

Part I

Energy Conservation

Abstract Introduction

1 KINETIC ENERGY

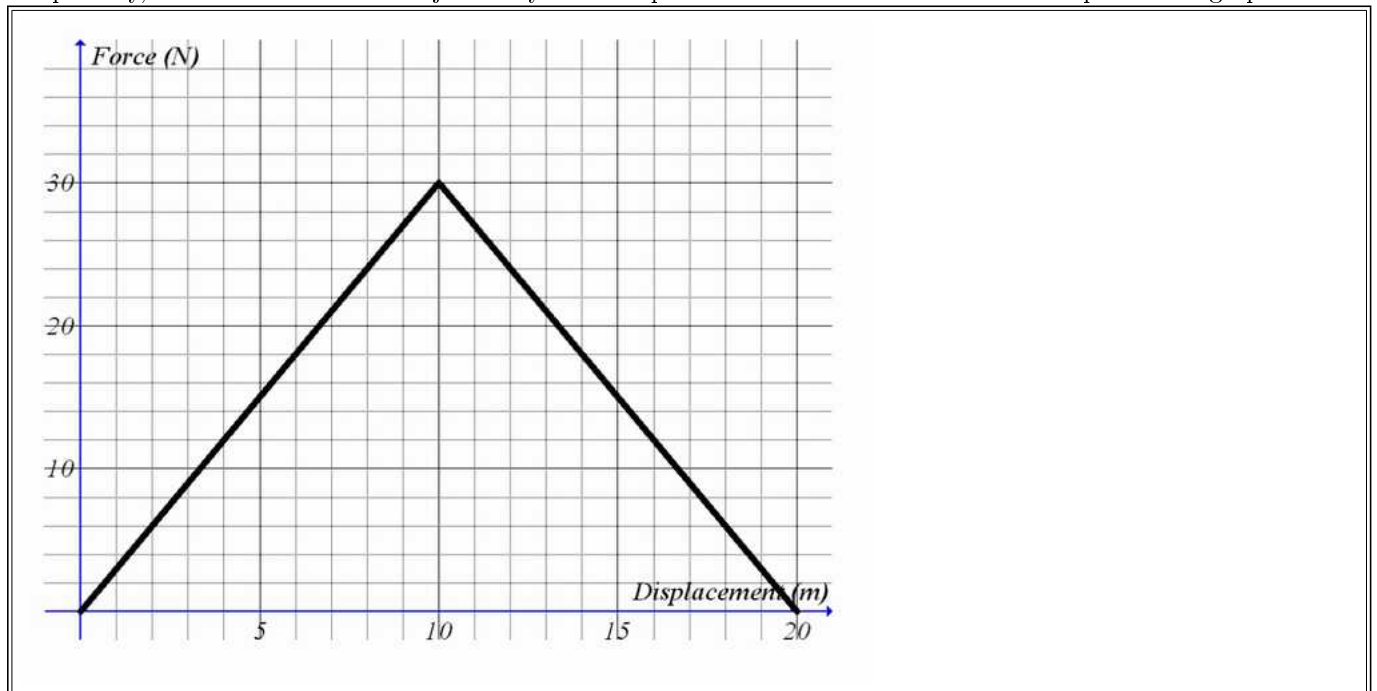
Objects have energy because of their motion; this energy is called kinetic energy. Kinetic energy of the objects having mass m and velocity v can be calculated with the formula given below;

$$E_k = \frac{1}{2}mv^2$$

As you see from the formula, kinetic energy of the objects is only affected by the mass and velocity of the objects. The unit of the E_k is again from the formula $\text{kg}\cdot\text{m}^2/\text{s}^2$ or in general use joule.

2 Work Done by a Variable Force

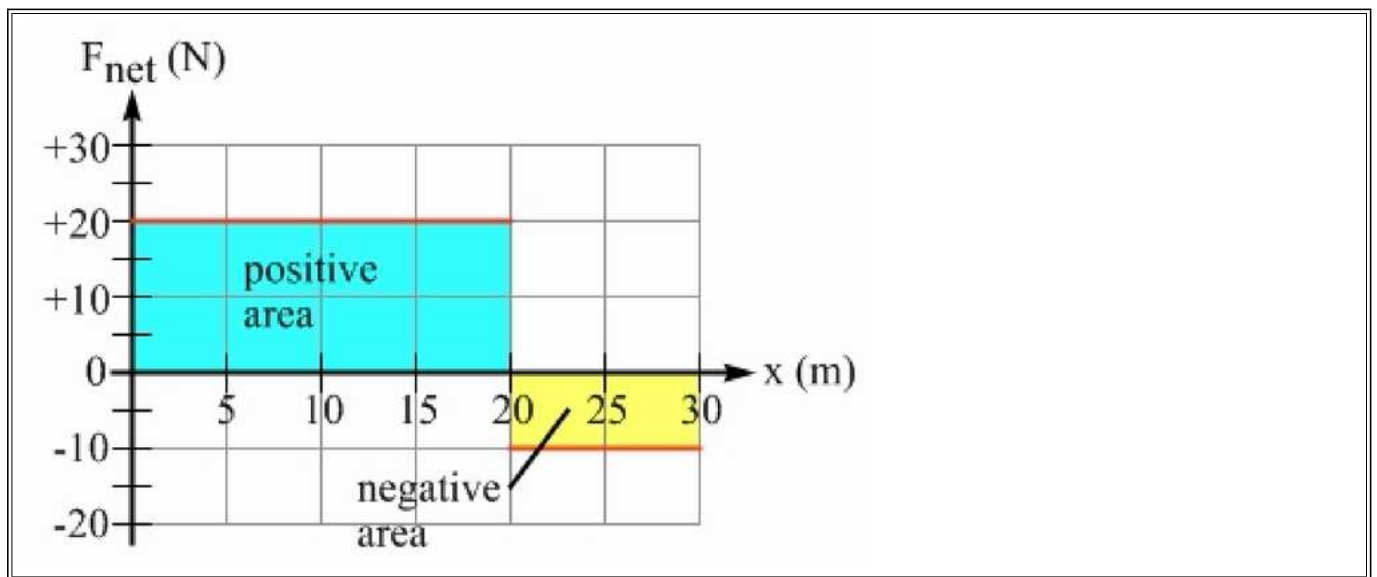
Graphically, the work done on an object or system is equal to the area under a Force vs. displacement graph:



The area under the graph from zero to 20 meters is 300 N m. Thus, the force represented by the graph does 300 J of work. This work is also a measure of the energy which was transferred while the force was being applied

3 The net force vs. position graph

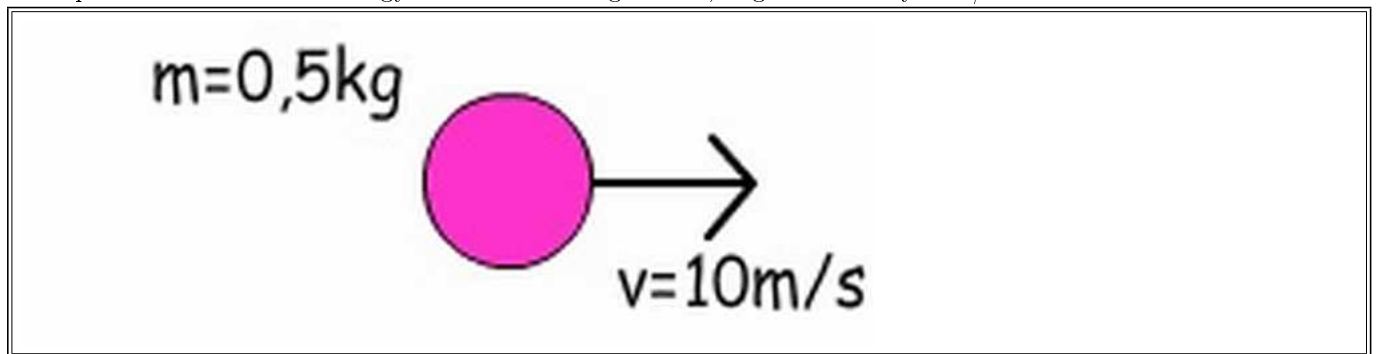
The area under the net force vs. position graph represents the change in kinetic energy (also known as the net work).



ory and Problems

4 Force vs. Distance graph.

Example : Find the kinetic energy of the ball having mass 0,5 kg and velocity 10m/s.

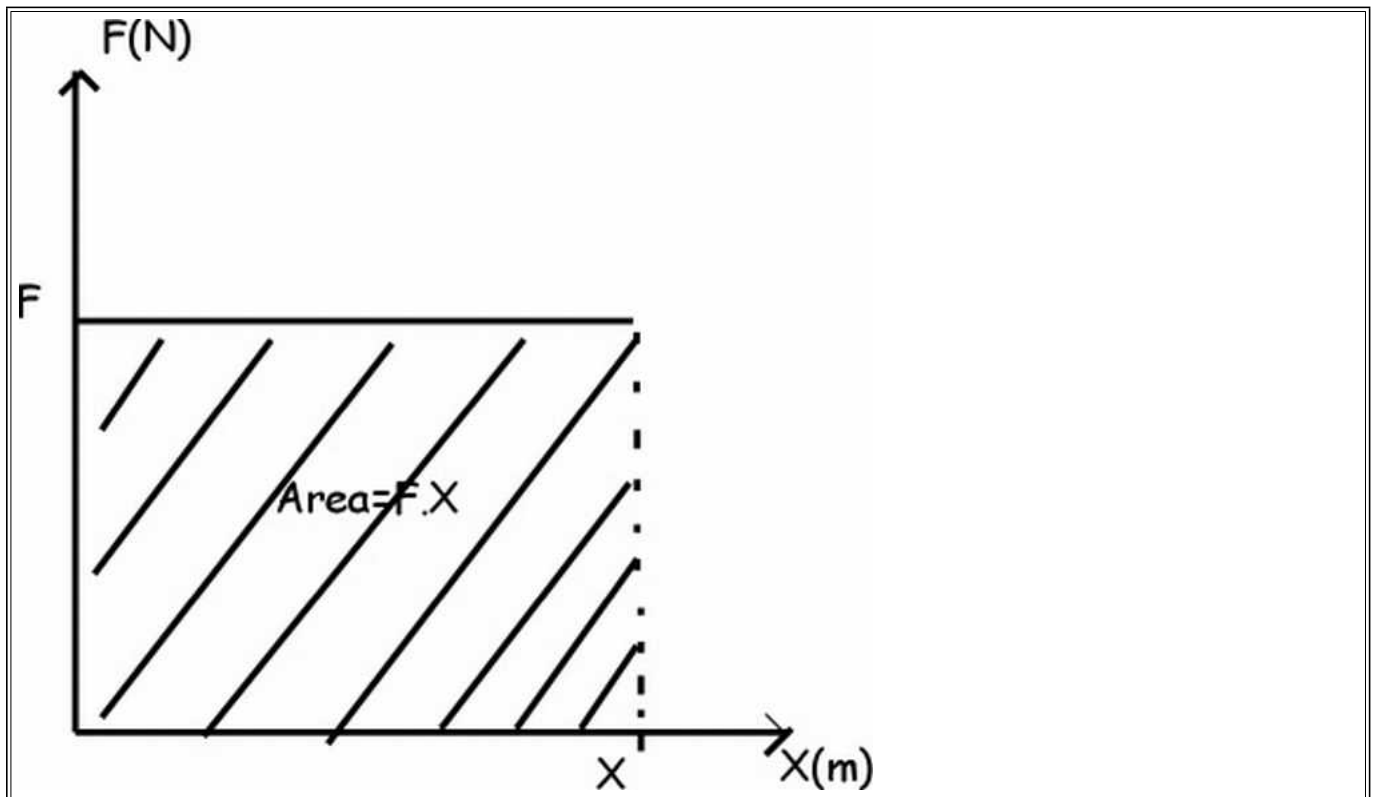


$$E_k = \frac{1}{2}mv^2$$

$$E_k = \frac{1}{2} \cdot 0,5 \cdot (10)^2$$

$$E_k = 25 \text{ joule}$$

As in the case of Kinematics we can use graphs to show the relations of the concepts here. Look at the given graph of

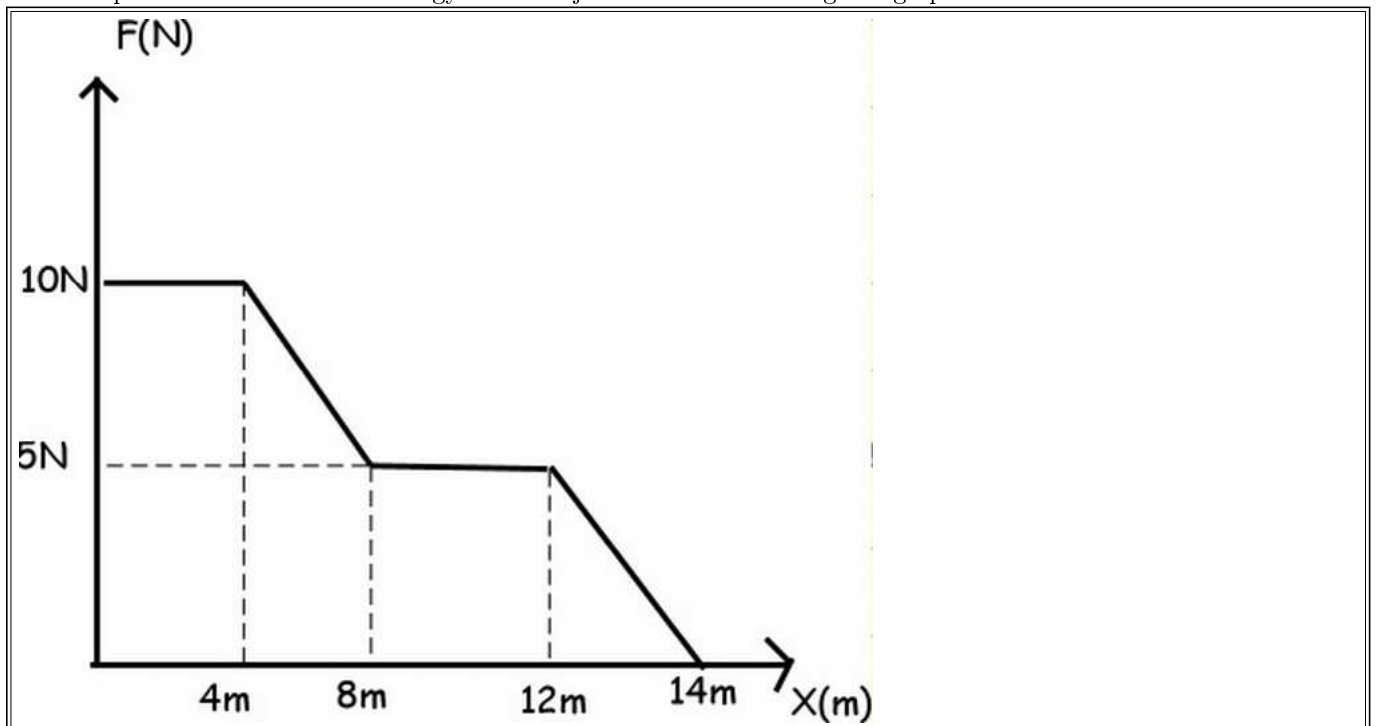


Area under the force vs. distance graph gives us work

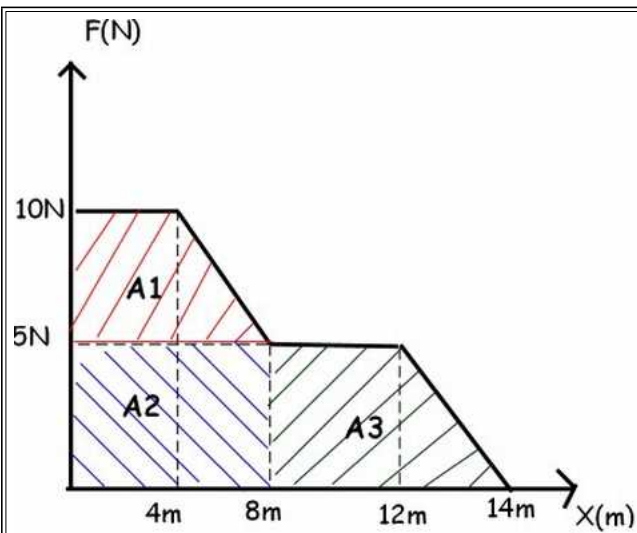
Work = Force. Distance = Area = $F.X$ (distance)

We can find energy of the objects from their Force vs. Distance graph.

Example : Find the Kinetic Energy of the object at 14m from the given graph below.



We can find the total kinetic energy of the object after 14m from the graph; we use area under it to find energy.



$$A1 = \frac{(8+4) \cdot 5}{2} = 30 \quad A3 = \frac{(6+4) \cdot 5}{2} = 25$$

$$A2 = 5 \cdot 8 = 40$$

$$\text{Total Area} = A1 + A2 + A3$$

$$\text{Total Area} = 30 + 40 + 25 = 95$$

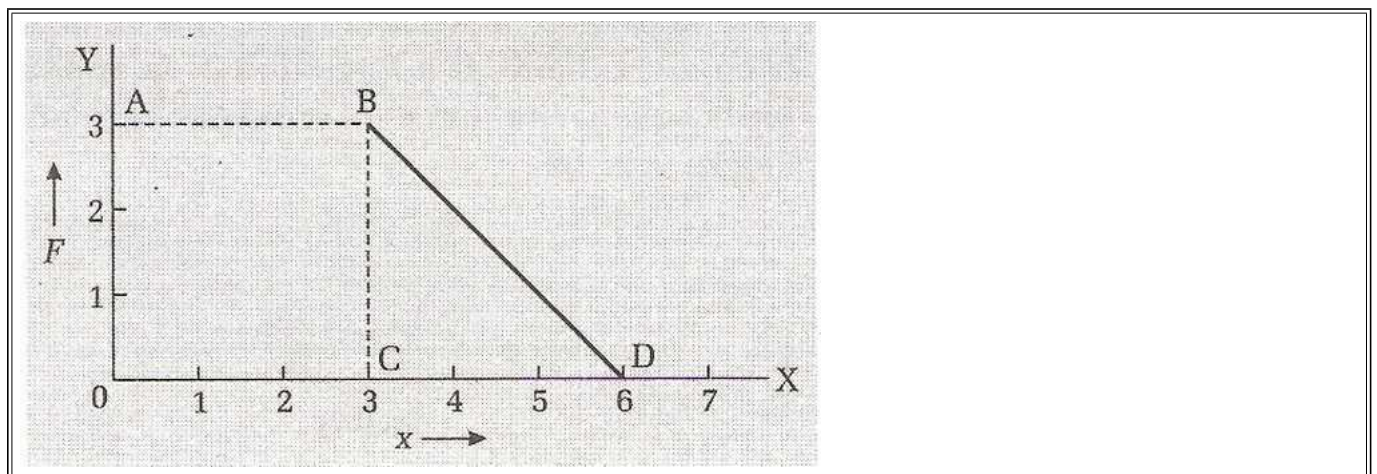
$$E_k = \text{Total Area} = 95 \text{ joule}$$

Practice Problems

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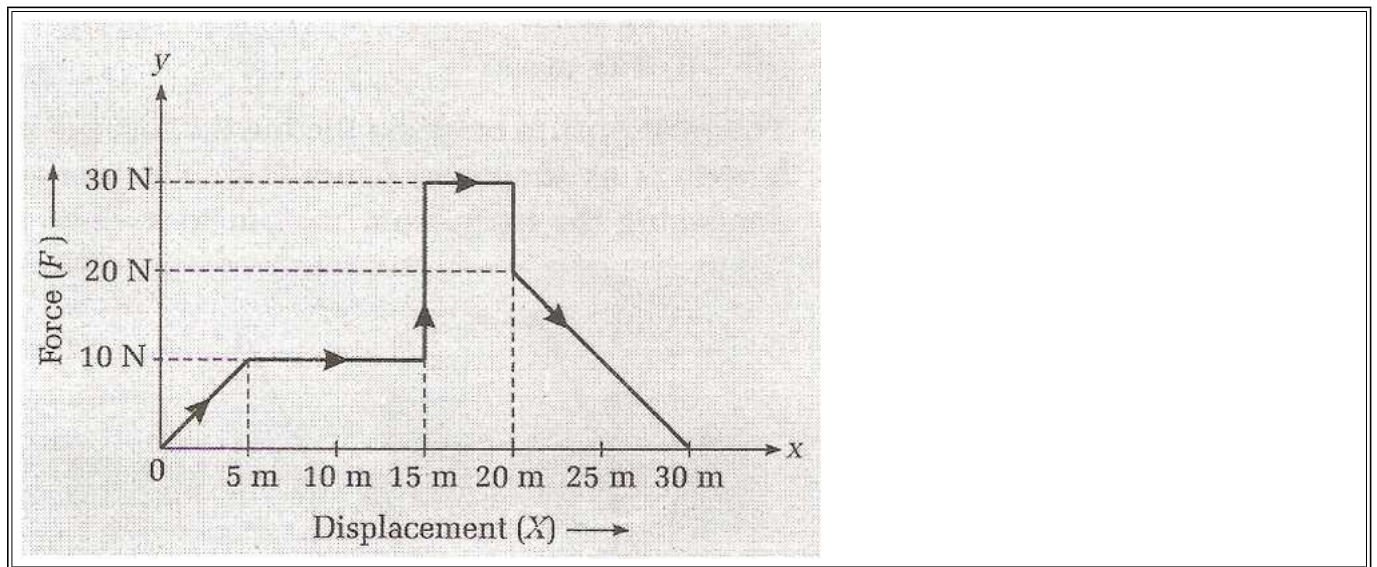
5 General Problem Set

Example : A force F acting on an object varies with distance x as shown in Figure. The force is in newton (N) and the distance (x) in metre. The work done by the force in moving from $x=0$ to $x=6\text{m}$ is



- a) 4.5 J
- b) 9.0 J
- c) 14.5 J
- d) 15 J

Example : Given below is a graph between a variable force (F) (along y-axis) and the displacement (X) (along x-axis) of a particle in one dimension. The work done by the force in the displacement interval between 0 m and 30 m is

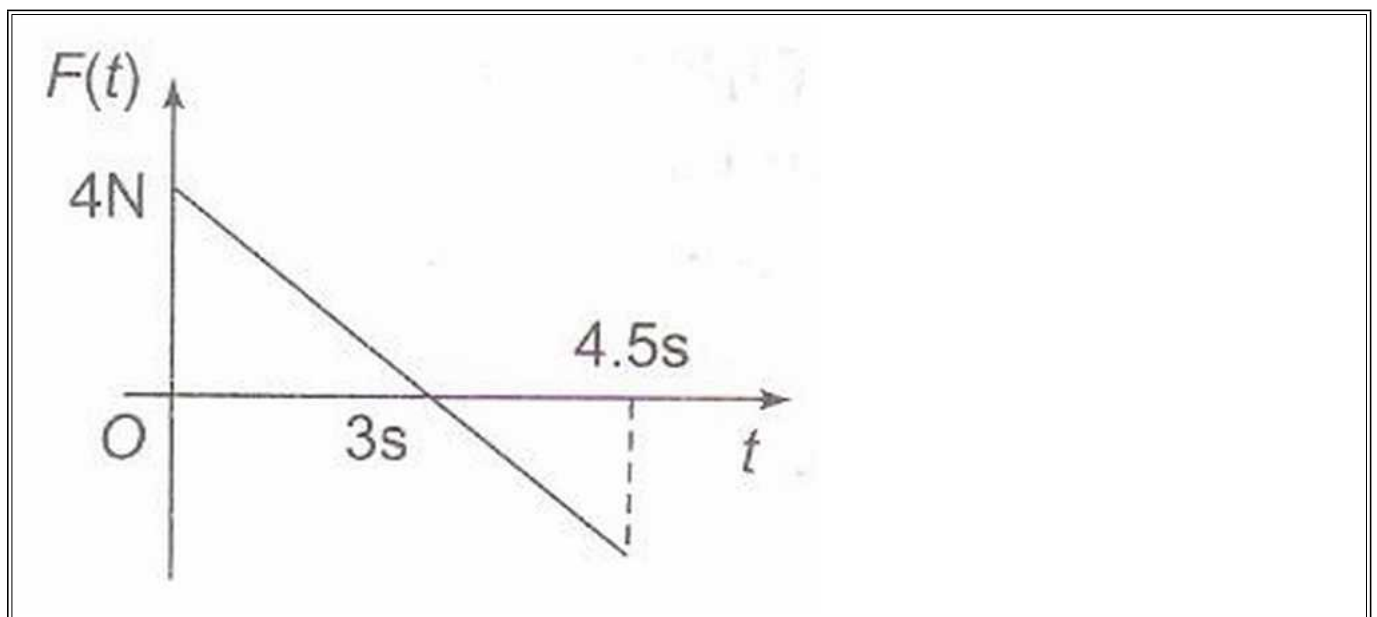


- a) 275 J
- b) 375 J
- c) 400 J
- d) 300 J

6 Previous Years IIT Problems

6.1 Single Answer

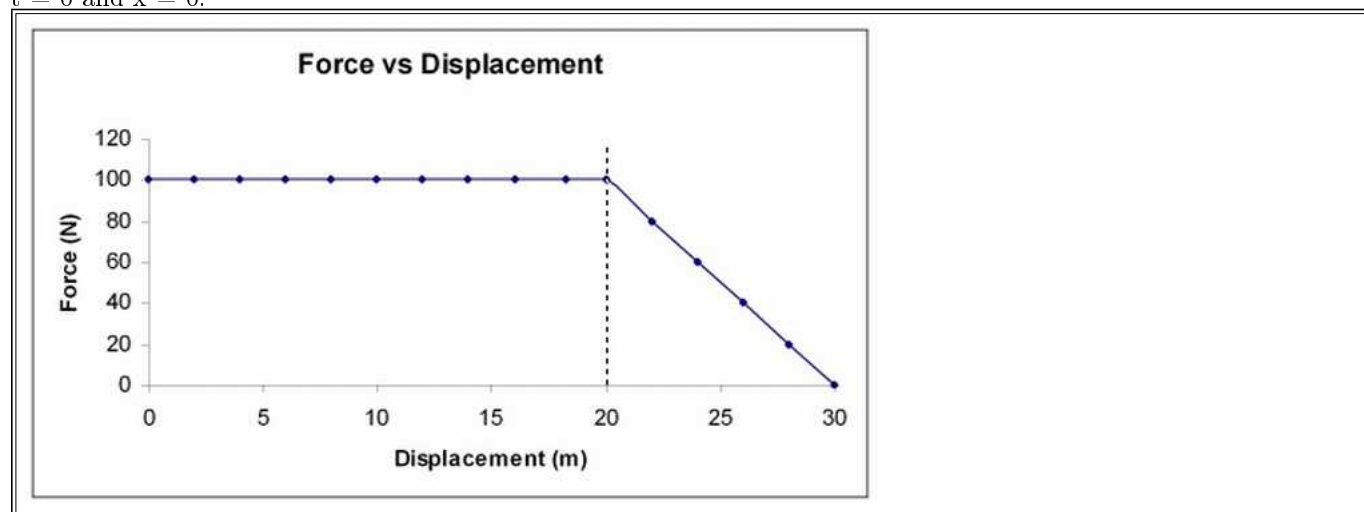
Example: A block of mass 2 kg is free to move along the x-axis. It is at rest and from $t=0$ onwards it is subjected to a time-dependent force $F(t)$ in the x direction. The force $F(t)$ varies with t as shown in the figure. The kinetic energy of the block after 4.5 seconds is



- a) 4.50J
- b) 7.50J
- c) 5.06J
- d) 14.06J

Review Questions I Refer to the following information for the next thirteen questions. [?]

A 5.0-kg mass is pushed along a straight line by a net force described in the graph below. The object is at rest at $t = 0$ and $x = 0$.



- During which displacement interval was the object's acceleration uniform?
- What acceleration did the object experience when $x = 10$ meters?
- How much work was done on the object during the first 20 meters?
- How much kinetic energy did the object gain during the first 20 meters?
- What was the object's instantaneous velocity at $x = 20$ meters?
- How much time was required to move it through the first 20 meters?
- How much did the object's momentum change in the first 20 meters?
- What was the object's instantaneous acceleration at $x = 22$ meters?
- Why can't the kinematics equations for uniformly accelerated motion be used to calculate the object's instantaneous velocity at $x = 30$ meters? What method should be used?
- How much work was done to move the object from 20 meters to 30 meters?
- What was the object's instantaneous speed at $x = 30$ meters?
- What was the total impulse delivered to the object from $x = 0$ to $x = 30$ meters?
- What percent of the impulse was delivered in the last 10 meters?