SOME PRACTICE PROBLEMS

1. If $u = x^2 + y^2 + z^2$, where, $x = e^t, y = e^t \sin t, z = e^t \cos t$ prove that $\frac{du}{dt} = 4e^{2t}$.
2. If $z = \sin^{-1}(x - y)$, $x = 3t$, $y = 4t^3$, prove that $\frac{dz}{dt} = \frac{3}{\sqrt{1-t^2}}$.
3. If $z = \tan^{-1}\left(\frac{x}{y}\right)$, $x = 2t$, $y = 1 - t^2$, prove that $\frac{dz}{dt} = \frac{2}{1+t^2}$.
4. If $u = f\,[e^{y-z}, e^{z-x}, e^{x-y}]$, then show that $u_x + u_y + u_z = 0$.
5. If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$.
6. If $u = f\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$, show that $x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} + z^2\frac{\partial u}{\partial z} = 0$.
7. If $u = f(x^n - y^n, y^n - z^n, z^n - x^n)$,

   prove that $\frac{1}{x^{n-1}}\frac{\partial u}{\partial x} + \frac{1}{y^{n-1}}\frac{\partial u}{\partial y} + \frac{1}{z^{n-1}}\frac{\partial u}{\partial z} = 0$.
8. If $u = f\,(x - y, y - z, z - x)$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.
9. If $u = f\,(2x - 3y, 3y - 4z, 4z - 2x)$, prove that $6\frac{\partial u}{\partial x} + 4\frac{\partial u}{\partial y} + 3\frac{\partial u}{\partial z} = 0$
10. If $x = u + v + w, y = uv + vw + wu, z = uvw$, and ϕ is a function of x, y & z,

    then prove that $x\frac{\partial \phi}{\partial x} + 2y\frac{\partial \phi}{\partial y} + 3z\frac{\partial \phi}{\partial z} = u\frac{\partial \phi}{\partial u} + v\frac{\partial \phi}{\partial v} + w\frac{\partial \phi}{\partial w}$.
11. If $x = \sqrt{vw}$, $y = \sqrt{wu}$, $z = \sqrt{uv}$ and ϕ is a function of x, y & z then prove that,

    $x\frac{\partial \phi}{\partial x} + y\frac{\partial \phi}{\partial y} + z\frac{\partial \phi}{\partial z} = u\frac{\partial \phi}{\partial u} + v\frac{\partial \phi}{\partial v} + w\frac{\partial \phi}{\partial w}$.
12. If $z = f\,(x, y)$, $x = r\cos\theta, y = r\sin\theta$, prove that $\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$.
13. If $z = f\,(x, y)$, $x = e^u + e^{-v}$, $y = e^{-u} - e^v$, then show that

    $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$.
14. If $w = \phi\,(u, v)$, $u = x^2 - y^2 - 2xy$, $v = y$, prove that $\frac{\partial w}{\partial v} = 0$ is equivalent

    to $(x + y)\frac{\partial w}{\partial x} + (x - y)\frac{\partial w}{\partial y} = 0$.
15. If $z = f\,(x, y), x = u\cosh v, y = u\sinh v$, prove that, $\left(\frac{\partial z}{\partial x}\right)^2 - \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial u}\right)^2 - \frac{1}{u^2}\left(\frac{\partial z}{\partial v}\right)^2$.

16. If $z = f(x, y)$, $x = e^u \cos v$, $y = e^u \sin v$, prove that

$$(i)\ x\frac{\partial z}{\partial v} + y\frac{\partial z}{\partial u} = e^{2u}\frac{\partial z}{\partial y} \qquad (ii)\ \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = e^{-2u}\left[\left(\frac{\partial z}{\partial u}\right)^2 + \left(\frac{\partial z}{\partial v}\right)^2\right]$$

17. If $z = f(x, y)$, $x = e^u \sec v$, $y = e^u \tan v$,

prove that $\left(\frac{\partial z}{\partial x}\right)^2 - \left(\frac{\partial z}{\partial y}\right)^2 = e^{-2u}\left[\left(\frac{\partial z}{\partial u}\right)^2 - cos^2 v\left(\frac{\partial z}{\partial v}\right)^2\right]$

18. If $z = f(u, v)$, $u = e^x, v = e^y$, prove that $\frac{\partial^2 z}{\partial x \partial y} = uv\frac{\partial^2 z}{\partial u \partial v}$.

19. If $z = f(u, v)$, $u = lx + my, v = ly - mx$,

prove that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = (l^2 + m^2)\left(\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2}\right)$.

20. If $z = f(u, v)$, $u = x^2 - y^2 - 2xy$, $v = y$,

prove that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 4\sqrt{u^2 + v^2}\left(\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2}\right)$