Cauchy's Homogeneous Linear Equation:

An equation of the form

$$a_0 x^n \frac{d^n y}{dx^n} + a_1 x^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + a_2 x^{n-2} \frac{d^{n-2} y}{dx^{n-2}} + \cdots + a_{n-1} x \frac{dy}{dx} + a_n y = X \quad \text{--(i)}$$

Where a_i's are constants and X is a function of x, is called Cauchy's homogeneous linear equation.

Such equations can be reduced to linear differential equations with constant coefficients by the substitution

$$x = e^z \quad \text{or} \quad z = \log x, \quad \text{so that}$$

$$\frac{dy}{dx} = \frac{dy}{dz} \cdot \frac{dz}{dx} = \frac{dy}{dz} \cdot \frac{1}{x}$$

$$\text{or } x \frac{dy}{dx} = \frac{dy}{dz} = Dy$$

$$\frac{d^2 y}{dx^2} = \frac{d}{dx}\left(\frac{1}{x}\frac{dy}{dz}\right) = \frac{-1}{x^2}\frac{dy}{dz} + \frac{1}{x} \cdot \frac{d^2 y}{dz^2} \cdot \frac{dz}{dx}$$

$$= -\frac{1}{x^2}\frac{dy}{dz} + \frac{1}{x^2}\frac{d^2 y}{dz^2}$$

$$\text{or } x^2 \frac{d^2 y}{dx^2} = \frac{d^2 y}{dz^2} - \frac{dy}{dz} = D^2 y - Dy = D(D-1)y$$

Similarly, $x^3 \frac{d^3 y}{dx^3} = D(D-1)(D-2)$ & so on.