# A Complete Course in Mathematics - Birthday Edition

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# Preface

We have reached the concept of Master Problems for Intermediate level Mathematics ( +1/+2) after long research and have put our complete experience in the form of our book.

This book was long pending to be written in the form of a complete bundle and now the job has finally been done. Manish Kalia has already mentored more than 150 students in top 500 ranks in these few years of his career. Manish Kalia has already been adjudged as the best teacher in and near chandigarh by all of the students who were able to make comparisions.

Alpha Classes started with a boom in 2013 with a catchy website showing all of the students internal testing process and internal rankings which finally went on the same way to the IITs etc exams. The concept got good reviews too.

# Acknowledgements

The complete family has been supportive, viz Manas, Neha , Malkit , all of them . We owe all of our book’s success to them.

# Part I. Calculus

# 1 Preliminaries

## 1.1 Relations

Q: Show that the relation R in the set of real numbers defined as *R* = {(*a*, *b*):√(*a*) + 3≤*b*} is neither reflexive nor symmetric nor transitive.

### 1.1.1 Problems for Practice

#### 1.1.1.1 Matrix match type problems

Matrix 1: Under column-I properties of relations are mentioned. Match the stated property with the relations listed under column-II , that satisfies it

|  |  |
| --- | --- |
| Column I | Column II |
| (A) Equivalence | (P) The relation R in the set of |
|  | natural numbers N, defined as |
|  | *R* = {(*a*, *b*):*a* > *b*2} |
| (B) Reflexive | (Q) Relation R defined in the |
|  | set {1, 2, 3, 4, 5} defined by |
|  | *R* = {(*a*, *b*):*a*2 + *ab* − 2*b*2 = 0} |
| (C) Symmetric | (R) Relation R defined on the set |
|  | of real numbers ℝ , defined by |
|  | *R* = {(*a*, *b*):√(*a*) + 5 < *b*} |
| (D) Transitive | (S) Relation R in the set |
|  | *A* = {*xϵ*ℤ;0 ≤ *x* ≤ 9},given by |
|  | R={(*a*, *b*):(*a* − *b*)4 is divisible by 3} |

## 1.2 Real Numbers

When we talk of functions, we talk of only real valued functions. So, it is important that we brush up our theory of real numbers done in junior classes.

**Definition**

Any decimal fraction, terminating or nonterminating, is called a [↓](" \l "index-Real-Numbers)real number.

The set of real numbers is represented by the symbol ℝ.

### 1.2.1 Rational Numbers

Every rational number[↓](" \l "index-Rational-numbers) can be written in the form (*p*)/(*q*) where p and q are integers. The set of rational numbers is reqresented by the symbol **ℚ**. i.e. **ℚ = {(*p*)/(*q*) | *q* ≠ 0,  *p* & *q* *ϵ* ℤ}** where ℤ is the set of Integers. The decimal representation of a rational number can be either [↓](" \l "index-Terminating-Decimal-Expansion)terminating or [↓](" \l "index-Non-Terminating-Recurring-Decimal)non-terminating reccuring. e.g.

3.24 is a terminating decimal represenation.

5.3 is a non-terminating recurring decimal representation.

**Properties**

* The set of rational numbers is **closed** with respect to the operations of addition, subtraction and product. i.e. If *p* and *q* are two rational numbers, then *p* + *q* , *p* − *q* and *pq* are rational. Also (*p*)/(*q*) is rational provided *q* ≠ 0.**[****[↓](" \l "index-Closure-Property-of-Rational-Numb)Closure Property]**
* Between any two rational numbers *p* & *q* , there exist infinite rational numbers.[**Property of Denseness**]

**Q. Find all the rational values of *x* for which *y* = √(*x*2 + *x* + 5) is a rational number.**

Sol. Suppose *x* and *y* are rational numbers . Then the sum *x* + *y* = *q* is a rational number.We now express *x* in terms of *q* .

*y* + *x* = √(*x*2 + *x* + 5) + *x* = *q*

√(*x*2 + *x* + 5) = *q* − *x*

*x*2 + *x* + 5 = *q*2 − 2*qx* + *x*2

*x* = (*q*2 − 5)/(1 + 2*q*), *q* ≠  − (1)/(2)

To prove the reverse i.e. *y* = √(*x*2 + *x* + 5) is a rational number if *x* = (*q*2 − 5)/(1 + 2*q*), *q* ≠  − (1)/(2)  
Its easy to check

*y* = √(*x*2 + *x* + 5) = √(((*q*2 − 5)/(1 + 2*q*))2 + ((*q*2 − 5)/(1 + 2*q*)) + 5)

 = √((*q*4 + 2*q*3 + 11*q*2 + 10*q* + 25)/((1 + 2*q*)2))

 = √(((*q*2 + *q* + 5)/(1 + 2*q*))2)

It may be observed that *q*2 + *q* + 5 is positive for all values of *q*.Hence

*y* = (*q*2 + *q* + 5)/(|1 + 2*q*|)

which is rational for *q* ≠  − (1)/(2).

Note : The sole purpose of adding *x* to *y* was to eliminate the square term of *x* . The same goal can be achieved by subtracting *x* from *y*.

### 1.2.2 Irrational Numbers

[↓](" \l "index-Non-terminiating-Non-recurring-de)Non-terminating non-repeating decimal fractions are called Irrational numbers . In fact, all real numbers which are not rational are Irrational numbers. The set of Irrational numbers can be represented by the symbol **T** or **ℝ − ℚ .**

**Properties**

* The sum,difference,product and quotient of two Irrational numbers may be rational or irrational.ie. The set **T** is not closed with respect to addition,subtraction,multiplication and division.**[Closure Property****[↓](" \l "index-Closure-Property-of-Irrational-Nu)]**
* The sum, difference,product and quotient of a rational number and an Irrational number is Irrational.
* There exist infinite rational and irrational numbers between two real numbers.**[Property of Denseness****[↓](" \l "index-Property-of-Denseness)]**

**Q. Prove that for every rational number *r* satisfying *r*2 < 3 one can always find a larger rational number *r* + *k*(*k* > 0)for which (*r* + *k*)2 < 3.**

Sol. We know, there exist infinite rational numbers between a rational number and an irrational number. So, there will be infinite rational numbers between *r* and √(3) . So, there exist infinite values of *k* satisfying the condition.Let us take the case when *r* is sufficiently close to √(3).In that case, we can chose a *k* such that *k* → 0. This implies *k*2 < *k*. i.e. (*r* + *k*)2 < *r*2 + 2*rk* + *k*. It is sufficient to put *r*2 + 2*rk* + *k* = 3.i.e.

*k* = (3 − *r*2)/(1 + 2*r*)

It may be observed that any rational number smaller than (3 − *r*2)/(1 + 2*r*) in magnitude will also satisfy the condition.

### 1.2.3 Problems for practice

#### 1.2.3.1 Subjective Problems

Q1. Find all the rational values of *x* for which *y* = √(49*x*2 + 5*x* + 3) is a rational number.

Q2. Prove that for every rational number *s* satisfying *s*2 > 5 one can always find a smaller rational number *s* − *k*(*k* > 0)for which (*s* − *k*)2 > 5.

Q3. Does 219.15155155551555555551……… represent a rational number?. Give reason.

Q4. Represent √(31) − 3 and √(52) on the number line.

#### 1.2.3.2 Multiple Answer MCQ’s

Q1. Which of the following are rational numbers.

a) (√(3))/(2 + √(3))   
b) (2)/(6)  
c) (√(5) − 2)(√(5) + 2)  
d) (11*π*)/(7*π*)

#### 1.2.3.3 Write the Answer Type Questions

Q1: The solution set of (7)/(*x* − 2) ≥ 5 is

#### 1.2.3.4 Assertion-Reason Type Questions

Each question in this section has four choices (a),(b),(c) and (d) out of which only one is correct. Mark your choices as follows:

(a) STATEMENT-1 is True, STATEMENT-2 is True, STATEMENT-2 is the correct explaination of STATEMENT-1.

(b) STATEMENT-1 is True, STATEMENT-2 is True, STATEMENT-2 is **NOT** the correct explaination of STATEMENT-1.

(c) STATEMENT-1 is True, STATEMENT-2 is False.  
(d) STATEMENT-1 is False, STATEMENT-2 is True

**Q1:** STATEMENT-1: For every rational number *r* satisfying *r*2 < 2 one can always find a larger rational number *r* + *k*(*k* > 0)for which (*r* + *k*)2 < 2.

STATEMENT-2: There exist infinite rational numbers between any two rational numbers.

#### 1.2.3.5 Hints and Solutions

**Subjective Problems**

Q1 *x* = (3 − *q*2)/(14*q* − 5) , *qϵ*ℚ& *q* ≠ (5)/(14) {Hint: Put *y* − 7*x* = *q* to get the answer. Also, equivalently if we had put *y* + 7*x* = *q* we would have got , *x* = (*q*2 − 3)/(14*q* + 5), *q* ≠  − (5)/(14)}

Q2 {Hint: Let us take *s*2 − 2*sk* = 5 which would imply *s*2 − 2*sk* + *k*2 > 5 . ie *k* = (*s*2 − 5)/(2*s*)}  
Q3 No , it is not a rational number. {Hint. 2*n*− 1fives between *n*th and (*n* + 1)th unity. The pattern is definite but non periodic.}  
Q4 {Hint. √(*n*) can be drawn by the following method of construction. Draw an arc from the origin of length (*n* − 1)/(2) . From the point where it cuts the y axis, draw an arc of length (*n* + 1)/(2) intersecting the x-axis at P. The distance OP is √(*n*) .}

**Multiple Answer MCQ’s**

Q1 B,C,D

**Write the Answer Type Questions**

Q1 the half open interval (2,17/5] [A]  [A] Intervals can be of the following kinds

1. Open Interval i.e of the kind (a,b)
2. Closed Interval i.e. of the kind [a,b]
3. Semi-Closed Interval(or Semi-open/Half-open) ie. of the kind (a,b] or [a,b)

**Assertion Reason Type Questions**

Q1 B

## 1.3 Absolute value of a real number

The magnitude of a number is called the [↓](" \l "index-Absolute-value-of-the-number)Absolute value of the number.Also,it is the distance of the point representing the number from origin on the number line . e.g. |2 − √(5)| = √(5) − 2.

**Mathematical Definition**

|***x***| = ⎧⎨⎩  − *x*  , *x* < 0        *x*  , *x*≥0

Some alternate definitions of |*x*| are

**|*x*|** =⎧⎪⎨⎪⎩ √(*x*2)     *max*{*x*,  − *x*}     *x*.*sgn*(*x*)

### 1.3.1 Properties of Absolute value

1. |*x*| = *α*(*α* > 0) ⇒ *x* = ±*α*
2. |*x*|≤*α*(*α* > 0) ⇒  − *α*≤*x*≤*α*
3. |*x*|≥*α*(*α* > 0) ⇒ *x*≥*α*or *x*≤ − *α*
4. |*x* + *y*|≤|*x*| + |*y*| **[↓](" \l "index-The-Triangle-Inequality)**{Equality occurs when *x* and *y* are of same sign. i.e. *xy*≥0 e.g.|( − 5) + ( − 2)| = | − 5| + | − 2| as (-5).(-2) is positive }[**The Triangle Inequality**] [B]  [B]

A proof of the Triangle Inequality  
|*x* + *y*|2 = (*x* + *y*)2  
 = *x*2 + 2*xy* + *y*2  
 ≤ *x*2 + 2|*x*||*y*| + *y*2  
 = |*x*|2 + 2|*x*||*y*| + |*y*|2  
 = (|*x*| + |*y*|)2  
⟹|*x* + *y*| ≤ |x|+|y|

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* + |*x* − *y*|≤|*x*| + |*y*| {Equality occurs when *x* and *y* are of opposite sign. i.e. *xy*≤0}
  + |*x* + *y*|≥||*x*| − |*y*|| {Equality occurs when *x* and *y* are of opposite sign. i.e. *xy*≤0}
  + |*x* + *y*|≥|*x*| − |*y*| {Equality occurs when *x* and *y* are of opposite sign i.e. *xy*≤0 and |*x*|≥|*y*|}
  + |*x* − *y*|≥||*x*| − |*y*|| {Equality occurs when *x* and *y* are of same sign. i.e. *xy*≥0}
  + |*x* − *y*|≥|*x*| − |*y*| {Equality occurs when *x* and *y* are of same sign . i.e. *xy*≥0 and |*x*|≥|*y*|}

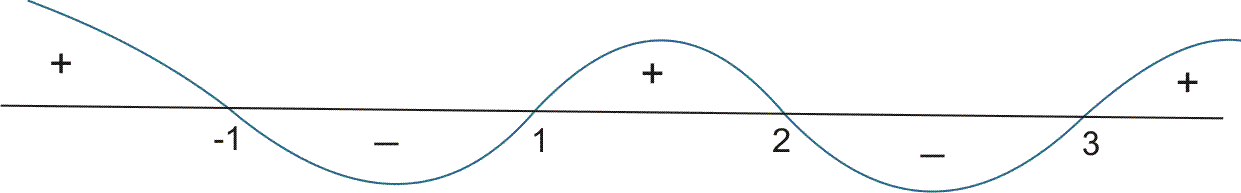
1. |*xy*| = |*x*||*y*|
2. |(*x*)/(*y*)| = (|*x*|)/(|*y*|),*y* ≠ 0

### 1.3.2 [↓](" \l "index-The-Wavey-Curve-Technique)The Wavey-Curve Technique

Let us consider, that we have an expression of the form

*e* = ((*x* + 1)(*x* − 1)(*x* − 3))/(*x* − 2)

and we have to find the values of *x* for which this expression is  > 0 ,  < 0 or  = 0 . To find the sign of the expression, we first of all mark these points on the number line . Then we draw a curve like the one shown in the figure



We start from the right hand side , draw a positive section of curve between ∞ and the right most point. Then, at the rightmost point( 3 in this case) , we switch the sign of the curve to negative and take it below the axis. Like this , we change sign of the curve at every [↓](" \l "index-Critical-Points-in-a-Wavey-Curve)Critical point( ie. at 3,2,1,-1 in this case). Now from this curve, we try to find out the sign of the expression

*e* = ((*x* + 1)(*x* − 1)(*x* − 3))/(*x* − 2)

*e* > 0 whenever + sign appears in the wavey-curve( i.e. when wavey-curve is above the axis)

 ⇒ *e* > 0 for *xϵ*( − ∞,  − 1)∪(1, 2)∪(3, ∞) [C]  [C]

The actual reason for the positive sign of *e* in the interval (3, ∞) is that in this interval the expressions *x* − 3 > 0 , *x* − 2>0 , *x* − 1 > 0 and also *x* + 1 > 0. Due to this reason, the expression *e* which is obtained by multiplying 3 positive quatities and dividing the resultant with a positive quantity is positive.  
Similarly, in the interval (2, 3) the quantities *x* − 2>0, *x* − 1 > 0 & *x* + 1 > 0 while the quantity *x* − 3 < 0 . Hence the expression *e* is negative in this interval.   
In this way we can investigate for other intervals also.

*e* < 0 whenever - sign appears in the wavey-curve( i.e. when wavey-curve is below the axis)

 ⇒ *e* < 0 for *xϵ*( − 1, 1)∪(2, 3)

*e* = 0 for all the critical points in the numerator( if and only if they don’t appear in the denominator too)

 ⇒ *e* = 0 for *xϵ*{ − 1, 1, 3}

*e* = Not defined for all the critical points in the denominator.

*e* = Not defined for *xϵ*{2}

{Note: It may be noted, that if we have to find *x* such that *e* ≥ 0 , then we will take the union of the values of *x* found out for *e* > 0 and *e* = 0

i.e. *e* ≥ 0 for *xϵ*(( − ∞,  − 1)∪(1, 2)∪(3, ∞))∪{ − 1, 1, 3} . The same expression can be written by using a closed interval at the points where *e* = 0 i.e. *e* ≥ 0 for *xϵ*( − ∞,  − 1]∪[1, 2)∪[3, ∞)

In a similar way, we can find the values of *x* for which *e* ≤ 0. The solution is *xϵ*[ − 1, 1]∪(2, 3]. }

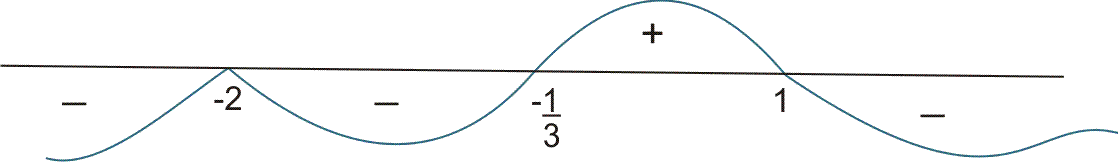
Example

We have to find the sign of *E* = ((*x* + 2)6(3*x* + 1)3)/((1 − *x*))

To find the sign of such an expression, we first of all make the coefficients of *x* as 1 in all terms. This gives

*E* =  − (27(*x* + 2)6(*x* + (1)/(3))3)/(*x* − 1)

The critical points of this expression are *x* = { − 2,  − (1)/(3), 1} . The wavey-curve will start with a negative sign at ∞ as there is a - sign in front of *E*. The curve will then change sign at the first critical point -1. Thereafter , the curve will change sign 3 times at the point *x* =  − (1)/(3). (It may be noted that if a curve changes sign an odd number of times, it is equivalent to change of sign just one time) . Then the curve changes sign 6 times at *x* =  − 2.(change of sign an even number of times means the sign is retained).Things will be more clear from the adjoining figure.



 ⇒ *E* > 0 for *xϵ*( − (1)/(3), 1)

Similarly, *E* < 0 for *xϵ*( − ∞,  − 2)∪( − 2,  − (1)/(3))∪(1, ∞).

Also, *E* = 0 for *xϵ*{ − 2,  − (1)/(3)} and *E* is not defined for *xϵ*{1}

**Q Solve for *x***

a) |2*x* + 5| = 6

b) |5*x* + 7| = |7*x* − 9 |

c) |3*x* − 5| < 1

d) |*x*2 − 5*x* + 6| > *x*2 − 5*x* + 6

e) |*x*| = *x* + 7

f) |*x*| = *x* − 5

g)**|((*x* − 5)3)/((*x* + 2)(*x* − 3))| > ((*x* − 5)3)/((*x* + 2)(*x* − 3))**

h) |cos*x*| = cos*x* + 1

i) |(*x*2 + 2*x* + 5) + (3 − 5*x*)| = |*x*2 + 2*x* + 5| + |3 − 5*x*|

j) |(*x*8 − 4) − (*x*4 + 2)| = |*x*8 − 4| − |*x*4 + 2|

k) *x*2 − |*x*| − 2 = 0

l) |*x* + 2| = |*x* − 2| + 2|*x*|

m) |(3*x* + 5)/(5*x* + 7)| − |(7*x* + 9)/(5*x* + 7)| = 1

n)**2|*x*+ 1| − |2*x* − 1| = 2*x* + 1**

Sol. a) 2*x* + 5 = ±6  
 ⇒ 2*x* = 1 or 2*x* =  − 11  
 ⇒ *x* = (1)/(2) or  − (11)/(2)  
b) |5*x* + 7| = |7*x* − 9 |  
 ⇒ (5*x* + 7) = ±(7*x* − 9)  
 ⇒ 5*x* + 7 = 7*x* − 9 or 5*x* + 7 =  − (7*x* − 9)  
 ⇒ 2*x* = 16 or 12*x* = 2  
 ⇒ *x* = 8 or *x* = (1)/(6) i.e. *xϵ*{8, (1)/(6)}

c) **Method 1**: |3*x* − 5| < 1

 ⇒  − 1 < 3*x* − 5 < 1 (Property 2)  
 ⇒ 4 < 3*x* < 6  
 ⇒ (4)/(3) < *x* < 2 i.e *xϵ*((4)/(3), 2)

**Method 2**: |3*x* − 5| < 1

Squaring both sides, we get  
(3*x* − 5)2 < 1  
 ⇒ 9*x*2 − 30*x* + 25 < 1  
 ⇒ 9*x*2 − 30*x* + 24 < 0  
 ⇒ 3*x*2 − 10*x* + 8 < 0  
 ⇒ (3*x* − 4)(*x* − 2) < 0  
 ⇒ *xϵ*((4)/(3), 2)  
d) |*x*2 − 5*x* + 6| > *x*2 − 5*x* + 6  
{Note: It can be observed that |*y*| is strictly greater than *y* if *y* < 0}  
 ⇒ *x*2 − 5*x* + 6 < 0  
 ⇒ (*x* − 2)(*x* − 3) < 0  
 ⇒ 2 < *x* < 3 i.e. *xϵ*(2, 3)

e) **Case I**: *x* ≥ 0

This assumption would imply that |*x*| = *x*  
 ⇒ *x* = *x* + 7 which is not true for any *x*. Hence , no solution exists.

**Case II**: *x* < 0

 ⇒  − *x* = *x* + 7  
 ⇒ 2*x* =  − 7 or *x* =  − (7)/(2) (which satisfies the condition *x* < 0)

f)**Case I**: *x* ≥ 0

 ⇒ *x* = *x* − 5 which is not true for any *x*. Hence , no solution exists for *x* ≥ 0.

**Case II**: *x* < 0

 ⇒  − *x* = *x* − 5  
 ⇒ 2*x* = 5 or *x* = (5)/(2)   
This value of *x* does not satisfy *x* < 0 . Hence , no solution exists for *x* < 0 also.  
g)|((*x* − 5)3)/((*x* + 2)2(*x* − 3))| > ((*x* − 5)3)/((*x* + 2)2(*x* − 3))  
It may be observed that |*y*| > *y* is only possible when *y* < 0  
 ⇒ ((*x* − 5)3)/((*x* + 2)2(*x* − 3)) < 0   
 ⇒ *xϵ*(3, 5)   
[Note:The solution to such equations can be found out with the help of **Wavey-Curve Technique** as discussed in the Lectures]  
h) |cos*x*| = cos*x* + 1

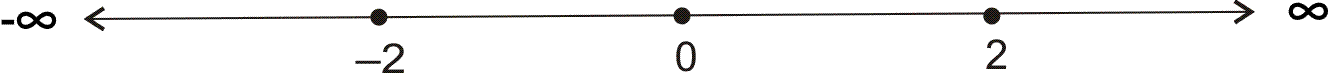
**Case I**: cos*x* ≥ 0

 ⇒ cos*x* = cos*x* + 1  
which is not possible for any *x*. Hence , no solution exists for cos*x* ≥ 0.

**Case II**:cos*x* < 0

 ⇒  − cos*x* = cos*x* + 1  
 ⇒ 2cos*x* =  − 1  
 ⇒ cos*x* =  − (1)/(2) (which satisies our assumption that cos*x* < 0)  
 ⇒ *x* = 2*nπ*±(2*π*)/(3) , *nϵZ*.  
i) |(*x*2 + 2*x* + 5) + (3 − 5*x*)| = |*x*2 + 2*x* + 5| + |3 − 5*x*|  
|*x* + *y*| = |*x*| + |*y*| ⇒ *xy* ≥ 0(equality condition in Property 4)  
i.e *x* & *y* are of the same sign  
Now *x*2 + 2*x* + 5 = (*x* + 1)2 + 4 > 0  
 ⇒ 3 − 5*x* ≥ 0  
i.e *x* ≤ (3)/(5) or *xϵ*(-∞,(3)/(5))  
j)|(*x*8 − 4) − (*x*4 + 2)| = |*x*8 − 4| − |*x*4 + 2|  
We know, |*x* − *y*| = |*x*| − |*y*| if *xy* ≥ 0 & |*x*|≥|*y*|  
Also , it may be noted that *x*4 + 4 > 0∀*x*  
 ⇒ Our conditions reduce to *x*8 − 4 ≥ *x*4 + 2  
 ⇒ *x*4 − 2 ≥ 1  
 ⇒ *x*4 ≥ 3  
 ⇒ *x* ≥ 4√(3) or *x* ≤  − 4√(3) i.e. *xϵ*( − ∞,  − 4√(3))∪(4√(3), ∞)  
k)*x*2 − |*x*| − 2 = 0  
 ⇒ |*x*|2 − |*x*| − 2 = 0  
 ⇒ (|*x*| − 2)(|*x*| + 1) = 0  
 ⇒ |*x*| = 2 or |*x*| =  − 1(Rejected)  
 ⇒ *x* = ±2 i.e *xϵ*{ − 2, 2}  
l)|*x* + 2| = |*x* − 2| + 2|*x*|

We first of all find the [↓](" \l "index-Critical-Points-in-an-Absolute-va)critical points. |*x* + 2| = 0 gives the point  − 2. Similarly, the points 2,0 are obtained by euating the other modulus terms to zero. So, there are 3 critical points ie at *x* = { − 2, 0, 2}. We plot these points on the number line and divide the whole number line at these critical points.



**Case I** : *x* <  − 2

The equation becomes  
 − (*x* + 2) =  − (*x* − 2) − 2*x*  
 ⇒ 2*x* = 4  
 ⇒ *x* = 2 (which is rejected as it does not belong to the interval ( − ∞,  − 2))

**Case II**:  − 2 ≤ *x* < 0

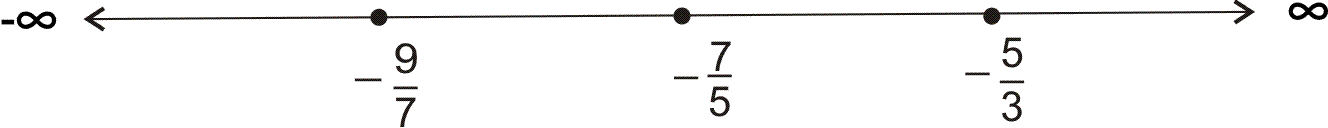
The equation becomes  
*x* + 2 =  − (*x* − 2) − 2*x*  
 ⇒ *x* = 0 (Which is rejected as it does not belong to the interval [ − 2, 0))

**Case III** : 0 ≤ *x* < 2

The equation becomes   
*x* + 2 =  − (*x* − 2) + 2*x*  
which is true for all *x* , but our interval is [0, 2)   
 ⇒ *xϵ*[0, 2) is a solution

**Case IV** : *x* ≥ 2

The equation in this case becomes  
*x* + 2 = (*x* − 2) + 2*x*  
 ⇒ *x* = 2 (which is accepted as it lies in the interval chosen i.e. [2, ∞))  
Hence *x* = 2 is a solution  
Now we combine all the solutions obtained from different intervals. So, the final solution is the union of all the cases i.e. *xϵ*[0, 2)∪{2}  
 ⇒ The complete solution set is *xϵ*[0, 2]  
m)|(3*x* + 5)/(5*x* + 7)| − |(7*x* + 9)/(5*x* + 7)| = 1  
First thing to be noted is that 5*x* + 7 ≠ 0  
i.e. *x* ≠  − (7)/(5)  
 ⇒ |3*x* + 5| − |7*x* + 9| = |5*x* + 7|  
Now in this case, the critical points are  − (9)/(7),  − (7)/(5) and  − (5)/(3). Plotting them on the number line, we get the intervals as shown below



**Case I**: *x* <  − (9)/(7)

 ⇒  − (3*x* + 5) + (7*x* + 9) =  − (5*x* + 7)

 ⇒ 9*x* =  − 11

 ⇒ *x* =  − (11)/(9) ( which lies in the interval defined by *x* <  − (9)/(7))

**Case II**:  − (9)/(7) ≤ *x* <  − (7)/(5)

 ⇒  − (3*x* + 5) − (7*x* + 9) =  − (5*x* + 7)

 ⇒  − 5*x* = 7

 ⇒ *x* =  − (7)/(5) (which is rejected because in the beginning we fixed *x* ≠  − (7)/(5) . Also the point  − (7)/(5) does not lie in the interval  − (9)/(7) ≤ *x* <  − (7)/(5) )

**Case III** :  − (7)/(5) ≤ *x* <  − (5)/(3)

 ⇒  − (3*x* + 5) − (7*x* + 9) =  + (5*x* + 7)

 ⇒  − 15*x* = 21

 ⇒ *x* =  − (7)/(5) (which is rejected because in the beginning we fixed *x* ≠  − (7)/(5) due to domain restrictions.)

**Case IV**: *x* ≥  − (5)/(3)

 ⇒ (3*x* + 5) − (7*x* + 9) = (5*x* + 7)

 ⇒  − 9*x* = 11

 ⇒ *x* =  − (11)/(9) (Rejected because  − (11)/(9) does not belong to the interval *x* ≥  − (5)/(3))

Hence, the only solution is *x* =  − (11)/(9).

n)2|*x*+ 1| − |2*x* − 1| = 2*x* + 1

The critical points lie at |*x* + 1| = 0 and |2*x* − 1| = 0 . i.e at *x* =  − 1 and *x* = 0 respectively. So , dividing the number line as done previously

**Case I**: *x* <  − 1

 ⇒ 2− (*x*+ 1) + (2*x* − 1) = 2*x* + 1

 ⇒ 2− (*x*+ 1) = 2

 ⇒  − (*x* + 1) = 1

 ⇒ *x* =  − 2 (Accepted )

**Case II**:  − 1 ≤ *x* < 0

 ⇒ 2*x*+ 1 + 2*x* − 1 = 2*x* + 1

 ⇒ 2*x*+ 1 = 2

 ⇒ *x* + 1 = 1

 ⇒ *x* = 0 (Rejected as it does not lie in the interval  − 1 ≤ *x* < 0 )

**Case III**: *x* ≥ 0

 ⇒ 2*x*+ 1 − (2*x* − 1) = 2*x* + 1

 ⇒ 2*x*+ 1 = 2.2*x* ( which is true for all *x* belonging to the interval *x* ≥ 0)

 ⇒ The solution to Case III is *xϵ*[0, ∞)

Hence, the general solution to this problem is *xϵ*{ − 2}∪[0, ∞).

### 1.3.3 Problems for Practice

#### 1.3.3.1 Subjective Problems

Q1: Solve for *x*

a) |sin*x*| =  − sin*x* + 2

b) |((*x* + 3)4(*x* − 3)3)/((*x* + 1)(*x* − 1))| <  − ((*x* + 3)4(*x* − 3)3)/((*x* + 1)(*x* − 1))

c) |*x*2 + 3*x* + 2| =  − (*x*2 + 3*x* + 2)

#### 1.3.3.2 Single Answer MCQ’s

Q1: The complete solution set of (1)/(|(4)/(*x*) − 10|) < 3 is

a) *xϵ*ℝ − ((12)/(31), (12)/(29))  
b) *xϵ*((12)/(31), (12)/(29)) − {(2)/(5)}  
c) *xϵ*ℝ − {0} − [(12)/(31), (12)/(29)]  
d) None of these

Q2: The point in the co-ordinate plane satisfying |7*x* + 5| =  − |3 − 3*y*| is

a)( − 2, 3)  
b)( − (5)/(7), 1)  
c) ( − 5,  − 3)  
d) None of these

Q3: The complete solution set of |cot*x*| = cot*x* + 2√(3) is

a){*x*:*x* = *nπ* + (*π*)/(3) , *nϵ*ℤ}  
b){*x*:*x* = *nπ* − (*π*)/(3) , *nϵ*ℤ}  
c){*x*:*x* = *nπ* + (*π*)/(6) , *nϵ*ℤ}  
d){*x*:*x* = *nπ* − (*π*)/(6) , *nϵ*ℤ}

Q4: The area bounded by the curves given by |*x*| + |2*y*| = 2 is

a) 2 units  
b) 4 units  
c) 6 units  
d) 8 units

#### 1.3.3.3 Multiple answer MCQ’s

Q1: The solution set of 2|tan*x*| = tan*x* + √(3) contains

a){*x*:*x* = *nπ* + (*π*)/(3) , *nϵZ*}  
b){*x*:*x* = *nπ* − (*π*)/(3) , *nϵZ*}  
c){*x*:*x* = *nπ* + (*π*)/(6) , *nϵZ*}  
d){*x*:*x* = *nπ* − (*π*)/(6) , *nϵZ*}

#### 1.3.3.4 Matrix Match Type Problems

Matrix 1: Under Column I , some equations are given . Under Column II, some solutions satisfying some of the equations are given. Match the entry in Column I with the solution satisfying it in Column II.{Note: [ ] is the Greatest Integer Function}

|  |  |
| --- | --- |
| Column I | Column II |
| (P)|sin− 1*x*| = sin− 1*x* + (*π*)/(6) | (A)-(1)/(2) |
| (Q)ln|*x*| = |ln*x*| | (B)1 |
| (R)|[*x*]| = [|*x*|] | (C)(3)/(2) |
| (S)|*x*2 − 3*x* + 2| > *x*2 − 3*x* + 2 | (D)2 |

#### 1.3.3.5 Linked Comprehension Type Problems

Comprehension 1: Consider the equation *y* = |(*x*4 − 9) − (*x*2 − 3)| − |*x*4 − 9| + |*x*2 − 3|. The sign of *y* depends on the value of *x*. Read the following questions based on this information and answer them carefully.

Q1: The value(s) of *x* for which *y* > 0 is/are :

a)*xϵR*  
b)*xϵ*( − √(3), √(3))  
c)*xϵ*( − 4√(3), √(3))  
d)None of these

Q2: The value(s) of *x* for which *y* ≤ 0 is/are :

a)*xϵR*  
b)*xϵ*( − √(3), √(3))  
c)*xϵ*( − 4√(3), √(3))  
d)None of these

Comprehension 2: The first 3 terms of an A.P. are |3*x*|, |3*x* − 1|and |3*x* + 1|. Based on this information, give the answers to the questions below

Q1: The value of *x* is

a) 1  
b) 2  
c) -1  
d) None of these

Q2: The 10th term of the A.P. is

a) (1)/(4)  
b) (5)/(4)  
c)(19)/(4)  
d) None of these

Q3: The sum of first 15 terms of the series is

a) (225)/(4)  
b)(359)/(4)  
c)(433)/(4)  
d) None of these

Comprehension 3: A function *fn*(*x*) is defined for all *nϵN* and *fn*+*m*(*x*) is defined as *fn*+*m*(*x*) = *fn*(*fm*(*x*)) where *f*1(*x*) = (2*x* − 1)/(*x* + 1) for *xϵR* − { − 1}. Using this definition , *f*2(*x*) = *f*1(*f*1(*x*)) = (*x* − 1)/(*x*) for *xϵR* − { − 1, 0}. Similarly *f*3(*x*) = *f*1(*f*2(*x*)) = (2 − *x*)/(1 − 2*x*) for *xϵR* − { − 1, 0, (1)/(2)}and so on. Based on the information, answer the questions below.

Q1: The domain of definition of *g*(*x*) = |ln(*f*34(*x*))| is

a) ( − ∞, 1) − { − 1, 0, (1)/(2)}  
b) ( − ∞, 1] − { − 1, 0, (1)/(2)}  
c) (1, ∞) − {2}  
d) None of these

Q2: The complete solution set of |*f*71(*x*)| > |(1)/(*f*73(*x*))| is

a) ( − 1, 1)  
b) ℝ − {2} − [ − 1, 1]  
c) ℝ − { − 1, 1, 2}  
d) None of these

Q3: The values of *x* for which |*f*56(*x*)| > *f*56(*x*) belong to the interval

a) ((1)/(2), 2)  
b) (2, ∞)  
c) (0, 1)  
d) None of these

#### 1.3.3.6 Hints and Solutions

**Subjective Questions**

Q1 a)*x* = 2*nπ* + (*π*)/(2) , *nϵZ*  
{Hint: The solution of the equation is all values of *x* which satisfy the equation , sin*x* = 1}  
b) No Solution i.e *xϵφ*  
{Hint: It may be observed that |*y*| <  − *y* is never true . Even |*y*| < *y* also gives no solution.}  
c) *xϵ*[ − 3,  − 2]  
{Hint: |*y*| =  − *y*  ⇒ *y* ≤ 0 }

**Single Answer MCQ’s**

Q1) C  
{Hint: (1)/(|(4)/(*x*) − 10|) < 3  
 ⇒ |(4)/(*x*) − 10| > (1)/(3)

**Case I** : (4)/(*x*) − 10 > (1)/(3)

 ⇒ (4)/(*x*) > (31)/(3)  
 ⇒ (1)/(*x*) > (31)/(12)  
 ⇒ 0 < *x* < (12)/(31)

**Case II:** (4)/(*x*) − 10 <  − (1)/(3)

 ⇒ (4)/(*x*) < (29)/(3)  
 ⇒ (1)/(*x*) < (29)/(12)  
 ⇒ (1)/(*x*) < 0 or 0 < (1)/(*x*) < (29)/(12)  
 ⇒ *x* < 0 or *x* > (12)/(29)  
Combining all cases, we get *xϵ*( − ∞, 0)∪(0, (12)/(31))∪((12)/(29), ∞) i.e. *xϵ*ℝ − {0} − [(12)/(31), (12)/(29)]}  
Q2) B   
Q3) D  
Q4) B

**Multiple Answer MCQ’s**

Q1) A,D

**Matrix Match Type Problems**

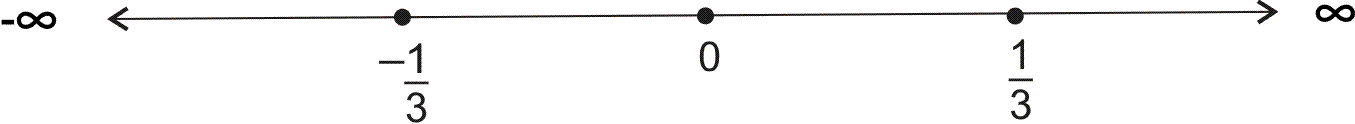
Matrix 1:  
*P* → *A*  
*Q* → *B*, *C*, *D*  
*R* → *B*, *C*, *D*  
*S* → *C*

**Linked Comprehension Type Problems**

**Comprehension 1**: Q1) D Q2)A

{Hint:a) *y* > 0   
 ⇒ |(*x*4 − 9) − (*x*2 − 3)| − |*x*4 − 9| + |*x*2 − 3| > 0  
 ⇒ |(*x*4 − 9) − (*x*2 − 3)| > |*x*4 − 9| − |*x*2 − 3|  
Now we know that |*x* − *y*|≥|*x*| − |*y*| {Equality occurs when *x* and *y* are of same sign . i.e. *xy*≥0 and |*x*|≥|*y*|}  
 ⇒ |(*x*4 − 9) − (*x*2 − 3)| = |*x*4 − 9| − |*x*2 − 3| when (*x*4 − 9)(*x*2 − 3)≥0 and |*x*4 − 9|≥|*x*2 − 3|  
i.e when (*x*2 − 3)2(*x*2 + 3)≥0 and |*x*2 − 3|(*x*2 + 3)≥|*x*2 − 3|  
i.e when (*x*2 − 3)2(*x*2 + 3)≥0 and |*x*2 − 3|(*x*2 + 2)≥0 which is true for all *x*.  
i.e ∀*xϵ*ℝ , y is identically equal to zero.  
Hence *y* > 0 has no solution  
and *y*≤0 is true for all *xϵ*ℝ. i.e (b) → A}

**Comprehension 2**: Q1) D Q2)C Q3)A

{Hint: a)|3*x*|, |3*x* − 1|and |3*x* + 1|are in A.P.  
 ⇒ 2 × |3*x* − 1| = |3*x*| + |3*x* + 1|  
The critical points in this case are { − (1)/(3), 0, (1)/(3)}   


**Case I** : *xϵ*( − ∞,  − (1)/(3))

In this case equation becomes  
2 ×  − (3*x* − 1) =  − 3*x* − (3*x* + 1)  
 ⇒  − 6*x* + 2 =  − 6*x* − 1 which gives no solution

**Case II**: *xϵ*[ − (1)/(3), 0)

In this case , the equation becomes  
2 × (3*x* − 1) =  − 3*x* − (3*x* + 1)  
 ⇒ 6*x* − 2 =  − 6*x* − 1  
 ⇒ *x* = (1)/(12) ( which is accepted as (1)/(12)*ϵ*[ − (1)/(3), 0))

**Case III** : *xϵ*[0, (1)/(3))

In this case , the equation becomes  
2 × (3*x* − 1) = 3*x* − (3*x* + 1)  
 ⇒ 6*x* − 2 =  − 1  
 ⇒ *x* = (1)/(6) (which is rejected as (1)/(6)∉[0, (1)/(3))

**Case IV** :*xϵ*[(1)/(3), ∞)

In this case, the equation becomes  
2 × (3*x* − 1) = 3*x* + (3*x* + 1)  
i.e 6*x* − 2 = 6*x* + 1 . Hence no solution in this case also.  
Combining all the cases, we get *x* = (1)/(12)  
b) Substituting the value of *x* , the series becomes {(1)/(4), (3)/(4), (5)/(4), ⋯⋯⋯}  
Hence, the series has first term *a* = (1)/(4) and common difference *d* = (1)/(2).  
*T*10 = *a* + (10 − 1)*d* = (1)/(4) + 9 × (1)/(2) = (19)/(4)  
c) The sum of first 15 terms of the series is  
*S*15 = (*n*)/(2)(2*a* + (*n* − 1)*d*) = (15)/(2)(2 × (1)/(4) + 9 × (1)/(2)) = (15)/(2) × (15)/(2) = (225)/(4)}

**Comprehension 3**: Q1) A Q2)B Q3)C

{Hint: *f*1(*x*) = (2*x* − 1)/(*x* + 1) for *xϵR* − { − 1}  
*f*2(*x*) = *f*1(*f*1(*x*)) = (*x* − 1)/(*x*) for *xϵR* − { − 1, 0}  
*f*3(*x*) = *f*1(*f*2(*x*)) = (2 − *x*)/(1 − 2*x*) for *xϵR* − { − 1, 0, (1)/(2)}  
On simlar lines, we can find *f*4(*x*) = *f*1(*f*3(*x*)) = (1)/(1 − *x*) for *xϵR* − { − 1, 0, (1)/(2), 1}  
*f*5(*x*) = *f*1(*f*4(*x*)) = (1 + *x*)/(2 − *x*) for *xϵR* − { − 1, 0, (1)/(2), 1, 2}  
*f*6(*x*) = *f*1(*f*5(*x*)) = *x* for *xϵR* − { − 1, 0, (1)/(2), 1, 2}  
It may be noted that this function repeats itself after a gap of 6 .i.e *f*6*n*+*r*(*x*) = *f*6(*f*6...*ntimes*(*fr*(*x*)))) = *fr*(*x*) for *xϵR* − { − 1, 0, (1)/(2), 1, 2}  
Q1: *f*34(*x*) = *f*4(*x*) for *xϵR* − { − 1, 0, (1)/(2), 1, 2}  
 ⇒ *f*34(*x*) = (1)/(1 − *x*) for *xϵR* − { − 1, 0, (1)/(2), 1, 2}  
 ⇒ *g*(*x*) = |ln(*f*34(*x*))| has domain given by   
(1)/(1 − *x*) > 0 for *xϵR* − { − 1, 0, (1)/(2), 1, 2}  
i.e *x* < 1 and *xϵR* − { − 1, 0, (1)/(2), 1, 2}  
i.e *xϵ*(( − ∞, 1) − { − 1, 0, (1)/(2)})  
Q2: |*f*71(*x*)| > |(1)/(*f*73(*x*))|   
i.e we have to solve |*f*5(*x*)| > |(1)/(*f*1(*x*))| for *xϵR* − { − 1, 0, (1)/(2), 1, 2}  
i.e |(1 + *x*)/(2 − *x*)| > |(1)/((2*x* − 1)/(*x* + 1))|for *xϵR* − { − 1, 0, (1)/(2), 1, 2}  
i.e |2*x* − 1| > |2 − *x*| for *xϵR* − { − 1, 0, (1)/(2), 1, 2}  
Squaring both sides we get  
4*x*2 − 4*x* + 1 > 4 + *x*2 − 4*x* for *xϵR* − { − 1, 0, (1)/(2), 1, 2}  
i.e. 3*x*2 > 3 for *xϵR* − { − 1, 0, (1)/(2), 1, 2}  
 ⇒ *xϵ*(ℝ − {2} − [ − 1, 1])  
Q3: |*f*56(*x*)| > *f*56(*x*)  
 ⇒ |*f*2(*x*)| > *f*2(*x*) for *xϵR* − { − 1, 0, (1)/(2), 1, 2}  
 ⇒ *f*2(*x*) < 0 for *xϵR* − { − 1, 0, (1)/(2), 1, 2}  
 ⇒ (*x* − 1)/(*x*) < 0 for *xϵR* − { − 1, 0, (1)/(2), 1, 2}  
 ⇒ *xϵ*((0, 1) − {(1)/(2)})  
These values are a substet of (0, 1).}

# 2 Functions

[↓](" \l "index-Functions)Functions are one of the most important areas of mathematics because they lie at the heart of much of mathematical analysis.

## 2.1 Domain of Definition and Range

Q: For the function *f*(*x*) = (*x*)/(*x* + 2) , find *f*(*x* + 3) , *f*(3*x*) ,3*f*(*x*) , 3*f*(3*x* + 3) , *f*(*x*3) , (*f*(*x*))3 with their respective domains of definition.

**Sol:** *f*(*x* + 3) = (*x* + 3)/(*x* + 5) where *x* + 5 ≠ 0 i.e *Df*(*x*+ 3) = ℝ − { − 5}

***f*(3*x*) = (3*x*)/(3*x* + 2)** where 3*x* + 2 ≠ 0 i.e. *Df*(3*x*) = ℝ − { − (2)/(3)}

**3*f*(*x*) = (3*x*)/(*x* + 2)** where *x* + 2 ≠ 0 i.e. *D*3*f*(*x*) = ℝ − { − 2}

**3*f*(3*x* + 3) = (3*x*)/(3*x* + 5)** where 3*x* + 5 ≠ 0 i.e. *D*3*f*(3*x*+ 3) = ℝ − { − (5)/(3)}

***f*(*x*3) = (*x*3)/(*x*3 + 2)** where *x*3 + 2 ≠ 0 i.e. *Df*(*x*3) = ℝ − { − 3√(2)}

**(*f*(*x*))3 = ((*x*)/(*x* + 2))3** where *x* + 2 ≠ 0 i.e. *D*(*f*(*x*))3 = ℝ − { − 2}

Q: Find the domains of definition of the following functions:

a) √(2 − *x*) − 3√(*x*) + 4√(2 + *x*)  
b) √(*x*2 + *x* − 2)  
c) (√(*x*2 − 3*x* + 2))/(√(4 − *x*2))  
d) √(tan*x*)  
e) √( − sin*x* − (1)/(2))  
f) √(1 − *cosecx*)  
g) log((*x*3(*x*2 − 4))/((*x*2 − 1)(*x* + 3)))  
h) √(log*x*)  
i) √(log*x*(2 − *x*))  
j) cos− 1((*x*)/(2) − 2) − log((*x*)/(2) − 2)  
k) √([*x*] − *x*) where [ ] represents the greatest integer function  
l) √(sin(([*x*])/(2)*π*))  
m) (1)/(√(|*x*| − *x*))

### 2.1.1 Problems for Practice

#### 2.1.1.1 Subjective Problems

Q1: (i) Prove that for the logarithmic function *y* = ln|*x*| , if the argument takes values in a G.P. , then the corresponding values of the function *y* are in A.P.\begin\_inset Separator latexpar\end\_inset

* (ii)Prove that for the exponential function *y* = *ex* , if the argument takes values in a A.P.. , then the corresponding values of the function *y* are in G.P.

#### 2.1.1.2 Hints and Solutions

**Subjective Problems**

Q1 : {Hint: (i) Let the arguments be defined by the general term *xn* = *arn*− 1 . Then the value of the function corresponding to it will be *yn* = ln|*arn*− 1| = ln|*a*| + (*n* − 1)ln|*r*|. This means that two consecutive terms *yn* and *yn*− 1 differ by a constant .i.e. *yn* − *yn*− 1 = ln|*r*|. Hence , these terms are in A.P.  
(ii)Proceeding in a similar manner , if we take the argument as a general term of an A.P. then the corresponding values of the function will have a constant common difference.}

## 2.2 Periodicity

### 2.2.1 Problems for practice

#### 2.2.1.1 Subjective Problems

Q1: Find the range of the function defined on **R** given by

***f*(*x*) = ⎧⎨⎩ *ex*−*n*  ;*n*≤*x*≤*n* + (2)/(3)        sin((3*π*)/(2)*x*)  ;*n* + (2)/(3) < *x* < *n* + 1**

where *nϵZ*.

## 2.3 Injectivity, Surjectivity and Bijectivity

### 2.3.1 Problems for Practice

#### 2.3.1.1 Subjective Problems

Q1: Check whether the function *f*:( − ∞,  − 3) → [ − (1)/(4), 0] given by *f*(*x*) = (*x* + 1)/(*x*2 + 2*x* + 5) is bijective in its domain or not. Given that *f*(*x*) is strictly decreasing in the domain.

#### 2.3.1.2 Single Answer MCQ’s

Q1: A function *f*:*R* → [1, 2] given by *f*(*x*) = log3(9 − 6|cos*x*|) is

a) One-one only  
b) Onto only  
c) One-one and onto  
d) None of these

Q2: Let *f*:[0, ∞) → *S* defined as *f*(*x*) = 2*x*(1 −*x*) be onto, then the set S is

a) [0, ∞)  
b) [1, ∞)  
c) [2− (1)/(4), 1]  
d) [2− (1)/(4), ∞)

#### 2.3.1.3 Hints and Solutions

**Single Answer MCQ’s**

Q1: B  
Q2: D

## 2.4 Inverse of a function

### 2.4.1 Problems for practice

#### 2.4.1.1 Subjective Problems

Q1: Check whether the function *f*:( − ∞, 0) → (0, ∞) given by *f*(*x*) = (1)/(√(|*x*| − *x*)) is one-one and onto. Find it’s inverse if possible.

Q2: A function *f*:*R* − { − 2, 2} → ( − ∞, (5)/(2)]∪(3, ∞) given by *f*(*x*) = 3 + (1)/(|*x*| − 2) . Divide the domain into minimum number of suitable subsets so that the function becomes invertible on each individual domain. Also find the inverse on each of these domains.

#### 2.4.1.2 Single Answer MCQ’s

Q1: A function on *f*:[2, 4) → ( − ∞, 0] is given by *f*(*x*) =  − 2 + log2( − *x*2 + 4*x*). Then *f*− 1(*x*) is given by

a) 2±2√(1 − 2*x*)  
b) 2 − 2√(1 − 2*x*)  
c)2 + 2√(4 − 2*x*)  
d) None of these

#### 2.4.1.3 Hints and Solutions

Single Answer MCQ’s  
Q1: D

## 2.5 End Chapter Problems

# 3 Limits

Q: Evaluate the limits

a)\underset*n* → ∞lim*cos*(√(*n*2 + *n*)*π*)

Sol. a) Case I :

# 4 Continuity & Diffrentiability

### 4.5.1 Problems for Practice

#### 4.5.1.1 Multiple answer MCQ’s

Q1: If *x* = 3√(*a*tan − 1*t*) , *y* = 3√(*a*cot − 1*t*) , then (*dy*)/(*dx*) =

a)  − (*y*)/(*x*)  
b)  − (*aπ*⁄ 6)/(*x*2)  
c) − (1)/(*x*2)  
d) − (*a*)/(*x*2)

# 5 Applications of Derivatives

## 5.1 Rate of Change

### 5.1.1 Problems for practice

**Subjective Problems**

Q1: A ladder 13*m* long is leaning against a wall. Its upper edge is pulled upwards at a rate of 12*cm* ⁄ *s*. Its lower edge slides on the ground. What is the rate of change of the angle *θ* ,which the ladder makes with the ground, w.rt. time , when the foot of the ladder is 12*m* away from the wall.

#### 5.1.1.1 Linked Comprehension Type Problems

Comprehension 1: A Scientist at CERN labs recently designed a gadget to measure the variations of pressure of Helium gas with change in its volume. The gadget consists of a hollow frustrum(of a cone) with constant base angle 45*o* and variable dimensions. The frustrum is closed at bottom and open at top. A spherical ball with variable radius *r* is dropped inside the frustrum from the open face. The ball expands to fit the frustrum and touches it at the top edge and the centre of the base{as shown in the Figure}. Helium gas is then put in the cavity between the ball and frustrum. The ball now contracts and the frustrum also contracts maintaining a constant angle 45*o* and touching the ball only at the top edge and the centre of the base.{Note: The area of the portion of sphere outside the frustrum is 2*πr*2(1 − (1)/(√(2)))}

Q1: Express the volume of the frustrum in terms of *r*.

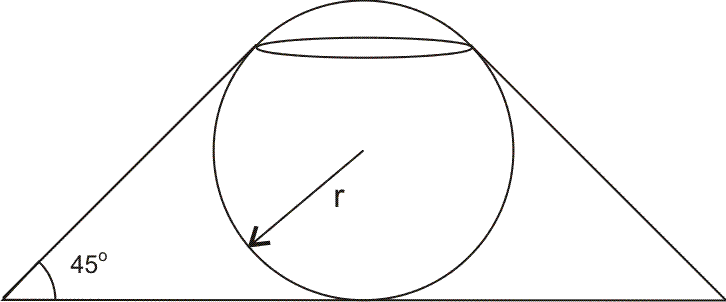
a)(1)/(3)*πr*3(2 + (1)/(√(2)))  
b)(1)/(3)*πr*3(2 − (1)/(√(2)))  
c)(1)/(3)*πr*3(7 + (19)/(4)√(2))  
d) None of these

Q2: The volume of the portion of sphere outside the frustrum is

a) (1)/(3)*πr*3(2 − (5)/(4)√(2))  
b) (1)/(3)*πr*3(2 − √(2))  
c)(1)/(3)*πr*3((2 − √(2))/(2))  
d) None of these

Q3: What is the rate of change of the volume of the cavity w.r.t. *r*

a) *πr*2((5)/(2) + (7)/(2)√(2))  
b) *πr*2(5 + (7)/(2)√(2))  
c) *πr*2(3 + 5√(2))  
d) None of these.



Q4: The pressure of the gas inside the cavity varies as *PVγ* = *k*(a constant) , then the rate of change of Pressure of the gas inside the cavity w.r.t. time is { at the instant when (*dr*)/(*dt*) =  − 2 units/sec , *r* = 3 units }

a) (2*γk*)/([*π*(3 + 5√(2))]*γ*+ 1).(1)/(32*γ*)  
b) (6*γk*)/([*π*(5 + (7)/(2)√(2))]*γ*).(1)/(32*γ*+ 1)  
c) (2*γk*)/((3 + 5√(2))*γ*− 1).(1)/(32*γ*+ 1)  
d) None of these

#### 5.1.1.2 Hints and Solutions

**Linked Comprehension Type Questions**

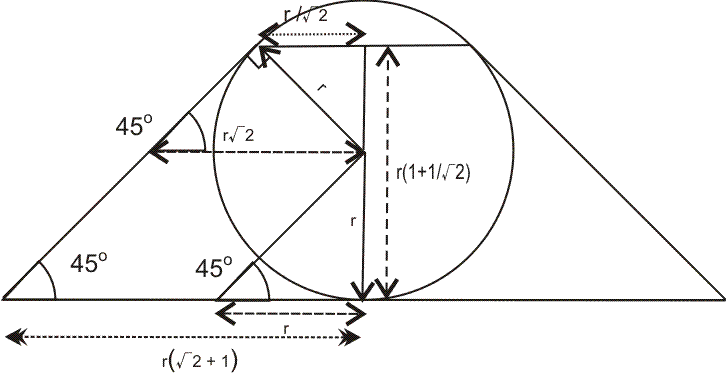
**Comprehension 1**

Answers: Q1) C , Q2) A ,Q3) B ,Q4) B.

{Hint: Q1) Let *r*1be the radius of the top opening, *r*2 be the radius of the base and *h* be the height of the frustrum.

From adjoining diagram, it is clear that

*r*1 = (*r*)/(√(2)) , *r*2 = *r*(√(2) + 1) & *h* = *r*(1 + (1)/(√(2)))

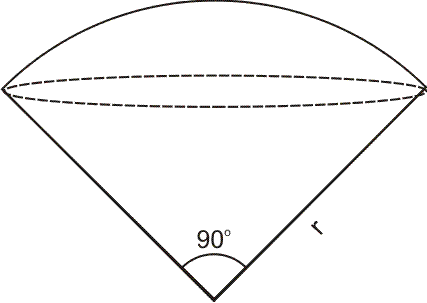


We know that the volume of a frustrum is *V* = (1)/(3)*πh*(*r*12 + *r*22 + *r*1*r*2)

 ⇒ *V* = (1)/(3)*πr*(1 + (1)/(√(2)))(((*r*)/(√(2)))2 + (*r*(√(2) + 1))2 + (*r*)/(√(2)).*r*(√(2) + 1))

 ⇒ *V* = (1)/(3)*πr*3(7 + (19)/(4)√(2))

Q2: To find the volume of the portion of sphere outside the frustrum, we first find the volume of 3Dimensional Portion of the sphere shown in figure



Let it’s volume be *v* and the upper Curved Surface Area be *a*

Using the concept of Solid Angle, (*v*)/(*V*) = (*a*)/(*A*) { = (Ω)/(4*π*)} (where *V* is the Volume of the Complete Sphere , *A* is the Surface Area of the Sphere and Ω is the solid angle)

 ⇒ (*v*)/((4)/(3)*πr*3) = (2*πr*2(1 − (1)/(√(2))))/(4*πr*2)

 ⇒ *v* = (2)/(3)*πr*3(1 − (1)/(√(2)))

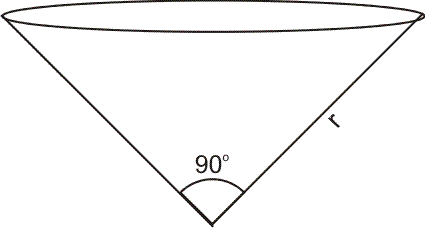
Alternatively, this Volume can be found out by generating diffrential cones of base area *dA* and slant height *r*.These diffrential cones are generated by joining the boundaries of the diffrential area with the centre of the sphere . The volume of one such cone will be

*dV* = (1)/(3)*rdA*

Adding these diffrential cones, we get the volume of the portion of sphere as

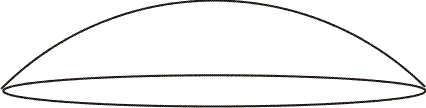
*v* = (1)/(3)*ra* = (1)/(3)*r*.2*πr*2(1 − (1)/(√(2)))=(2)/(3)*πr*3(1 − (1)/(√(2))) which is the same as found above.

Now we need to subtract the volume of the cone from this volume to get the required volume of the portion of sphere outside the frustrum.



*Vcone* = (1)/(3)*π*((*r*)/(√(2)))2.(*r*)/(√(2))

*Vportion* = *v* − *Vcone*



*Vportion* = (2)/(3)*πr*3(1 − (1)/(√(2))) − (1)/(3)*π*((*r*)/(√(2)))2.(*r*)/(√(2))

*Vportion*=(1)/(3)*πr*3(2 − (5)/(4)√(2))

{The student may note that the concept of Solid angle can be developed intuitively without any prior knowledge of the concept}

Q3: From the figure , it is clear that

*Vcavity* = *VFrustrum* + *VPortion* − *VSphere*

 ⇒ *VCavity*=(1)/(3)*πr*3(7 + (19)/(4)√(2)) +(1)/(3)*πr*3(2 − (5)/(4)√(2)) -(4)/(3)*πr*3

 ⇒ *VCavity* = (1)/(3)*πr*3(5 + (7)/(2)√(2))

 ⇒ (*dVCavity*)/(*dr*) = *πr*2(5 + (7)/(2)√(2))

Q4:*PVγ* = *k*

 ⇒ *P* = (*k*)/(*Vγ*)

 ⇒ *P* = (*k*)/(((1)/(3)*πr*3(5 + (7)/(2)√(2)))*γ*)

 ⇒ (*dP*)/(*dt*) = (*k*)/(((1)/(3)*π*(5 + (7)/(2)√(2)))*γ*).( − 3*γ*)/(*r*3*γ*+ 1).(*dr*)/(*dt*)=(6*γk*)/([*π*(5 + (7)/(2)√(2))]*γ*).(1)/(32*γ*+ 1)}

## 5.2 Maxima and Minima

### 5.2.1 Problems for Practice

#### 5.2.1.1 Subjective Problems

Q1: A thin rectangular sheet is inscribed in a sphere of radius *R* . What can be its maximum area.

Q2: A cylinder of height *h* is inscribed in a right circular cone having base radius *R* and semi verticlal angle *α* . What is the rate of change of volume of cylinder w.r.t. *α* at the instant when *α* = (*π*)/(4).

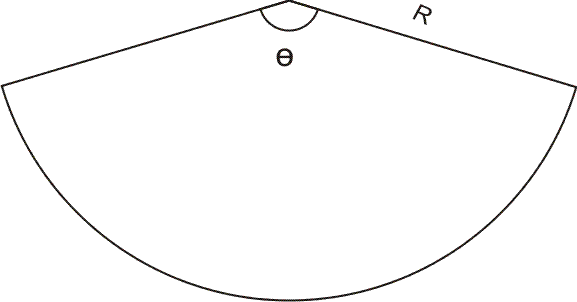
#### 5.2.1.2 Single Answer MCQ’s

Q1: From a military base located at the origin, a Surface-to-Surface Missile(STSM) was fired onto a target city located at (8,0) along the path *y* = 8*x* − *x*2. Sometime later, a Radar located in the target city detected the missile and an Anti-Ballistic Interceptor Missile(ABIM) was fired from the city along the path *y* = √(8*x* − *x*2)  [D]  [D] A guided missile may move along a non-parabolic path as it is propelled by rockets or jet enginesto intercept the missile. Ironically, the ABIM made substantial damage to the military base and the target city was also destroyed. What was the maximum distance between the trajectories of the two missiles?

a) 4 units  
b) 8 units  
c) 12 units  
d) 16 units

#### 5.2.1.3 Linked Comprehension Type Problems

Comprehension 1: A circular sheet of radius *R* is taken and a sector of angle *θ* is cut out of it. A cone is made of this cut-out sector(Curved Surface only). The volume of the cone depends on the angle *θ* of the sector.



Q1: The angle *θ* for which the volume of the cone generated is the maximum is

A)(1)/(√(2)).2*π*  
B)√((2)/(3)).2*π*  
C)√((3)/(4)).2*π*  
D)None of these

Q2: The Volume of the cone with the maximum volume is

A)(√(3))/(4)*πR*3  
B)(1)/(6√(2))*πR*3

C)(2)/(9√(3))*πR*3

D)None of these

Q3: The semi-vertical angle of the cone with maximum volume is

A)(*π*)/(6)  
B)(*π*)/(4)  
C)(*π*)/(3)  
D)None of these

Comprehension 2: S is an ellipse in the Cartesian plane with its major axis parallel to x-axis, having centre at ((1)/(2), 1) , eccentricity (1)/(√(2)) and passing through the intersection of *y* = *f*(*x*) and the line 2*x* + *y* = 1 . Further *f*(*x*) is a polynomial satisfying the property relation *f*(*x* + *y*) = *f*(*x*) + *f*(*y*) ∀*x*, *y* and *f*(1) = *c* , where 0 ≤ *c* ≤ 32

Q1: The minimum possible area of the ellipse is

a) (*π*√(2))/(11)  
b) (2√(2)*π*)/(11)  
c) (*π*√(2))/(9)  
d) None of these

Q2: The equation of auxillary circle for the ellipse of maximum area, is

a) (*x* − (1)/(2))2 + (*y* − 1)2 = (1)/(4)  
b) (*x* − (1)/(2))2 + (*y* − 1)2 = 2  
c) (*x* − (1)/(2))2 + (*y* − 1)2 = (17)/(9)  
d) None of these

Q3: \underset*c* → 0+lim{*f*(1 + *f*⁄(*x*))}*f*(*x*) for *x* > 0 , is

a) 1  
b) 0  
c) 17  
d) None of these

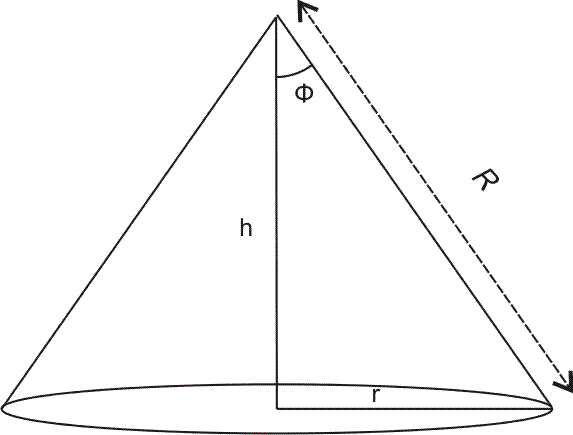
#### 5.2.1.4 Hints and Solutions

Q1) C

**Linked Comprehension Type Problems**

**Comprehension 1**:

Answers: Q1) B Q2)C Q3)D  
{Hints:Q1) The area of the sector is *A* = (*θ*)/(2*π*).*πR*2=(*θ*)/(2)*R*2



When the cone is generated from the sector, its Curved Surface Area will be equal to the area of the sector.  
 ⇒ *πrR* = (*θ*)/(2)*R*2  
 ⇒ *r* = (*θ*)/(2*π*)*R*  
Also *h* = √(*R*2 − *r*2)=√(*R*2 − ((*θ*)/(2*π*)*R*)2)  
*V*(Volume of the Cone) =(1)/(3)*πr*2*h*  
 ⇒ *V* = (1)/(3)*π*((*θ*)/(2*π*)*R*)2√(*R*2 − ((*θ*)/(2*π*)*R*)2)  
 ⇒ *V*=(1)/(3)*π*(*R*3)/((2*π*)3)*θ*2√((2*π*)2 − *θ*2)  
 ⇒ (*dV*)/(*dθ*)=(1)/(3)*π*(*R*3)/((2*π*)3)[2*θ*√((2*π*)2 − *θ*2) + (*θ*2( − 2*θ*))/(2√((2*π*)2 − *θ*2))]  
 = (1)/(3)*π*(*R*3)/((2*π*)3)[(2*θ*((2*π*)2 − *θ*2) + ( − *θ*3))/(√((2*π*)2 − *θ*2))]   
As is clear, the points *θ* = 0 & *θ* = 2*π* will yield a minima each. There will be an intermediate maxima between these two points.  
 ⇒ The point of maxima will lie at *θ*corresponding to   
2((2*π*)2 − *θ*2) − *θ*2 = 0  
 ⇒ 2(2*π*)2 − 3*θ*2 = 0  
 ⇒ *θ* = √((2)/(3)).2*π*  
Q2: Volume of the cone is given by  
*V* = (1)/(3)*π*((*θ*)/(2*π*)*R*)2√(*R*2 − ((*θ*)/(2*π*)*R*)2)  
Substituting *θ* = √((2)/(3)).2*π*  
*V* = (2)/(9√(3))*πR*3units  
Q3: It can be observed from the figure that for the semi-vertical angle *φ*,  
sin*φ* = (*r*)/(*R*)  
 ⇒ sin*φ* = (((*θ*)/(2*π*)*R*))/(*R*)  
 ⇒ sin*φ* = (*θ*)/(2*π*) = √((2)/(3))  
 ⇒ *φ* = sin− 1√((2)/(3))}

**Comprehension** 2:

Answers: Q1) C Q2)B Q3)A  
{ Hint: Q1: It can be easily verified that *f*(*x*) = *cx*  
Solving it with 2*x* + *y* = 1 gives  
*x* = (1)/(*c* + 2) , *y* = (*c*)/(*c* + 2)  
The equation of ellipse will be  
((*x* − (1)/(2))/(*a*))2 + ((*y* − 1)/(*b*))2 = 1  
Now, since the eccentricity is (1)/(√(2)) , *b*2 = *a*2(1 − *e*2) i.e. *b*2 = (*a*2)/(2)  
Hence the equation of ellipse becomes  
((*x* − (1)/(2))2)/(*a*2) + ((*y* − 1)2)/((*a*2)/(2)) = 1  
Now, *x* = (1)/(*c* + 2) , *y* = (*c*)/(*c* + 2) lies on it  
 ⇒ (((1)/(*c* + 2) − (1)/(2))2)/(*a*2) + (((*c*)/(*c* + 2) − 1)2)/((*a*2)/(2)) = 1  
 ⇒ *a*2 = (*c*2 + 32)/(4(*c* + 2)2)  
Area of ellipse =*πab* = (*πa*2)/(√(2))  
Area → *min* ⇒ (*d*)/(*dc*)((*πa*2)/(√(2))) = 0  
i.e. (*d*)/(*dc*)(*π*(*c*2 + 32)/(4√(2)(*c* + 2)2)) = 0  
 ⇒ (*π*(*c* − 16))/(√(2)(*c* + 2)3) = 0  
This gives *c* = 16 . Also it may be noted that the derivative is negative in the left neighbourhood of *c* = 16 and positive on the right neighbourhood of *c* = 16 . Hence, *c* = 16 is a point of minima.  
 ⇒ *Amin* = (*πa*2)/(√(2)) = (*π*√(2))/(9)  
Q2: It is easy to observe that maxima of A can occur at *c* = 0 or *c* = 32   
At *c* = 0 , *a*2 = 2  
At *c* = 32 , *a*2 = (32 × 33)/(4 × (34)2) < (1)/(4)   
Hence, the maximum occurs at *c* = 0   
The equation of auxillary circle is  
(*x* − (1)/(2))2 + (*y* − 1)2 = *a*2  
i.e. (*x* − (1)/(2))2 + (*y* − 1)2 = 2  
Q3: \underset*c* → 0+lim{*f*(1 + *f*⁄(*x*))}*f*(*x*) for *x* > 0 , is  
 = \underset*c* → 0+lim(*c*(1 + *c*))*cx*  
 = *e*\underset*c*→ 0 + limln(*c*(1 +*c*)).*cx*  
 = *e*\underset*c*→ 0 + lim(ln(*c*(1 +*c*)))/((1)/(*c*)).*x*  
Applying L’Hospital Rule  
 = *e*\underset*c*→ 0 + lim((2*c*+ 1)/((*c*(1 +*c*))))/( − (1)/(*c*2)).*x*  
 = *e*0 = 1}

## 5.3 Tangents and Normals

### 5.3.1 Problems for Practice

#### 5.3.1.1 Multiple Answer MCQ’s

Q1: The point(s) on the curve *y*2 + 4 = 8*x* , where the tangent makes equal intercepts with the axes , is/are

a)((5)/(2), 4)  
b)((5)/(2),  − 4)  
c)((5)/(4), 4)  
d)((5)/(4), 2)

#### 5.3.1.2 Matrix Match Type Problems

Matrix 1: Under Column I, equations of some curves are listed. Under Column II, equations of some lines are listed. An entry in Column I is linked to an entry in column II , if the entry in Column II is either a tangent or a normal to the curve given in the entry in Column I.

|  |  |
| --- | --- |
| Column I | Column II |
| P)*y*2 − 6*y* − 4*x* + 21 = 0 | A)*x* = 2 |
| Q)*x*2 − 4*x* + 4*y* − 4 = 0 | B)*y* = 3 |
| R)*x*2 + *y*2 − 4*x* − 6*y* + 5 = 0 | C)*y* = *x* + 1 |
| S)*x*2 + *y*2 − 6*y* + 7 = 0 | D)*y* = *x* + 5 |
|  | E)*y* + *x* = 5 |

#### 5.3.1.3 Answers

3.3.1.1(Multiple Answer MCQ’s)  
Q1: A,B  
3.3.1.2(Matrix Match Type Problems)  
Matrix 1:   
(P)⟼B,C,E   
(Q)⟼A,C,E   
(R)⟼A,B,C,D,E   
(S)⟼B,C,D,E

# 6 Indefinite Integrals

Q1: Evaluate the following Integrals

a)∫(*x*2*ex*)/((*x* + 2)2)*dx*

Sol: a) ∫(*x*2*ex*)/((*x* + 2)2)*dx*

 = ∫((*x*2 − 4 + 4))/((*x* + 2)2)*exdx*  
 = ∫((*x* − 2)/(*x* + 2) + (4)/((*x* + 2)2))*exdx*  
Now we know, if *f*(*x*) = (*x* − 2)/(*x* + 2) , then *f*⁄(*x*) = (4)/((*x* + 2)2)  
Hence, ∫(*x*2*ex*)/((*x* + 2)2)*dx* = ((*x* − 2)/(*x* + 2))*ex* + *C*

#### 6.3.1.1 Subjective Problems

Q1: Evaluate the Integral

**∫(*x*4*m* + *x*2*m* + *xm*)((3)/(2)*x*3*m* + 3*xm* + 6)(1)/(*m*)*dx*** for *x* > 0

Q2: Evaluate the Integral

**∫*cosec*− 1((√(9*x*2 + 12*x* + 29))/(3*x* + 2))*dx***

#### 6.3.1.2 Single Answer MCQ’s

Q1: The integral ∫(*dx*)/(sin(*x* − *a*)cos(*x* − *b*)) equals

a)(1)/(cos(*a* − *b*))ln|(sin(*x* − *a*))/(cos(*x* − *b*))| + *C*  
b)(1)/(sin(*a* − *b*))ln|(cos(*x* − *a*))/(cos(*x* − *b*))| + *C*  
c)(1)/(cos(*a* − *b*))ln|(cos(*x* − *a*))/(sin(*x* − *b*))| + *C*  
d) None of these

Q2: If ∫(4*x*2 + 2*x* + 1)/(*x*(2*x* − 1)(2*x* + 1))*dx* = *A*ln|*x*| + *B*ln|2*x* − 1| + *C*ln|2*x* + 1| + *D* . Then, *A* + *B* − *C* equals

a) 0  
b) 1  
c) 3  
d) None of these

#### 6.3.1.3 Hints and Solutions

**Single Answer MCQ’s**

Q1: Answer : A  
{Hint: ∫(*dx*)/(sin(*x* − *a*)cos(*x* − *b*)) = (1)/(cos(*a* − *b*))∫(cos((*x* − *b*) − (*x* − *a*)))/(sin(*x* − *a*)cos(*x* − *b*))*dx* =  (1)/(cos(*a* − *b*))∫(cos(*x* − *a*)cos(*x* − *b*) + sin(*x* − *a*)sin(*x* − *b*))/(sin(*x* − *a*)cos(*x* − *b*))*dx* =   
(1)/(cos(*a* − *b*))(∫(cos(*x* − *a*))/(sin(*x* − *a*))*dx* + ∫(sin(*x* − *b*))/(cos(*x* − *b*))*dx*) = (1)/(cos(*a* − *b*))(ln|sin(*x* − *a*)| − ln|cos(*x* − *b*)|) + *C* = (1)/(cos(*a* − *b*))ln|(sin(*x* − *a*))/(cos(*x* − *b*))| + *C* }  
Q2:Answer : A  
{Hint:We first of all make the partial fractions of (4*x*2 + 2*x* + 1)/(*x*(2*x* − 1)(2*x* + 1)) . If we want to do it with the method of vedic mathematics, we must make the coefficients of all the *x* equal in all the fractions. We make it equal to 2 for our convenience.  
i.e (2(4*x*2 + 2*x* + 1))/(2*x*(2*x* − 1)(2*x* + 1)) = (*a*)/(2*x*) + (*b*)/(2*x* − 1) + (*c*)/(2*x* + 1)  
 ⇒ *a* = (2(1))/(( − 1)(1)) =  − 2 , *b* = (2(3))/((1)(2)) = 3 and *c* = (2(1))/(( − 1)( − 2)) = 1  
 ⇒ ∫(4*x*2 + 2*x* + 1)/(*x*(2*x* − 1)(2*x* + 1))*dx* = ∫( − (2)/(2*x*) + (3)/(2*x* − 1) + (1)/(2*x* + 1))*dx*  
 =  − ln|*x*| + (3)/(2)ln|2*x* − 1| + (1)/(2)ln|2*x* + 1| + Integration Constant  
Comparing this with the given equation *A*ln|*x*| + *B*ln|2*x* − 1| + *C*ln|2*x* + 1| + *D* , we get *A* =  − 1, *B* = (3)/(2) and *C* = (1)/(2)  
 ⇒ *A* + *B* − *C* =  − 1 + (3)/(2) − (1)/(2) = 0}

# 7 Definite Integrals

### 7.3.1 Problems for Practice

#### 7.3.1.1 Single Answer MCQ’s

Q1: ∫02tan− 1((4*x* − 4)/(4 + 2*x* − *x*2))*dx* equals

a) 0  
b) 1  
c)  − (*π*)/(4)  
d) (*π*)/(3)

Q2: If for *t* > 0 , the definite integral ∫0*t*2*x*(3)/(2)*f*(*x*)*dx* = 2*t*5. Then *f*(√(2)) equals

a) (2(√(2)))/()

Q3: ∫−*e*2 −*e*− 2|(log|*x*|)/(*x*)|*dx* equals

a) 2  
b) -2  
c) 4  
d) -4

Q4: \underset*x* → 0lim∫*x*tan*x*(1)/(*t*3)*dt* equals

a) 0  
b) (1)/(2)  
c) (1)/(3)  
d) None of these

Q5: If ∫1log*xt*5*f*(*t*)*dt* = log(*x*) − 1 , then *f*(2)equals

a) 0  
b) 1  
c) (1)/(2)  
d) (1)/(32)

#### 7.3.1.2 Multiple Answer MCQ’s

Q1: *f*(*x*) is a twice diffrentiable function on ( − ∞, ∞) such that *f*(*x*) = *f*(2 − *x*) and *f*⁄((2)/(√(7))) = 0 , then

A) *f*⁄(1) = 0  
B ) *f*⁄(*x*) vanishes at least thrice in [0, 2].  
C) ∫− 11*f*(*x* + 1)tan*xdx* = 0  
D) ∫01*f*(*t*)*e*sin(*π*)/(2)*tdt* = ∫12*f*(2 − *t*)*e*sin(*π*)/(2)*tdt*

#### 7.3.1.3 Matrix Match type Problems

Matrix 1: In Column I, some expressions containing Integrals are given.In Column II, some values are given . Match the expression in Column I with the values in Column II.

|  |  |
| --- | --- |
| Column I | C II |
| (P)(∫545ln(*x*(3)/(2))*dx*)/(∫√(5)3√(5)*x*ln(*x*2)*dx*) | (A)0 |
| (Q) 16∫0(1)/(√(2))*x*3*dx* − ∫− √(2)7√(2)([(*x* + √(2))/(16)])3*dx* | (B)1 |
| where [ ] is the greatest integer function. |  |
| (R) 5∫(2)/(5)(3)/(5)*e*(5*x*− 2)3*dx* + ∫− 2− 3*e*− (*t*+ 2)3*dt* | (C)2 |
| (S)(2)/(3)∫− 12cos2(((*x* + 1)5)/(243))*dx* + 4∫0(1)/(2)sin2(32*x*5)*dx* | (D)3 |

#### 7.3.1.4 Hints and Solutions

**Single Answer MCQ’s**

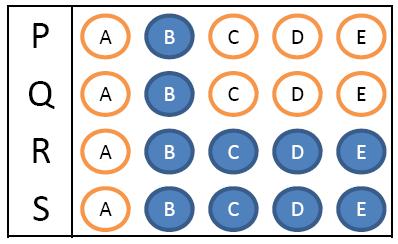
Q1 a) A  
{ Hint : a)Let *I* = ∫02tan− 1((4*x* − 4)/(4 + 2*x* − *x*2))*dx*   
 ⇒ *I* = ∫02tan− 1((*x* − 1)/(1 + (*x*)/(2) − (*x*2)/(4)))*dx*  
 ⇒ *I* = ∫02tan− 1(((*x*)/(2) − (1 − (*x*)/(2)))/(1 + (*x*)/(2)(1 − (*x*)/(2))))*dx*  
Now , we know that ∀*xϵ*(0, 2), both (*x*)/(2) and (1 − (*x*)/(2)) lie in the interval (0, 1).  
 ⇒ *I* = ∫02(tan− 1((*x*)/(2)) − tan− 1(1 − (*x*)/(2)))*dx*  
 ⇒ *I* = ∫02tan− 1((*x*)/(2))*dx* − ∫02tan− 1(1 − (*x*)/(2))*dx*  
Now, applying the property, ∫*abf*(*x*)*dx* = ∫*abf*(*a* + *b* − *x*)*dx* on the second integral, we get  
 ⇒ *I* = ∫02tan− 1((*x*)/(2))*dx* − ∫02tan− 1(1 − ((2 − *x*))/(2))*dx*  
 ⇒ *I* = ∫02tan− 1((*x*)/(2))*dx* − ∫02tan− 1((*x*)/(2))*dx* = 0

**Multiple Answer MCQ’s**

Q1: A, B, C ,D

**Matrix Match type Problems**

Matrix 1:



# 8 Applications of Integrals

## 8.1 Areas

### 8.1.1 Problems for Practice

#### 8.1.1.1 Subjective Problems

Q1: For *x* ≥ 0 , the curve *y* = *x*2sin*x* forms alternate humps and ditches with respect to the x axis. Find the ratios of the areas of the second hump and the first ditch.

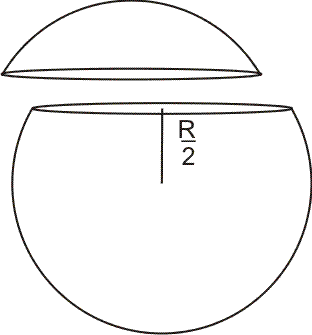
### 8.1.2 Hints and Solutions

**Subjective Problems**

Q1:

## 8.2 Curved Surface Area and Volume

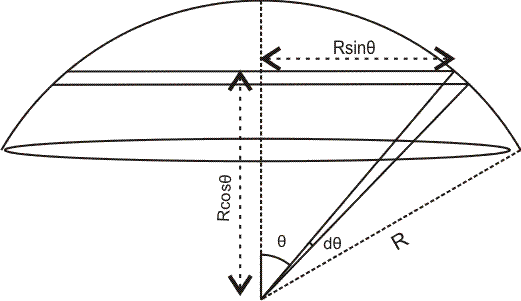
Q: A cork ball of radius *R* is taken and it is cut with a knife at a distance (*R*)/(2) from the centre . Find the volume of the smaller of the cut portions.



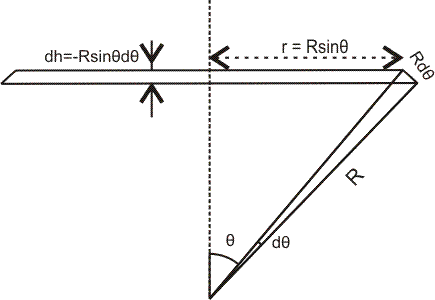
Sol: To calculate the volume of the portion of ball, we divide it into diffrential cylinders. Let us keep a single parameter *θ* to exress the radius and height of the diffrential cylinders. The parameter *θ* varies from (*π*)/(3) to 0 as the distance of the cylinder from the centre varies from 0 to (*R*)/(2) and the radius varies from (*R*√(3))/(2) to 0. The distance *h* of the diffrential cylinder from the centre is given by

***h* = *R*cos*θ***

**⇒ *dh* =  − *R*sin*θdθ***( The height of the diffrential cylinder)



Also, *r* = *R*sin*θ*(Radius of the diffrential cylinder)



Note that *dθ* is so small that the curve on the edges will vanish and the figure will be a cylinder.

Hence we can find the volume of the diffrential cylinder

 ⇒ *dV* = *πr*2*dh* = *π*(*R*sin*θ*)2( − *R*sin*θdθ*)

 ⇒ *dV* =  − *πR*3sin3*θdθ*

 ⇒ *V* = ∫(*π*)/(3)0 − *πR*3sin3*θdθ*

 ⇒ *V* = *πR*3∫0(*π*)/(3)sin3*θdθ*

 ⇒ *V* = *πR*3∫0(*π*)/(3)sin*θ*(1 − cos2*θ*)*dθ*

 ⇒ *V* = *πR*3(∫0(*π*)/(3)sin*θdθ* − ∫0(*π*)/(3)sin*θ*cos2*θdθ*)

 ⇒ *V* = *πR*3([ − cos*θ*]0(*π*)/(3) − [( − cos3*θ*)/(3)]0(*π*)/(3))

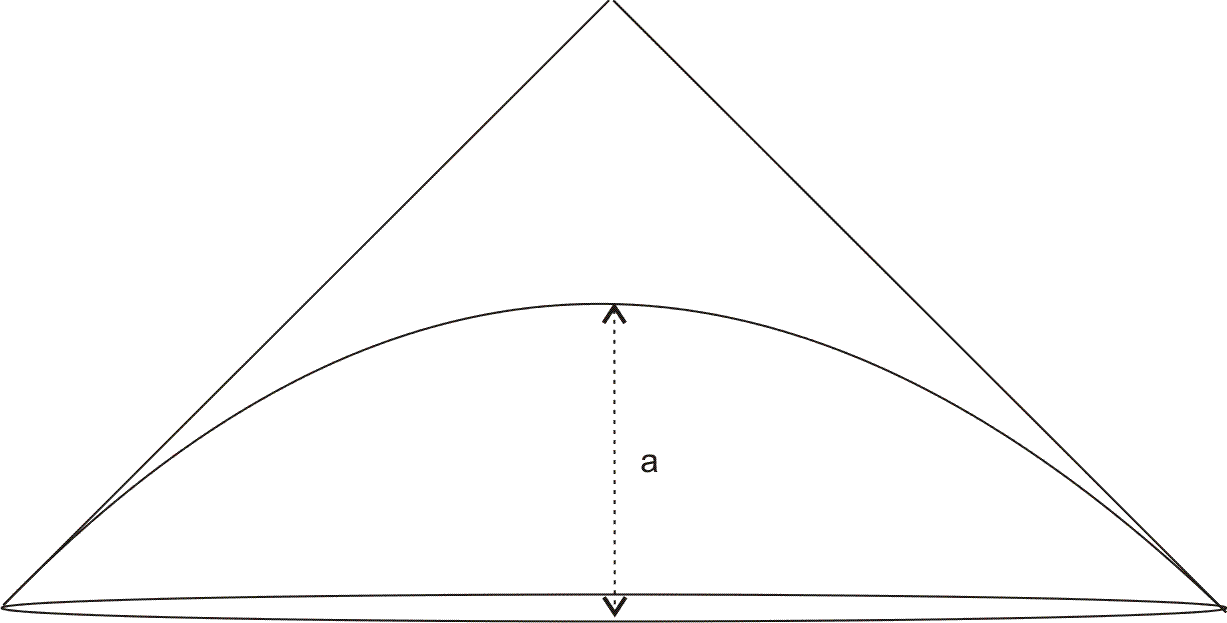
 ⇒ *V* = *πR*3( − ((1)/(2) − 1) + (1)/(3)((1)/(8) − 1))

 ⇒ *V* = (5)/(24)*πR*3

### 8.2.1 Problems for Practice

#### 8.2.1.1 Linked Comprehension Type Problems

Comprehension 1: An unnamed space project, by a major space organization is in the form of a cone with a paraboloid cavity at the bottom. The parabolic cavity has its focus at the centre of the base of the cone and it touches the the outer curved surface of the cone at the base edge. The height of the cavity is ’*a*’. The body of the project is made of solid Lead and a coating of carbon of thickness 2*μm* is made on both the curved surfaces(CSA of cone and the paraboloid cavity). A diagram is given to make the situation more clear.



Q1: The height of the space project is

a) (3)/(2)*a*  
b) 2*a*  
c) 4*a*  
d) None of these.

Q2: The volume of lead in the project is

a) (2)/(3)*πa*3  
b) *πa*3  
c) (4)/(3)*πa*3  
d) None of these

Q3: The total amount of Carbon used in the project is

a)(8)/(3)*πa*2(3√(2) − 2) × 10− 6*m*3

b)(8)/(3)*πa*2(5√(2) − 2) × 10− 6*m*3  
c)(8)/(3)*πa*2(7√(2) − 2) × 10− 6*m*3  
d) None of these

Comprehension 2: Two adjacent BSNL towers have the following specifications. One is located in Sector-36, Chandigarh and forms a Hemi-Spherical cell of radius 300m . Second one is located in Village Attawa and forms a hemi-spherical cell of radius 400m. Both the towers are at a distance of half a kilometer.[It may be assumed that the Transmitters are at ground level.]

Q1: The angle of intersection of the cells is

a) (*π*)/(6)  
b) (*π*)/(3)  
c) (*π*)/(2)  
d) None of these

Q2: The maximum height at which a person carrying a mobile phone can stand to recieve signals from both the towers is

a)90m  
b)160m  
c)240m  
d) None of these

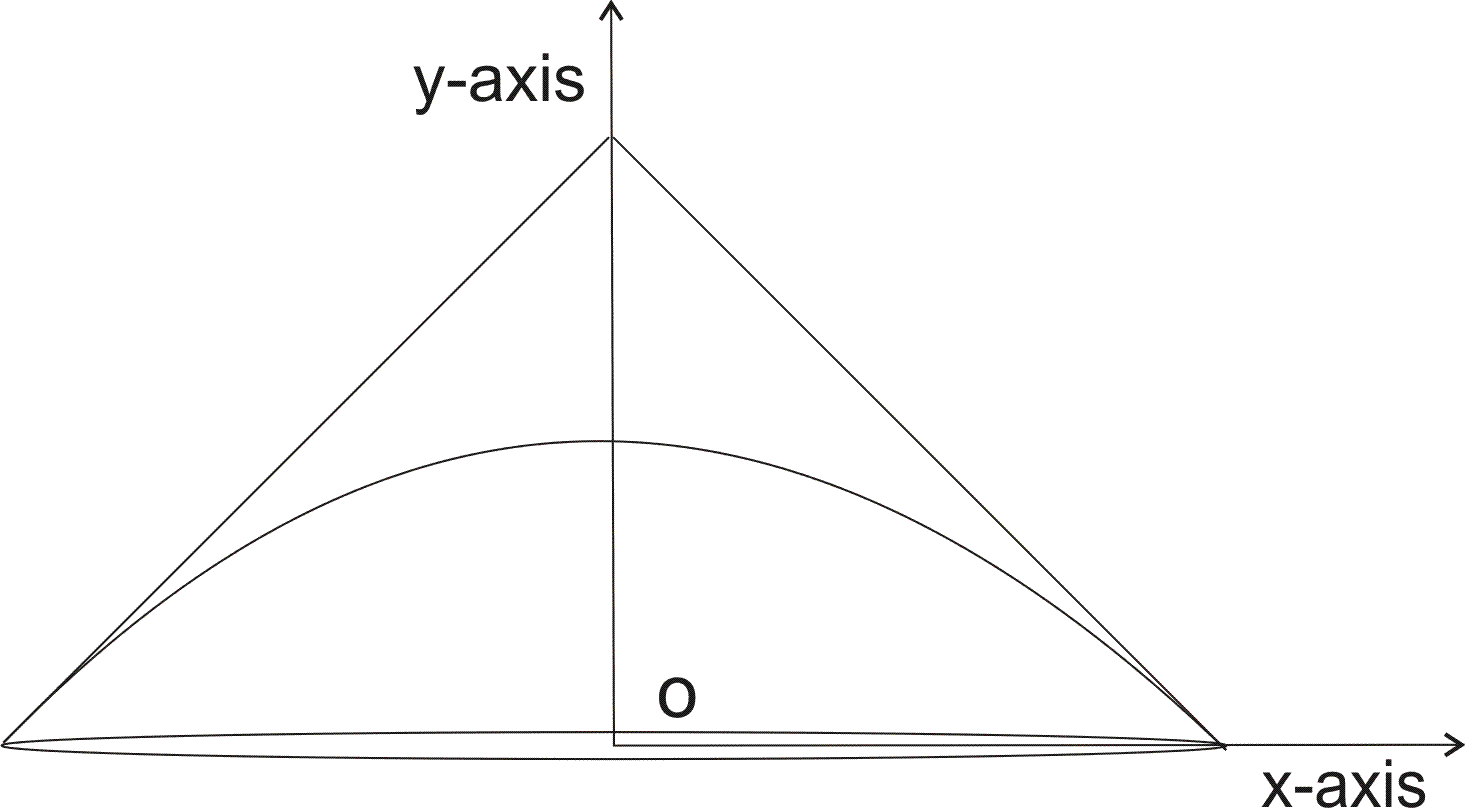
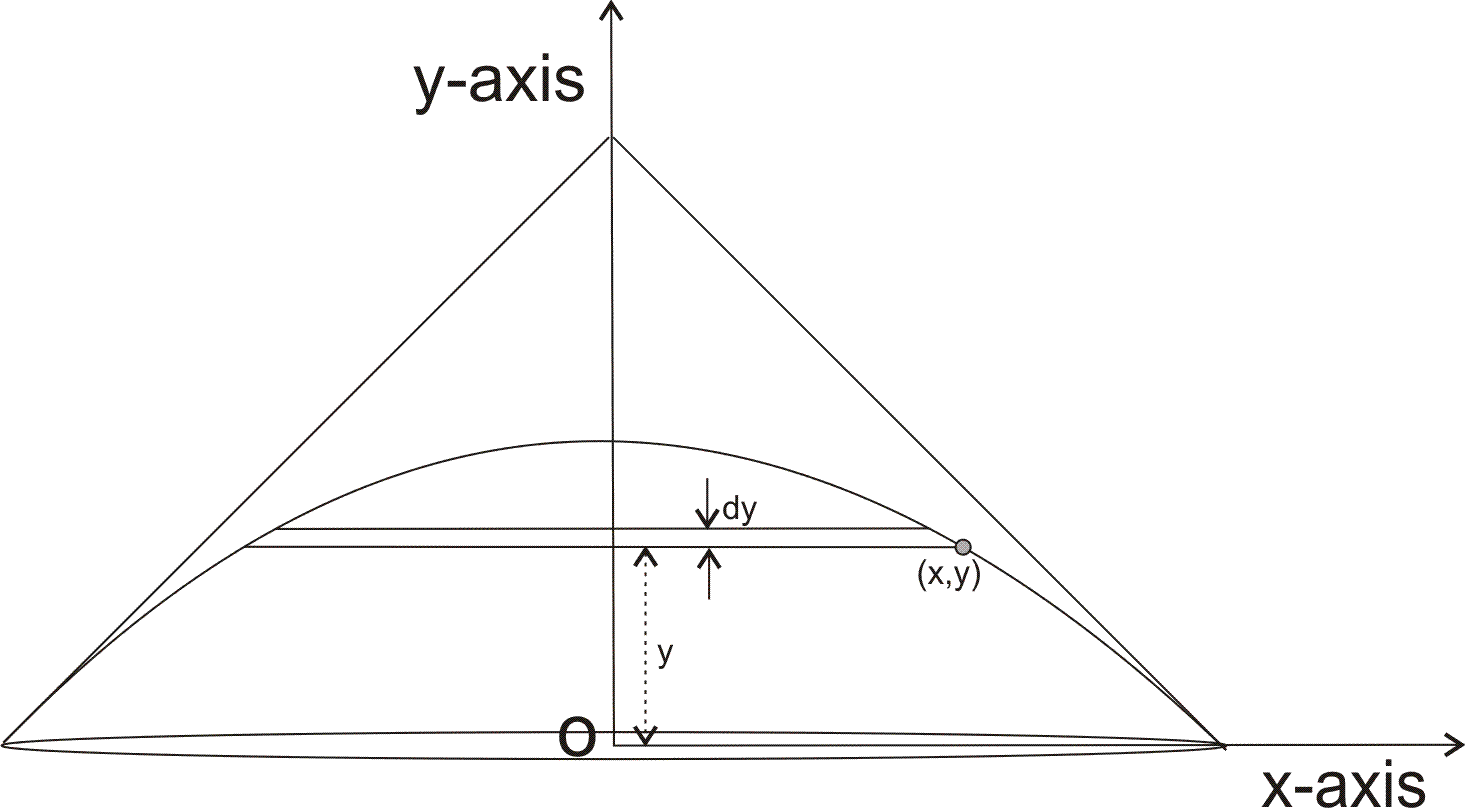
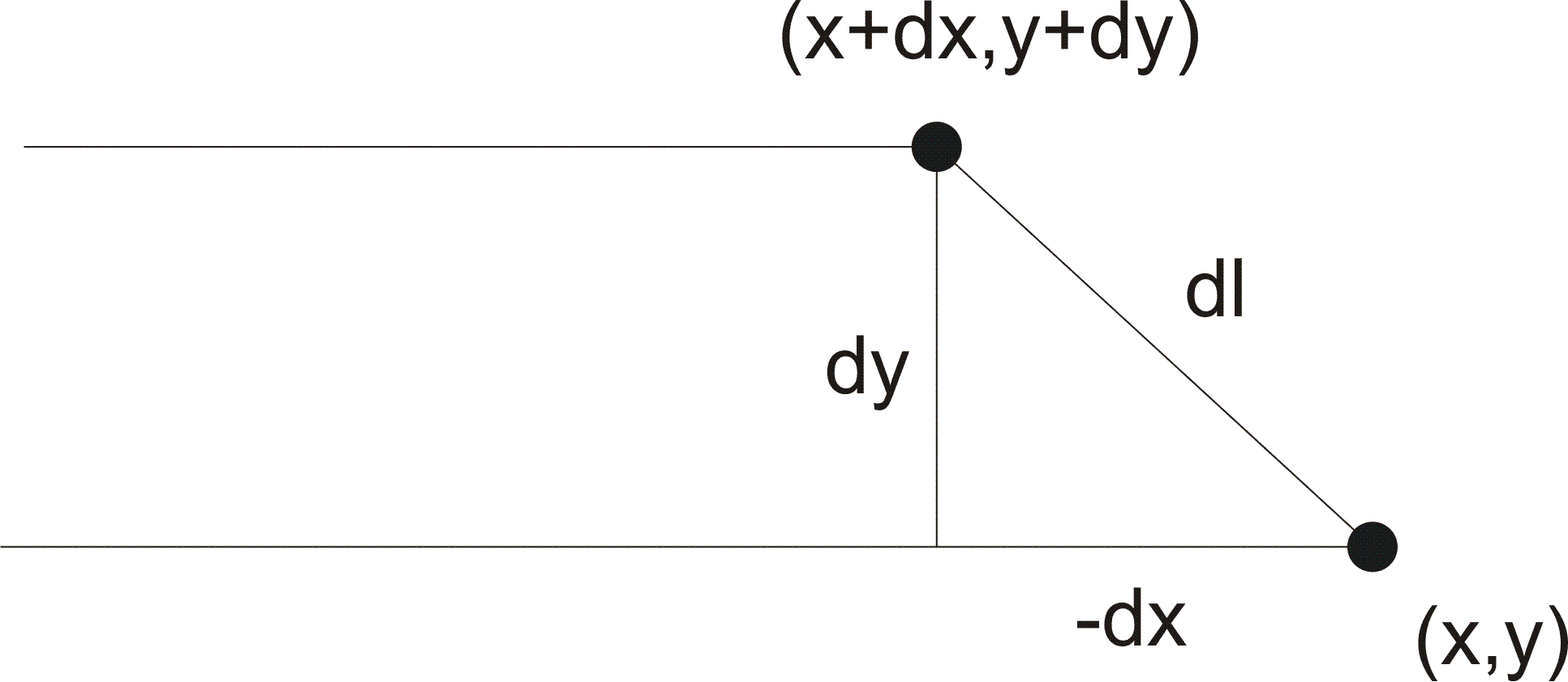
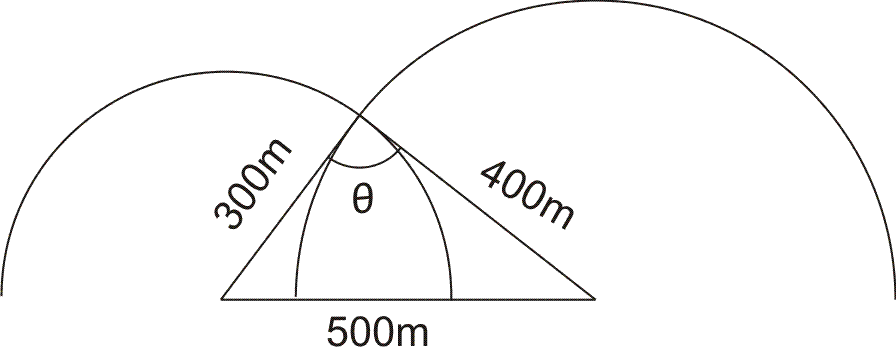
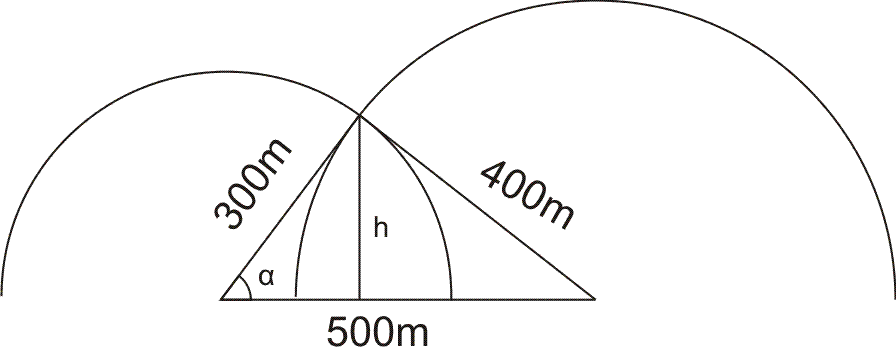
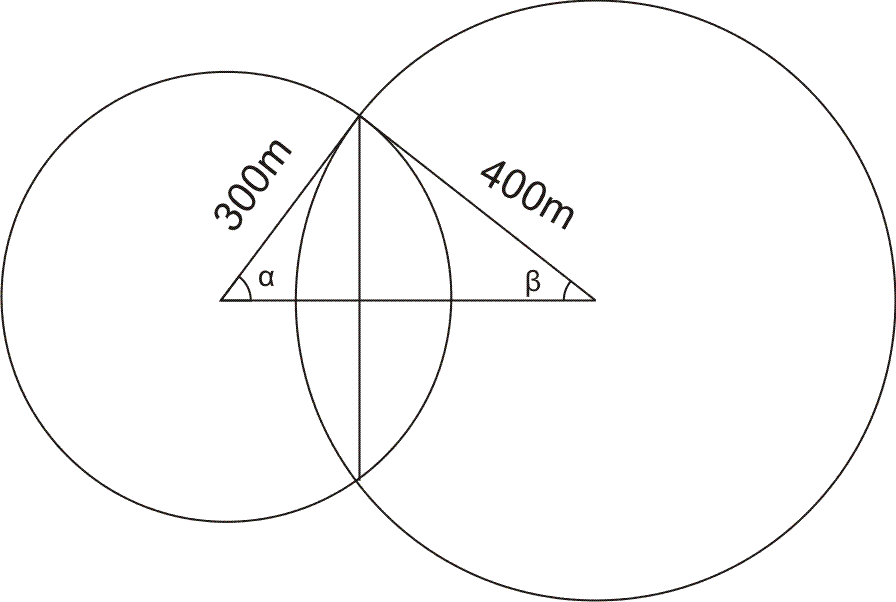
Q3: The volume of the portion which recieves signals from both the towers is

a)(92)/(3)*π* × 105*m*3

b)92*π* × 105*m*3  
c)(92)/(5)*π* × 105*m*3  
d)None of these

#### 8.2.1.2 Hints and Solutions

**Linked Comprehension type problems**

Comprehension 1: Answers Q1) B Q2) A Q3) C  
{Hint: Let us choose a coordinate axis as shown in the figure.  
  
a) The parabola is a downward facing parabola with with vertex at (0, *a*) and focus at O(0, 0). Its equation is given by *x*2 =  − 4*a*(*y* − *a*).[ We are working in the cross-sectional plane only.] . We know that the latus rectum of the parabola is of length 4*a*. This means the points where the parabola touches the cone have co-ordinates (2*a*, 0) and ( − 2*a*, 0). The slope of tangent at these two points can be found out by diffrentiating the curve  
2*x* =  − 4*a*(*dy*)/(*dx*)  
 ⇒ (*dy*)/(*dx*) =  − (*x*)/(2*a*)  
 ⇒ (*dy*)/(*dx*)|(2*a*, 0) =  − 1  
Hence the cone has a base angle of 45*o* .i.e the height of the project is 2*a*.  
b) The volume of the cone is equal to *VCone* = (1)/(3)*πrbase*2*h* = (1)/(3)*π*(2*a*)2(2*a*) = (8)/(3)*πa*3  
Now our concern is to find the volume of the cavity. To do this, we devide the cavity into diffrential cylinders of radius *x* and height *dy*.  
  
The volume of a diffrential cylinder is   
*dV* = *πx*2*dy*  
 ⇒ *VCavity* = ∫0*aπx*2*dy*  
 ⇒ *VCavity* = *π*∫0*a*( − 4*a*(*y* − *a*))*dy* [ We know the equation of the parabola as *x*2 =  − 4*a*(*y* − *a*)]  
 ⇒ *VCavity* =  − 4*aπ*∫0*a*(*y* − *a*)*dy* =  − 4*aπ*[(*y*2)/(2) − *ay*]0*a* =  − 4*aπ*( − (*a*2)/(2)) = 2*πa*3  
Hence, *VLead* = *VCone* − *VCavity* = (8)/(3)*πa*3 − 2*πa*3 = (2)/(3)*πa*3  
c) The curved surface area of the cone is *πrconelcone* = *π*(2*a*)(2√(2)*a*) = 4√(2)*πa*2  
The curved surface area of the cavity can be found out by taking diffrential elements as shown in the figure.  
  
It may be noted that *x* co-ordinate is decreasing, i.e. *dx* is negative.  
Diffrential surface area  = 2*πx*√(( − *dx*)2 + (*dy*)2) = 2*πx*√(*dx*2 + *dy*2) = 2*πx*|*dx*|√(1 + ((*dy*)/(*dx*))2) = 2*πx*( − *dx*)√(1 + ( − (*x*)/(2*a*))2) =  − (*π*)/(2*a*)2*x*√(4*a*2 + *x*2)*dx*  
Integrating this diffrential area, we get the Curved Surface Area of the Parabolic Cavity as  
C.S.Area  = ∫2*a*0( − (*π*)/(2*a*)2*x*√(4*a*2 + *x*2)*dx*) = ∫02*a*(*π*)/(2*a*)2*x*√(4*a*2 + *x*2)*dx* = (*π*)/(2*a*)∫02*a*2*x*√(4*a*2 + *x*2)*dx* = ↓(*π*)/(2*a*)((4*a*2 + *x*2))/((3)/(2))(3)/(2)]02*a* = (*π*)/(3*a*)[(8*a*2)(3)/(2) − (4*a*2)(3)/(2)] = *πa*2[(16√(2) − 8)/(3)]  
Hence, the total C.S.A is 4√(2)*πa*2 + *πa*2[(16√(2) − 8)/(3)] = *πa*2[(28√(2) − 8)/(3)] = (4)/(3)*πa*2(7√(2) − 2)   
 ⇒ The amount of carbon used is C.S.A  × 2*μm*3 = (4)/(3)*πa*2(7√(2) − 2) × 2 × 10− 6*m*3 = (8)/(3)*πa*2(7√(2) − 2) × 10− 6*m*3}  
Comprehension 2 : Answers : Q1)C Q2)C Q3) A  
{Hint: The adjoining figure clearly explains all the parameters required for finding the angle between the cells  
  
a)It may be noted that (300)2 + (400)2 = (500)2. Hence, the intervening angle is 90*o*  
b) We may redraw the figure as shown below  
  
cos*α* = (300)/(500) = (3)/(5)  
 ⇒ *h* = 300 × sin*α* = 300 × (4)/(5) = 240*m*  
c) If we complete the spheres , the required volume will be half of the intersecting portions of the spheres.  
*Vreqd* = (1)/(2) × Common portion of the complete spheres  
  
We know cos*α* = (3)/(5) . On similar lines , we can find cos*β* = (4)/(5)  
 ⇒ *Vreqd* = (1)/(2)(*Vleft* + *Vright*)[ where *Vleft* is the volume of the portion of right cell subtending an angle 2*β* at the center of the right cell. Similar is the case with *Vright*.  
From the cork ball example done earlier, we know that *Vleft* = ∫0*βπRright*3sin3*θdθ* = *πRright*3([ − cos*θ*]0*β* − [( − cos3*θ*)/(3)]0*β*) = *πRright*3( − ((4)/(5) − 1) + (1)/(3)((64)/(125) − 1)) = *π*(400)3((14)/(375))   
On similar grounds *Vright* = ∫0*απRleft*3sin3*θdθ* = *πRleft*3([ − cos*θ*]0*α* − [ − (cos3*θ*)/(3)]0*α*) = *πRleft*3( − ((3)/(5) − 1) + (1)/(3)((27)/(125) − 1)) = *π*(300)3((52)/(375))   
 ⇒ *Vreqd* = (1)/(2)(*Vleft* + *Vright*) = (1)/(2)((*π*(100)3)/(375))[43 × 14 + 33 × 52] = (92)/(3)*π* × 105*m*3}

# 9 Diffrential Equations

#### 9.2.1.1 Matrix Match type Problems

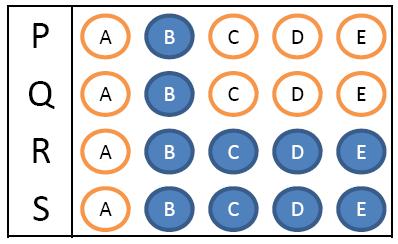
Matrix 1: Under Column I, some families of curves are mentioned. Under Column II, the diffrential equations reprsenting them are given. Match the curves in Coloumn I with the diffrential equations which can possibly represent them under Column II.

|  |  |
| --- | --- |
| **Column I** | **Column II** |
| (P) A circle of arbitrary radius | (A) (*x* − *y*)2(1 + (*y*⁄)2) |
| ’a’ in the second quadrant tou- | = (*x* + *yy*⁄)2 |
| ching both the coordinate axes. |  |
| (Q) A circle of arbitrary radius | (B) (*x* + *y*)2(1 + (*y*⁄)2) |
| ’b’ in the fourth quadrant tou- | = (*x* + *yy*⁄)2 |
| ching both the coordinate axes. |  |
| (R) A circle of arbitrary radius | (C) (*x* + *y*)2(1 + (*y*⁄)2) |
| ’c’ , touching the lines *x* =  − 2*c* | = (1)/(9)(*x* + *yy*⁄)2 |
| and *y* = 2*c* |  |
| (S) A circle of arbitrary radius | (D) (3*x* + *y*)2(1 + (*y*⁄)2) |
| ’d’ touching the lines *x* = 2*d* | = (*x* + *yy*⁄)2 |
| and *y* =  − 2*d* |  |
|  | (E) (*x* + 3*y*)2(1 + (*y*⁄)2) |
|  | = (*x* + *yy*⁄)2 |

#### 9.2.1.2 Hints and Solutions

**Matrix Match type Problems**

Matrix 1:



# Part II. Algebra

# 10 Complex Numbers

Matrix 1: A line *az* + *az* + *c* + *c* = 0 is taken in the argand plane. The equation of line after applying the transformation given, is to be found. Column I contains the transformation while column II contains the resulting equations.

|  |  |
| --- | --- |
| Column I | Column II |
| (P)The line is translated a distance *Re*(*a*) | (A)*az* + *az* = *c* + *c* + 2*Re*(*ia*2) |
| along the positive x-axis , *Im*(*a*) along |  |
| positive y-axis , and then reflected in |  |
| the real axis |  |
| (Q)The line is translated a distance *Re*(*a*) | (B)*az* + *az* + *c* + *c* = 0 |
| along the positive x-axis , *Im*(*a*) along |  |
| positive y-axis , and then reflected in |  |
| imaginary axis |  |
| (P)The line is translated a distance *Im*(*a*) | (C)*az* + *az* + *c* + *c* − 2|*a*|2 = 0 |
| along the positive x-axis , *Re*(*a*) along |  |
| negative y-axis , and then reflected in |  |
| the real axis |  |
| (P)The line is translated a distance *Im*(*a*) | (D)*az* + *az* + 2|*a*|2 = *c* + *c* |
| along the positive x-axis , *Re*(*a*) along |  |
| positive y-axis , and then reflected in |  |
| imaginary axis |  |
|  | (E) *az* + *az* + 2*Im*(*a*2) = *c* + *c* |

# 11 Quadratic Equations

#### 11.2.1.1 Linked Comprehension type Problems

Comprehension 1: Let a , b , c be three distinct real numbers and *f*(*x*) be a quadratic polynomial satisfying the equation

**[ 9*a*2 9*a* 1       9*b*2 9*b* 1       9*c*2 9*c* 1 ][ *f*( − 1)   *f*( − 2)   *f*( − 3) ] = [ 16*a*2 + 10*a*   16*b*2 + 10*b*   16*c*2 + 10*c* ]** . Let *V* be a point of local maxima of *y* = *f*(*x*) and *A* be the point where *y* = *f*(*x*) meets the x-axis and *B* be a point on *y* = *f*(*x*) such that *AB* subtends a right angle at *V*

Q1: The curve *f*(*x*) is given by

A) *f*(*x*) =  − (2)/(9)*x*2 + 2  
B) *f*(*x*) =  − (*x*2)/(10) + 16  
C) *f*(*x*) =  − (*x*2)/(16) + 10  
D) None of these

Q2: The equation of chord *AB* can be

A) 5*x* + 6*y* + 15 = 0  
B) 6*x* + 5*y* + 15 = 0  
C) 65*x* + 27*y* + 106 = 0  
D) None of these

Q3: The area of the region lying between the curve and the chord *AB* is

A) (3445)/(8)*unit*2  
B) (3445)/(64)*unit*2  
C) (3445)/(128)*unit*2  
D) None of these

# 12 Probability

#### 12.2.1.1 Matrix Match Type Problems

Matrix 1: A student goes to exam on a bicycle or scooter or car or on foot. The probabilities of his using bicycle, scooter car or being on foot are (1)/(10) , (1)/(5) , (3)/(10) or (2)/(5) respectively. The probability of reaching late at the examination centre by using these modes are (1)/(3),(1)/(4),(1)/(12) and (1)/(24) respectively. The student reached the exam centre on time. Under Column I, the modes of travel are given and under Column II, the probabilies that the student used the particular mode of transport provided that he reached the exam centre on time. Match the corresponding entries.

|  |  |
| --- | --- |
| Column I | Column II |
| (P) Bicycle | (A) (2)/(5) |
| (Q)Scooter | (B) (1)/(5) |
| (R)Car | (C) (2)/(15) |
| (S)On Foot | (D) (4)/(15) |

#### 12.2.1.2 Linked Comprehension Type Questions

Comprehension 1: There are *n* biased coins and *n* boxes numbered 1 to *n*. The coins are tossed and each put into its corresponding box. For the *kth* coin, the odds in favour of appearance of head are *k* and the odds against are 2*n* − *k* . Let *En* denote the event of selecting the *kth* box and *H* denote the event that the selected box contains a Head.

Q1: If *P*(*Ek*) = *c*∀*k* , then *P*(*Em*|*H*) where *mϵ*{1, 2, 3, ......*n*} is\begin\_inset Separator latexpar\end\_inset

* A) (*m*)/(*n*(*n* + 1))  
  B) (2*m*)/(*n*(*n* + 1))  
  C) (*c*)/(*n* + 1)  
  D) (2)/(*n* + 1)

Q2: If *P*(*Ek*) ∝ *k* for *k* = 1, 2, 3........*n* . Find *P*(*H*)\begin\_inset Separator latexpar\end\_inset

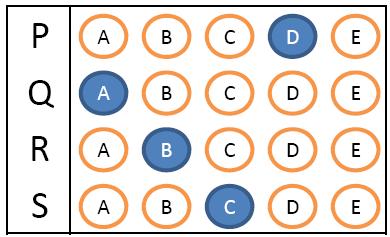
* A) (2*n* + 1)/(6*n*)  
  B) (2*n* + 1)/(4*n*)  
  C) (2)/(3)  
  D) (1)/(3)

Q3: Evaluate the probability *P* that only the *mth* box contains a head .Its value for *m* = 3, *n* = 5 is\begin\_inset Separator latexpar\end\_inset

* A) 0.12  
  B) 0.0832  
  C) 0.648  
  D) None of these

#### 12.2.1.3 Hints and Solutions

Matrix Match Type Problems  
Matrix 1:Answers



Linked Comprehension Type Questions  
Comprehension 1 Answers Q1) B Q2) A Q3) C

# 13 Logarithms Worksheet

Q1: Find the domains of definition of the following functions:

a)log((*x*3(*x*2 − 4))/((*x*2 − 1)(*x* + 3)))  
b)√(log*x*)  
c) √(log*x*(2 − *x*))

Q2 Matrix 1: Under Column I , some equations are given . Under Column II, some solutions satisfying some of the equations are given. Match the entry in Column I with the solution satisfying it in Column II.{Note: [ ] is the Greatest Integer Function}

|  |  |
| --- | --- |
| Column I | Column II |
| (P)|sin− 1*x*| = sin− 1*x* + (*π*)/(6) | (A)-(1)/(2) |
| (Q)ln|*x*| = |ln*x*| | (B)1 |
| (R)|[*x*]| = [|*x*|] | (C)(3)/(2) |
| (S)|*x*2 − 3*x* + 2| > *x*2 − 3*x* + 2 | (D)2 |

Q3 Comprehension 1: A function *fn*(*x*) is defined for all *nϵN* and *fn*+*m*(*x*) is defined as *fn*+*m*(*x*) = *fn*(*fm*(*x*)) where *f*1(*x*) = (2*x* − 1)/(*x* + 1) for *xϵR* − { − 1}. Using this definition , *f*2(*x*) = *f*1(*f*1(*x*)) = (*x* − 1)/(*x*) for *xϵR* − { − 1, 0}. Similarly *f*3(*x*) = *f*1(*f*2(*x*)) = (2 − *x*)/(1 − 2*x*) for *xϵR* − { − 1, 0, (1)/(2)}and so on. Based on the information, answer the questions below.

1: The domain of definition of *g*(*x*) = |ln(*f*34(*x*))| is

a) ( − ∞, 1) − { − 1, 0, (1)/(2)}  
b) ( − ∞, 1] − { − 1, 0, (1)/(2)}  
c) (1, ∞) − {2}  
d) None of these

2: The complete solution set of |*f*71(*x*)| > |(1)/(*f*73(*x*))| is

a) ( − 1, 1)  
b) ℝ − {2} − [ − 1, 1]  
c) ℝ − { − 1, 1, 2}  
d) None of these

3: The values of *x* for which |*f*56(*x*)| > *f*56(*x*) belong to the interval

a) ((1)/(2), 2)  
b) (2, ∞)  
c) (0, 1)  
d) None of these

Q4: (i) Prove that for the logarithmic function *y* = ln|*x*| , if the argument takes values in a G.P. , then the corresponding values of the function *y* are in A.P.\begin\_inset Separator latexpar\end\_inset

* (ii)Prove that for the exponential function *y* = *ex* , if the argument takes values in a A.P.. , then the corresponding values of the function *y* are in G.P.

# 14 Matrices

Q: Invent a 3 × 3 magic matrix M with the entries 1 , 2 , 3 , ........9. All rows and columns and diagonals add to 15 . The first row could be ( 8, 3, 4) . What could be M.

Q: If *A* = [ 1 *i*      − *i* 1 ] . Find *ATA*.

Q: Find the adjoints of the following matrices

a) [ 15 7     8 14 ]  
b) [ 7 0     0 7 ]  
c) [ 1 2     3 4 ]

Q: If *A* = [ 2*x* + 6   26   2*z* + 48   *t*2 + 41 ]

**= [ *x*2  − *x* 1       *y*2  − *y* 2       *z*2  − *z* 3       *t*2  − *t* 4 ][ 1 2     2 3     3 4 ][ 1   2 ]**

Then , *A* can be

a) [ 10   26   46   47 ]  
b) [ 10   26   54   42 ]  
c) [ 10   26   46   42 ]  
d) [ 10   26   54   41(9)/(25) ]

#### 14.2.1.1 Linked Comprehension Type Questions

Comprehension 1: Two vectors *v* = (1, 2, 3) and *w* = (1, 3, 4) are taken and put in the columns of a matrix as

*M* = [ 1 1     2 3     3 4 ]

The linear combination of these two vectors can be expressed as

*c*[ 1   2   3 ] + *d*[ 1   3   4 ] = [ 1 1     2 3     3 4 ][ *A*   *B* ]

Q1: A relation between A , B , c and d is \begin\_inset Separator latexpar\end\_inset

* a) 6*c* + 8*d* = *A* + *B*  
  b) 8*c* + 6*d* = *A* + *B*  
  c) 6*c* + 8*d* = 6*A* + 8*B*  
  d) None of these

Q2: The matrix [ *A*   *B* ] in the equation   
[ 1 1     2 3     3 4 ][ *A*   *B* ] = [ 2   5   8 ] is given by\begin\_inset Separator latexpar\end\_inset

* a) [ 1   1 ]  
  b) [ 1   2 ]  
  c) [ 2   2 ]  
  d) None of these

#### 14.2.1.2 Matrix Match Type Problems

**Matrix 1 :** Three elementary matrices *E* = [ 1 0 0       2 1 0       0 0 1 ] , *F* = [ 1 0 0       0 1 0        − 1 0 1 ] , *G* = [ 1 0 0       0 1 0       0  − 1 1 ] are given. Under column I , resultant matrices (lower triangular) obtained from *E*, *F*, *G* or their inverses are given. Match the matrices given in column I with their corresponding expressions given under column II :

|  |  |
| --- | --- |
| Column I | Column II |
| (P)[ 1 0 0        − 2 1 0        − 1 1 1 ] | (A)*EFG* |
| (Q)[ 1 0 0       2 1 0        − 1  − 1 1 ] | (B)*EF*− 1*G* |
| (R)[ 1 0 0        − 2 1 0       1 1 1 ] | (C)*E*− 1*FG*− 1 |
| (S)[ 1 0 0       2 1 0       1  − 1 1 ] | (D)*E*− 1*F*− 1*G*− 1 |
|  | (E)*G*− 1*F*− 1*E*− 1 |

# Part III. Trigonometry

# 15 Trigonometric Identities

### 15.2.1 Problems for Practice

#### 15.2.1.1 Subjective Problems

Q1: Prove the following Identities

a)**(2cos2*nθ* + 1)/(2cos*θ* + 1) = (2cos*θ* − 1)(2cos2*θ* − 1)(2cos22*θ* − 1)………(2cos2*n*− 1*θ* − 1)**

#### 15.2.1.2 Single Answer MCQ’s

Q1: If *x* = *r*sin*α*sin*β*cos*γ* , *y* = *r*sin*α*sin*β*sin*γ* , *z* = *r*sin*α*cos*β* , *w* = *r*cos*α* then *x*2 + *y*2 + *z*2 + *w*2 is independent of

A) *r*, *α*, *β*, *γ*  
B) *r*, *α*, *β*  
C) *α*, *β*, *γ*  
D) None of these

Q2: If tan*θ* = (√(6)*a*)/(2) − (1)/(2√(6)*a*) , then sec*θ* − tan*θ* is equal to

(A) √(6)*a*, (1)/(√(6)*a*)  
(B)  − √(6)*a*, (1)/(√(6)*a*)  
(C) √(6)*a*,  − (1)/(√(6)*a*)  
(D) None of these

Q1: tan(*π*)/(96) + 2tan(*π*)/(48) + 4tan(*π*)/(24) + 8 + 8√(3) is equal to

A) cot(*π*)/(96) − 6  
B) cot(*π*)/(96)  
C) 32  
D) None of these

#### 15.2.1.3 Multiple Answer Type Questions

# 16 Trigonometric Equations

#### 16.2.1.1 Single Answer MCQ’s

Q1: The number of solutions of 8*cosec*2*x* + 8− cot2*x* = (33)/(2) , 0 ≤ *x* ≤ 2*π* is

A) 4  
B) 6  
C) 8  
D) None of these

Q1: The number of distinct real solutions of

(*x*2 + 3)(*cosecx* + cot*x*) − 3*x* + √(3)[*x*(*cosecx* + cot*x*) − (*x*2 + 3)] = 0 on  − 2*π* ≤ *x* ≤ 2*π* is  
A) 1  
B) 2  
C) 4  
D) None of these

Q3 : If  − *π* ≤ *θ* ≤ *π* and *r*, *θ* satisfy *r*sin*θ* = 3 and *r* = 3(2 + 3sin*θ*) , then the number of ordered pairs (*r*, *θ*), which are solutions is/are

A) 0  
B) 1  
C) 2  
D) None of these

Q4: If 0 ≤ *x*, *y*, *z*, *t* ≤ 2*π* and sin*x* + (sin*y*)/(2) + (sin*z*)/(3) =  − *t*2 + 2*t* − (17)/(6) , then the value of *x* + *y* + *z* is

A) (3*π*)/(2)  
B) (5*π*)/(2)  
C) (7*π*)/(2)  
D) (9*π*)/(2)

#### 16.2.1.2 Multiple Answer Type Questions

Q1: If cos*θ* = *a* for exactly one value of *θϵ*[0, (7*π*)/(6)] , then the value of ’a’ can be

A) 0  
B) (√(3))/(2)  
C)  − 1  
D)  − (1)/(2)

Q2: If sin9*x* + cos6*x* = 1 in the interval  − 3*π* ≤ *x* ≤ 2*π*, then which of these statements is/are correct

A) The number of roots of the equation is 7.  
B) The sum of roots is  − 4*π*  
C) The ratio of the number of roots on the left side of zero to the number of roots on the right side on the right side of zero, on the number line is (4)/(3).  
D) None of these

# 17 Properties of Triangle

#### 17.2.1.1 Single Answer MCQ’s

Q1: The angles of a right angled triangle are in A.P. The ratio of the area of circumcircle of the triangle to the area of the triangle is

A) (2*π*)/(√(3))  
B) (*π*)/(√(3))  
C) (*π*)/(3)  
D) None of these

Q1: In a △*ABC* , with sides a,b,c and ∠*B* = (*π*)/(6) , ∠*C* = (*π*)/(4) . The area of the triangle is

A) ((√(3) − 1))/(4)*a*2  
B) (1)/(2)*a*2  
C) ((√(3) + 1))/(4)*a*2  
D) None of these

Q2. The area of △*ABC* is (*b* + *c*)2 − *a*2 . Then which of following statements is true

A) ∠*A* is acute  
B) ∠*A* is right angle  
C) ∠*A* is obtuse  
D) None of these

#### 17.2.1.2 Multiple Answer Type Questions

#### 17.2.1.3 Assertion-Reason Type Questions

Q1: Statement 1: If the sides of a triangle are 3,4,5 then the altitudes of the triangle are (1)/(3), (1)/(4), (1)/(5).

Statement 2: If the sides of a triangle are in A.P. then the altitudes of the triangle are in H.P.

#### 17.2.1.4 Linked Comprehension Type Questions

Comprehension 1: For a quadrilateral with sides a, b , c , d , semi-perimeter ’s’ and sum of any two opposite angles  = 2*α* , the area △ is given by

△2 = (*s* − *a*)(*s* − *b*)(*s* − *c*)(*s* − *d*) − *abcd*cos2*α*

Now, we take a special quadrilateral which can be inscribed in a circle *C*1 and a circle *C*2 is inscribed in it

Q1: The area of a special quadrilateral △ is given by

A) √(*s*(*s* − *a*)(*s* − *b*)(*s* − *c*))  
B) (*s*2)/(3)  
C) √(*abcd*)  
D) √(3*abcd*)

Q2: The radius of the inscribed circle *C*2 is given by

A) (△)/(*s*)  
B) (2△)/(*s*)  
C) (3△)/(*s*)  
D) (4△)/(*s*)

Q3: Cosine of the angle between the diagonals of the special quadrilateral is given by

A) ±(*ab* − *cd*)/(*s*2)  
B) ±(*ac* − *bd*)/(△)  
C) ±(*ab* − *cd*)/(*ab* + *cd*)  
D) ±(*ac* − *bd*)/(*ac* + *bd*)

Q4: If B is the angle between sides *a* and *b* of the special quadrilateral then *tan*(*B*)/(2) is given by

A) √((*ab*)/(*cd*))  
B) √((*cd*)/(*ab*))  
C) (*ab*)/(*cd*)  
D) (*cd*)/(*ab*)

# 18 Inverse Trigonometry

### 18.2.1 Problems for Practice

#### 18.2.1.1 Single Answer MCQ’s

Q1: The number of solutions of the equation sin− 1(1 − *x*) − 2sin− 1(*x*) = (*π*)/(2) is/are

a) One  
b) Two  
c) Three  
d) None of these

Q2: If (1)/(√(2)) < *x* < 1 , the number of solutions of the equations

tan− 1((1)/(*x*) − 1) + tan− 1((1)/(*x*)) + tan− 1((1)/(*x*) + 1) = tan− 1(3*x*) is  
A) 0  
B) 3  
C) 4  
D) None of these

Q3: The equation cos− 1*x* = 3cos− 1*a* has a solution for

A) all real values of *a*  
B) all real values of *a* satisfying *a* ≤ 1  
C) all real values of *a* satisfying (1)/(2) ≤ *a* ≤ 1  
D) all real values of *a* satisfying (1)/(√(2)) ≤ *a* ≤ (√(3))/(2)

#### 18.2.1.2 Multiple Answer Type Questions

#### 18.2.1.3 Linked Comprehension Type Problems

Comprehension 1: A function *f*:((5*π*)/(2), (7*π*)/(2)) → ( − 1, 1) is defined by *f*(*x*) = sin*x*. Its inverse is denoted by *f*− 1(*x*) = sin− 1*x* . Another function *g*:(2*π*, 3*π*) → ( − 1, 1) is defined by *g*(*x*) = cos*x* . Its inverse is denoted by *g*− 1(*x*) = cos− 1*x* . An onto function *h*:( − 1, 0) → *Rh* is given by *h*(*x*) = *f*− 1(*x*) − *g*− 1(*x*) . Now answer the following questions.

Q1: *g*− 1( − *x*) is given by

a) *π* − *g*− 1(*x*)  
b) 5*π* − *g*− 1(*x*)  
c) (5*π*)/(2) − *g*− 1(*x*)  
d) None of these

Q2: Set ’*Rh*’ is

a) {(*π*)/(2)}  
b) ((5*π*)/(2), 3*π*)  
c) (0, (*π*)/(2))  
d) None of these

Q3: If a function *v*:( − ∞, 0] → [3*π*, (7*π*)/(2)) is defined by *v*(*x*) = *f*− 1((2*x* − 2−*x*)/(2*x* + 2−*x*)) , then *v*(*x*) is

a) Injective Only  
b) Surjective Only  
c) Bijective  
d) None of these.

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