

Graphs MPEGc (Mathematics/Physics/Economics/Geography/computing)

Manas Kalia and Rajat Kalia

25 December , 2023

Contents

I Introduction 19

1 Theory 21

1.1 Introduction **www.lhup.edu** 21

1.2 Elements of a good Graph 21

1.3 Graphical Representation of Uncertainties 21

1.4 Curve Fitting 21

1.5 Uncertainty in a Slope 21

1.6 Graphical Analysis of Data 22

1.7 EXERCISES. 22

2 Common Graph Forms in Physics 23

2.1 Linear Relationship 23

2.2 Inverse Relationship 23

2.3 Inverse-Square Relationship 23

2.4 Double-Inverse Relationship 23

2.5 Power Relationship 23

2.6 Power Relationship 2 23

2.7 Polynomial of Second Degree 23

2.8 Exponential Relationship 23

2.9 Natural Log (LN) Relationship 23

II Kinematics 25

3 Abstract Introduction 27

3.1 Displacement-Time Graph 27

3.2 Velocity-time graph 27

3.3 Acceleration-time graph 27

3.4 One More Go in a different perspective 28

3.4.1 Distance-Time Graphs 28

3.4.2 Displacement-time graphs 28

3.4.3 Velocity-time graphs 28

3.4.4 Acceleration-time graphs 28

3.4.5 Self-Test Questions**www.miniphysics.com** 28

3.5 Revision 28

3.5.1 Position vs. Time Graphs 28

3.5.2 Average Velocity 28

3.5.3 Average Speed 28

3.5.4 Curved Position vs. Time Graphs 28

3.5.5 Calculating Displacement 29

3.5.6 Curved Velocity vs. Time Graphs 29

3.5.7 Acceleration vs. Time Graphs 29

3.5.8 Calculating Change in Velocity 29

3.6 Exercises 29

3.6.1 Problems on Position-Time Graph 29

3.6.1.1 Subjectives 29

3.6.2 Problems on Velocity-Time Graph 30

3.6.2.1 Subjective 30

3.6.2.2 Miscellaneous 31

3.6.3 Problems on Area Under v-t graph 31

3.6.4 Problems on Average Velocity 31

4 Few Basic Problems 33

4.1 Exercises 36

5 Classical Approach 39

5.1 The Equations of motion and the origin of Graph Handling 39

5.1.1 The First Equation 39

5.1.2 The Second Equation 39

5.1.3 The Acceleration-Position Graph Variate 40

5.1.4 Sign of Acceleration from Position-Time Graph 40

5.1.5 The Average-Velocity / Instantaneous Velocity , Equal Case 41

5.1.6 The Velocity-Displacement Case 41

5.1.7 Motion Under Free Fall due to Gravity 41

5.1.8 Projectile Motion 42

5.1.9 Miscellaneous 42

5.2 Question Types 42

5.2.1 Passage Type 42

5.2.2 Matching 43

5.3 Previous Years IIT Problems 43

6 Exercises 45

6.1 Review Exercise I 45

6.2 Review Exercise II 46

III Laws of Motion 49

7 Abstract Introduction 51

7.1 Force – Time Graphs 51

7.2 Change in Momentum or the "Impulse" 51

7.2.1 Force vs Time Graphs 51

7.3 Impulse graphs (A Case Study) **bbc.co.uk** 51

8	Friction	53
8.1	Static Friction hyperphysics.phy-astr.gsu.edu	53
8.2	Kinetic Friction	53
8.3	Friction Plot	53
8.4	Rolling Friction	53
8.5	Few problems related to Friction	53
9	Theory and Problems	55
9.1	Impulse as Force-time Graph	55
9.2	Force Time graph with respect to momentum	55
10	Problems for Practice	57
10.1	General Problem Set	57
10.1.1	Single Answer Type	57
10.2	Previous Years IIT Problems	57
10.2.1	Single Answer	57
IV	Energy Conservation	59
11	Abstract Introduction	61
11.1	KINETIC ENERGY	61
11.2	Work Done by a Variable Force	61
11.3	The net force vs. position graph	61
12	Theory and Problems	63
12.1	Force vs. Distance graph.	63
13	Practice Problems	65
13.1	General Problem Set	65
13.2	Previous Years IIT Problems	65
13.2.1	Single Answer	65
14	Review Questions I	67
V	Rotatory Motion	69
15	Problems for Practice	71
15.1	General Problem Set	71
15.1.1	Single Answer Type	71
16	Angular Momentum Conservation	73
VI	Gravitation	75
17	Basics	77
17.1	Variation of “g”	77
18	Problems for Practice	79
18.1	General Problem Set	79
18.1.1	Single Answer	79
18.1.2	Multiple Answer	79
18.1.3	Comprehension Type	79
18.1.3.1	Concept 1 Gravitational potential inside a spherical shell is constant and outside the shell it varies as $V \propto \frac{1}{r}$ (with negative sign). Here r is the distance from centre.	79
18.2	Previous Years IIT Problems	79
18.2.1	Single Answer	79
18.2.2	Matching	80
VII	Periodic Motion	81
19	Abstract Introduction (SHM)	83
19.1	Position vs time	83
19.2	Velocity vs time	83
19.3	Acceleration vs time	83
19.3.1	Peak Height	83
19.4	Graphing position, velocity, and acceleration with respect to each other	83
19.5	Position and acceleration verses velocity	83
20	Problems	85
20.1	General Problem Set	85
20.1.1	Single Answer Type	85
20.1.2	Multiple Answer Type	85
20.1.3	Matching Type Questions	85
20.1.4	Comprehension Type Questions	85
20.1.4.1	Comprehension 1	85
20.1.4.2	Comprehension 2	85
20.2	Previous Years IIT Problems	86
20.2.1	Paragraph	86
VIII	Statics	87
21	Modulii of Elasticity	89
21.1	General Problem Set	89
21.1.1	Single Answer Type	89
21.1.2	Comprehension Type	89
21.1.2.1	Comprehension 1	89

CONTENTS	5
IX Thermodynamics	91
22 Practice Problems	93
22.1 General Problem Set	93
22.1.1 Single Answer Type	93
22.1.2 Multiple Answer Type	95
22.1.3 Matching Type	95
22.1.3.1 Matrix Match 1	95
22.1.3.2 Matrix Match 2	95
22.1.4 Comprehension Type	96
22.1.4.1 Comprehension 1	96
22.1.4.2 Comprehension 2	96
22.1.5 Subjective Problems	96
22.2 Previous Years IIT Problems	96
22.2.1 Multiple Answer	96
22.2.2 Paragraph	96
22.2.3 Matching	96
X Mechanical Waves	97
23 General Problem Set	99
23.1 Single Answer Questions	99
XI Sound	101
24 Practice Problems	103
24.1 General Problem Set	103
24.1.1 Single Answer Type	103
24.1.2 Multiple Answer Type	103
24.2 Previous Years IIT Problems	103
24.2.1 Passage	103
XII Electrostatics	105
25 Problems for Practice	107
25.1 General Problem Set	107
25.1.1 Single Answer Type	107
XIII Current Electricity	109
26 Basics	111
26.1 Theory	111
26.1.1 Ohm-meter	111
26.1.2 Power Transfer to a load	111
26.2 Problems	111
26.2.1 Objective Type Questions	111
26.2.2 Subjective	111
27 Capacitors	113
27.1 Discharge of a capacitor	113
27.2 Charging a capacitor	113
XIV Magnetic Field	115
28 Problems for Practice	117
28.1 General Problem Set	117
28.2 IIT Previous Years Problems	117
XV Lattice	119
29 Experiments	121
29.1 Photoelectric Effect	121
29.1.1 An Experiment vlab.amrita.edu	121
29.1.1.1 The work function for some metals are listed in the table.	121
29.1.1.2 Graph connecting photocurrent and applied reverse potential :	121
29.1.2 Original Einstien Experiment physics.info	121
29.1.2.1 Discussion	121
29.1.3 Graphs of Photoelectric Current vs Voltage	122
29.1.3.1 For radiation of different Intensities ($I_1 > I_2$) but the same frequency.	122
29.1.3.2 For radiation of different frequencies ($\nu_1 > \nu_2$) but of the same intensity.	122
30 Problems	123
30.1 Previous Years IIT Problems	123
30.1.1 Single Answer Questions	123
XVI Nucleus	125
31 Problems	127
31.1 Previous Years IIT Problems	127
31.1.1 Multiple Answer	127
31.1.2 Integer Type	127
31.1.3 Paragraph	127
XVII Ray Optics	131
32 Theory	133
32.1 Graphs for convex and concave lenses	133
32.1.1 Enlarged view of an object distance (u) against image distance (v) graph	133
32.1.2 Minimum distance	133

33 Problems for Practice	135
33.1 General Problem Set	135
33.1.1 Single Answer Type	135
33.1.2 Multiple Answer Type	135
33.1.3 Comprehension Type	135
33.1.3.1 Comprehension 1	135
 XVIII Semiconductors	 137
 34 Diode	 139
34.1 Rectifier	139
34.2 Junction Diode	139
 35 Triode	 141
 36 p-n Junction	 143
 37 Transistors	 145
 XIX Logic Gates	 147
 38 Problems	 149
 XX Microeconomics	 153
 39 The Complete Theory in three graphs ¹	 155
39.1 The Indian Economics	155
39.2 Outside India Economics	155
39.3 India to Outside and reverse relations in Market	155
 40 Conventional Terminology	 157
40.1 Production Frontier tutor2u.net	157
40.1.1 Opportunity Cost and the PPF	157
40.1.2 PPF and Economic Efficiency	157
40.1.2.1 Production Possibilities	157
40.2 Budget Line, Constraints map and the condition of tangency courses.byui.edu	157
40.2.1 Maximizing Utility	158
40.2.2 Practice	158
40.2.3 Deriving Demand	159
40.2.4 How Businesses React	159
40.2.5 Indifference Curves	160
40.2.6 Marginal Rate of Substitution	160
40.2.7 Budget Constraint	161
40.2.8 Utlity Maximization	161
40.2.9 Income and Substitution Effects	162
40.2.10 Extreme Cases	162
40.2.11 Government Food Stamps Example	162
40.3 Supply and Demand Curves (Understanding Price and Quantity in the Marketplace)	163
40.3.1 The Law of Demand	163
40.3.2 Price Elasticity	163
40.3.3 The Law of Supply	163
40.3.4 Equilibrium: Where Supply Meets Demand	164
40.3.5 Changes in Demand and Supply	164
40.3.6 Changes in any of the following factors can typically cause demand to shift:	164
 XXI Marketing	 167
 41 Consumer Marketing	 169
 42 Industrial Marketing	 171
 43 Services Marketing	 173
 XXII Civil Administration	 175
 44 Police Services	 177
44.1 Police in Countries other than India	177
44.1.1 LAPD of America	177
44.1.2 NLEA of Britain	177
44.2 Indian Police	177
44.3 Punjab Police	177
 45 Administration Services	 179
45.1 Indian Foreign Services	179
45.2 IAS	179
45.3 Engineering Services	179
 XXIII The Market	 183
 46 The Indian Context	 185
46.1 Eatables (Kiryana, Mandi, Halwai, Confectionary and Restuarants)	185
46.1.1 Unprocessed	185
46.1.2 Semi-Processed (For furthur processing at home)	185
46.1.3 Ready to Eat	185
46.1.3.1 Ice Creams , Flavoured Milk	185

¹These concepts were first disclosed to me by my mom in childhood, then again told to me by the economics teacher in SMU MBA which I was doing. Evidently both of them probably read from this book as the source. So, cycles go on and knowledge remains in loop.

XXIV The Media	187
47 International News Channels	189
47.1 CNBC	189
47.2 CNN	189
47.3 MSN News (via windows e-print media)	189
48 Prominent Indian News Channels	191
48.1 DD Broadcast (accross various of its channels)	191
48.2 Aaj Tak	191
48.3 Zee News	191
48.3.1 The Delimma	191
48.4 NDTV	191
48.5 Other Smaller Channels	191
XXV The Cream vs the bearers (in Indian Context and beyond)	193
49 Introduction to the discussion “The Cream vs the bearers”	195
XXVI Possibility of Nuclear Holocaust and UN’s non-proliferation treaty (which India doesn’t sign due to security concerns)	199
50 Pakistan vs India, Area of high sensitivity	201
51 Possibility of US using the nukes again , low	203
52 The Role of Sci-Fi (The Phoenix writeup in Xth book, World at arms game by Gameloft, one MIT OCW Course etc. SCI-FI stuff predicting the near future)	205
52.1 The Phoenix	205
52.2 World at Arms	205
52.3 MIT OCW	205
XXVII Scientology	209
53 Physics in a nutshell	211
54 Something more Concrete (than merely panchtantra stories)	213
XXVIII Introduction to Similar Sound and Meaning words used in All (or most) languages Worldover	219
55 Mathematics Related	221
55.1 Tri	221
55.2 Jama	221
55.3 Taksim	221
55.4 Calculus	221
55.5 Jacobian	221
55.5.1 What’s it like in the flight experience?	221
55.5.2 Is your own education complete ? I mean telling your wife to complete her education before showing up?	221

Initiation

Preface

Acknowledgements

I would like to thank my son Manas for his love and support. Moreover , to my father for the day to day needs.

Mathematics

Physics

Part I

Introduction

Chapter 1

Theory

1.1 Introduction www.lhup.edu

A graph is an accurate pictorial representation of data. The accuracy of data in physics requires that graphs be made on good quality graph paper. Nearly all graphs in physics are smooth line graphs; broken line (connect the dots) graphs and bar graphs are seldom appropriate.

The style and format of a graph will depend upon its intended purpose. Three types are common in physics

1. **PICTORIAL GRAPHS.** These are the kind found in mathematics and physics textbooks. Their purpose is to simply and clearly illustrate a mathematical relation. No attempt is made to show data points or errors on such a graph.

2. **DISPLAY GRAPHS.** These present the data from an experiment. They are found in laboratory reports, research journals, and sometimes in textbooks. They show the data points as well as a smooth line representing the mathematical relation.

3. **COMPUTATIONAL GRAPHS.** These are drawn for the purpose of extracting a numerical result from the data. An example is the calculation of the slope of a straight line graph, or its intercepts.

1.2 Elements of a good Graph

Certain informational and stylistic features are required in all graphs:

1. The graph must have a descriptive title or caption, clearly stating what the graph illustrates.

2. Data points are plotted as small dots with a sharp pencil, or as pinpricks. Some method should be used to emphasize the location of the points, for example, a neat circle drawn around each point.

Images/CC/P-Graphs/5e/Images/Introduction/G01.jpg

Figure : Good and bad choices of geometric slope of a graph.

3. Curves drawn through the points should be smooth (use ¹ French curves if your hand is not steady). The curve should stand out clearly.

4. Choose scales that are convenient to plot and easy to read.

5. Choose scales such that the graph occupies most of the page. The two scales need not have the same size units. Also, the scales need not begin at zero.

6. Indicate the name, letter symbol and units of each variable plotted on each axis.

7. All text (title, labels, etc.) should be printed.

PHYSICAL SLOPE AND GEOMETRIC SLOPE

Slope. When textbooks refer to the "slope" of a plotted graph line we mean the "physical slope"

$$\text{physical slope} = \frac{\Delta y}{\Delta x}$$

where Δy and Δx are expressed in the physical units of the x and y axes. This slope has physical significance in describing the physical data.

Geometric slope. A line which makes a 45° angle with an axis will not necessarily have a physical slope of size 1. Some authors introduce the term "geometric slope" to describe the tilt of the line on the page. This is a ratio of lengths of the legs of the triangle, without reference to the units plotted on the axes.

There is seldom (probably never) any need to calculate the geometric slope of a line on a graph. The idea is only useful when describing the appearance of the graph on the page. One rule of graph construction states that the graph should occupy most of the page. For square graph paper this suggests a geometric slope of 45°. See Fig for examples of good and bad choices of geometric slope.

THE APPEARANCE OF THE GRAPH

Images/CC/P-Graphs/5e/Images/Introduction/G02.jpg

Figure : Elements of a graph.

Use quality graph paper, size 8.5 by 11 inches only.

The left margin is largest, for binding or stapling.

Nothing should be in the white margins except a page number. Axes, lettering and labeling should all be within the printed grid area. [The grid lines serve as guide lines for neat, uniform printed lettering.]

The title must be descriptive.

Both axes are labeled with the full name of the quantity (not merely its symbol), and its units.

The plotted points and curve should occupy most (more than half) of the area of the graph paper.

¹A French curve is a template usually made from metal, wood or plastic composed of many different curves. It is used in manual drafting to draw smooth curves of varying radii. The shapes are segments of the Euler spiral or clothoid curve. The curve is placed on the drawing material, and a pencil, knife or other implement is traced around its curves to produce the desired result.

Modern computer-aided design (CAD) systems use vector-based graphics to achieve a precise radius, so no template is required. Digital computers can also be used to generate a set of coordinates that accurately describe an arbitrary curve, and the points can be connected with line segments to approximate the curve with a high degree of accuracy. Some computer-graphics systems make use of Bézier curves, which allow a curve to be bent in real time on a display screen to follow a set of coordinates, much in the way a French curve would be placed on a set of three or four points on paper.

Sometimes a small sketch of the experimental situation may be included, located where it will not confuse the interpretation of the graph. In the same manner an equation, or short explanatory comment, may be included.

1.3 Graphical Representation of Uncertainties

Display graphs and computational graphs should clearly show the size of the experimental uncertainties (errors) in each plotted point. There are several conventional ways to do this, the commonest being the use of error bars illustrated below:

Images/CC/P-Graphs/4.1e/Images/Introduction/G03.jpg

Figure : Various styles of error bars.

The plotted point is represented as a dot, and the range of uncertainty is shown by the extent of the bars on either side. The types shown in (a) are suitable where the error is entirely in one variable, or where the errors in both variables have been lumped together. The types shown in (b) are preferred where it is necessary to show the error in each variable explicitly.

When the uncertainties have a symmetric distribution about the mean, the error bars extend equally on either side of the points. If the data distributions are not symmetric, the plotted points will not be centrally located in the range of uncertainty and the error bars might look like those in Figure. part (c).

Error bars may not be necessary when the data points are so numerous that their scatter is clearly shows the uncertainty. In these cases error bars would clutter the graph making it difficult to interpret. Another situation where error bars are inappropriate is when the scale of the graph is such that the bars would be very small. In this case, it may be possible to indicate the uncertainty by the size of the circle or rectangle surrounding each point.

1.4 Curve Fitting

The curve drawn through plotted data need not pass exactly through every data point. But usually the curve should pass within the uncertainty range of each point, that is, within the error bars, if the bars represent limits of error.

One principle of curve fitting is also a fundamental rule of science itself:

Assume the simplest relation consistent with the data. We are not justified in assuming a more complex relation than can be demonstrated by the data. If a curve were drawn with detail smaller than the data uncertainty, that detail would be only a guess.

This rule of simplicity may also be expressed mathematically. The mathematical relations encountered in physics may often be represented by power series such as

$$y = A + Bx + Cx^2 + Dx^3 + Ex^4 + \dots$$

where A, B, C, ... are constants.

For very "wiggly" curves, many terms of this equation, including high powers of x, might be required to express the equation of the relation. The simplest relations are those which contain the smallest powers of x. The simplest relations of all are

$$y = a \text{ or } y = a + bx$$

which describe straight lines. Many relations in physics are, fortunately, of this form. Others only include the x^2 term, describing a parabolic curve. Note that double valued curves, sometimes encountered in physics, cannot be represented by Equation.

When sizable amounts of data are taken, standard mathematical methods are available which generate the equation of the simplest curve which statistically "best fits" the data.

The student may wonder how one can be certain that the curve fitted to the data is the "correct" curve. The answer is that relations are never known with certainty. The uncertainty of available data always limits the certainty of the results. Someday someone may obtain more accurate data and be able to show that the old relations are slightly incorrect, and provide us with better ones. As data improves, so does our understanding of relations—this is the way of scientific progress. But we never should claim to know a relation better than the data allows.

1.5 Uncertainty in a Slope

One use of a computational graph is to determine the slope of a straight line. This is illustrated in Figure. Eight data points are shown with error bars on each. If these bars represent maximum error, any line drawn to represent this data should pass within all bars.

If the error bars represent error estimates smaller than the maximum (average deviation, standard deviation, etc.), then the fitted curve need not pass within all of the error bars, just most of them.

Images/CC/P-Graphs/5e/Images/Introduction/G04.jpg

Figure : Fitting a curve.

Even a simple "manual" curve fit with a ruler can reveal the uncertainties in the slope resulting from uncertainties in the data. Figure. illustrates this process.

The dotted lines A and C fall within the error bars, and represent the maximum and minimum slope one could justify from this data. The "best" value of slope might be that of solid line B.

The third point from the left seems to limit the slope the most, and would appear to be "suspect." But one ought not to "throw it out" without better reason, based on further investigation.

1.6 Graphical Analysis of Data

Graphs can be a valuable tool for determining or verifying functional relations between variables. Many special types of graph paper are available for handling the most frequently encountered relations. You are probably already familiar with linear graph paper and polar coordinate paper.

You may have purchased a packet of graph paper for this course. It includes samples of graph papers you will use in this course, and a few other types. As you read the material below, examine the corresponding papers from your packet.

LINEAR RELATIONS are those which satisfy the equation

$$y = mx + b$$

where the variables are x and y , and m and b are constants. When y is plotted against x on ordinary Cartesian (linear) graph paper, the points fall on a straight line with slope m and a y -intercept b , as shown in Fig. 1.5.

The slope of an experimental relation is often physically significant. It is obtained by choosing two well-separated points on the line (x_1, y_1) and (x_2, y_2). From Eq. 7-3:

$$y_1 = mx_1 + b$$

$$\text{and } y_2 = mx_2 + b$$

Subtract the first from the second.

$$(y_2 - y_1) = m(x_2 - x_1) .$$



Therefore,

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$

The slope of a straight line is a ratio of the "lengths" of two legs of a right triangle constructed with the legs parallel to the graph axes and with the graph line along the hypotenuse. Figure. illustrates this, the slope being $\Delta y / \Delta x$.

So far this discussion has been strictly mathematical. Now let's consider a fairly realistic physical example. Figure. shows the curve from measurements of the velocity of a moving body as a function of time.

If we use letters v for velocity and t for time, we'd expect this curve to be described by the relation:

$$v = v_o + at$$

Here the constant a (acceleration) is the slope of the line, while v_o plays the role of b in Equation. These two constants are physically significant, and we wish to find their values from the graph.

We choose two points on the line at $t = 4.25$ and 8.75 , with corresponding values of velocity: 42 cm/s and 98 cm/s . Mark these points on Figure. , to confirm these values. The slope of the line is therefore:

$$a = \frac{(98 \text{ cm/s} - 42 \text{ cm/s})}{8.75 \text{ s} - 4.25 \text{ s}} = \frac{56}{4.5} \text{ cm/s}^2 = 56/4.5 \text{ cm/s}^2 = 12.44 \text{ cm/s}^2$$

When calculating this ratio do not use ruler-measured lengths. The lengths are expressed in the units marked on the graph axes. The calculated slope is therefore independent of the particular choice of units, of the way you choose to label the graph scale divisions, and is also independent of the size of the graph paper.

INTERCEPTS: The values of the intercepts are often physically significant. They can be simply read from the graph—if the $x = 0$ and $y = 0$ axes happen to be within the graph's boundaries. In the equation $y = mx + b$, the y intercept is b .

The v intercept of Figure. is the value of v when $t = 0$. It has the same units and dimensions as y . If, as in this case, the v intercept does not lie within the area of the graph, it may be calculated using the slope and one value taken from a point on the fitted line. Take the point $v = 98 \text{ cm/s}$ when $t = 8.75 \text{ sec}$.

$$v = v_o + at, \text{ in our case, } v = v_o + 12.44 t$$

so,

$v = v_o - 12.44 t = 98 - 12.44(8.75) = -10.89 \text{ cm/s}$ A check of the graph, Figure. , shows that this looks reasonable.

STRAIGHTENING A CURVE. When it is possible to convert an experimental relation to a straight line graph it is usually useful to do so. Look for such opportunities. For example, when studying gases at constant temperature we find that

$$PV = C$$

where P is pressure, V is the gas volume and C is constant. The graph of P vs. V is a branch of an hyperbola. But if we graph P vs. $1/V$, or V vs. $1/P$, the data would fall on a straight line.

$$P = \left(\frac{1}{V} \right) C$$

One reason for doing this is that it is easier to fit the experimental data with a ruler-drawn straight line, than to draw the best hyperbola on a PV graph. Another advantage is that the P vs. $1/V$ graph has a slope

$$C = \frac{\Delta P}{\Delta \left(\frac{1}{V} \right)}$$

Therefore the constant C is easily determined from the straight line. This constant was not evident, nor was it easy to determine from the PV graph!

Inexpensive electronic calculators make it so easy to manipulate data that there is no good excuse to pass up an opportunity to "linearize" experimental graphs.

1.7 EXERCISES.

In each case state how you could plot (x, y) data on linear paper to obtain a straight line graph. What quantity in the equation is determinable from the slope of the straight line? What quantity is determinable from an intercept?

(1.1) $x(y + 1) = 3$

(1.2) $1/x + 1/y = 5$

(1.3) $y = Ae^{-x}$

(1.4) $y = \sqrt{A - x}$

(1.5) $y^2 + x^2 = 7$

Chapter 2

Common Graph Forms in Physics

Working with graphs – interpreting, creating, and employing – is an essential skill in the sciences, and especially in physics where relationships need to be derived. As an introductory physics student you should be familiar with the typical forms of graphs that appear in physics. Below are a number of typical physical relationships exhibited graphically using standard X-Y coordinates (e.g., no logarithmic, power, trigonometric, or inverse plots, etc.). Study the forms of the graphs carefully, and be prepared to use the program Graphical Analysis to formulate relationships between variables by using appropriate curve-fitting strategies. Note that all non-linear forms of graphs can be made to appear linear by “linearizing” the data. Linearization consists of such things as plotting X versus Y^2 or X versus $1/Y$ or Y versus $\log(X)$, etc. Note: While a 5th order polynomial might give you a better fit to the data, it might not represent the simplest model.

2.1 Linear Relationship

What happens if you get a graph of data that looks like this? How does one relate the X variable to the Y variable? It’s simple, $Y = A + BX$ where B is the slope of the line and A is the Y-intercept. This is characteristic of Newton’s second law of motion and of Charles’ law:

$$F = ma$$

$$P/T = \text{const.}$$

Images/CC/P-Graphs/5e/Images/Kinematics/graph1_cgf.jpg

2.2 Inverse Relationship

This might be a graph of the pressure and temperature for a changing volume constant temperature gas. How would you find this relationship short of using a computer package? The answer is to simplify the plot by manipulating the data. Plot the Y variable versus the inverse of the X variable. The graph becomes a straight line. The resulting formula will be $Y = A/X$ or $XY = A$. This is typical of Boyle’s law:

$$PV = \text{const}$$

Images/CC/P-Graphs/5e/Images/Kinematics/graph2_cgf.jpg

2.3 Inverse-Square Relationship

Of the form $Y = A/X^2$. Characteristic of Newton’s law of universal gravitation, and the electrostatic force law:

$$F = \frac{Gm_1m_2}{r^2}$$

$$F = \frac{kq_1q_2}{r^2}$$

Images/CC/P-Graphs/4.1e/Images/Kinematics/graph3_cgf.jpg

In the latter two examples above there are only subtle differences in form. Many common graph forms in physics appear quite similar. Only by looking at the “RMSE” (root mean square error provided in Graphical Analysis) can one conclude whether one fit is better than another. The better fit is the one with the smaller RMSE. See below for more examples of common graph forms in physics.

2.4 Double-Inverse Relationship

Of the form $1/Y = 1/X + 1/A$. Most readily identified by the presence of an asymptotic boundary ($y = A$) within the graph. This form is characteristic of the thin lens and parallel resistance formulas.

$$\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}$$

and

$$\frac{1}{R_t} = \frac{1}{R_1} + \frac{1}{R_2}$$

Images/CC/P-Graphs/5e/Images/Kinematics/graph4_cgf.jpg

2.5 Power Relationship

Top opening parabola. Of the form $Y = AX^2$. Typical of the distance-time relationship:

$$d = \frac{1}{2}at^2$$

Images/CC/P-Graphs/5e/Images/Kinematics/graph5_cgf.jpg

2.6 Power Relationship 2

Side opening parabola. Of the form $Y^2 = A_1X$ or $Y = A_2X^{1/2}$. Typical of the simple pendulum relationship: $P^2 = k_1l$ or $P = k_2\sqrt{l}$

Images/CC/P-Graphs/4.1e/Images/Kinematics/graph6_cgf.jpg

2.7 Polynomial of Second Degree

Of the form $Y = AX + BX^2$. Typical of the kinematics equation:

$$d = v_ot + \frac{1}{2}at^2$$

Images/CC/P-Graphs/5e/Images/Kinematics/graph7_cgf.jpg

2.8 Exponential Relationship

Of the form $Y = A * \exp(BX)$. Characteristic of exponential growth or decay. Graph to left is exponential growth. The graph of exponential decay would look not unlike that of the inverse relationship. Characteristic of radioactive decay.

$$N = N_0e^{-\lambda t}$$

Images/CC/P-Graphs/4.1e/Images/Kinematics/graph8_cgf.jpg

2.9 Natural Log (LN) Relationship

Of the form $Y = A \ln(BX)$. Characteristic of entropy change during a free expansion:

$$S_f - S_i = nR \ln(V_f/V_i)$$

Images/CC/P-Graphs/5e/Images/Kinematics/graph9_cgf.jpg

Part II

Kinematics

Chapter 3

Abstract Introduction

Images/CC/P-Graphs/4.1e/Images/Kinematics/4791850.jpg

Figure : Graphical Representation of what we are going to learn in this chapter.

Images/CC/P-Graphs/4.1e/Images/Kinematics/9327449.jpg

Figure : Some Graphs

Images/CC/P-Graphs/4.1e/Images/Kinematics/6331664.jpg

Figure : Few more graphs

Images/CC/P-Graphs/4.1e/Images/Kinematics/5128203.jpg

Figure : Few more

Images/CC/P-Graphs/4.1e/Images/Kinematics/1922956.jpg

Figure : Just last in this category.

If we have a high velocity, the graph has a steep slope. If we have a low velocity the graph has a shallow slope (assuming the vertical and horizontal scale of each graph is the same).

Images/CC/P-Graphs/4.1e/Images/Kinematics/Image13.jpg

Figure : Graph comparison of high speed vs low speed. [It may be noted that in Distance-Time Graphs , the slope is Speed (Distance and corresponding Speed both being Scalars), while in Displacement-Time Graphs, the slope is velocity(Displacement and corresponding Velocity both being Vectors).]

Images/CC/P-Graphs/4.1e/Images/Kinematics/KinGraphSummary.jpg

Figure : Graphical representation of the paradigms of this chapter.

3.1 Displacement-Time Graph

A displacement-time graph shows the positions of a moving object at different times. Fig. shows the displacement-time graph of a car. From time $t = 0$ to $t = 5$ s , the car moves forward, and at $t = 5$ s it has a displacement of 60 m. Then it remains stationary there for 5 s, and finally moves back to its starting position in another 5 s. www.hk-phy.org

Images/CC/P-Graphs/4.1e/Images/Kinematics/5-01.jpg

Figure : Displacement-Time Graph of the problem being discussed.

The velocity of motion can be determined from the slope of the displacement-time graph. The velocity of the car is $\frac{60}{5} = 12 \text{ m/s}$ from $t=0$ to $t=5$ s , it is zero (the car is at rest) from $t=5$ s to $t=10$ s and is $(0-60)/(15-10)$ from $t=10$ s to $t=15$ s. The negative slope in the last 5 s indicates that the car is moving backwards. Note that the slope of the graph in each of the time intervals is a constant, showing that the car is in a uniform motion (constant velocity) in each interval.

Question : An object starts from rest and undergoes a positive, constant acceleration for ten seconds, it then continues on with constant velocity. Which of the following graphs correctly describes the situation?

Images/CC/P-Graphs/Common/Reduced/reviewkinematics/ImagesReduced/Tugk9-mod

www.physport.org

3.2 Velocity-time graph

A velocity-time graph shows the velocities of a moving object at different times. Figure. shows three velocity-time graphs. www.hk-phy.org

Fig. (a) represents motion at a constant velocity, the acceleration is zero and thus the slope of the graph is zero. Fig. represents a uniform acceleration given by

$$a = \frac{v - u}{t} = \frac{6 - 0}{15} = 0.4 \text{ ms}^{-2}$$

This is represented by the slope of the velocity-time graph. Fig. represents a uniform deceleration given by ,

$$a = \frac{v - u}{t} = \frac{0 - 4}{15} \approx -0.27 \text{ ms}^{-2}$$

which is also represented by the slope of the graph. The negative slope indicates that the object decelerates. Note that the steeper the slope, the larger is the magnitude of the acceleration.

Images/CC/P-Graphs/4.1e/Images/Kinematics/5-05.jpg

Fig. For the uniform motion shown in Fig. (a), the displacement is given by velocity \times time $= 4 \times 15 = 60$. This is represented by the area of the rectangle under the graph. In fact, for any velocity-time graph, the displacement at a certain time can be calculated from the area under the graph. Following this rule, we can work out the displacement for an object in uniform acceleration or deceleration. For Fig. (b) , the displacement is $6 \times 15/2 = 45 \text{ m}$, while for Fig. (c), the displacement is $4 \times 15/2 = 30 \text{ m}$

Fig. shows the velocity-time graph of an elevator moving upwards. The elevator is initially at rest on the ground floor. It accelerates from rest for 2.5 s, reaching a velocity of 1 ms^{-1} , then it moves at this constant velocity for 5 s, and finally decelerates to rest in 2.5 s.

Images/CC/P-Graphs/4.1e/Images/Kinematics/5-06.jpg

The acceleration of the elevator at each time interval can be deduced from the slope of the graph. In the first 2.5 s, the elevator accelerates at $1/2.5 = 0.4 \text{ ms}^{-2}$. The acceleration is zero for the next 5 s, and in the last 5 s the elevator decelerates at $(0-1)/(10-7.5) = -0.4 \text{ ms}^{-2}$ to rest (the negative slope indicates a deceleration). Note that for the whole trip, the elevator is going upwards.

The total displacement can be calculated from the area under the graph. In this example, the total displacement of the elevator is the area of the trapezium under the graph, i.e., total displacement $= (5+10) \times 1/2 = 7.5 \text{ m}$.

3.3 Acceleration-time graph

Fig. An acceleration-time graph shows the accelerations of a moving object at different times. Fig. shows the acceleration-time graph constructed from the velocity-time graph in Fig. . It can be seen that during the first 2.5 s and the last 2.5 s, the elevator is moving with a constant acceleration and deceleration respectively, and in between it moves at a constant velocity (zero acceleration).

Images/CC/P-Graphs/4.1e/Images/Kinematics/5-11.jpg

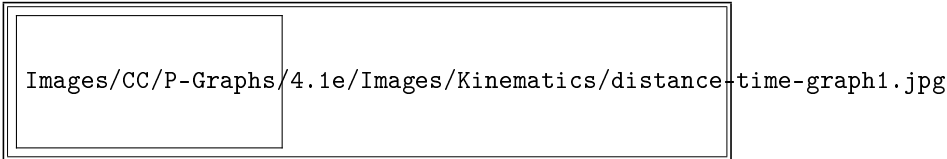
Question : Acceleration vs time graphs for five objects are shown below. All axes have the same scale. Which object has the greatest change in velocity during the interval?

Images/CC/P-Graphs/Common/Reduced/reviewkinematics/ImagesReduced/TUGK2.jpg

www.physport.org

3.4 One More Go in a different perspective

3.4.1 Distance-Time Graphs

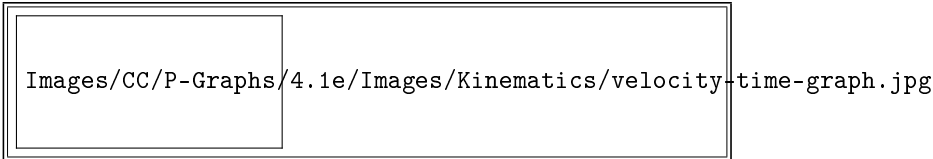


For a distance-time graph, the distance never decreases. When the object is stationary, the distance-time graph will be horizontal. The gradient of a distance-time graph is the instantaneous speed of the object. For straight line with positive gradient, it means that the object is travelling at uniform speed There is no straight line with negative gradient (as the distance never decreases) For curves, it means that the object is travelling at non-uniform speed www.miniphysics.com

3.4.2 Displacement-time graphs

The details are similar as distance-time graphs, except that the distance is now displacement, and speed is now velocity. One minor difference: There is a straight line with negative gradient, it means that the object is travelling at uniform velocity in the opposite direction.www.miniphysics.com

3.4.3 Velocity-time graphs

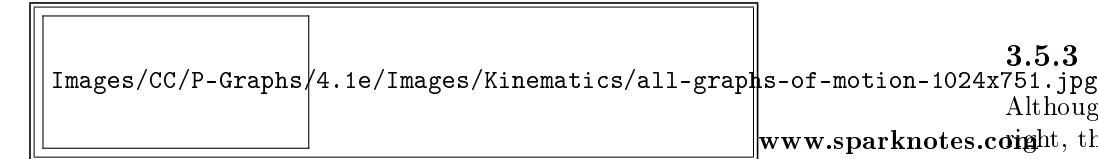


When the object is stationary, it is a straight horizontal line at 0. When the object is undergoing uniform motion, it is a straight horizontal line at v m s⁻¹ where v is the velocity of the object. For straight line with positive gradient, it means that the object is accelerating. For straight line with negative gradient, it means that the object is decelerating. For curves, it means that the acceleration of the object is changing. The area under the graph is the change in displacement of the object

3.4.4 Acceleration-time graphs

Area under graph is the change in velocity www.miniphysics.com

The figure below shows the displacement-time graph, velocity-time graph and acceleration-time graph for the respective state of motion. It serves as a summary of the text above.

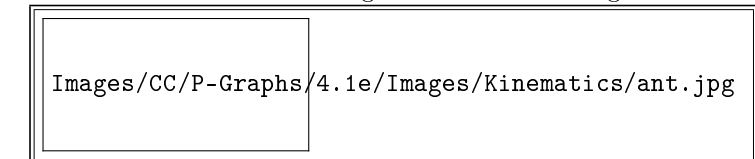


3.4.5 Self-Test Questionswww.miniphysics.com

- Q Can you tell from a displacement-time graph whether an object is stationary?
- Answer Yes. If the object is stationary, it will appear as a horizontal line on a displacement-time graph.
- Q How can you obtain the average velocity and instantaneous velocity from a displacement-time graph.
- Answer The average velocity can be found by using $\frac{\text{total displacement}}{\text{total time taken}}$
- The instantaneous velocity at a point in time can be found from the gradient of the tangent to that point in time.
- Q Can you tell from a velocity-time graph whether an object is stationary?
- Answer Yes. If the object is stationary, the velocity-time graph will be a horizontal line at $v=0$.
- Q How would you obtain the acceleration of an object from a velocity-time graph? What does the area under a velocity-time graph represent?
- Answer The acceleration of an object at a point in time can be obtained from the gradient of the tangent to that point in time.
- The area under a velocity-time graph represents the total distance traveled.
- Q Can you tell from an acceleration-time graph whether an object is stationary?
- Answer No, you cannot. Do you know why?

3.5 Revision

Since you are not allowed to use calculators, SAT II Physics places a heavy emphasis on qualitative problems. A common way of testing kinematics qualitatively is to present you with a graph plotting position vs. time, velocity vs. time, or acceleration vs. time and to ask you questions about the motion of the object represented by the graph. Because SAT II Physics is entirely made up of multiple-choice questions, you won't need to know how to draw graphs; you'll just have to interpret the data presented in them. Knowing how to read such graphs quickly and accurately will not only help you solve problems of this sort, it will also help you visualize the often-abstract realm of kinematic equations. In the examples that follow, we will examine the movement of an ant running back and forth along a line.www.sparknotes.com



3.5.1 Position vs. Time Graphs

Position vs. time graphs give you an easy and obvious way of determining an object's displacement at any given time, and a subtler way of determining that object's velocity at any given time. Let's put these concepts into practice by looking at the following graph charting the movements of our friendly ant.www.sparknotes.com



Any point on this graph gives us the position of the ant at a particular moment in time. For instance, the point at (2,-2) tells us that, two seconds after it started moving, the ant was two centimeters to the left of its starting position, and the point at (3,1) tells us that, three seconds after it started moving, the ant is one centimeter to the right of its starting position. Let's read what the graph can tell us about the ant's movements. For the first two seconds, the ant is moving to the left. Then, in the next second, it reverses its direction and moves quickly to $y = 1$. The ant then stays still at $y = 1$ for three seconds before it turns left again and moves back to where it started. Note how concisely the graph displays all this information. Calculating Velocity We know the ant's displacement, and we know how long it takes to move from place to place. Armed with this information, we should also be able to determine the ant's velocity, since velocity measures the rate of change of displacement over time. If displacement is given here by the vector y , then the velocity of the ant is

$$v = \frac{\Delta y}{\Delta t}$$

If you recall, the slope of a graph is a measure of rise over run; that is, the amount of change in the y direction divided by the amount of change in the x direction. In our graph, is the change in the y direction and is the change in the x direction, so v is a measure of the slope of the graph. For any position vs. time graph, the velocity at time t is equal to the slope of the line at t . In a graph made up of straight lines, like the one above, we can easily calculate the slope at each point on the graph, and hence know the instantaneous velocity at any given time. We can tell that the ant has a velocity of zero from $t = 3$ to $t = 6$, because the slope of the line at these points is zero. We can also tell that the ant is cruising along at the fastest speed between $t = 2$ and $t = 3$, because the position vs. time graph is steepest between these points. Calculating the ant's average velocity during this time interval is a simple matter of dividing rise by run, as we've learned in math class.

3.5.2 Average Velocity

$$\begin{aligned} \text{velocity} &= \frac{y_{\text{final}} - y_{\text{initial}}}{t_{\text{final}} - t_{\text{initial}}} \\ &= \frac{1 - (-2) \text{ cm}}{3 - 2 \text{ s}} \\ &= 3 \text{ cm/s to the right} \end{aligned}$$

How about the average velocity between $t = 0$ and $t = 3$? It's actually easier to sort this out with a graph in front of us, because it's easy to see the displacement at $t = 0$ and $t = 3$, and so that we don't confuse displacement and distance.

3.5.3 Average Speed

Although the total displacement in the first three seconds is one centimeter to the right, the total distance traveled is two centimeters to the left, and then three centimeters to the right, for a grand total of five centimeters. Thus, the average speed is not the same as the average velocity of the ant. Once we've calculated the total distance traveled by the ant, though, calculating its average speed is not difficult:

$$5\text{cm}/3\text{s} = 1.67\text{cm/s}$$

3.5.4 Curved Position vs. Time Graphs

This is all well and good, but how do you calculate the velocity of a curved position vs. time graph? Well, the bad news is that you'd need calculus. The good news is that SAT II Physics doesn't expect you to use calculus, so if you are given a curved position vs. time graph, you will only be asked qualitative questions and won't be expected to make any calculations. A few points on the graph will probably be labeled, and you will have to identify which point has the greatest or least velocity. Remember, the point with the greatest slope has the greatest velocity, and the point with the least slope has the least velocity. The turning points of the graph, the tops of the "hills" and the bottoms of the "valleys" where the slope is zero, have zero velocity.www.sparknotes.com



In this graph, for example, the velocity is zero at points A and C, greatest at point D, and smallest at point B. The velocity at point B is smallest because the slope at that point is negative. Because velocity is a vector quantity, the velocity at B would be a large negative number. However, the speed at B is greater even than the speed at D: speed is a scalar quantity, and so it is always positive. The slope at B is even steeper than at D, so the speed is greatest at B. Velocity vs. Time Graphs Velocity vs. time graphs are the most eloquent kind of graph we'll be looking at here. They tell us very directly what the velocity of an object is at any given time, and they provide subtle means for determining both the position and acceleration of the same object over time. The "object" whose velocity is graphed below is our ever-industrious ant, a little later in the day.



We can learn two things about the ant's velocity by a quick glance at the graph. First, we can tell exactly how fast it is going at any given time. For instance, we can see that, two seconds after it started to move, the ant is moving at 2 cm/s. Second, we can tell in which direction the ant is moving. From $t = 0$ to $t = 4$, the velocity is positive, meaning that the ant is moving to the right. From $t = 4$ to $t = 7$, the velocity is negative, meaning that the ant is moving to the left. Calculating Acceleration We can calculate acceleration on a velocity vs. time graph in the same way that we calculate velocity on a position vs. time graph. Acceleration is the rate of change of the velocity vector, a , which expresses itself as the slope of the velocity vs. time graph. For a velocity vs. time graph, the acceleration at time t is equal to

the slope of the line at t . What is the acceleration of our ant at $t = 2.5$ and $t = 4$? Looking quickly at the graph, we see that the slope of the line at $t = 2.5$ is zero and hence the acceleration is likewise zero. The slope of the graph between $t = 3$ and $t = 5$ is constant, so we can calculate the acceleration at $t = 4$ by calculating the average acceleration between $t = 3$ and $t = 5$:

$$\begin{aligned} \text{velocity} &= \frac{y_{\text{final}} - y_{\text{initial}}}{t_{\text{final}} - t_{\text{initial}}} \\ &= \frac{-2 - (2) \text{ cm/s}}{5 - 3 \text{ s}} \\ &= -2 \text{ cm/s}^2 \end{aligned}$$

The minus sign tells us that acceleration is in the leftward direction, since we've defined the y -coordinates in such a way that right is positive and left is negative. At $t = 3$, the ant is moving to the right at 2 cm/s , so a leftward acceleration means that the ant begins to slow down. Looking at the graph, we can see that the ant comes to a stop at $t = 4$, and then begins accelerating to the right. www.sparknotes.com

3.5.5 Calculating Displacement

Velocity vs. time graphs can also tell us about an object's displacement. Because velocity is a measure of displacement over time, we can infer that:

$$\text{displacement} = \text{velocity} \times \text{time}$$

Graphically, this means that the displacement in a given time interval is equal to the area under the graph during that same time interval. If the graph is above the t -axis, then the positive displacement is the area between the graph and the t -axis. If the graph is below the t -axis, then the displacement is negative, and is the area between the graph and the t -axis. Let's look at two examples to make this rule clearer. First, what is the ant's displacement between $t = 2$ and $t = 3$? Because the velocity is constant during this time interval, the area between the graph and the t -axis is a rectangle of width 1 and height 2.

Images/CC/P-Graphs/4.1e/Images/Kinematics/VvsTgraph_shade1.jpg

The displacement between $t = 2$ and $t = 3$ is the area of this rectangle, which is $2 \text{ cm/s} \cdot 1 \text{ s} = 2 \text{ cm}$ to the right. Next, consider the ant's displacement between $t = 3$ and $t = 5$. This portion of the graph gives us two triangles, one above the t -axis and one below the t -axis.

Images/CC/P-Graphs/4.1e/Images/Kinematics/VvsTgraph_shade2.jpg

Both triangles have an area of $\frac{1}{2}(1 \text{ s})(2 \text{ cm/s}) = 1 \text{ cm}$. However, the first triangle is above the t -axis, meaning that displacement is positive, and hence to the right, while the second triangle is below the t -axis, meaning that displacement is negative, and hence to the left. The total displacement between $t = 3$ and $t = 5$ is **ZERO**.

In other words, at $t = 5$, the ant is in the same place as it was at $t = 3$. www.sparknotes.com

3.5.6 Curved Velocity vs. Time Graphs

As with position vs. time graphs, velocity vs. time graphs may also be curved. Remember that regions with a steep slope indicate rapid acceleration or deceleration, regions with a gentle slope indicate small acceleration or deceleration, and the turning points have zero acceleration.

3.5.7 Acceleration vs. Time Graphs

After looking at position vs. time graphs and velocity vs. time graphs, acceleration vs. time graphs should not be threatening. Let's look at the acceleration of our ant at another point in its dizzy day.

Images/CC/P-Graphs/4.1e/Images/Kinematics/AvsTgraph.jpg

Acceleration vs. time graphs give us information about acceleration and about velocity. SAT II Physics generally sticks to problems that involve a constant acceleration. In this graph, the ant is accelerating at 1 m/s^2 from $t = 2$ to $t = 5$ and is not accelerating between $t = 6$ and $t = 7$; that is, between $t = 6$ and $t = 7$ the ant's velocity is constant. www.sparknotes.com

3.5.8 Calculating Change in Velocity

Acceleration vs. time graphs tell us about an object's velocity in the same way that velocity vs. time graphs tell us about an object's displacement. The change in velocity in a given time interval is equal to the area under the graph during that same time interval. Be careful: the area between the graph and the t -axis gives the change in velocity, not the final velocity or average velocity over a given time period. What is the ant's change in velocity between $t = 2$ and $t = 5$? Because the acceleration is constant during this time interval, the area between the graph and the t -axis is a rectangle of height 1 and length 3. www.sparknotes.com

Images/CC/P-Graphs/4.1e/Images/Kinematics/AvsTgraph_shade1.jpg

The area of the shaded region, and consequently the change in velocity during this time interval, is $1 \text{ cm/s}^2 \cdot 3 \text{ s} = 3 \text{ cm/s}$ to the right. This doesn't mean that the velocity at $t = 5$ is 3 cm/s ; it simply means that the velocity is 3 cm/s greater than it was at $t = 2$. Since we have not been given the velocity at $t = 2$, we can't immediately say what the velocity is at $t = 5$. Summary of Rules for Reading Graphs You may have trouble recalling when to look for the slope and when to look for the area under the graph. Here are a couple handy rules of thumb: The slope on a given graph is equivalent to the quantity we get by dividing the y -axis by the x -axis. For instance,

the y -axis of a position vs. time graph gives us displacement, and the x -axis gives us time. Displacement divided by time gives us velocity, which is what the slope of a position vs. time graph represents. The area under a given graph is equivalent to the quantity we get by multiplying the x -axis and the y -axis. For instance, the y -axis of an acceleration vs. time graph gives us acceleration, and the x -axis gives us time. Acceleration multiplied by time gives us the change in velocity, which is what the area between the graph and the x -axis represents. We can summarize what we know about graphs in a table:

Images/CC/P-Graphs/4.1e/Images/Kinematics/graphsummary.jpg

3.6 Exercises

3.6.1 Problems on Position-Time Graph

3.6.1.1 Subjectives

Example : A boy walks at a velocity of 0.5 m/s along a street for 50 s , suddenly he remembers that he has to buy something in a shop that he has passed by, so he turns around and walks at a velocity of -1 m/s for 10 s . He then stops for 30 s at the shop, and finally walks forward again at 0.5 m/s for another 20 s . Plot a displacement-time graph for the motion of the boy. www.hk-phy.org

Solution : See Fig. 5-4.

Images/CC/P-Graphs/4.1e/Images/Kinematics/5-04.jpg

Example : A marathon runner runs at a constant 12 km/h . intmath.com

- Express her displacement travelled as a function of time.
- Graph the motion for $0 \leq t \leq 4h$

Solution : a. $s = 12t$, for displacement s and time t .

- Graph of $s = 12t$.

Images/CC/P-Graphs/4.1e/Images/Kinematics/kinematics4-g.jpg

We stop the graph at $(4, 48)$.

Example : This is the graph of a journey by sports car: intmath.com

Images/CC/P-Graphs/4.1e/Images/Kinematics/kinematics2-g.jpg

- What is the velocity for each stage of the journey?
- What is the average (mean) velocity for the whole journey?

Solution : a. This table outlines the stages of the journey.

12:00 to 12:30	Travelled 50 km in 30 minutes, so 100 km/h
12:30 to 13:00	Stopped
13:00 to 14:00	Travelled 50 km in 60 minutes, so 50 km/h
14:00 to 14:30	Stopped
14:30 to 15:00	Travelled 100 km back towards the starting point in 30 minutes, so -100 km/h

b. Even though the whole journey was 200 km (100 km out and 100 km back) in 3 hours , the displacement for the journey (the distance from the starting point) is 0 km .

So the average velocity is 0 km/h .

On the other hand, the average speed was $\frac{200}{3} = 66.7 \text{ km/h}$

In summary,

$$\text{ave velocity} = \frac{\text{displacement}}{\text{time}}$$

$$\text{ave speed} = \frac{\text{distance}}{\text{time}}$$

Example : A particle in a magnetic field moves as follows: intmath.com

Images/CC/P-Graphs/4.1e/Images/Kinematics/kinematics3-g.jpg

Find the velocity for each part of the motion.

Solution :

$$\text{For } t=0 \text{ to } 2: \frac{\Delta s}{\Delta t} = \frac{4}{2} = 2 \text{ ms}^{-1}$$

$$\text{For } t=2 \text{ to } 7: \frac{\Delta s}{\Delta t} = \frac{0}{5} = 0 \text{ ms}^{-1}$$

$$\text{For } t=7 \text{ to } 8: \frac{\Delta s}{\Delta t} = \frac{-4}{2} = -4 \text{ ms}^{-1}$$

(The velocity is negative, since the particle is going in the opposite direction.)

Example : An object's position during a 10 second time interval is shown by the graph below:

Images/CC/P-Graphs/4.1e/Images/Kinematics/ex22.jpg

- Determine the object's total distance traveled and displacement.
- What is the object's velocity at the following times: $t = 1$, $t = 3$, and $t = 6$.
- Determine the object's average velocity and average speed from $t = 0$ to $t = 10$.
- What is the object's acceleration at $t = 5$?

Solution : a.) The total distance traveled by the object is the sum of all the distances it traveled during the time interval. In the first two seconds it traveled 3 m . Then it traveled 0 m in the next two seconds. Then over the next five seconds,

the object moved 5 m, then remained at rest. so the total distance is $3 + 5 = 8$ m. The displacement of the object is simply the final position minus the initial position, or $-2 - 0 = -2$ m.

b.) Notice that each of these points is in the middle of a line segment on the graph. Because of this, the instantaneous velocity at these points is the same as the average velocity over the time intervals represented by each segment, so: $v(t) = (x_f - x_i)/(t_f - t_i)$ $v(1) = (3 - 0)/(2 - 0) = 3/2 = 1.5$ m/s $v(3) = (3 - 3)/(4 - 2) = 0/2 = 0$ m/s $v(6) = (-2 - 3)/(9 - 4) = -5/5 = -1$ m/s

Notice that the formula $(x_f - x_i)/(t_f - t_i)$ is the same as the slope formula for this graph. The velocity at any point on a position vs. time graph is simply the slope of the graph at that point. By this definition, we also know that the velocity of any position function is its derivative with respect to time. You can also go from a velocity function to a position function using integration. Go to calculus notes

c.) Average velocity is displacement divided by time. We found in part a that the object's displacement is -2 m, so: $v_{avg} = -2/10 = -0.2$ m/s

Average speed is total distance divided be time, and we found in part a that the object's total distance traveled is 8 m. so: $8/10 = 0.8$ m/s

d.) We determined in part b that the object's velocity is represented by the slope of the line segment on the graph. Since the slope of this segment is constant, the object's velocity at $t = 5$ is constant. Since constant velocity means there is no acceleration, $a = 0$.

3.6.2 Problems on Velocity-Time Graph

3.6.2.1 Subjective

Example : Fig. shows the velocity-time graph of a train travelling from station 1 to station 2. The train moves along a straight railway.



Describe the motion of the train, stating the acceleration and velocity in each stage. What is the displacement of station 2 from station 1?

Solution : From $t=0$ to $t=10$ s , the train accelerates from $v=0$ to $v=20$ m/s .

$$a=20/10=2 \text{ m s}^{-2}$$

From $t=10$ s to $t=60$ s , the train moves at a constant velocity $v=20$ m/s. $a=0$.

From $t=60$ s to $t=80$ s , the train decelerates from $v=20$ m/s back to $v=0$ m/s .

$$a=(0-20)/(80-60)=-1 \text{ m s}^{-2}$$

Total displacement = the area under the velocity-time graph

$$s = \frac{(80 + 50)}{2} \times 20 = 1300m$$

Example : A car is initially moving at a velocity of 36 . Suddenly the driver sees a girl running across of the road at 13 m in front of the car. It takes 0.5 s for the driver to react and start braking the car. The car then takes one more second to stop. Plot the velocity-time graph for the car, starting from the time when the driver sees the girl. What is the deceleration of the car? Would the car hit the girl?



Solution : See Fig. . The car decelerates at 10 ms^{-2} . It travels a total distance of $(0.5+1.5)*10/2=10$ m , so it would not hit the girl, but stop at a distance of 3 m from the girl.

Note: The total distance travelled by the car after the driver seeing the danger is called the stopping distance. See the next chapter for more analysis on car braking.

Example ; Fig. shows the displacement-time graph of an object. Plot its velocity-time graph. Find the average velocity of the object.



Solution : See Fig. .



Since the total displacement of the object is zero, the average velocity is also zero.

Note: The object moves backwards from $t=4$ s to $t=6$ s . In this interval, the area bounded by the velocity-time graph is negative (i.e., the area is under the time axis), indicating that the object has a displacement in the opposite direction.

Example : A particle in a generator is accelerated from rest at the rate of 55 ms^{-2} .

a. What is the velocity at $t=3$ s?

b. What is the acceleration at $t=3$ s?

c. What is the distance travelled in 3 seconds?

d. Graph the acceleration (as a $v - t$ graph) for $0 \leq t \leq 3$ s $0 \leq t \leq 3$ s.

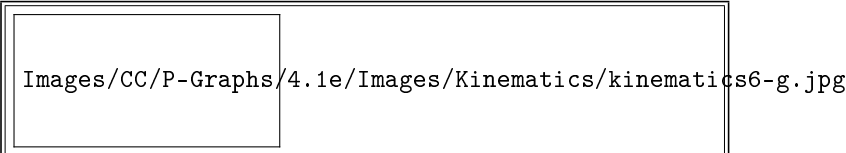
Solution : a. Velocity $=55 \times 3=165 \text{ ms}^{-1}$

b. The acceleration is a constant 55 ms^{-2} , so at $t=3$ s, the acceleration will be 55 ms^{-2} .

c. The distance travelled in $\displaystyle 3$ seconds is $165 \times 1.5=247.5$ m. We obtain this from the area under the line between 0 and 3 (i.e. the area of the shaded triangle below).

d. Note in the graph that we have velocity on the vertical axis, and the units are m/s.

The graph finishes at (3, 165).



Example : A girl starts from rest and travels along a straight line. The diagram below shows the velocity-time graph of the girl from 0 s to 90 s.



(a) Describe the motion of the girl from 0 s to 40 s.

(b) Find the average velocity of the girl in the first 70 s.

(c) Draw the acceleration-time graph of the girl from 0 s to 90 s.

(d) Find the displacement of the girl from the starting point to the position at 90 s.

Solution : (a) From 0 s to 20 s, she travels with constant acceleration of 0.25 ms^{-2} ; From 20 s to 40 s, she travels at constant velocity of 5 m/s.

(b) Total displacement = area under $v-t$ graph $= \frac{1}{2} (30 + 70) * 5 = 250$ m Average velocity $= 250/70 = 3.57$ m/s

(c) Acceleration is the gradient of $v-t$ graph.



(d) Displacement = area under $v-t$ graph $= 250 - \frac{1}{2} * 20 * 5 = 200$ m

Tips: (1) It is very easy to mix up kinematics graphs. The only way to differentiate these graphs is to look at the y-axis!

(2) When describing the motion, do it region by region! And always talk about acceleration or speed / velocity with values if applicable. Refer to example above.

Example : Two sport cars start from rest at the same place. One of them, colored red, accelerates at 0.90 ms^{-2} for 15 s, and continues at constant speed thereafter. The other car, colored blue, accelerates at 0.85 ms^{-2} for 20 s and then remains at that speed.

Draw both journeys on the same velocity-time graph and determine the time and distance that the second car overtakes the first car.

Solution :



The velocities they reach are $0.9 \times 15 = 13.5 \text{ ms}^{-1}$ and $0.85 \times 20 = 17 \text{ ms}^{-1}$ respectively.

Similar to the problem above, we have:

First car's (red) distance at time t is found by finding the area of the trapezoid whose boundary is the t -axis, the red lines and the vertical line representing time at t seconds. This of course assumes $t > 15$ (otherwise, we have negative distances, and the trapezoid only starts at $t=15$.)

$$\text{So the distance (at time } t) = \frac{1}{2} \times 13.5 (t + t - 15)$$

= The distance of the second car (blue) is found by finding the area of the blue trapezoid, bounded by the blue lines, the t -axis and the vertical line. (And this one assumes $t > 20$, for the same reasons as above.)

$$\text{So the distance (at time } t) = \frac{1}{2} \times 17 (t + t - 20)$$

The distance when they meet is the same, so:

$$\frac{1}{2} \times 13.5 \times (2t - 15) = \frac{1}{2} \times 17 \times (2t - 20)$$

Solving gives:

$$\text{Time} = 19.643 \text{ s}$$

$$\text{Distance} = 163.93 \text{ m}$$

Dilemma

However, this solution gives us a dilemma. As mentioned above, the expressions we found for the area of the trapezoids required t to be more than 15 s and 20 s, respectively.

We can confirm the blue car overtakes the red car before 20 seconds, by calculating the distances travelled at that time.

After 20 seconds:

$$\text{Red car has travelled } 0.5 \times (20 + 5) \times 13.5 = 168.75 \text{ m}$$

Blue car has travelled $0.5 \times 20 \times 17 = 170$ m The blue car has travelled further than the red car, so it means it has already overtaken the red car.

Back to the drawing board

So, in fact, for this question we need to consider the area of the blue triangle, not the blue trapezoid.

This is the correct diagram for finding t :



The red car's distance is fine from before (it's a trapezoid):

$$d = \frac{1}{2} \times 13.5 (t + t - 15)$$

But the blue car's distance at t is the area of the triangle (not trapezoid) bounded by the blue line and the vertical line at t (we are taking $\frac{1}{2}$ base times height):

$$\text{distance} = \frac{1}{2} \times t \times 0.85 \times t$$

Equating these gives:

$$6.75(2t - 15) = 0.425t^2$$

Solving gives:

$$t = 12.14 \text{ s}, t = 19.63 \text{ s}$$

The first solution has no practical meaning, since $t > 15$ for the expression to work.

So we conclude the time taken for the blue car to overtake the red car is 19.63 s and the distance travelled at that time is 163.7 m.

Further Information

To illustrate we have found the correct answer, below is the graph of distance against time. Labelling of axes is very important in this work!

The red curve corresponds to the acceleration portion of the first race car. At $t=15$, or point C(15,101.25), the red car stops accelerating and continues at a constant speed (CA is no longer a curve, but a straight line).

The blue curve corresponds to the second (blue) car. It doesn't accelerate as hard (its curve is below the red curve, indicating it covers less distance in the same time), but it accelerates for longer. At $t=20$, or point D (20,170), it stops accelerating and its graph is now a straight line.

We can see the curves intersect at time $t=19.63$ s, or point A (19.63,163.7), as we found in the calculation above. This means they have covered the same distance (163.7 m) at that time, so that's when the blue car passes the red car.

Images/CC/P-Graphs/4.1e/Images/Kinematics/race-cars2.jpg

3.6.2.2 Miscellaneous

Example : You drive a car in such a way that its motion is described by the velocity-time graph shown here. Draw the displacement-time and acceleration-time graphs that correspond to this motion, and describe in words how the car moves. www.uwgb.edu

Images/CC/P-Graphs/4.1e/Images/Kinematics/fig1.jpg

Solution : (Hint : In this problem, you are asked to describe the motion of the car. Whenever you are asked to describe the motion of an object without worrying about the cause of that motion, you have a kinematics problem. This problem is different from most kinematics problems, however, in that you are not asked for a numerical description but rather to use words and graphs to describe how the car moves.)

(Queries : How did you know that $x = 0$ at $t = 0$?

You don't! If you are given a velocity-time graph, you know the initial speed but not the initial location of the object. (Calculus students, remember $v = dx/dt$, and the derivative of a constant—your initial location—is zero.) I chose to call the location of the car at $t = 0$ the origin, but you could start your graph at any point. The shape of the graph, however, should look the same.

Why can't I treat the straight line from t_1 to t_3 in a single step?

Any time velocity is above the t -axis, it has a positive value and so slope of the x - t graph is also positive. A positive slope means that the line or curve is in such a direction as to make between a 0 and a 90° angle above the $+x$ axis. Any time velocity is below the t -axis, it has a negative value and so the slope of the x - t graph is also negative. A negative slope means that the line or curve is in such a direction as to make between a 0 and a 90° angle below the $+x$ axis.

Why doesn't the line from t_3 to t_4 look more curved?

As the velocity-time curve gets closer and closer to $v = 0$ (the t -axis), velocity's value is getting smaller regardless of direction. As the value of v decreases, so does the slope of the x - t graph. A decreasing slope means that the slope gets smaller and smaller—the line gets becomes more horizontal.

The curve from t_3 to t_4 on the x - t graph shown here doesn't look very curved. This is because its slope goes from the same value as it ended with on the t_2 to t_3 curve to almost zero (a horizontal line) and so not a lot of change as I have drawn it. Any line you draw that curves down and to the right becoming more horizontal as it goes is fine.)

Select a relation

To go between a velocity-time graph and a displacement-time or acceleration-time graph, you need to understand how velocity, displacement and acceleration are related to each other. In other words, you need to use the definitions of velocity and acceleration:

$v = \Delta x / \Delta t$ In words, the value of velocity = the slope of the x - t graph.

$a = \Delta v / \Delta t$ In words, the value of acceleration = the slope of the v - t graph.

Hint: Graphing problems seem like they should be straightforward, and the equations that you need are only those given above. It is very, very common, however, to make mistakes on these problems because it feels like the graphs should be pictures of the motion and they are not. In order to avoid those mistakes, make a table based on the sentences above and then draw the graph from the table.

Displacement-Time Graph

Images/CC/P-Graphs/4.1e/Images/Kinematics/solve.jpg

Images/CC/P-Graphs/4.1e/Images/Kinematics/fig2.jpg

Once you understand the displacement-time graph, continue down to the acceleration-time graph.

Acceleration-time Graph

Images/CC/P-Graphs/4.1e/Images/Kinematics/solve1.jpg

Understand

In this problem, you are given the velocity-time graph for the motion of a car. By relating the value of velocity to the slope of the x - t graph (this is just the definition of velocity) you are able to draw the x - t graph corresponding to this motion.

By relating the slope of the v - t graph to the value of acceleration (this is just the definition of acceleration) you are able to draw the a - t graph corresponding to this motion.

You can describe the motion looking at any of the three graphs. From $t = 0$ to t_1 : The car travels forward (+ direction) with a constant speed. There is no acceleration and the car moves away from its starting point at a constant rate.

From t_1 to t_2 : The car slows to a stop at a constant rate. It is still moving forward, but the amount of distance it covers in each second is decreasing. Acceleration acts against the motion of the car, or in the negative direction.

From t_2 to t_3 : The car reverses direction, moving faster and faster (at a constant acceleration) in the negative direction. Acceleration is acting with the motion of the car, so it is also in the negative direction. The amount of distance the car covers each second increases.

From t_3 to t_4 : The car continues to move in the negative direction but at a decreasing speed. The rate at which the speed decreases is getting greater—the driver is braking harder as the car stops—and so acceleration increases. Acceleration is acting against the motion of the car, or in the positive direction. The car covers less and less distance each second.

3.6.3 Problems on Area Under v - t graph

Example : A charged particle in an accelerator starts from rest, accelerates at 1.5 ms^{-2} for 3 s and then continues at a steady speed for a further 6 s.

Draw the v - t graph and find the total distance travelled.

Solution:

Images/CC/P-Graphs/4.1e/Images/Kinematics/ex.jpg

Total distance travelled is the area under the graph (in this case we need to find the area of a trapezium).

$$\begin{aligned} \text{distance} &= \frac{(a+b)h}{2} \\ &= \frac{(9+6) \times 4.5}{2} \\ &= 33.75 \text{ m} \end{aligned}$$

Example : A car is travelling at a constant speed of 72 km/h and passes a stationary police car. The police car immediately gives chase, accelerating uniformly to reach a speed of 90 km/h in 10 s and continues at this speed until he overtakes the other car. Find:

- the time taken by the police to catch up with the car,
- the distance travelled by the police car when this happens.

Solution :

Images/CC/P-Graphs/4.1e/Images/Kinematics/ex3a.jpg

The v - t curve for the car is represented by the red line, while that v - t curve for the police car is the blue line.

We need to find the unknown time t (in seconds), when the police catch up to the car. We find this by comparing the distance travelled by each (it will be the same distance at the overtaking point.) So we need to set the area under the v - t curve for the car (the pink shaded area) to be equal to the area under the v - t curve for the police car (the blue shaded area).

a. The area under the curve for the car at time t (in seconds) is simply $72t$. (It is a rectangle, 72 high and width t).

The area under the trapezium (trapezoid) for the police car at unknown time t , using $A = (a+b) \frac{h}{2}$ is:

$$(t+t-10) \times \frac{90}{2} = 45(2t-10)$$

We set these equal to find the required time: $72t = 45(2t-10)$ That is, when $72t = 90t - 450$ So $t = 25$ s will be the time the police car catches up.

b. Both of the cars have travelled $72 \times \left(\frac{25}{3600}\right) \times 1000 = 500$ m during those 25 s. [We have used $d = v \times t$ and converted from seconds to hours (since the velocities are given in km/h).]

3.6.4 Problems on Average Velocity

Example : An object's velocity during a 10 second time interval is shown by the graph below:

Images/CC/P-Graphs/4.1e/Images/Kinematics/ex23.jpg

- Determine the object's total distance traveled and displacement.
- At $t = 0$, the object's position is $x = 2$ m. Find the object's position at $t = 2$, $t = 4$, $t = 7$, and $t = 10$.
- What is the object's acceleration at the following times: $t = 1$, $t = 3$, and $t = 6$.
- Sketch the corresponding acceleration vs. time graph from $t = 0$ to $t = 10$.

Solution : a.) Recall that the equation for velocity is $v = x/t$. If we solve this for x , we get $x = vt$. Notice that this is the same as the area of a rectangles whose sides are lengths v and t , so we can determine that the displacement is the area enclosed by the velocity vs. time graph. So, we will find the area of each section under the graph:

Images/CC/P-Graphs/4.1e/Images/Kinematics/ex23a.jpg

The total distance traveled by the object is simply the sum of all these areas: $3 + 6 + 4.5 + 2 + 2 = 17.5$ m The displacement is found in a similar fashion, except areas below the x -axis are considered negative: $3 + 6 + 4.5 - 2 - 2 = 9.5$ m Interestingly enough, the area enclosed by any function can be represented by a definite integral. For example, if this graph were defined as a function $v(t)$, then the displacement

would be the integral from 0 to 10 of $v(t)dt$, and the total distance traveled would be the integral from 0 to 10 of $|v(t)|dt$ Go to calculus notes

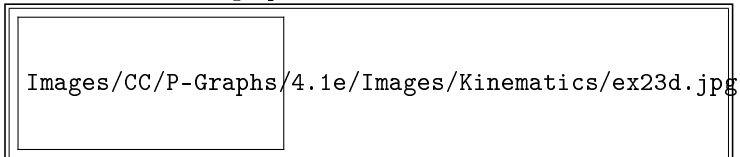
b.) The position of the object at a given point in time can be found in much the same way we found the displacement in part a, except this time we must also add in the initial value given. So: $x(2) = 2 + 3 = 5$ m $x(4) = 2 + 3 + 6 = 11$ m $x(7) = 2 + 3 + 6 + 4.5 = 15.5$ m $x(10) = 2 + 3 + 6 + 4.5 - 2 - 2 = 11.5$ m

Notice that this can also be done by adding the integral from 0 to t of $v(t)dt$ to the initial value of 2. Go to calculus notes

c.) Like velocity in part b of problem 22, the instantaneous acceleration at any point along one of the graph's line segments is the same as the average acceleration across that line segment. The formula for acceleration is $a_{avg} = \Delta v / \Delta t = (v_f - v_i) / (t_f - t_i)$, so: $a(t) = (v_f - v_i) / (t_f - t_i)$ $a(1) = (3 - 0) / (2 - 0) = 3/2 = 1.5$ m/s² $a(3) = (3 - 3) / (4 - 2) = 0/2 = 0$ m/s² $a(6) = (-2 - 3) / (9 - 4) = -5/5 = -1$ m/s²

Similarly to the relationship between velocity and position, the formula for acceleration is the same as the slope formula for a velocity vs. time graph. So, we can say that the slope of any velocity vs. time graph is its acceleration. Notice that this definition defines acceleration as the derivative of velocity. So, it is true that for any velocity function $v(t)$, its derivative is an acceleration function $a(t)$. Also, integration can be used to go from an acceleration function to a position function. Go to calculus notes

d.) We know that the acceleration along each line segment of this velocity vs. time graph is equal to the slope of the line segment. We determined these slopes in part c, so the acceleration graph would look like:



This graph uses horizontal lines instead of points to represent that the acceleration is defined at that value at any point along that section. The open circles at the end of each line segment simply indicate that at those time values, acceleration is not defined at either value represented by the horizontal lines. At these points, acceleration is undefined because it changes instantaneously from one value to the next, which cannot be represented numerically.

Chapter 4

Few Basic Problems

Problem Set 1 Use the following graph to answer Questions #1 - #7.

Images/CC/P-Graphs/4.1e/Images/Kinematics/graph01.gif

1. Which object(s) is(are) maintaining a state of motion (i.e., maintaining a constant velocity)?

Answer ; Objects A, B, D, and E.

Objects A, B, D, and E are maintaining a state of motion (i.e., remaining with constant velocity) as demonstrated by the constant slope. If the slope is constant, then the velocity is constant.

2. Which object(s) is(are) accelerating?

Answer : Object C

Object C is accelerating. An accelerating object has a changing velocity. Since the slope of a p-t graph equals the velocity, an accelerating object is represented by a changing slope.

3. Which object(s) is(are) not moving?

Answer : Objects A and E.

Objects A and E are not moving. An object which is not moving has a zero velocity; this translates into a line with zero slope on a p-t graph.

4. Which object(s) change(s) its direction?

Answer : None of the objects change direction.

None of these objects change direction. An object changes its direction if it changes from a + to a - velocity (or vice versa). This translates into a p-t graph with a + slope and then a - slope (or vice versa).

5. On average, which object is traveling fastest?

Answer : Object B.

Object B is traveling fastest. To be traveling fastest is to have the greatest speed (or greatest magnitude of velocity). This translates into the line on a p-t graph with the greatest slope.

6. On average, which moving object is traveling slowest?

Answer : Object D.

Object D is traveling slowest. To be traveling slowest is to have the smallest speed (or smallest magnitude of velocity). This translates into the line on a p-t graph with the smallest slope.

7. Which object has the greatest acceleration?

Answer ; Object C.

Object C has the greatest acceleration. It is the only object with an acceleration. Accelerated motion on a p-t graph is represented by a curved line.

Problem Set 2 Use the following graph to answer Questions #8 - #13.

Images/CC/P-Graphs/4.1e/Images/Kinematics/graph08.gif

8. Which object(s) is(are) maintaining its state of motion?

Answer : Objects A and E.

Objects A and E are maintaining their state of motion. To maintain the state of motion is to keep a constant velocity (i.e., to have a zero acceleration). This translates into a zero slope on a v-t graph.

9. Which object(s) is(are) accelerating?

Answer : Objects B and C (and D during the first part of its motion).

Objects B and C are accelerating (and for a while, object D). Accelerated motion is indicated by a sloped line on a v-t graph.

10. Which object(s) is(are) not moving?

Answer : Each of the objects are moving.

Each of the objects are moving. If an object were not moving, then the v-t graph would be a horizontal line along the axis ($v = 0$ m/s).

11. Which object(s) change(s) its direction?

Answer : Objects B and C.

Objects B and C change their direction. An object that is changing its direction is changing from a + to a - velocity. Thus, the line on a v-t graph will pass from the + to the - region of the graph. Object D is not changing its direction; object D first moves in the - direction with increasing speed and then maintains a constant speed.

12. Which accelerating object has the smallest acceleration?

Answer : Object B.

Object B has the smallest acceleration. Acceleration is indicated by the slope of the line. The object with the smallest acceleration is the object with the smallest slope.

13. Which object has the greatest velocity?

Answer : Object A (E is a close second place).

Object A has the greatest velocity (and object E is a "close second"). The velocity is indicated by how far above or how far below the axis the line is. Object A has a large + velocity. Object E has a large (but not as large) - velocity.

Problem Set 3 14. Sketch a position-time graph for an object which is moving with a constant, positive velocity.

Answer : A position-time graph for an object which is moving with a constant, positive velocity is shown below. A positive, constant velocity is represented by a line with constant slope (straight) and positive slope (upwards sloping).

Images/CC/P-Graphs/4.1e/Images/Kinematics/graph14.gif

Graph of Question 14

15. Sketch a position-time graph for an object which is moving with a constant, negative velocity.

Answer : A position-time graph for an object which is moving with a constant, negative velocity is shown below. A negative, constant velocity is represented by a line with constant slope (straight) and negative slope (downwards sloping).

Images/CC/P-Graphs/4.1e/Images/Kinematics/graph15.gif

Graph of Question 15

16. Sketch a position-time graph for an object moving in the + dir'n and accelerating from a low velocity to a high velocity.

Answer : A position-time graph for an object moving in the + dir'n and accelerating from a low velocity to a high velocity is shown below. If the object is moving in the + dir'n, then the slope of a p-t graph would be +. If the object is changing velocity from small to large values, then the slope must change from small slope to large slope.

Images/CC/P-Graphs/4.1e/Images/Kinematics/graph16.gif

Graph of Question 16

17. Sketch a position-time graph for an object moving in the + dir'n and accelerating from a high velocity to a low velocity.

Answer : A position-time graph for an object moving in the + dir'n and accelerating from a high velocity to a low velocity is shown below. If the object is moving in the + dir'n, then the slope of a p-t graph would be +. If the object is changing velocity from high to low values, then the slope must change from high slope to low slope.

Images/CC/P-Graphs/4.1e/Images/Kinematics/graph17.gif

Graph of Question 17

18. Sketch a position-time graph for an object moving in the - dir'n and accelerating from a high velocity to a low velocity.

Answer : A position-time graph for an object moving in the - dir'n and accelerating from a high velocity to a low velocity is shown below. If the object is moving in the - dir'n, then the slope of a p-t graph would be -. If the object is changing velocity from high to low values, then the slope must change from high slope to low slope.

Images/CC/P-Graphs/4.1e/Images/Kinematics/graph18.gif

Graph of Question 18

19. Sketch a position-time graph for an object moving in the - dir'n and accelerating from a low velocity to a high velocity.

Answer : A position-time graph for an object moving in the - dir'n and accelerating from a low velocity to a high velocity is shown below. If the object is moving in the - dir'n, then the slope of a p-t graph would be -. If the object is changing velocity from low to high values, then the slope must change from low slope to high slope.

Images/CC/P-Graphs/4.1e/Images/Kinematics/graph19.gif

Graph of Question 19

20. Sketch a position-time graph for an object moving in the + dir'n with constant speed; first a slow constant speed and then a fast constant speed.

Answer : A position-time graph for an object moving in the + dir'n with constant speed; first a slow constant speed and then a fast constant speed is shown below. If an object is moving in the + dir'n, then the slope of the line on a p-t graph would be +. At first, the line has a small slope (corresponding to a small velocity) and then the line has a large slope (corresponding to a large velocity).

Images/CC/P-Graphs/4.1e/Images/Kinematics/graph20.gif

Graph of Question 20

21. Sketch a position-time graph for an object moving in the + dir'n with constant speed; first a fast constant speed and then a slow constant speed.

Answer : A position-time graph for an object moving in the + dir'n with constant speed; first a fast constant speed and then a slow constant speed is shown below. If an object is moving in the + dir'n, then the slope of the line on a p-t graph would be +. At first, the line has a large slope (corresponding to a large velocity) and then the line has a small slope (corresponding to a small velocity).

Images/CC/P-Graphs/4.1e/Images/Kinematics/graph21.gif

Graph of Question 21

22. Sketch a position-time graph for an object moving in the - dir'n with constant speed; first a slow constant speed and then a fast constant speed.

Answer : A position-time graph for an object moving in the - dir'n with constant speed; first a slow constant speed and then a fast constant speed is shown below. If an object is moving in the - dir'n, then the slope of the line on a p-t graph would be -. At first, the line has a small slope (corresponding to a small velocity) and then the line has a large slope (corresponding to a large velocity).

Images/CC/P-Graphs/4.1e/Images/Kinematics/graph22.gif

Graph of Question 22

23. Sketch a position-time graph for an object moving in the - dir'n with constant speed; first a fast constant speed and then a slow constant speed.

Answer : A position-time graph for an object moving in the - dir'n with constant speed; first a fast constant speed and then a slow constant speed is shown below. If an object is moving in the - dir'n, then the slope of the line on a p-t graph would be -. At first, the line has a large slope (corresponding to a large velocity) and then the line has a small slope (corresponding to a small velocity).

Images/CC/P-Graphs/4.1e/Images/Kinematics/graph23.gif

Graph of Question 23

24. Sketch a position-time graph for an object which moves in the + direction at a slow constant speed and then in a - direction at a fast constant speed.

Answer : A position-time graph for an object which moves in the + direction at a slow constant speed and then in a - direction at a fast constant speed is shown below. The object must first have a + slope (corresponding to its + velocity) then it must have a - slope (corresponding to its - velocity). Initially, the slope is small (corresponding to a small velocity) and then the slope is large (corresponding to a large velocity).

Images/CC/P-Graphs/4.1e/Images/Kinematics/graph24.gif

Graph of Question 24

25. Sketch a position-time graph for an object which moves in the + direction at a fast constant speed and then in a - direction at a slow constant speed.

Answer : A position-time graph for an object which moves in the + direction at a fast constant speed and then in a - direction at a slow constant speed is shown below. The object must first have a + slope (corresponding to its + velocity) then it must have a - slope (corresponding to its - velocity). Initially, the slope is large (corresponding to a large velocity) and then the slope is small (corresponding to a small velocity).

Images/CC/P-Graphs/4.1e/Images/Kinematics/graph25.gif

Graph of Question 25

26. Sketch a position-time graph for an object which moves in the - direction at a slow constant speed and then in a + direction at a fast constant speed.

Answer : A position-time graph for an object which moves in the - direction at a slow constant speed and then in a + direction at a fast constant speed is shown below. The object must first have a - slope (corresponding to its - velocity) then it must have a + slope (corresponding to its + velocity). Initially, the slope is small (corresponding to a small velocity) and then the slope is large (corresponding to a large velocity).

Images/CC/P-Graphs/4.1e/Images/Kinematics/graph26.gif

Graph of Question 26

27. Sketch a velocity-time graph for an object moving with a constant speed in the positive direction.

Answer : A velocity-time graph for an object moving with a constant speed in the positive direction is shown below. To have "a constant speed in the positive direction" is to have a + velocity which is unchanging. Thus, the line on the graph will be in the + region of the graph (above 0). Since the velocity is unchanging, the line is horizontal. Since the slope of a line on a v-t graph is the object's acceleration, a horizontal line (zero slope) on a v-t graph is characteristic of a motion with zero acceleration (constant velocity).

Images/CC/P-Graphs/4.1e/Images/Kinematics/graph27.gif

Graph of Question 27

28. Sketch a velocity-time graph for an object moving with a constant speed in the negative direction.

Answer : A velocity-time graph for an object moving with a constant speed in the negative direction is shown below. To have "a constant speed in the negative direction" is to have a - velocity which is unchanging. Thus, the line on the graph will be in the - region of the graph (below 0). Since the velocity is unchanging, the line is horizontal. Since the slope of a line on a v-t graph is the object's acceleration, a horizontal line (zero slope) on a v-t graph is characteristic of a motion with zero acceleration (constant velocity).

Images/CC/P-Graphs/4.1e/Images/Kinematics/graph28.gif

Graph of Question 28

29. Sketch a velocity-time graph for an object which is at rest.

Answer : A velocity-time graph for an object which is at rest is shown below. To be "at rest" is to have a zero velocity. Thus the line is drawn along the axis ($v=0$).

Images/CC/P-Graphs/4.1e/Images/Kinematics/graph29.gif

Graph of Question 29

30. Sketch a velocity-time graph for an object moving in the + direction, accelerating from a slow speed to a fast speed.

Answer : A velocity-time graph for an object moving in the + direction, accelerating from a slow speed to a fast speed is shown below. An object which is moving in the + direction and speeding up (slow to fast) has a + acceleration. (If necessary, review the dir'n of the acceleration vector in the Physics Classroom Tutorial.) Since the slope of a line on a v-t graph is the object's acceleration, an object with + acceleration is represented by a line with + slope. Thus, the line is a straight diagonal line with upward (+) slope. Since the velocity is +, the line is plotted in the + region of the v-t graph.

Images/CC/P-Graphs/4.1e/Images/Kinematics/graph30.gif

Graph of Question 30

31. Sketch a velocity-time graph for an object moving in the + direction, accelerating from a fast speed to a slow speed.

Answer : A velocity-time graph for an object moving in the + direction, accelerating from a fast speed to a slow speed is shown below. An object which is moving in the + direction and slowing down (fast to slow) has a - acceleration. (If necessary, review the dir'n of the acceleration vector in the Physics Classroom Tutorial.) Since the slope of a line on a v-t graph is the object's acceleration, an object with - acceleration is represented by a line with - slope. Thus, the line is a straight diagonal line with downward (-) slope. Since the velocity is +, the line is plotted in the + region of the v-t graph.

Images/CC/P-Graphs/4.1e/Images/Kinematics/graph31.gif

Graph of Question 31

32. Sketch a velocity-time graph for an object moving in the - direction, accelerating from a slow speed to a fast speed.

Answer : A velocity-time graph for an object moving in the - direction, accelerating from a slow speed to a fast speed is shown below. An object which is moving in the - direction and speeding up (slow to fast) has a - acceleration. (If necessary, review the dir'n of the acceleration vector in the Physics Classroom Tutorial.) Since the slope of a line on a v-t graph is the object's acceleration, an object with - acceleration is represented by a line with - slope. Thus, the line is a straight diagonal line with downward (-) slope. Since the velocity is -, the line is plotted in the - region of the v-t graph.

Images/CC/P-Graphs/4.1e/Images/Kinematics/graph32.gif

Graph of Question 32

33. Sketch a velocity-time graph for an object moving in the - direction, accelerating from a fast speed to a slow speed.

Answer : A velocity-time graph for an object moving in the - direction, accelerating from a fast speed to a slow speed is shown below. An object which is moving in the - direction and slowing down (fast to slow) has a + acceleration. (If necessary, review the dir'n of the acceleration vector in the Physics Classroom Tutorial.) Since the slope of a line on a v-t graph is the object's acceleration, an object with + acceleration is represented by a line with + slope. Thus, the line is a straight diagonal line with upward (+) slope. Since the velocity is -, the line is plotted in the - region of the v-t graph.

Images/CC/P-Graphs/4.1e/Images/Kinematics/graph33.gif

Graph of Question 33

34. Sketch a velocity-time graph for an object which first moves with a slow, constant speed in the + direction, and then with a fast constant speed in the + direction.

Answer : A velocity-time graph for an object which first moves with a slow, constant speed in the + direction, and then with a fast constant speed in the + direction is shown below. Since there are two parts of this object's motion, there will be two distinct parts on the graph. Each part is in the + region of the v-t graph (above 0) since the velocity is +. Each part is horizontal since the velocity during each part is constant (constant velocity means zero acceleration which means zero slope). The second part of the graph will be higher since the velocity is greater during the second part of the motion.

Images/CC/P-Graphs/4.1e/Images/Kinematics/graph34.gif

Graph of Question 34

35. Sketch a velocity-time graph for an object which first moves with a fast, constant speed in the + direction, and then with a slow constant speed in the + direction.

Answer : A velocity-time graph for an object which first moves with a fast, constant speed in the + direction, and then with a slow constant speed in the + direction is shown below. Since there are two parts of this object's motion, there will be two distinct parts on the graph. Each part is in the + region of the v-t graph (above 0) since the velocity is +. Each part is horizontal since the velocity during each part is constant (constant velocity means zero acceleration which means zero slope). The first part of the graph will be higher since the velocity is greater during the first part of the motion.

Images/CC/P-Graphs/4.1e/Images/Kinematics/graph35.gif

Graph of Question 35

36. Sketch a velocity-time graph for an object which first moves with a constant speed in the + direction, and then moves with a positive acceleration.

Answer : A velocity-time graph for an object which first moves with a constant speed in the + direction, and then moves with a positive acceleration is shown below. Since there are two parts of this object's motion, there will be two distinct parts on the graph. Each part is in the + region of the v-t graph (above 0) since the velocity is +. The slope of the first part is zero since constant velocity means zero acceleration and zero acceleration is represented by a horizontal line on a v-t graph (slope = acceleration for v-t graphs). The second part of the graph is an upward sloping line since the object has + acceleration (again, the slope = acceleration for v-t graphs).

Images/CC/P-Graphs/4.1e/Images/Kinematics/graph36.gif

Graph of Question 36

37. Sketch a velocity-time graph for an object which first moves with a constant speed in the + direction, and then moves with a negative acceleration.

Answer : A velocity-time graph for an object which first moves with a constant speed in the + direction, and then moves with a negative acceleration is shown below. Since there are two parts of this object's motion, there will be two distinct parts on the graph. Each part is in the + region of the v-t graph (above 0) since the velocity is +. The slope of the first part is zero since constant velocity means zero acceleration and zero acceleration is represented by a horizontal line on a v-t graph (slope = acceleration for v-t graphs). The second part of the graph is a downward sloping line since the object has - acceleration (again, the slope = acceleration for v-t graphs).

Images/CC/P-Graphs/4.1e/Images/Kinematics/graph37.gif

Graph of Question 37

4.1 Exercises

Example : Which position-versus-time graph represents the motion shown in the motion diagram?



Solution :

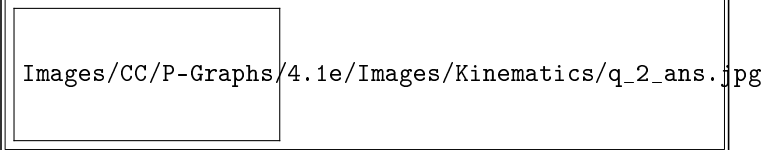


Example : Which velocity-versus-time graph goes with this position-versus-time graph on the left?



Note that the variable “s” denotes a generic Cartesian coordinate. It could be x or y.

Solution :



The velocity graph must match the slope of the position graph. The position graph starts with a constant positive slope. Then the slope decreases to zero.

Example : Which position-versus-time graph goes with this velocity-versus-time graph on the left? The particle’s position at $t_i = 0$ s is $x_i = -10$ m .

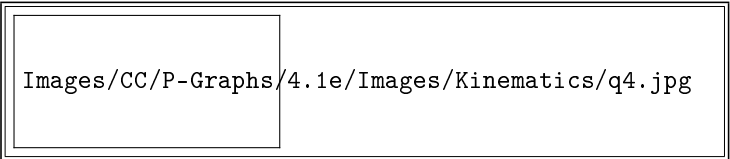


Solution :



The velocity graph must match the slope of the position graph. The intercept of the position graph is arbitrary.

Example : Which velocity-versus-time graph or graphs goes with this acceleration-versus-time graph? The particle is initially moving to the right and eventually to the left.



Solution :



The slope of the velocity graph must match the acceleration graph. The intercept is based on the direction information.

Example : The ball rolls up the ramp, then back down. Which is the correct acceleration graph?



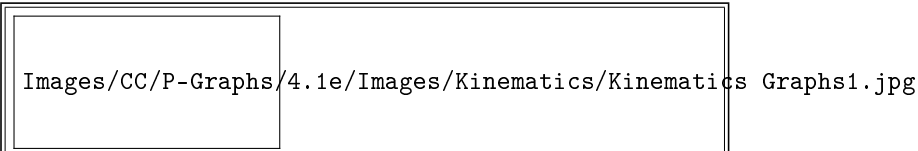
Solution :



The ball will move up the ramp while slowing down, then it will reach a turnaround point and begin to move down the ramp with increasing speed. The velocity graph at right is consistent with that description. The acceleration graph shown in graph d is consistent with the requirement that it match the slope of the velocity graph.

Example : An expectant father paces back and forth producing the potison-versus-time graph shown here.

(a) Without performing a calculation indicate whether the father’s velocity is positive, negative, or zero on the segments of the graph labeled A, B, C, and C.



Segment A:

Segment B:

Segment C:

Segment D:

(b) Calculate the average velocity for each segment and show that your results verify your answers to part (a).

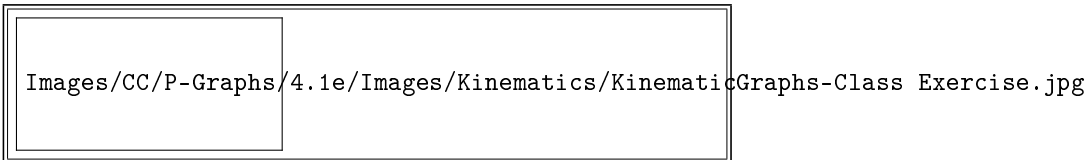
Segment A:

Segment B:

Segment C:

Segment D:

Example : 2. A motorcycle moves according to the velocity-versus-time graph shown. Find the displacement of the motorcycle for each of the segments A, B, and C.

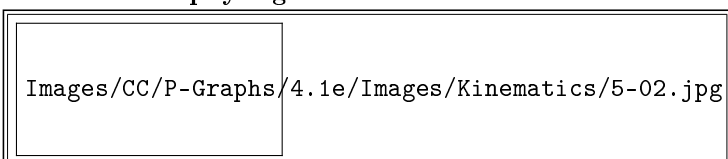


Segment A:

Segment B:

Segment C:

Example : Fig. shows the displacement-time graph of a car during parking. Describe the motion of the car and find the velocity in each time interval. www.hk-phy.org



Solution: Stage 1: The car moves forwards from the origin to $s=4$ m in the first 5 s.

$v=4/5=0.8$ m/s (forwards)

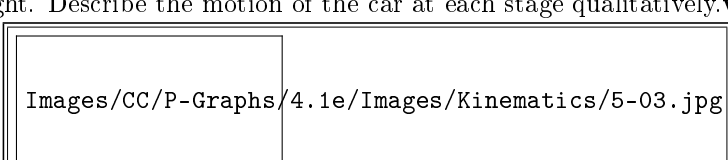
Stage 2: The car moves backwards, passes the origin, to $s=-4$ m in the next 5 s.

$v=(-4-4)/(10-5)=-1.6$ m/s (backwards)

Stage 3: The car remains at rest in the last 5 s.

$v=0$ m/s

Example : Fig. is the displacement-time graph for a car encountering a traffic light. Describe the motion of the car at each stage qualitatively. www.hk-phy.org



Solution : Stage 1: moving with constant velocity; stage 2: decelerating; stage 3: at rest; stage 4: accelerating; stage 5: moving with the same constant velocity as in stage 1.

Note: In stage 2 the slope of the curve is decreasing, while in stage 4 the slope is increasing. This indicates that the car is decelerating in stage 2 and accelerating in stage 4.

Example : An object’s position during a given time interval is shown by the graph below:



- a.) At which of the marked points is the object’s velocity the greatest? The least?
- b.) Is the object’s acceleration positive or negative between points A and B?
- c.) Suppose this curve can be modeled by the function $x(t) = t^3 - 9.5t^2 + 23t - 9$. Find the object’s velocity and acceleration at $t = 1$, $t = 3$, and $t = 5$.
- d.) Using the function from part c, determine the object’s maximum and minimum positions and velocities within the interval from $t = 1$ to $t = 6$.

Solution : a.) Remember from problem 22 that velocity is the slope of a position vs. time graph such as this. By looking at lines tangent to the curve, we can see which point has the highest and lowest slope:



Looking at the red tangent lines, we can immediately eliminate point B as a candidate for both the maximum and minimum velocity, as its tangent is horizontal and thus has a slope of 0. Point C is the only marked point whose tangent line has a negative slope, so point C has the lowest velocity. Looking at points A and D, point A’s tangent line has a steeper positive slope so point A has the highest velocity.

b.) We know that acceleration is a change in velocity, so by asking whether acceleration is positive or negative, we are asking if the velocity is increasing or decreasing. Since velocity is the slope of this graph, we must determine how the slope of the curve is changing between points A and B. Looking at the diagram in part a, we see that the slope at point A is positive, and the slope at point B is 0. As such, the slope, and thus the velocity, must be decreasing. Therefore, the object’s acceleration is negative in this interval.

c.) We know from problem 22 that velocity is the derivative of position, and from problem 23 that acceleration is the derivative of velocity. So, we will begin by differentiating the position function twice: $x(t) = t^3 - 9.5t^2 + 23t - 9$ $v(t) = 3t^2 - 19t + 23$ $a(t) = 6t - 19$

Now that we know the velocity and acceleration functions, all that is left is to plug the values of t into these functions and simplify: $v(1) = 3 * 1^2 - 19 * 1 + 23 = 3 - 19 + 23 = 7$ m/s $v(3) = 3 * 3^2 - 19 * 3 + 23 = 27 - 57 + 23 = -7$ m/s $v(5) = 3 * 5^2 - 19 * 5 + 23 = 75 - 95 + 23 = 3$ m/s

$a(1) = 6 * 1 - 19 = 6 - 19 = -13$ m/s² $a(3) = 6 * 3 - 19 = 18 - 19 = -1$ m/s² $a(5) = 6 * 5 - 19 = 30 - 19 = 11$ m/s²

d.) Thinking logically about the graph, the possible candidates for maximum and minimum position are at the end points of the interval and at the spots, like point B, where the slope of the graph is 0. So, first we set the velocity function from part c equal to 0 and solve for t: $v(t) = 3t^2 - 19t + 23 = 0$ $t = 1.63008$ s or $t = 4.70326$ s

Note that this was solved using a graphing calculator. The AP exam will not ask you to solve a quadratic this complicated by hand, however you may have to solve a simpler function using the quadratic formula. Also, we keep as many decimal places as we can at this stage in order to maintain accuracy. Now that we know all the possible times at which the position could be at a maximum or minimum within the interval, we simply plug these t values into x(t). Don’t forget to check the end points:

$$x(t) = t^3 - 9.5t^2 + 23t - 9 \quad x(1) = 5.5 \text{ m} \quad x(1.63008) = 7.58 \text{ m} \quad x(4.70326) = -6.93 \text{ m} \\ x(6) = 3 \text{ m}$$

We see that the minimum position is -6.93 m, and the maximum position is 7.58 m. Finding the maximum and minimum velocities is achieved in the same manner, except we set the acceleration function equal to 0 and plug the t values into the velocity function: $a(t) = 6t - 19 = 0 \quad 6t = 19 \quad t = 19/6 = 3.16667 \text{ s}$

$$v(t) = 3t^2 - 19t + 23 \quad v(1) = 7 \text{ m/s} \quad v(3.16667) = -7.08 \text{ m/s} \quad v(6) = 17 \text{ m/s}$$

So the minimum velocity is -7.08 m/s, and the maximum velocity is 17 m/s.

Chapter 5

Classical Approach

5.1 The Equations of motion and the origin of Graph Handling

5.1.1 The First Equation

The Equation $v = \frac{dx}{dt}$ in linear motion implies

- i) The **Slope of Position-Time Graph** is **Instantaneous Velocity**.
 - ii) The **Area under the Velocity-Time Graph** is **Change in Position**.
- { The second one requires the manipulation , $dx = vdt$ i.e. $\int dx = \int vdt$ }

The equations can be further manipulated to obtain the Speed Time Graph , where
speed = rate of change of distance wrt time

Few of the following examples illustrate this concept:

Example 1: On a displacement-time graph, two straight lines make angles of 30° and 60° with the time-axis. The ratio of the velocities represented by them is

- a) $1 : \sqrt{3}$
- b) 1:3
- c) $\sqrt{3} : 1$
- d) 3:1

{ Hint: The velocity in a displacement-time is given by the slope of the curve. Slope = tan(gent) of angle of inclination of s-t graph. This gives the respective ratios $\tan 30^\circ / \tan 60^\circ$

Answer: b) is the correct answer. }

Example 2: A body is moving in a straight line as shown in velocity-time graph. The displacement and distance travelled by body in 8 second are respectively:

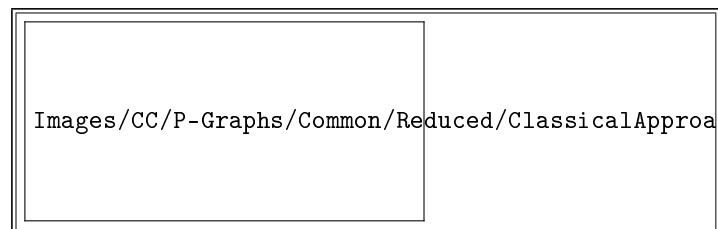


- a) 12 m, 20 m
- b) 20 m, 12 m
- c) 12 m, 12 m
- d) 20 m, 20 m

{ Hint: The displacement in a velocity-time graph is given by the area under the graph with proper signs. From 0s - to 2s , the area is 8m . From 2s - to 4s , the area is -4m . From 4s - to 8s , the area is 8m. Adding these 3 values , we get $8m + (-4m) + 8m = 12m$. } The distance in a v-t graph is given by the absolute area under the graph. So, taking the absolute values of individual area divisions, we get $8m + 4m + 8m = 20m$

Answer: a) is the correct answer. }

Example 3: The graph shows position as a function of time for two trains running on parallel tracks. Which statement is true?

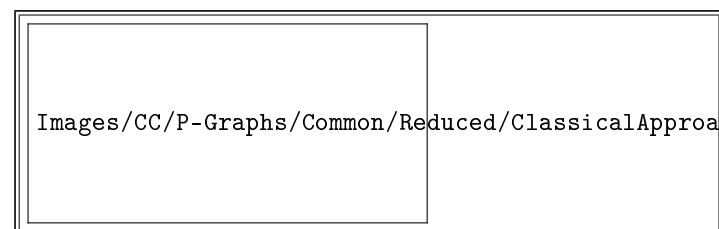


- a) At time t_B both trains have the same velocity.
- b) Both trains have the same velocity at some time after t_B .
- c) Both trains have the same velocity at some time before t_B .
- d) Somewhere on the graph, both trains have the same acceleration.

{ Hint: Depending on the question requirements, we'll have to check all the assertions one by one.

a) In a position time graph, the slope gives velocity. It can be clearly seen that Graph B has a much lower slope than Graph A at time t_B . So, the assertion is wrong.

b,c) By drawing a line parallel to the line A which is a tangent to Graph B , it can be seen where the two graphs have same slope. It is clear that the graphs have same slope between 0 and t_B as noted from the figure. So, assertion b is wrong while c is correct.

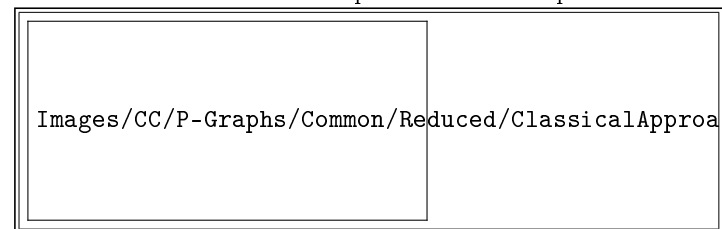


d) As the Graph A has a constant slope, so the acceleration of body A is zero. Whereas Graph B is constantly turning, so the slope can be assumed to be non-zero throughout. According to some revelations, however it is noted that the figure is not clear enough to show whether Graph B is straight after t_B or bending. In case it is assumed to be straight, then after t_B both trains will have same (zero) acceleration. Also at start both have large (infinite) acceleration, in which case the ratio of the two large (infinite) values may be calculated if initial conditions are mentioned and is required.

At our level we would assume this assertion to be wrong, however making a note that the image should have been more clearly presented.

Answer: c) is the correct assertion. }

Example 4: The velocity-time graph of a particle in linear motion is as shown. Both v and t are in SI units. The displacement of the particle is



- a) 6 m
- b) 8 m
- c) 16 m
- d) 18 m

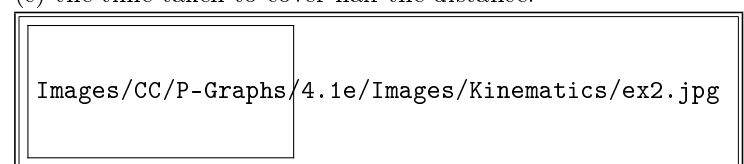
{ Hint : For displacement calculations, between 0 - to 4 , area of the positive trapezium = $\frac{1}{2} \times (2 + 4) \times 4 = 12$

between 4 - to 8, area of negative trapezium = $\frac{1}{2} \times (2 + 4) \times (-2) = -6$.

So, the answer is +6
So , a is the correct option. }

Example 5 : A speedboat starts from rest, accelerating at 2 ms^{-2} for 20 s. It then continues at a steady speed for a further 30 s and decelerates to rest in 30 s. Find:

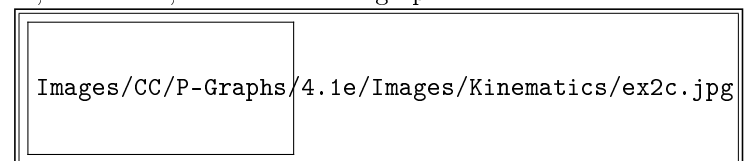
- (a) the distance travelled in m,
- (b) the average speed in ms^{-1} and,
- (c) the time taken to cover half the distance.



Solution : a) distance=area of trapezium = $\frac{(a+b)h}{2}$
= $\frac{(80+30)(40)}{2}$
= 2200 m

b) average speed = $\frac{\text{distance travelled}}{\text{time taken}}$
= $\frac{2200}{80}$
= 27.5 ms^{-1}

c) We need to find the time when the area of the trapezium is half of its original area, or 1100m , as shown in the graph.



The base of this unknown trapezium has length t , and the top of the trapezium will have length $t-20$. So we have:

area of trapezium = $\frac{(a+b)h}{2}$
 $1100 = \frac{(t + [t - 20]) 40}{2} = 20(2t - 20)$

$55 = 2t - 20$

$75 = 2t$

$t = 37.5 \text{ s}$

So it will take 37.5 s to cover half the distance.

5.1.2 The Second Equation

Proceeding similar to above, the equation $a = \frac{dv}{dt}$ implies

- i) The **Slope of Velocity-Time Graph** is **Instantaneous Acceleration**.
 - ii) The **Area under Acceleration-Time Graph** is **Change in Velocity**.
- { The second one requires the manipulation , $dv = a dt$ i.e. $\int dv = \int a dt$ }

A few of the following examples illustrate it.

Example 1: A car starts from rest acquires a velocity v with uniform acceleration $2ms^{-2}$ then it comes to stop with uniform retardation $4ms^{-2}$. If the total time for which it remains in motion is 3 sec, the total distance travelled is:

- a) 2 m
- b) 3 m
- c) 4 m
- d) 6 m

{Hint: For solving this problem, we draw the graph of the problem,

Images/CC/P-Graphs/4.1e/Images/KinematicsClassicalApproach/ImagesReduced/Q1-secondequation-Answers.jpg

According to graph, let the time when it reaches maximum velocity be T , and the maximum velocity be V .

$$\Rightarrow V = 2XT \text{ and also } V = 4X(3-T)$$

Equating the equations,

$$2T = 12-4T = V$$

$$\Rightarrow 6T = 12$$

$$\Rightarrow T = 2$$

$$\Rightarrow V = 2T = 4$$

Calculating the area under the graph using the calculated parameters, Area = $\frac{1}{2} \times 4 \times 3 = 6m$

So, area under the graph is $6m = \text{displacement}$. Also, as all the area is on the positive side, so distance = $6m$.

}

Example 2: A particle starts from rest. Its acceleration (a) vs time (t) is as shown in the Figure. The maximum speed of the particle will be

Images/CC/P-Graphs/Common/Reduced/ClassicalApproach/accelerationvstime.jpg

- a) 110 m/s
- b) 55 m/s
- c) 550 m/s
- d) 660 m/s

{ Hint : Writing the equation of the graph, we get $\frac{a}{10} + \frac{t}{11} = 1$

$$\Rightarrow a = \frac{10}{11}(11 - t)$$

Integrating, (we will assume initial velocity to be zero as the body starts from rest.)

$$v = \frac{10}{11}(11t - \frac{1}{2}t^2)$$

Substituting $t = 11s$

$$v_{11s} = 55m/s$$

Answer: b) is the correct answer }

Example 3: Acceleration-time graph of a particle moving in a straight line is shown in Figure. The velocity of particle at time $t = 0$ is 2 m/s. Velocity at the end of fourth second is

Images/CC/P-Graphs/Common/Original/ClassicalApproach/acc-t-graph-3.jpg

- a) 8 m/s
- b) 10 m/s
- c) 12 m/s
- d) 14 m/s

{ Hint: Area under the acceleration-time graph is change in velocity. Area of the triangle is Half (into) base (into) altitude = $8m/s$. Adding the initial value of $2m/s$, we get $2 + 8 = 10m/s$.

Answer: b) is the correct answer }

Example 4: Each of the three graphs represents acceleration vs time for an object that already has a positive velocity at time t_1 . Which graph/graphs show an object whose speed is increasing for the entire time interval between t_1 and t_2 ?

Images/CC/P-Graphs/Common/Original/ClassicalApproach/ncertproblem.jpg

- a) Graph I only
- b) Graphs I and II
- c) Graphs I and III
- d) Graphs I, II and III

{Hint: Area under the acceleration time graphs give change in velocity. In all the three figures, the area under the graphs are +ve, hence velocity is increasing in all cases. As the initial velocity is +ve, in all three cases the velocity remains throughout positive. So, the speed is also increasing in all cases.}

Example 5: A body starts from rest and moves along a straight line with constant acceleration. The variation of speed v with distance s is given by the graph

Images/CC/P-Graphs/Common/Original/ClassicalApproach/Untitled2.jpg

{ Hint: The problem given has $v_o = 0$

Now acceleration = constant (lets say k) = $\frac{dv}{dt}$

$$\Rightarrow k = v \frac{dv}{ds}$$

$$\Rightarrow \int_0^v v dv = \int_0^s k ds$$

$$\Rightarrow v = \sqrt{2ks}$$

The graph is proportional to square root function

Hence, b (as it is the only graph with such a property)

Answer : b) is the correct graph }

5.1.3 The Acceleration-Position Graph Variate

This kind of graph requires the manipulation of the Equation $a = \frac{dv}{dt}$ as follows

$$a = \frac{dv}{dx} \cdot \frac{dx}{dt}$$

$$\Rightarrow a = \frac{dv}{dx} \cdot v$$

$\Rightarrow adx = vdv$ and integration can be performed to further solve it.

Example 1: A body, initially at rest, starts moving along x-axis in such a way that its acceleration vs displacement plot is as shown in the Figure. The maximum velocity of the particle is

Images/CC/P-Graphs/Common/Original/ClassicalApproach/accelerationposition1.jpg

- a) 1 m/s
- b) 6 m/s
- c) 2 m/s
- d) None of these

{ Hint : The area under the a-x graph gives change in $\frac{v^2}{2}$ as can be evaluated by Integration method. The area of triangle is $(0.5) \times (1) \times (1) = 0.5$
 $\Rightarrow v = 1m/s$. Initial position is given to be 0 in the graph. Hence, we take only the positive sign.

Answer : a) is the correct answer. }

Example 2 : A particle initially at rest, it is subjected to a non-uniform acceleration a , as shown in the gure. The maximum speed attained by the particle is

Images/CC/P-Graphs/Common/Original/ClassicalApproach/acclnvdisplacement.jpg

- a) 605 m/s
- b) 110 m/s
- c) 55 m/s
- d) 110 m/s

{ Hint: The answer is $55m/s$ as calculated in the example above(In the second equation section). Hence, c) is the correct response.

Answer : c) is the correct answer. }

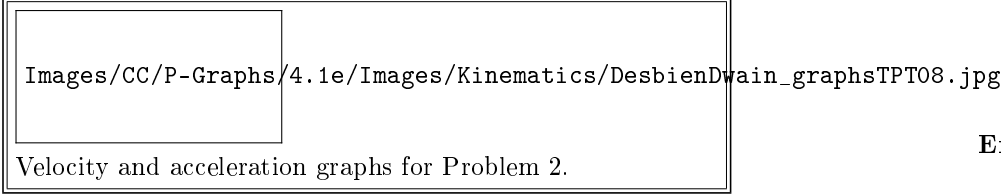
5.1.4 Sign of Acceleration from Position-Time Graph

The sign of Acceleration can be determined from the Position-Time Graph. The methodology involves of looking at the Concavity of the Graph

- i) If the graph is Concave-Up, the Acceleration is Positive.
- ii) If the graph is Concave-Down, the Acceleration is Negative.
- iii) If the graph is a straight line, the Acceleration is ZERO. { Irrespective of any other factor, such as the slope or direction of line}

Example 1: The graph given below is a plot of distance vs time. For which labelled region is the “Velocity Positive and the Acceleration Negative”

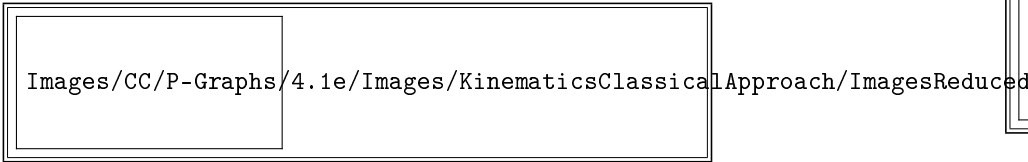
the base (time) for area 1 is five times the base (time) for area 2. Therefore, area 1 must be five times larger than area 2. This means that area 2 must be 1/6 the total height of the building, or 92.2 m.



After studying this solution for a long time and finding no flaws in the physics, I pulled out my physics book and tried solving other one-dimensional kinematics problems in a similar manner. It quickly became obvious that they could be done using graphs, and in most cases this is the easier method. I was hooked and began teaching one-dimensional problem solving this way in all my classes. The next class period I asked the student whose solution this was why he chose to do the problem this way. He explained that we had emphasized graphs so much in class that they must be more useful than something we are simply supposed to sketch for the problems. I was amazed and appreciative as I now had a new way to teach problem solving. This kind of solution is more visual and helps get students away from hunting for the “correct equation.” It has worked well for me at all levels of introductory physics, conceptual through calculus based. If you find other interesting problems that are especially suited to the above method, I would appreciate your sharing them.

5.1.8 Projectile Motion

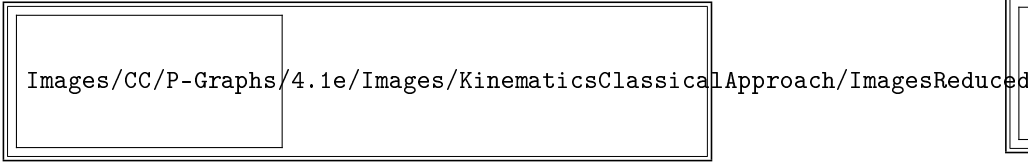
Example 1: A shell is fired from a gun at an angle to the horizontal. Graphs are drawn for its horizontal component of velocity v_x and its vertical component of velocity v_y .



{ Hint: v_x would remain constant as $u\cos\theta$ and v_y would linearly decrease and go negative after maximum height is attained(if the fire angle is +ve).
So, a) is the requisite graph. }

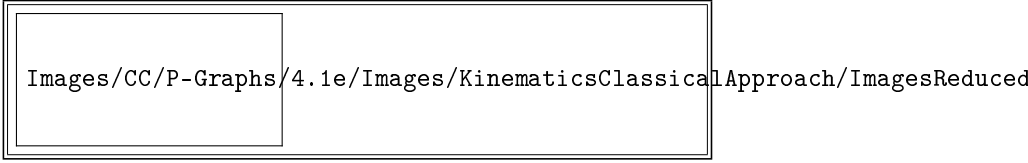
5.1.9 Miscellaneous

Example 1: Figure shows the displacement-time (x-t) graph of body moving in a straight line. Which one of the graphs shown in Fig. represents the velocity-time (v-t) graph of the motion of the body.



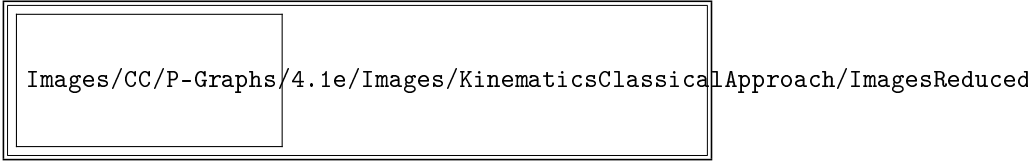
{ Hint: The graph has initially (0-5) a +ve slope, then zero slope and then (15-20) an equal -ve slope
So, d) is the correct answer. }

Example 2: Which of the displacement-time (x–t) graphs shown in Fig. can possibly represent one dimensional motion of a particle?



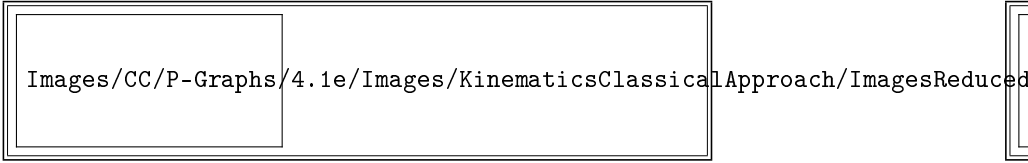
{ Hint: The object cannot be at two positions on a single time instant,
So, d) is the correct option. }

Example 3: A car starts from rest, accelerates uniformly for 4 seconds and then moves with uniform velocity. Which of the (x-t) graphs shown in Fig. represents the motion of the car upto t = 7 s?



{ Hint: As the car is accelerating upto 4 seconds, the graph should be concave up. Further ahead , it moves with constant velocity , so a positive slope ahead of 4 seconds.
d) is the correct answer. }

Example 4: Two stones are thrown up simultaneously with initial speeds of u_1 and u_2 , ($u_2 > u_1$) . They hit the ground after 6 s and 10 s respectively. Which graph in Fig. correctly represents the time variation of $\Delta x = x_2 - x_1$, the relative position of the second stone with respect to the first upto t = 10 s? Assume that the stones do not rebound after hitting the ground.



{ Hint : While both are in air, the differece of their velocity vectors would be a constant vector , so $\Delta x = c_1 t$. Also, after the first stone hits ground, $\Delta x = c_2 t - x_o$ as the velocity component in x direction of second stone is constant. There is no discontinuity. So, both before and after one hits the ground , it would be a straight line.

Hence, d) is the correct answer. }

Example 5: Figure shows the velocity-time (v - t) graphs for one dimensional motion. But only some of these can be realized in practice. These are

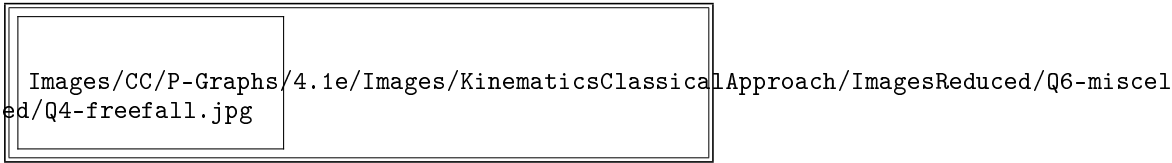


- a) (i), (ii) and (iv) only
- b) (i), (ii) and (iii) only
- c) (ii) and (iv) only
- d) all

{ Hint: At one particular instant, the object cannot have two different velocities.

Hence, c) is the correct answer. }

Example 6: Which of the velocity-time (v-t) graphs shown in Fig. can possibly represent one-dimensional motion of a particle?

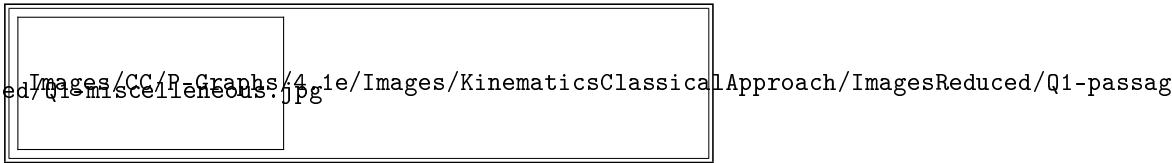


{Hint: In c) , the particle is constantly increasing in one dimension, In d) , it stops near the end.
c),d) are the correct answers. }

5.2 Question Types

5.2.1 Passage Type

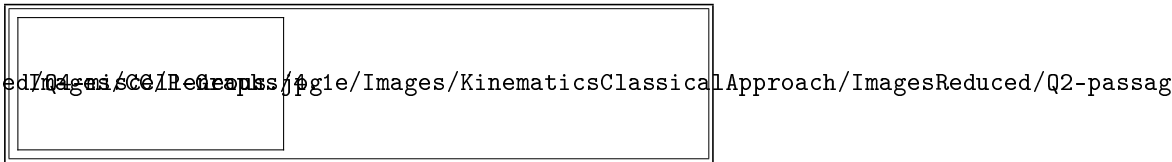
Example: The speed-time graph of the motion of a body is shown in Fig.



1. The accelerations of the body during the last 2 second is
 - a) $\frac{20}{3}ms^{-2}$
 - b) $-\frac{20}{7}ms^{-2}$
 - c) $-10ms^{-2}$
 - d) Zero
2. The ratio of distance travelled by the body during the last 2 seconds to the total distance travelled by it is
 - a) 1/9
 - b) 2/9
 - c) 3/9
 - d) 4/9
3. The average speed of the car during the whole journey is
 - a) 10 m/s
 - b) 20 m/s
 - c) 90/7 m/s
 - d) 40/7 m/s

{Hint: 1. As no contradictory statements are present, we would take velocity as the value of speed only. Actually velocity is required for calculation of acceleration, speed doesn’t contradict anything so we’ll use it’s value for velocity.
-10 from slope, c)
2. Speed time graph’s area is distance. From area calculation, in last 2 seconds, the area is 20 and total area is 90.
So, 2/9 is the required ration b)
3. Total distance from area is 90, and total time is 7.
So, 90/7. c)}

Example: A body starts from rest at time t = 0 and undergoes an acceleration as shown in Fig.Which of the graphs shown in Fig. represents the velocity-time (v-t) graph of the motion of the body from t = 0 s to t = 4 s?



Images/CC/P-Graphs/4.1e/Images/KinematicsClassicalApproach/ImagesReduced/Q2-passagetype_im2.jpg

{ Hint: d) is the v-t graph by calculating area function. }

- In Question above, what is the velocity of the body at time $t = 2.5$ s?
 - 2.5 m/s
 - 3.5 m/s
 - 4.5 m/s
 - 5.5 m/s
- In above question, how much distance does the body cover from $t = 0$ s to $t = 4$ s?
 - 6 m
 - 9 m
 - 12 m
 - 15 m
- In above question, which of the graphs shown in Fig. represents the displacement-time (x-t) graph of the motion of the body from $t = 0$ s to $t = 4$ s?

Images/CC/P-Graphs/4.1e/Images/KinematicsClassicalApproach/ImagesReduced/Q2-passagetype-Answers.jpg

{ Hint: 1. The area under the accln-time graph gives change in velocity. Till 2.5s, $6-1.5=4.5$ is the required area.

So, c)

2. Area under the v-t graph d) is 12 till 4 seconds.

So, c)

3. Till 2 seconds, the graph should be concave-up while from 2 to 4 it should be concave-down.

So, b)

}

5.2.2 Matching

- Match the graphs (a), (b), (c) and (d) shown in Fig. with the types of motions (p), (q), (r) and (s) that they represent

Images/CC/P-Graphs/4.1e/Images/KinematicsClassicalApproach/ImagesReduced/Q4-matchingtype.jpg

- Motion with non-uniform acceleration
- Motion of a body covering equal distances in equal intervals of time
- Motion having a constant retardation
- Uniformly accelerated motion.

{ Hint: p->d, q->a, r ->c, s->a,b,c }

- Figure shows the displacement -time (x - t) graph of the m-tion of a body.

Images/CC/P-Graphs/4.1e/Images/KinematicsClassicalApproach/ImagesReduced/Q2-matchingtype.jpg

{ Hint p->c, q->a,c, r-> d, s->b }

5.3 Previous Years IIT Problems

- Q1:** A particle starts from rest. Its acceleration (a) versus time (t) is as shown in the gure. The maximum speed of the particle will be

Images/CC/P-Graphs/Common/Reduced/ClassicalApproach/accelerationvstime.jpg

- 110 m /s
- 55 m /s
- 550 m /s
- 660 m /s

{ Hint : See In chapter examples for solution. }

- Q2:** If graph of velocity vs. distance is as shown, which of the following graphs correctly represents the variation of acceleration with displacement ?

Images/CC/P-Graphs/Common/Reduced/ClassicalApproach/velocitydisplacement.jpg

Images/CC/P-Graphs/4.1e/Images/KinematicsClassicalApproach/ImagesReduced/Q2-passagetype-Answers.jpg

{ Hint : The graph of the question is a straight line with the equation, $\frac{x}{x_o} + \frac{v}{v_o} = 1$

This gives, $v = v_o(1 - x/x_o)$

So, diffrentiating it, we get

$a = -v_o/x_o$ which is a constant. Only in a) it is shown to be a constant.

So, a) }

Passage A lift is going up. The variation in the speed of the lift is as given in the graph.

Images/CC/P-Graphs/4.1e/Images/KinematicsClassicalApproach/ImagesReduced/Q3-passagetype-Answers.jpg

- Q3:** What is the height to which the lift takes the passengers ?

- 3.6 m
- 28.8 m
- 36 m
- cannot be calculated from the above graph

- Q4:** In the above graph, what is the average velocity of the lift?

- 1 m/s
- 2.88 m/s
- 3.24 m/s
- 3 m/s

- Q5:** In the above graph, what is the average acceleration of the lift?

- $1.8m/s^2$
- $-1.8m/s^2$
- $0.3m/s^2$

{Hint: Q3: Area under the graph is 36. Taking initial position as zero, c) is the best fit answer.

Q4. For av. velocity, total displacement / total time should be calculated. which is $36/12 = 3m/s$. d) is the required answer.

Q5. The accln is 1.8 from 0s to 2s, 0 from 2s to 10s and -1.8 from 10s to 12s. So, area under the a-t graph is zero. So av. accln is zero d)

}

- Q6:** Four persons K, L, M and N are initially at the corners of a square of side of length d. If every person starts moving with velocity v such that K is always headed towards L, L towards M, M towards N and N towards K, then the four persons will meet after

- d^2 / v
- $d / 2v$
- $d / 2v$
- $d / 2v$

{Hint: Not a graphs question, we'll discuss it in kinematics book. Although an easy one. It was mistakenly added to graphs book. Let's use it to signify the fact that even if a diagram is made, it is still not a graph.}

Example. A ball is dropped vertically from a height h above the ground. It hits the ground and bounces up vertically to a height h/2. Neglecting subsequent motion and air resistance, its velocity v varies with the height h as (see Fig.) (I.I.T. 2000)

Images/CC/P-Graphs/4.1e/Images/KinematicsClassicalApproach/ImagesReduced/e

{ Hint : Case 1: Thrown from height h with zero initial velocity, $v = -gt$ and $h = h_o - 1/2gt^2$, so eliminating t, we get $h = h_o - v^2/2g$. So, $v = -\sqrt{2g(h_o - h)}$

Case 2: We assume the opposite for solving purpose that the ball is now thrown from a height $h_o/2$ and replace h_o only and see the effect. Taking upward velocity as positive, we get $v = \sqrt{2g(h_o/2 - h)}$

a) is the required answer. }

Chapter 6

Exercises

6.1 Review Exercise I

Question 1. A spark timer/air table produced the tape pictured below.

Images/CC/P-Graphs/4.1e/Images/Kinematics/22.jpg

The object, moving to the right, was

- a. moving with uniform motion
- b. speeding up
- c. slowing down
- d. travelling with constant speed

{ Hint: c }

Question 2. Study the position-time graph pictured below and select the statement that is true.

Images/CC/P-Graphs/Common/Reduced/reviewkinematics/positionvstime.jpg

- a. The object accelerates, stops, then accelerates in the opposite direction.
- b. The object's speed is greatest during the first segment.
- c. The object's acceleration is greatest during the last segment.
- d. The object's average velocity is zero.
- e. The object travels a greater distance in the first segment than in the last segment.

{ Hint : D }

Question 3. The position-time graph pictured below represents the motions of two objects, A and B. Which of the following statements concerning the objects' motions is true?

Images/CC/P-Graphs/Common/Reduced/reviewkinematics/32.jpg

- a. Object B travels the greater distance.
- b. Object A has the greater speed.
- c. Object A leaves the reference point at an earlier time.
- d. Both objects have the same speed at the point where the lines cross.
- e. Object A is travelling for a longer period of time.

{ Hint : B }

Question 4. The position-time graph pictured below represents a race between three contestants A, B, and C. The race begins at time zero at the sound of the starter's pistol. Which of the following statements is true?

Images/CC/P-Graphs/4.1e/Images/Kinematics/33.jpg

- a. The runner who started last finished first.
- b. The fastest runner won the race.
- c. The runner with a head start won the race.
- d. Only one runner began at the sound of the starter's pistol.
- e. All runners ran the same distance.

{ Hint : C }

Question 5. The position-time graph pictured below depicts a person, P, running to catch a bus, B, that has just begun to pull away. Which of the following statements is true?

Images/CC/P-Graphs/4.1e/Images/Kinematics/34.jpg

- a. The person has no chance of catching the bus.
- b. The person's acceleration is greater than that of the bus.
- c. The person has two opportunities to catch up to the bus.
- d. The speed of the bus is always greater than that of the person.
- e. The person's speed is always greater than that of the bus.

{ Hint : C }

Question 6. The position-time graph that depicts a ball thrown vertically upward that returns to the same position is

Images/CC/P-Graphs/Common/Reduced/reviewkinematics/ImagesReduced/35.jpg

- a. A
- b. B
- c. C
- d. D

e. E

{ Hint : B }

Question 7. The position-time graph that represents "uniform motion" is

Images/CC/P-Graphs/Common/Reduced/reviewkinematics/ImagesReduced/36.jpg

- a. A
- b. B
- c. C
- d. D
- e. E

{ Hint : A }

Question 8. The velocity-time graph that represents "uniform motion" is

Images/CC/P-Graphs/Common/Reduced/reviewkinematics/ImagesReduced/37.jpg

- a. A
- b. B
- c. C
- d. D
- e. E

{ Hint : E }

Question 9. The velocity-time graph pictured below depicts the motion of a motorcycle. Which of the following statements is true?

Images/CC/P-Graphs/4.1e/Images/Kinematics/38.jpg

- a. The motorcycle is always experiencing an acceleration.
- b. The motorcycle's greatest speed occurs toward the end of the recorded time interval.
- c. The motorcycle's average acceleration is zero.
- d. The motorcycle eventually reaches uniform motion.
- e. The motorcycle accelerates until it reaches a constant speed.

{ Hint : A }

Question 10. The velocity-time graph pictured below represents the motion of a police car, P, in pursuit of a motorcycle, M. The motorcycle has just passed the police car. Which of the following statements is true?

Images/CC/P-Graphs/4.1e/Images/Kinematics/39.jpg

- a. Both vehicles are at rest when the pursuit begins.
- b. The police car eventually catches the motorcycle.
- c. The motorcycle accelerates and then slows down.
- d. At the end of the recorded time interval, the police car has yet to catch the motorcycle.
- e. The police car passes the motorcycle.

{ Hint : D }

Question 11. Consider the following velocity-time graph and select the statement that is true.

Images/CC/P-Graphs/Common/Reduced/reviewkinematics/velocityvstime.jpg

- a. At no time can the motion be considered "uniform."
- b. The object returns to its original position.
- c. The object travels in one direction and then the other.
- d. The object is accelerating throughout the entire recorded time.
- e. The object speeds up and later slows down.

{ Hint : E }

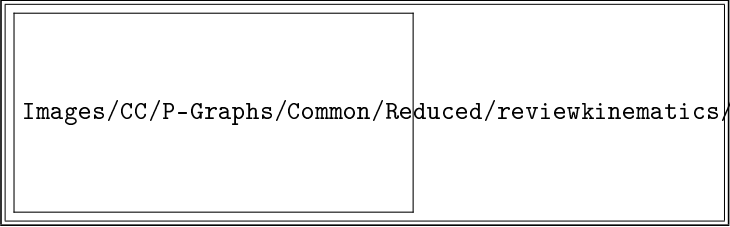
____ 41. Which of the following velocity-time graphs represents the motion of a ball thrown vertically upward?

Images/CC/P-Graphs/Common/Reduced/reviewkinematics/ImagesReduced/41.jpg

- a. A
- b. B
- c. C
- d. D
- e. E

{ Hint : B }

_____ 42. The following velocity-time graph depicts the motions of two objects, A and B. Which of the statements describing the graph is true?



- a. Both objects are accelerating uniformly.
- b. The two objects are travelling in opposite directions.
- c. Both objects start from rest.
- d. Object A travels farther than object B.
- e. Object B travels farther than object A.

{ Hint : A }

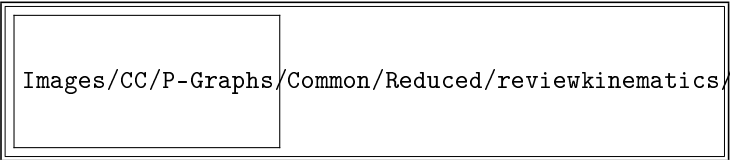
_____ 43. Which statement describes the motion represented by the following acceleration-time graph?



- a. The object is moving with uniform motion.
- b. The object has a constant velocity.
- c. The object has a uniform acceleration.
- d. The object is stopped.
- e. The object has a changing acceleration.

{ Hint : C }

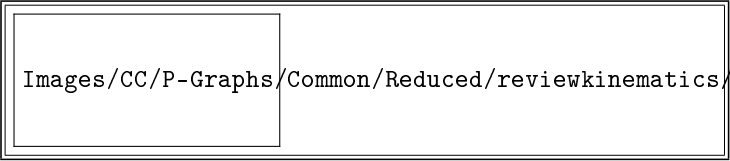
_____ 44. A ball is thrown vertically upward into the air. Which of the following acceleration-time graphs represents the ball’s motion?



- a. A
- b. B
- c. C
- d. D
- e. E

{ Hint : D }

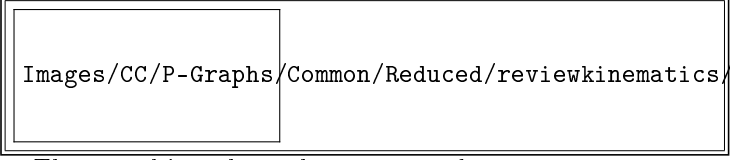
_____ 45. Four of the five graphs pictured below could all represent the same motion. Which graph does not belong to this group?



- a. A
- b. B
- c. C
- d. D
- e. E

{ Hint : C }

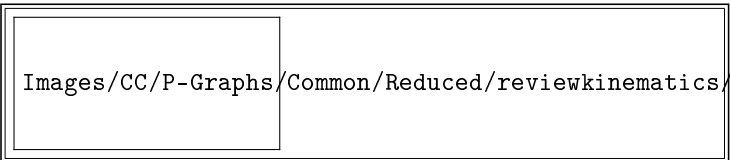
_____ 49. The position-time graph below depicts the motions of two objects, A and B. Which of the following statements concerning the objects’ motions is NOT true?



- a. The two objects have the same speed.
- b. The two objects travel the same distance.
- c. The two objects travel with uniform motion.
- d. The two objects travel for the same amount of time.
- e. The two objects have the same velocity.

{ Hint : E }

_____ 50. What type of motion is depicted by the following acceleration-time graph?



- a. constant velocity
- b. non-uniformly changing acceleration
- c. constant acceleration
- d. uniformly changing acceleration
- e. uniform motion

{ Hint: D }

_____ 51. The diagram below shows the first three legs of a trip: A to B, B to C, and C to D. If a person returns from point D to point A, what is the displacement for this fourth and final leg?

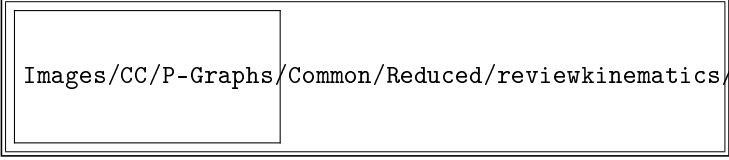


- a. 7 km [37° W of N]
- b. 5 km [37° W of N]
- c. 5 km [37° E of S]
- d. 7 km [37° E of S]
- e. 5 km [37° N of E]

{ Hint : B }

6.2 Review Exercise II

1. A cart travels with a constant nonzero acceleration along a straight line. Which graph best represents the relationship between the distance the cart travels and time of travel?



{ Hint: First of all the graph should be a parabola. Secondly, it can’t be (3) as distance can only increase. }

Base your answers to questions 2 through 4 on the information below.

A car on a straight road starts from rest and accelerates at 1.0 meter per second² for 10 seconds. Then the car continues to travel at constant speed for an additional 20 seconds.

2. Determine the speed of the car at the end of the first 10 seconds.

{ Hint : v=at, so v at 10 s = 10 m/s }

3. On the grid at below, use a ruler or straightedge to construct a graph of the car’s speed as a function of time for the entire 30-second interval.



{ Hint: Try out yourself. }

4. Calculate the distance the car travels in the first 10 seconds. [Show all work, including the equation and substitution with units.]

{ Hint: s=1/2at² = 50m }

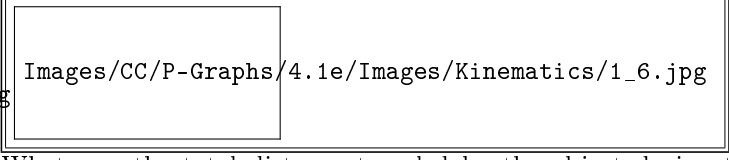
5. A student throws a baseball vertically upward and then catches it. If vertically upward is considered to be the positive direction, which graph best represents the relationship between velocity and time for the baseball? [Neglect friction.]



{ Hint : v=u-gt, so velocity is continually decreasing with 9.81 m/s and even turns negative, equal to the initial value in magnitude on return and opposite direction if air drag is neglected. }

D) is the correct option. }

6. The graph below represents the displacement of an object moving in a straight line as a function of time.



What was the total distance traveled by the object during the 10-second time interval?

- 1. 0 m
- 2. 8 m
- 3. 16 m
- 4. 24 m

{ Hint: The object moves 16m forward and 8m backward. So, distance is 24 m. }

4) is the correct option. }

7. Which graph best represents the relationship between the acceleration of an object falling freely near the surface of Earth and the time that it falls?



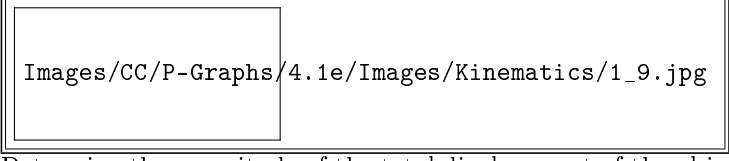
{ Hint : Acceleration due to gravity near the earth surface is constant, being 9.81 m/s² }

8. Which pair of graphs represent the same motion of an object?



{ Hint : A }

9. The graph below represents the velocity of an object traveling in a straight line as a function of time.



Determine the magnitude of the total displacement of the object at the end of the first 6 seconds.


{Hint 50m, area under the graph till 6s.: }

10. Which graph best represents the motion of a block accelerating uniformly down an inclined plane?



{Hint: (4) as the graph would be a function of t², calculated via the usual equations of motion for uniformly accelerated motion. }

Base your answers to questions 11 and 12 on the graph below, which represents the motion of a car during a 6-second time interval.



Images/CC/P-Graphs/4.1e/Images/Kinematics/1_11plus12.jpg

11. What is the acceleration of the car at $t=5.0$ seconds?

1. 0.0 m/s^2
2. 2.0 m/s^2
3. 2.5 m/s^2
4. 10 m/s^2


{Hint : It is the slope of v-t graph, at $t=5.0$, the graph has zero slope. So, (1)}

12. What is the total distance traveled by the car during this 6-second interval?

1. 10 m
2. 20 m
3. 40 m
4. 60 m

{ Hint : Area under the v-t curve, 40m, So, (3)}


13. Which graph best represents the relationship between the velocity of an object thrown straight upward from Earth's surface and the time that elapses while it is in the air? [Neglect friction.]



Images/CC/P-Graphs/4.1e/Images/Kinematics/1_13.jpg

{ Hint : (4) }

14. The graph below shows the relationship between the speed and elapsed time for an object falling freely from rest near the surface of a planet.




Images/CC/P-Graphs/4.1e/Images/Kinematics/1_14.jpg

What is the total distance the object falls during the first 3 seconds?

1. 12 m
2. 24 m
3. 44 m
4. 72 m

{ Hint : Area under the graph between 0 and 3, 12m }

15. The graph below represents the relationship between speed and time for an object moving along a straight line.




Images/CC/P-Graphs/4.1e/Images/Kinematics/1_15.jpg

What is the total distance traveled by the object during the first 4 seconds?

1. 5 m
2. 20 m
3. 40 m
4. 80 m

{ Hint : 40m , So, 3 }

Base your answers to questions 16 and 17 on the graph below, which shows the relationship between speed and elapsed time for a car moving in a straight line.



Images/CC/P-Graphs/4.1e/Images/Kinematics/1_16plus17.jpg


16. Determine the magnitude of the acceleration of the car.

{ Hint : Slope of the line , 1.25 m/s^2 }

17. Calculate the total distance the car traveled during the time interval 4.0 seconds to 8.0 seconds. [Show all work, including the equation and substitution with units.]

{ Hint : It's the area of trapezium with parallel sides , 5 and 10 and distance between them as 4 , = 30m }

Base your answers to questions 18 through 20 on the graph below, which represents the relationship between velocity and time for a car moving along a straight line, and your knowledge of physics.



Images/CC/P-Graphs/4.1e/Images/Kinematics/1_18to20.jpg

18. Determine the magnitude of the average velocity of the car from $t=6.0$ seconds to $t=10.0$ seconds.

{ Hint : Velocity is constant between 6 and 10 , ie 15 m/s }

19. Determine the magnitude of the car's acceleration during the first 6.0 seconds.

{ Hint : Slope of the graph is 1.5 m/s^2 }

20. Identify the physical quantity represented by the shaded area on the graph.

{ Hint : Displacement of the car from initial position by $t=10.0 \text{ s}$, calculated to be 105m }

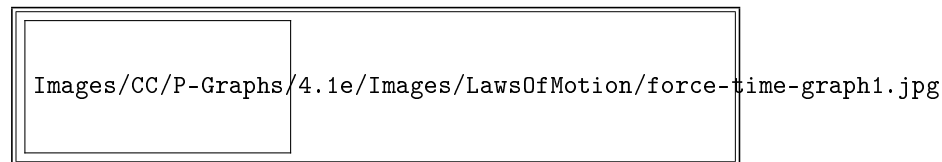
Part III

Laws of Motion

Chapter 7

Abstract Introduction

7.1 Force – Time Graphs



The area under a force – time graph gives us the impulse of the force applied (and hence the change in momentum of the object). For this graph the impulse (the area under the graph) is 2000 kg ms⁻¹. physicsnet.co.uk

7.2 Change in Momentum or the "Impulse"

The change in the momentum of a system (or the impulse delivered by the net force) is given mathematically by the Momentum Principle,

$$\Delta \vec{p} = \vec{F}_{net} \Delta t$$

In this form, the change in momentum is calculated over a “discrete” time step. That is, the calculation is done over a known or determined time interval. If the force is non-constant (i.e., depends on location or velocity), this calculation is not exact. In fact, in this case, the net force is the average net force over the time interval. So that a better definition is this:

$$\Delta \vec{p} = \vec{F}_{net,avg} \Delta t$$

This definition works well for case where you might use iterative procedures to determine the change in momentum over small time intervals. If on the other hand, you can analytically integrate the force (e.g., it is or can be put into a form which is time dependent), then you can use the derivative form of the Momentum Principle,

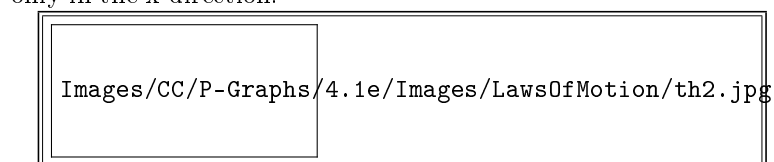
$$\Delta \vec{p} = \int_{t_i}^{t_f} \vec{F}_{net} dt$$

In any event, either (or both) can be useful to think about graphs of force vs time.

7.2.1 Force vs Time Graphs

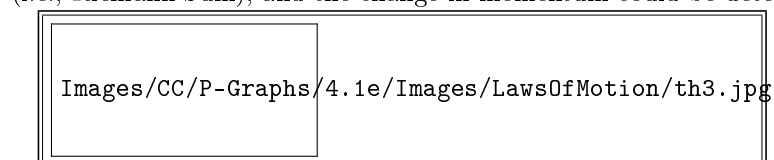
In some situations, it is easier to empirically measure force versus time graphs because the situations lend themselves more easily to these empirical measurements rather than what might be more complex physical theories. This is true in different engineering contexts (e.g., impact design and the flow of fluids). In these cases, you are interested in determining the change in momentum (and thus the velocity) of the system in question1).

Below is a force vs time graph where the “area under the curve” has been highlighted. In this example, we are only looking at the component of the net force in the xx-direction. Such graphs can be produced for each component of the net force, but let’s say that for this system, there was a non-zero component of the net force only in the x-direction.



For the above figure, the momentum change over the complete time interval can be determined in a straightforward way due to the simple geometric shapes produced. Area above the zero line are positive momentum changes, and area below are negative. By adding up the “area under the curve” in this way, we obtain a momentum change of 7 Ns.

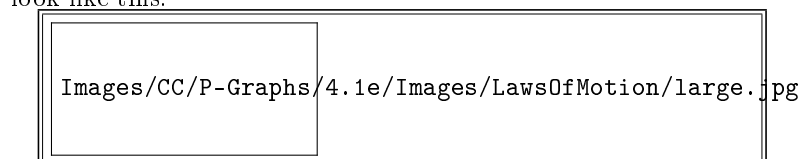
The figure below shows the force vs time graph for another system. In this case, the graph has a smooth form, which doesn’t appear to be analytic. The “area under the curve” for this graph could be analyzed computationally, by taking small steps (i.e., Riemann Sum), and the change in momentum could be determined.



1) It is possible to determine the displacement of such systems as well. This can be done using velocity vs time graphs that are produced from the analysis of force vs time graphs.

7.3 Impulse graphs (A Case Study) bbc.co.uk

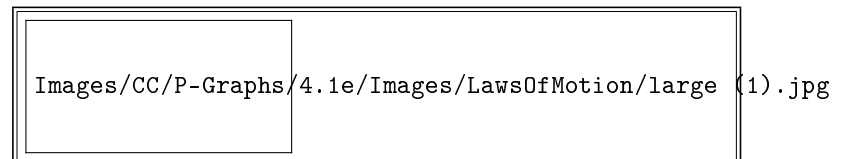
The force on the squash ball in the previous question is an average force and often the force changes during the collision. For this example the force–time graph could look like this.



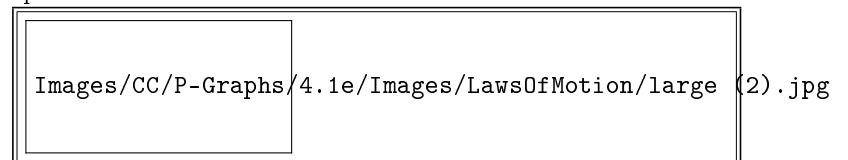
Notice the peak force is greater than the average force calculated.

The area under a force time graph is equal to the impulse. For any collision with a fixed change in momentum, if the time of contact can be increased, the peak force is reduced:

For example if the squash ball was replaced with a softer version of same mass the collision graph would look like this:



If the squash ball was replaced with a harder version of same mass the collision graph would look like this:



Question Modern cars are designed to crumple on impact in a collision. How does this help to protect the occupants from harm?

Answer The change in momentum (area under the force time graph) can’t be changed at the time of the accident (mass is fixed and it is too late for the driver to slow down!) By increasing the time of collision the peak force is less and hopefully lets the occupants come to less harm as a result.

Question : How do I find "Velocity" from "Force vs. Time" graph?

Solution : You need two additional pieces of information: the mass of the object and its initial velocity. Given those, the relation is:

$$v(t) = v_o + \frac{1}{m} \int_0^t F(t) dt$$

Chapter 8

Friction

8.1 Static Friction hyperphysics.phy-astr.gsu.edu

Static frictional forces from the interlocking of the irregularities of two surfaces will increase to prevent any relative motion up until some limit where motion occurs. It is that threshold of motion which is characterized by the coefficient of static friction. The coefficient of static friction is typically larger than the coefficient of kinetic friction.

Images/CC/P-Graphs/4.1e/Images/Friction/ImagesReduced/fsta.jpg

In making a distinction between static and kinetic coefficients of friction, we are dealing with an aspect of "real world" common experience with a phenomenon which cannot be simply characterized. The difference between static and kinetic coefficients obtained in simple experiments like wooden blocks sliding on wooden inclines roughly follows the model depicted in the friction plot from which the illustration above is taken. This difference may arise from irregularities, surface contaminants, etc. which defy precise description. When such experiments are carried out with smooth metal blocks which are carefully cleaned, the difference between static and kinetic coefficients tends to disappear. When coefficients of friction are quoted for specific surface combinations are quoted, it is the kinetic coefficient which is generally quoted since it is the more reliable number.

8.2 Kinetic Friction

When two surfaces are moving with respect to one another, the frictional resistance is almost constant over a wide range of low speeds, and in the standard model of friction the frictional force is described by the relationship below. The coefficient is typically less than the coefficient of static friction, reflecting the common experience that it is easier to keep something in motion across a horizontal surface than to start it in motion from rest.

Images/CC/P-Graphs/4.1e/Images/Friction/ImagesReduced/fkin.jpg

8.3 Friction Plot

Static friction resistance will match the applied force up until the threshold of motion. Then the kinetic frictional resistance stays about constant. This plot illustrates the standard model of friction.

Images/CC/P-Graphs/4.1e/Images/Friction/ImagesReduced/frict1.jpg

The above plot, though representing a simplistic view of friction, agrees fairly well with the results of simple experiments with wooden blocks on wooden inclines. The experimental procedure described below equates the vector component of the weight down the incline to the coefficient of friction times the normal force produced by the weight on the incline.

Images/CC/P-Graphs/4.1e/Images/Friction/ImagesReduced/frictcoef.jpg

Having taken a large number of students through this experiment, I can report that the coefficient of static friction obtained is almost always greater than the coefficient of kinetic friction. Typical results for the woods I have used are 0.4 for the static coefficient and 0.3 for the kinetic coefficient.

When carefully standardized surfaces are used to measure the friction coefficients, the difference between static and kinetic coefficients tends to disappear, indicating that the difference may have to do with irregular surfaces, impurities, or other factors which can be frustratingly non-reproducible. To quote a view counter to the above model of friction: "Many people believe that the friction to be overcome to get something started (static friction) exceeds the force required to keep it sliding (sliding friction), but with dry metals it is very hard to show any difference. The opinion probably arises from experiences where small bits of oil or lubricant are present, or where blocks, for example, are supported by springs or other flexible supports so that they appear to bind." R. P. Feynman, R. P. Leighton, and M. Sands, The Feynman Lectures on Physics, Vol. I, p. 12-5, Addison-Wesley, 1964.

8.4 Rolling Friction

A rolling wheel requires a certain amount of friction so that the point of contact of the wheel with the surface will not slip. The amount of traction which can be obtained for an auto tire is determined by the coefficient of static friction between the tire and the road. If the wheel is locked and sliding, the force of friction is determined by the coefficient of kinetic friction and is usually significantly less.

Assuming that a wheel is rolling without slipping, the surface friction does no work against the motion of the wheel and no energy is lost at that point. However, there is some loss of energy and some deceleration from friction for any real wheel, and this is sometimes referred to as rolling friction. It is partly friction at the axle and can be partly due to flexing of the wheel which will dissipate some energy. Figures of 0.02 to 0.06 have been reported as effective coefficients of rolling friction for automobile tires, compared to about 0.8 for the maximum static friction coefficient between the tire and the road.

8.5 Few problems related to Friction

Example : A block on the horizontal table is acted upon by a force F. The graph of frictional force against F is

Images/CC/P-Graphs/4.1e/Images/LawsOfMotion/ImagesReduced/Scan_20170409 (5

Example: A block rests on a rough plane whose inclination θ to the horizontal can be varied. Which of the following graphs indicates how the frictional force F between the block and plane varies as θ is increased ?

Images/CC/P-Graphs/4.1e/Images/LawsOfMotion/ImagesReduced/Scan_20170409 (5

Example : Block A is placed on block B, whose mass is greater than that of A.

Images/CC/P-Graphs/4.1e/Images/LawsOfMotion/ImagesReduced/Scan_20170418 (2

There is friction between the blocks, while the ground is smooth. A horizontal force P, increasing linearly with time, begins to act on A. The accelerations a_1 and a_2 of A and B respectively are plotted against time (t). Choose the correct graph.

Images/CC/P-Graphs/4.1e/Images/LawsOfMotion/ImagesReduced/Scan_20170418

Example : A body moves with uniform speed on a rough surface. If force F of dynamic friction is plotted with time t as shown in figure, the graph will be

Images/CC/P-Graphs/4.1e/Images/LawsOfMotion/ImagesReduced/Scan_20170409 (4

Chapter 9

Theory and Problems

9.1 Impulse as Force-time Graph

Example : For the graph shown above, assume that it shows a constant force of 25 N acting over a 10 s period of time. Determine the impulse.

Images/CC/P-Graphs/4.1e/Images/LawsOfMotion/im1.jpg

Illustration : Graph of Example (Force as a function of Time)

Solution : (Theory : So far we've implied some things about what is constant and what can change in the impulse formula $F \Delta t = m \Delta v$.

We look at situations where we expect the mass of the object will stay constant.

- The velocity will change, and that's why we put a delta in front of it.
- Time is changing (sort of) as we measure it over a period of time.
- Force must be a constant.

We assume that the force being exerted on the object was always the same, causing a constant acceleration. If we are looking at a simple impulse question (where the force is constant), we can figure out exactly what we can interpret from a graph.

- Later this may help us to figure out a more complicated question, like if the force changes. The following graph is an example of one of those simple situations where the force remains constant during the entire time.
- If we look at what the slope might represent, we get...

$$\text{slope} = \text{rise} / \text{run}$$

$$\text{slope} = F / \Delta t$$

Since nothing in the impulse formula can be rearranged to give us force over time, the slope doesn't mean anything to us in this situation.

If we look at the area under the line, we get something a bit better...

$$\text{Area} = lw = F \Delta t = \Delta p$$

Since area under the line is equal to impulse...

$$\text{Area} = lw$$

$$\text{Area} = 25 \times 10$$

$$\text{Area} = 2.5e2$$

$$p = 2.5e2 \text{Ns}$$

If we really wanted to, we could have simply use $\Delta p = F \Delta t$ to figure out the impulse. We could do this in this situation because the force is constant.

- If we need to do a question where the force is not constant, we can still use the area under the line to get the impulse, even though the formula $\Delta p = F \Delta t$ can not be used.

Example : I am in a car that is accelerating from rest at a red light. I want to calculate the impulse that is acting on the car during the first 5.78s. If I know that the force on the car steadily increases from 0 N to 3012 N over this time, determine the impulse. If the mass of the car is 1500 kg, also determine the final velocity of the car.

Images/CC/P-Graphs/4.1e/Images/LawsOfMotion/im2.jpg

Illustration : Graph for Example (Force as a function of Time)

Solution : Let's start by graphing the information we were given. We will get a nice linear graph, since it said that the force steadily increases.

If we calculate the area under the graph (a triangle) we will know what the impulse is.

$$A = \frac{1}{2} bh$$

$$= \frac{1}{2} (5.78 \text{ s})(3012 \text{ N})$$

$$= 8704.68 \text{ A}$$

$$= 8.70e3 \text{ kgm/s}$$

To calculate the final velocity, we can use the value for the impulse we just got with the right hand side of the impulse formula. Remember that the initial velocity (sitting at the light) is zero...

$$\Delta p = m \Delta v$$

$$\Delta p = m(v_f - v_i)$$

$$\Delta p = mv_f$$

$$v_f = \Delta p / m$$

$$v_f = 8704.68 / 1500$$

$$v_f = 5.80312$$

$$v_f = 5.80 \text{m/s}$$

The graph that we make does not have to be a pretty right angle triangle either. We can also do some crazy stuff with what we are looking for in the question, as the next example shows.

Example : This graph shows the result of applying 500 kgm/s of impulse to an object as it moved across the floor for 10.0 s. Determine the maximum force that was exerted.

Images/CC/P-Graphs/4.1e/Images/LawsOfMotion/im3.jpg

Illustration 3: Pushing object across the floor.

Solution : Even though it is not a right angle triangle, this graph still shows a triangle that we can use the regular area formula with. In this case, we already know

the area (the impulse is 500 kgm/s) and we know the base (10.0 s). All we want is the height of the triangle, since that is the magnitude of the maximum force.

$$\text{Area} = bh/2$$

$$\Delta p = F \Delta t/2$$

$$F = 2 \Delta p / \Delta t = 2 \times 500 / 10.0 \text{ F} = 100 \text{N}$$

Even if the graph is a curved line, you can still at least estimate the area under the graph.

- Although this will only be an approximate area, without getting into calculus it's as good as you'll get and as good as you need.
- On the graph shown below we have an s-curve that would be difficult to calculate the exact area of.
- Instead, we just look at the triangle drawn in red. For the little bit extra it has near the beginning, it misses a bit later on. These two parts should more or less make up for each other, so that the area of the triangle will be about the same as the area under the curve.

Images/CC/P-Graphs/4.1e/Images/LawsOfMotion/im4.jpg

Illustration 4: Instead of trying to figure out the area of the curve exactly, we just use the area of the triangle as an approximation.

9.2 Force Time graph with respect to momentum

Images/CC/P-Graphs/4.1e/Images/LawsOfMotion/graph.jpg

Example : If the mass of the object is 3.0 kg, what is its final velocity over the 8.0 s time period? **physicsforums.com**

Solution : You need the area under the curve, and you do not need calculus, as that is a trapezoid. The area of a trapezoid is the average of the bases times the height, which is $(4 + 8) \text{seconds} / 2 \times 100 \text{ N} = 600 \text{ N*s}$. Set this to $mv - mv_o$, and assuming v_o is zero get $v_{\text{final}} = 200 \text{ m/s}$.

Example : The graph given below belongs to an object having mass 2kg and velocity 10m/s. It moves on a horizontal surface. If a force is applied to this object between (1-7) seconds find the velocity of the object at 7 seconds. **physicstutorials.org**

Images/CC/P-Graphs/4.1e/Images/LawsOfMotion/eample46.jpg

Solution : Area under the graph gives us impulse. First, we find the total impulse with the help of graph given above then total impulse gives us the momentum change. Finally, we find the final velocity of the object from the momentum change.

Images/CC/P-Graphs/4.1e/Images/LawsOfMotion/eample46solution.jpg

Example : Suppose a force, $F(t) = 6t^2 - 3t + 1$, acts on an 7-kg mass for three seconds.

Images/CC/P-Graphs/4.1e/Images/LawsOfMotion/14472c80-cdb1-4390-ba60-eb4a

- What impulse will the 7-kg object receive in the first three seconds?
- If the mass started from rest, what is its final velocity?

Solution : Force as the rate of change of momentum The impulse equation $J = (\text{net } F)t = \Delta p$ where $p = mv$ can be rearranged to state that the applied net force applied to an object equals the rate of change of the its momentum.

$$\text{net } F = \frac{\Delta p}{\Delta t}$$

$$\text{net } F = \frac{m \Delta v}{\Delta t}$$

$$\text{net } F = ma$$

Images/CC/P-Graphs/4.1e/Images/LawsOfMotion/68302de1-5791-4eab-a050-7194

That is, the net force acting on an object can be calculated as the slope of a momentum vs time graph. In terms of the calculus, this result equates to taking the derivative.

$$\frac{dp(t)}{dt} = F(t)$$

Notice that force must expressed as a function in terms of time, not displacement. Calculus will allow us to determine expressions for instantaneous, non-constant forces and thus is applicable to a wider range of situations. Let's work an example using this relationship.

Using the graph provided below, determine the instantaneous force acting on the 7-kg mass at each of the specified times:



- t = -2 seconds,
- t = 0 seconds,
- t = 3.5 seconds.



Example : The force shown in the force vs time graph below acts on a 1.7 kg object.



- (a) Find the magnitude of the impulse of the force.
Ans : 8 kg m/s.
- (b) Find the final velocity of the object if the object was initially at rest.
Ans : 2.76 m/s
- (c) Find the final velocity of the object if the object was initially moving along the x axis with a velocity of -1.7 m/s.
Ans : “ “

Note : The force shown in the force vs time graph below acts on a 1.7 kg object. Find the final velocity of the object if the object was initially at rest. Find the final velocity of the object if the object was initially moving along the x axis with a velocity of -1.7 m/s.

Example : Relating Momentum and Impulse

EXPLORATION – An impulsive bike ride Suki is riding her bicycle, in a straight line, along a flat road. Suki and her bike have a combined mass of 50 kg. At t = 0, Suki is traveling at 8.0 m/s. Suki coasts for 10 seconds, but when she realizes she is slowing down, she pedals for the next 20 seconds. Suki pedals so that the static friction force exerted on the bike by the road increases linearly with time from 0 to 40 N, in the direction Suki is traveling, over that 20-second period. Assume there is constant 10 N resistive force, from air resistance and other factors, acting on her and the bicycle the entire time. Step 1 - Sketch a diagram of the situation. The diagram is shown in Figure 6.2, along with the free-body diagram that applies for the first 10 s and the free-body diagram that applies for the 20second period while Suki is pedaling.



Figure : A diagram of (a) Suki on her bike, as well as free-body diagrams while she is (b) coasting and while she is (c) pedaling. Note that in free-body diagram (c), the static friction force \vec{F}_S gradually increases because of the way Suki pedals.

Step 2 - Sketch a graph of the net force acting on Suki and her bicycle as a function of time. Take the positive direction to be the direction Suki is traveling. In the vertical direction, the normal force exactly balances the force of gravity, so we can focus on the horizontal forces. For the first 10 seconds, we have only the 10 N resistive force, which acts to oppose the motion and is thus in the negative direction. For the next 20 seconds, we have to account for the friction force that acts in the direction of motion and the resistive force. We can account for their combined effect by drawing a straight line that goes from -10 N at t = 10 s, to +30 N (40 N - 10N) at t = 30 s. The result is shown in Figure 6.3.

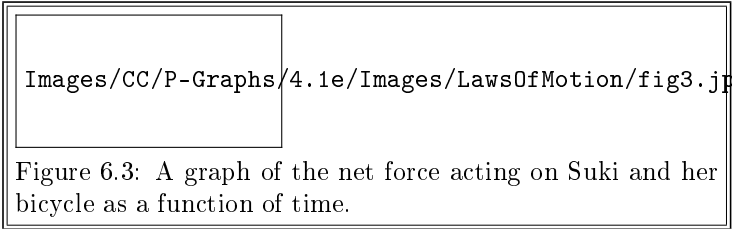


Figure 6.3: A graph of the net force acting on Suki and her bicycle as a function of time.

Step 3 - What is Suki's speed at t = 10 s? Let's apply Equation 6.3, which we can write as:

$$F_{net}\Delta t = \Delta(m\vec{v}) = m\Delta\vec{v} = m(\vec{v}_{10s} - \vec{v}_i) .$$

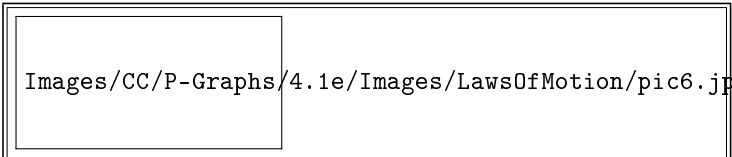
Solving for the velocity at t = 10 s gives:

$$\vec{v}_{10s} = \vec{v}_i + \frac{F_{net}\Delta t}{m} = +8.0m/s + \frac{(-10N)(10s)}{50kg} = +8.0m/s - 2.0m/s = +6.0m/s$$

Thus, Suki's speed at t = 10 s is 6.0 m/s. We can also obtain this result from the forceversus-time graph, by recognizing that the impulse, $\vec{F}_{net}\Delta t$, represents the area under this graph over some time interval Δt . Let's find the area under the graph, over the first 10 seconds, shown highlighted in green in Figure 6.4. The area is negative, because the net force is negative over that time interval. The area under the graph is the impulse:

$$\vec{F}_{net}\Delta t = -10N \times 10s = -100Ns = -100kg\ m/s$$

Figure 6.4: The green rectangle represents the area under the graph for the first 10 s. The area is negative, because the force is negative. From Equation 6.3, we know the impulse is equal to the change in momentum.



Suki's initial momentum is $m\vec{v}_i = 50kg \times 8.0m/s = +400kg\ m/s$. Her momentum at t = 10 s is therefore + 400kg m/s - 100kg m/s = + 300kg m/s. Dividing this by the mass to find the velocity at t = 10 s confirms what we found above:

$$\vec{v}_{10s} = \frac{\vec{p}_{10s}}{m} = \frac{\vec{p}_i + \Delta\vec{p}}{m} = \frac{+400kgm/s - 100kgm/s}{50kg} = \frac{+300kgm/s}{50kg} = +6.0m/s$$

Step 4 - What is Suki's speed at t = 30 s? Let's use the area under the force-versus-time graph, between t = 10 s and t = 30 s, to find Suki's change in momentum over the 20-second period. This area is highlighted in Figure 6.5, split into a negative area for the time between t = 10 s and t = 15 s, and a positive area between t = 15 s and t = 30 s. These regions are triangles, so we can use the equation for the area of a triangle, $0.5 \times \text{base} \times \text{height}$. The area under the curve, between 10 s and 15 s, is $0.5 \times (5.0s) \times (-10\text{ N}) = -25kg\ m/s$. The area between 15 s and 30 s is $0.5 \times (15s) \times (30\text{ N}) = +225kg\ m/s$. The total area (total change in momentum) is +200 kg m/s.

Note that another approach is to multiply the average net force acting on Suki and the 50 kg mass over this interval, by the time interval (20 s), for a +200 kg m/s change in momentum.

Figure 6.5: The shaded regions correspond to the area under the curve for the time interval from t = 10 s to t = 30 s.



In step 3, we determined that Suki's momentum at t = 10 s is +300 kg m/s. With the additional 200 kg m/s, the net momentum at t = 10 s is +500 kg m/s. Dividing by the 50 kg mass gives a velocity at t = 30 s of +10 m/s.

Key idea for the graphical interpretation of impulse: The area under the net force versus time graph for a particular time interval is equal to the change in momentum during that time interval.

Chapter 10

Problems for Practice

10.1 General Problem Set

10.1.1 Single Answer Type

Example : The relationship between the force F and position x of a body is as shown in figure. The work done in displacing the body from $x=1\text{m}$ to $x=5\text{m}$ will be

Images/CC/P-Graphs/4.1e/Images/LawsOfMotion/ImagesReduced/Scan_20170409 (2).jpg

- a) 30 J
- b) 15 J
- c) 25 J
- d) 20 J

{ Hint : Area under the graph from 1 to 5 taking signs , 15 J }

Example: A particle of mass m , initially at rest, is acted upon by a variable force F for a brief interval of time T . It begins to move with a velocity u after the force stops acting. F is shown in the graph as a function of time. The curve is a semicircle

Images/CC/P-Graphs/4.1e/Images/LawsOfMotion/ImagesReduced/Scan_20170418.jpg

- a) $u = \frac{\pi F_0^2}{2m}$
- b) $u = \frac{\pi T^2}{8m}$
- c) $u = \frac{\pi F_0 T}{4m}$
- d) $u = \frac{F_0 T}{2m}$

{ Hint : From impulse relation , $mv_f = \frac{\pi F_0^2}{2}$. So, a, b,c }

10.2 Previous Years IIT Problems

10.2.1 Single Answer

Example: A block of mass m is on an inclined plane of angle θ . The coefficient of friction between the block and the plane is μ and $\tan\theta > \mu$. The block is held stationary by applying a force P parallel to the plane. The direction of force pointing up the plane is taken to be positive. As P is varied from $P_1 = mg(\sin\theta - \mu\cos\theta)$ to $P_2 = mg(\sin\theta + \mu\cos\theta)$, the frictional force f versus P graph will look like

Images/CC/P-Graphs/4.1e/Images/LawsOfMotion/ImagesReduced/001q.jpg

Images/CC/P-Graphs/4.1e/Images/LawsOfMotion/ImagesReduced/001.jpg

{ Solution: Case I : $P_1 = mg(\sin\theta - \mu\cos\theta)$

Case II: $P_2 = mg(\sin\theta + \mu\cos\theta)$

Images/CC/P-Graphs/4.1e/Images/LawsOfMotion/ImagesReduced/001ans.jpg

In case a), frictional force f is positive and in case b), f is negative. When $P = mgsin\theta, f = \mu mgcos\theta = 0$

When $P < mgsin\theta, f = mgsin\theta - P$

When $P > mgsin\theta, f = P - mgsin\theta$

Thus f varies linearly with P , is positive when $P = P_1$ and negative when $P = P_2$. So the correct option is a)

}

Part IV

Energy Conservation

Chapter 11

Abstract Introduction

11.1 KINETIC ENERGY

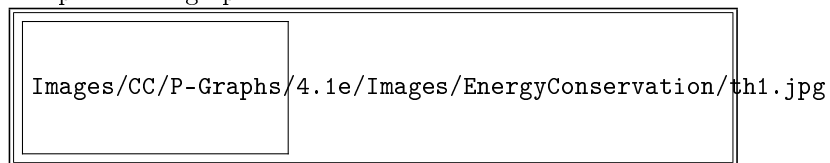
Objects have energy because of their motion; this energy is called kinetic energy. Kinetic energy of the objects having mass m and velocity v can be calculated with the formula given below;

$$E_k = \frac{1}{2}mv^2$$

As you see from the formula, kinetic energy of the objects is only affected by the mass and velocity of the objects. The unit of the E_k is again from the formula $\text{kg.m}^2/\text{s}^2$ or in general use joule.

11.2 Work Done by a Variable Force

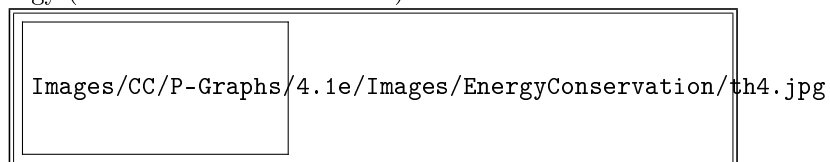
Graphically, the work done on an object or system is equal to the area under a Force vs. displacement graph:



The area under the graph from zero to 20 meters is 300 N m. Thus, the force represented by the graph does 300 J of work. This work is also a measure of the energy which was transferred while the force was being applied

11.3 The net force vs. position graph

The area under the net force vs. position graph represents the change in kinetic energy (also known as the net work).



Chapter 12

Theory and Problems

12.1 Force vs. Distance graph.

Example : Find the kinetic energy of the ball having mass 0,5 kg and velocity 10m/s.

Images/CC/P-Graphs/4.1e/Images/EnergyConservation/topornek.jpg

$$Ek = \frac{1}{2}mv^2$$

$$Ek = \frac{1}{2} \cdot 0,5 \cdot (10)^2$$

$$Ek = 25 \text{ joule}$$

As in the case of Kinematics we can use graphs to show the relations of the concepts here. Look at the given graph of

Images/CC/P-Graphs/4.1e/Images/EnergyConservation/graphforce.jpg

Area under the force vs. distance graph gives us work

Work = Force · Distance = Area = F · X (distance)

We can find energy of the objects from their Force vs. Distance graph.

Example : Find the Kinetic Energy of the object at 14m from the given graph below.

Images/CC/P-Graphs/4.1e/Images/EnergyConservation/example28.jpg

We can find the total kinetic energy of the object after 14m from the graph; we use area under it to find energy.

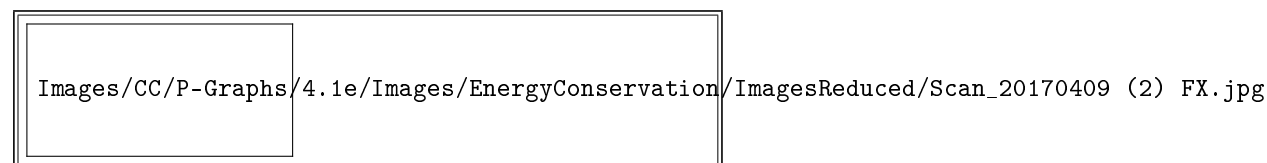
Images/CC/P-Graphs/4.1e/Images/EnergyConservation/example28solutionson.jpg

Chapter 13

Practice Problems

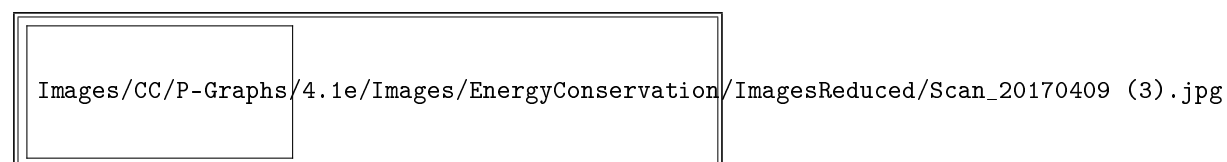
13.1 General Problem Set

Example : A force F acting on an object varies with distance x as shown in Figure. The force is in newton (N) and the distance (x) in metre. The work done by the force in moving from $x=0$ to $x=6\text{m}$ is



- a) 4.5 J
- b) 9.0 J
- c) 14.5 J
- d) 15 J

Example : Given below is a graph between a variable force (F) (along y-axis) and the displacement (X) (along x-axis) of a particle in one dimension. The work done by the force in the displacement interval between 0 m and 30 m is

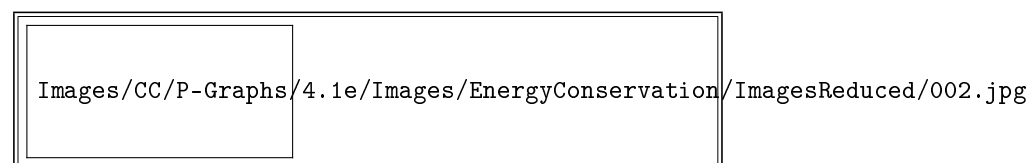


- a) 275 J
- b) 375 J
- c) 400 J
- d) 300 J

13.2 Previous Years IIT Problems

13.2.1 Single Answer

Example: A block of mass 2 kg is free to move along the x-axis. It is at rest and from $t=0$ onwards it is subjected to a time-dependent force $F(t)$ in the x direction. The force $F(t)$ varies with t as shown in the figure. The kinetic energy of the block after 4.5 seconds is



- a) 4.50J
- b) 7.50J
- c) 5.06J
- d) 14.06J

Chapter 14

Review Questions I

Refer to the following information for the next thirteen questions.
dev.physicslab.org

A 5.0-kg mass is pushed along a straight line by a net force described in the graph below. The object is at rest at $t = 0$ and $x = 0$.

Images/CC/P-Graphs/4.1e/Images/EnergyConservation/0406768d-b992-4a57-9d87-a50f2b36ebc3.jpg

- During which displacement interval was the object's acceleration uniform?
- What acceleration did the object experience when $x = 10$ meters?
- How much work was done on the object during the first 20 meters?
- How much kinetic energy did the object gain during the first 20 meters?
- What was the object's instantaneous velocity at $x = 20$ meters?
- How much time was required to move it through the first 20 meters?
- How much did the object's momentum change in the first 20 meters?
- What was the object's instantaneous acceleration at $x = 22$ meters?
- Why can't the kinematics equations for uniformly accelerated motion be used to calculate the object's instantaneous velocity at $x = 30$ meters? What method should be used?
- How much work was done to move the object from 20 meters to 30 meters?
- What was the object's instantaneous speed at $x = 30$ meters?
- What was the total impulse delivered to the object from $x = 0$ to $x = 30$ meters?
- What percent of the impulse was delivered in the last 10 meters?

Part V

Rotatory Motion

Chapter 15

Problems for Practice

15.1 General Problem Set

15.1.1 Single Answer Type

Example : The angular velocity of a rotating disc decreases linearly with angular displacement from 60 rev/min. to zero during 10 rev as shown. Determine the angular velocity of the disc 3 sec after it begins to slow down

Images/CC/P-Graphs/Common/Reduced/RotatoryMotion/rotation-u1.jpg

- (a) $\frac{20\pi}{10}$
- (b) $\frac{17\pi}{10}$
- (c) $\frac{7\pi}{3}$
- (d) $\frac{10\pi}{3}$

{ Hint: Answer C }

Example: Moment of Inertia I of a solid sphere about an axis parallel to a diameter and at a distance x from its centre of mass varies as

Images/CC/P-Graphs/4.1e/Images/RotatoryMotion/ImagesReduced/007.jpg

Example: A wheel is rolling without sliding on a horizontal surface. The centre of the wheel moves with a constant speed v_o . Consider a point P on the ri which is at the top at time t=0. The square of speed of point P is plotted against time t. The correct plot is (R is radius of the wheel)

Images/CC/P-Graphs/4.1e/Images/RotatoryMotion/ImagesReduced/008.jpg

Example: A rod of mass m and length l is hinged at one of its end A as shown in figure.

Images/CC/P-Graphs/4.1e/Images/RotatoryMotion/ImagesReduced/009-a.jpg

A force F is applied at a distance x from A. The acceleration of centre of mass (a) varies with x as

Images/CC/P-Graphs/4.1e/Images/RotatoryMotion/ImagesReduced/009.jpg

Example: A circular platform is free to rotate in a horizontal plane about a vertical axis passing through its centre. A tortoise is sitting at the edge of the platform. Now the platform is given an angular velocity ω_o . When the tortoise move along a chord of the platform with a constant velocity (with respect to the platform). The angular velocity of the platform $\omega(t)$ will vary with time t as

Images/CC/P-Graphs/4.1e/Images/RotatoryMotion/ImagesReduced/010.jpg

Example: The curve for the moment of inertia of a sphere of constant mass M versus its radius of gyration K will be

Images/CC/P-Graphs/4.1e/Images/RotatoryMotion/ImagesReduced/Scan_20170409 (2).jpg

{ Hint : a) }

Example : The graph between rotational energy E_r and angular velocity ω is represented by which curve

Images/CC/P-Graphs/4.1e/Images/RotatoryMotion/ImagesReduced/Scan_20170409

{ Hint : a) }

Example : The curve between angular momentum L and angular velocity ω will be

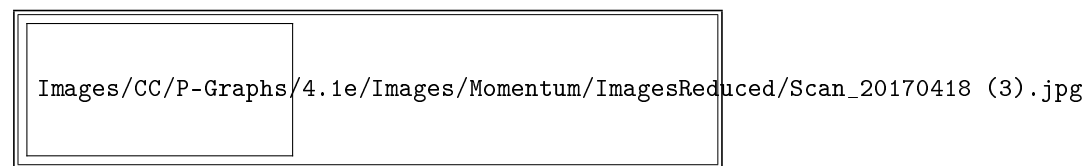
Images/CC/P-Graphs/4.1e/Images/RotatoryMotion/ImagesReduced/Scan_20170409

{ Hint : a) }

Chapter 16

Angular Momentum Conservation

Example : A block slides on a rough horizontal ground from point A to point B. Point C is midway between A and B. The coefficient of friction between the block and the ground is constant. Its angular momentum L about C is plotted against time t . Which of the following curves is correct?



Part VI

Gravitation

Chapter 17

Basics

17.1 Variation of “g”

Outside the earth $g = g_o \left(\frac{R}{r} \right)^2$

where g_o is the acceleration due to gravity at the surface of earth.



$$g_o = \frac{GM}{R^2}, \text{ R = radius of earth}$$

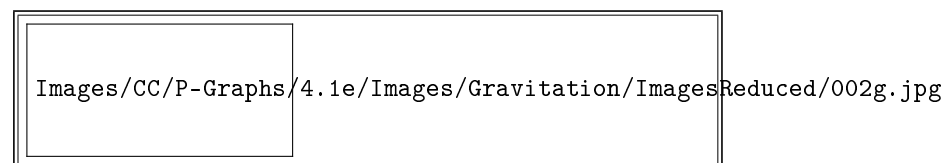
Chapter 18

Problems for Practice

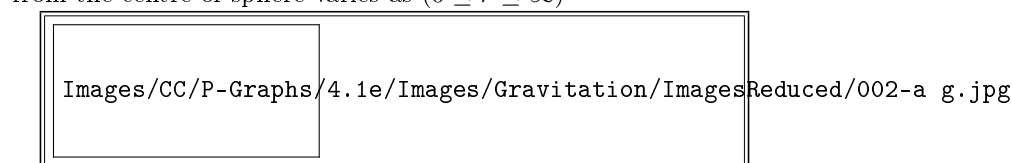
18.1 General Problem Set

18.1.1 Single Answer

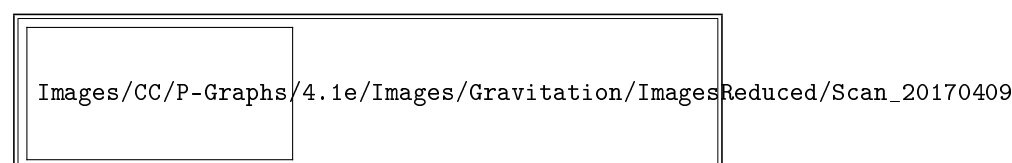
Example: A sphere of mass M and radius R_2 has a concentric cavity of radius R_1 as shown in figure.



The force F exerted by the sphere on a particle of mass m located at a distance r from the centre of sphere varies as ($0 \leq r \leq \infty$)

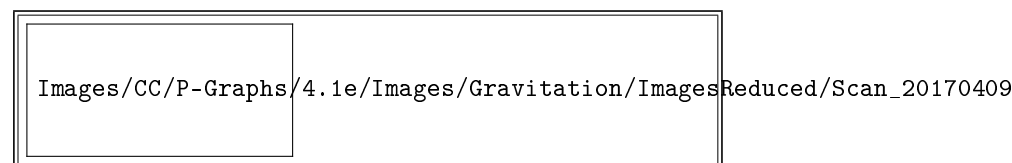


Example : The variation of acceleration due to gravity as one moves away from earth's centre is given by the graph



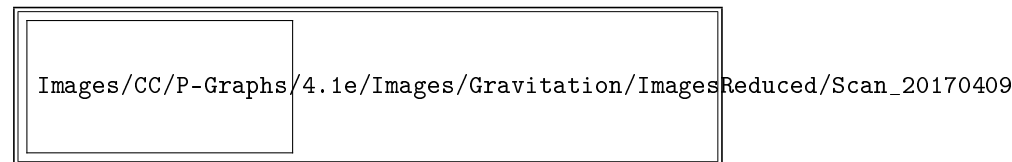
{ Hint: c) }

Example : Which of the following graphs represents the motion of a planet moving about the sun?



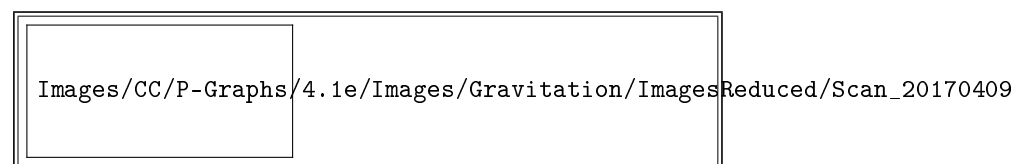
{ Hint : a) }

Example : P is a point at a distance r from the centre of a solid sphere of radius a . The gravitational potential at P is V . If V is plotted as a function of r , which is the correct curve?



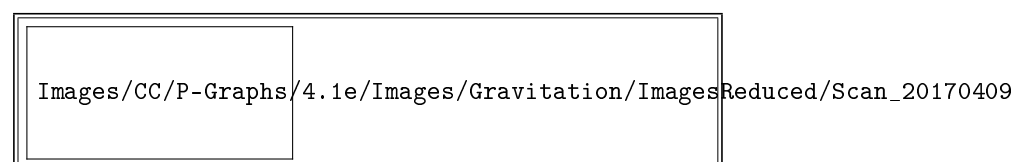
{ Hint : c) }

Example : Which of the graphs represents correctly the variation of intensity of gravitational field I with the distance r from the centre of a spherical shell of mass M and radius a ?



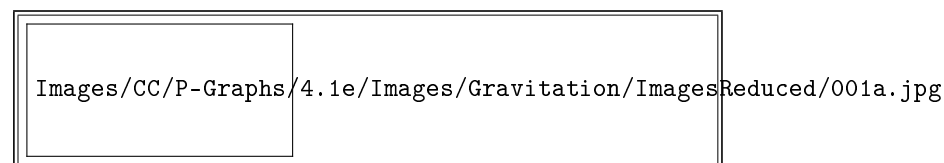
{ Hint : d) }

Example : The dependence of acceleration due to gravity g on the distance r from the center of the earth, assumed to be a sphere of radius R of uniform density is as shown in figure below



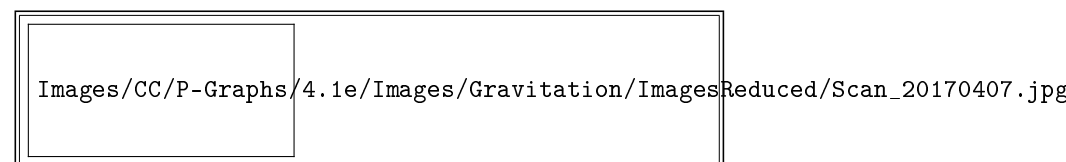
18.1.2 Multiple Answer

Example: Two concentric spherical shells are as shown in figure. The magnitude of gravitational potential (V) and field strength (E) vary with distance (r) from centre as

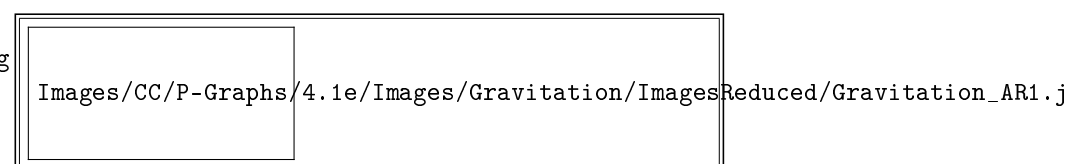


18.1.3 Comprehension Type

18.1.3.1 Concept 1 Gravitational potential inside a spherical shell is constant and outside the shell it varies as $V \propto \frac{1}{r}$ (with negative sign). Here r is the distance from centre.



Example1: Two concentric spherical shells are as shown in figure. The V - r graph will be as



Statement : 1: Gravitational Field at distance x from the centre on the axis of a ring is given as

$$E = \frac{Gmx}{(R^2 + x^2)^{3/2}}$$

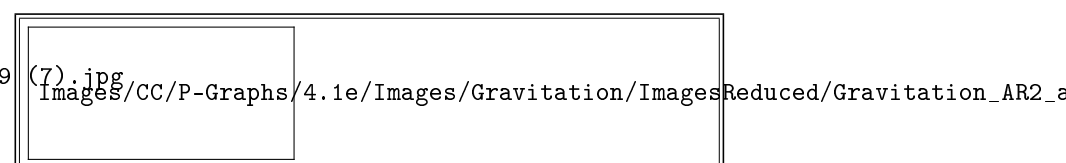
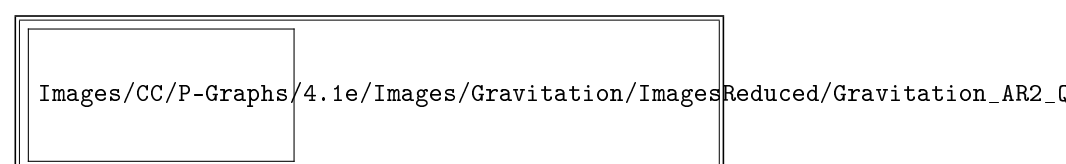
Here m is the mass of ring and R its radius.

2: Gravitational field at distance $x (\geq R)$ from centre of a solid sphere is given as,

$$E = \frac{Gm}{x^2}$$

Here m is the mass of solid sphere.

Example2: One ring of radius R and mass m and one solid sphere of same mass m and same radius R are placed with their centres on positive x -axis. We are moving from some finite distance on negative x -axis towards positive x -axis. Plane of the ring is perpendicular to x -axis. How will the net gravitational field vary with distance moved on x -axis. We move only up to surface of solid sphere. O is the origin,



Statement: Change in potential energy when a mass m is taken to a height h from the surface of earth is given by

$$\Delta U = \frac{mgh}{1 + h/R}$$

18.2 Previous Years IIT Problems

18.2.1 Single Answer

A spherically symmetric gravitational system of particles has a mass density

$$\rho = \begin{cases} \rho_o & \text{for } r \leq R \\ 0 & \text{for } r > R \end{cases}$$

where ρ_o is a constant. A test mass can undergo circular motion under the influence of the gravitational field of particles. Its speed v as a function of distance r ($0 < r < \infty$) from the centre of the system is represented by



{Solution: If M is the total mass of the system of particles, the orbital speed of the test mass is

$$v = \sqrt{\frac{GM}{r}}$$

For $r \leq R$, $v = \sqrt{\frac{G \times \frac{4\pi}{3} r^3 \rho_o}{r}}$ which gives $v \propto r$

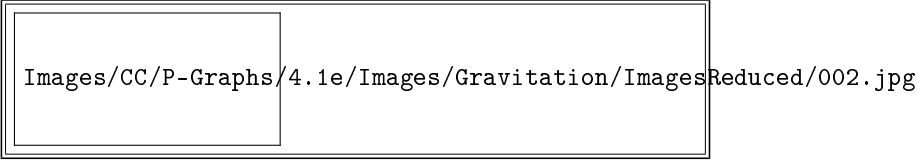
i.e. v increases linearly with r up to $r=R$. Hence choices b) and d) are wrong.

For $r > R$, the whole mass of the system is $M = \frac{4\pi}{3} R^3 \rho_o$, which is constant. Hence for $r > R$,

$$v = \sqrt{\frac{GM}{r}}$$

i.e. $v \propto \frac{1}{\sqrt{r}}$. Hence, the correct choice is c). }

18.2.2 Matching



Part VII

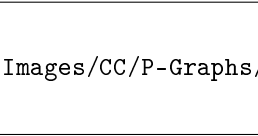
Periodic Motion

Chapter 19

Abstract Introduction (SHM)

19.1 Position vs time

The graph of position verse time is a sine wave with a possible phase shift. The phase shift is how much the ahead or behind the position is on the sine wave.
ipodphysics.com



Images/CC/P-Graphs/4.1e/Images/SHM/Sine wave.JPG

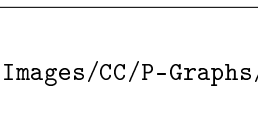
Consider this graph, if the "clock" is started at 0.05 (where the mass is at it's maximum stretch) seconds then there would be a phase shift of 90 degrees (or we could replace the sine function for a cosine). If the "clock" is started at 0.1 seconds (where mass moves down instead of up; left instead of right) then the phase shift would be 180 degrees (a negative sine function).



Images/CC/P-Graphs/4.1e/Images/SHM/ellipse.JPG

19.2 Velocity vs time

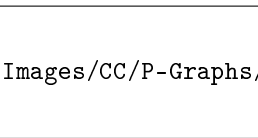
Consider the position verses time graph, at any point were the mass has reached the amplitude (maximum distance from from the equilibrium point) the speed of the mass at these point is zero. When the position is at zero then the speed is at a maximum (if you don't believe it, consider conservation of energy). This "shifts" the position graph by 90 degrees "creating" a cosine graph for velocity. The other way of thinking about is velocity is the change in position with respect to time, the change in a sine wave with respect to time is a cosine graph.



Images/CC/P-Graphs/4.1e/Images/SHM/cosine wave.JPG

19.3 Acceleration vs time

The acceleration verse time graph is the easiest of the graphs to make. The simple harmonic motion is based on a relationship between position and acceleration; $x = -Ka$. So the graph of position and acceleration should look alike, except for the negative sign. In fact position and acceleration are the same shape just mirror copies of each other.



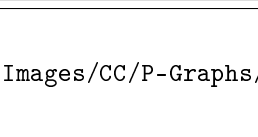
Images/CC/P-Graphs/4.1e/Images/SHM/-sinewave.JPG

19.3.1 Peak Height

It is important to note that the shapes of each graph is similar, that each graph has the same frequency, period and wavelength, but they don't have the same amplitude, for common simple harmonic motion, the height the peak of the function (not to confused with the amplitude, amplitude refers to the height of the position graph alone, i'm talking about the height of the position, velocity and acceleration graphs) tend to get smaller and smaller starting with the position graph being the tallest and the acceleration being the shortest.

19.4 Graphing position, velocity, and acceleration with respect to each other

Graphing position to the other functions can be complicated and when tested on it, most student are unable to give the right answer. First consider this, for simple harmonic motion position and acceleration are proportional. $x = -ka$ this is a linear relationship so the graph is a line, the slope is negative so the line is heading down.



Images/CC/P-Graphs/4.1e/Images/SHM/linear.jpg

19.5 Position and acceleration verses velocity

The position and acceleration verse velocity graph look entirely different. First off the straight line test fails when plotting position vs velocity or acceleration verse velocity. Take the point were x and a are zero, there are two possible answers for the point (the speed maybe at maximum) the object could be moving down or up at the point. That means the velocity can be a positive maximum or a negative maximum, two separate values. Looking a the points were the velocity is equal to zero, there are two possible answers, either at the top or at the bottom. If you continue to plot data points graph that is developed is a ellisipe

Chapter 20

Problems

20.1 General Problem Set

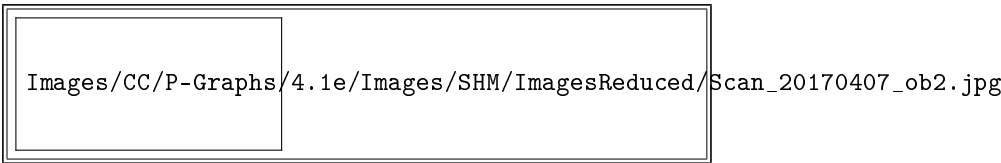
20.1.1 Single Answer Type

Example 1: Acceleration-displacement graph of a particle executing SHM is as shown in given figure. The time period of its oscillation is (in sec)

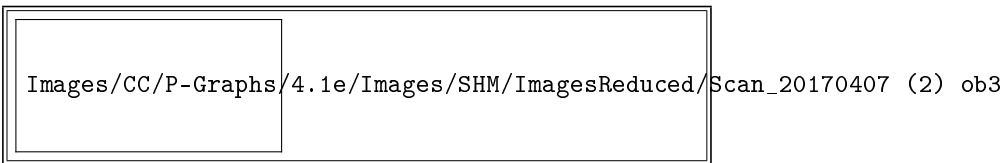


- a) $\pi/2$
- b) 2π
- c) π
- d) $\pi/4$

Example 2: Displacement-time graph of a particle executing SHM is as shown.



Example 3: For a particle executing SHM the displacement x is given by $x = A \cos \omega t$. Identify the graph which represents the variation of potential energy (PE) as a function of time t and displacement x



- a) I, III
- b) II, IV
- c) II, III
- d) I, IV

20.1.2 Multiple Answer Type

Example 1: Velocity-time graph of a particle executing SHM is shown in figure. Select the correct alternative(s).



- a) At position 1 displacement of particle may be positive or negative.
- b) At position 2 displacement of particle is negative.
- c) At position 3 acceleration of particle is positive.
- d) At position 4 acceleration of particle is positive.

Example 2: Acceleration-time graph of a particle executing SHM is as shown in figure. Select the correct alternative(s).



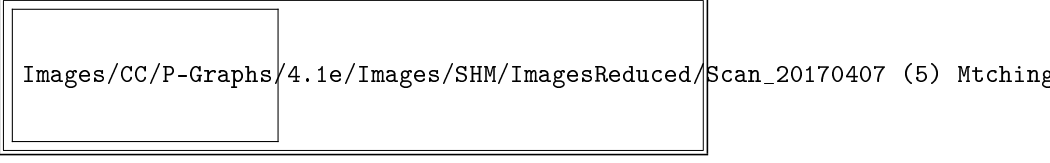
- a) Displacement of particle at 1 is negative.
- b) Velocity of particle at 2 is positive.
- c) Potential energy of particle at 3 is maximum.
- d) Speed of particle at 3 is decreasing.

20.1.3 Matching Type Questions

Example 1: Velocity-time graph of a particle in SHM is as shown in figure. Match the following



Example 2: F-x and x-t graph of a particle in SHM are as shown in figure. Match the following



20.1.4 Comprehension Type Questions

20.1.4.1 Comprehension 1

Concept : In SHM, force $\left(F = -\frac{dU}{dr} \right)$ on the particle at mean position is zero. Potential energy at extreme position is maximum
Example: U-r graph of a particle which can be under SHM is as shown in figure. What conclusion cannot be drawn from the graph?



- a) Mean position of the particle is at $r=2\text{m}$.
- b) Potential energy of particle at mean position is 10 J.
- c) Amplitude of oscillation is 1 m.
- d) None of these.

20.1.4.2 Comprehension 2

Statement : In case of pure rolling $a = R\alpha$, where a is the linear acceleration and α the angular acceleration.

Question : A disc of mass m and radius R is attached with a spring of force constant k at its centre as shown in figure. At $x=0$, spring is unstretched. The disc is moved to $x=A$ and then released. There is no slipping between disc and ground. Let f be the force of friction on the disc from the ground.



Example 1 : f versus t (time) graph will be as



Example 2 : In the problem if $k = 10 \text{ N/m}$, $m = 2 \text{ kg}$, $R = 1 \text{ m}$ and $A = 2 \text{ m}$. Find linear speed of the disc at mean position

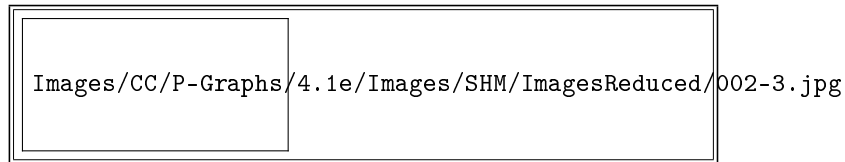
- a) $\sqrt{\frac{40}{3}} \text{ m/s}$
- b) $\sqrt{20} \text{ m/s}$
- c) $\sqrt{\frac{10}{3}} \text{ m/s}$
- d) $\sqrt{\frac{50}{3}} \text{ m/s}$

20.2 Previous Years IIT Problems

20.2.1 Paragraph

Paragraph 1: When a particle of mass m moves on the x -axis in a potential of the form $V(x) = kx^2$ it performs simple harmonic motion. The corresponding time period is proportional to $\sqrt{\frac{m}{k}}$, as can be seen easily using dimensional analysis.

However, the motion of a particle can be periodic even when its potential energy increases on both sides of $x=0$ in a way different from kx^2 and its total energy is such that the particle does not escape to infinity. Consider a particle of mass m moving on the x -axis. Its potential energy is $V(x) = \alpha x^4$ ($\alpha > 0$) for $|x|$ near the origin and becomes a constant equal to V_0 for $|x| \geq X_0$ (see figure)



1. If the total energy of the particle is E , it will perform periodic motion only if
 - a) $E < 0$
 - b) $E > 0$
 - c) $V_0 > E > 0$
 - d) $E > V_0$
2. For periodic motion of small amplitude A , the time period T of this particle is proportional to
 - a) $A\sqrt{\frac{m}{\alpha}}$
 - b) $\frac{1}{A}\sqrt{\frac{m}{\alpha}}$
 - c) $A\sqrt{\frac{\alpha}{m}}$
 - d) $\frac{1}{A}\sqrt{\frac{\alpha}{m}}$
3. The acceleration of this particle for $|x| > X_0$ is
 - a) Proportional to V_0
 - b) Proportional to V_0/mX_0
 - c) Proportional to $\sqrt{V_0/mX_0}$
 - d) Zero

Part VIII

Statics

Chapter 21

Modulii of Elasticity

21.1 General Problem Set

21.1.1 Single Answer Type

Example 1 : The graph shows the extension (Δl) of a wire of length 1.0m suspended from the top of a roof at one end and with a load W connected to the other end. If the cross-sectional area of the wire is $10^{-6}m^2$, calculate the Young's modulus of the material of the wire

Images/CC/P-Graphs/4.1e/Images/ModuliiOfElasticity/ImagesReduced/Scan_20170407.jpg

- a) $2 \times 10^{11} N/m^2$
- b) $2 \times 10^{10} N/m^2$
- c) $2 \times 10^{12} N/m^2$
- d) $2 \times 10^{13} N/m^2$

Example : The graph shown was obtained from experimental measurements of the period of oscillation T for different masses M placed in the scale pan on the lower end of the spring balance. The most likely reason for the line not passing through the origin is that the

Images/CC/P-Graphs/Common/Reduced/Elasticity/elasticity-e1.jpg

- (a) Spring did not obey Hooke's law
 - (b) Amplitude of oscillation was too large
 - (c) Clock used needed regulating
 - (d) Mass of the pan was neglected
- { Hint: Answer D }

21.1.2 Comprehension Type

21.1.2.1 Comprehension 1

Concept : Free body diagram of liquid can be drawn in similar manner as we draw the free body diagram of a solid. The only difference is, what we call the normal reaction between solid-solid boundary, we here call it pressure X area in case of liquids. Both are perpendicular to the surface.

Example : Equal amounts of liquid are filled in two vessels of different shapes as shown in figure. Let F_1 be the force by the base on liquid in case (i) and F_2 in case (ii) Then

Images/CC/P-Graphs/4.1e/Images/ModuliiOfElasticity/ImagesReduced/Scan_20170407_comp1.jpg

- a) $F_1 > F_2$
- b) $F_1 < F_2$
- c) $F_1 = F_2$
- d) Data insufficient

Question : A cube (side = 10 cm) of density $0.5 g/cm^3$ is placed in a vessel of base area 20 cm X 20 cm. A liquid of density $1.0 g/cm^3$ is gradually filled in the vessel at a constant rate $Q = 50 cm^3/s$.

Images/CC/P-Graphs/4.1e/Images/ModuliiOfElasticity/ImagesReduced/Scan_20170407_comp1_2.jpg

Example : If we plot a graph between the normal reaction on cube by the vessel versus time. The graph will be like

Images/CC/P-Graphs/4.1e/Images/ModuliiOfElasticity/ImagesReduced/Scan_20170407_comp1_3.jpg

- Example: The cube will leave contact with the vessel after time $t = \dots\dots\dots$ s
- a) 30
 - b) 40
 - c) 60
 - d) 20

Part IX

Thermodynamics

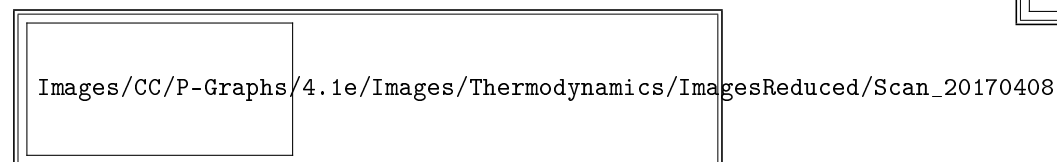
Chapter 22

Practice Problems

22.1 General Problem Set

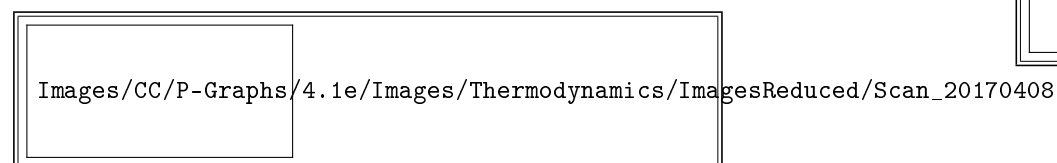
22.1.1 Single Answer Type

Example : Pressure versus temperature graphs of an ideal gas are as shown in figure. Choose the wrong statement

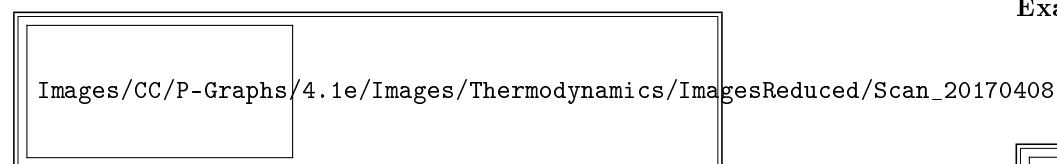


- a) Density of gas is increasing in graph (i)
- b) Density of gas is decreasing in graph (ii)
- c) Density of gas is constant in graph (iii)
- d) None of the above

Example : In a cyclic process shown in the figure an ideal gas is adiabatically taken from B to A, the work done on the gas during the process B->A is 30J, when the gas is taken from A->B the heat absorbed by the gas is 20 J. The change in internal energy of the gas in the process A-> B is

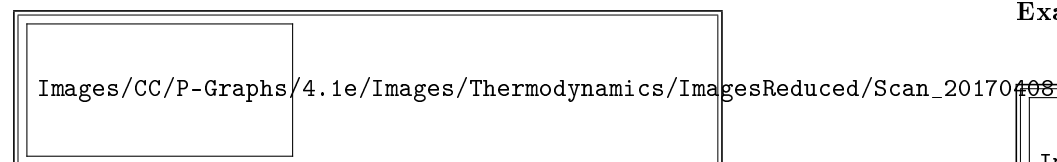


Example : An ideal monoatomic gas undergoes a cyclic process ABCA as shown in the figure. The ratio of heat absorbed during AB to the work done on the gas during BC is



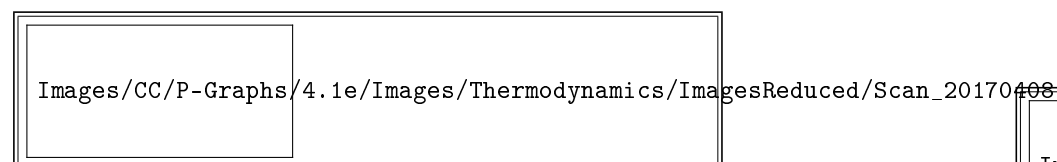
- a) $5/2 \ln 2$
- b) $5/3$
- c) $5/4 \ln 2$
- d) $5/6$

Example : Three moles of an ideal monoatomic gas performs a cycle 1->2->3->4->1 as shown. The gas temperatures in different states are $T_1=400\text{K}$, $T_2=800\text{K}$, $T_3=2400\text{K}$ and $T_4=1200\text{K}$. The work done by the gas during the cycle is (2-3 and 4-1 are isobaric)

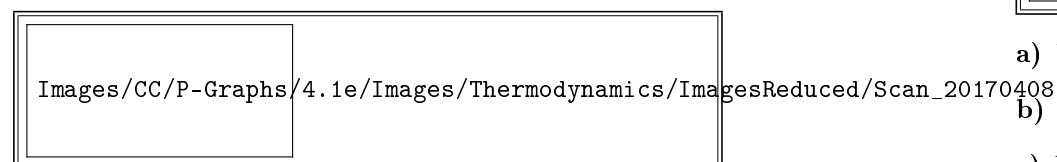


- a) 1200 R
- b) 3600 R
- c) 2400 R
- d) 2000 R

Example : Temperature of a body θ is slightly more than the temperature of the surrounding θ_0 . Its rate of cooling (R) versus temperature of body (θ) is plotted, its shape would be

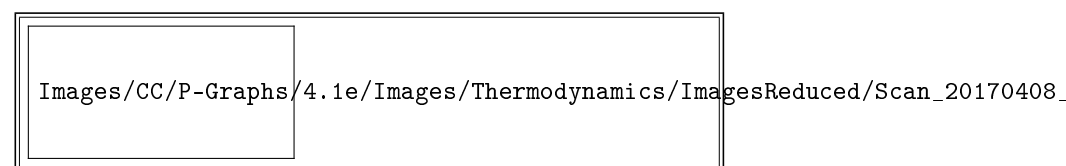


Example : Pressure versus density graph of an ideal gas is shown in figure



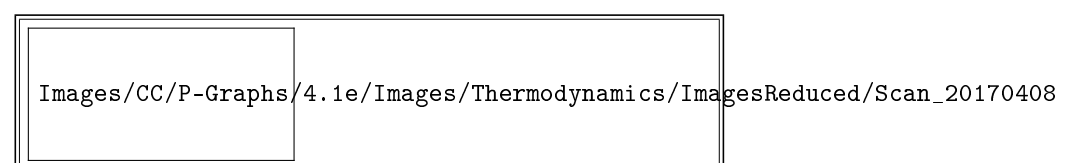
- a) during the process AB work done by the gas is positive
- b) during the process AB work done by the gas is negative
- c) during the process BC internal energy of the gas is increasing
- d) None of the above

Example : Ideal gas is taken through the process shown in the figure



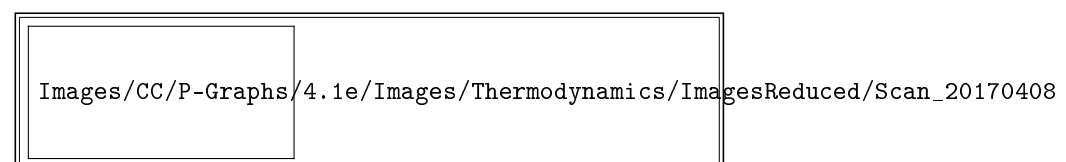
- a) In process AB, work done by system is positive
- b) In process AB, heat is rejected
- c) In process AB, internal energy increases
- d) In process AB internal energy decreases and in process BC, internal energy increases

Example : A gas is expanded from volume V_0 to $2V_0$ under three different processes. Process 1 is isobaric, process 2 is isothermal and process 3 is adiabatic. Let $\Delta U_1, \Delta U_2$ and ΔU_3 be the change in internal energy of the gas in these three processes. Then



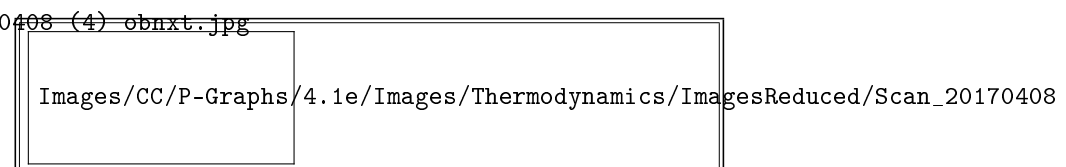
- a) $\Delta U_1 > \Delta U_2 > \Delta U_3$
- b) $\Delta U_1 < \Delta U_2 < \Delta U_3$
- c) $\Delta U_2 < \Delta U_1 < \Delta U_3$
- d) $\Delta U_2 < \Delta U_3 < \Delta U_1$

Example : Pressure versus temperature graph of an ideal gas at constant volume V is shown by the straight line A. Now mass of the gas is doubled and the volume is halved, then the corresponding pressure versus temperature graph will be shown by the line



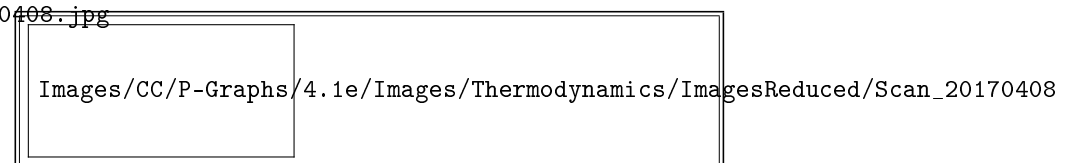
- a) A
- b) B
- c) C
- d) None of these

Example : P-V diagram of an ideal gas is as shown in figure. Work done by the gas in the process ABCD is



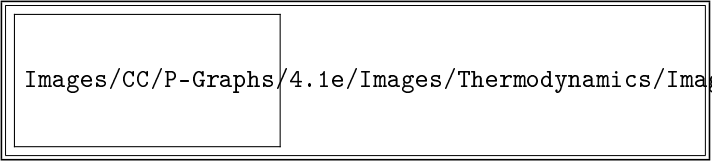
- a) $4P_0V_0$
- b) $2P_0V_0$
- c) $3P_0V_0$
- d) P_0V_0

Example : Pressure versus temperature graph of an ideal gas of equal number of moles of different volumes are plotted as shown in figure. Choose the correct alternatives.



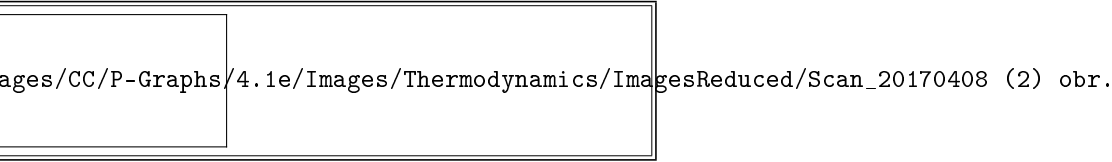
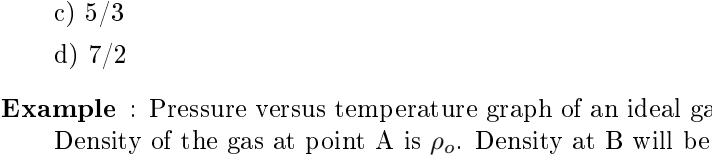
- a) $V_1 = V_2, V_3 = V_4$ and $V_2 > V_3$
- b) $V_1 = V_2, V_3 = V_4$ and $V_2 < V_3$
- c) $V_1 = V_2 = V_3 = V_4$
- d) $V_4 > V_3 > V_2 > V_1$

Example : Volume versus temperature graph of two moles of helium gas is as shown in figure. The ratio of heat absorbed and the work done by the gas in process 1-2 is

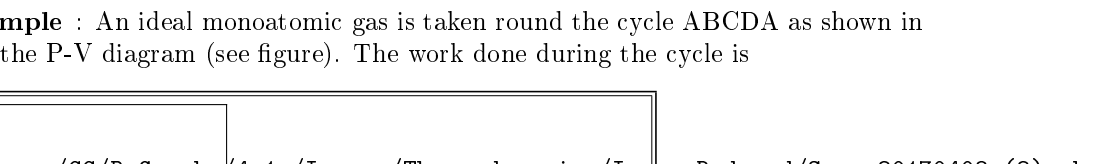


- a) 1590 J
- b) 1620 J
- c) 1540 J
- d) 1570 J

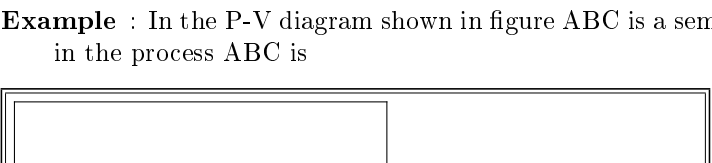
Example : One mole of a monoatomic ideal gas undergoes the process A -> B in the given P-V diagram. The specific heat for this process is



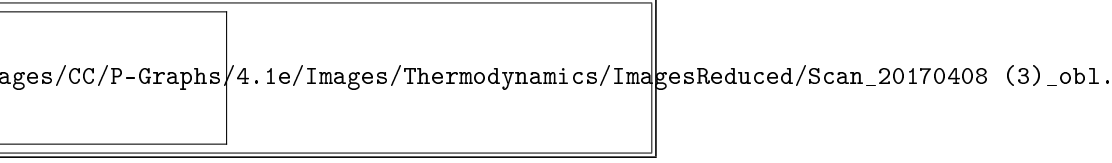
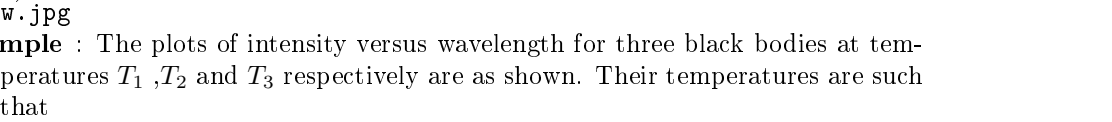
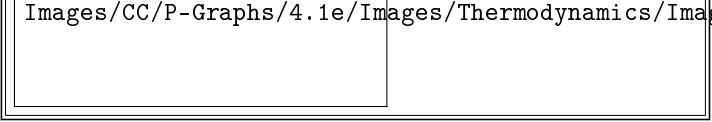
- a) $3R/2$
- b) $13R/6$
- c) $5R/2$
- d) $2R$



Example : An ideal monoatomic gas is taken round the cycle ABCDA as shown in the P-V diagram (see figure). The work done during the cycle is

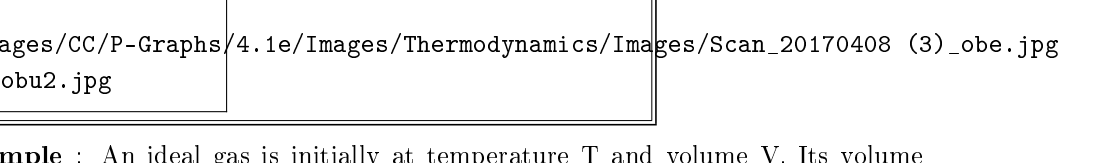
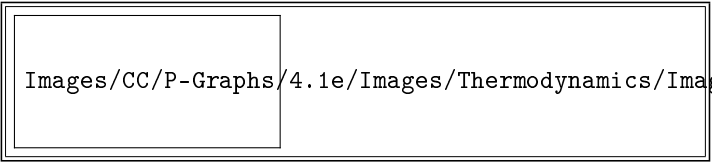
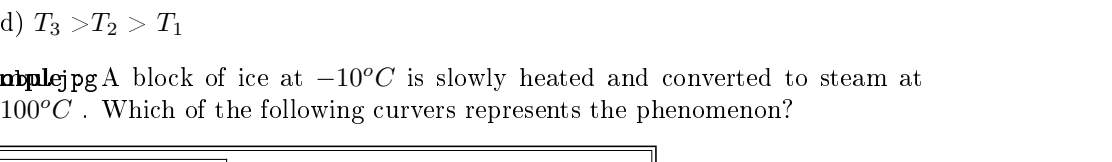


Example : The plots of intensity versus wavelength for three black bodies at temperatures T_1 , T_2 and T_3 respectively are as shown. Their temperatures are such that

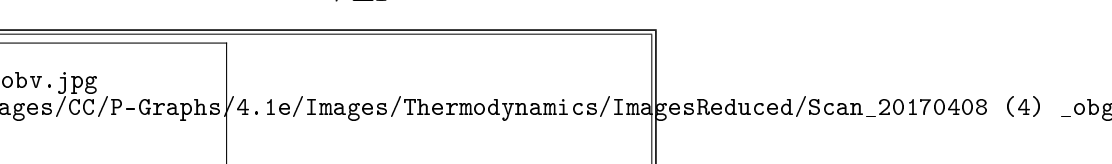


- a) zero
- b) $\frac{\pi}{2}$ atm-L
- c) $-\frac{\pi}{2}$ atm-L
- d) 4 atm-L

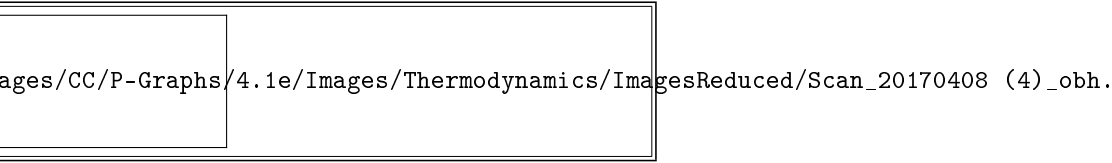
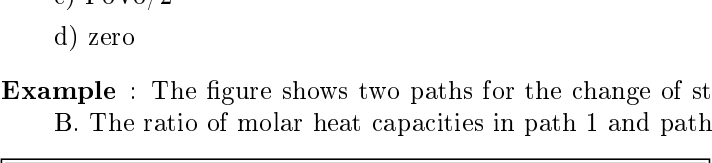
Example : Pressure versus temperature graph of an ideal gas is as shown in figure corresponding density (ρ) versus volume (V) graph will be



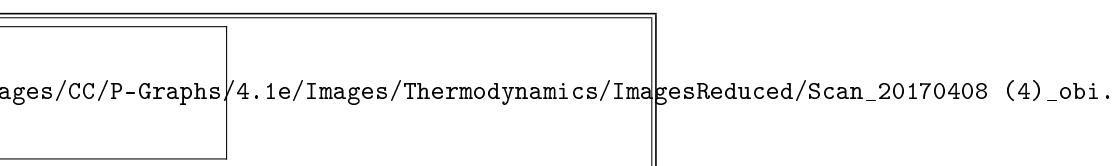
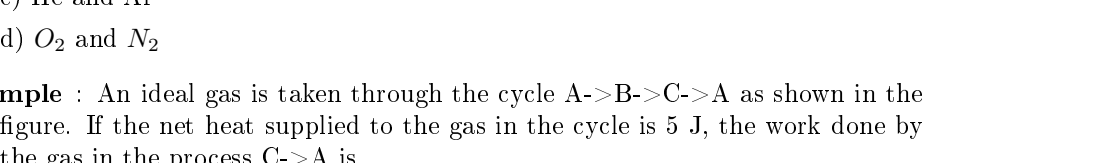
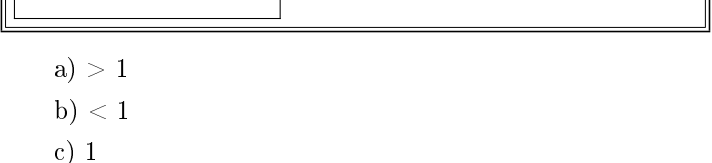
Example : A block of ice at $-10^{\circ}C$ is slowly heated and converted to steam at $100^{\circ}C$. Which of the following curvers represents the phenomenon?



Example : An ideal gas is initially at temperature T and volume V. Its volume is increased by ΔV due to an increase in temperature ΔT , pressure remaining constant. The quantity $\delta = \frac{\Delta V}{V\Delta T}$ varies with temperature as

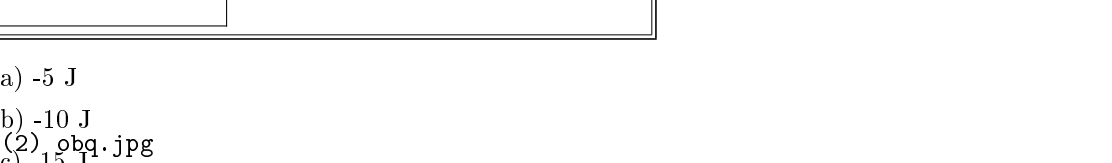
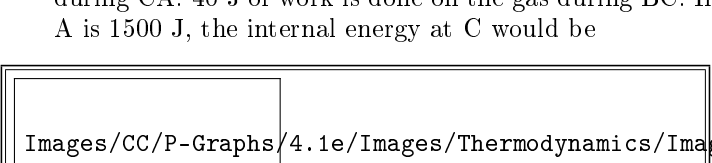


Example : P-V plots for two gases during adiabatic processes are shown in the figure. Plots 1 and 2 should correspond respectively to

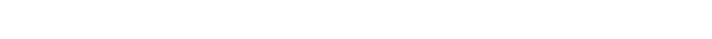


- a) PoVo
- b) 2PoVo
- c) PoVo/2
- d) zero

Example : The figure shows two paths for the change of state of a gas from A to B. The ratio of molar heat capacities in path 1 and path 2 is



Example : A sample of an ideal gas is taken through a cycle as shown in figure. It absorbs 50 J of energy during the process AB, no heat during BC, rejects 70J during CA. 40 J of work is done on the gas during BC. Internal energy of gas at A is 1500 J, the internal energy at C would be



- a) > 1
- b) < 1
- c) 1
- d) Data insufficient

- a) He and O_2
- b) O_2 and He
- c) He and Ar
- d) O_2 and N_2

Example : An ideal gas is taken through the cycle A->B->C->A as shown in the figure. If the net heat supplied to the gas in the cycle is 5 J, the work done by the gas in the process C->A is

Example : An ideal gas is taken through the cycle A->B->C->A as shown in the figure. If the net heat supplied to the gas in the cycle is 5 J, the work done by the gas in the process C->A is

- a) -5 J
- b) -10 J
- c) -15 J
- d) -20 J

Example : Which of the following graphs correctly represents the variation of $\beta = -\frac{dV/dP}{V}$ with P for an ideal gas at constant temperature?

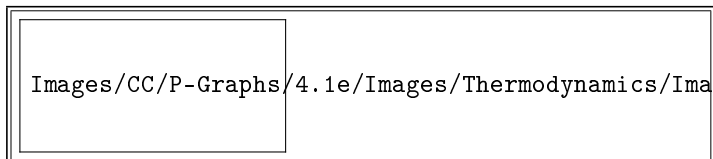


Example : The graph, shown in the diagram, represents the variation of temperature (T) of the bodies, x and y having same surface area, with time (t) due to the emission of radiation. Find the correct relation between the emissivity and absorptivity of the two bodies



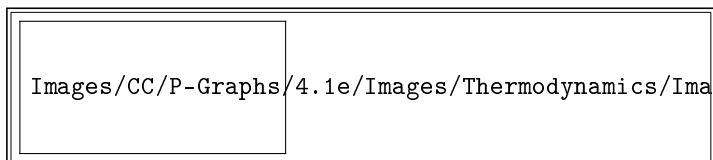
- a) $E_x > E_y$ and $a_x < a_y$
- b) $E_x < E_y$ and $a_x > a_y$
- c) $E_x > E_y$ and $a_x > a_y$
- d) $E_x < E_y$ and $a_x < a_y$

Example : Liquid oxygen at 50 K is heated to 300K at constant pressure of 1 atm. The rate of heating is constant. Which of the following graphs represent the variation of temperature with time?



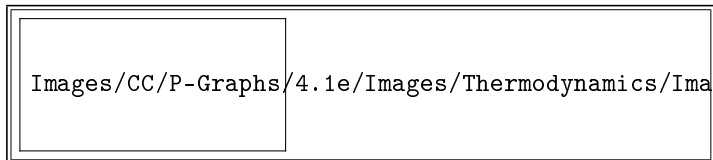
22.1.2 Multiple Answer Type

Example : P-V diagram of a cyclic process ABCA is as shown in figure. Choose the correct statement (s)



- a) $\Delta Q_{A \rightarrow B}$ = negative
- b) $\Delta U_{B \rightarrow C}$ = positive
- c) $\Delta U_{C \rightarrow A}$ = negative
- d) ΔW_{CAB} = negative

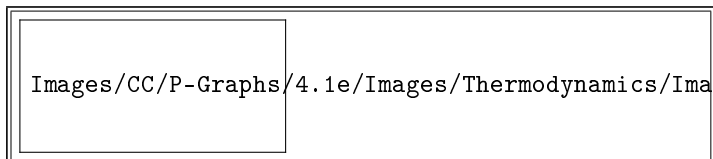
Example : A gas undergoes the change in its state from position A to position B via three different paths as shown in figure.



Select the correct alternative (s).

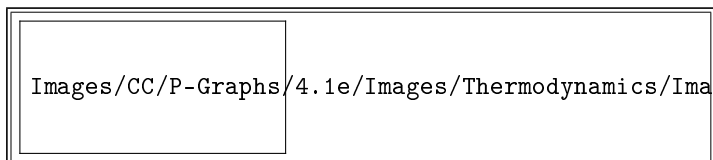
- a) Change in internal energy in all the three paths is equal.
- b) In all the three paths heat is absorbed by the gas.
- c) Heat absorbed / released by the gas is maximum in path 1
- d) Temperature of the gas first increases and then decreases in path 1

Example : During the process A-B of an ideal gas



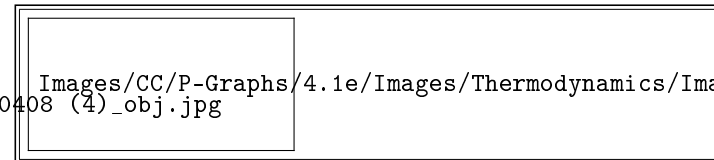
- a) work done on the gas is zero
- b) density of the gas is constant
- c) slope of line AB from the T-axis is inversely proportional to the number of moles of the gas
- d) slope of line AB from the T-axis is directly proportional to the number of moles of the gas

Example : Temperature versus pressure graph of an ideal gas is shown in figure. During the process AB



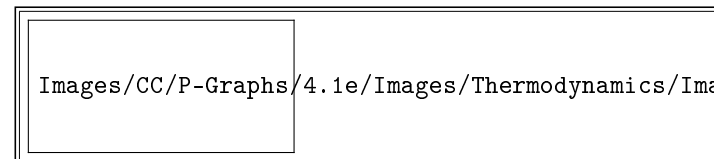
- a) internal energy of the gas remains constant
- b) volume of the gas is increased
- c) work done on the gas is positive
- d) pressure is inversely proportional to volume

Example : One mole of an ideal monochromatic gas is taken from A to C along the path ABC. The temperature of the gas at A is T_o . For the process ABC



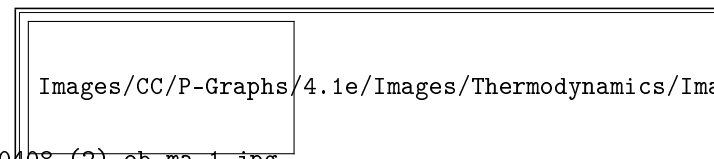
- a) work done by the gas is RT_o
- b) change in internal energy of the gas is $\frac{11}{2}RT_o$
- c) heat absorbed by the gas is $\frac{11}{2}RT_o$
- d) heat absorbed by the gas is $\frac{13}{2}RT_o$

Example : n moles of a monoatomic gas undergo a cyclic process ABCDA as shown in figure. Process AB is isobaric, BC is adiabatic, CD is isochoric and DA is isothermal. The maximum and minimum temperature in the cycle are $4T_o$ and T_o respectively. Then



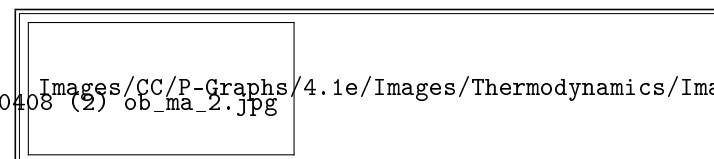
- a) $T_B > T_C > T_D$
- b) heat is released by the gas in the process CD
- c) heat supplied to the gas in the process AB
- d) total heat supplied to the gas is $2nRT_o \ln(2)$

Example : The density (ρ) of an ideal gas varies with temperature T as shown in figure. Then



- a) the product of P & V at A is equal to the product of P & V at B
- b) pressure at B is greater than the pressure at A
- c) work done by the gas during the process AB is negative
- d) the change in internal energy from A to B is zero

Example : One mole of an ideal monoatomic gas (initial temperature T_o) is made to go through the cycle abca shown in the figure. If U denotes the internal energy, then choose the correct alternatives

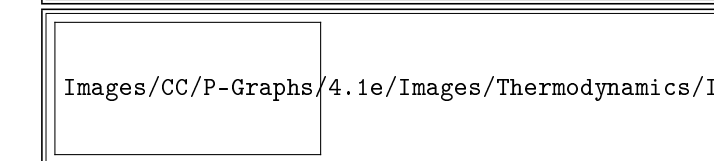
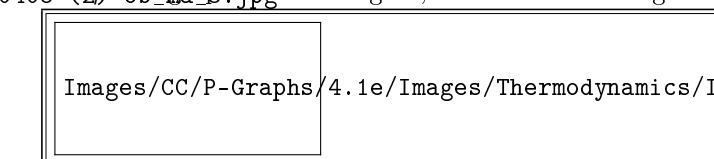


- a) $U_c - U_a = 10.5RT_o$
- b) $U_b - U_a = 4.5RT_o$
- c) $U_c > U_b > U_a$
- d) $U_c - U_b = 6RT_o$

22.1.3 Matching Type

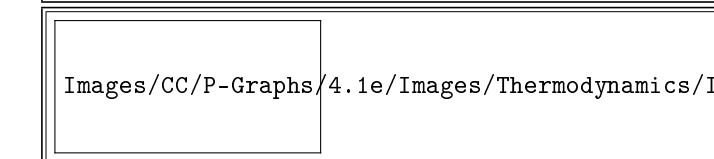
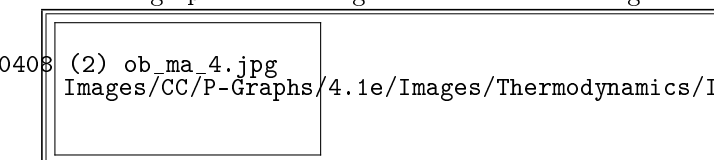
22.1.3.1 Matrix Match 1

In the P-T graph shown in figure, match the following



22.1.3.2 Matrix Match 2

In the V-T graph shown in figure match the following



22.1.4 Comprehension Type

22.1.4.1 Comprehension 1

Statement : V-T graph of an ideal gas is as shown in figure.



Question : Work done by the gas in complete cyclic process abcd is

- a) zero
- b) positive
- c) negative
- d) Data is insufficient

Question : Heat is supplied to the gas in process (es)

- a) da, ab and bc
- b) da and ab only
- c) da only
- d) ab and bc only

22.1.4.2 Comprehension 2

Two moles of a monoatomic gas are taken from a to c, via three paths abc, ac and adc.



Question : Work done by the gas in process ac is

- a) 1000 R
- b) 900 R
- c) 600 R
- d) 1500 R

Question : If work done by the gas in abc is W_1 , in ac work done is W_2 and in adc work done is W_3 , then

- a) $W_2 > W_3 > W_1$
- b) $W_1 > W_2 > W_3$
- c) $W_2 > W_1 > W_3$
- d) $W_3 > W_2 > W_1$

22.1.5 Subjective Problems

Example: The moles of an ideal monoatomic gas undergoes a cyclic process as shown in the figure.



The temperatures in different states are $6T_1 = 3T_2 = 2T_4 = T_3 = 1800\text{K}$. Determine the work done by the gas during the cycle.

Example: A fixed mass of oxygen gas performs a cycle ABCA as shown. Find efficiency of the process.



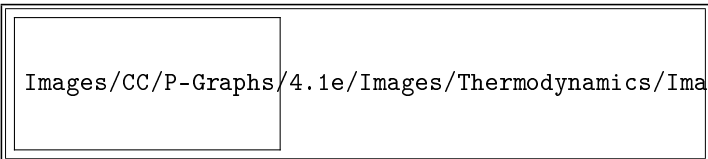
Example: A fixed mass of gas is taken through a process $A \rightarrow B \rightarrow C \rightarrow A$. Here $A \rightarrow B$ is isobaric, $B \rightarrow C$ is adiabatic and $C \rightarrow A$ is isothermal. Find efficiency of process. (Take $\gamma = 1.5$)



Example: At a temperature of $T_o = 273^\circ\text{K}$, two moles of an ideal gas undergoes a process as shown. The total amount of heat imparted to the gas equals $Q = 27.7\text{kJ}$. Determine the ratio of molar specific heat capacities.



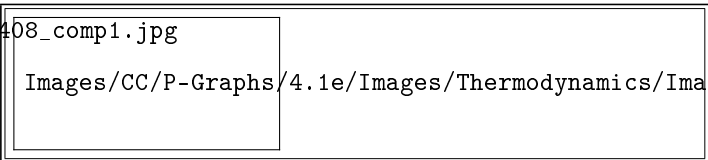
Example: n moles of an ideal gas is made to undergo the cycle 1-2-3-4-1 as shown in the figure. Process 3-4 is a straight line. The gas temperatures in states 1,2 and 3 are T_1 , T_2 and T_3 respectively. Temperature at 3 and 4 are equal. Determine the work done by the ga during the cycle.



22.2 Previous Years IIT Problems

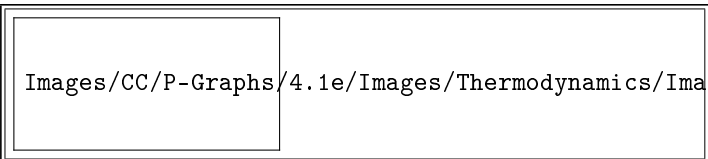
22.2.1 Mutliple Answer

Example: The figure shows the P-V plot of an ideal gas taken through a cycle ABCDA. The part ABC is a semi-circle and CDA is half of an ellipse. Then,



- a) the process during the path A->B is isothermal.
- b) heat flows out of the gas during the path B->C->D
- c) work done during the path A->B->C is zero.
- d) positive work is done by the gas in the cycle ABCDA

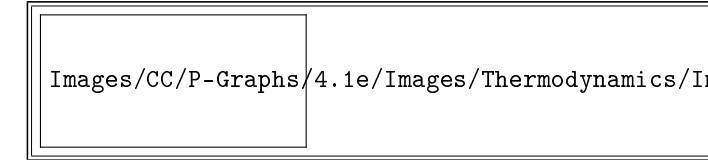
Example: One mole of an ideal gas in initial state A undergoes a cyclic process ABCA, as shown in the figure. Its pressure at A is P_o . Choose the correct option(s) from the following



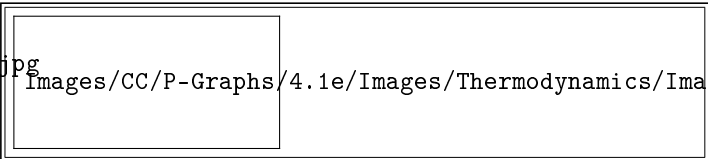
- a) Internal energies at A and B are the same
- b) Work done by the gas in process AB is $P_o V_o \ln 4$
- c) Presssure at C is $\frac{P_o}{4}$.
- d) Temperature at C is $\frac{T_o}{4}$.

22.2.2 Paragraph

Paragraph 1: Electrical resistance of certain materials, known as superconductors, changes abruptly from a nonzero value to zero as their temperature is lowered below a critical temperature $T_c(0)$. An intersting property of superconductors is that their critical temperatre becomes smaler than $T_c(0)$ if they are placed in a magnetic field, i.e., the critical temperature $T_c(B)$ is a function of the magnetic field strength B. The dependence of $T_c(B)$ on B is shown in the figure.



1: In the graphs below, the resistance R of a superconductor is shown as a function of its temperature T for two different magnetic fields B1 (solid line) and B2 (dashed line). If B2 is larger than B1 which of the following graphs shows the correct variation of R with T in these fields?

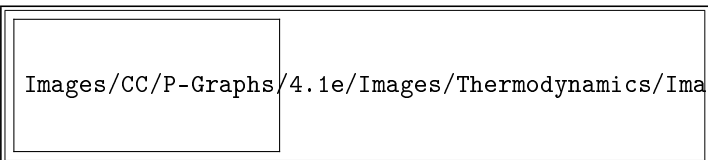


2: A superconductor has $T_c(0) = 100\text{K}$. When a magnetic field of 7.5 Tesla is applied, its T_c decreases to 75 K. For this material one can definitely say that when

- a) B = 5 Tesla, $T_c(B) = 80\text{ K}$
- b) B = 5 Tesla, $75\text{ K} < T_c(B) < 100\text{K}$
- c) B = 10 Tesla, $75\text{K} < T_c < 100\text{K}$
- d) B = 10 Tesla, $T_c = 70\text{ K}$

22.2.3 Matching

Example:



Part X

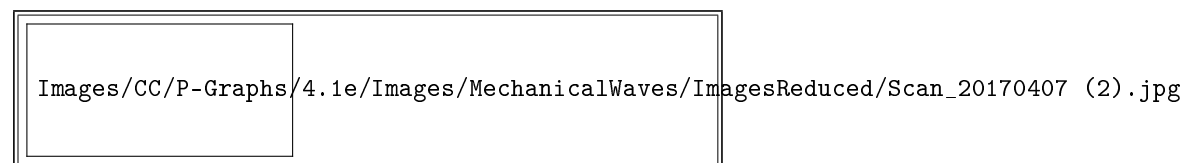
Mechanical Waves

Chapter 23

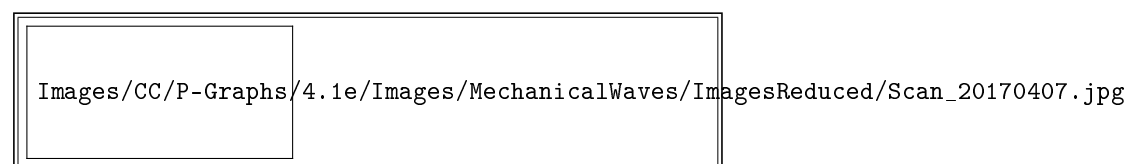
General Problem Set

23.1 Single Answer Questions

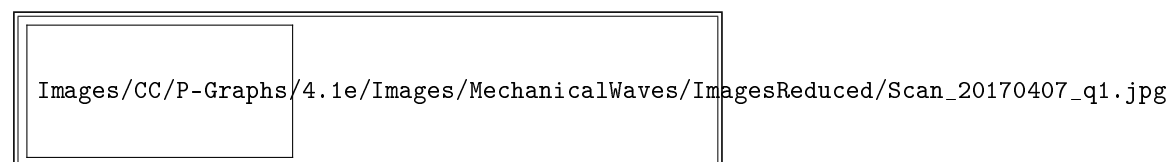
Example 1: A uniform rope having mass m hangs vertically from a rigid support. A transverse wave pulse is produced at the lower end. The speed ν of wave pulse varies with height h from the lower end as



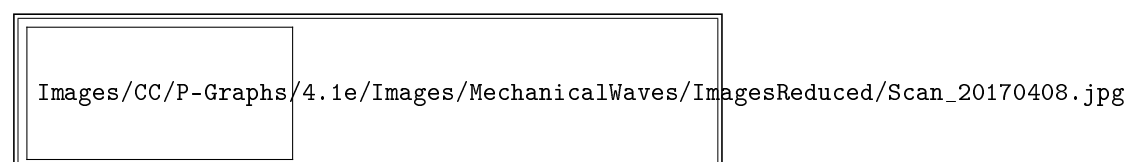
Example 2: A wave pulse on a string has the dimension shown in figure. The wave speed is $v=1 \text{ cm/s}$. If point O is a free end.



- (i) The shape of wave at time $t=3\text{s}$ is
(ii) The shape of the wave at time $t=3\text{s}$ if O is a fixed end will be
{ Both answers from the image below }



Example 3 : Two pulses in a stretched string, whose centres are initially 8 cm apart, are moving towards each other as shown in the figure. The speed of each pulse is 2 cm/s. After 2s the total energy of the pulses will be



- a) zero
b) purely kinetic
c) purely potential
d) partly kinetic and partly potential

Part XI

Sound

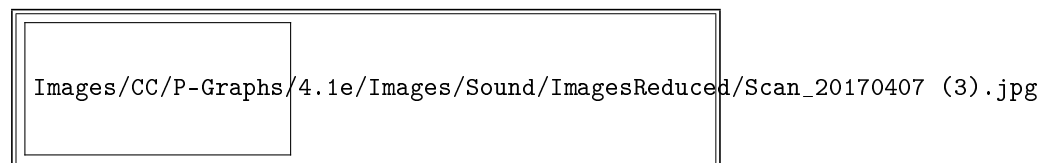
Chapter 24

Practice Problems

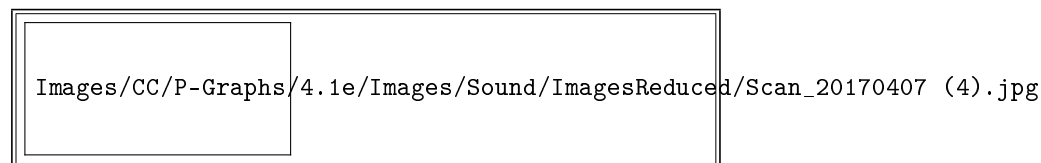
24.1 General Problem Set

24.1.1 Single Answer Type

Example 1 : Source and observer both start moving simultaneously from origin one along x-axis and the other along y-axis with speed of source = 2 (speed of observer). The graph between the apparent frequency observed by observer (f) and time (t) would be



Example 2: An observer starts moving with uniform acceleration a towards a stationary sound source of frequency f_o . As the observer approaches the source, the apparent frequency f heard by the observer varies with time t as



24.1.2 Multiple Answer Type

Example 1 : A stationary observer receives a sound of frequency $f_o = 2000$ Hz. Source is moving with constant velocity on a road at some non-zero perpendicular distance from observer. The apparent frequency f varies with time as shown in figure. Speed of sound = 300 m/s. Choose the correct alternative(s).

- a) Speed of source is 66.7 m/s
- b) f_m shown in figure cannot be greater than 2500 Hz
- c) Speed of source is 33.33 m/s
- d) f_m shown in figure cannot be greater than 2250 Hz

24.2 Previous Years IIT Problems

24.2.1 Passage

Two trains A and B are moving with speed 20 m/s and 30 m/s respectively in the same direction on the same straight track, with B ahead of A. The engines are at the front ends. The engine of train A blows a long whistle. Assume that the sound of the whistle is composed of components varying in frequency from $f_1 = 800$ Hz to $f_2 = 1120$ Hz, as shown in the figure. The spread in the frequency (highest frequency-lowest frequency) is thus 320 Hz. The speed of sound in still air is 340 m/s.



1: The speed of sound of the whistle is

- a) 340 m/s for passengers in A and 310 m/s for passenger in B
- b) 360 m/s for passengers in A and 310 m/s for passenger in B
- c) 310 m/s for passengers in A and 360 m/s for passenger in B
- d) 340 m/s for passengers in both the trains

{ Solution: The speed of sound depends only on the modulus of elasticity and the density of the medium in which it travels. The speed of sound does not depend on the speed of the source of sound or of the observer. Hence the correct option is d) }

2: The distribution of the sound intensity of the whistle as observed by the passengers in train A is best represented by



{ Solution: For train A, there is no relative motion between the source and the passengers. Hence the frequency of sound heard by passengers in train A will be the same as the frequency of sound emitted by the whistle. Therefore, the correct choice is a). }

3: The spread of frequency as observed by the passenger in train B is

- a) 310 Hz
- b) 330 Hz
- c) 350 Hz
- d) 290 Hz

{ Solution: The apparent frequency of sound as heard by passengers in train B is given by

$$f' = f_o \left(\frac{v - u_B}{v - u_A} \right)$$

where f_o = actual frequency, v = speed of sound, u_B = speed of train B and u_A = speed of train A.

$$f' \text{ (For } f_o = 800 \text{ Hz)} = 800 \times \left(\frac{340 - 30}{340 - 20} \right) = 775 \text{ Hz}$$

$$f' \text{ (For } f_o = 1120 \text{ Hz)} = 1120 \times \left(\frac{340 - 30}{340 - 20} \right) = 1085 \text{ Hz}$$

\therefore Spread of frequency = 1085 - 775 = 310 Hz Hence the correct choice is a)

}

Part XII

Electrostatics

Chapter 25

Problems for Practice

25.1 General Problem Set

25.1.1 Single Answer Type

Example : The distance between plates of a parallel plate capacitor is $5d$. The positively charged plate is at $x=0$ and negatively charged plate is at $x=5d$.

Images/CC/P-Graphs/4.1e/Images/Electrostatics/ImagesReduced/Scan_20170409 (3) _ob1.jpg

Two slabs one of conductor and the other of a dielectric of same thickness d are inserted between the plates as shown in figure. Potential (V) versus distance x graph will be

Images/CC/P-Graphs/4.1e/Images/Electrostatics/ImagesReduced/Scan_20170409 (3) _ob1q.jpg

Example : A particle of mass m and charge q is projected vertically upwards. A uniform electric field \vec{E} is acted vertically downwards. The most appropriate graph between potential energy U (gravitational plus electrostatic) and height h (\ll radius of earth) is (assume U to be zero on surface of earth)

Images/CC/P-Graphs/4.1e/Images/Electrostatics/ImagesReduced/Scan_20170409.jpg

Example : A conducting shell of radius R carries charge $-Q$. A point charge $+Q$ is placed at the centre. The electric field E varies with distance r (from the centre of the shell) as

Images/CC/P-Graphs/4.1e/Images/Electrostatics/ImagesReduced/Scan_20170419.jpg

Example : Two identical positive charges are fixed on the y -axis, at equal distances from the origin O . A particle with a negative charge starts on the negative x -axis at a large distance from O , moves along the x -axis, passed through O and moves far away from O . Its acceleration a is taken as positive along its direction of motion. The particle's acceleration a is plotted against its x -coordinate. Which of the following best represents the plot ?

Images/CC/P-Graphs/4.1e/Images/Electrostatics/ImagesReduced/Scan_20170419_2.jpg

Example: Two identical point charges are placed at a separation of l . P is a point on the line joining the charges, at a distance x from any one charge. The field at P is E . E is plotted against x for values of x from close to zero to slightly less than l . Which of the following best represents the resulting curve?

Images/CC/P-Graphs/4.1e/Images/Electrostatics/ImagesReduced/Scan_20170419 (2).jpg

Example : Four equal charges of magnitude q each are placed at four corners of a square with its centre at origin and lying in y - z plane. A fifth charge $+Q$ is moved along x -axis. The electrostatic potential energy (U) varies on x -axis as

Images/CC/P-Graphs/4.1e/Images/Electrostatics/ImagesReduced/Scan_20170419 (2)_2.jpg

Example : A circular ring carries a uniformly distributed positive charge. The electric field (E) and potential (V) varies with distance (r) from the centre of the ring along its axis as

Images/CC/P-Graphs/4.1e/Images/Electrostatics/ImagesReduced/Scan_20170419 (3)_3.jpg

Part XIII

Current Electricity

Chapter 26

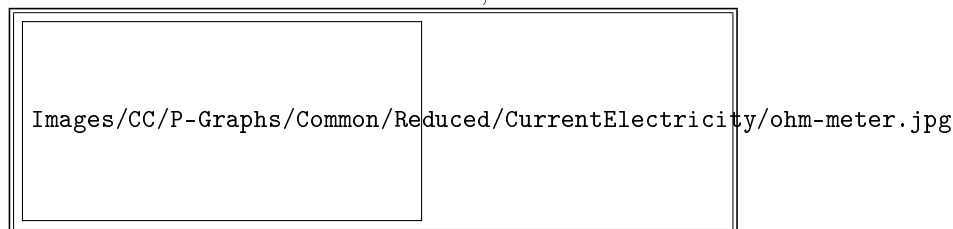
Basics

26.1 Theory

26.1.1 Ohm-meter

It is an instrument designed to measure resistance. It contains an Ammeter, a Battery and a Rheostat as shown in Figure. The terminals A and B are first short circuited and the Rheostat is adjusted to show full deflection on Ammeter. The full scale deflection corresponds to zero external resistance.

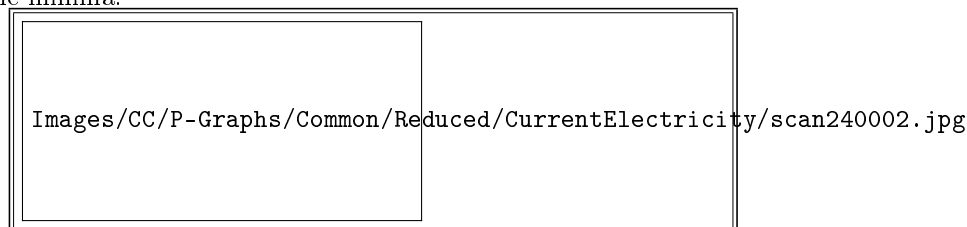
Now, connecting a resistance box between points A and B, ammeter deflection θ is noted for different values of R and a graph is plotted between θ and R. The graph is called Calibration Curve and is shown in Figure. Now the resistance box is removed and an unknown resistance is connected to the circuit. The deflection is noted down and from the calibration curve, the value of R is found out.



26.1.2 Power Transfer to a load

The power transfer to the load by the cell will be $P = I^2 R = \frac{E^2 R}{(R + r)^2}$

From the equation, it is clear that Power would be zero, if $R=0$ or ∞ and gives the minima.



$\frac{dP}{dR} = 0$ i.e. $\frac{d}{dR} \left[\frac{E^2 R}{(R + r)^2} \right] = 0$ will give any other local maxima / local minima

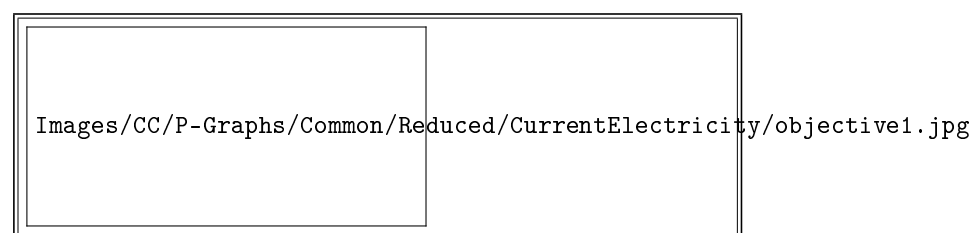
It is zero at $R=r$ which gives a maxima

i.e. power transfer to the load by a cell is maximum when $R=r$ and $P_{max} = \frac{E^2}{4r}$

26.2 Problems

26.2.1 Objective Type Questions

Example: Resistance as shown in Figure is negative at

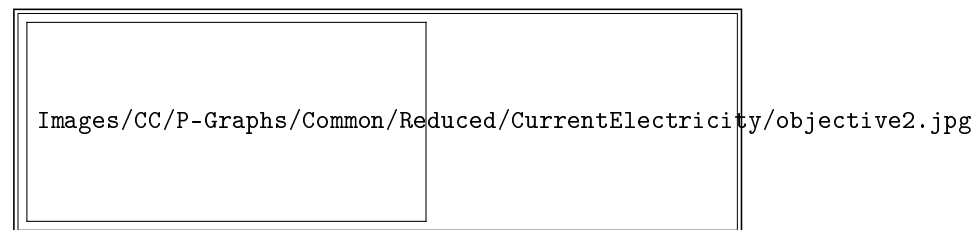


- a) A
- b) B
- c) C
- d) None of these

{Hint: The resistance is given by V/I and NOT dV/dI . V and I are shown positive, so R (their ratio) is also +ve.

d) is the correct answer. }

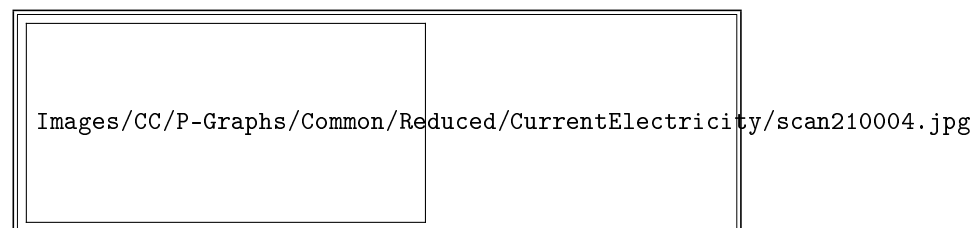
Example: A cell of EMF E having an Internal Resistance r is connected to an External Resistance R. The potential difference V across the resistance R varies with R as the curve :



- a) A
- b) B
- c) C
- d) D

{ Hint: $V = ER/(r+R)$, So, B is the required curve as of $y = xc'/(x+c'')$ }

Example: 10 cells, each of EMF E and internal resistance r are connected in series to a variable external resistance. Figure shows the variation of terminal potential difference with the current drawn from the combination. EMF of each cell is :



- a) 1.6 V
- b) 3.6 V
- c) 1.4 V
- d) 4.2 V

{Hint : The equation of the line is $V/42 + i/14 = 1$

Also, $V = 10ER/10r+R$

So, $10ER/42(10r+R) + i/14 = 1$

$10ER + 3i(10r+R) = 42(10r+R)$

As R is Variable, setting R as infinity

$10E + 3i = 42$

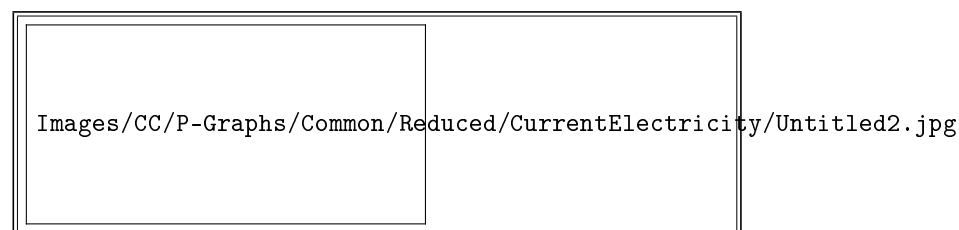
$E = (42 - 3i) / 10 = 4.2 - 0.3i$

Also at R infinity i would be zero as it is a series connected circuit

So, $E = 4.2$ V

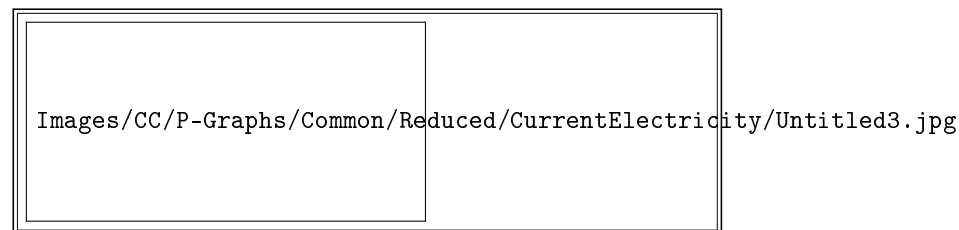
d) is the correct answer }

Example: In the circuit shown in figure, R3 is a variable resistance. As the value of R3 is changed, current I through the cell varies as shown. Obviously, the variation is asymptotic, i.e. $I \rightarrow 6A$ as $R_3 \rightarrow \infty$. Resistance R_1 and R_2 are respectively, :



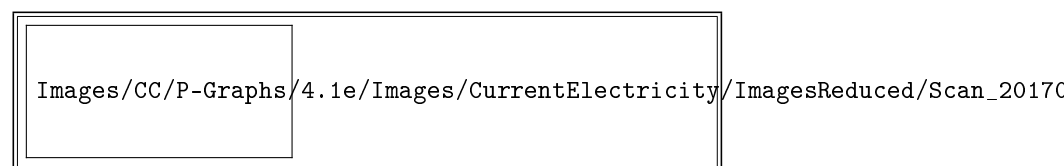
- a) $4\Omega, 2\Omega$
- b) $2\Omega, 4\Omega$
- c) $2\Omega, 2\Omega$
- d) $1\Omega, 4\Omega$

Example: The variation of current with potential difference is as shown in Figure. The resistance of the conductor is :



- a) 1Ω
- b) 2Ω
- c) 3Ω
- d) 4Ω

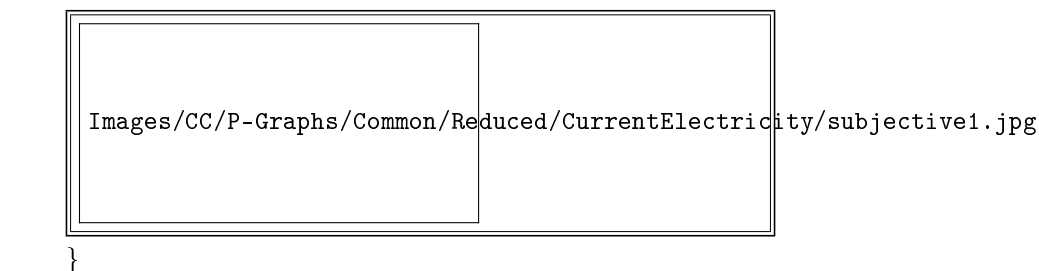
Example : The two ends of a uniform conductor are joined to a cell of emf E and some internal resistance. Starting from the midpoint P of the conductor, we move in the direction of the current and return to P. The potential V at every point on the path is plotted against the distance covered (x). Which of the following best represents the resulting curve?



26.2.2 Subjective

Example: Draw a) I vs R b) V vs I, characteristics for a cell.

{ Hint: For a cell, as $I = E/(R+r)$ and $V = E - Ir$,



Chapter 27

Capacitors

Mathematical treatment of charging and discharging a capacitor
schoolphysics.co.uk

Images/CC/P-Graphs/4.1e/Images/CurrentElectricity/ImagesReduced/1.jpg

27.1 Discharge of a capacitor

The area under the current-time discharge graph gives the charge held by the capacitor. The gradient of the charge-time graph gives the current flowing from the capacitor at that moment.

Images/CC/P-Graphs/4.1e/Images/CurrentElectricity/ImagesReduced/3.jpg

In Figure let the charge on a capacitor of capacitance C at any instant be q , and let V be the potential difference across it at that instant.

The current (I) in the discharge at that instant is therefore: $I = -dq/dt$

But $V = IR$ and $q = CV$ so $dq/dt = d(CV)/dt = C dV/dt$ Therefore we have $V = -CR dV/dt$ Rearranging and integrating gives:

Capacitor discharge (voltage decay): $V = V_0 e^{-(t/RC)}$

where V_0 is the initial voltage applied to the capacitor. A graph of this exponential discharge is shown below in Figure

Since $Q = CV$ the equation for the charge (Q) on the capacitor after a time t is therefore:

Capacitor discharge (charge decay): $Q = Q_0 e^{-(t/RC)}$

$V = V_0 e^{-(t/RC)}$ and also $I = I_0 e^{-(t/RC)}$ $Q = Q_0 e^{-(t/RC)}$

You should realise that the term RC governs the rate at which the charge on the capacitor decays.

When $t = RC$, $V = V_0/e = 0.37 V_0$ and the product RC is known as the time constant for the circuit. The bigger the value of RC the slower the rate at which the capacitor discharges.

The value of C can be found from this discharge curve if R is known.

27.2 Charging a capacitor

When a capacitor (C) is being charged through a resistance (R) to a final potential V_0 the equation giving the voltage (V) across the capacitor at any time t is given by:

Capacitor charging (potential difference): $V = V_0[1 - e^{-(t/RC)}]$

and the variation of potential with time is shown in Figure

Images/CC/P-Graphs/4.1e/Images/CurrentElectricity/ImagesReduced/7.jpg

As the capacitor charges the charging current decreases since the potential across the resistance decreases as the potential across the capacitor increases.

Figure shows how both the potential difference across the capacitor and the charge on the plates vary with time during charging.

The charging current would be given by the gradient of the curve in Figure at any time and the graph of charging current against time is shown in next Figure.

Images/CC/P-Graphs/4.1e/Images/CurrentElectricity/ImagesReduced/8.jpg

The area below the current-time curve in both charging and discharging represents the total charge held by the capacitor.

Part XIV

Magnetic Field

Chapter 28

Problems for Practice

28.1 General Problem Set

Example : A current i is uniformly distributed over the cross section of a long hollow cylindrical wire of inner radius R_1 and outer radius R_2 . Magnetic field B varies with distance r from the axis of the cylinder as

Images/CC/P-Graphs/4.1e/Images/MagneticField/ImagesReduced/Scan_20170419 (5).jpg

28.2 IIT Previous Years Problems

Example: A magnetic field $\vec{B} = B_o \hat{j}$ exists in the region $a < x < 2a$ and $\vec{B} = -B_o \hat{j}$ in the region $2a < x < 3a$ where B_o is a positive constant. A positive point charge moving with a velocity $\vec{v} = v_o \hat{i}$, where v_o is a positive constant, enters the magnetic field at $x=a$. The trajectory of the charge in this region can be like

Images/CC/P-Graphs/4.1e/Images/MagneticField/ImagesReduced/005 IIT20071.jpg

Images/CC/P-Graphs/4.1e/Images/MagneticField/ImagesReduced/005 IIT20071-Answer.jpg

Solution: Force experienced by the charge q is given by

$$\vec{F} = q (\vec{v} \times \vec{B})$$

In the region from $x=a$ to $x=2a$, the force is $\vec{F}_1 = q (v_o \hat{i} \times B_o \hat{j}) = qv_o B_o \hat{k}$ directed along the positive z -axis.

In the region from $x=2a$ to $x=3a$, the force is

$$\vec{F}_2 = q (v_o \hat{i} \times (-B_o) \hat{j}) = -qv_o B_o \hat{k}$$

directed along the negative z -axis.

Since force \vec{F}_1 and \vec{F}_2 are perpendicular to velocity \vec{v} , the correct trajectory is as shown in option a).

Part XV

Lattice

Chapter 29

Experiments

29.1 Photoelectric Effect

29.1.1 An Experiment vlab.amrita.edu

Aim:

1. To understand the phenomenon Photoelectric effect as a whole.
2. To draw kinetic energy of photoelectrons as a function of frequency of incident radiation.
3. To determine the Planck's constant from kinetic energy versus frequency graph.
4. To plot a graph connecting photocurrent and applied potential.
5. To determine the stopping potential from the photocurrent versus applied potential graph.

Theory:

During his experiments on electromagnetic radiation (to demonstrate light consists of e-m waves), Hertz noticed a spark between the two metallic balls when a high frequency radiation incident on it. This is called photoelectric effect. Photoelectric effect is the emission of electrons when electromagnetic radiations having sufficient frequency incident on certain metal surfaces. We call the emitted electrons as photoelectrons and the current they constitute as photocurrent. The phenomenon was first observed by Heinrich Hertz in 1880 and explained by Albert Einstein in 1905 using Max Planck's quantum theory of light. As the first experiment which demonstrated the quantum theory of energy levels, photoelectric effect experiment is of great historical importance.

The important observations on Photoelectric effect which demand quantum theory for its explanation are:

1. The Photoelectric effect is an instantaneous phenomenon. There is no time delay between the incidence of light and emission of photoelectrons.
2. The number of photoelectrons emitted is proportional to the intensity of incident light. Also, the energy of emitted photoelectrons is independent of the intensity of incident light.
3. The energy of emitted photoelectrons is directly proportional to the frequency of incident light. The basic experimental set up which explains Photoelectric effect is as given below,

Images/CC/P-Graphs/4.1e/Images/PhotoElectricEffect/ImagesReduced/PhotoelectricEffect(1).ip

It has been observed that there must be a minimum energy needed for electrons to escape from a particular metal surface and is called work function 'W' for that metal. The work function can be expressed in terms of frequency as,

$$W = h\nu_o \text{ --- (1)}$$

Where h is the Planck's constant and

ν_o is the threshold frequency (minimum frequency for photoelectric effect).

29.1.1.1 The work function for some metals are listed in the table.

Images/CC/P-Graphs/4.1e/Images/PhotoElectricEffect/ImagesReduced/workfunction(1).ip

29.1.1.2 Graph connecting photocurrent and applied reverse potential :

For constant intensity and different frequencies

Images/CC/P-Graphs/4.1e/Images/PhotoElectricEffect/ImagesReduced/photowave

For constant frequency and different intensities

Images/CC/P-Graphs/4.1e/Images/PhotoElectricEffect/ImagesReduced/11(2).jpg

29.1.2 Original Einstien Experiment physics.info

29.1.2.1 Discussion

dilemma Under the right circumstances light can be used to push electrons, freeing them from the surface of a solid. This process is called the photoelectric effect (or photoelectric emission or photoemission), a material that can exhibit this phenomena is said to be photoemissive, and the ejected electrons are called photoelectrons; but there is nothing that would distinguish them from other electrons. All electrons are identical to one another in mass, charge, spin, and magnetic moment.

The photoelectric effect was first observed in 1887 by Heinrich Hertz (1857–1894) during experiments with a spark-gap generator (the earliest form of radio receiver). In these experiments, a spark is generated between two small metal spheres in the transmitter to induce a similar spark to jump between two different metal spheres in the receiver. Compared to later radio devices, the spark-gap generator was notoriously difficult to work with. The air gap would often have to be smaller than a

millimeter for a the receiver to consistently reproduce the spark of the transmitter. Hertz found that he could increase the sensitivity of his spark-gap device by illuminating it with visible or ultraviolet light. Later studies by J.J. Thomson (1856–1940) showed that this increased sensitivity was the result of light pushing on electrons (which he discovered in 1897).

While this is interesting, it is hardly amazing. All forms of electromagnetic radiation transport energy and it is quite easy to imagine this energy being used to push tiny particles of negative charge free from the surface of a metal where they are not all that strongly confined in the first place. The era of modern physics is one of completely unexpected and inexplicable discoveries, however. Subsequent investigations into the photoelectric effect yielded results that did not fit with the classical theory of electromagnetic radiation. When it interacted with electrons, light just didn't behave like it was supposed to. Repairing this tear in theory required more than just a patch. It meant rebuilding a large portion of physics from the ground up.

Images/CC/P-Graphs/4.1e/Images/PhotoElectricEffect/ImagesReduced/circuit

It was Philipp Lenard (1862–1947), an assistant of Hertz, who performed the earliest, definitive studies of the photoelectric effect. Lenard used metal surfaces that were first cleaned and then held under a vacuum so that the effect might be studied on the metal alone and not be affected by any surface contaminants or oxidation. The metal sample was housed in an evacuated glass tube with a second metal plate mounted at the opposite end. The tube was then positioned or constrained in some manner so that light would only shine on the first metal plate — the one made out of photoemissive material under investigation. Such a tube is called a photocell (formally) or an electric eye (informally). Lenard connected his photocell to a circuit with a variable power supply, voltmeter, and microammeter as shown in the schematic diagram above. He then illuminated the photoemissive surface with light of differing frequencies and intensities.

Knocking electrons free from the photoemissive plate would give it a slight positive charge. Since the second plate was connected to the first by the wiring of the circuit, it was at the same electric potential, which would then attract the photoelectrons floating freely through the vacuum where they would land and return back to the plate from which they started. Keep in mind that this experiment doesn't create electrons out of light, it just uses the energy of the light to push electrons that are already there around the circuit. The photoelectric current generated by this means was quite small, but could be measured with the microammeter (a sensitive galvanometer with a maximum deflection of only a few microamps). It also serves as a measure of the rate at which photoelectrons are leaving the surface of the photoemissive material.

Note how the power supply is wired into the circuit — with its negative end connected to the plate that isn't illuminated. This sets up a potential difference that tries to push the photoelectrons back into the photoemissive surface. When the power supply is set to a low voltage it traps the least energetic electrons, reducing the current through the microammeter. Increasing the voltage drives increasingly more energetic electrons back until finally none of them are able to leave the metal surface and the microammeter reads zero. The potential at which this occurs is called the stopping potential. It is a measure of the maximum kinetic energy of the electrons emitted as a result of the photoelectric effect.

What Lenard found was that the intensity of the incident light had no effect on the maximum kinetic energy of the photoelectrons. Those ejected from exposure to a very bright light had the same energy as those ejected from exposure to a very dim light of the same frequency. In keeping with the law of conservation of energy, more electrons were ejected by a bright source than a dim source.

Later experiments by others, most notably the American physicist Robert Millikan (1865–1953), found that light with frequencies below a certain cutoff value, called the threshold frequency, would not eject photoelectrons from the metal surface no matter how bright the source. These result were completely unexpected. Given that it is possible to move electrons with light and given that the energy in a beam of light is related to its intensity, classical physics would predict that a more intense beam of light would eject electrons with greater energy than a less intense beam no matter what the frequency. This was not the case, however.

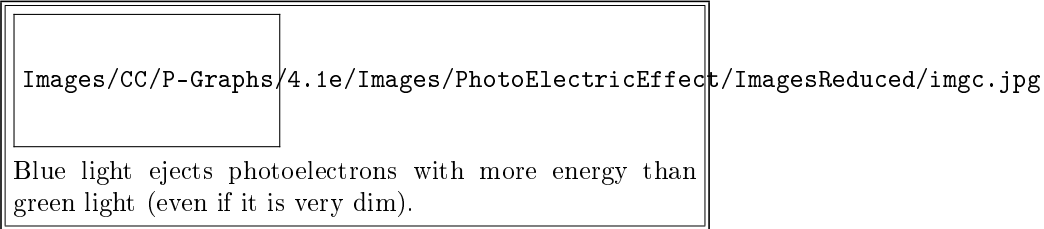
Results of a Typical Photoelectric Experiment

Images/CC/P-Graphs/4.1e/Images/PhotoElectricEffect/ImagesReduced/imga.jp

Red light does not eject photoelectrons (even if it is very bright).

Images/CC/P-Graphs/4.1e/Images/PhotoElectricEffect/ImagesReduced/imgb.jp

Green light does eject photoelectrons (even if it is very dim).

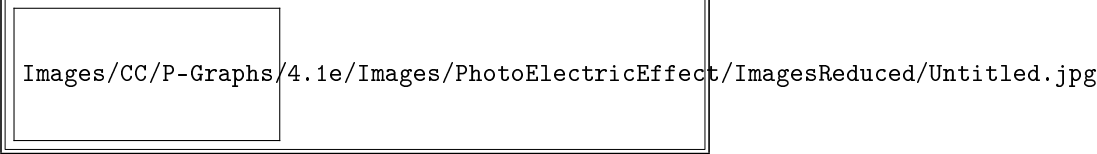


Actually, maybe these results aren't all that typical. Most elements have threshold frequencies that are ultraviolet and only a few dip down low enough to be green or yellow like the example shown above. The materials with the lowest threshold frequencies are all semiconductors. Some have threshold frequencies in the infrared region of the spectrum.

The classical model of light describes it as a transverse, electromagnetic wave. Of this there was very little doubt at the end of the Nineteenth Century. The wave nature of light was confirmed when it was applied successfully to explain such optical phenomena as diffraction, interference, polarization, reflection and refraction. If we can imagine light as waves in an electromagnetic ocean and be quite successful at it, then it wouldn't be much of a stretch for us to image electrons in a metal surface as something like tethered buoys floating in an electromagnetic harbor. Along come the waves (light) which pull and tug at the buoys (electrons). Weak waves have no effect, but strong ones just might yank a buoy from their mooring and set it adrift. A wave model of light would predict an energy–amplitude relationship and not the energy–frequency relationship described above. Photoelectric experiments describe an electromagnetic ocean where monstrous swells wouldn't tip over a canoe, but tiny ripples would fling you into the air.

If that wasn't enough, the photoelectrons seem to pop out of the surface too quickly. When light intensities are very low, the rate at which energy is delivered to the surface is downright sluggish. It should take a while for any one particular electron to capture enough of this diffuse energy to free itself. It should, but it doesn't. The instant that light with an appropriate frequency of any intensity strikes a photoemissive surface, at least one electron will always pop out immediately ($t < 10^{-9}$ s). Continuing with the ocean analogy, imagine a harbor full of small boats (electrons). The sea is calm except for tiny ripples on the surface (low intensity, short wavelength light). Most of the boats in the harbor are unaffected by these waves, but one is ripped from the harbor and sent flying upward like a jet aircraft. Something just ain't right here. No mechanical waves behave like this, but light does.

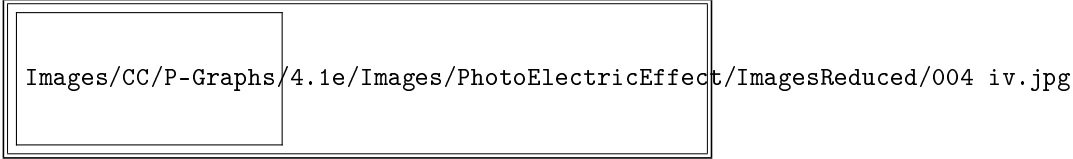
new idea The two factors affecting maximum kinetic energy of photoelectrons are the frequency of the incident radiation and the material on the surface. As shown in the graph below, electron energy increases with frequency in a simple linear manner above the threshold. All three curves have the same slope (equal to Planck's constant) which shows that the energy–frequency relation is constant for all materials. Below the threshold frequency photoemission does not occur. Each curve has a different intercept on the energy axis, which shows that threshold frequency is a function of the material.



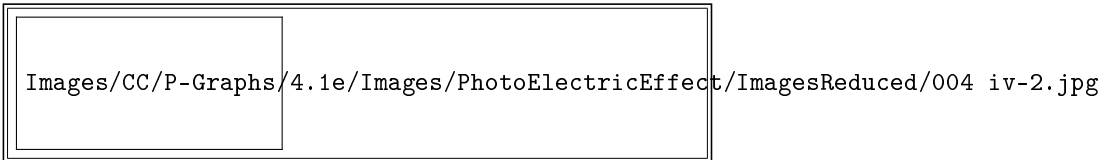
The genius that figured out what was going on here was none other than the world's most famous physicist Albert Einstein. In 1905, Einstein realized that light was behaving as if it was composed of tiny particles (initially called quanta and later called photons) and that the energy of each particle was proportional to the frequency of the electromagnetic radiation that it was a part of. Recall from the previous section of this book that Max Planck invented the notion of quantized electromagnetic radiation as a way to solve a technical problem with idealized sources of electromagnetic radiation called blackbodies. Recall also that Planck did not believe that radiation was actually broken up into little bits as his mathematical analysis showed. He thought the whole thing was just a contrivance that gave him the right answers. The genius of Einstein was in recognizing that Planck's contrivance was in fact a reasonable description of reality. What we perceive as a continuous wave of electromagnetic radiation is in reality a stream of discrete particles.

29.1.3 Graphs of Photoelectric Current vs Voltage

29.1.3.1 For radiation of different Intensities ($I_1 > I_2$) but the same frequency.



29.1.3.2 For radiation of different frequencies ($\nu_1 > \nu_2$) but of the same intensity.



Chapter 30

Problems

30.1 Previous Years IIT Problems

30.1.1 Single Answer Questions

Example: Photoelectric effect experiments are performed using three different metal plates p,q and r having work functions $\phi_p = 2.0\text{eV}$. $\phi_q = 2.5\text{eV}$. $\phi_r = 3.0\text{eV}$ respectively. A light beam containing wavelengths of 550nm, 450nm and 350nm with equal intensities illuminates each of the plates. The correct I-V graph for the experiment is [Take $hc=1240\text{ eV nm}$]



Part XVI

Nucleus

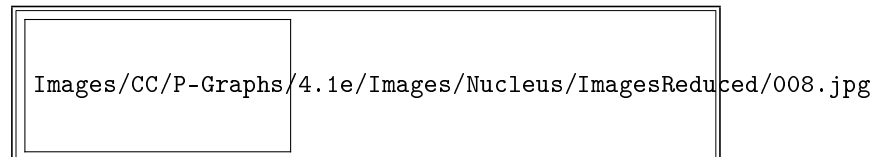
Chapter 31

Problems

31.1 Previous Years IIT Problems

31.1.1 Multiple Answer

Example: Assume that the nuclear binding energy per nucleon (B/A) versus mass number is as shown in the figure. Use this plot to choose the correct choice (s) given below.

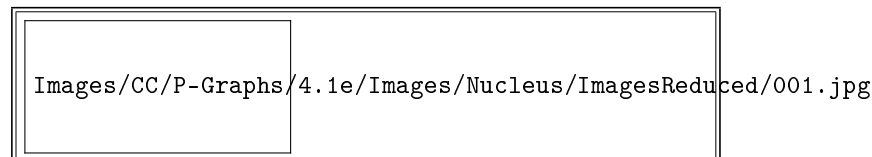


- a) Fusion of two nuclei with mass numbers lying in the range $1 < A < 50$ will release energy
- b) Fusion of two nuclei with mass numbers lying in the range of $51 < A < 100$ will release energy
- c) Fission of a nucleus lying in the mass number range of $100 < A < 200$ will release energy when broken into equal fragments
- d) Fission of a nucleus lying in the mass number range of $200 < A < 260$ will release energy when broken into equal fragments

{ Solution: Energy is released if the total binding energy of the products is greater than the total binding energy of the reactants. This is not possible in choices a) and c). The correct choices are b) and d). }

31.1.2 Integer Type

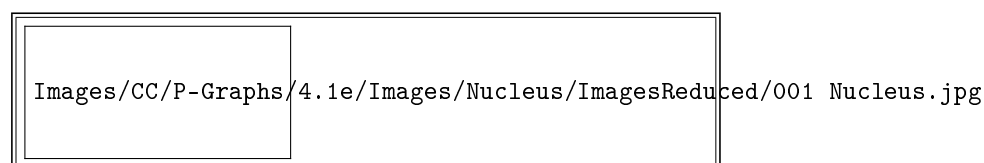
Question : To determine the half life of radioactive element, a student plots a graph of $\ln \left| \frac{dN(t)}{dt} \right|$ vs t. Here $\frac{dN(t)}{dt}$ is the rate of radioactive decay at time t. If the number of radioactive nuclei of this element decreases by a factor of p after 4.16 years, find the value of p.



{ Answer: 8 }

31.1.3 Paragraph

Question: The nuclear charge (Ze) is non-uniformly distributed within a nucleus of radius R. The charge density (ρ) (charge per unit volume) is dependent only on the radial distance r from centre of the nucleus as shown in figure. The electric field is only along the radial direction.



1. The electric field at $r=R$ is

- a) independent of a
- b) directly proportional to a
- c) directly proportional to a^2
- d) inversely proportional to a

{ Solution: The charge $q=Ze$ can be assumed to be concentrated at the centre of the nucleus. The electric field at $r=R$ is

$$E = \frac{q}{4\pi\epsilon_0 R^2} = \frac{Ze}{4\pi\epsilon_0 R^2}$$

which is a constant. Hence the correct choice is a) . }

2. For $a=0$, the value of d (maximum value of ρ as shown in the figure) is

- a) $\frac{3Ze}{4\pi R^3}$
- b) $\frac{3Ze}{\pi R^3}$
- c) $\frac{4Ze}{3\pi R^3}$
- d) $\frac{Ze}{3\pi R^3}$

{ Solution: Total charge is

$$\begin{aligned} q &= \int_0^R 4\pi r^2 \left(d - \frac{d}{R}r \right) dr \\ &= 4\pi \left[d \int_0^R r^2 dr - \frac{d}{R} \int_0^R r^3 dr \right] \\ &= \frac{\pi d R^3}{3} \end{aligned}$$

$$\text{Now that } d = \frac{3Ze}{\pi R^3}$$

So, the correct choice is b) }

3. The electric field within the nucleus is generally observed to be linearly dependent on r. This implies

- a) $a=0$
- b) $a=R/2$
- c) $a=R$
- d) $a=2R/3$

{ Solution: For spherical charge distribution, the electric field is linearly dependent on r if the charge density ρ is uniform, i.e. $a=R$. Hence the correct choice is c). }

Part XVII

Ray Optics

Chapter 32

Theory

32.1 Graphs for convex and concave lenses

(Real is positive sign convention) schoolphysics.co.uk

Figure shows a graph where the reciprocal of the image distance is plotted against the reciprocal of the object distance. The graph is a straight line that intercepts both axes at $1/f$ where f is the focal length of the lens. Since the object is real this graph is for a convex lens.

Images/CC/P-Graphs/4.1e/Images/RayOptics/1.jpg

Figure shows a graph of the object distance plotted against the image distance for both convex and concave lenses. Both real and virtual objects and images are shown.

Images/CC/P-Graphs/4.1e/Images/RayOptics/2.jpg

32.1.1 Enlarged view of an object distance (u) against image distance (v) graph

The focal length of the lens is 10 cm.

The graph is completely symmetrical so that when $u = 2f$, v also equals $2f$.

Images/CC/P-Graphs/4.1e/Images/RayOptics/3.jpg

32.1.2 Minimum distance

The next graph shows the distance between the object and image ($u+v$) plotted against the object distance (u) (it could equally well have been v).

The minimum value for ($u+v$) is $4f$ when $u=v=2f$. This means that no image can be formed with a convex lens of focal length 10 cm if the object and the screen are closer than 40 cm.

Images/CC/P-Graphs/4.1e/Images/RayOptics/4.jpg

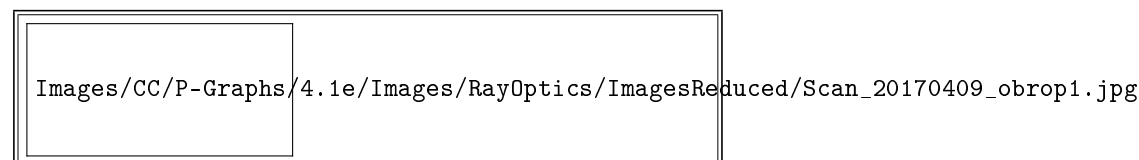
Chapter 33

Problems for Practice

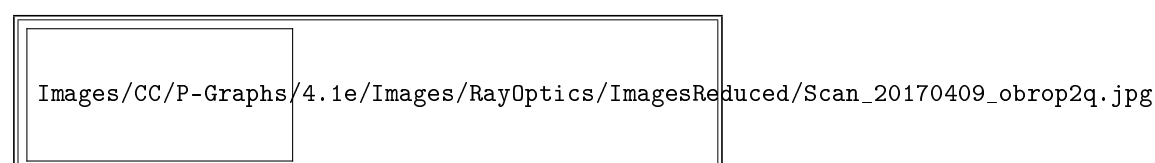
33.1 General Problem Set

33.1.1 Single Answer Type

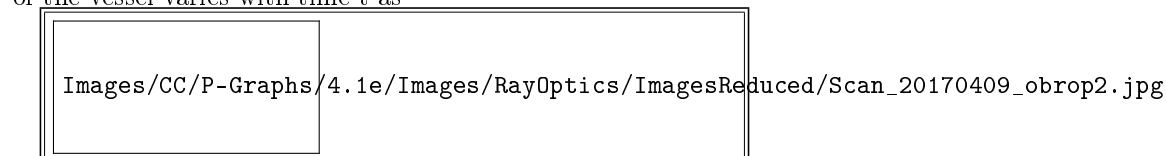
Example : As the position of an object (u) reflected from a concave mirror is varied, the position of the image (v) also varies. By letting the u change from 0 to $+\infty$ the graph between v versus u will be



Example : A point source S is placed at a height h from the bottom of a vessel of height H ($< h$). The vessel is polished at the base. Water is gradually filled in the vessel at a constant rate $\alpha m^3/s$.

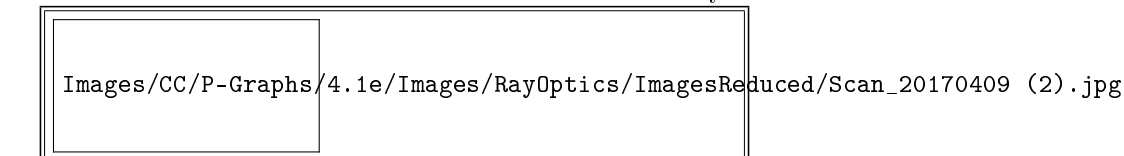


The distance d of image of the source after reflection from mirror from the bottom of the vessel varies with time t as

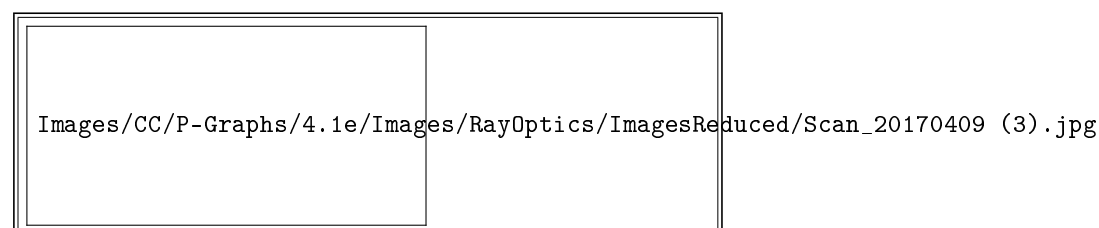


33.1.2 Multiple Answer Type

Example 1: The image distance (v) is plotted against the object distance (u) for a concave mirror of focal length f . Which of the graphs shown in Figure represents the variation of v versus u as u is varied from zero to infinity?



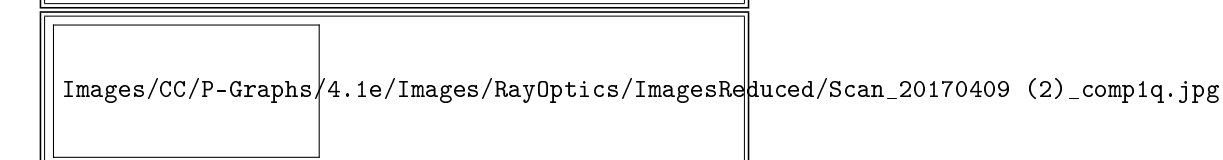
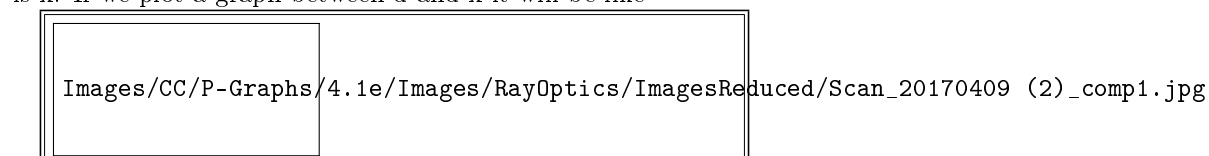
Example : A student measures the focal length of a convex lens by putting an object pin at a distance U from the lens and measuring the distance v of the image pin. Graph between u and v plotted by the student should look like :



33.1.3 Comprehension Type

33.1.3.1 Comprehension 1

Liquid is filled in a vessel of height $2H/3$. At the bottom of the vessel there is a spot P and a hole from which liquid is coming out. Let d be the distance of image of P from an eye at height H from bottom at an instant when level of liquid in the vessel is x . If we plot a graph between d and x it will be like



Part XVIII

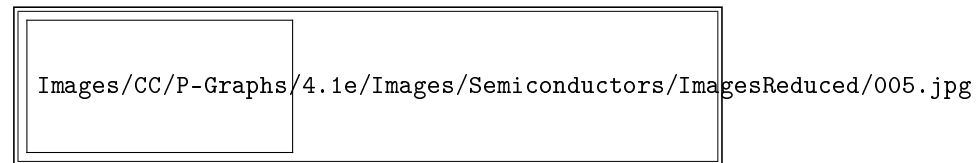
Semiconductors

Chapter 34

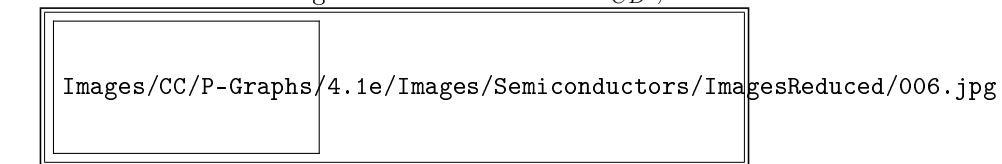
Diode

34.1 Rectifier

Example: In the half-wave rectifier circuit shown:

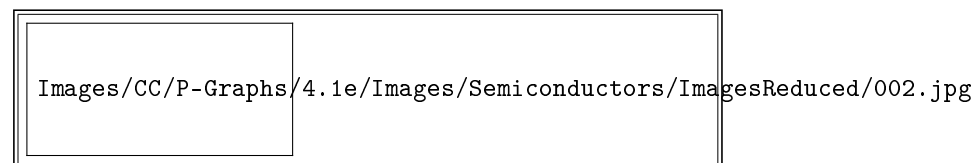


Which one of the following wave forms is true for V_{CD} , the



34.2 Junction Diode

Example: The resistance of a germanium junction diode, whose V-I is shown in figure is: ($V_k = 0.3V$)

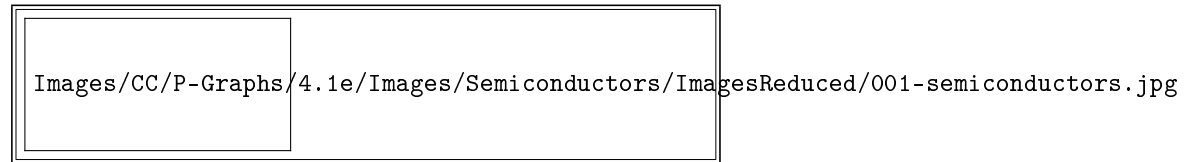


- a) $5\ k\Omega$
- b) $0.2\ k\Omega$
- c) $2.3\ k\Omega$
- d) $\left(\frac{10}{2.3}\right)k\Omega$

Chapter 35

Triode

Example: The characteristic of triode shown in Figure. is known as :

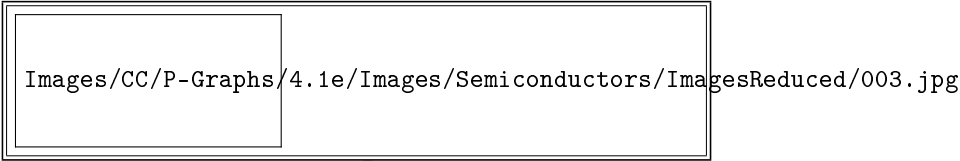


- a) mutual characteristic
- b) transfer characteristic
- c) static plate characteristic
- d) voltage transfer characteristic

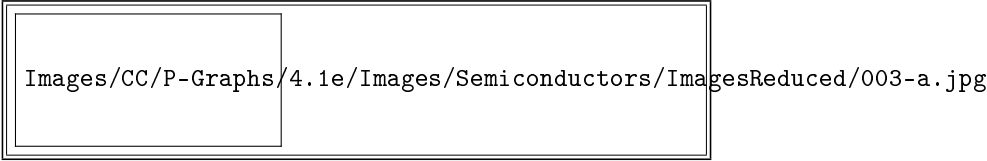
Chapter 36

p-n Junction

If in a p-n junction diode, a square input signal of 10 V is applied as shown



Then the output signal across R_L will be:



{Answer: a) }

Chapter 37

Transistors

Example: The output current versus time curve of a rectifier is shown in figure.
The average value of the output current in this case is:



- a) 0
- b) $\frac{I_0}{2}$
- c) $\frac{2I_0}{\pi}$
- d) I_0

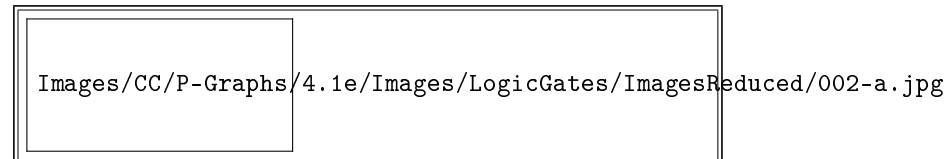
Part XIX

Logic Gates

Chapter 38

Problems

Example: The following figure shows a logic gate circuit with two inputs A and B and the output C. The voltage wave forms of A, B and C are as shown below:



The logic circuit gate is

- a) AND gate
- b) NAND gate
- c) NOR gate
- d) OR gate

{ Answer: a) }

Economics

Part XX

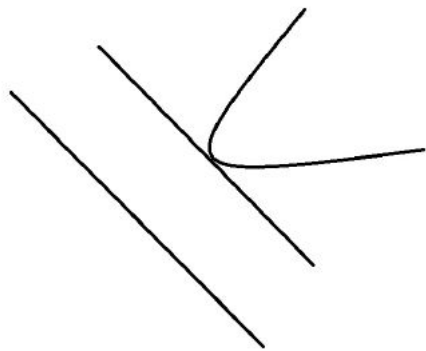
Microeconomics

Chapter 39

The Complete Theory in three graphs ¹

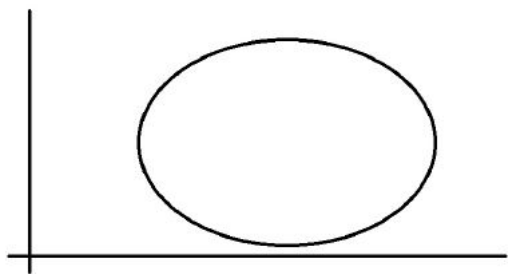
39.1 The Indian Economics

Indian economics and all it's relations get a graph of this form . It's called the Caviat.



39.2 Outside India Economics

It's called the BLOB. This rule governs all the outside india economics. It's strage the economics in India is completely different from that of Outside India.



39.3 India to Outside and reverse relations in Market

It's the Starfish relation. Draw a starfish in the middle of a paper and you would get to know about it. Plus it can lose a leg too.

Don't worry , right now my scanner is not working, I'd have to buy a new one soon. Because of scanner inavailability my Physics Graphs book is also not proceeding furthur. So, keep a eye on furthur editions , I'll make microeconomics an elaborate section with plenty of examples and also be giving proofs to the Caviat and BLOB and Starfish theormys.

Chapter 40

Conventional Terminology

40.1 Production Frontier [tutor2u.net](#)

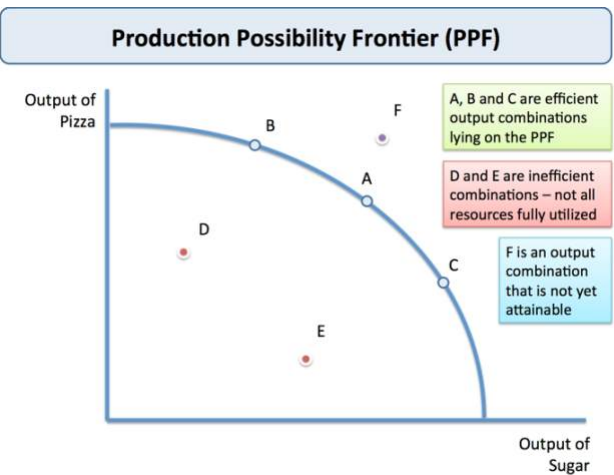
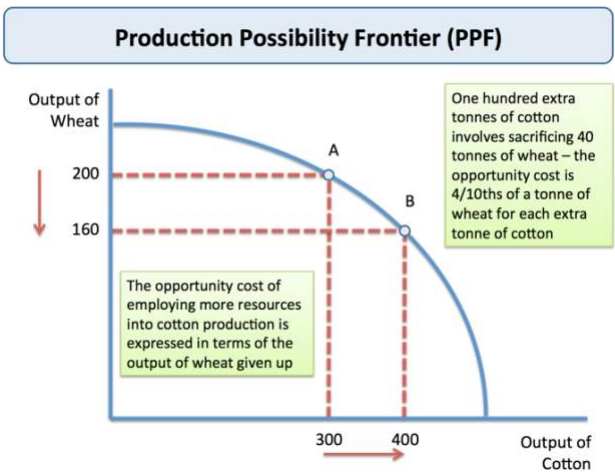
A production possibility frontier (PPF) shows the maximum possible output combinations of two goods or services an economy can achieve when all resources are fully and efficiently employed

40.1.1 Opportunity Cost and the PPF

Reallocating scarce resources from one product to another involves an opportunity cost

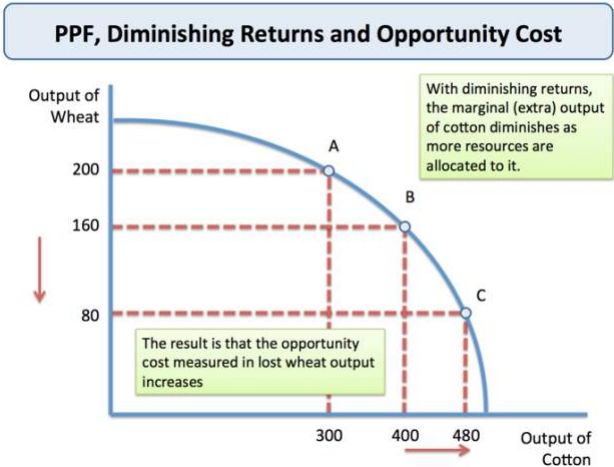
If we increase our output of consumer goods (i.e. moving along the PPF from point A to point B) then fewer resources are available to produce capital goods'

If the law of diminishing returns holds true then the opportunity cost of expanding output of X measured in terms of lost units of Y is increasing.



We normally draw a PPF on a diagram as concave to the origin i.e. as we move down the PPF, as more resources are allocated towards Good Y the extra output gets smaller – so more of Good X has to be given up in order to produce Good Y

This is an explanation of the law of diminishing returns and it occurs because not all factor inputs are equally suited to producing items



40.1.2 PPF and Economic Efficiency

40.1.2.1 Production Possibilities

A production possibility frontier is used to illustrate the concepts of opportunity cost, trade-offs and also show the effects of economic growth.

Points within the curve show when a country's resources are not being fully utilised

Combinations of the output of consumer and capital goods lying inside the PPF happen when there are unemployed resources or when resources are used inefficiently. We could increase total output by moving towards the PPF

Combinations that lie beyond the PPF are unattainable at the moment

A country would require an increase in factor resources, an increase in the productivity or an improvement in technology to reach this combination.

Trade between countries allows nations to consume beyond their own PPF.

Producing more of both goods would represent an improvement in welfare and a gain in what is called allocative efficiency.

40.2 Budget Line, Constraints map and the condition of tangency [courses.byui.edu](#)

In this section, we are going to take a closer look at what is behind the demand curve and the behavior of consumers. How does a consumer decide to spend his/her income on the many different things that he/she wants, i.e., food, clothing, housing, entertainment? We assume that the goal of the consumer is to maximize his/her level of satisfaction or joy, constrained by his/her income.

Economists use the term utility as a measure of satisfaction, joy, or happiness. How much satisfaction does a person gain from eating a pizza or watching a movie? Measuring utility is based solely on the preferences of the individual and has nothing to do with the price of the good. Let's do an experiment in utility.

Step 01: Get some of your favorite candy, pastries, or cookies.

Step 02: Take a bite and evaluate, on a scale from 0 to 100 (with 100 being the greatest utility), the level of utility from that bite. Record the marginal utility of that bite (i.e., how much you get from that one additional bite).

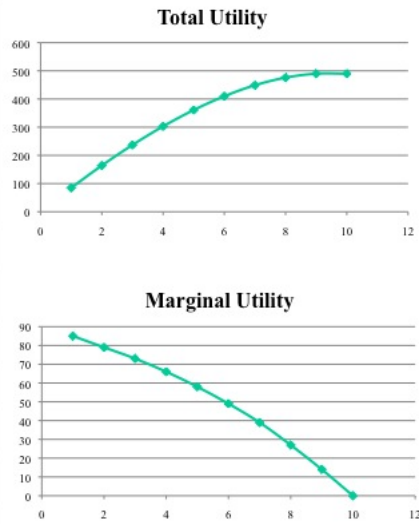
Step 03: Repeat step 02. It is important to be consistent with each unit consumed, i.e., the same size and no drinking milk or water part way though. When you run out of candy or your marginal utility goes to zero you can stop.

Law of Diminishing Marginal Utility

The law of diminishing marginal utility states that as more of the good is consumed, the additional satisfaction from another bite will eventually decline. The marginal utility is the satisfaction gained from each additional bite. As more of the good is consumed, we gain less additional satisfaction from consuming another unit. Thus even if a good were free and you could consume as much as you wanted, there would be a limit to the amount you would consume due to the law of diminishing marginal utility.

Summing the marginal utilities gives us the total utility. For example, let's say the first chocolate was an 85 and the second chocolate had a marginal utility of 79, then the total utility from consuming two chocolates is 164. The total utility from consuming three chocolates is $85 + 79 + 73 = 237$. As long as our marginal utility is positive our total utility increases although with diminishing marginal utility it increases at a decreasing rate.

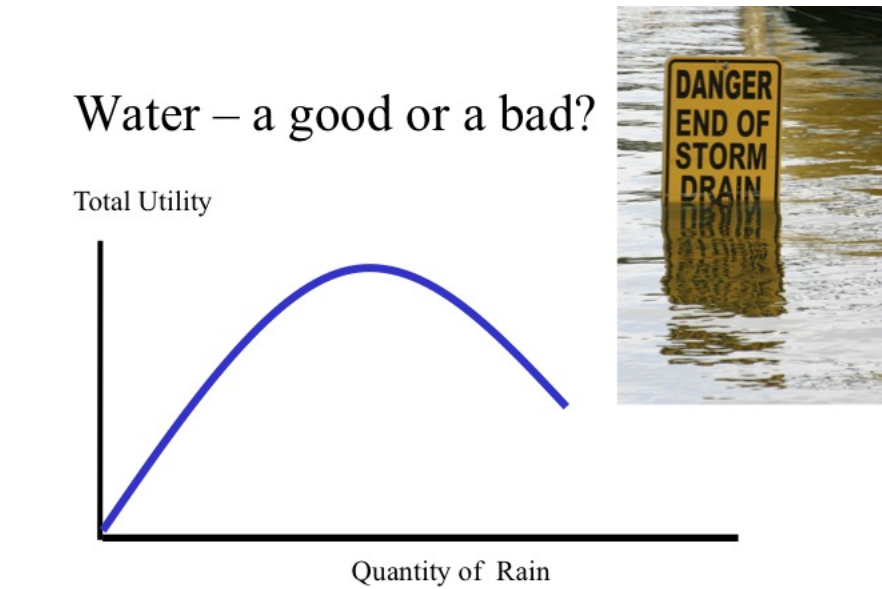
Quantity	Marginal Utility	Total Utility
1	85	85
2	79	164
3	73	237
4	66	303
5	58	361
6	49	410
7	39	449
8	27	476
9	14	490
10	0	490



Can marginal utility be negative? Yes. At a holiday dinner, you may overeat and suffer from indigestion afterwards to a point where you regret having eaten too much, but at the time of the dinner, you expected greater utility from eating the last of the meal. We would not willingly consume an item that gave us negative marginal utility. Then why would an individual stuff themselves during a hot dog eating contest where clearly the last hot dogs consumed are making them worse off? Although the marginal utility from the last hot dog itself makes the person worse off, the utility from winning the contest is greater making the marginal utility positive.

The marginal utility of an item can change. For example, during a drought water provides a high positive marginal utility, and with more rain the marginal utility declines. At some point, there is too much rain, it turns from being a good utility

to a bad one and the marginal utility of more rain, when it is already flooding, is negative.



40.2.1 Maximizing Utility

Utility values can be determined by an individual ranking his/her preferences from least preferred to most preferred. The resulting ranking or utility values are subjective or individual. They are also ordinal rather than cardinal. Ordinal means that the utility values simply define a ranking of preferences rather than an actual cardinal measurement.

Imagine a class has 10 students in the class and the teacher lined the students up according to height. He then numbered them off according to height, assigning the shortest student a 1 and the tallest student a 10. Is it true that student number 4 is twice tall as student number 2? Of course not. All we know from the ranking is that student number 4 is taller than student number 2. Now, imagine that in another class another teacher has also ranked 10 students according to height. Is it true that student number 10 from the second classroom is taller than student 1 from the first classroom? We cannot say since the ranking is only valid within a particular course. In order to say a student is twice as tall as another student or to be able to compare students between classes, we would need a cardinal measure of height like inches or centimeters.

Since utility is ordinal and not cardinal we cannot make interpersonal comparisons of utility. Does a rich person value a dollar more or less than a poor person? While some would say that a poor person likely has more unmet needs and thus he would value the dollar more. The answer is that it all depends on the preferences of the individuals. A poor person may prefer to live a more simplistic life and place a lower value on having an extra dollar than a rich person who has a “love of money” (1 Timothy 6:10). We simply cannot make such an interpersonal comparison of utility.

So how does the consumer decide what to purchase? Unfortunately everything has a price and consumers only have so much money to spend. Consequently consumers try to spend the limited money they have on what will give them the greatest amount of satisfaction. The decision rule for utility maximization is to purchase those items that give the greatest marginal utility per dollar and are affordable or within the budget. Many grocery stores provide a tag that indicates the price per pound for the good. This allows consumers to compare the cost per pound for different brands or different sizes. The same concept is used for maximizing utility but we divide the marginal utility by the price to get the marginal utility per dollar.

Maximizing Utility

UNIT PRICE

\$1.75

PER POUND

RETAIL PRICE

\$2.19

20 oz.

SB Raisin Bran

20290 ||||| 09/04

Let’s say that we eat only two goods: milk shakes and pizza where the price of each slice of pizza is \$2 and the price of each shake is \$1 and we only have \$11 to spend. Since the price of each good is different we need to divide the marginal utility by the price to allow for a common comparison. We then compare the marginal utility per dollar for pizzas verses shakes. For the first unit the marginal utility per dollar of a shake is 50 compared to only 45 for the pizza, so we would purchase the first shake. We then compare the marginal utility per dollar of the first pizza (45) to the marginal utility per dollar of the second shake (40) and purchase the first slice of pizza. If the marginal utility per dollar is the same for the two goods and we have income to purchase both then we would do so, as seen in the second slice of pizza and the second shake.

Income = \$11

$$\frac{MU_{pizza}}{P_{pizza}} = \frac{MU_{shakes}}{P_{shakes}}$$

Price:	\$2.00			\$1.00
Quantity	MU of Pizza	MU _{pz} /P _{pz}	MU of Shakes	MU _{shakes} /P _{shakes}
1	90	45	50	50
2	80	40	40	40
3	70	35	30	30
4	60	30	20	20
5	50	25	15	15
6	40	20	10	10

We continue to allocate on budget on those goods that yield the highest marginal utility per dollar. In this example, we would purchase four slices or pizza and three milkshakes and spend our entire budget of \$11. The total utility from this purchase would be the sum of the marginal utilities: 50 + 90 + 80 + 40 + 70 + 60 + 30 = 420. At the last items purchased the marginal utility per dollar spent on the two goods is the same, no other combination of pizzas and milk shakes will give us greater utility given our budget.

Income = \$11

$$\frac{MU_{pizza}}{P_{pizza}} = \frac{MU_{shakes}}{P_{shakes}}$$

Price:	\$2.00			\$1.00
Quantity	MU of Pizza	MU _{pz} /P _{pz}	MU of Shakes	MU _{shakes} /P _{shakes}
1	90	(2 nd) 45	50	(1 st) 50
2	80	(3 rd or 4 th) 40	40	(3 rd or 4 th) 40
3	70	(5 th) 35	30	(6 th or 7 th) 30
4	60	(6 th or 7 th) 30	20	20
5	50	25	15	15
6	40	20	10	10

40.2.2 Practice

Here is your chance to practice. Holly has \$20 to spend on either movies or bowling and wants to maximize her utility. Complete the table and determine how many movies and rounds of bowling will maximize her utility.

Practice - Income = \$20

Price:	\$8.00			\$4.00
Quantity	MU of Movies	MU/P _{movies}	MU of Bowling	MU/P _{bowling}
1	100		60	
2	80		40	
3	60		30	

Our first step is to divide the marginal utility of each item by the price. With an income of \$20 she is limited in what she can purchase. Since the marginal utility per dollar for bowling (15) is greater than the marginal utility of the first movie (12.5) she would initially go bowling, spending \$4. Comparing the first movie (12.5) to the second round of bowling (10) she would go to the movie spending an additional \$8 and a total of \$12. The decision is a little harder. Holly has eight dollars still to spend and the marginal utility per dollar is the same for each good. If she chooses the movie she will spend all eight dollars, but if she goes bowling she will spend four dollars and still have four to spend. We have assumed that she wants to spend all her money and gains no utility from holding the cash. Since she can’t afford to buy another movie but would instead by a third round of bowling, which only has a marginal utility per dollar of 7.5. Thus we see that to maximize her utility, she would purchase one round of bowling and two movies giving her a total utility of (60 + 100 + 80) = 240. Recall our decision rule is to have the marginal utility per dollar spent on the last items be the same for all goods. In this case, we are unable to have that exactly but we try to get as close as possible. There is no other combination that would give us greater utility given our income.

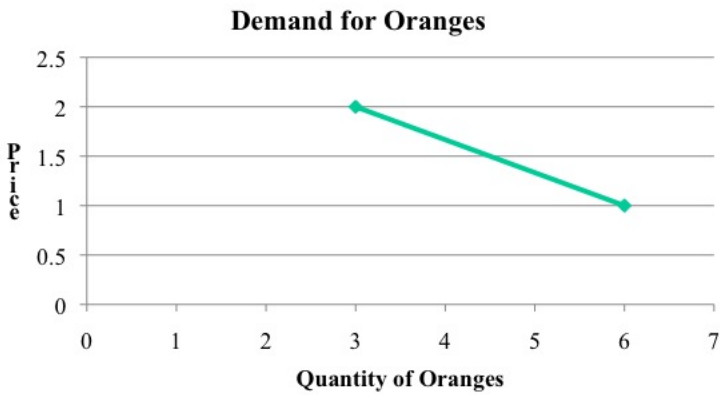
Practice Answer:

Income \$20

$$\frac{MU_{movies}}{P_{movies}} = \frac{MU_{bowling}}{P_{bowling}}$$

Price:	\$8.00			\$4.00
Quantity	MU of Movies	MU/P _{movies}	MU of Bowling	MU/P _{bowling}
1	100	12.5	60	15
2	80	10	40	10
3	60	7.5	30	7.5

Deriving the Demand Curve



The demand curve can be seen in the diamond-water paradox. Why does water that is essential to sustain life cost so much less than diamonds that are atheistically pleasing, but are relatively unnecessary? Recall that price reflects the scarcity of a good. Overall, the supply of water is relatively abundant while the supply of diamonds is relatively limited. Thus the price we pay for water is low compared to the price of diamonds.

Is it logical for someone who is maximizing his utility to purchase both water and diamonds? When deciding what to purchase we compare the marginal utility divided by the price. With lots of water consumption, the total utility of water is very large but the marginal utility of the last gallon consumed is relatively low. Few diamonds are purchased so while the marginal utility is very large, say the diamond ring you just purchased for your future spouse, the total utility is low since few diamonds are purchased.

40.2.3 Deriving Demand

Knowing how the consumer behaves allows us to derive a demand curve. Let's say that Suzette eats either an apple or an orange as a snack. She has \$12 to spend. Given that each fruit costs two dollars, she will maximize her utility by purchasing 3 apples and 3 oranges. If we are looking at the demand for oranges, this will give us one point on the demand curve. At a price of \$2.00, the quantity demanded of oranges is 3.

Deriving Demand: Income = \$12

Price:	\$2.00		\$2.00	
Quantity	MU of Apples	MU/P _{apples}	MU of Oranges	MU/P _{oranges}
1	90	(1 st)	45	70 (2 nd or 3 rd)
2	70	(2 nd or 3 rd)	35	60 (4 th)
3	50	(5 th or 6 th)	25	50 (5 th or 6 th)
4	30		15	40
5	20		10	30
6	10		5	20
7	5		2.5	12
8	0		0	5

Recall that as we move along the demand curve, the only thing that changes is the price of the good (ceteris paribus or holding all else constant). If the price of oranges decreases to \$1, the quantity of oranges demanded increases to 6.

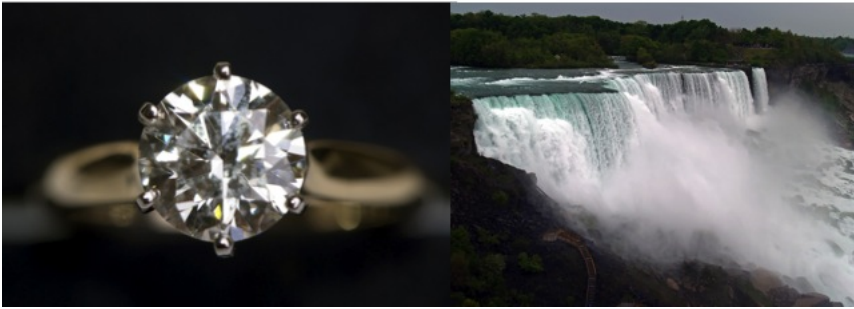
Deriving Demand: Income = \$12

Price:	\$2.00		\$1.00	
Quantity	MU of Apples	MU/P _{apples}	MU of Oranges	MU/P _{oranges}
1	90	(4 th)	45.00	70 (1 st)
2	70	(5 th)	35.00	60 (2 nd)
3	50	(8 th)	25.00	50 (3 rd)
4	30		15.00	40 (6 th)
5	20		10.00	30 (7 th)
6	10		5.00	20 (9 th)
7	5		2.50	12
8	0		-	5

We can plot the two points and create a demand curve for oranges. At a price of \$2 the quantity demanded is 3 and at a price of \$1 the quantity demanded is 6. Recall that the demand curve reflects the marginal benefit or the willingness to pay of the consumer.

Diamond-Water Paradox

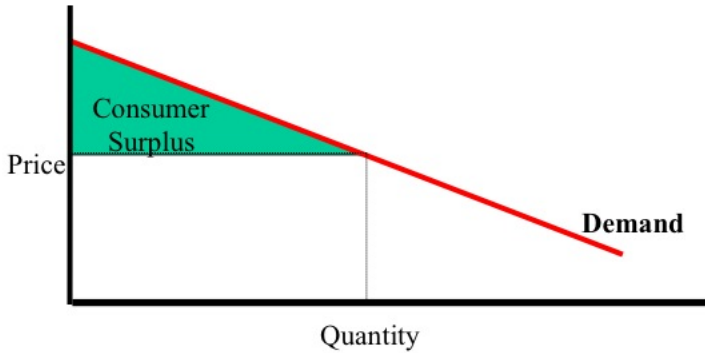
$$\frac{MU_{diamonds}}{P_{diamonds}} = \frac{MU_{water}}{P_{water}}$$



40.2.4 How Businesses React

Knowing that individuals experience diminishing marginal utility, how do businesses react? Recall that consumer surplus is the area below the demand curve but above the price. Think of some examples of how businesses react given the law of diminishing marginal utility.

Think



One example is the price per unit based on package size. An ice cream store has three different serving sizes - a 6, 10, and 12 ounce cup. The price of the smallest size, "Like It," is \$4.29 or 71.5 cents per ounce. For just 32 cents more, one can have four more ounces, "Love It," making the marginal cost per ounce 8 cents and the average cost per ounce 46 cents. Upgrading to the "Gotta Have It" size adds an additional two ounces with only 15.5 cents per ounce more and an average cost per ounce of only 41 cents. Certainly the large size is cheaper per ounce, but not everyone wants to eat that large of a serving. For those only wanting a small serving, the store takes advantage of their greater willingness to pay for that portion size. Whether its ice cream, eggs, milk, popcorn, or cereal, it is common practice to charge a higher price per unit for a smaller package size. However it pays for consumers to do the math since businesses will at times charge a higher price on the larger packages size. If customers believe that bigger is always cheaper and fail to do the math, they may get caught paying a higher price per unit.

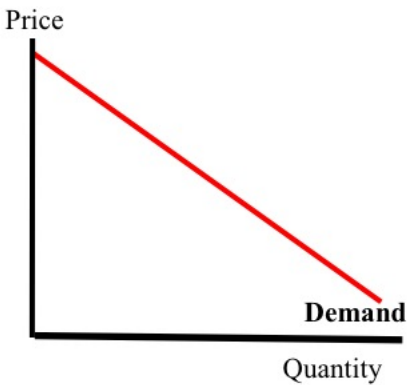
Ice Cream Store

			Marginal	Average
Size	Oz	Price	\$/oz.	\$/oz
Like It	6	4.29	0.715	0.715
Love It	10	4.61	0.08	0.461
Gotta Have It	12	4.92	0.155	0.41

Services often follow a similar pricing scheme with lower average prices for more frequent attendance. For example, below are the prices for the “hopper pass” at Disneyland. Tickets to sporting events follow a similar pricing approach with the per game price being lower if multiple games are purchased, such as the season pass.

Disneyland Passes

Days	Price	Price per day
1	\$97.00	\$97.00
2	\$151.00	\$75.50
3	\$204.00	\$68.00
6	\$254.00	\$42.33



Consider this example. You are on a long airplane ride, seated next to an eccentric looking woman and a businessman. Halfway into the flight, the woman says to you and the businessman, that she is very rich and bored of flying. To break up the monotony, she offers you and the businessman a chance to split \$5,000. The rules are as follows: the businessman makes an offer of how to split the money and you either accept or reject. If you accept, you get the agreed upon split. If you don’t, you both get nothing. This is a one time offer. The businessman thinks and offers the following split \$4,995 for him and \$5 for you. Do you accept or reject the offer? Why?

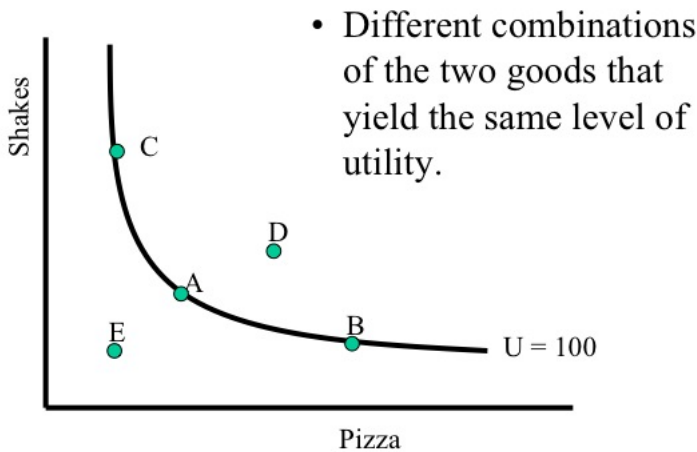
The answer to these questions will vary among individuals. Some will accept stating they have five dollars more than they did before. Others will reject the offer, saying that it is worth at least five dollars to them to deny they businessman the \$4,995. Remember that when we talk about utility, it includes not only monetary items but also the nonmonetary.

In The Theory of Moral Sentiments, Adam Smith wrote: "How selfish soever man may be supposed, there are evidently some principles in his nature which interest him in the fortune of others and render their happiness necessary to him though he derives nothing from it except the pleasure of seeing it." Remember that utility is derived from many different areas including service and philanthropic acts.

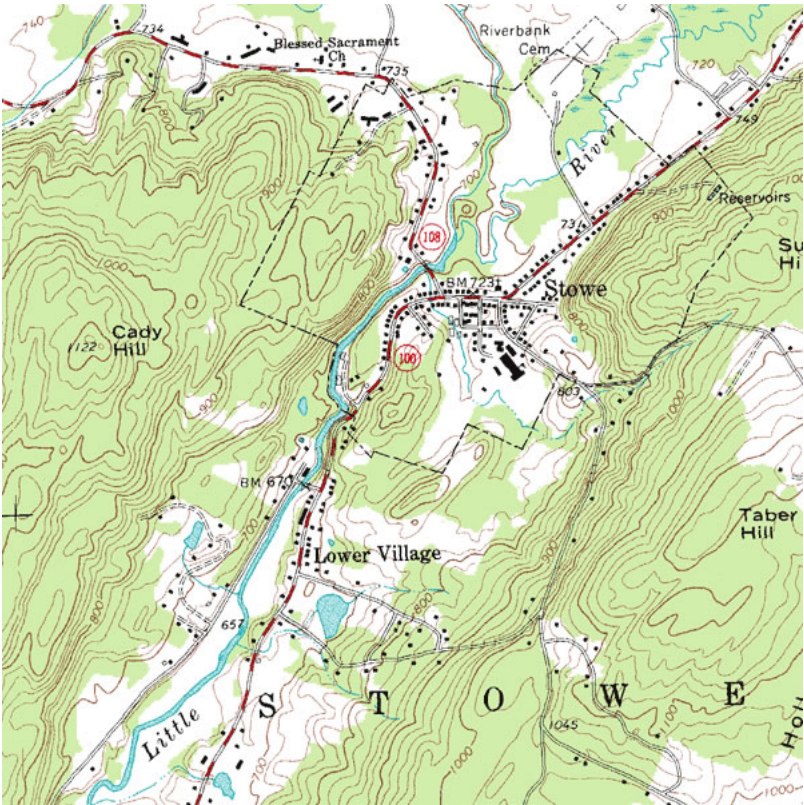
40.2.5 Indifference Curves

Indifference curves and budget constraints allow for a more in-depth analysis of demand. For modeling purposes we will look at the two goods. An indifference curve shows the different combinations of the two goods that yield the same level of utility, independent of the price of the goods. Due to the law of diminishing marginal utility, the indifference curve between the two goods is convex to the origin. All combinations of the two goods (pizza and shakes) that are on the indifference curve (A, B, and C) yield the same level of utility, say Utility = 100. Having more of good, yields a higher level of utility (combination D) and having less of the goods yields a lower level of utility (combination E).

Indifference Curves

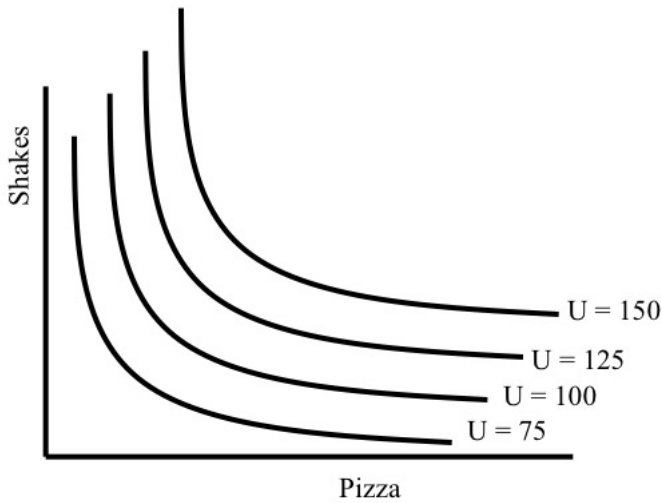


An indifference curve map shows the family of indifference curves. There could be an infinite number of indifference curves that would reflect the level of utility at different combinations of the two goods. Just as a line on a topographical map indicates the different points that are at the same elevation, the different points along an indifference curve, indicate that same level of utility.



Source: U.S. Geological Survey

Indifference Curve Map

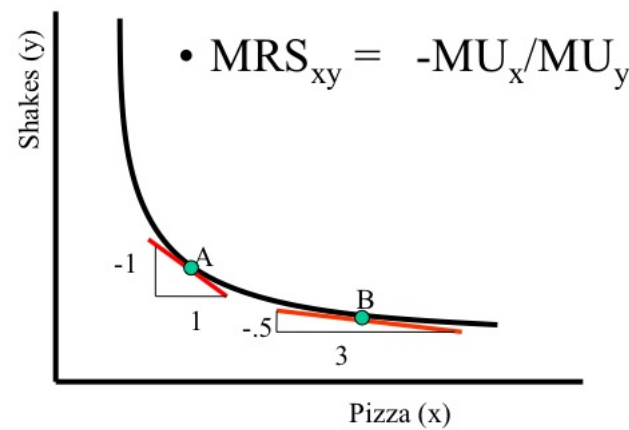


40.2.6 Marginal Rate of Substitution

The marginal rate of substitution is the slope of the curve and measures the rate at which the consumer would be willing to give up one good for the other while maintaining the same level of utility. Thus the marginal rate of substitution reflects the ratio of marginal utilities between the two goods.

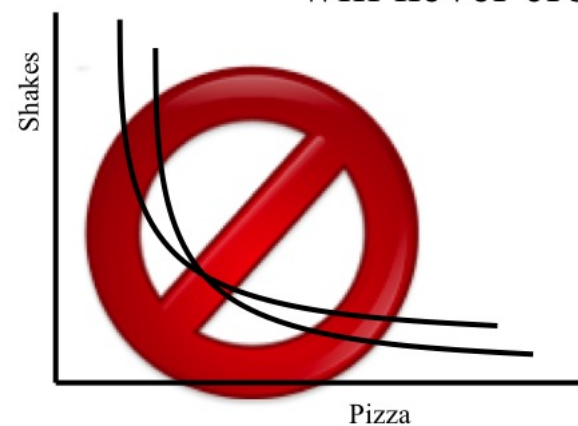
For example, at point A, the consumer would be willing to trade one shake for one additional slice of pizza. At point B, the consumer already has a lot of pizza but few shakes so the marginal utility from an additional pizza is relatively lower and the marginal utility from the shake he would have to give up would be relatively large, thus to maintain the same level of utility he would have to gain 3 pizzas to willingly give up one half a shake.

Marginal Rate of Substitution



Since any combination of the two goods will only yield one level of utility at a particular point in time, indifference curves will never cross each other.

Indifference Curves will never cross



40.2.8 Utility Maximization

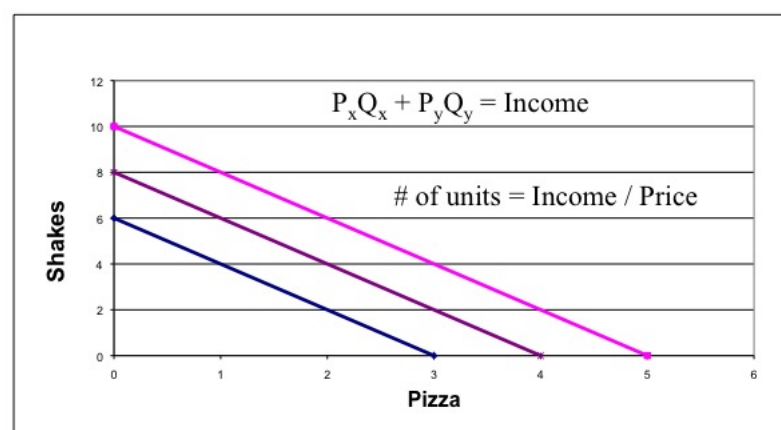
Given the goal of consumers is to maximize utility given their budget constraints, they seek that combination of goods that allows them to reach the highest indifference curve given their budget constraint. This occurs where the indifference curve is tangent to the budget constraint (combination A). Note that combinations B and C cost the same amount as A; however, A is on a higher indifference curve. Combination D yields that same utility as C and B but doesn't use all of the income, thus the consumer can increase utility by consuming more. Combination E is preferred to combination A, but is unattainable given the budget constraint.

40.2.7 Budget Constraint

The budget constraint indicates the combinations of the two goods that can be purchased given the consumer's income and prices of the two goods. The intercept points of the budget constraint are computed by dividing the income by the price of the good. For example, if the consumer had \$8 to spend and the price of pizza was \$2 and shakes were \$1, then the consumer could buy four pizzas (\$8/\$2) or eight shakes (\$8/\$1). Any combination of the two goods that are on or beneath the budget constraint are affordable, while those to the outside (farther from the origin) are unaffordable.

A greater income will cause a parallel shift rightward of the budget constraint while a decrease in income will cause a parallel shift leftward.

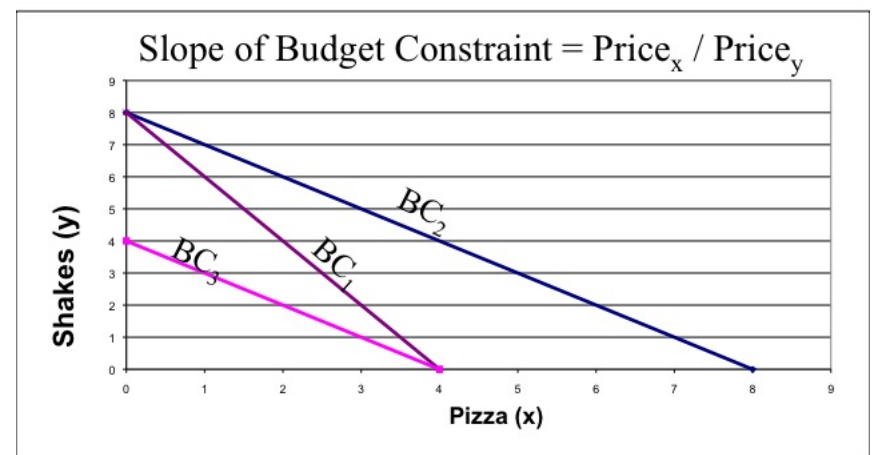
Budget Constraint



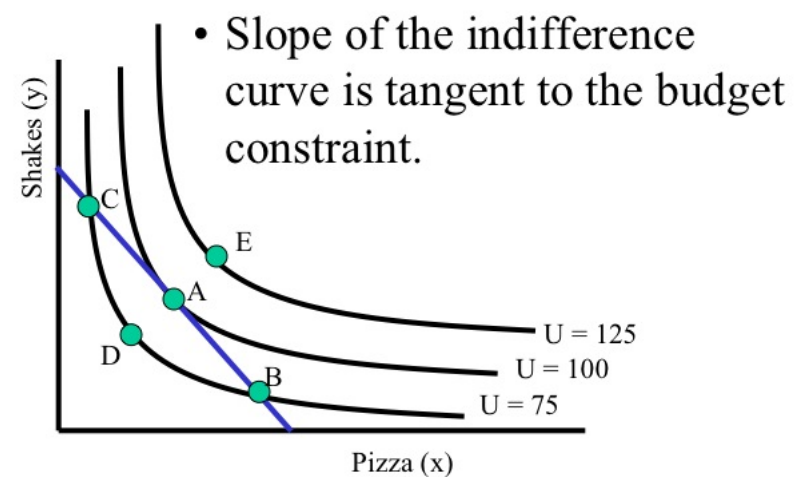
Changing the prices of the goods changes the slope of the budget constraint. If the consumer's income is \$8 and the price of pizza is \$2 and the price of shakes is \$1, then the budget constraint would be BC1. If the price of pizza drops to \$1, then the budget constraint would rotate out on the x-axis to BC2. Alternatively, if the price of shakes increased to two dollars then the budget constraint would become BC3.

The slope of the budget constraint is the negative ratio of the prices ($-P_x/P_y$). For example, given the price of pizza (on the x-axis) is \$2 and the price of shakes (on the y-axis) is \$1, then the slope of the budget constraint would be -2.

Price changes



Utility Maximization



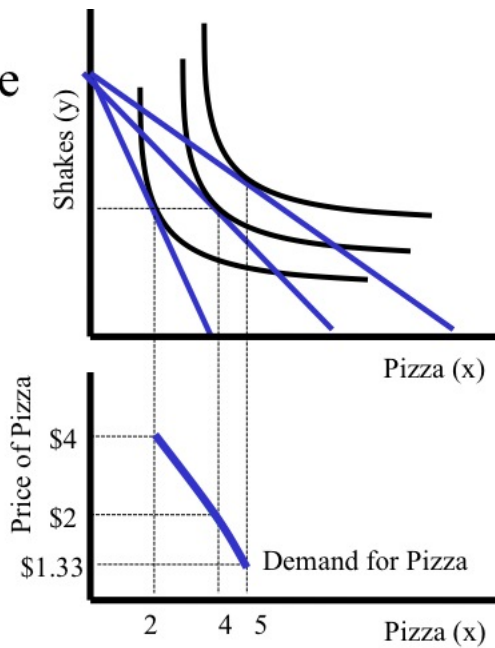
We previously mentioned that utility is maximized where the marginal utility per dollar spent is the same for each of the goods. At the point where the indifference curve is tangent to the budget constraint, the slope of the indifference curve which is the ratio of marginal utilities ($-MU_x/MU_y$) is equal to the slope of the budget constraint ($- \text{Price } x / \text{Price } y$). This equation can be rewritten to show that the marginal utility per dollar spent will be the same for both goods.

Utility Maximization

$$-\frac{MU_x}{MU_y} = -\frac{P_x}{P_y} \Rightarrow \frac{MU_x}{P_x} = \frac{MU_y}{P_y}$$

The demand curve can be derived from the indifference curves and budget constraints by changing the price of the good. For example, if the price of pizza is \$4, the quantity demanded of pizza is two. If the price of pizza decreases, the budget constraint becomes flatter and the consumer can purchase more pizza, say the price of pizza drops to \$2 and consumer purchases 4 units. If the price drops to \$1.33, the quantity demanded increases to 5. Plotting each of the price and quantity demanded points creates the demand curve for pizza.

Deriving the Demand

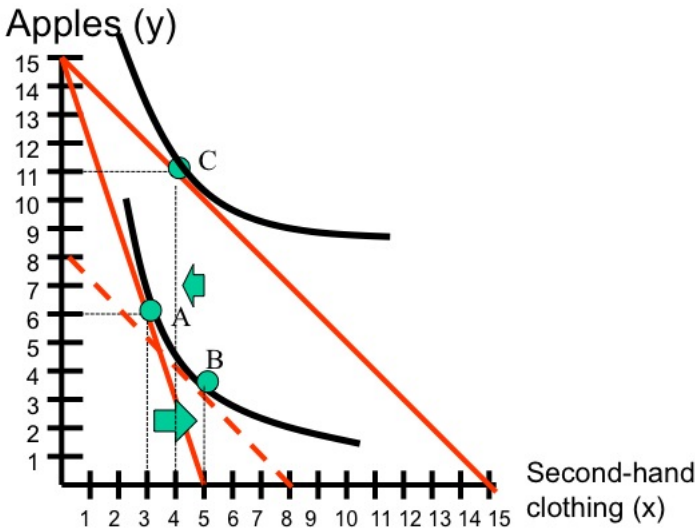


40.2.9 Income and Substitution Effects

When discussing why the demand curve is downward sloping, we outlined the substitution effect and income effect. We can observe the changes in quantity demanded along the demand curve due to the change in price; however, the indifference curves and budget constraints can help us analyze the size of the income and substitution effects.

For example, say the consumers income is \$15 and the price of apples is \$1 and the price of oranges is \$3. At these prices the consumer purchases six apples and three oranges. When the price of oranges falls to \$1, the consumer purchases eight apples and seven oranges. Thus on the demand curve for oranges, the consumer purchases three oranges when the price is three dollars and seven oranges when the price is one dollar.

Income & Substitution Effects

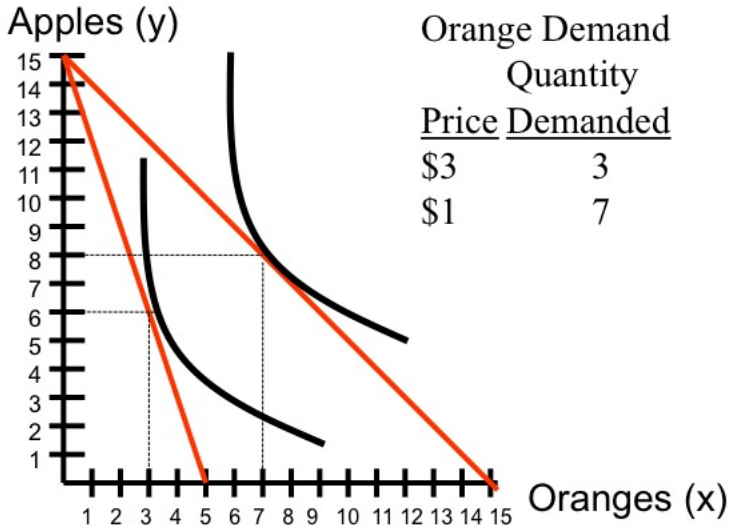


40.2.10 Extreme Cases

When examining indifference curves and budget constraints, we can look at a few extremes. One extreme case would be if the two goods are perfect complements. For example, you do not get additional satisfaction from having another right shoe, unless you have a left shoe to go with it. In the case of perfect complements, you always consume at the minimum combination of the two goods.

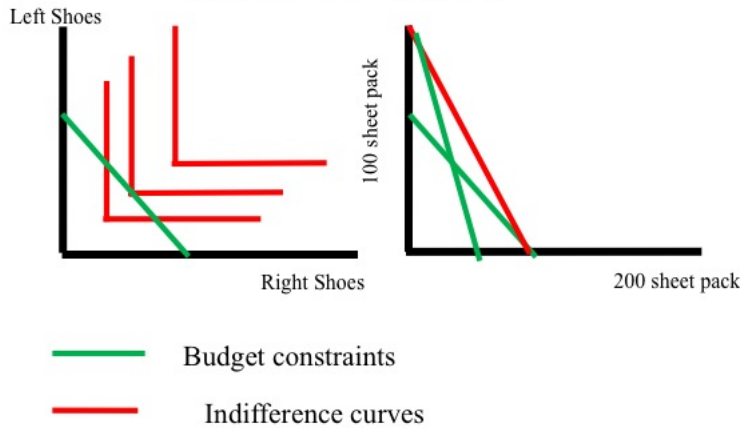
Another extreme is perfect substitutes. You purchase paper in either the 100 or 200 sheet packs and only value the number of sheets. You are indifferent between having two one-hundred sheet packages or one two-hundred sheet package. In the case of perfect substitutes, there are three different outcomes that will maximize utility. If the price of one package, yields a lower per sheet cost, the consumer will buy only that good, so consumption will take place at one of the two intercepts. The third outcome is when the budget constraint has the same slope as the indifference curve. In this case, any combination along the budget constraint will yield the same level of utility.

Income & Substitution Effects



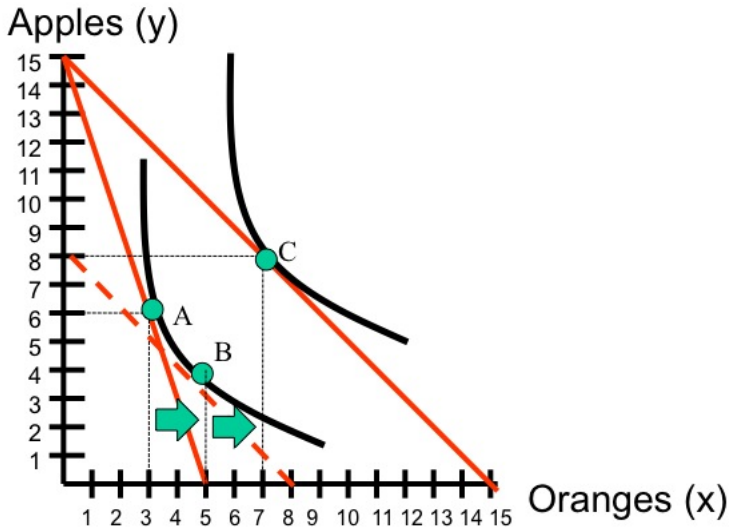
Orange Demand	
Price	Quantity Demanded
\$3	3
\$1	7

Extreme Cases



Bringing the new budget constraint back to the original indifference curve allows us to break down the income and substitution effects. Since the slope of the budget constraint reflects the ratio of prices, the substitution effect is the increase in the number of oranges that would be purchased given the new prices, while staying on the original indifference curve that is moving from point A to point B. The movement from point B to point C is the income effect, the additional consumption of oranges due to the increased purchasing power. With a decrease in the price of oranges, the relative price of apples has increased and fewer apples would be consumed due to the substitution effect; however, due to increased purchasing power, more apples are purchased as well as more oranges.

Income & Substitution Effects



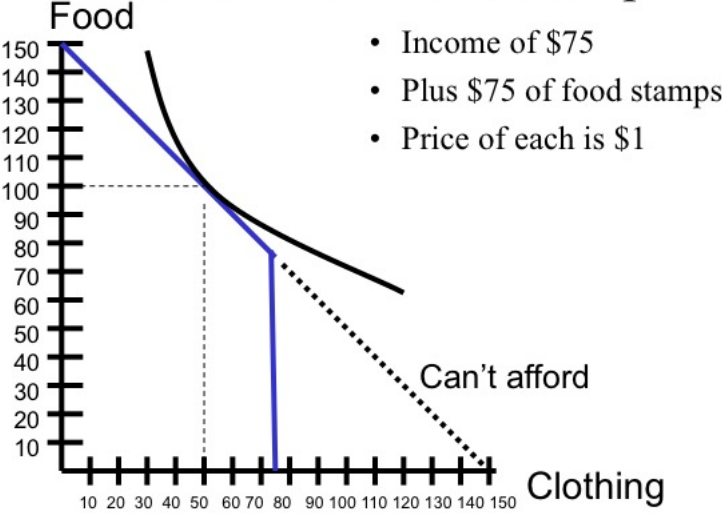
Recall from our elasticity discussion that the income elasticity for an inferior good is negative. For example, as income rises the demand for used clothing decreases. Looking at second-hand clothing on the x-axis, as the price declines the substitution will be positive (movement from point A to point B); however, the income effect (movement from B to C) will be negative.

40.2.11 Government Food Stamps Example

Why does the government give welfare recipients food stamps instead of cash? Why are food stamps sold on the black market for a discount? Let's assume a person has \$75 of income and receives \$75 of food stamps from the government. For simplicity, we will assume that the price of each unit of food and the price of clothing are each one dollar. The budget constraint allows the consumer to purchase up to 150 units of food, but since food stamps can only be used to purchase food, the consumer is limited to only being able to purchase 75 units of other items.

The amount of food and other goods, the individual will purchase depends on the shape of his indifference curve. Provided that the person spends at least 75 dollars on food, he is not constrained by receiving food stamps instead of cash.

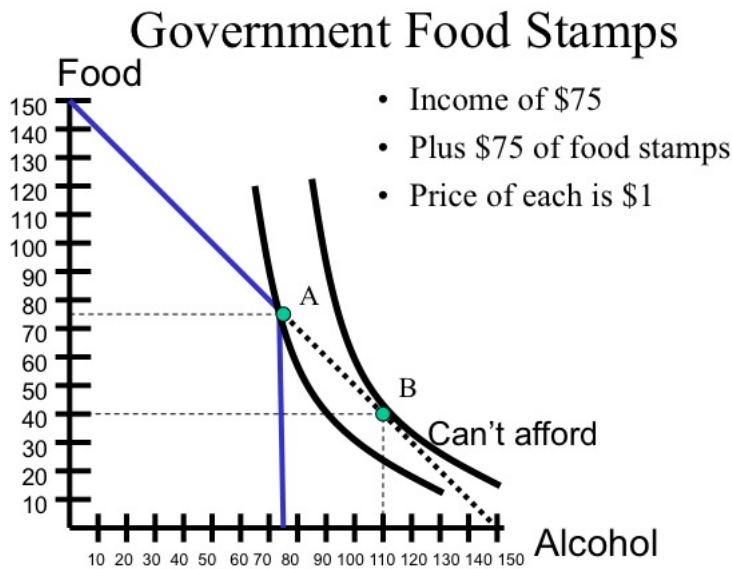
Government Food Stamps



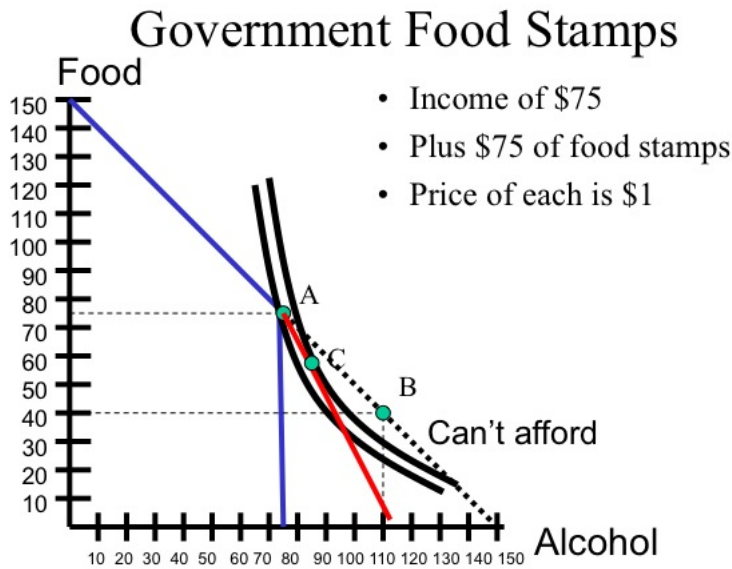
- Income of \$75
- Plus \$75 of food stamps
- Price of each is \$1

Unfortunately, for those individuals with strong addictions, such as cigarettes or alcohol, their indifference curves reflect the greater value received from the addict substance. Since food stamps only apply to purchasing food, the consumer is not able to get as much utility and is restricted to point A. If the food stamps had been

in the form of cash, the consumer would have purchased 40 units of food and 110 units of alcohol, point B, which would yield a higher level of utility.

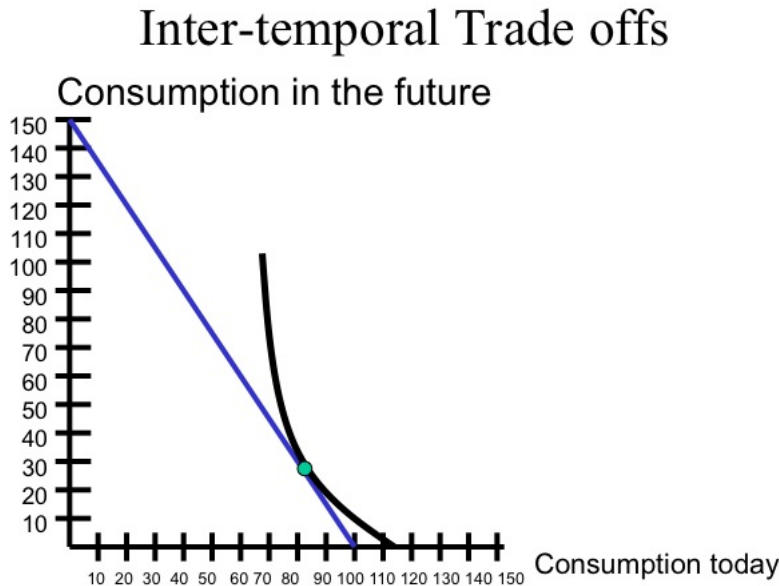


If the consumer could convert a portion of the food stamps to cash, even at a discount, he would be able to reach a higher indifference curve. Assume that he can trade food stamps on the black market for 50 cents on the dollar which extends his budget constraint increasing the amount of alcohol that can be purchased. Selling food stamps at a discount allows the consumer to move to point C, which yields a higher utility than point A.



We can also model why individuals purchase items today and carry the debt on their credit cards paying a high interest rate. For example, let's say the price of each good is \$1. Assume Will has \$100 of income which he could spend today or invest the money and receive \$150 in the future. By waiting, Will would have a greater purchasing power, but his consumption bundle will depend on his preferences. If he has a strong preference for having consumption today, he would be willing to pay a higher price for those goods today. The same is true for those who buy items on credit. They are willing to pay the purchase price plus all the interest, so that they can have it today (so the indifference curve touches the budget constraint at a point closer to the x-axis as seen in the figure).

Similarly, some individuals are willing to pay to go to the expensive theaters to see a movie when it is first released. Others will wait until it comes to the cheap theaters or even until it comes out on DVD/Blu-ray. Although individuals are paying for the "movie experience" and not just the film itself, we can see the time preferences of individuals and the price differentials over time (as represented by different placements of the indifference curves).



40.3 Supply and Demand Curves (Understanding Price and Quantity in the Marketplace)

Why do parents – and stores – behave this way? The answer is in the laws of supply and demand. Together, these laws give us strong clues about what to produce, how much to produce, and how much to charge. Because supply and demand play such a central role in our economy, it's important to understand how they operate – and

how you can use them to analyze decisions about price and quantity.

40.3.1 The Law of Demand

Demand, in economic terms, shows how much of a product consumers are willing to purchase, at different price points, during a certain time period.

After all, we all have limited resources, and we all have to decide what we're willing and able to purchase – and at what price. As an example, let's look at a simple model of the demand for a good – let's say, gasoline. (Note that this example is illustrative only, and not a description of the real gasoline market.)

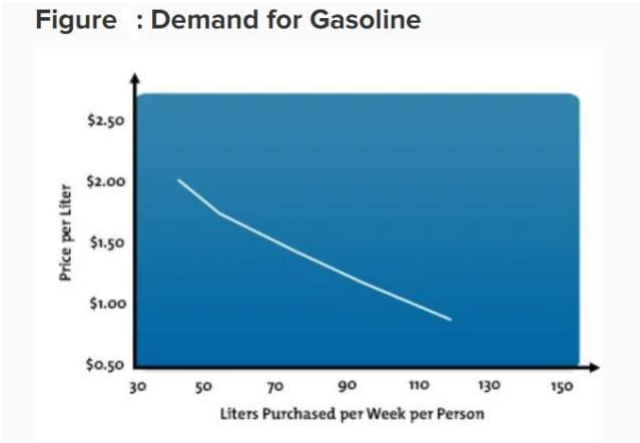
If the price of gas is \$2.00 per liter, people may be willing and able to purchase 50 liters per week, on average. If the price drops to \$1.75 per liter, they may be able to buy 60 liters. At \$1.50 per liter, they may be prepared to purchase 75 liters. Note that while some gas usage is essential – driving to work, for example – some use is optional. Therefore, as gas prices drop, people may choose to make more optional trips during weekends, and so on.

The resulting demand schedule for gas might look like this.

Buyer Demand per Consumer	
Price per liter	Quantity (liters) demanded per week
\$2.00	50
\$1.75	60
\$1.50	75
\$1.25	95
\$1.00	120

This schedule, and probably your own experience as a consumer, illustrates the law of demand: as price falls, the corresponding quantity demanded tends to increase. Since price is an obstacle, the higher the price of a product, the less it is demanded. When the price is reduced, demand increases.

So, there is an "inverse" relationship between price and quantity demanded. When you graph the relationship, you get a downward-sloping line, like the one shown in figure 1, below:



To create a market demand curve for gasoline, individual demand is totaled and combined.

40.3.2 Price Elasticity

The extent to which demand changes with price is known as "price elasticity of demand."

Inelastic products tend to be those that people must have, but they use only a fixed quantity of it. Electricity is an example: if power companies lower the price of electricity, consumers may be happy, but they probably won't use a lot more power in their homes, because they don't need much more than they already use. However, demand for luxury goods, such as restaurant meals, is extremely elastic – consumers quickly choose to stop going to restaurants if prices go up.

Price elasticity also affects supply. Products with an inelastic supply usually have a long lead time, with little control over the quantity produced. Farm crops are one example, because if there's a price change, farmers can't decide halfway through the growing season to produce more or less of a certain crop. On the other hand, products with a high elasticity of supply tend to come from industries that can change their production levels more quickly – for example, oil (although the oil industry may be operating close to full capacity, right now.).

40.3.3 The Law of Supply

While demand explains the consumer side of purchasing decisions, supply relates to the producer's desire to make a profit. A supply schedule shows the amount of product that suppliers are willing and able to produce and make available to the market, at specific price points, during a certain time period. In short, it shows us the quantities that suppliers are willing to offer at various prices.

This happens because suppliers tend to have different costs of production. At a low price, only the most efficient producers can make a profit, so only they produce. At a high price, even high cost producers can make a profit, so everyone produces.

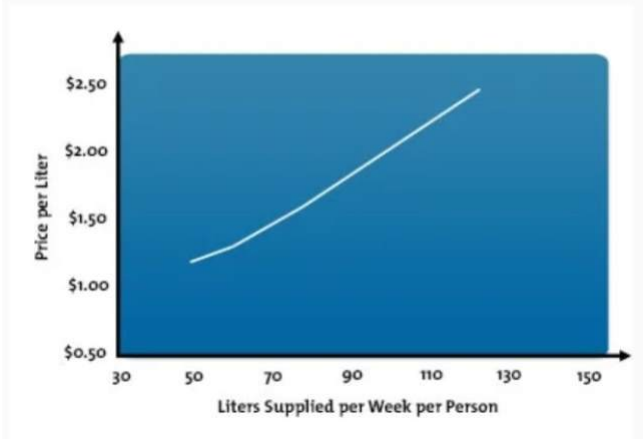
Using our gasoline example, we find that oil companies are willing and able to supply certain amounts of gas at certain prices, as seen below. (Note: we've assumed a simple economy in which gas companies sell directly to consumers.)

Gas Supply per Consumer	
Price per liter	Quantity (liters) supplied per week
\$1.20	50
\$1.30	60
\$1.50	75
\$1.75	95
\$2.15	120

At a low price of \$1.20 per liter, suppliers are willing to provide only 50 liters per consumer per week. If consumers are willing to pay \$2.15 per liter, suppliers will provide 120 liters per week. The question is this: what prices are needed to convince producers to offer various quantities of a product or service?

As price rises, the quantity supplied rises as well. As price falls, so does supply. This is a "direct" relationship, and the supply curve has an upward slope.

Figure : Example Supply Schedule for Gasoline



Because suppliers want to provide their products at high prices, and consumers want to purchase the products at low prices, how is the price of goods actually set? Let's go back to our gas example. If oil companies try to sell their gas at \$2.15 per liter, do you think they'll sell as much? Probably not. Yet, if oil companies lower the price to \$1.20 per liter, consumers will be very happy, but will there be enough profit? And furthermore, will there be enough supply to meet the higher demand by consumers? No, and no again.

To determine the price and quantity of goods in the market, we need to find the price point where consumer demand equals the amount that suppliers are willing to supply. This is called the market "equilibrium."

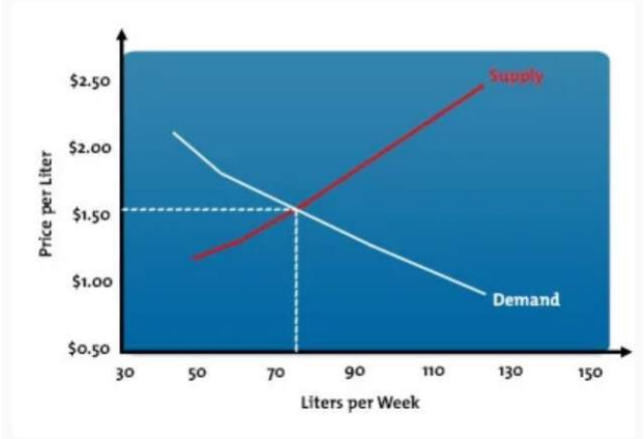
40.3.4 Equilibrium: Where Supply Meets Demand

Equilibrium is the point where the quantity demanded equals the quantity supplied. This means that there's no surplus of goods and no shortage of goods. A shortage occurs when demand is greater than supply – in other words, when the price is too low. A surplus occurs when the price is too high, and therefore consumers don't want to buy the product.

The great thing about the free market system is that prices and quantities tend to move toward equilibrium and, for the most part, keep the market stable.

Consider our example. At \$1.20 per liter, consumer demand exceeds supply, and there's a shortage of gas in the market. Shortages tend to drive up the price, because consumers compete to purchase the product. However, when prices go up too much, demand decreases, even though the supply may be available. Consumers may start to purchase substitute products, or they simply may not purchase anything. This creates a surplus. To eliminate the surplus, the price goes down and consumers start buying again. In this manner, equilibrium is usually maintained quite efficiently.

Figure : Market Equilibrium



In our gas example, the market equilibrium price is \$1.50, with a supply of 75 liters per consumer per week, as shown in figure 3.

Market equilibrium explains movement along the supply and demand curves. However, it doesn't explain changes in total demand and total supply.

40.3.5 Changes in Demand and Supply

A change in price initially results in a movement along a demand or supply curve, and it leads to a change in the quantity demanded or supplied.

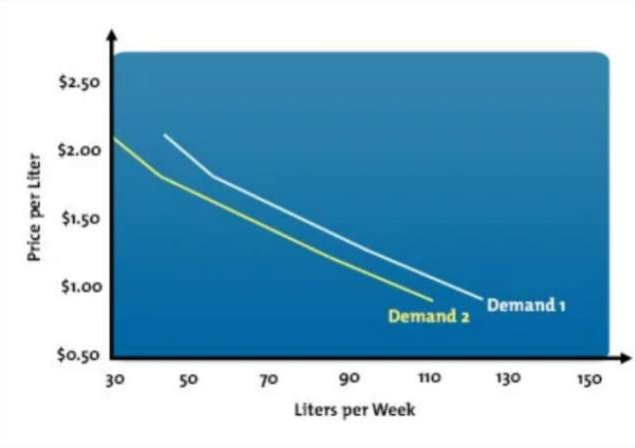
But what happens when there's a long-term change in price?

If consumers are faced with an extreme change in the price of gas, their pattern of demand for gas changes. They not only start choosing different types of transportation – like taking the bus or riding a bicycle to work – but they also start buying more gas-efficient vehicles – like compact cars, motorcycles, or scooters. The effect is a major change in total demand and a major shift in the demand curve. The new schedule for demand is now Demand 2, shown below.

Price per liter	Quantity (liters) per week	
	Demand 1	Demand 2
\$2.00	50	30
\$1.75	60	40
\$1.50	75	55
\$1.25	95	75
\$1.00	120	100

You can see this in the graph in figure 4, below. At each price point, the total demand is less, and the demand curve shifts.

Figure : Demand Shifts



40.3.6 Changes in any of the following factors can typically cause demand to shift:

- Consumer income.
- Consumer preference.
- Price and availability of substitute goods.
- Population.

With a shift in demand, the equilibrium point also completely shifts.

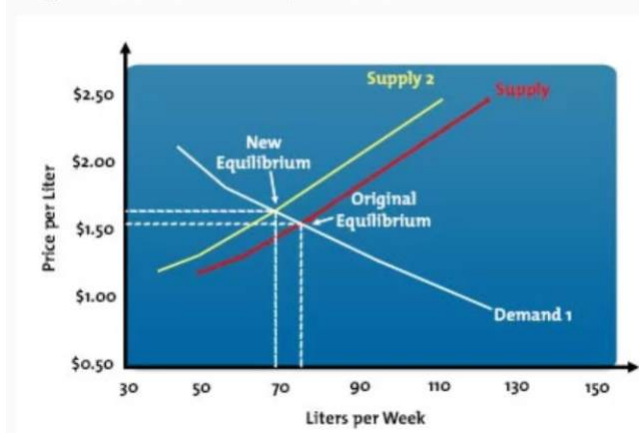
Demand 2		Demand 1		Supply	
Quantity (liters) per week	Price per liter	Quantity (liters) per week	Price per liter	Quantity (liters) per week	Price per liter
30	\$2.00	50	\$2.00	50	\$1.20
40	\$1.75	60	\$1.75	60	\$1.30
55	\$1.50	75	\$1.50	75	\$1.50
75	\$1.25	95	\$1.25	95	\$1.75
100	\$1.00	120	\$1.00	120	\$2.15

The same type of shift can occur with supply. If the price of drilling for and refining gas increases, or if political events cause suppliers to decrease their output, the supply curve can move. The result is that for the same price, the quantity supplied will be either higher or lower than the current supply curve.

A common complaint with the oil and gas industry is that suppliers deliberately manipulate price by shifting the supply curve. The result is an equilibrium price that's higher and at a lower quantity, as in the following example.

Demand 1		Supply 1		Supply 2	
Quantity (liters) per week	Price per liter	Quantity (liters) per week	Price per liter	Quantity (liters) per week	Price per liter
50	\$2.00	50	\$1.20	40	\$1.20
60	\$1.75	60	\$1.30	50	\$1.30
75	\$1.50	75	\$1.50	65	\$1.50
95	\$1.25	95	\$1.75	85	\$1.75
120	\$1.00	120	\$2.15	120	\$2.15

Figure 5, below, shows the curves that result from this schedule.

Figure : Market Equilibrium

When supply decreases, the supply curve shifts to the left. When supply increases, the supply curve shifts to the right.

Changes in supply can result from events like the following:

Change in production costs. Improved technology that makes production more efficient. Industry growth and shrinkage.

Key Points Although the phrase "supply and demand" is commonly used, it's not always understood in proper economic terms. The price and quantity of goods and services in the marketplace are largely determined by (a) consumer demand and (b) the amount that suppliers are willing to supply.

Demand and supply can be graphed as curves – and the two curves meet at the equilibrium price and quantity. The market tends to naturally move toward this equilibrium – and when total demand and total supply shift, the equilibrium moves accordingly. It's an interesting relationship that determines much of what happens in a free market economy. If you understand how these factors influence pricing, supply, and purchasing decisions, it will help you analyze the market and make better price and supply decisions for your company.

Apply This to Your Life Consider these questions in relation to your life and business:

Are your production costs higher or lower than those of your competitors? What does this mean for your business if supply or demand curves shift? Are you charging as much as you could for your product, particularly if demand is strong and supply is weak? Conversely, are you charging too much? Are you paying too much for your inputs, particularly if demand for these inputs is weakening? If demand for your inputs is likely to strengthen, or supply weaken, in the future, how can you protect yourself against this? Are there any other questions you should be asking yourself as a result of knowing about supply and demand curves, equilibria, and shifts in supply and demand curves?

Part XXI

Marketing

Chapter 41

Consumer Marketing

For doing consumer marketing first of all the organization should have funds.

In case of sufficient funds, depending on budget, one can go for

1. TV Channels for Advertisement
2. Newspapers
3. Radio
4. Phone Calls
5. Door to door visits by sales members.
6. Pamphlets and Street Stickers (Although this has been declared illegal some time before while my brother was in full form and me in small case was practicing via this way and no other way. It led to friction with the govt. and my subsequent captivation by the mental asylum of Chandigarh. Although I now don't remember which govt. was present during that time when my pamphlets and street stickers were declared illegal . I was married with wife staying with me and finding it difficult to fulfill her needs in terms of money and govt. (or say the local adminsitration) was making it more difficult for me to satisfy her. She was not working ofcourse.). It seems there were miscreants out there with dirty eye on my wife. But the question which arises whether they have a real eye and how long would they not get a real eye. Interestingly Amit pasted a street sticker outside our house speaking something about mahamurkh, probably telling about himself. It can be congress , it can be shopkeeprs or their BJP , it can be bhapas. Many contenders of the competition. Whoever did it would be thrown out of India.India is my body itself. There are organs, bones, inside my body but then waste also accumulates down to my stomach in the intestine and in the bladder which needs to be excreted or uninated from time to time. It is in reality those people who exist as waste inside my body, those who say lies like it's illegal to post pamphlets or street stickers and also add shit to my food if serving food somewhere and when these people are excreted from my body(ie india) can be seen inside the sewage tank. And the real problem is that they know that they are shit or urine in reality or the third form, the fart and don't shy away from doing such tantrums.And it's also known that no legitimate resident of India would ever be a traitor of this kind and these people which are being thrown out of india as waste are probably not worth earth and would probably grow as a miniature lifeforms used as food by aliens as usually nobody on earth consumes shit, urine or farts as a regular diet. Usually I have set it a rule for myself that if somebody targets my son manas, i'd let him defend his own self. If somebody targets my limp brother however, he/she would pay serious price. Mom has given all my ideology or the ethical understanding of the world. Though a very clever person herself and a cheapster, I have always kept her at a higher position than my wife.

Chapter 42

Industrial Marketing

When a large organization builds some product, it needs to sell it either to the general public or the secondary industry dependent on it. The industrial ingredient produced by the organization can be large and expensive or small and cheap or various combinations. Moreover there can be organizations importing semi finished products from as many as 100 or more different industries. There is no upper and lower limit to how many input sources of semi finished products can be required for one product of the organization.

Chapter 43

Services Marketing

I'll have to search the internet or see some courses , kindly keep open to furthur editions of the book.

Part XXII

Civil Administration

Chapter 44

Police Services

44.1 Police in Countries other than India

44.1.1 LAPD of America

We see them in the movies, slightly bad dressed than the FBI . In the Rambo movie we saw them in bad light. In Matrix, not doint much and not strong too. However, they are the common law enforcing agency in the US.

44.1.2 NLEA of Britain

We see them in the british movies and recently in news was one Brit Officer who targetted the three attackers with his batton , stopped them but was hit in the face with a knife. But definitely not like the MI-6 James bond we see in 007 movies.

44.2 Indian Police

I encountered the Chandigarh police and somewhat the Hydrabad police . With one or two (probably rajput guys) misbehaving with me in the police everywhere, others stay calm and don't see me as a target. However the local people don't see them as very nice people .

44.3 Punjab Police

The most strict of the Police forces in India, led to the end of militancy completely and now maintaining peace properly too.

Chapter 45

Administration Services

45.1 Indian Foreign Services

They usually take care of all the inter-nation diplomatic talks. Usually as the team of the Ambassador.

45.2 IAS

Though they see themselves as the rulers of India, I don't have a very good feeling about them. One Rajat Saini at IIT Delhi, praised them but they don't appear to be that good a stuff to discuss here. However, recently there was a case too in Chandigarh, with an IAS's daughter of Chandigarh taking stand against a minister's son (of Haryana) for eve teasing, there are still higher importance cases like the 10 and 12 year old girls rape cases pending in the court of law to which Chandigarh should look forward to for proper justice.

45.3 Engineering Services

They are very popular amongst Indian masses, e.g. after the complete revamp of IAS exam, which doesn't ask any technical questions now. The IES exams ask for the complete Engineering you have studied with an IPS/IAS like interview. However, critics to this exam comment and question whether an Engineer can ever be a real administrator or he should prefer PHD and join as a Professor somewhere. However, this is a useful exam one can look forward to if he/she has both technical/administration aptitude.

Sociology

Part XXIII

The Market

Chapter 46

The Indian Context

46.1 Eatables (Kiryana, Mandi, Halwai, Confectionary and Restuarants)

46.1.1 Unprocessed

Wheat or other grains in cereal form.

46.1.2 Semi-Processed (For furthur processing at home)

Wheat Flour

Rice

Vegetables

Dals

46.1.3 Ready to Eat

46.1.3.1 Ice Creams , Flavoured Milk

Amul and Mother Dairy are the popular National Brands. Verka is the local brand available in Chandigarh and parts of Punjab.

Part XXIV

The Media

Chapter 47

International News Channels

47.1 CNBC

It has a Divya Sharma (My top student at Alpha Classes) Lookalike Anchor and mostly dicussing Business.

47.2 CNN

It's not available in the Channels present in our Airtel Digital Tv Subscription although earlier when Digital TV hadn't been a norm , it used to come on our TV.

47.3 MSN News (via windows e-print media)

Very Elaborative. It's an experience , but then sometimes windows people start demanding upgrade to windows 10 killing all our news updates and games with closeure of windows store. 3 of my PC's had original Windows, one of them gifted to my wife, other now with my brother and this is something of a sort showing that I am a rich person. However her other PC via HCL didn't have original windows. Apart from it my Macbook Air had orgininal Operating system too. It appears that as a system rule, if somebody uses a pirated software, it's normalized price needs to be paid to the software owners during system audit. So, with lots of people using Microsoft Office, Photoshop etc softwares would be audited when a System Audit Cheque is accepted anywhere into the System. And people don't fear being audited too, they say we are not afraid to pay the price as it would be lesser than what it is on the web if we buy the original one under the agreement. And mind you , Bill Gates is born rich due to the Microsoft Office Licences Audit by the System. As of today I am using all legal softwares with this document being written in Open Source Lyx for Latex file editing as a word like environment. Though my Reports at the time of my degree, i presented in hardcore latex, however over the time , i have shifted to using the editor.

Chapter 48

Prominent Indian News Channels

48.1 DD Broadcast (accross various of its channels)

48.2 Aaj Tak

Aajtak initially started with a small 2 to 4 minutes fast news on DD Metro with brilliant team members and way ahead of it's times presentation. Later it became a base for the Aam aadmi party as it's very brilliant members like Ashutosh leaving reporting and goint to join the National Theme Party. I don't know if the present day team of Aaj tak has any links with Aam Aadmi Party or not. Usually the channels with political bias lose their spirit in small run, even in this special case when the party has emerged of this channel to some extent. News channels like ABP when they first came on TV, they were fast but it was clear that they were party supportive with ABP supporting congress and also defending it. Though at one time it became my favourite News channel but i don't think it deserves a mention here do to the issues of unclean nutrality.

48.3 Zee News

They started as Zee TV, the only channel which used to come via the expensive Cable Connection while DD (Doordarshan) Channels like Doordarshan, DD Jalandhar , DD Metro (this required a special antenna) used to come free via the antenna. Then subsiquently more channels joined the bandwagon ,and side by side Zee grew the number of it's channels. Howev ere, they were traditionally not a news channel but but had more of saas bahu serials as their top item.

48.3.1 The Delimma

It's presently the biggest news channel in terms of the Channel group's infrastructure capable of covering the news of whole India and many places of world, maintaining neutrality, keeping ethics too. But somehow the news are non-chrispy non-masala mixed though good for a person who wants to listen to true news but not much useful for someone who wants to enjoy the telivision for say half an our to get a break from work etc.

48.4 NDTV

It was sued by the BJP , the present ruling party , for giving some controversial news.

48.5 Other Smaller Channels

Now there are lots of Channels with the word News in their name, covering various segments of news. They are playing as a group probably to fend from the larger fish in the territory.

Part XXV

The Cream vs the bearers (in Indian Context and beyond)

Chapter 49

Introduction to the discussion “The Cream vs the bearers”

India starts with the President of India, the moral authority (though a low caste nowadays) and then the Prime Minister , the authority on State’s Power . And then we have the Cheif ministers looking after their individual territories. Governors, Mayors etc. Heads of Police. etc people exist who are complete incharge of India. But these people in total would be only a few thousands at the most as far as india is concerned.

Rest of the people, even the rich , the other politicians who are down in the line, the businessmen (although there is some negative perception about the market , introduced by my mom to me) and the common masses , would come under the bearers banner IMO.

Ecology

Part XXVI

Possibility of Nuclear Holocaust and UN's non-proliferation treaty (which India doesn't sign due to security concerns)

Chapter 50

Pakistan vs India, Area of high sensitivity

One major place where there can be a possible Nuclear war is the Pakistan vs India , sensitive area. From my meditation experience , I know that pakistan fires nuclear missile on India in future but at that time India has sheldon shield, a type of nuclear shield and the missile pollutes only pakistan area and not India's. Later however many pakistanis cross to india in an attempt to find cleaner area for survival , that's near the occurance of Kalki (Kalgi) Avtar near 10000 AD in time(The attack probably occurs in 8000-8500 AD).

Chapter 51

Possibility of US using the nukes again , low

It has been criticized hugely for the bad lives of bomb victims in Japan. Even as till our period the bomb victims's lives have not been extensively filmed, but still there is some awareness in the educated world that the bombs were indeed dropped on Japan. Believe me, US doesn't want to play the bad guy again. It has only kept few nukes for it's own protection and for maintining it's watchdog reputation for the present world.

Chapter 52

The Role of Sci-Fi (The Phoenix writeup in Xth book, World at arms game by Gameloft, one MIT OCW Course etc. SCI-FI stuff predicting the near future)

52.1 The Phoenix

In the novel, the plot is such that the Nuclear Holocaust had already taken place and one man Akshay ,who also has a good quality robot friend , survives it with few other fellows and they are trying to unite. And akshay , a good techie , somehow manages to get a branded screwdriver from a fellow survivor and with it's use his robot starts behaving more humanly and intelligently.

52.2 World at Arms

The plot in the game is also after the Holocaust and in not very distant future but probably after our lives we can hope so, maybe in the 2400's . So, there are small territories owned by groups of people at various places, they either have the nuclear technology or can buy it for small size nuclear attacks and moreover machines have been developing and turning into organic bodies , playing their politics , so , kind of before the “Matrix Movie” plot in the game.

52.3 MIT OCW

I haven't seen this course yet. Other realistic courses, I have started liking though, MIT, NPTEL, Coursera , etc and even youtube offering plenty of good courses, I have watched bits and pieces here and there but they are a huge knowledge store.

Myth and Miracle

Part XXVII

Scientology

Chapter 53

Physics in a nutshell

Now I won't start with the statement "Physics is everywhere". It's an old statement. I would instead start with let's say , what's a blackhole or a quasar or a pulsar.

Now you are in +1/+2 so mostly you know about gravitation , current , pressure etc. So, in a blackhole, let's see at the centre first, one possibility is that there's extreme pressure, temprature gravitation and current. Second possibility is that above the centre the forces balance out, like in case of sun, it's being said that even though above it's extremely hot with plasma and neuclear fusion reactions going on , in the inside it is not so and there are people out there trying to scientifically prove it. There are sci-fi movies like the "The Core" which say that earths centre is also the same as sun and there is no hot core and it's a primitive civilization living inside the earth, however in case of earth, these claims have been widely rejected, maybe possible in case of somewhere else in the universe's earth, i mean some other earth.

So, what is a blackhole. Is it something to be inquisitive of. Is it something to be afraid of?? Or is it the solution to all our problems. And why am i interested in blackhole and how can i help in learning about it.

{Gandhiji's 3 monkeys : Don't see bad, don't speak bad , don't chat bad. Now every mental patient can be classified in one of these three types, the ones who use their vision to see bad things etc. and in return lose both their eyes in old age as a punishment and then there are other two types of monkeys}

So , due to my disease which is a form of Paranoid Scizophrenia , i can look into matter (where i am guided to see it) . I can use it to peek into some girl's bathroom or else i can see and show a blackhole's structure. Don't worry I am not a dangerous person who can spy on you, I take medicine now. These things which i am showing here are what I saw before taking medicine and now i have faint memory of them. Gandhiji claimed that every person in the world is one of these three types of monkeys and has one of these special gifts which connects them to God and after 35 years of age everybody has old age protection from god's side.

So what exactly the use of these blackholes , quasars and pulsar's etc.

They can create heavier elements than our normal 118 elements . But then why don't we find these elements created by blackholes anywhere.

Now I will tell what I have seen , then later I'll tell the fantasy. Blackholes are the excretion points of the universe where they consume worn out planets and stars and exit them out of this universe by feeding them to the other end of the opening of other universes about to be created . Now a question arises, if this much mass was teleported out of the universe, where does the universe balance it's mass from. The answer is again what I can see , the universe boundries have a shell like covering made of faint white and black salt which creates life and matter. from there the material enters , from blackholes it exits. But then we know of the nutron burst which created the universe? Don't blackholes consume all mass and all planets and stars and grow big to become the neutron star? and then it bursts again to create the univese again . The answer is , yes this is one alternate mechanism. I told our universe's mechanism and you are telling the one connected to yours.

Million dollar question? What happens to US when these blackholes consume everything.

And there is a 2 Rupee answer (provided free of cost or covered in the price of this book). We have already moved to alternate universes and blackholes consume only completely dead planets and solar systems. So when the universe is completely dead , then the neutron star is formed.

Another irritation in brain. What is a universe according to hindu religion? It's the body of bhagwan in spirtual domain. Like we have a body, now we are on the outside , we can't take care or look inside , so we are the devil who is greater being then god. Under our skin , there is a force always looking inwards and providing all the necessary things to all cells who are inside and taking care of them . Now we see these cells on operation or a cut in physical domain , but if we close our eyes and look inside we can see a similar univese to ours from a position outside it , near our head , our eyes. So universe is nothing but a person's body, inwardly controlled by God and not that person, however that person is the one who provides food and takes work out of the body. Interestingly what we see outside is also another universe with all the people and we are a small being in this universe while that inner universe is so cool we are even bigger than it. So what are we? we are at the boundary. Why we see both differently is because , like we say that the universe has all the planets and solar system and so frightning things and inside our body there are the cells the tissues and the brain , heart etc frightening things. Why do we see them different, This is because we are seeing the outside planets and solar systems etc in the spiritual domain , similarly in our body we can look in spiritual domain by closing our eyes and looking inwards. While the cells and tissues and organs etc are in Physical domain . Our universe also has it's physical domain similar to us and we tried to visualize the blackhole as an excretory organ. There is another domain, the mechanical domain one of the three domains which i have seen and it has all the medicine reservoirs in our body and we simply refill them or change the medicine already present in our bodies in these reservoirs. There can be other domains in auditory and speech sections, I myself haven't seen them how they work, but i hope there would be atleast 3 domains there too in each method.

The fantasy

But some people say a blackhole is extremely dangerous, tell some horror stories or other interesting stories.

Fruits and vegetables

Now there is an intersting story going, and I have seen it with my eyes as well. 10000 years punishment in a blackhole. It occurs when someone becomes a vegetable or a fruit. Now everyday the birth begins , there is very faint light inside the blackhole. We can move slightly in the room , the blade starts and we are cut completely and then we die. Then we wake up and again have a fresh body and it's again cut and then we are left to die, it's repeated 84 lakh times in a total time span of 10000 years. This punishment starts when 84 lakh vegetables or fruits are consumed by one person. Interstingly a chicken on the other hand has relatively much more freedom than this case . So, a chicken consuming person can be in much better state than this person, however traditional hindu religion says not to consume non-veg. Possibly the removal of shell of the vegetable , and offering the first piece to god etc are possible escape routes from this punishment, while Guru Nanak Dev ji says "sayanpan lakh howen , ik na chale naal ", meaning , that in front of almighty no cleverness works. **(Don't worry, this doesn't happen ever. I'll tell later why.)**

Howcome?

The universe is only a small part of the world without much significance. Now if the complete logic of everything fails, then say a billion universe group fails, in that case the error recovery runs. I have seen this happeing in my vision (Live) , I know the pain too as it happened to me too and also saw the difference in magnitude of pain in blackhole, quasar, pulsar etc. Blackhole is amongst the easiest as an enemy punishes there. In a pulsar , you keep thinking all the time that it's your father who is running it. All are extremely painfull.

Joke???

Well there are people out there who think that if everything fails it reestablishes in no time but in fact it reestablishes in 10000 years.

What's the best part.

The best part is that I am a mad person. Mostly my sayings are not meant to be believed and what's the use also to be getting scared without much reason. Thinking about this punishment can give us extreme courage in very dangerous situations as any situation can be very very better than this situation.

Other kind of blackhole

There is a very funny blackhole created in the name of one person named Krishan (not to be confused with Lord Krishna). In this blackhole there are in infinite number of machine buffalows with plastic casings as skin. This person krishan has to milk them to survive and he lives in a room at the center of that blackhole.

Any other kind

One normal blackhole similar to a mother's womb had a person like me at it's centre and as that person is big in size, the currents appear small , the pressure appears not much etc. But then that blackhole starts to end and persons dig into it to live there and in the centre that person wakes up as a Black monkey. When he woke up the last time there was ola shower in chandigarh, we call it mullets or what???

Other intersting visions

There are visions of Durga with 1 tiger and 999 tigresses.

Another is Durga with 8 billion tigers and tigresses

Another is Kali standing on a Damped wave with infinite tigers and tigresses.

Now these three visions, the scenario is seen at the beginning and starts, later what happens is very intersting.

In one tiger and 999 tigresses, the tiger is able to save durga and takes her away. He is Man (the inner heart) and durga is probably saved in this scenario.

In 2nd 8 billion tigers and tigresses, all of the tigers and tigresses are born out of durga's womb with an explosion and she gets red clothing due to the blood which came out along with the birth and due to the explosion it spreads on her body. In this scenario , durga is later killed by the tigers while some tigers try to save her too and fight for righteousness. There is an invisible tiger too in this case.

In the Kali one , she looks like priyanka chopra.

Then there is a 13 member family , where one boy is born at the 7th place and he has a sister. He kills the sister with her own nail and she later becomes goddess and creates the world with one tiger . Now in that case none of the 13 family members experienced any physical pain and the brother who had killed the girl and tiger later gets left alone and with one word Aum he sets into meditation and he is one person in that world and takes all births of humans and animals etc all alone. Now there was a very massive operation which took place when that guy killed the girl and the tiger and implanted them in his body. It was a perfect operation. That guy is dil and there is a floor also where he meditated and that was also dil. (It's the heart in hindi). Had that person sinned or not is not known but he is the normal living thing we see ourselves and around us. Moreover this girl looks a bit clever in the vision and probably knew or was pretending that she was not getting harmed in

the operation (atleast that's what appears from the facial expressions which I saw.). Aah I remember, before finally setting into meditation, he bent his body into shapes thinking that the tiger's body shape was different than his and he bent his body into 8400000 shapes too(Or probably 8.4 , i.e. 8 or 9 shapes, or best just 8 shapes for ease.). So, for example if he bent his body like a mosquito , the mosquito he made would be as big as a human. The girl's name was Suman Radaa (similar sounding to modern day Radha) , boy's name was Rahul .

The bible's begining of the world

In one of the most popular version's of bible , it is said that the earth was already present in the beginning, it had matter and water , these two components and probably no air . Then the holy spirit (probably god) roamed near it in space and with one command all the living beings were created.

Now I want to tell what happened in the delta time that command was being spoken, we know that before it no living being was there and after it all the living beings came to existance. Now lets see what happened in that small time. Now before that time the matter looks completely calm and dark and water was also calm and only spilt when the holy spirit passed by (sort of primitive ocean waves) . The matter was clear in it's mind that it is the strongest and it can't be beaten, while the water looks extremely happy and enjoyed even more when holy spirit came near and sort of played with it. In the place of contact of water and matter, there was some sort of contact which was not a matter of conflict between matter and water till the holy spirit came to picture. Now when the command was spoken, water was the first to get frightened and it immediately realised that it's not a single entity and composed of smaller particles and they tried to come near each other in a state of fear created by the command and disturbances were set in water. As the intensity of shrill of the shriek kept on increasing, the disturbances further increased and vapour was created. Matter was still like a single entity and it was feeling like a pain in head (it's body) due to the sudden noice and wanted to reply back but didn't have a tongue or face for that matter. It started withering near the contact , not due to water causing itching but because the noice was so painfull to it. Later we know that now earth has a hot core and an atmosphere and probably formed in the beginning itself . All the living things were created after the peak of the delta was reached (similar to birth case) and slightly after the peak noice . All living beings were created, now where did the holy spirit go, it was also a human in the plane where lots of other humans were standing and he brought out a mic from his pocket and made the first announcement and explained the key things to everyone just born. Now he was completely mad while everyone else just born was partially mad. It's not known what happened to him afterwards but then we know of the Adam and eve story and see that he was still present or maybe some of his descendents or he himself. Then probably Christ was the last in their lineage and now there further cults like illuminati claiming that marovengians in france is a group of people who have christ's descendents and lineage, and many movies like Da Vinci Code showing the legend.

Now here , from physics point of view , what's important is that matter is present, water is separate from matter and then all the charge is concentrated in one supernatural being , the holy spirit . Now every human has a soul which is small chage with shiny black colour or blue color matter in the interior. So we see that Physics mostly is dealing in the Spiritual domain of the world and not physical or mechanical domain as much.

Criticism Now this is a partially mad man's view , i.e. me . I can se souls and male souls have mostly black condensed liquid in the interior , probably a mixture of matter and water for completeness.

However , drug addicts on the other hand think that this cant be the case with them. They say that their soul is completely white , they say that matter and liquid combination can be created as a grass mixture too and can be fed inside the charge outer shell of the soul. Now interestingly I have myself seen souls which are dense white and are very big in size (3 to 4 cm dia compared to usual 2mm dia(meter)) . They look like a spherical egg. There are other smaller completely white souls too (with 2.5 cm dia kindaa) , now i don't know whose souls these are in all but some people have really peculiar souls. It's possible that I am consuming excessive amount of coca cola coke and it's visible in my soul. In other people's case it can be some other substance they are consuming heavily .

Chapter 54

Something more Concrete (than merely panchtantra stories)

Now, we know of Hydrogen. What's it like ? does it smell good or bad ? (Well actually hydrogen has a punchy smell, you can buy uncle chips and it is packed in Nitrogen getting punchy odour due to the chips and the masala) We know it can be used as fuel for state of the art combustion cells, and is seen as a modern alternative to petroleum. We know fusion bombs (also some are hydrogen bombs) work on this principle , that a nuclear reaction hydrogen atoms combine to form Helium and produce lots of energy. Hitler's Archeotype which was a balloon like aeroplane having hydrogen caught fire and all people inside died (America was an enemy though but also the technology was flawed too). Modern cars and space shuttles might benefit from it.

Now hydrogen done. Do we discuss all elements or few of them as we are not a chemistry course.

Hmmm. Hydrogen has importance to physics. Then which other elements . Let's see . Ummm... let have a brief look at all.

Helium, it's the product of a fusion reaction, and an inert gas. Used by scuba divers when mixed with oxygen in some particular ratio, it has upper hand to nitrogen due to non formation of compounds with blood under large pressure.

Lithium, used in some logic gates and transistors (probably ic chips too), first alkali , light in weight atoms.

Berillium, well benadryl cough medicine tastes like it. lots reactive.

Boron, a nuclear product and having nuclear active isotopes too. Having interesting reactions in chemistry salt analysis.

Carbon, Now this deserves some space. Your complete organic chemistry is compounds of carbon. Human body has mostly organic compounds as it's living. Then we gift diamonds , they are pure carbon. What else, we have the carbon contact points in the dc generator motor etc. We have charcoal , a type of slightly impure carbon used as the bed in chemistry salt analysis. We have lots of electrical equipments using carbon parts , batteries and all, even cells and high voltage ones too have large percentage of carbon present. Hmmm. Is that all, we see that all things which are black are not carbon. So we couldn't find as many uses of carbon as it appeared , a blackhole is not made of carbon , it can have probably very little percentage of it. But the human organs are all carbon based organic compounds made . We see that Carbon has ample applications in Organic chemistry and biology but not as much in Physics but still it can have say few usefull applications we can say. Mostly coz we talk electricity , we need metals, we talk machines we need metals, we talk say civil engineering , we have asphalt there , a carbon compound and oil's last residue but otherwise civil engineering has dams and large constructions and carbon can have small applications somewhere or the other in it but not a massively important material . Actually why i am speaking a bit anti is coz since childhood i have been told that it's extremely important material and now that i am trying to write a paragraph , i find not so many uses to be able to write a good paragraph. But the atomic number 6 and mass number 12 can say something. But then these are human relation talk and not physics talk. Let's stay in physics domain only and the judgement is , it's another normal element like the rest of the elements, not too useless , not too usefull.

Nitrogen, It's odourless . Air is odourless and has large percentage of Nitrogen. It is used by plants and nitrogenous manures are considered extremely good for crops. Nitrogen fixing bacteria fix them for plants. And I have already said we have nitrogen packaging of food too. In a sci-fi movie terminator 2 , the enemy gets nearly killed by the accident with a liquid nitrogen tank and few minutes later gets completely killed too due to some other reason.

Oxygen, the massively important material for breathing widely portrayed in sci-fi that people in mars and the rebellion there are getting lesser of it. Mostly kept and extracted for artificial respiration systems. Oxides are a trouble as they are to be broken back to get the pure element and the pure element sometimes comes in contact with air and forms oxide again. No special use as far as physics is concerned. If we spell it as Oak-season then it refers to the dirtiest enemy in the Mythology movies. And they keep on attacking in their season. Now clean air is devtas according to some people(according to me too as i have seen them) and bhagwan is foul air (clean air which has gathered some foul and is smelling bad now) , now what is oxygen it remains to be questioned as it's a part of both clean and foul air .

Fluorine, I have my toothpaste which has fluorine, close up is the name of the paste. I don't have very good reviews for my teeth, but that's mostly due to lesser hardwork from my side . The toothpaste is amongst the best or I would say, the best toothpaste.

Neon, earlier neon lamps were made for light bulbs. Then we had neon gas leakage in large hadron collider due to which it didn't start on the recommended date. These are the only two instances i heard of neon. Inert gas.

Sodium, now Natrium (Na) . Kept in kerosene so that it doesn't catch fire in air. In water if it's kept it burns beautifully . A component of caustic soda and baking powder. 23 it's atomic mass and 11 it's atomic number. Sodna means a very bad term in punjabi sexual terminology. It's interesting that the english terminology has chosen the name sodium and not Natrium completely ignoring the punjabi meaning. As far as physics is concerned , in present times we don't see much use except compounds of it can be usefull. Direct Sodium we don't use much except state of art projects i think.

Magnesium, magnisum ribbon burning in air and MgO_2 ash collection experiments we read in 8th class. It's interesting that the ash is not MgO but with two oxygen atoms even though both O and Mg are divalent.

Aluminium, aha , 13 atomic number and 27 mass number. We used to have aluminium wires in electrical transmission earlier and now we have copper wires

instead. Aluminium sheets are used widely in cutting industry. Aluminium parts used in machinery. Aluminium has good ductility , malleability etc.

Silicon, 14 atomic number. Used in most semiconductor devices. Widely available in earth's crust as silicon dioxide. Artificial chests for ladies in modelling careers. In physics, semiconductors is a vast topic and extends to electronics engineering apart from connecting to various other disciplines. In a computer the Processor and Motherboard are both silicon based.

Phosphorus, 15 atomic number. In this particular substance , we can say we know one natural application. We see bodies burning in shamshan ghat in night due to the phosphorus in bones catching fire. This makes shamshan ghats really haunted places as they create misconception of ghosts behind these burning bones.

Sulphur, 16 atomic number. Sulphur dioxide has fart like pungent smell. Sulphur is supposed to be present in alien clothes which sci-fi movies show have smell. Then sulphur is a component of tubeless tyres etc. Also present in some chemicals. Match stick's burning masala's component etc are few of it's uses.

Chlorine, 17 atomic number but less reactive than fluorine in group 17 elements. Common Salt is abundant in nature and can be used to create chlorine using electrolysis. Detective television series show that it could be used in killings back in 1940's and then it used to be to be very hard to trap them due to weak medical technology for ascertaining the cause of death, however these days it's not used for killing as medical science has grown strong.

Argon, 18 atomic number, (not to be confused with Aragorn of LOTR, he was not inert and was linked to many women and few elves too) . Inert gas.

Potassium, also known as Kalium, 19 Atomic number, My wife was 19 years old when she married me, She was duggal before marriage and became Kalia after marriage with me , then later she left me and married someone else.

Introduction to Vocabulary and Language as Science

Preface

We will be using English Script while describing vocabulary from a few languages, English (The European and Christian Language), Hindi (Modernised Version of the Language of Magadh Kingdom), Punjabi(Indus Ctivilization Language), Sanskrit (Traditional Indian Language under the popular Vedic Civilizations like that of Rama and Krishna) etc.

Part XXVIII

Introduction to Similar Sound and Meaning words used in All (or most) languages Worldover

Chapter 55

Mathematics Related

55.1 Tri

Meanings It can mean three , or Gravy, or to use hit-and-trial or work harder, or a bigger plant with a thick stem(e.g. a Pandori)

Other Related Words tati - 30 (one tenth of 300), excreta(or those who work in this domain, maybe studying it.) etc. , a textile engineer girl. , stammering badly when asking for tea.)

55.2 Jama

Meanings Addition, Just born

Derived Meanings Jamadar - The person who knows just addition, or is new in the clan. Although meant for the low education people, this word is used as a slang for the people doing punitive work as cleaners of toilet , floor managers etc implying that Punit Duggal was low in Education. Similarly Umesh Duggal also did Engineering from a lowly college from reservation from his quota for diploma holders and now their sister has gone totally unsensitive towards the failed courses and they do not matter any more to her even when she probably began the degree in 2012 probably. Duggal himself had a job somehow although he didn't look educated. Raj, everybody knows , didn't need and didn't care for any education.

55.3 Taksim

Meanings High order mathematical operation of Division, The Mobile has an internal calculator while we used it for talking more, judgement on educational status on the basis of fast calculation of division.

55.4 Calculus

Meaning Short form for We are Kal's Kul's people. or the Devil(it's hindi slang Kal) Group people. (Here US is not used but us is used meaning We, although some Punjabi songs refer my family as Pakke Amrika wala), means those knowing this type of mathematics naturally and keep experimenting coolly with it are Devil's children. Basically it would be extremely uneconomical for a computer to do one or 10's of additions, subtractions . Or even 30's or so. So, it deals with higher order calculations and doesn't choose additive mathematics.

Derived Words Calculator - Unless you buy a scientific calculator , it cannot do Calculus, 8th member in the family of Devil, or Neha. Calculus is much higher than Calculator in age and the whole family knows like newton that there should be some method to find the volume of a sphere and naturally comes to it before it is taught in intermediate.

55.5 Jacobian

Meaning Matka(With a master's degree) or People doing higher studies than encountered by us. or Ye Cobi Han . Or this is a copy.(or these are Cobras) Although I have already said in RN's excerpt in my resume , that it's possible that these extreme knowledge tools have some use for God. But not for normal humans who can't feel or interpret these tools physically as to what they mean. But definitely they can be very useful at God's level when whole of the knowledge is required for him. But there can be pitfalls for us like the Maxwell theory which has been classified completely rubbish by me so far as it cannot be verified, but it seems later in life I prove it's some part as right, the way they are presently given in books is wrong probably. Though Maxwell's tools those 4 equations if physically interpreted as waves can be very useful in understanding physics, but maybe I'm wrong and those equations don't mean anything . But I remember something of previous births and probably did some matlab work to prove those equations right for my own use. But I have already failed the course in IIT, proving that what was taught in the course was wrong. So, really lots of work pending for me to do in life, proving maxwell's equations and interpreting them physically for my own use. Probably someone explained the meaning of Maxwell's equations to me from my own book in previous births, that's why I was able to fly then and not now in this birth. But most of the times I flied in Previous births, I had to exit the body immediately afterwards in most cases, simply making it more safe to learn such a knowledge just before death or when one's term as a being is about to end and not before that so that one starts flying and loses his cool about the fact that anyone should not fly without proper protection or it can be dangerous.

55.5.1 What's it like in the flight experience?

One low lying jet of air comes, and you can adjust your hand to ride on it, it's similar to swimming except the fact that it's much easier than swimming as you don't have to apply any thrust but simply fly with the jet's thrust. But if the jet somehow ends in mid air, you simply fall without getting hurt, but there can be crash landings in case you don't land properly . If however some obstacle comes, it's not to be worried of as the jet would find another passage and not end in the obstacle, and we are simply riding the jet already present in air instead of creating our own jet(the

way aeroplanes fly). The only problem is that you probably need to understand the aerodynamics of it using Maxwell's equations and interpret them physically before it actually works. And believe me , all objects are living, so my underwear would learn from seeing me fly and it would be able to fly on itself and can carry a human with it later too. While some people like amit might say that they already fly as birds, but we say it's in a bird's birth not human birth with human unabridged body.

55.5.2 Is your own education complete ? I mean telling your wife to complete her education before showing up?

It's not M.Tech. It's limiting case of Masters audited by God probably and you should pass it at death only with the honours going on after restart. It involves some extreme arts like being able to fly coz u have studied it, being able to swim because your feet can touch the bottom of any deep reservoir as one ligament is broken , being able to slide down any wall coz your wrist bone is broken. Amongst these only flying is something to be spoken of highly others are merely luck related. So, God would be probably a PHD if he is higher than us.

Geography

computing

