Batch: C7-3 Roll No.: 57
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Course: APPLIED MATHEMATICS-I
~~Experiment / assignment~~ / tutorial No. 1
Grade: Signature of the Faculty with date

SOMAIYA VIDYAVIHAR UNIVERSITY

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$\frac{28}{25}$

Q 1. Given: $A = \begin{bmatrix} 2i & 2+i & 1-i \\ -2+i & -i & 3i \\ -1-i & 3i & 0 \end{bmatrix}$

So, $\bar{A} = \begin{bmatrix} -2i & 2-i & 1+i \\ -2-i & i & -3i \\ -1+i & -3i & 0 \end{bmatrix}$

We are required to express the given matrix as—

$$A = \frac{1}{2}(A+\bar{A}) + i\left(\frac{1}{2i}(A-\bar{A})\right)$$

Where; $P = \frac{1}{2}(A+\bar{A})$ is real-skew symmetric matrix.

and $Q = \frac{1}{2i}(A-\bar{A})$ is real symmetric matrix.

Thus,

$$P = \frac{1}{2}\left(\begin{bmatrix} 2i & 2+i & 1-i \\ -2+i & -i & 3i \\ -1-i & 3i & 0 \end{bmatrix} + \begin{bmatrix} -2i & 2-i & 1+i \\ -2-i & i & -3i \\ -1+i & -3i & 0 \end{bmatrix}\right)$$

$\therefore P = \begin{bmatrix} 0 & 2 & 1 \\ -2 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}$ $\because P' = \begin{bmatrix} 0 & -2 & 1 \\ 2 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} = -P$

$\therefore$ P is a real skew-symmetric matrix

Similarily,

$$Q = \frac{1}{2i}\left(\begin{bmatrix} 2i & 2+i & 1-i \\ -2+i & -i & 3i \\ -1-i & 3i & 0 \end{bmatrix} - \begin{bmatrix} -2i & 2-i & 1+i \\ -2-i & i & -3i \\ -1+i & -3i & 0 \end{bmatrix}\right)$$

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$Q = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & 3 \\ -1 & 3 & 0 \end{bmatrix}$ $\because Q' = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & 3 \\ -1 & 3 & 0 \end{bmatrix} = Q$

$\therefore$ Q is real-symmetric matrix

$\therefore$ [A = P + iQ ⇒ where, P is real-skew symmetric matrix
Q is real-symmetric matrix.]

Thus expressed.

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Q2. Given: $A = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\therefore A^T = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$

To verify if A is an orthogonal matrix –
It should satisfy the following equation/condition:

$$\Rightarrow \underline{AA^T = A^TA = I}$$

Now,

$$\Rightarrow AA^T = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\phi + \sin^2\phi & \sin\phi\cos\phi - \sin\phi\cos\phi & 0 \\ \sin\phi\cos\phi - \sin\phi\cos\phi & \sin^2\phi + \cos^2\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3$$

Similarly,

$$\Rightarrow A^TA = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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$$= \begin{bmatrix} \cos^2\phi + \sin^2\phi & -\sin\phi\cos\phi + \sin\phi\cos\phi & 0 \\ -\sin\phi\cos\phi + \sin\phi\cos\phi & \sin^2\phi + \cos^2\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3$$

Therefore, $AA^T = A^TA = I$ //

Thus, $A^T = A^{-1} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$

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Q 3 Given:

$$A = \begin{bmatrix} -1 & 2 & 3 & -2 \\ 2 & -5 & 1 & 2 \\ 3 & -8 & 5 & 2 \\ -5 & -12 & -1 & 6 \end{bmatrix}$$

$$R_1 \to -R_1 : \begin{bmatrix} 1 & -2 & -3 & 2 \\ 2 & -5 & 1 & 2 \\ 3 & -8 & 5 & 2 \\ -5 & -12 & -1 & 6 \end{bmatrix}$$

$$R_2 \to R_2 - 2R_1 : \begin{bmatrix} 1 & -2 & -3 & 2 \\ 0 & -1 & +7 & -2 \\ 0 & -2 & 14 & -4 \\ 0 & -2 & -16 & -4 \end{bmatrix}$$

$$R_2 \to -R_2 : \begin{bmatrix} 1 & -2 & -3 & 2 \\ 0 & 1 & -7 & 2 \\ 0 & -2 & 14 & -4 \\ 0 & -2 & 16 & -4 \end{bmatrix}$$

7 $R_3, R_4 \to R_3, R_4 + 2R_2$:

i.e., $\begin{cases} R_3 \to R_3 + 2R_2 \\ R_4 \to R_4 + 2R_2 \end{cases}$

$$\begin{bmatrix} 1 & -2 & -3 & 2 \\ 0 & 1 & -7 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$R_1 \to R_1 + 3R_4$,
$R_2 \to R_2 + 7R_4$:

$$\begin{bmatrix} 1 & -2 & 0 & 2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -2 & 0 & 2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ Reduced to Row-Echelon form!