Module 4

Single Phase Transformer

6

Syllabus :

Construction, Working principle, e.m.f. equation, Ideal and practical transformer, Phasor diagrams, Equivalent circuit, O.C. and S.C. tests, Efficiency and regulation, All day efficiency.

6.1 Introduction :

- The **transformer** is a static device (i.e. the one which does not contain any rotating or moving parts) which is used to transfer electrical energy from one ac circuit to another ac circuit, with increase or decrease in voltage / current but **without any change in frequency**.
- This is shown in Fig. 6.1.1.

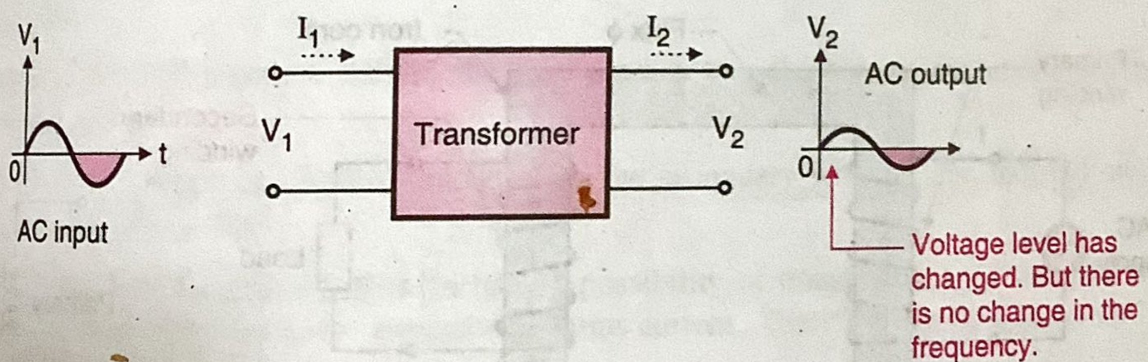


Fig. 6.1.1

- It is important to remember that input to a transformer and output from a transformer both are alternating (AC) quantities.
- The electrical energy is generated and transmitted at an extremely high voltages. The voltage is to be then reduced to a lower value for its domestic and industrial use.
- This is done by using a transformer. Thus it is possible to reduce the voltage level using a transformer (then the transformer is called as a step down transformer).
- On the other hand, we can also use the transformer to increase the voltage level (step up transformer).
- The power transmission system using transformers is shown in Fig. 6.1.2. When the transformer changes the voltage level, it changes the current level also.

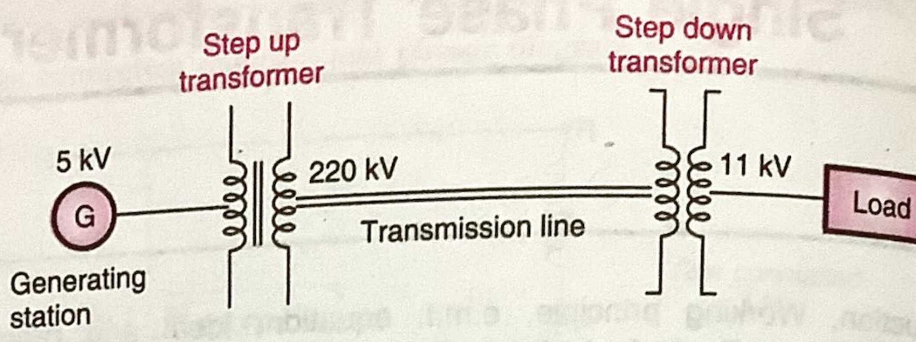


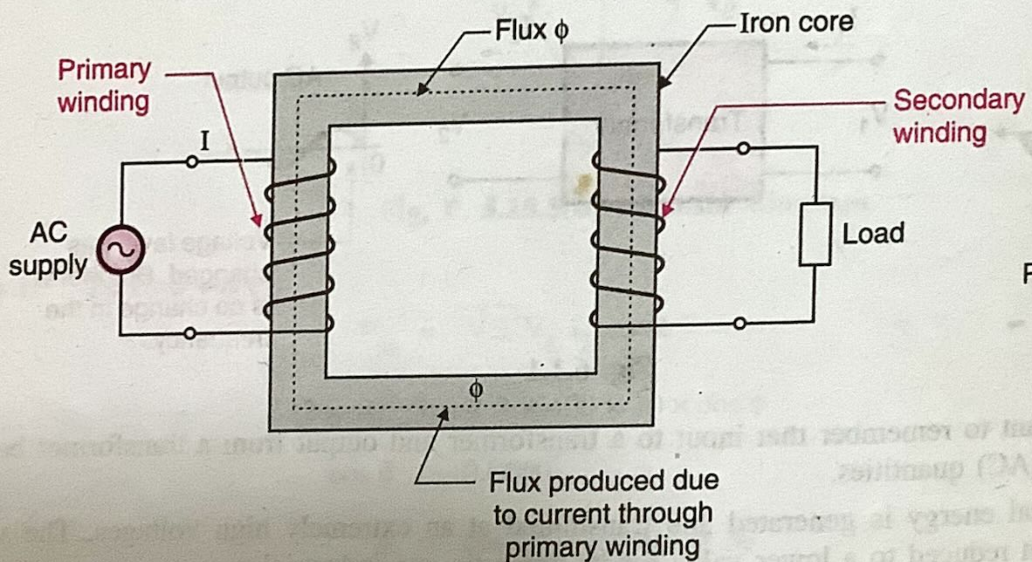
Fig. 6.1.2 : Transmission system

6.1.1 Types of Transformer :

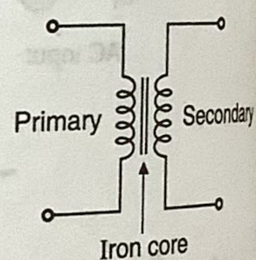
- Transformers are designed for either single-phase or three-phase supply. Accordingly they are called as **single-phase transformers or three-phase transformers.**
- However the principle of operation for both the types is same.

6.1.2 Principle of Operation :

- The construction of a single-phase transformer is as shown in Fig. 6.1.3(a). It consists of two highly inductive coils (windings) wound on an iron or steel core.
- The winding (coil) connected to the ac supply is called as **primary winding** whereas the other one is called as the **secondary winding**. The ac supply is connected to the primary winding whereas the load is connected to the secondary winding.



(a) Elementary transformer



(b) Symbol of transformer

Fig. 6.1.3

- The primary and secondary windings are isolated from each other as well as from the iron core. Thus there is absolutely no physical connection between the primary and secondary windings. The symbolic representation of the transformer is shown in Fig. 6.1.3(b).
- The principle of operation of a transformer has been explained in the Table 6.1.1.

Table 6.1.1 : Operating principle of a transformer

1. As soon as the primary winding is connected to the signle - phase ac supply, an ac current starts flowing throught it.

↓

2. The ac primary current produces an alternating flux ϕ in the core.

↓

3. Most of this changing flux gets linked with the secondary winding thorugh the core.

↓

4. The varying flux will induce voltage into the secondary winding according to the Faraday's laws of electromagnetic induction.

- Thus due to primary current, there is an induced voltage in the secondary winding due to mutual induction.
- Hence the emf induced in the secondary is called as the mutually induced emf.

Can the transformer operate on DC ?

- The answer is no. The transformer action does not take place with a direct current of constant magnitude.
- Because with a DC primary current, the flux produced in the core is not alternating but it is of constant value.
- As there is no change in the flux linkage with the secondary winding, the induced emf in the secondary winding is zero.
- If dc is applied to the primary then there is a possibility of transformer core saturation. If core saturates the primary will draw excessively large current. Therefore application of DC should be avoided.

6.2 Construction of a Transformer :

- The most important parts of a transformer are the windings (coils) and the core.
- However for the large capacity transformers, some other parts such as suitable tank, conservator, bushings, breather, explosion vent etc. are also used alongwith the core and windings.
- The construction of a large single phase transformer is shown in Fig. 6.2.1.

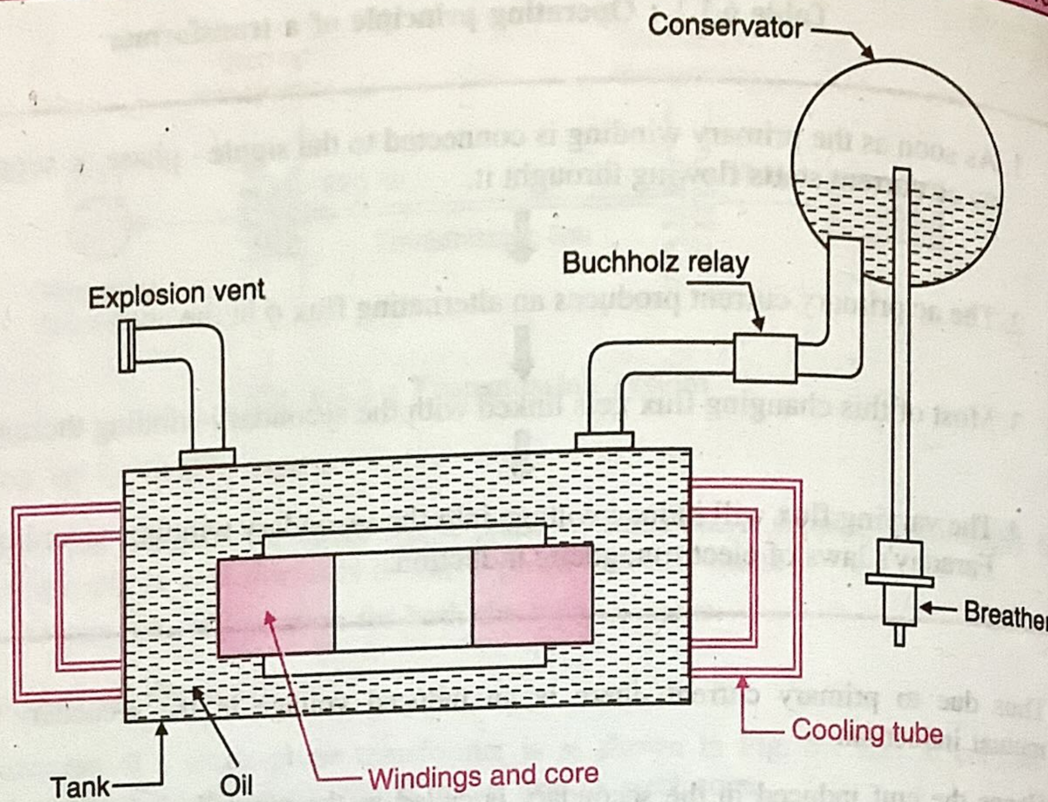


Fig. 6.2.1 : Construction of a single phase transformer

6.2.1 Laminated Steel Core :

- The material used for the construction of the transformer core is **silicon steel**. It is used for its high permeability and low magnetic reluctance. Due to this the magnetic field produced in the core is very strong.
- The core is in the form of stacks of laminated thin steel sheets which are electrically isolated from each other. The laminations are typically 0.35 to 0.5 mm thick.
- The various ways of core construction are shown in Fig. 6.2.2. The core is assembled in such a way, that the assembly provides a continuous path for the magnetic flux, with a minimum air gap.

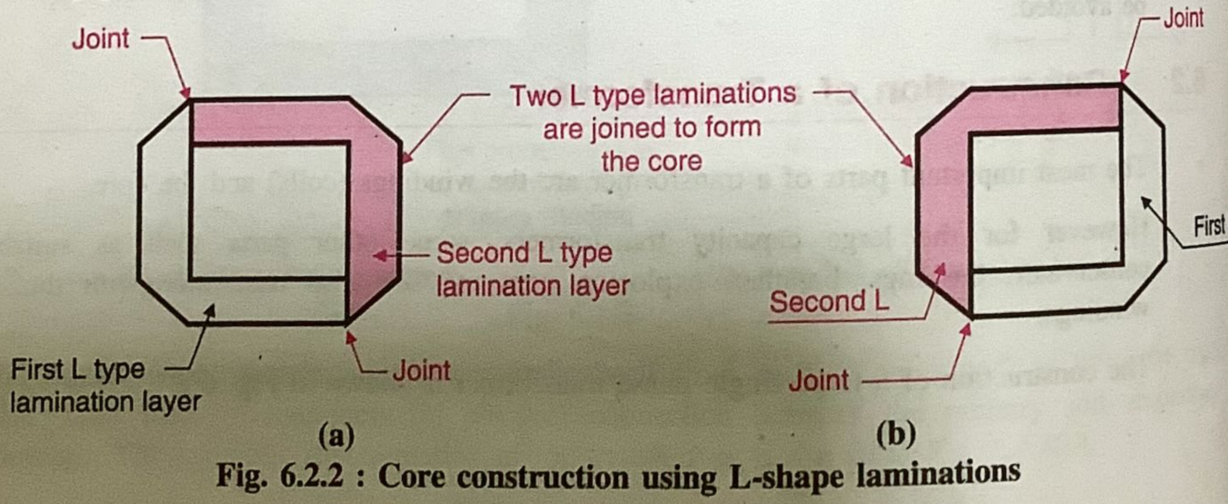


Fig. 6.2.2 : Core construction using L-shape laminations

- Figs. 6.2.2(a) and (b) shows the arrangement of two L-shaped laminations whereas Figs. 6.2.2(c) and (d) shows the adjacent layers of I-shaped laminations.

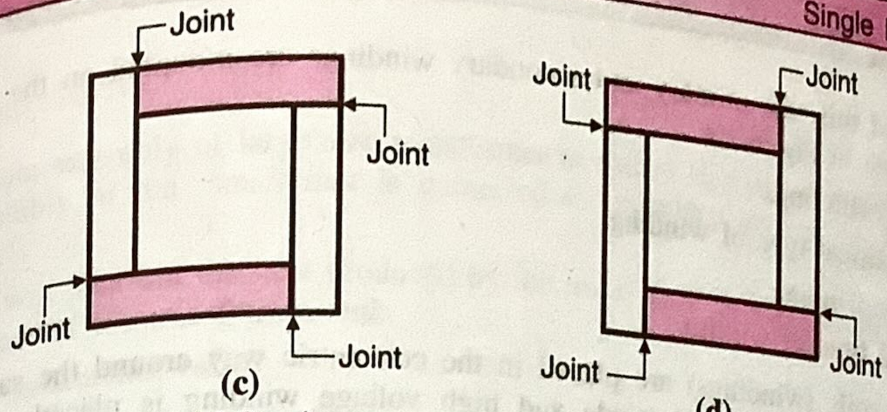


Fig. 6.2.2 : Arrangement of I-shaped laminations

6.2.2 Different Cross-sections for Transformer Limbs :

- The cross-section of the limb of the core of small transformer is rectangular as shown in Fig. 6.2.3(a) and the windings wound around it are also rectangular.

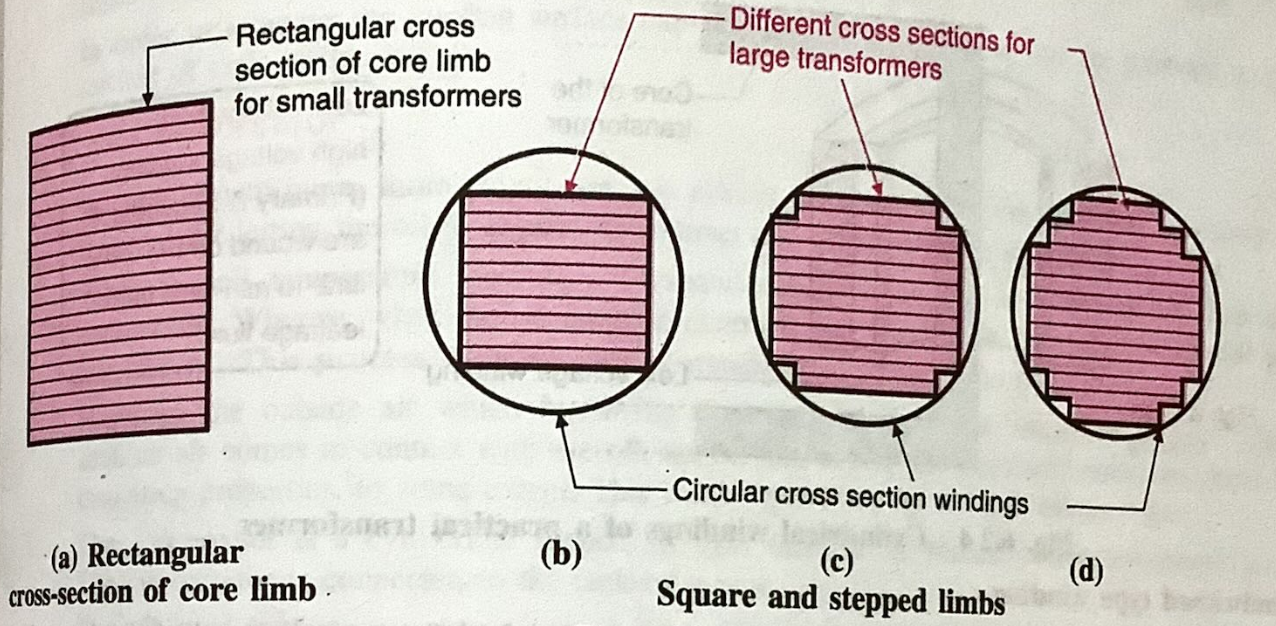


Fig. 6.2.3

- But as the size of the transformers increases, use of rectangular cross-section limb and rectangular windings become wasteful. Hence circular cross-section windings are generally preferred.
- The cross-section of the limb of core of such transformers is either square or stepped as shown in Figs. 6.2.3(b), (c) and (d).
- With increase in the number of steps, the cross-section of the windings will be more and more close to circular cross-section and less copper is required to wind these coils.
- But due to the stepped structure of the limb of the core the labour charges to construct the core increases.

6.2.3 Windings of the Transformer :

- In Fig. 6.2.4 we have shown the primary and secondary windings to be on two different limbs of the core.
- But if such an arrangement is made practically, then a part of the flux produced in the core will not be linked to the secondary winding at all. This is called as the leakage flux.

- In order to avoid this, the primary and secondary windings are mounted on the same limb of the core as shown in Fig. 6.2.4.

There are two types of windings :

- Concentric cylindrical type of winding
- Sandwiched type winding.

Concentric cylindrical type of winding :

- The cylindrical coils (windings) are placed in the concentric way around the same limb with the low voltage winding placed inside and high voltage winding is placed outside it with proper insulation between the windings as shown in Fig. 6.2.4.
- Both the windings are insulated from the core as well.
- Why is the low voltage winding is placed close to the core ? Because it is easy to insulate a low voltage winding from the core rather than insulating the high voltage winding from the core.

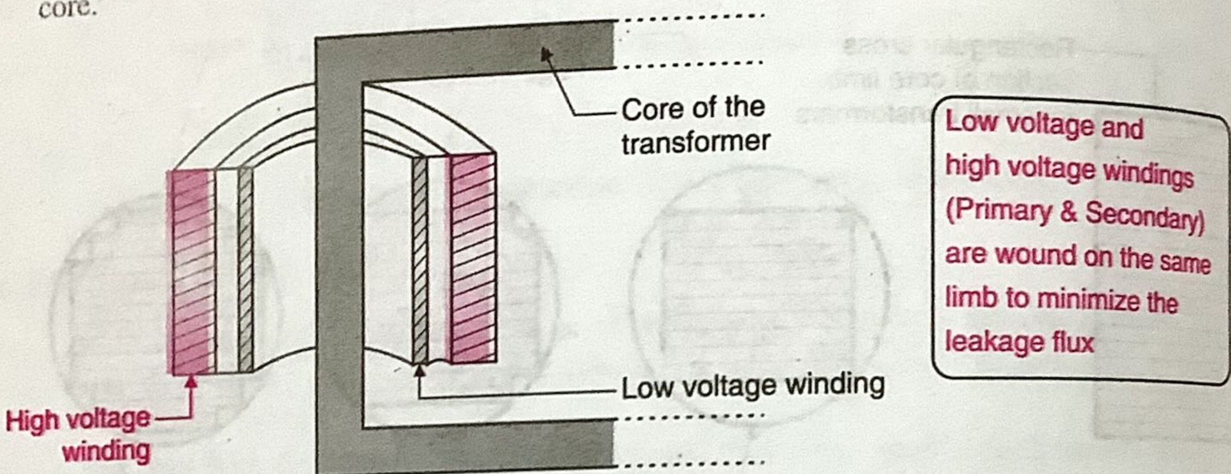


Fig. 6.2.4 : Cylindrical windings of a practical transformer

Sandwiched type winding :

- The other type of windings is called as the **sandwiched type winding** which is shown in Fig. 6.2.5.

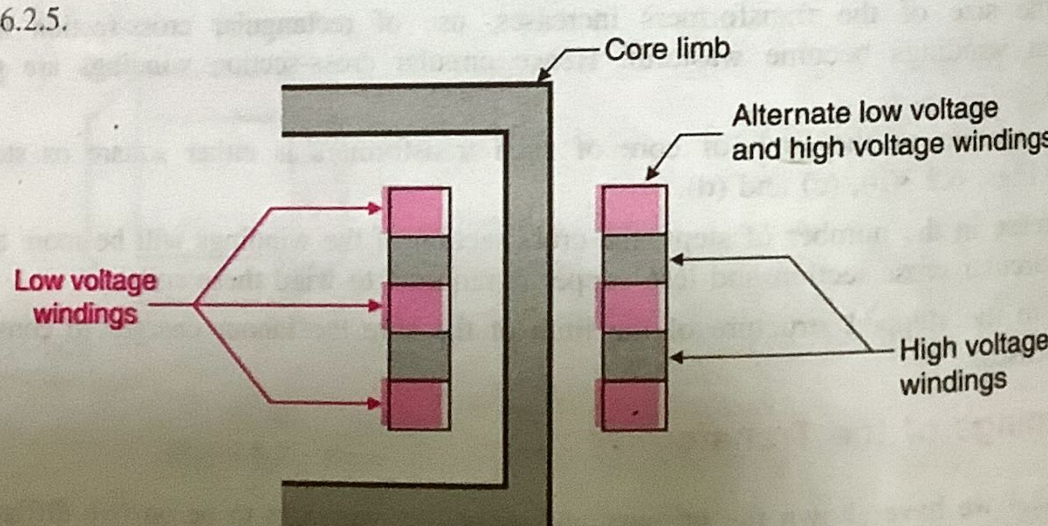


Fig. 6.2.5 : Sandwich type winding

- Here the high voltage and low voltage windings are divided into a number of small coils and then these small windings are interleaved as shown in Fig. 6.2.5.
- The top and bottom windings are low voltage coils because they are close to the core.

6.2.4 Transformer Tank :

- The whole assembly of large size transformer is placed in a **sheet metal tank** Inside the tank the assembly of the transformer is immersed in **oil** which acts as an **insulator** as well as a **coolant**
- The oil will take out the heat produced by the transformer windings and core and transfer it to the surface of the transformer tank.

Function of transformer oil :

- The consturction of the transformer should be such that the heat generated at the core and at the windings should be removed efficiently.
- Moreover, in order to avoid the insulation deterioration, the moisture should not be allowed to creep into the insulation.
- Both these objectives can be achieved by immersing the built up transformer in a closed tank filled with noninflammable insulting oil called transformer oil.
- In order to increase the cooling surface exposed to ambient, tubes or fins are provided on the outside of tank walls.

6.2.5 Conservator :

- In large transformers, some empty space is always provided above the oil level. This space is essential for letting the oil to expand or contract due to the temperature changes.
- When the oil temperature increases, it expands and the air will be expelled out from the conservator. Whereas when the oil cools, it contracts and the outside air gets sucked inside the conservator. **This process is called as the breathing of the transformer.**
- However, the outside air which has being drawn in can have the **moisture content** When such an air comes in contact with the oil, the oil will absorb the moisture content and loses its insulating properties, to some extent. This can be prevented by using a **conservator**
- The conservator is a cylindrical shaped air tight metal drum placed on the transformer tank. The conservator is connected to the tank by a pipe.
- The oil level in the conservator is such that, always some empty space is available above the oil. Due to the use of conservator, the main tank will be always full with oil and the surface of oil in the tank will not be exposed directly to the air.

6.2.6 Breather :

- The apparatus through which breathing of the transformer takes place is called as "Breather".
- The air goes in or out through the breather. To reduce the moisture content of this air, some drying agent (material that absorbs moisture) such as silica gel or calcium chloride is used in the breather. The dust particles present in the air are also removed by the breather.

6.2.7 Buccholz Relay :

- There is a pipe connecting the tank and conservator. On this pipe a protective device called **Buccholz Relay** is mounted.
- When the transformer is about to be faulty and draws large currents, the oil becomes very hot and decomposes.
- During this process different types of gases are liberated. The Buccholz relay get operated by these gases and gives an alarm to the operator. If the fault continues to persist, then the relay will trip off the main circuit breaker to protect the transformer.

6.2.8 Explosion Vent :

- An explosion vent or relief valve is the bent up pipe fitted on the main tank.
- The explosion vent consists of a glass diaphragm or aluminium foil. When the transformer becomes faulty, the cooling oil will get decomposed and various type of gases are liberated.
- If the gas pressure reaches a certain level then the diaphragm in the explosion vent will burst to release the pressure. This will save the main tank from getting damaged.

6.3 Transformer Types :

The transformers are of different types depending on the arrangement of the core and the windings as follows :

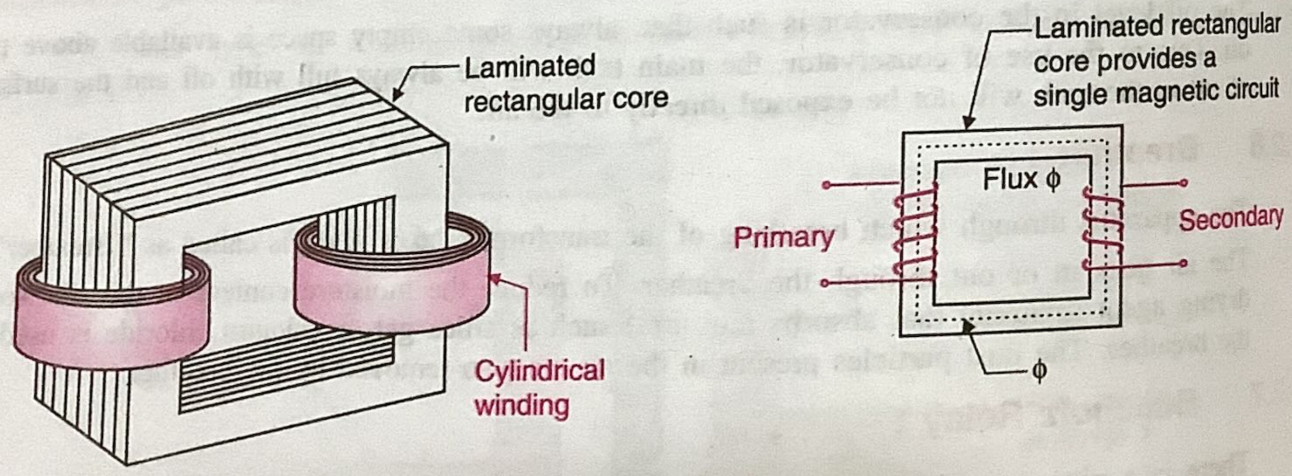
Transformers

- Core type
- Shell type
- Berry type

6.3.1 Core Type Transformer :

The construction of core type transformer is shown in Fig. 6.3.1(a).

- The core of this transformer is in the form of a rectangular frame made from laminations. It provides a single magnetic circuit as shown in Fig. 6.3.1(b).
- The primary and secondary windings are uniformly distributed on two limbs of the core.
- Both the windings (Fig. 6.3.1(a)) are of cylindrical shape and they are arranged in a concentric manner with the low voltage winding placed near the core.

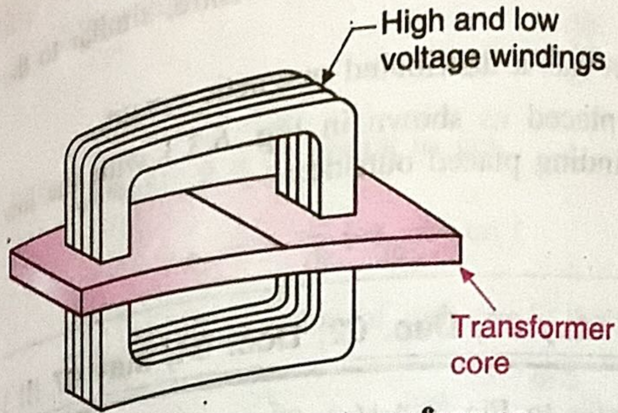


(a) Core type transformer **(b) The rectangular core provides a single magnetic circuit**

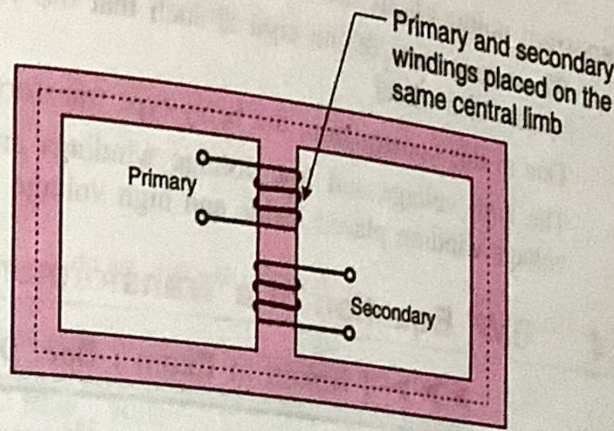
Fig. 6.3.1

6.3.2 Shell Type Transformer :

Fig. 6.3.2(a) shows the construction of a shell type transformer.



(a) Shell type transformer



(b) Simplified diagram

Fig. 6.3.2

Important points about the shell type transformer are as follows :

- The primary and secondary windings are placed on the central limb of the core. The high voltage and low voltage windings are of sandwich type, which are in the form of interleaved pancakes.
- This type of core provides double magnetic circuit. This type of core provides a better mechanical support and protection for the windings.

Table 6.3.1 : Comparison of core type and shell type transformers

Sr. No.	Core type transformer	Shell type transformer
1.	The core has only one window.	The core has two windows.
2.	Windings encircle the core.	Core encircles the windings.
3.	Cylindrical windings are used.	Sandwich type windings are used.
4.	This transformer is easy to repair.	It is not so easy to repair.
5.	Better cooling since more surface is exposed to the atmosphere.	Cooling is not very effective.
6.	Less mechanical protection to the coils (windings).	Better mechanical protection to the coils (windings).

6.3.3 Berry Type Transformers :

The berry type transformer is shown in Fig. 6.3.3.

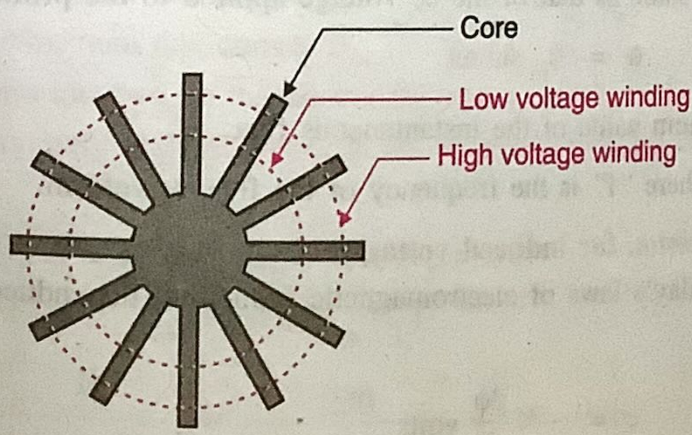


Fig. 6.3.3 : Plan view of berry type transformer

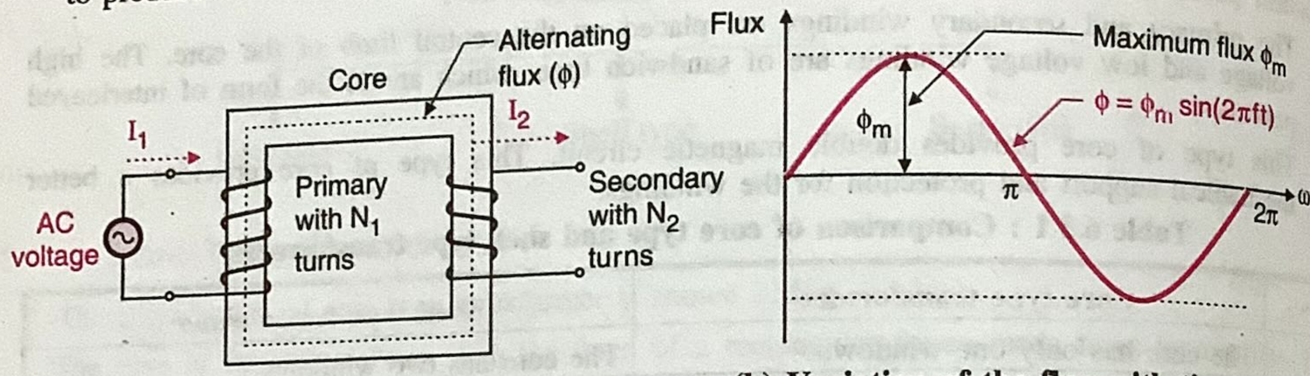
Important points about the berry type transformer are as follows :

- The construction of the core is such that the yoke radiates out from the centre, similar to the spokes of a wheel.
- Due to this construction, the berry type transformer has a distributed magnetic circuit.
- The high voltage and low voltage windings are placed as shown in Fig. 6.3.3 with the low voltage winding placed inside and high voltage winding placed outside.

6.4 EMF Equation of a Transformer :

>>> [Asked in Exam : Dec. 01, May 02, Dec. 02, Dec. 04, May 07 !!!]

- For deriving the emf equation of a transformer refer to Fig. 6.4.1(a). The primary winding is connected across the ac supply. This forces an alternating current through the primary winding to produce an alternating flux (ϕ) in the core.



(a) Elementary transformer (b) Variation of the flux with time

Fig. 6.4.1

- This varying flux gets linked with the secondary and primary windings to induce the mutually induced and self induced emfs in the secondary and primary windings respectively.

6.4.1 Expressions for the Induced Voltages :

Let us now obtain the expressions for the induced voltages in the primary and secondary windings.

Step 1 : Expression for the instantaneous flux ϕ :

As shown in Fig. 6.4.1(b), the instantaneous flux changes in a sinusoidal manner with respect to time. Its frequency "f" is same as that of the ac voltage applied to the primary winding.

$$\therefore \quad \phi = \phi_m \sin \omega t \qquad ...(6.4.1)$$

Where ϕ_m = Maximum value of the instantaneous flux.

ω = $2 \pi f$ where "f" is the frequency of the flux waveform.

Step 2 : Obtain the expression for induced voltage :

According to the Faraday's laws of electromagnetic induction, the induced emf due to varying flux is given by,

$$e = -N \frac{d\phi}{dt} \text{ volts}$$

Step 3 : Obtain the maximum value of "e" per turn :

The value of induced emf per turn can be obtained by substituting N = 1.

$$\therefore \quad e = -\frac{d\phi}{dt}$$

Substitute $\phi = \phi_m \sin \omega t$ to get,

$$e = -\frac{d}{dt}[\phi_m \sin \omega t] \qquad \therefore \quad e = -\phi_m \omega \cdot \cos \omega t \qquad ...(6.4.2)$$

The maximum value of induced voltage per turn is given by substituting $\cos \omega t = \pm 1$.

$$\therefore \quad e_{max} = \omega \phi_m = 2\pi f \phi_m \text{ volts} \qquad ...(6.4.3)$$

Step 4 : Obtain the rms value of "e" per turn :

$$\text{RMS value of "e" per turn} = \frac{e_{max}}{\sqrt{2}} = \frac{2\pi f \phi_m}{\sqrt{2}} = \sqrt{2}\,\pi f \phi_m = 4.44\, f \phi_m \qquad ...(6.4.4)$$

Step 5 : Obtain the expressions for induced voltages E_1 and E_2 :

Let E_1 be the rms induced voltage in the primary winding with N_1 turns and E_2 be the rms induced voltage in the secondary winding having N_2 turns.

$\therefore$ RMS value of induced voltage in primary is,

$$E_1 = \text{RMS value of "e" per turn} \times \text{Number of primary turns}$$

$$\therefore \quad E_1 = 4.44\, f \phi_m \times N_1 = 4.44\, f \cdot N_1 \cdot \phi_m \text{ volts} \qquad ...(6.4.5)$$

Similarly rms value of induced voltage in the secondary winding is,

$$E_2 = \text{RMS value of "e" per turn} \times \text{Number of secondary turns}$$

$$\therefore \quad E_2 = 4.44\, f N_2 \phi_m \text{ volts} \qquad ...(6.4.6)$$

Equations (6.4.5) and (6.4.6) represent the e.m.f. equations.

Ex. 6.4.1 : A step down transformer operates on a 50 Hz ac supply with a primary voltage of 230 V. The cross sectional area of the core is 50 cm^2. Calculate,

1) The maximum flux ϕ_m.
2) The maximum flux density B_m.
3) Voltage induced on the secondary side.

Assume primary and secondary turns to be 500 and 250 respectively.

Soln. :

Given : $E_1 = 230$ V, $A = 50\ cm^2$, $N_1 = 500$, $N_2 = 250$.

1) Maximum flux ϕ_m :

$$E_1 = 4.44\, \phi_m N_1 f$$

$$\therefore \quad \phi_m = \frac{E_1}{4.44 \times N_1 \times f} = \frac{230}{4.44 \times 500 \times 50} = 2.07 \text{ mWb} \qquad ...\text{Ans.}$$

2) **Maximum flux density B_m :**

$$B_m = \frac{\phi_m}{A} = \frac{2.07 \times 10^{-3}}{50 \times 10^{-4}} = \mathbf{0.4144\ Tesla}$$ ...Ans.

3) **Secondary induced voltage E_2 :**

$$E_2 = 4.44\,\phi_m N_2 f = 4.44 \times 2.07 \times 10^{-3} \times 250 \times 50 = \mathbf{114.88\ volts}$$ ...Ans.

6.5 Voltage and Current Ratios of a Transformer :

>>> [Asked in Exam : Dec. 02 !!!]

For obtaining the voltage and current ratios of a transformer, consider the elementary transformer shown in Fig. 6.5.1.

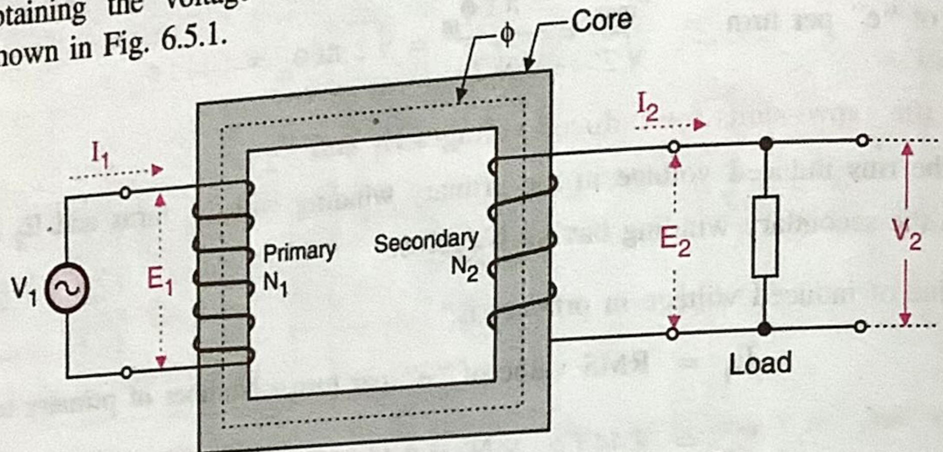


Fig. 6.5.1 : Elementary single phase transformer

6.5.1 Voltage Ratios of the Transformer with Load :

- Let N_1 be the number of turns of the primary and let N_2 be the number of turns of the secondary winding. Also let the rms induced voltage in the primary winding be E_1 volts and the rms secondary induced voltage be E_2 volts.
- Referring to Equations (6.4.5) and (6.4.6) we can write that,

$$E_1 = 4.44\,\phi_m f N_1 \text{ volts}$$

and $$E_2 = 4.44\,\phi_m f N_2 \text{ volts}$$

- Taking the ratio of these expressions we get,

$$\frac{E_1}{E_2} = \frac{N_1}{N_2} \text{ or } \frac{E_2}{E_1} = \frac{N_2}{N_1} \quad ...(6.5.1)$$

- These are the ratios of emfs induced in the two windings of the transformer.

6.5.2 Voltage Ratios for the Transformer without Load :

- Assume the load on the secondary winding in Fig. 6.5.1 is now disconnected. Hence the secondary current $I_2 = 0$. Hence the load terminal voltage V_2 of Fig. 6.5.1 is equal to secondary induced voltage E_2.

$$\therefore \quad V_2 = E_2 \qquad \text{...(6.5.2)}$$

- The primary current I_1 on no load is very small. Hence the ac supply voltage V_1 and primary induced voltage E_1 can be equated.

$$\therefore \quad V_1 \approx E_1 \qquad \text{...(6.5.3)}$$

- Substituting Equations (6.5.2) and (6.5.3) into Equation (6.5.1) we get,

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} \quad \text{or} \quad \frac{V_2}{V_1} = \frac{N_2}{N_1} \qquad \text{...(6.5.4)}$$

- Thus the primary and secondary terminal voltages are proportional to the number of turns of the respective windings.

Definition of voltage ratio :

The ratio of the primary and secondary terminal voltages (i.e. V_1 and V_2) is called as the voltage ratio.

$$\therefore \quad \text{Voltage ratio} = \frac{\text{Primary terminal voltage } (V_1)}{\text{Secondary terminal voltage } (V_2)} \qquad \text{...(6.5.5)}$$

6.5.3 Transformation Ratio (K) :

[Asked in Exam : May 06 !!!]

The transformation ratio for voltage is defined as the ratio of secondary voltage to the primary voltage of a transformer. It is denoted by K.

$$\therefore \quad \text{Transformation ratio K} = \frac{V_2}{V_1} = \frac{E_2}{E_1} = \frac{N_2}{N_1} \qquad \text{...(6.5.6)}$$

6.5.4 Turns Ratio of the Transformer :

The turns ratio of a transformer is defined as the ratio of the number of primary turns to the number of secondary turns.

$$\therefore \quad \text{Turns ratio} = \frac{N_1}{N_2} \qquad \text{...(6.5.7)}$$

6.5.5 Types of Transformers based on the Value of K :

Based on the value of the transformation ratio K, the transformers are classified as, step up, step down or one as to one transformers.

Step up transformer :

The transformer having $K > 1$ or $V_2 > V_1$ is called as the step up transformer. We get higher secondary voltage as compared to the primary winding, hence the name step up.

7

Step down transformer :

The transformer having $K < 1$ or $V_2 < V_1$ is called as the step down transformer. We get lower secondary voltage than the primary voltage, hence the name step down.

One-to-one transformer :

The transformer having $K = 1$ or $V_1 = V_2$ is called as a one-to-one transformer.

6.5.6 Current Ratios :

- The transformer transfers electrical power from one side to the other (primary to secondary) with a very high efficiency (η). If we assume that the power loss taking place in the transformer is very low ($\eta \approx 100\,\%$) then, we can write that

$$\text{Power input} = \text{Power output}$$

$$\therefore \quad V_1 I_1 \cos\phi_1 = V_2 I_2 \cos\phi_2 \qquad ...(6.5.8)$$

Where I_1 and I_2 are the RMS values of the primary and secondary currents of the transformer.

- $\cos\phi_1$ and $\cos\phi_2$ are the power factors of the primary and secondary sides of the transformer. Practically they are of same values.

$$\therefore \quad \cos\phi_1 = \cos\phi_2 \qquad ...(6.5.9)$$

$$\therefore \quad V_1 I_1 = V_2 I_2 \qquad ...(6.5.10)$$

$$\therefore \quad \frac{I_1}{I_2} = \frac{V_2}{V_1} \qquad ...(6.5.11)$$

$$\text{But} \quad \frac{V_2}{V_1} = \frac{N_2}{N_1} = K \quad \therefore \quad \frac{I_1}{I_2} = \frac{N_2}{N_1} = K \qquad ...(6.5.12)$$

- This expression shows that the primary and secondary currents are inversely proportional to the number of turns of the corresponding windings.
- Equation (6.5.11) states that $\frac{I_1}{I_2} = \frac{V_2}{V_1}$, hence the winding currents are inversely proportional to the corresponding winding voltages. i.e. I_1 is inversely proportional to V_1 and I_2 to V_2.

Ex. 6.5.1 : A single-phase transformer has 350 primary and 1050 secondary turns. The primary is connected to a 400 V, 50 Hz supply. If the net cross-sectional area of core is 50 cm^2, find : i) The maximum value of flux density in the core ii) The voltage induced in the secondary winding.

Soln. :

i)
$$E_1 = 4.44\,\phi_m . f . N \text{ V} \; ; \quad \text{where } B_m = \frac{\phi_m}{A_i}$$

$$\therefore \quad E_1 = 4.44\, B_m \times A_i \times f \times N_1 \text{ volts}$$

$$\therefore \quad 400 = 4.44 \times B_m \times 50 \times 10^{-4} \times 50 \times 350$$

$$\text{Giving} \quad B_m = 1.0296 \text{ T}$$

ii)

$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$

$$\therefore \quad E_2 = 400 \times \frac{1050}{350} = 1200 \text{ V}$$

Ex. 6.5.2 : A single-phase transformer has 1000 turns on its primary and 400 turns on secondary side. An a.c. voltage of 1250 V, 50 Hz is applied to its primary side with secondary open circuited. Find : i) The emf on the secondary side ii) The peak value of flux density. Take effective area of core as 60 cm^2.

Soln. :

i)

$$E_2 = E_1 \times \frac{N_2}{N_1} = 1250 \times \frac{400}{1000} = 500 \text{ V}$$

ii)

$$E_1 = 4.44 \times \phi_m f N_1$$

$$E_1 = 4.44 \times B_m \times A_i \times f \times N_1 \text{ volts} \quad \text{...Because } \phi_m = B_m A_i$$

$$\therefore \quad 1250 = 4.44 \times B_m \times 60 \times 10^{-4} \times 50 \times 1000$$

$$\therefore \quad B_m = 0.9384 \text{ Tesla}$$

6.6 Rating of Transformer :

- Generally the rating of a machine should indicate the power supplied by it. But in case of a transformer, the output power is not constant.
- It keeps changing with the load. The output power factor is also a function of load.
- Hence the rating of a transformer is expressed in terms of voltage and current as follows :

$\therefore$ Rating of a transformer = Primary voltage × Primary current

Or = Secondary voltage × Secondary current

- As the voltage and current may or may not be in phase, the units of transformer ratings are Volt Ampere (VA) or kilo Volt Ampere (kVA) or Mega Volt Ampere (MVA).

$$\therefore \text{Rating in VA or kVA or MVA} = V_1 \times I_1 = V_2 \times I_2 \quad \text{....(6.6.1)}$$

- From the given ratings of the transformer we can obtain the values of primary and secondary currents as :

$$I_1 = \frac{\text{kVA} \times 10^3}{V_1} \quad \text{...(6.6.2)}$$

$$\text{and} \quad I_2 = \frac{\text{kVA} \times 10^3}{V_2} \quad \text{....(6.6.3)}$$

- If the rating is specified in MVA then the multiplying factor will be 10^6 rather than being 10^3.

The complete ratings of a transformer :

The complete ratings of a transformer includes the ratio of primary and secondary voltages, kVA rating and supply frequency as follows :

3300 V/240 V, 5 kVA, 50 Hz

Where 3300 V is the primary voltage V_1.

240 V is the secondary voltage V_2.

5 kVA is the kVA rating and 50 Hz is the supply frequency.

Ex. 6.6.1 : A 3300 V / 200 V, 50 Hz, 100 kVA transformer has its low voltage winding with 80 turns. Calculate :

i) The currents in both the windings.

ii) Number of turns of high voltage winding and

iii) Maximum value of the flux. Given that the transformer is loaded to it full capacity.

Soln. :

Given : Rating of the transformer 3300 V / 200 V, 100 kVA, frequency f = 50 Hz, $N_2 = 80$

Part I : Primary and secondary currents :

$$\text{Primary current } I_1 = \frac{\text{kVA} \times 10^3}{V_1} = \frac{100 \times 10^3}{3300} = 30.303 \text{ A}$$ ...Ans.

$$\text{Secondary current } I_2 = \frac{\text{kVA} \times 10^3}{V_2} = \frac{100 \times 10^3}{200} = 500 \text{ A}$$ ...Ans.

Part II : Number of turns of high voltage winding :

Primary is the high voltage winding.

$$\therefore \frac{V_1}{V_2} = \frac{N_1}{N_2}, \qquad \therefore N_1 = \frac{V_1}{V_2} \times N_2$$

$$\therefore N_1 = \frac{3300}{200} \times 80 = 1320$$ ...Ans.

Part III : Maximum value of flux :

$$E_1 = 4.44 \phi_m f N_1$$

$$\therefore \phi_m = \frac{E_1}{4.44 f N_1} = \frac{3300}{4.44 \times 50 \times 1320} = 11.26 \text{ mWb}$$ ...Ans.

Ex. 6.6.2 : A 200 kVA, 3300/240 V, 50 Hz single phase transformer has 80 turns on secondary winding. Calculate :

(i) Primary and secondary currents on full load.

(ii) Maximum value of flux

(iii) Number of primary winding turns.

Soln. :

Given : kVA rating = 200 kVA, V_1 = 3300 V, V_2 = 240 V, f = 50 Hz, N_2 = 80

Part I : Primary and secondary currents on full load :

$$kVA = V_1 I_1 = V_2 I_2$$

$$\therefore \quad I_1 = \frac{VA}{V_1} = \frac{200 \times 10^3}{3300} = 60.6 \text{ A}$$ ...Ans.

$$\text{and} \quad I_2 = \frac{VA}{V_2} = \frac{200 \times 10^3}{240} = 833.33 \text{ A}$$ ...Ans.

Part II : Maximum flux :

We know that $E_2 = 4.44 \phi_m f N_2$

$$\therefore \quad \phi_m = \frac{E_2}{4.44 \times f \times N_2} = \frac{240}{4.44 \times 50 \times 80} = 13.51 \text{ mWb}$$ ...Ans.

Part III : Number of primary turns :

$$\frac{N_1}{N_2} = \frac{3300}{240}$$

$$\therefore \quad N_1 = \frac{3300}{240} \times 80 = 1100$$ ...Ans.

Ex. 6.6.3 : Estimate the number of turns of primary and secondary windings of a 1 phase, 50 Hz transformer with a maximum value about 0.05 Wb and voltage ratio of 11000 V / 110 V.

Soln. :

Given : f = 50 Hz, ϕ_m = 0.05 Wb

Voltage ratio = $\frac{11000 \text{ V}}{110 \text{ V}}$, E_1 = 11000 V, E_2 = 110 V

To find : i) Number of turns of primary winding N_1

ii) Number of turns of secondary winding N_2.

Steps to be followed :

Step 1 : To find N_2 :

$$E_2 = 4.44 \phi_m f N_2$$

$$110 = 4.44 \times 0.05 \times 50 \times N_2$$

$$\therefore \quad N_2 = \frac{110}{4.44 \times 0.05 \times 50} = 9.9 \text{ turns}$$ ...Ans.

Step 2 : To find N_1 :

$$\frac{E_1}{E_2} = \frac{N_1}{N_2}$$

$$\therefore \quad N_1 = \frac{11000}{110} \times 9.9 = \mathbf{990\ turns}$$

...Ans.

6.7 Losses in a Transformer :

>>> [Asked in Exam : Dec. 04 !!!]

- An ideal transformer is loss free. But in the practical transformer there are following losses taking place.

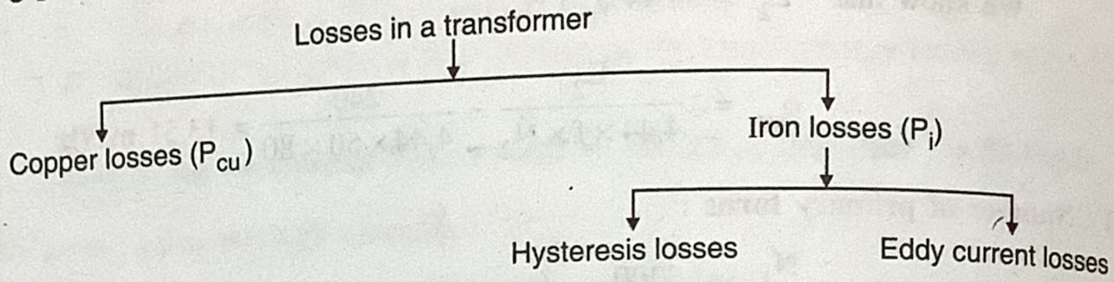


Fig. 6.7.1 : Different losses taking place in a transformer

- As shown in Fig. 6.7.1, the total loss in a transformer can be divided into two types namely the **copper loss** and the **iron loss**.
- The iron loss is further classified into two types namely the **hysteresis loss** and **eddy current loss**.

6.7.1 Copper Loss (P_{cu}) :

- The total power loss taking place in the winding resistances of a transformer is known as the copper loss.

  $\therefore$ Copper loss = Power loss in the primary resistance + Power loss in the secondary resistance.

- The copper loss is denoted by P_{cu}.

$$\therefore \quad P_{cu} = I_1^2 R_1 + I_2^2 R_2 \qquad \text{...(6.7.1)}$$

Where $I_1^2 R_1$ = Power loss taking place in the primary resistance R_1.

and $I_2^2 R_2$ = Power loss taking place in the secondary resistance R_2.

- The copper loss should be kept as low as possible to increase the efficiency of the transformer. To reduce the copper loss, it is essential to reduce the resistance R_1 and R_2 of the primary and secondary windings.

- Copper losses are also called as **variable losses** as they are dependent on the square of load current. The relation between copper loss at full load and that at half load is as follows :

$$P_{cu\,(HL)} = \left(\frac{1}{2}\right)^2 P_{cu\,(FL)}$$

$$\therefore \quad P_{cu\,(HL)} = \frac{P_{cu\,(FL)}}{4} \qquad ...(6.7.2)$$

Where $P_{cu\,(FL)}$ = Copper loss at full load and

$P_{cu\,(HL)}$ = Copper loss at half load.

6.7.2 Iron Loss (P_i) :

- Iron loss P_i is the power loss taking place in the iron core of the transformer.
- It is equal to the sum of two components called hystersis loss and eddy current loss.

$$P_i = \text{Hysteresis Loss} + \text{Eddy current loss}$$

Hysteresis losses :

- We have already discussed the hysteresis loss taking place in a magnetic material.
- The area enclosed by the hysteresis loop of a material represents the hysteresis loss.
- Hence special magnetic materials should be used in order to reduce the hysteresis loss.
- Materials such as silicon steel has hysteresis loops with very small area.
- Hence such materials are preferred for the construction of core. **Commercially such steel is called as Lohys, means low hysteresis materials.**
- The hysteresis loss is frequency dependent. As we increase the frequency of operation, the hysteresis loss will increase proportionally.

Eddy current losses :

- Due to the time varying flux, there is some induced emf in the transformer core.
- This induced emf causes some currents to flow through the core body.
- These currents are known as the **eddy currents**.
- The core is made of steel and has some finite resistance. Hence due to the flow of eddy currents, heat will be produced. The power loss due to the eddy currents is given by :

$$\textbf{Eddy current loss} = (\textbf{Eddy current})^2 \times r \qquad ...(6.7.3)$$

Where r = Resistance of the core.

- The eddy current losses are **minimized by using the laminated core.**
- The core is manufactured as a stack of laminations rather than a solid iron core.
- These laminations are insulated from each other by means of a varnish coating on all the laminations.
- Hence each lamination acts as a separate core with a small cross-sectional area, providing a large resistance to the flow of eddy currents.
- The eddy current loss also is frequency dependent. It is directly proportional to the square of operating frequency.

- Hence the iron loss P_i of the total loss is dependent on the frequency but the copper loss P_{cu} is constant irrespective of frequency.

The iron loss is denoted by P_i. It is the sum of hysteresis and eddy current loss. Iron loss is a constant loss which does not depend on the level of load.

6.8 An Ideal Transformer :

[Asked in Exam : May 03 !!!]

An ideal transformer is the transformer having the following characteristics :

- The losses are zero (No iron loss, no copper loss).
- The primary and secondary winding resistances are zero.
- The leakage flux is zero. Therefore all the flux produced by the primary winding is coupled to the secondary.
- A small current is required to develop flux inside the core. This happens because the permeability of the core is very large.
- The external voltage applied to the primary, V_1 is same as the primary induced voltage E_1. This is because the primary winding resistance is zero and so there is no voltage drop across it.

$$\therefore \quad E_1 = V_1$$

- Similarly the voltage induced in the secondary winding (E_2) will be equal to the load voltage V_2, because the secondary resistance is zero.

$$\therefore \quad E_2 = V_2$$

- The transformation ratio for an ideal transformer is given by,

$$K = \frac{E_2}{E_1} = \frac{V_2}{V_1}$$

- Efficiency of an ideal transformer is 100%. This is because there are no losses taking place.
- The voltage regulation is 0%. That means the secondary voltage will remain constant irrespective of the load current.

6.8.1 Ideal Transformer on No Load :

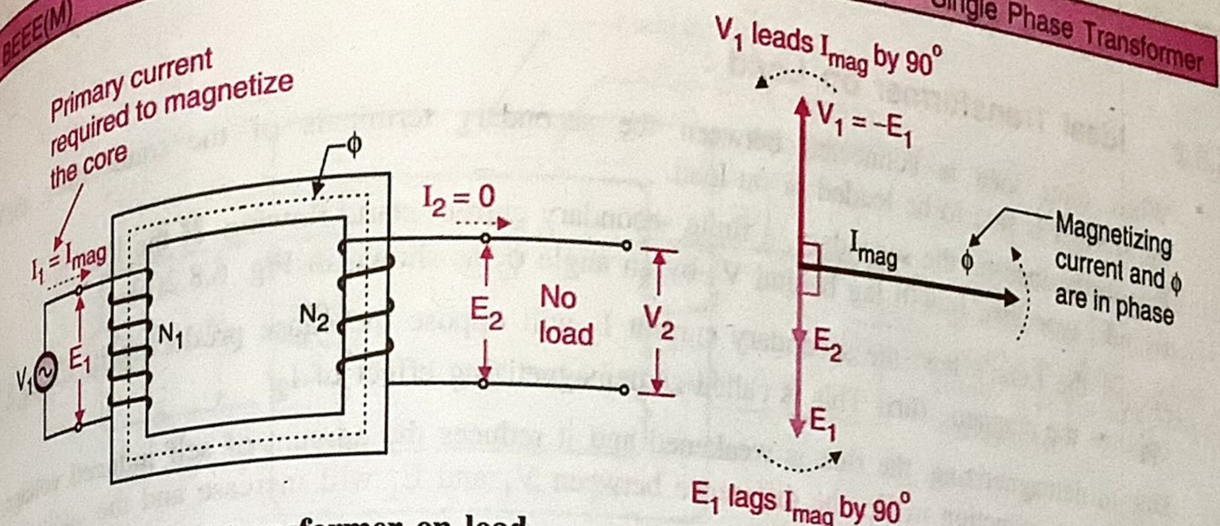
An ideal transformer is as shown in Fig. 6.8.1(a). An ideal transformer is the one which has no power losses. In order to have a transformer with zero loss, the following conditions should be satisfied.

Conditions for an ideal or lossfree transformer :

i) The primary and secondary windings do not have any resistance. i.e. winding resistance of primary secondary should be zero.

ii) The losses taking place in the core i.e. hysteresis loss and eddy current loss should be zero.

iii) There should not be any leakage flux.



(a) Ideal transformer on load

Fig. 6.8.1

(b) Phasor diagram

Operation and phasor diagram :

- An ac voltage V_1 is applied across the primary winding of the transformer. As the load on the transformer is zero i.e. $R_L = \infty$, ideally the primary current $I_1 = 0$. But practically a small current called the magnetizing current $I_{mag.}$ flows through the primary winding.
- The magnetizing current is used for magnetizing the transformer core. As the primary winding is assumed to be purely reactive (R = 0) the magnetizing current lags behind the primary induced voltage by 90°, as shown in Fig. 6.8.1(b).
- Due to the sinusoidal magnetizing current, a sinusoidally varying magnetic flux is produced in the iron core. The flux is in time phase with $I_{mag.}$ as shown in Fig. 6.8.1(b).
- Due to this varying flux, emfs are induced in the primary (self induced voltage) E_1 and secondary (mutually induced voltage) E_2 respectively.

$$E_1 = -V_1 \quad \text{and} \quad E_2 = V_2 \qquad \text{...(6.8.1)}$$

- The magnitudes of E_1 and E_2 are proportional to the number of turns N_1 and N_2 respectively.
- The secondary induced voltage E_2 will also oppose V_1. So E_2 also appears in phase opposition with V_1. The magnitude of E_2 however is dependent on the turns ratio N_2 / N_1.
- E_1 and E_2 appear in phase with each other and in phase opposition with V_1.

Power input on no load :

- The input power to the ideal transformer on no load is given by,

$$\therefore \quad P_0 = V_1 I_m \cos\phi_0 \qquad \text{...(6.8.2)}$$

where ϕ_0 = Angle between V_1 and I_m which is 90°

$$\therefore \quad P_0 = V_1 I_m \cos 90^\circ = 0 \text{ W} \qquad \text{...(6.8.3)}$$

- Thus the input power to an ideal transformer on no load is zero. The output power is zero and there are no losses taking place in the ideal transformer.

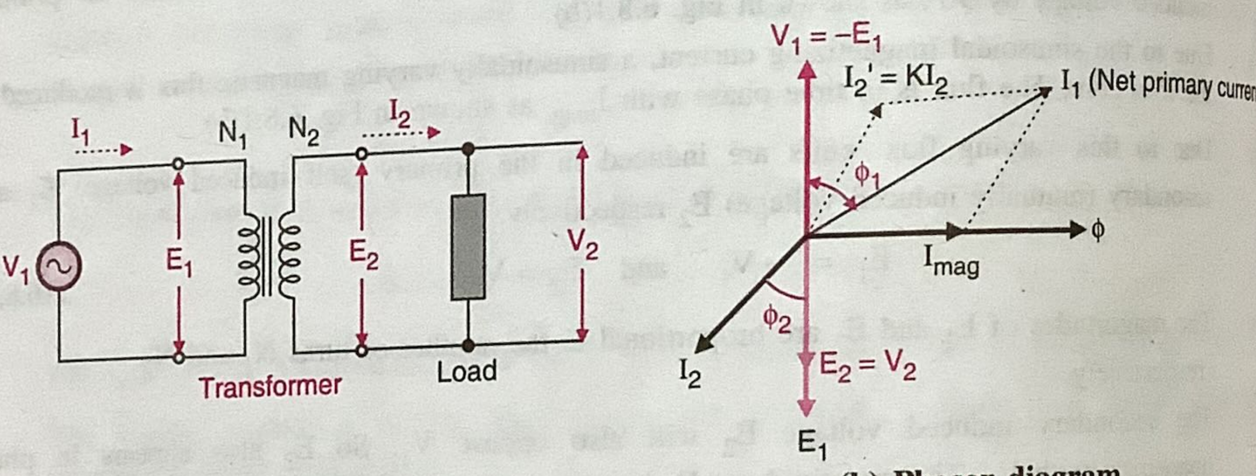
6.8.2 Ideal Transformer on Load :

- When some load is connected between the secondary terminals of the transformer, the transformer is said to be loaded or on load.
- Due to the load on the secondary, a finite secondary current starts flowing. If the load is (R + L) type then I_2 will lag behind V_2 by an angle ϕ_2 as shown in Fig. 6.8.2(b).
- As per the Lenz's law, the secondary current I_2 will oppose the cause producing it. Hence it opposes the magnetic flux. This is called as demagnetizing effect of I_2.
- Due to demagnetizing, the flux is weakened and it reduces the amount of self induced voltage E_1. Due to reduction in E_1, the difference between V_1 and E_1 will increase and the additional primary current I_2' called as load component starts flowing as shown in Fig. 6.8.2(b).

$$I_2' = I_2 \times \frac{N_2}{N_1} = K I_2 \quad \text{...(6.8.4)}$$

- The current $I_2' = K I_2$ and it is 180° out of phase with the current I_2. The net primary current I_1 is the phasor sum of I_2' and I_{mag} as shown in Fig. 6.8.2(b).

$$\therefore \quad \bar{I}_1 = \bar{I}_2' + \bar{I}_{mag} \quad \text{...(6.8.5)}$$



(a) Ideal transformer on load

(b) Phasor diagram

Fig. 6.8.2

- Thus due to load on secondary side, the primary current of the transformer increases to supply the additional power to the load.
- The angle between V_1 and I_1 is ϕ_1 as shown in Fig. 6.8.2(b). Hence the primary power factor is $\cos \phi_1$.

Why does primary current increase when the load current is increased ?

- When the transformer is loaded, the load current I_2 will start flowing. Due to increase in load current I_2 the secondary ampere turns $N_2 I_2$ will also increase.
- This increased secondary mmf $(N_2 I_2)$ will increase the flux ϕ_2 set up by the secondary current.

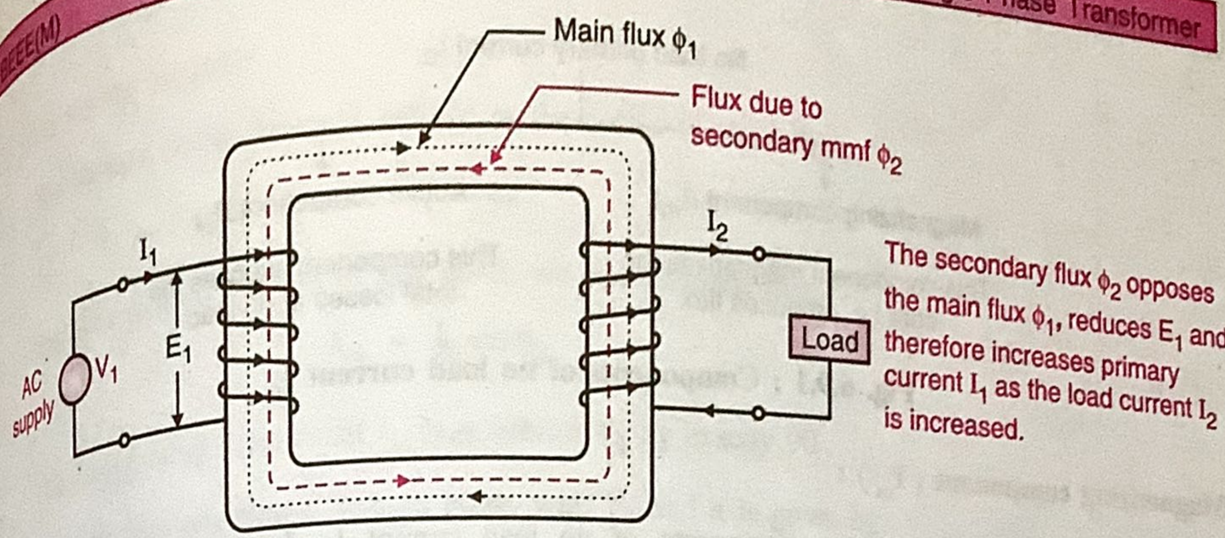


Fig. 6.8.3 : Primary current increases with increase in the secondary current

- This flux opposes the main flux ϕ_1 set up in the core by the current flowing through the primary winding. Hence the secondary mmf $N_2 I_2$ is called as the demagnetizing ampere turns.
- Due to reduction in the main flux ϕ_1, the induced emf in the primary winding E_1 will also reduce. Hence the difference between V_1 and E_1 will increase and the primary current will increase.

6.9 Practical Transformer on No Load :

- The major difference between the ideal and practical transformer is that we have to take into account various "losses".
- Some of losses taking place in the practical transformer are as follows :

1. Hysteresis loss.
2. Eddy current loss.
3. Iron losses.
4. Copper losses taking place in the winding resistances.

- When the practical transformer is on no load, the secondary current will be zero. Hence the copper loss in the secondary winding is zero.
- However a small primary current does flow under the no load condition. Due to the small primary resistance, a small primary copper loss takes place even at no load.
- The primary current under no load condition has to supply the iron losses (hysteresis loss and eddy current loss) and a small primary copper loss.
- The primary current under the no load condition is denoted by I_0. The no load primary current has two main functions to perform as follows :

(1) To magnetize the core i.e. to produce flux and

(2) To supply for the total losses under the no load condition.

- The no load primary current I_0 can be thought of made up of two components I_m and I_c which perform the two functions mentioned above.

No load primary current I_0

Magnetizing component (I_m)
This component magnetizes the core i.e. produces flux

Active component (I_c)
This component supplies the total losses at no load

Fig. 6.9.1 : Components of no load current I_0

Magnetizing component (I_m) :

- This is the purely reactive component of no load current I_0. It magnetizes the core and produces flux in the core.
- This component is at 90° with respect to E_1 as shown in the phasor diagram shown in Fig. 6.9.2. This component (I_m) is also called as wattless component.

Active component I_c :

- The active component I_c is the component of no load current I_0. Its job is to supply the total loss under no load conditions.
- It is therefore called as the power component or core loss component of I_0. It is at 90° with respect to the magnetizing current I_m as shown in Fig. 6.9.2.

6.9.1 Phasor Diagram of Practical Transformer on No Load :

➤➤➤ [Asked in Exam : Dec. 06 !!!]

- The phasor diagram of a practical transformer on no load is as shown in Fig. 6.9.2.
- The two components I_m and I_c are 90° phase shifted with respect to each other and I_0 is the resultant of the two.
- Hence the total no load current I_0 is the phasor addition of I_c and I_m.

$$\therefore \quad \bar{I}_0 = \bar{I}_m + \bar{I}_c \qquad ...(6.9.1)$$

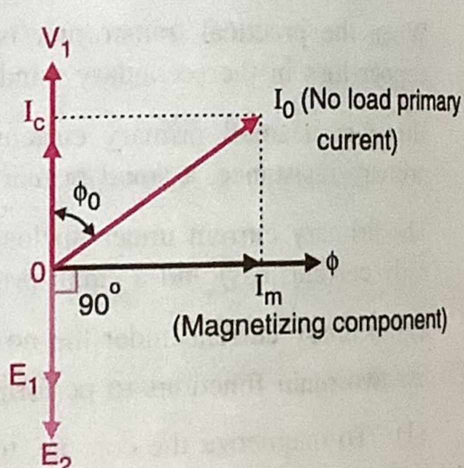


Fig. 6.9.2 : Phasor diagram of practical transformer on no load

- In the practical transformer, the no load current I_0 does not have a 90° phase shift with respect to V_1. But now it lags V_1 by an angle ϕ_0 which is smaller than 90°.

No load power factor :

- The no load power factor is defined as the cosine of angle ϕ_0.

$\therefore$ No load power factor $= \cos\phi_0$

- From the phasor diagram of Fig. 6.9.2, we can conclude that,

$$I_m = I_0 \sin\phi_0 \quad ...(6.9.2)$$

- The magnetizing component I_m lags behind V_1 by exactly 90°.
- The core loss component I_c is in phase with V_1 and it is given by,

$$I_c = I_0 \cos\phi_0 \quad ...(6.9.3)$$

- The magnitude of no load primary current is therefore given by,

$$I_0 = \sqrt{I_m^2 + I_c^2} \quad ...(6.9.4)$$

And the no load power factor angle ϕ_0 is given by,

$$\phi_0 = \tan^{-1}(I_m / I_c) \quad ...(6.9.5)$$

- The total power input on no load is denoted by W_0 and it is given by,

$$W_0 = V_1 I_0 \cos\phi_0 \quad ...(6.9.6)$$

But $I_0 \cos\phi_0 = I_c$, Hence

$$W_0 = V_1 I_c \quad ...(6.9.7)$$

- The value of no load primary current I_0 is very small, of the order of 3 to 5% of the rated full load current.
- Therefore the primary copper loss is very very small. Hence I_c is called as core loss or iron loss component.
- Hence W_0 represents the core loss or iron loss as the copper is small.

$$\therefore \quad W_0 = \text{Iron loss} = V_1 I_c \quad ...(6.9.8)$$

6.9.2 Practical Transformer on Load :

- When some load is connected between the secondary terminals of the transfer, the transformer is said to be load or it is said to be on load.
- Due to the load on the secondary, a finite secondary current starts flowing. Depending on the type of load (resistive, inductive or capacitive) the secondary current I_2 will be in phase with or lag or lead the load voltage V_2.

	Type of load	Load current
1.	Purely resistive (R)	I_2 is in phase with V_2
2.	(R + L) type	I_2 lags behind V_2
3.	(R + C) type	I_2 lead V_2

- Due to loading of the transformer, the primary current increases above its no load value. The increase in primary current takes place due to following sequence of operation.
- When the transformer is loaded, the load current I_2 will start flowing. Due to increase in load current I_2 the secondary ampere turns $N_2 I_2$ will also increase.
- This increased secondary mmf $(N_2 I_2)$ will increase the flux ϕ_2 set up by the secondary current.

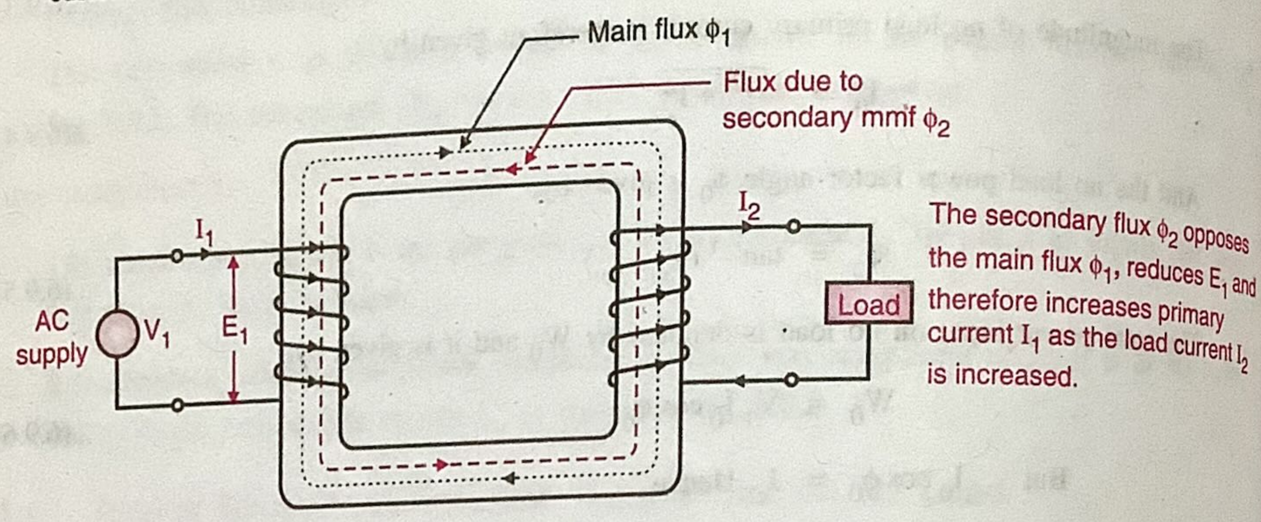


Fig. 6.9.3 : Primary current increases with increase in the secondary current

- This flux opposes the main flux ϕ_1 set up in the core by the current flowing through the primary winding. Hence the secondary mmf $N_2 I_2$ is called as the demagnetizing ampere turns.
- Due to reduction in the main flux ϕ_1, the induced emf in the primary winding E_1 will also reduce. Hence the difference between V_1 and E_1 will increase and the primary current will increase.

Effect of additional primary current :

- The additional current drawn by the primary winding due to the loading is called as the load component I_2'.
- This current (I_2') is 180° out of phase with the load current I_2. The current I_2' develops its own magnetic flux ϕ_2' as shown in Fig. 6.9.4.
- ϕ_2' is in the opposite direction to that of ϕ_2 as shown in Fig. 6.9.4. Hence it helps the main flux ϕ_1 as ϕ_1 and ϕ_2' are in the same direction.

Thus the reduction in the main flux due to ϕ_2 is compensated by ϕ_2' and the core flux ϕ_1 will almost remains constant. Therefore for any load between no load to full load, the core flux will always remain constant.

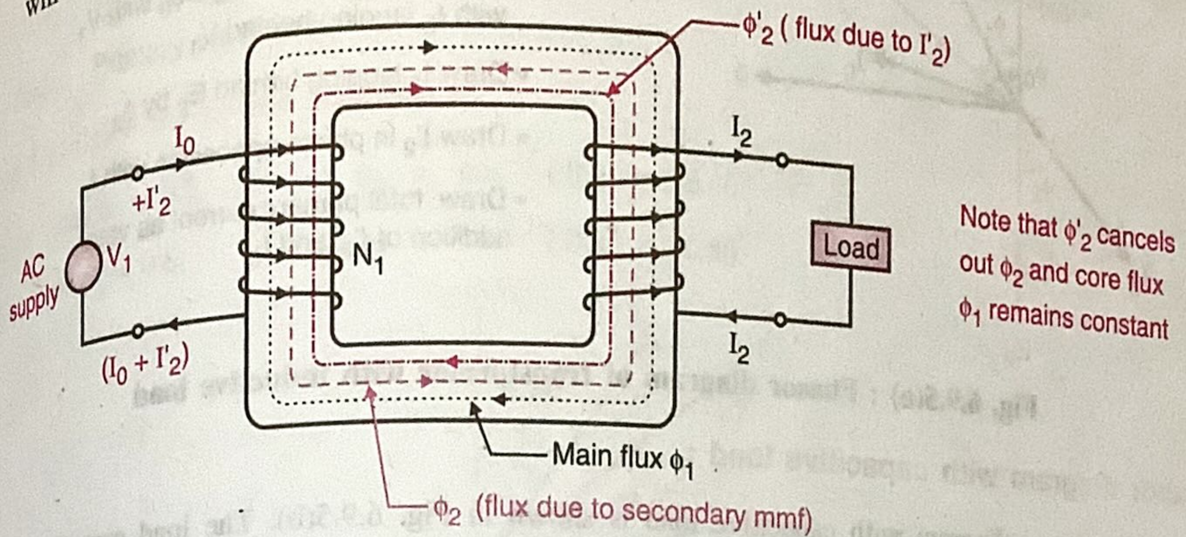


Fig. 6.9.4 : Effect of load component I_2'

6.9.3 Phasor Diagram of Transformer on Load :

▶▶▶ [Asked in Exam : Dec. 01, Dec. 03, May 04, May 05, Dec. 05 !!!]

- The flux ϕ_2' balances out the flux ϕ_2. That means the ampere turns corresponding to ϕ_2' will be equal to the ampere turns corresponding to ϕ_2.

$$\therefore \underbrace{N_1 I'_2}_{\text{Ampere turns corresponding to } \phi_2'} = \underbrace{N_2 I_2}_{\text{Ampere turns corresponding to } \phi_2}$$

$$\therefore \quad I_2' = \frac{N_2}{N_1} I_2 = K\, I_2 \qquad \text{...(6.9.9)}$$

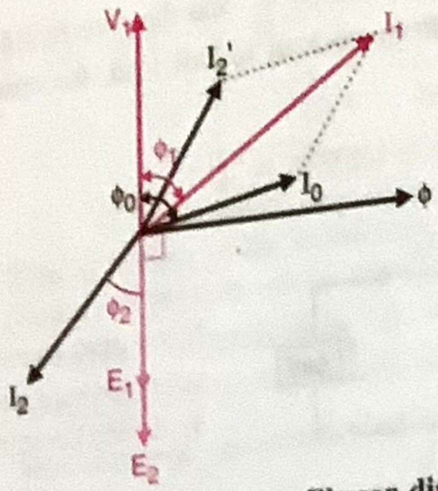
- This current (I_2') flows in addition with I_0 through the primary winding as shown in Fig. 6.9.4.

Total primary current :

- Thus total primary current is given by vector addition of no load current I_0 and the load component I_2'.

$$\overline{I_1} = \overline{I_0} + \overline{I_2'} \qquad \text{...(6.9.10)}$$

- The phasor diagram of a transformer with inductive load is shown in Fig. 6.9.5(a) along with the necessary explanation.

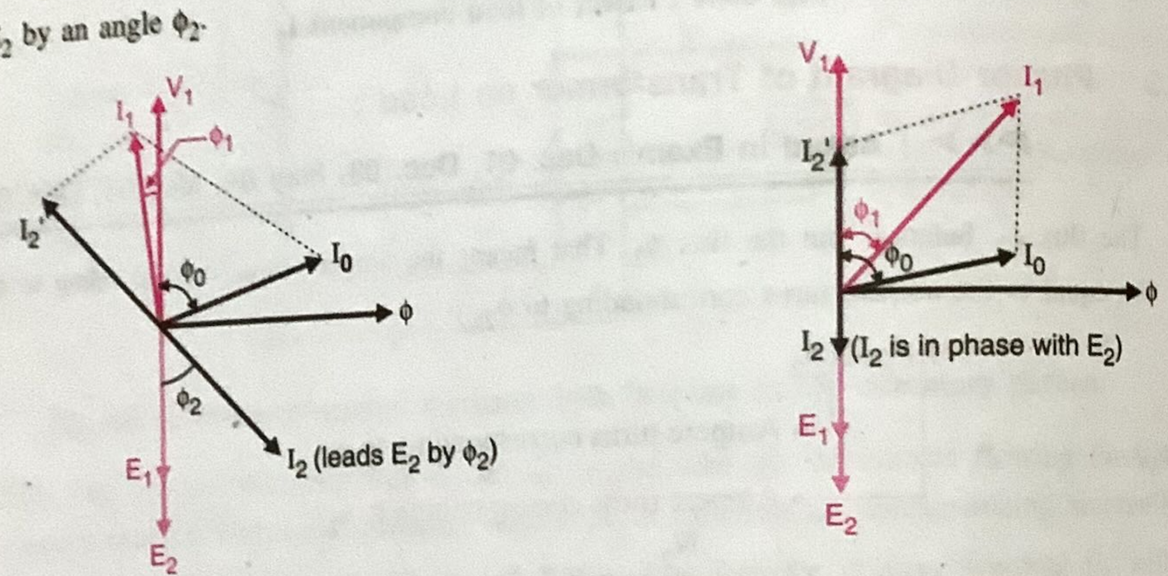


- Draw ϕ as reference phasor.
- Draw V_1, E_1, E_2 as before.
- Draw I_0 making an angle ϕ_0 with V_1 with I_0 lagging behind V_1
- Draw I_2 lagging behind E_2 by ϕ_2
- Draw I'_2 in phase opposition with I_2
- Draw total primary current as vetor addition of I'_2 and I_0

Fig. 6.9.5(a) : Phasor diagram of transformer with inductive load

Phasor diagram with capacitive load :

- The phasor diagram with capacitive load is shown in Fig. 6.9.5(b). The load current I_2 leads E_2 by an angle ϕ_2.



(b) Capacitive load **(c) Resistive load**

Fig. 6.9.5 : Phasor diagram of the transformer on load

- The other steps to be followed to draw the phasor diagram are exactly same as those followed for the inductive load.

Phasor diagram with resistive load :

- The phasor diagram with a resistive load is shown in Fig. 6.9.5(c). The load current I_2 is in phasor with E_2.
- The other steps to be followed to draw the phasor diagram are exactly same as those followed for the inductive load.

Ex. 6.9.1 : A 400 V/100 V transformer takes a no load current of 5 A at 0.2 lagging p.f. Secondary winding supplies a load of 100 A at a p.f. of 0.8 lagging. Find the primary input current.

Soln. :

Given : $\frac{E_1}{E_2} = \frac{400\text{ V}}{100\text{ V}}$, $I_0 = 5$ A, $\cos \phi_0 = 0.2$ lagging, $I_2 = 100$ A, $\cos \phi_2 = 0.8$ lagging.

To find : Primary current I_1

Step 1 : Draw the phasor diagram :

$$\text{Given : } \cos \phi_0 = 0.2, \quad \therefore \quad \phi_0 = \cos^{-1}(0.2) = 78.463^\circ$$

$$\text{and } \cos \phi_2 = 0.8, \quad \therefore \quad \phi_2 = \cos^{-1}(0.8) = 36.86^\circ$$

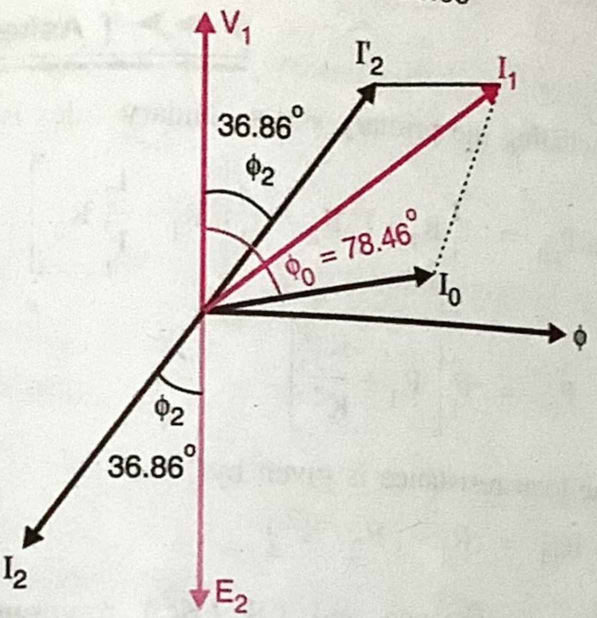


Fig. P. 6.9.1 : Phasor diagram

The phasor diagram is as shown in Fig. P. 6.9.1.

Step 2 : To find I_1 :

$$\frac{E_2}{E_1} = n \qquad n = \frac{100}{400}$$

$$n = 0.25 \qquad I_2' = n I_2$$

$$\therefore I_2' = 0.25 \times 100 \qquad \therefore I_2' = 25 \text{ A}$$

By Parallelogram Law,

$$I_1^2 = I_2'^2 + I_0^2 + 2 \times I_2' \times I_0 \times \cos(\phi_0 - \phi_2)$$

$$I_1^2 = (25)^2 + (5)^2 + [(2 \times 25 \times 5) \times \cos(78.46 - 38.86)]$$

$$I_1^2 = 625 + 25 + 250 \times \cos(41.59) = 836.98$$

$$\therefore I_1 = \mathbf{28.93\ A}$$

...Ans.

6.10 Equivalent Resistance :

- In order to make the calculations easy, we can transfer the resistance from primary side to secondary side or vice versa.
- Let R_1 and R_2 be the winding resistances of the primary and the secondary windings of a transformer, and let K be the transformation ratio.

$$K = \frac{V_2}{V_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2}$$

6.10.1 Resistance Transferred to Primary Side :

➤➤➤ [Asked in Exam : Dec. 05 !!!]

- The total copper loss including the primary and secondary sides is given by,

$$\text{Total copper loss } P_{cu} = I_1^2 R_1 + I_2^2 R_2 = I_1^2 \left[R_1 + \frac{I_2^2}{I_1^2} R_2 \right]$$

- But $\left(\frac{I_2}{I_1}\right)^2 = \frac{1}{K^2}$ $\quad \therefore \quad P_{cu} = I_1^2 \left[R_1 + \frac{R_2}{K^2} \right]$ ...(6.10.1)

- In Equation (6.10.1), the total resistance is given by ,

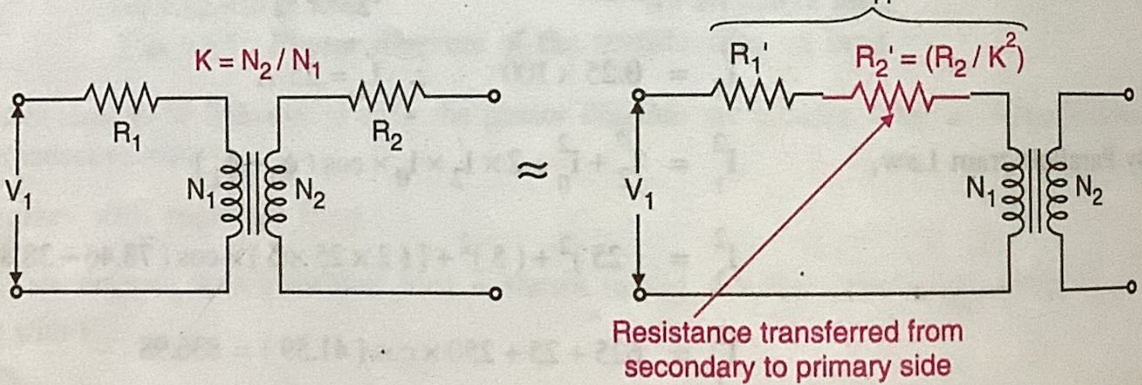
$$R_{1T} = R_1 + (R_2 / K^2)$$

Where R_1 is the primary resistance and (R_2 / K^2) represents the secondary resistance transferred to the primary side. Let us denote it by R_2'.

$\therefore$ Secondary resistance transferred to primary side is given by,

$$R_2' = \frac{R_2}{K^2} \qquad \text{...(6.10.2)}$$

This is shown in Fig. 6.10.1.



(a) Transformer with individual primary and secondary resistances (b) Transfer of resistance from secondary to primary

Fig. 6.10.1