2D

14.3.1 2D Projectile Motion

14.3.1.1 Projection at an angle to the Horizontal



From Figure, we get

$$v_x = u \cos \theta \dots\dots\dots (1)$$

$$v_y = u \sin \theta - gt \dots\dots\dots (2)$$

$$x = u \cos \theta t \dots\dots\dots (3)$$

$$y = u \sin \theta t - \frac{1}{2} g t^2 \dots\dots\dots (4)$$

Case I : Maximum height, H

At maximum height, v_y is zero.

This gives the time at maximum height, (equating equation (2) to zero)

$$T_H = \frac{u \sin \theta}{g} \quad \text{---(Supplementary Result)}$$

Substituting this value of T_H in y-> equation (4), we get maximum height (H)

$$H = u \sin \theta \cdot \frac{u \sin \theta}{g} - \frac{1}{2} g \left(\frac{u \sin \theta}{g} \right)^2$$

$$H = \frac{u^2 \sin^2 \theta}{2g} \quad \text{---(Primary Result) [Maximum Height]}$$

Also , the value of x at this point

$$X_H = u \cos \theta \cdot T_H = \frac{u^2 \sin 2\theta}{2g} \quad \text{---(Supplementary Result)}$$

Case II: Range, R and Time of Flight, T

At R, y=0.

Substituting this value in equation (4), we get

$$T = \frac{2u \sin \theta}{g} \quad \text{---(Primary Result) [Time of Flight]}$$

Interestingly, $T = 2T_H$ ---(Supplementary Result)

Substituting the time of flight in x, we get R

$$R = u \cos \theta \cdot T = u \cos \theta \cdot \frac{2u \sin \theta}{g}$$

$$R = \frac{u^2 \sin 2\theta}{g} \quad \text{---(Primary Result) [Range]}$$

Also, $R = 2X_H$ implying that Maximum height occurs at half the Flight range.

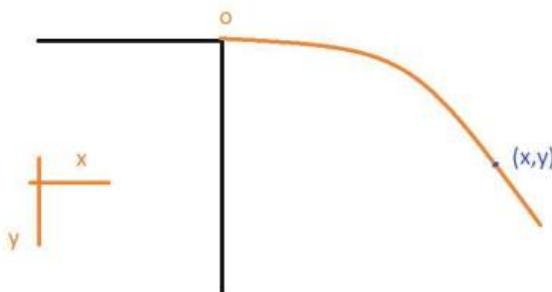
Case III : Equation of Trajectory

Eliminating t from the equations of x and y (3 and 4) we get

$$y = u \sin \theta \cdot \left(\frac{x}{u \cos \theta} \right) - \frac{1}{2} g \cdot \left(\frac{x}{u \cos \theta} \right)^2$$

$$y = x \tan \theta - \frac{1}{2} g \frac{x^2}{u^2 \cos^2 \theta} \quad \text{---(Primary Result) [Equation of Trajectory]}$$

14.3.1.2 Horizontal Projection (Corollary)



In the case of Horizontal Projection

$$v_x = u$$

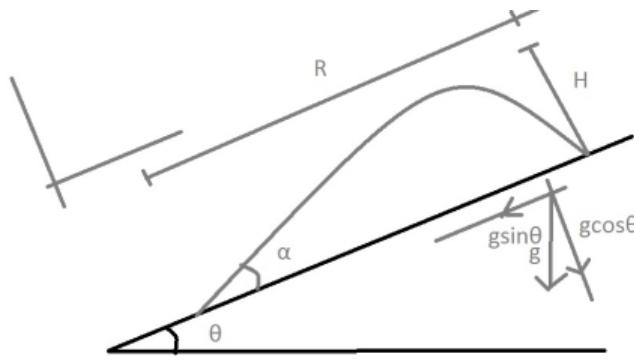
$$v_y = gt$$

$$x = ut$$

$$y = \frac{1}{2}gt^2$$

14.3.1.3 Projection on Inclined Plane

For an inclined plane, we shift the co-ordinate axis x along the plane and y perpendicular to it. This is not necessary, but convenient usually. However, there are cases when it's useful to treat a projectile on an inclined plane as a normal projectile of the above two cases.



$$v_x = u \cos \alpha - g \sin \theta t$$

$$v_y = u \sin \alpha - g \cos \theta t$$

$$x = u \cos \alpha t - \frac{1}{2} g \sin \theta t^2$$

$$y = u \sin \alpha t - \frac{1}{2} g \cos \theta t^2$$

Case I : Maximum Distance from the plane

At this Distance, $v_y = 0$

$$\text{Solving we get , } t_H = \frac{u \sin \alpha}{g \cos \theta}$$

Substituting in y, we get

$$H = u \sin \alpha \cdot \frac{u \sin \alpha}{g \cos \theta} - \frac{1}{2} g \cos \theta \left(\frac{u \sin \alpha}{g \cos \theta} \right)^2$$

$$\Rightarrow H = \frac{u^2 \sin^2 \alpha}{2g \cos \theta}$$

Case II : Range along the plane, Time to hit the plane

At R, $y = 0$

$$t_R = \frac{2u \sin \alpha}{g \cos \theta} = 2t_H \text{ (Even when the maximum distance occurs at an unsymmetrical point)}$$

Substituting in x,

$$R = u \cos \alpha \cdot \frac{2u \sin \alpha}{g \cos \theta} - \frac{1}{2} g \sin \theta \left(\frac{2u \sin \alpha}{g \cos \theta} \right)^2$$

$$\Rightarrow R = \frac{2u^2 \sin \alpha \cos (\alpha + \theta)}{g \cos^2 \theta}$$

Chapter 15

Laws of Motion

15.1 Problem solving techniques

15.1.1 Free Body Diagrams

15.1.2 The Constraint equation

TO BE CONTINUED.....

Index

2D Projectile Motion, 165

A

Angular Momentum Conservation, 75

C

Calibration Curve, 128

Capacitors, 133

Common Graph Forms in Physics, 32

Crossing the River problems (Theory), 162

Current Electricity, 128

Curve Fitting, 29

D

Derivation of Newton's Equations of Motion from basic forms and graphs, 159

Double-Inverse Relationship, 34

E

Electromagnetism, 125

Electrostatics, 125

Elements of a good Graph, 27

Energy Conservation, 66

Exponential Relationship, 36

F

Friction, 52

G

Graphical Analysis of Data, 30

Graphical Representation of Uncertainties, 29

Gravitation, 75

H

Heat, 97

I

Inverse Relationship, 33

Inverse-Square Relationship, 33

L

Lattice, 139

Laws of Motion, 49

Linear Relationship, 32

Logic Gates, 154

M

Magnetic Field, 135

Matter, 139

Mechanical Waves, 121

Modulii of Elasticity, 92

Motion in 2D, 165

N

Natural Log (LN) Relationship, 36

Newton's first equation of motion, 159

Newton's second equation of motion, 160

Newton's third equation of motion, 161

Nucleus, 140

O

Optics, 143

P

Periodic Motion, 84

Photoelectric Effect, 139

p-n Junction, 153

Polynomial of Second Degree, 35

Power Relationship, 35

Power Relationship 2, 35

Previous Years IIT Problems, 47, 65, 69, 82, 91, 117, 123, 139, 140

R

Ray Optics, 143

Relative Velocity, 162

Rotatory Motion, 71

S

Semiconductors, 151

SHM, 84

Sound, 122

Statics, 92

T

The Average-Velocity / Instantaneous Velocity , Equal Case, 45

The First Equation, 41

The Second Equation, 43

Thermodynamics, 97

Transistors, 153

Triode, 152

U

Uncertainty in a Slope, 30

W

Waves, 121