Question 1

Complete

Mark 3.00 out of 6.00

Consider $u = \cos^{-1}\left(\frac{y^3}{x^2 - y^2}\right)$

For the given problem, which of the following function is homogeneous?

none of the given options | cos u | u | cos^-1(u)

The correct answer is: cos u

deg of homogeneous function is 1

Then By corollary of Euler's theorem,

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$$

none of the given options | cos u | tan u | -cot u

The correct answer is: -cot u

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} =$$

(a) $\tan^3 u$ (b) $\cos u \tan^2 u$ (c) $-\cot^3 u$ (d) *none of the given options*

d | a | c | b

The correct answer is: c

Hence from above results the value of

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} + \left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right) =$$

(a) $-\cos u \, \mathrm{cosec}^2 u$ (b) $\cos u \tan^2 u$ (c) $\tan^3 u + \sec u$ (d) *none of the given options*

a | d | b | c

The correct answer is: d

Question 2

Complete

Mark 3.00 out of 6.00

Consider $u = \tan^{-1}(y^2 - x^2 + 5xy)$

For the given function which of the following is homogeneous?

u none of the given options tan u tanx

The correct answer is: tan u

deg of homogeneous function is 0

Then By corollary of Euler's theorem,

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$$

u tan u sin 2u none of the given options

The correct answer is: sin 2u

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} =$$

sin 4u sin 4u - sin 2u u none of the given options

The correct answer is: sin 4u - sin 2u

Hence from above results the value of

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} + \left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right) =$$

u none of the given options sin 4u +2 sin 2u sin 4u

The correct answer is: sin 4u

Question 3

Complete

Mark 1.33 out of 4.00

Consider $u=\frac{(x^6+y^6)}{(x^2y^2)}+x^6\tan^{-1}\left(\frac{x^2+y^2}{x^2+2xy}\right)$

Whether u is homogeneous?

- YES
- (•) No

The correct answer is: No

If we write u = v + w where $v=\frac{(x^6+y^6)}{(x^2y^2)}$ and $w=x^6\tan^{-1}\left(\frac{x^2+y^2}{x^2+2xy}\right)$

If v is homogeneous, deg of v is 2

If w is homogeneous, deg of w is undefined

Then By Euler's theorem, $x\frac{\partial v}{\partial x}+y\frac{\partial v}{\partial y}=$

- none of other options
- -2u
- 6v
- (•) 2v

The correct answer is: 2v

$x^2\frac{\partial^2 v}{\partial x^2}+2xy\frac{\partial^2 v}{\partial x\partial y}+y^2\frac{\partial^2 v}{\partial y^2}=$

- (•) 6v
- 2v
- -2u
- none of other options

The correct answer is: 2v

$x\frac{\partial w}{\partial x}+y\frac{\partial w}{\partial y}=$

- 6w
- 2w
- -2v
- 6u
- (•) none of other options

The correct answer is: 6w

$$x^2\frac{\partial^2 w}{\partial x^2}+2xy\frac{\partial^2 w}{\partial x\partial y}+y^2\frac{\partial^2 w}{\partial y^2}=$$

○ 30w ○ none of other options ◉ 42w ○ 6w

The correct answer is: 30w

Hence from above results the value of

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}-\left(x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}\right)=$$

○ 24w ○ 4v+36w ◉ 36w ○ 24u ○ none of other options

The correct answer is: 24w

Putting x = 1 and y = 1 in above result, which of the following is numerical value of expression

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}-\left(x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}\right)$$

(a) 9π (b) 6π (c) $24\tan^{-1}\left(\frac{2}{3}\right)$ (d) $36\tan^{-1}\left(\frac{2}{3}\right)$ (e) none of other options

○ a ○ c ◉ d ○ b ○ e

The correct answer is: c

Question 4

Complete

Mark 3.20 out of 4.00

Consider $u = x^3\left[e^{-\frac{y}{x}}\right] + \frac{1}{y^3}\left[\sin\left(\frac{x}{y}\right)\right]$

Whether u is homogeneous?

YES No

The correct answer is: No

If we write u = v + w where $v = x^3\left[e^{-\frac{y}{x}}\right]$ and $w = \frac{1}{y^3}\left[\sin\left(\frac{x}{y}\right)\right]$

If v is homogeneous, deg of v is 3

If w is homogeneous, deg of w is -3

Then By Euler's theorem, $x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} =$

-3v 3u 6v none of other options 3v

The correct answer is: 3v

$x^2\frac{\partial^2 v}{\partial x^2} + 2xy\frac{\partial^2 v}{\partial x \partial y} + y^2\frac{\partial^2 v}{\partial y^2} =$

6v 12v none of other options 6u -6v

The correct answer is: 6v

$x\frac{\partial w}{\partial x} + y\frac{\partial w}{\partial y} =$

-3w -3u -6w none of other options 3w

The correct answer is: -3w

$$x^2\frac{\partial^2 w}{\partial x^2}+2xy\frac{\partial^2 w}{\partial x\partial y}+y^2\frac{\partial^2 w}{\partial y^2}=$$

12u | none of other options | -12w | 6w | 12w

The correct answer is: 12w

Hence from above results the value of

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}+\left(x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}\right)=$$

3v+9w | 3v-15w | none of other options | 9v-15w | 9u

The correct answer is: 9u