# A Complete Course in Physics ( Graphs ) - First Edition

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# Preface and Acknowledgements

This book is a research evaluate stuff on Kinematics covering graphs. The material has got acclaim locally and now looking for it internationally.

I would like to thank my family, well wishers for their support and consistent encouragement and most of all God for his grace and Guidance.

# 1 Concepts of Graphs

## 1.1 The Equations of motion and the origin of Graph Handling

### 1.1.1 [↓](" \l "index-The-First-Equation)The First Equation

The Equation *v* = (*dx*)/(*dt*) in linear motion implies

i) The **Slope** of **Position-Time Graph** is **Instantaneous Velocity.**

ii) The **Area** under the **Velocity-Time Graph** is **Change in Position**.

{ The second one requires the manipulation , *dx* = *vdt* i.e. ∫*dx* = ∫*vdt* }

The equations can be further manipulated to obtain the Speed Time Graph , where

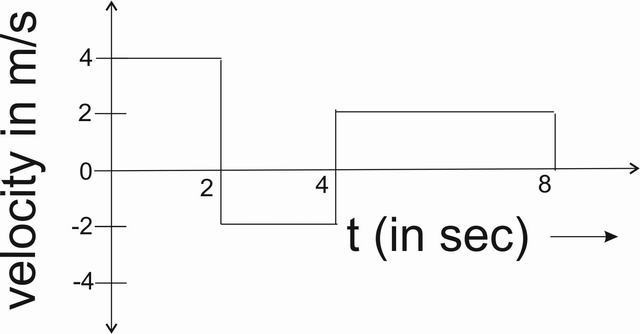
speed = rate of change of distance wrt time

Few of the following examples illustrate this concept:

Example: On a displacement-time graph, two straight lines make angles of 30° and 60° with the time-axis. The ratio of the velocities represented by them is

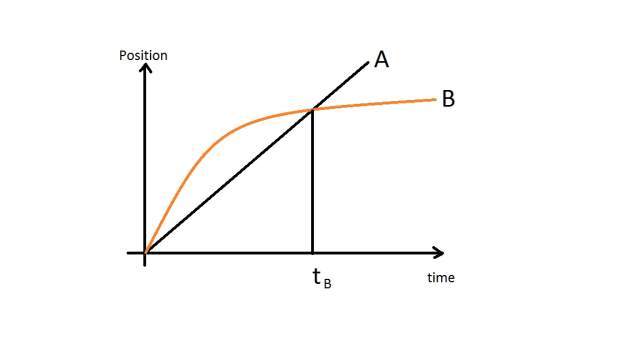
a) 1:√(3)  
b) 1:3  
c) √(3):1  
d) 3:1

Example: A body is moving in a straight line as shown in velocity-time graph. The displacement and distance travelled by body in 8 second are respectively:



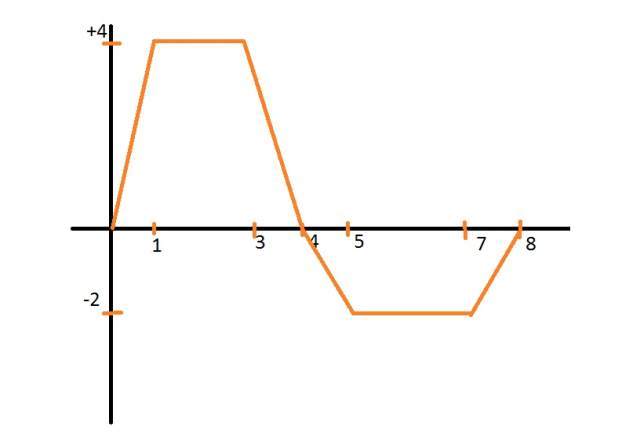
a) 12 m, 20 m  
b) 20 m, 12 m  
c) 12 m, 12 m  
d) 20 m, 20 m

Example: The graph shows position as a function of time for two trains running on parallel tracks. Which statement is true?



a) At time t*B* both trains have the same velocity.  
b) Both trains have the same velocity at some time after t*B* .  
c) Both trains have the same velocity at some time before t*B* .  
d) Somewhere on the graph, both trains have the same acceleration.

Example: The velocity-time graph of a particle in linear motion is as shown. Both v and t are in SI units. The displacement of the particle is



a) 6 m  
b) 8 m  
c) 16 m  
d) 18 m

### 1.1.2 [↓](" \l "index-The-Second-Equation)The Second Equation

Proceeding similar to above, the equation *a* = (*dv*)/(*dt*) implies

i) The **Slope** of **Velocity-Time Graph** is **Instantaneous Acceleration**.

ii) The **Area** under **Acceleration-Time Graph** is **Change in Velocity**.

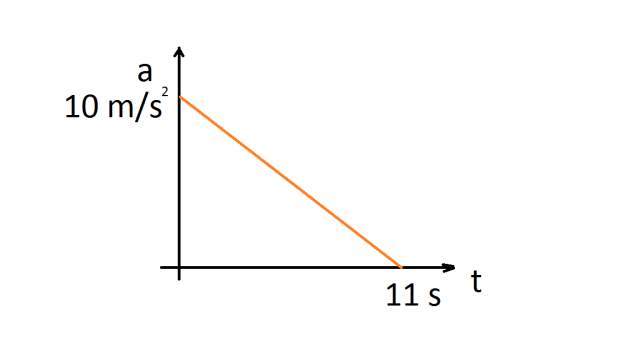
{ The second one requires the manipulation , *dv* = *adt* i.e. ∫*dv* = ∫*adt* }

A few of the following examples illustrate it.

Example: A car starts from rest acquires a velocity v with uniform acceleration 2*ms*− 2 then it comes to stop with uniform retardation 4*ms*− 2 . If the total time for which it remains in motion is 3 sec, the total distance travelled is:

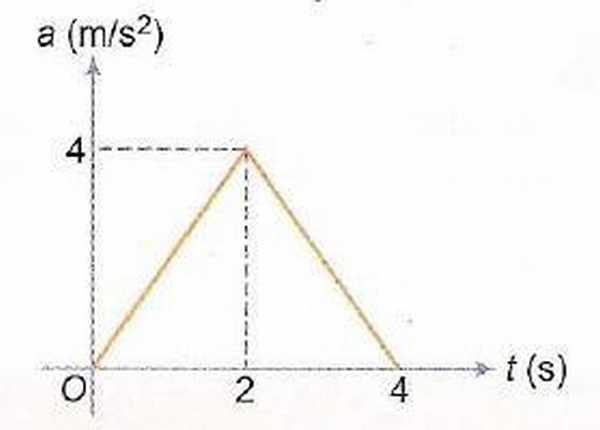
a) 2 m  
b) 3 m  
c) 4 m  
d) 6 m

Example: A particle starts from rest. Its acceleration (a) vs time (t) is as shown in the Figure. The maximum speed of the particle will be

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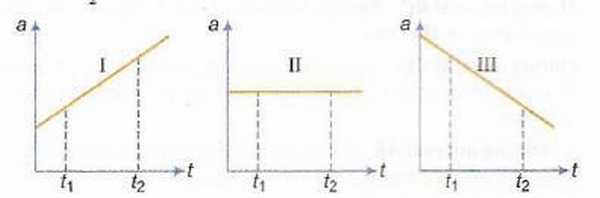
a) 110 m/s  
b) 55 m/s  
c) 550 m/s  
d) 660 m/s

Example: Acceleration-time graph of a particle moving in a straight line is shown in Figure. The velocity of particle at time t = 0 is 2 m/s. Velocity at the end of fourth second is

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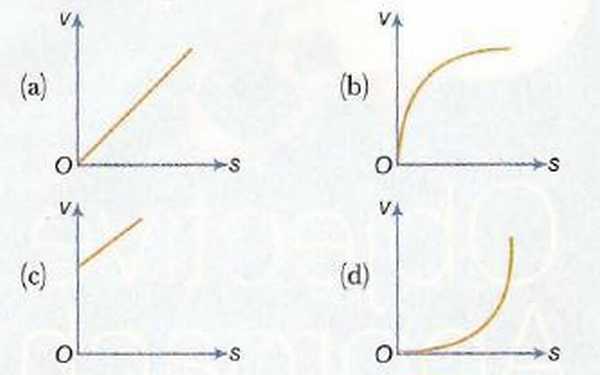
a) 8 m/s  
b) 10 m/s  
c) 12 m/s  
d) 14 m/s

Example: Each of the three graphs represents acceleration vs time for an object that already has a positive velocity at time *t*1 . Which graph/graphs show an object whose speed is increasing for the entire time interva1 between *t*1 and *t*2 ?

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a) Graph I only  
b) Graphs I and II  
c) Graphs I and III  
d) Graphs I, II and III

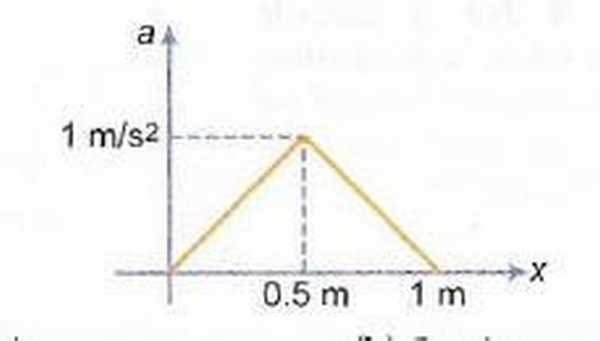
Example: A body starts from rest and moves along a straight line with constant acceleration. The variation of speed v with distance s is given by the graph

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### 1.1.3 The Acceleration-Position Graph Variate

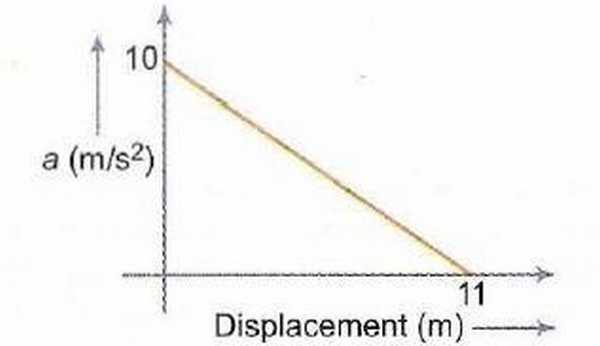
This kind of graph requires the manipulation of the Equation *a* = (*dv*)/(*dt*) as follows  
*a* = (*dv*)/(*dx*).(*dx*)/(*dt*)  
 ⇒ *a* = (*dv*)/(*dx*).*v*  
 ⇒  *adx* = *vdv* and integration can be performed to further solve it.

Example: A body, initially at rest, starts moving along x-axis in such a way that its acceleration vs displacement plot is as shown in the Figure. The maximum velocity of the particle is

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a) 1 m/s  
b) 6 m/s  
c) 2 m/s  
d) None of these

Example : A particle initially at rest, it is subjected to a non-uniform acceleration a, as shown in the figure. The maximum speed attained by the particle is

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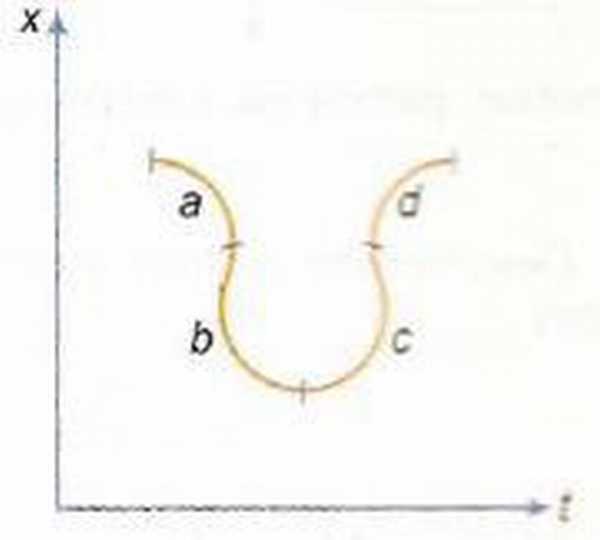
a) 605 m/s  
b) 110 m/s  
c) 55 m/s   
d) 110 m/s

## 1.2 Other Types of Graphs

### 1.2.1 Sign of Acceleration from Position-Time Graph

The sign of Acceleration can be determined from the Position-Time Graph. The methodology involves of looking at the Concavity of the Graph  
i) If the graph is Concave-Up, the Acceleration is Positive.  
ii) If the graph is Concave-Down, the Acceleration is Negative.  
iii) If the graph is a straight line, the Acceleration is ZERO. { Irrespective of any other factor , such as the slope or direction of line}

Example: The graph given below is a plot of distance vs time. For which labelled region is the “Velocity Positive and the Acceleration Negative”

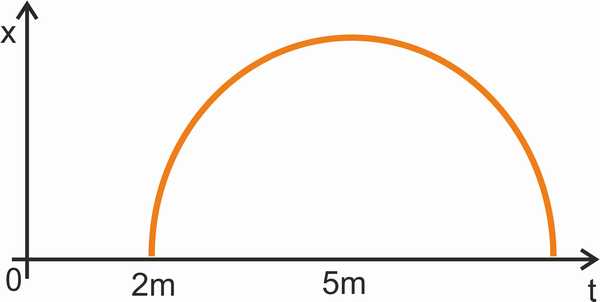


a) a  
b) b  
c) c  
d) d

### 1.2.2 [↓](" \l "index-The-Average-Velocity-/-Instantane)The Average-Velocity / Instantaneous Velocity , Equal Case

We know , that ( in a x-t graph) the slope of the Secant is the Average Velocity , whereas the slope of Tangent is the Instaneous Velocity. The point where these two lines coincide, is the point where Average Velocity is equal to Instantaneous Velocity.

Example: Position-time graph is shown which is a semicircle from t = 2 to t = 8 s. Find time t at which the instantaneous velocity is equal to average velocity over first t seconds,

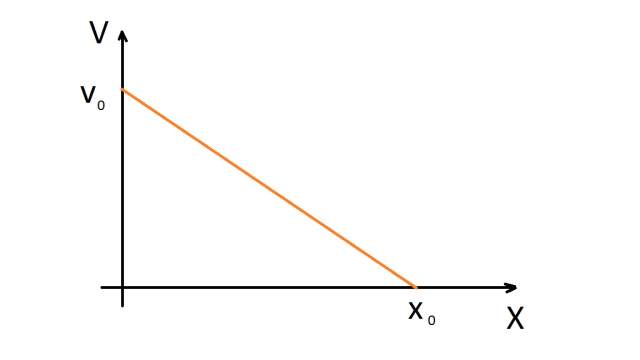


a) 4.8 s  
b) 3.2 s  
c) 2.4 s  
d) 5 s

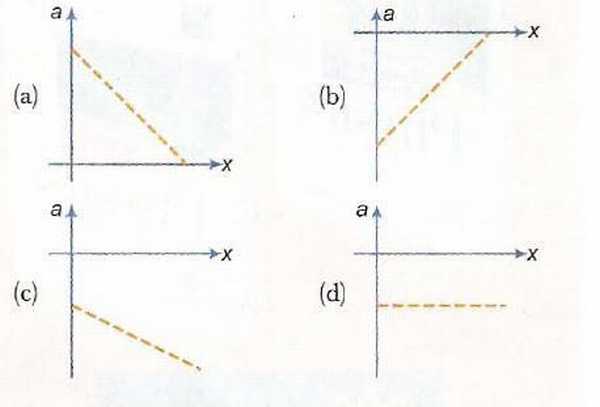
### 1.2.3 The Velocity-Displacement Case

This can be handled in a similar way as Acceleration-Displacement case by integrating the respective equation. Here the problem is of v = f(x) type, which can be integrated by writing (*dx*)/(*dt*) = *f*(*x*)  
i.e. dx = f(x)dt

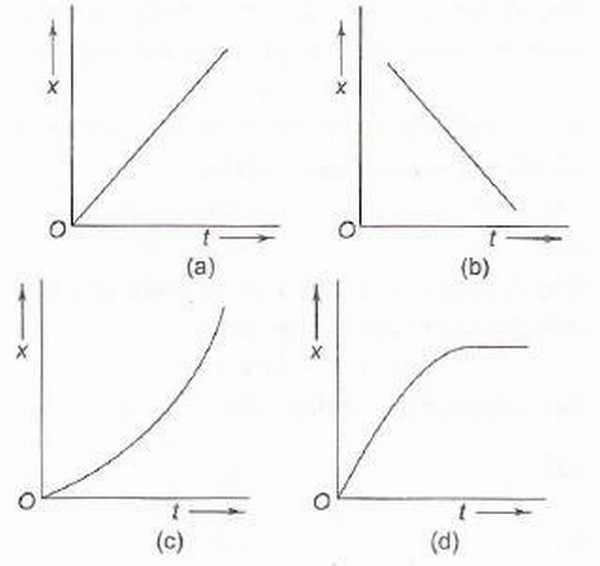
Example: The velocity-displacement graph of a particle moving along a straight line is shown here.

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The most suitable acceleration-displacement graph will be



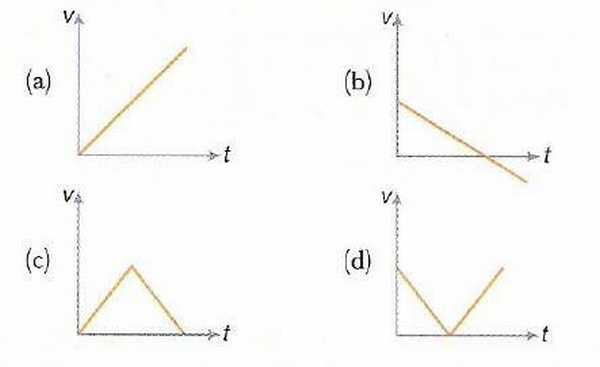
Example. The velocity (v) of a body moving along the postive x-direction varies with displacement (x) from the origin as p v = k x , where k is a constant. Which of the graphs shown in Fig. correctly represents the displacement-time (x - t) graph of the motion?

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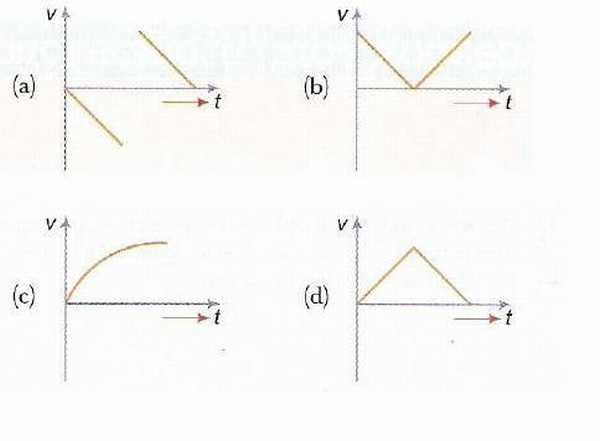
### 1.2.4 [↓](" \l "index-Motion-Under-Free-Fall)Motion Under Free Fall due to Gravity

In such examples, the governing equations rule and the coordi-nate system needs to be properly chosen.

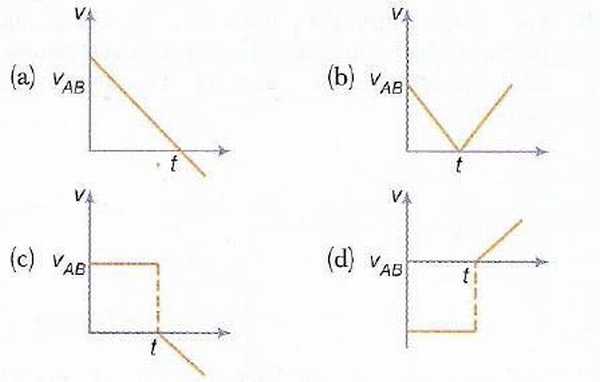
Example: Which of the following graphs correctly represents velocity-time relationship for a particle released from rest to fall freely under gravity?

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Example: The velocity-time graph of a body falling from rest under gravity and rebounding from a solid surface is represented by which of the following graphs?

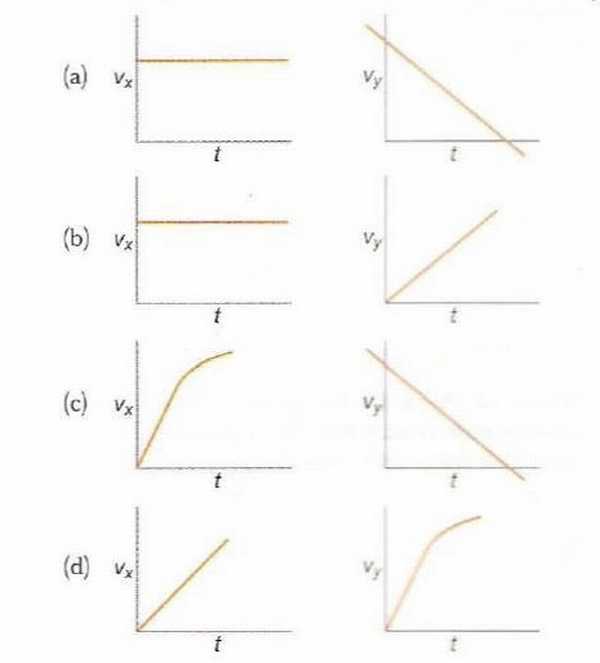
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Example: A body A is thrown vertically upwards with such a velocity that it reaches a maximum height of h. Simultaneously another body B is dropped from height h. It strikes the ground and doesn’t rebound. The velocity of A relative to B vs time graph is best represented by (upward direction is positive.)

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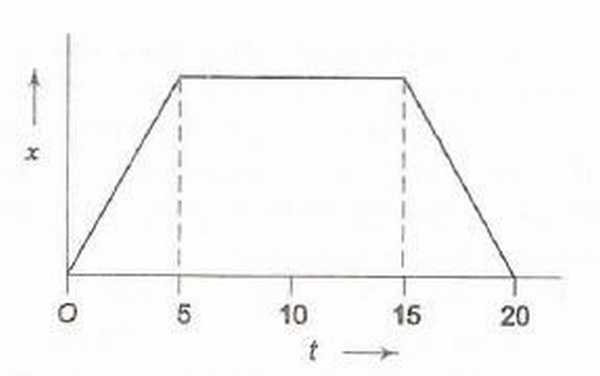
### 1.2.5 Projectile Motion

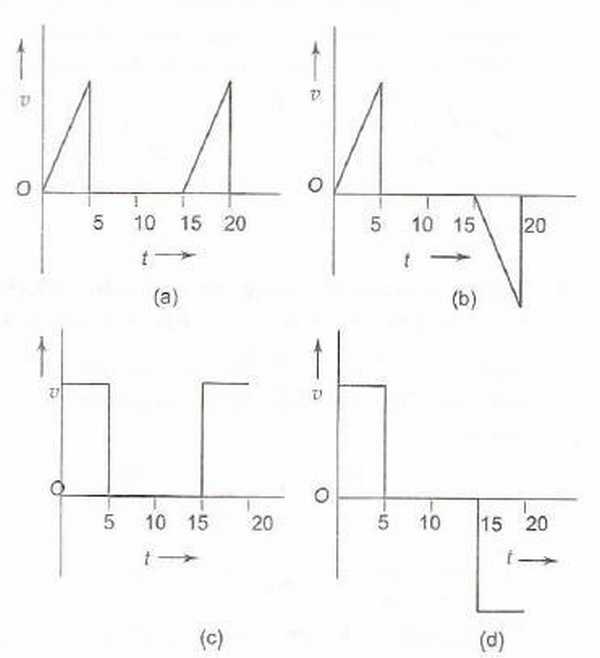
Example: A shell is fired from a gun at an angle to the horizontal. Graphs are drawn for its horizontal component of velocity *vx* and its vertical component of velocity *vy*.

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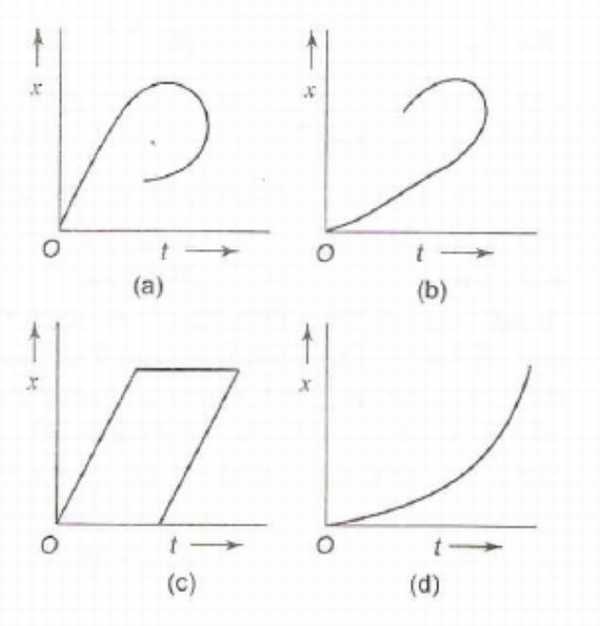
### 1.2.6 Miscelleneous

Example. Figure shows the displacement-time ( x-t ) graph of body moving in a straight line. Which one of the graphs shown in Fig. represents the velocity- time ( v-t ) graph of the motion of the body.

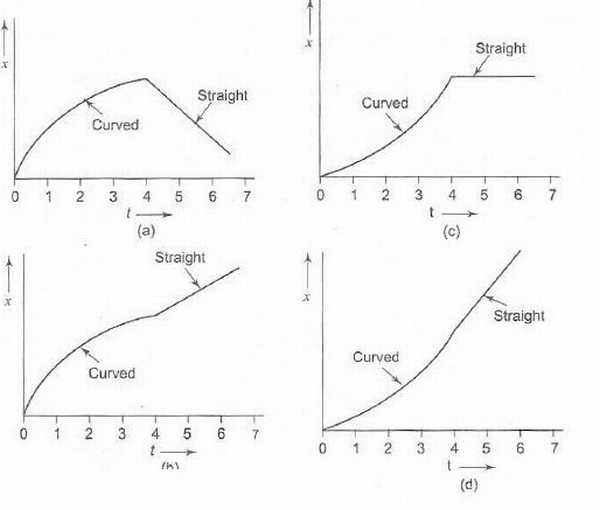
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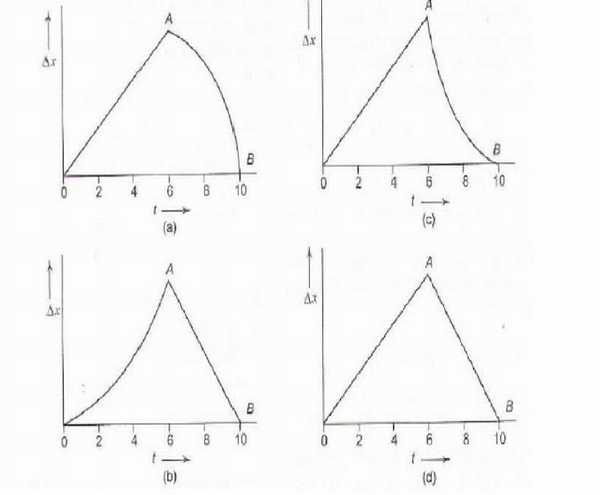
Example. Which of the displacement-time (x−t) graphs shown in Fig. can possibly represent one dimensional motion of a particle?

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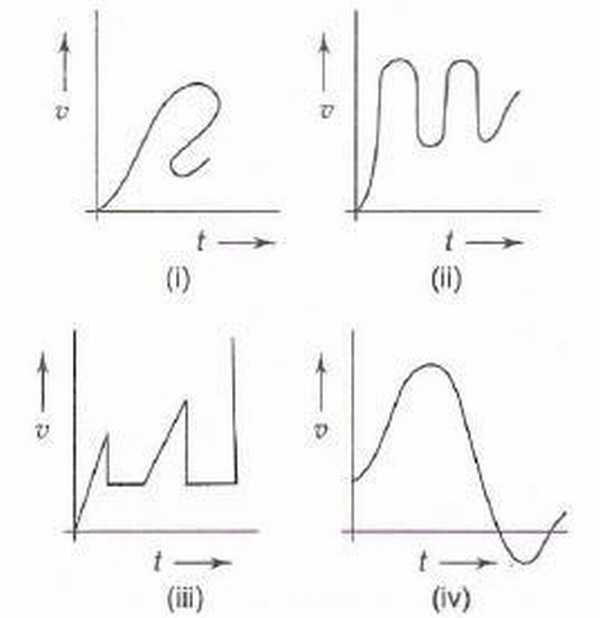
Example. A car starts from rest, accelerates uniformly for 4 seconds and then moves with uniform velocity. Which of the (x-t) graphs shown in Fig. represents the motion of the car upto t = 7 s?

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Example. Two stones are thrown up simultaneously with initial speeds of *u*1 and *u*2, (*u*2 > *u*1) . They hit the ground after 6 s and 10 s respectively. Which graph in Fig. correctly represents the time variation of △*x* = *x*2 − *x*1, the relative position of the second stone with respect to the first upto t = 10 s? Assume that the stones do not rebound after hitting the ground.

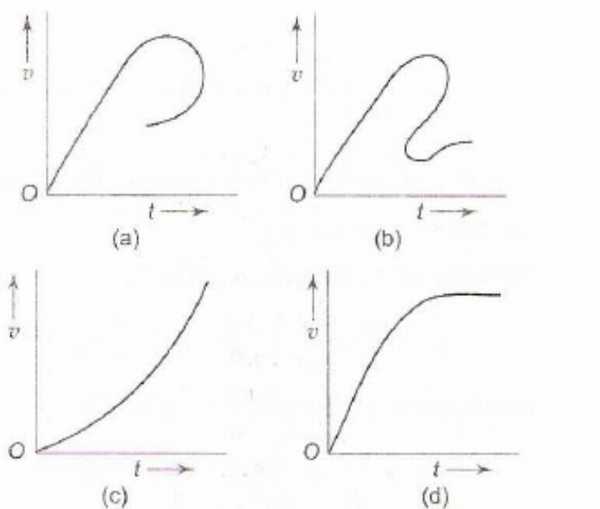
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Example. Figure shows the velocity-time (v - t) graphs for one dimensional motion. But only some of these can be realized in practice. These are

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a) (i), (ii) and (iv) only  
b) (i), (ii) and (iii) only  
c) (ii) and (iv) only  
d) all

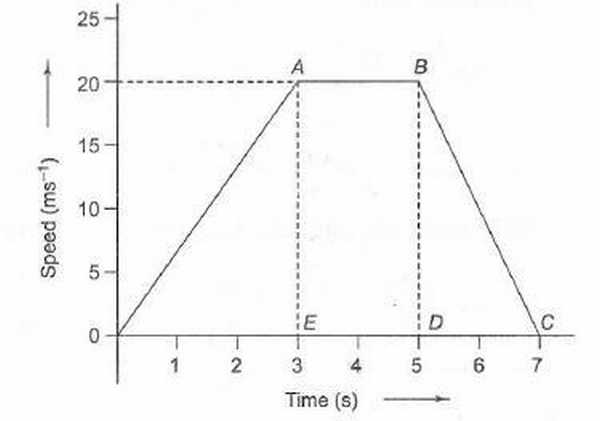
Example. Which of the velocity-time (v-t) graphs shown in Fig. can possibly represent one-dimensional motion of a particle?

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## 1.3 Question Types

### 1.3.1 Passage Type

Example: The speed-time graph of the motion of a body is shown in Fig.

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1. The accelerations of the body during the last 2 second is

a) (20)/(3)*ms*− 2  
b)  − (20)/(7)*ms*− 2  
c)  − 10*ms*− 2  
d) Zero

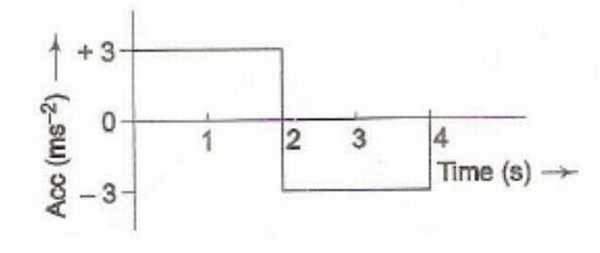
2. The ratio of distance travelled by the body during the last 2 seconds to the total distance travelled by it is

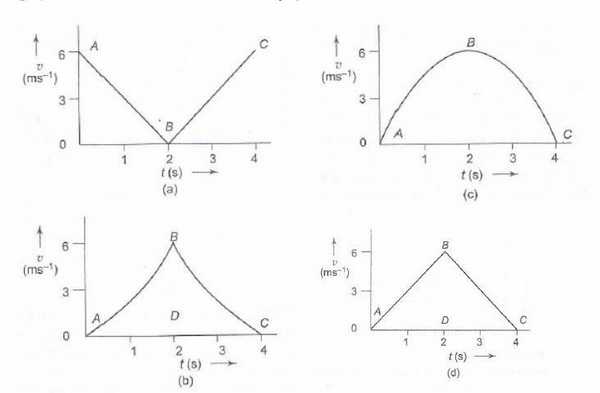
a) 1/9   
b) 2/9   
c) 3/9   
d) 4/9

3. The average speed of the car during the whole journey is

a) 10 m/s   
b) 20 m/s   
c) 90/7 m/s   
d) 40/7 m/s

Example: A body starts from rest at time t = 0 and undergoes an acceleration as shown in Fig.Which of the graphs shown in Fig. represents the velocity-time (v-t) graph of the motion of the body from t = 0 s to t = 4 s?

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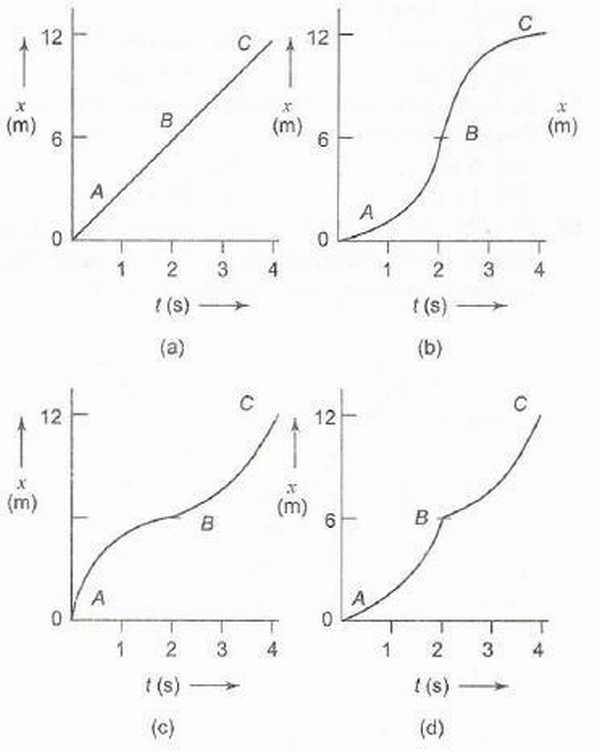
1. In Question above, what is the velocity of the body at time t = 2.5 s?

a) 2.5 m/s  
b) 3.5 m/s  
c) 4.5 m/s  
d) 5.5 m/s

2. In above question, how much distance does the body cover from t = 0 s to t = 4 s?

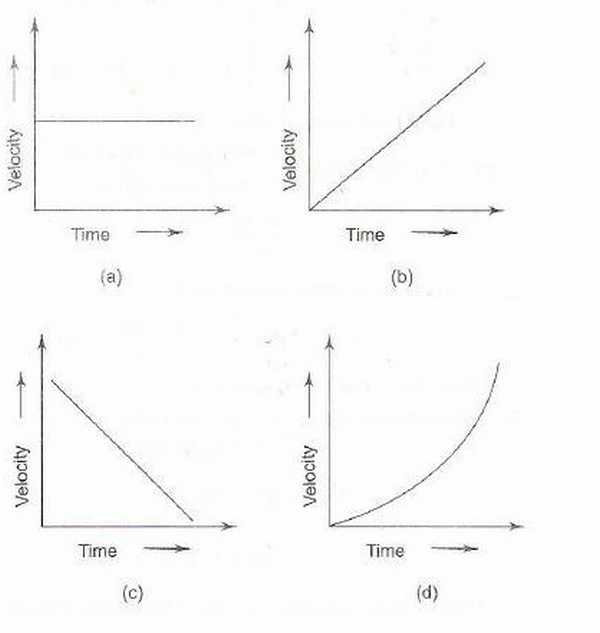
a) 6 m  
b) 9 m  
c) 12 m  
d) 15 m

3 In above question, which of the graphs shown in Fig. represents the displacement-time (x-t) graph of the motion of the body from t = 0 s to t = 4 s?

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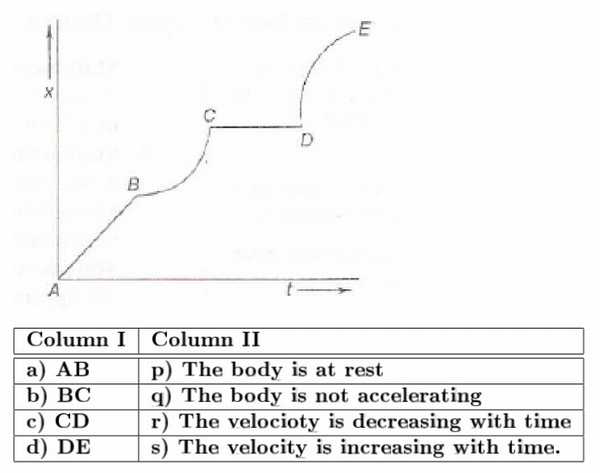
### 1.3.2 Matching

1. Match the graphs (a), (b), (c) and (d) shown in Fig. with the types of motions (p), (q), (r) and (s) that they represent

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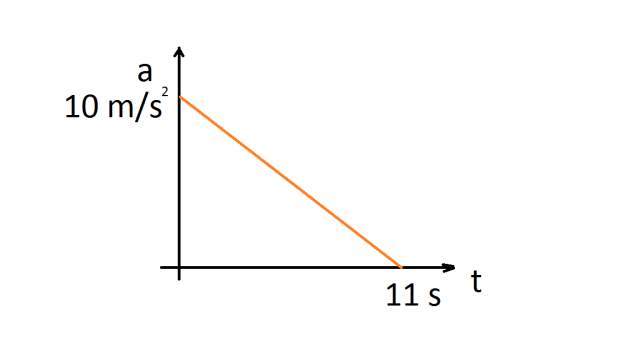
p) Motion with non-uniform acceleration  
q) Motion of a body covering equal distances in equal intervals of time  
r) Motion having a constant retardation  
s) Uniformly accelerated motion.

2. Figure shows the displacement ·time (x - t) graph of the m-tion of a body.

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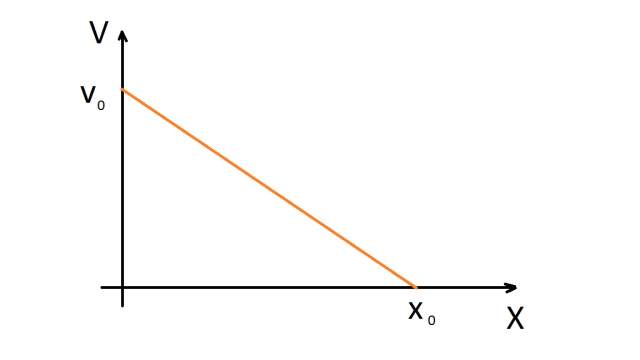
## 1.4 [↓](" \l "index-Previous-Year-Problems-IIT)Previous Year Problems IIT

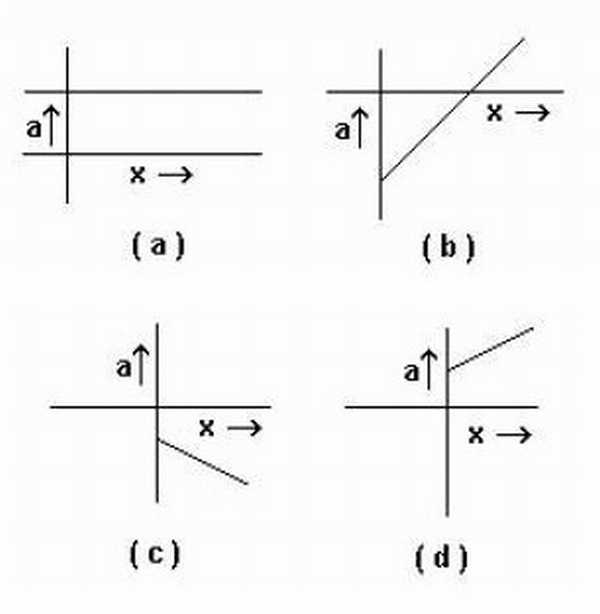
Q1: A particle starts from rest. Its acceleration ( a ) versus time ( t ) is as shown in the gure. The maximum speed of the particle will be

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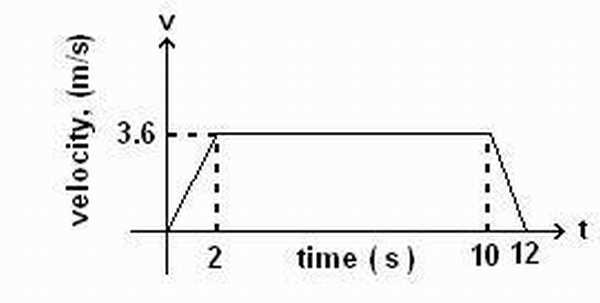
a ) 110 m /s   
b ) 55 m /s   
c ) 550 m /s   
d ) 660 m /s

Q2: If graph of velocity vs. distance is as shown, which of the following graphs correctly represents the variation of acceleration with displacement ?

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Passage A lift is going up. The variation in the speed of the lift is as given in the graph.

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Q3: What is the height to which the lift takes the passengers ?

a ) 3.6 m   
b ) 28.8 m   
c ) 36 m  
d ) cannot be calculated from the above graph

Q4: In the above graph, what is the average velocity of the lift?

a ) 1 m/s   
b ) 2.88 m/s  
c ) 3.24 m/s   
d ) 3 m/s

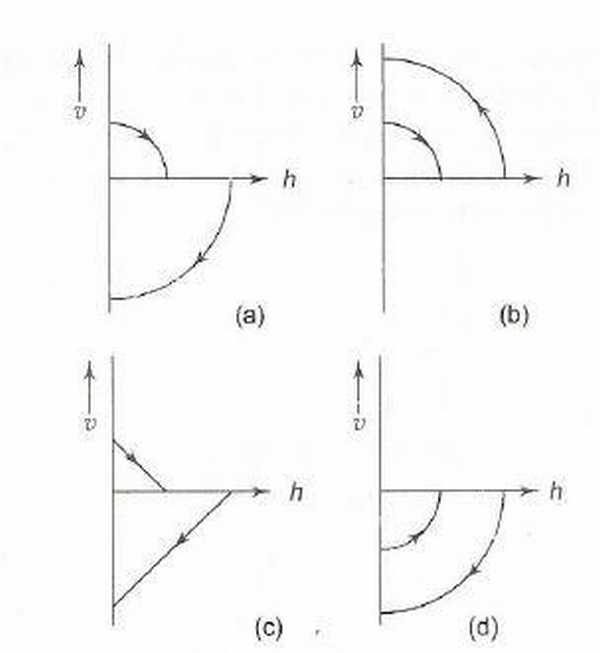
Q5: In the above graph , what is the average acceleration of the lift?

a ) 1.8*m* ⁄ *s*2   
b ) −1.8*m* ⁄ *s*2   
c ) 0.3*m* ⁄ *s*2   
d ) zero

Q6: Four persons K, L, M and N are initially at the corners of a square of side of length d. If every person starts moving with velocity v such that K is always headed towards L, L towards M, M towards N and N towards K, then the four persons will meet after

a ) d/v s   
b ) d2/ v s   
c ) d / 2v s   
d ) d / 2v s

Example. A ball is dropped vertically from a height h above the ground. It hits the ground and bounces up vertically to a height h/2. Neglecting subsequent motion and air resistance, its velocity v varies with the height h as (see Fig.) (l.l.T. 2000)

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