Ex: Solve $\dfrac{dx}{y+z} = \dfrac{dy}{z+x} = \dfrac{dz}{x+y}$ ——— (1).

Sol: choosing $1,-1,0$ & $0,1,-1$ as multipliers, each fn

$$= \frac{dx - dy}{(y+z)-(z+x)} = \frac{dy - dz}{(z+x)-(x+y)} \quad \text{——— (2)}$$

So $\dfrac{dx-dy}{-(x-y)} = \dfrac{dy-dz}{-(y-z)}$ or $\dfrac{dx-dy}{x-y} = \dfrac{dy-dz}{y-z}$.

$\Rightarrow \quad \log(x-y) = \log(y-z) + \log c_1$

$\Rightarrow \quad (x-y)/(y-z) = c_1$ ——— (3)

Choosing $1,1,1$ as multipliers, each fraction of (1).

$$= \frac{dx+dy+dz}{2(x+y+z)} \quad \text{——— (4)}$$

Combining the first fraction in (2) with fracti (4),

$$\frac{dx-dy}{-(x-y)} = \frac{dx+dy+dz}{2(x+y+z)}$$

$\Rightarrow \quad (x-y)(x+y+z)^{1/2} = c_2$ ——— (5)

The reqd. soln is given by (3) & (5)

Ex: Solve $dx/y = dy/x = dz/z$ ——— (1)

Sol: From first two fractions, we obtain $x^2 - y^2 = c_1$ —(2)

Choosing $1,1,0$ as multipliers, each fraction of (1)

$$= \frac{dx+dy}{y+x}$$

Combining this fraction with last fnd of (1), we get

$\Rightarrow \quad \dfrac{dx+dy}{x+y} = \dfrac{dz}{z} \Rightarrow \log(x+y) - \log z = \log c_2$

$\Rightarrow \quad (x+y)/z = c_2$ ——— (3)

The complete soln is given by (2) & (3)