SOME PRACTICE PROBLEMS

1. If $x = r\cos\theta, y = r\sin\theta$, find $\frac{\partial(x,y)}{\partial(r,\theta)}$.

2. If $x = uv$, $y = \frac{u+v}{u-v}$, find $\frac{\partial(u,v)}{\partial(x,y)}$.

3. If $u = \frac{x+y}{1-xy}, v = \tan^{-1}x + \tan^{-1}y$, find $\frac{\partial(u,v)}{\partial(x,y)}$.

4. If $x = \frac{u^2-v^2}{2}, y = uv, z = w$, find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.

5. If $u = 1 - x, v = x(1 - y), w = xy(1 - z)$, show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = -x^2y$.

6. If $u = x + y + z, v = x^2 + y^2 + z^2, w = xy + yz + zx$, show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = 0$.

7. If $u_1 = \frac{x_2 x_3}{x_1}, u_2 = \frac{x_3 x_1}{x_2}, u_3 = \frac{x_1 x_2}{x_3}$, find the value of $\frac{\partial(u_1,u_2,u_3)}{\partial(x_1,x_2,x_3)}$.

8. If $x = e^v \sec u, y = e^v \tan u$, find $\frac{\partial(u,v)}{\partial(x,y)}$

9. If $x = r^2 \cos 2\theta, y = r^2 \sin 2\theta$, find $\frac{\partial(x,y)}{\partial(r,\theta)}$.

10. If $x = a\cosh u\cos v, y = a\sinh u\sin v$, show that $\frac{\partial(x,y)}{\partial(u,v)} = \frac{a^2(\cosh 2u - \cos 2v)}{2}$.

11. If $ux = yz, vy = zx, wz = xy$, find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.

12. Show that $JJ' = 1$ where $x = e^v \sec u, y = e^v \tan u$.

13. Show that $JJ' = 1$ where $x = uv, y = \frac{u}{v}$.

14. If $x = v^2 + w^2, y = w^2 + u^2, z = u^2 + v^2$, prove that $JJ' = 1$.

15. If $x = u\cos v, y = u\sin v$, show that $\frac{\partial(x,y)}{\partial(u,v)} \cdot \frac{\partial(u,v)}{\partial(x,y)} = 1$.

16. $u = f(x), v = f(x,y), w = f(x,y,z)$, prove that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = \frac{\partial u}{\partial x}\frac{\partial v}{\partial y}\frac{\partial w}{\partial z}$

17. Hence find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ if $u = e^x, v = e^{x+y}, w = e^{x+y+z}$.