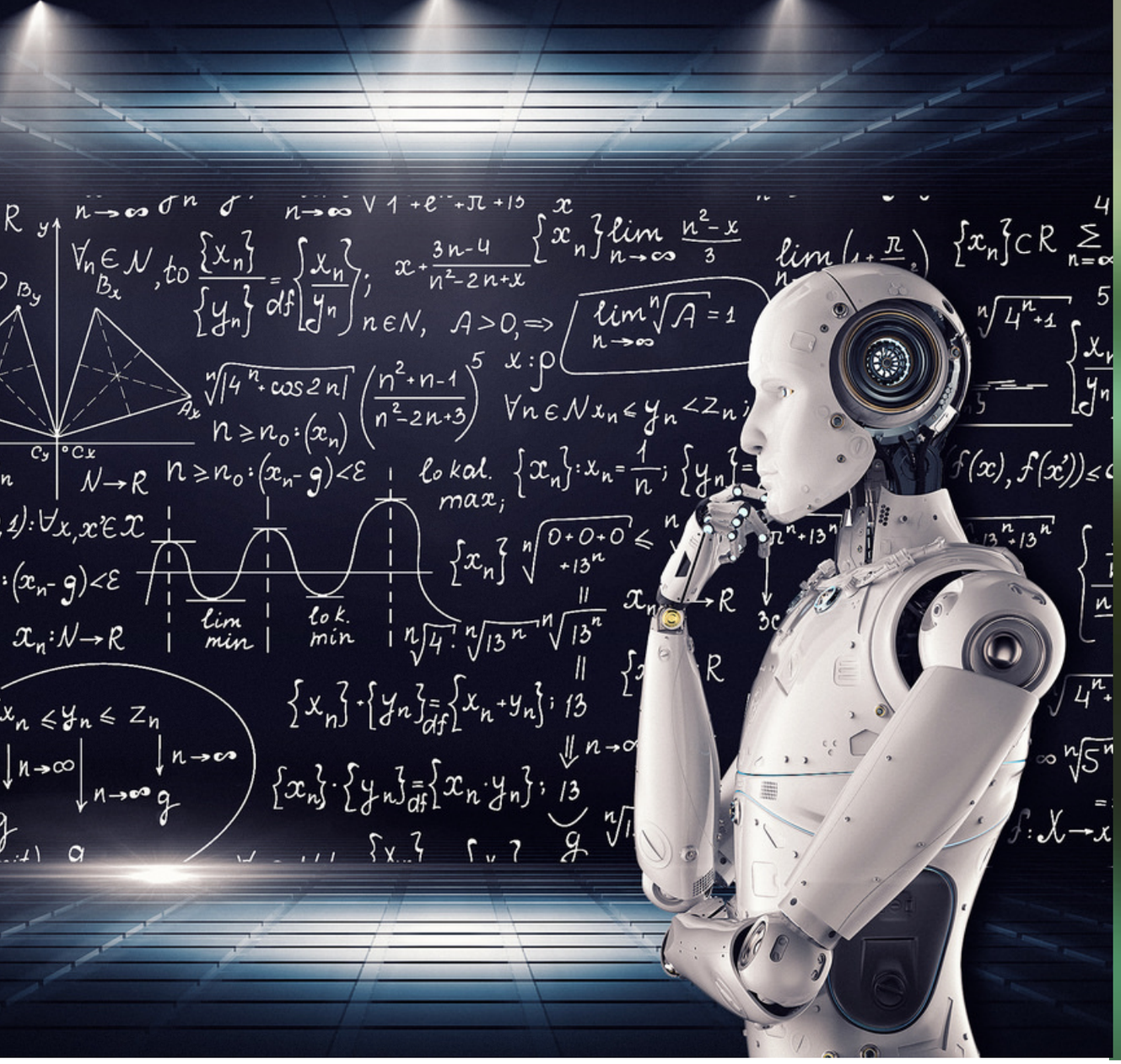
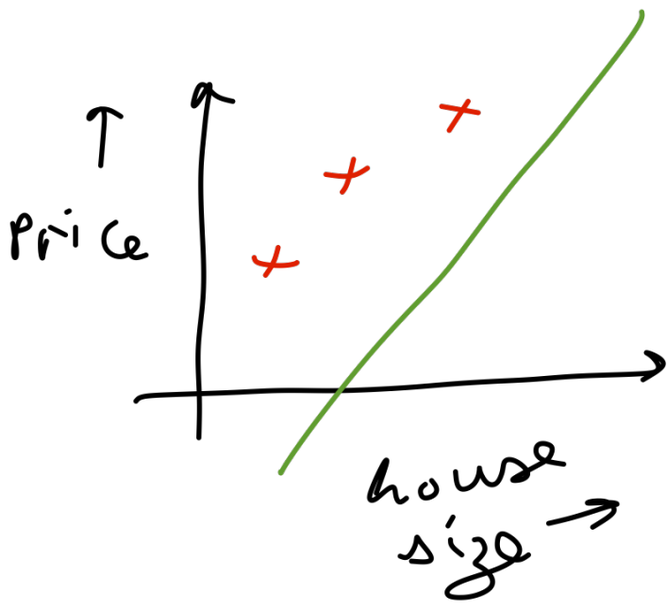


Cost Function-A Graphical Analysis(2-D)

Rajat Modi

RECAP.....



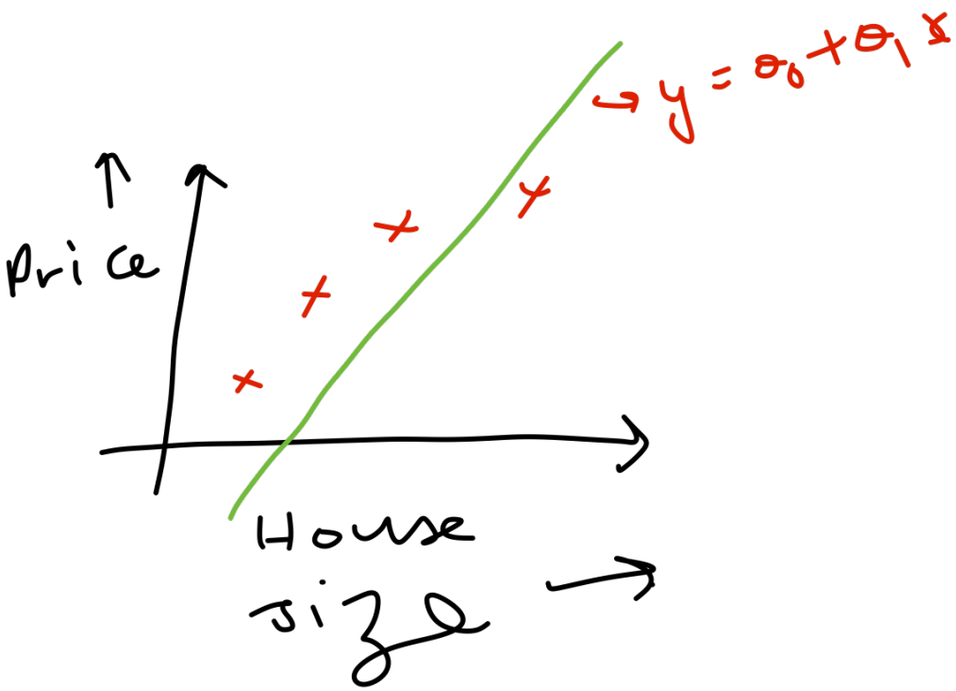
$$h_\theta(x) = \theta_0 + \theta_1 x$$

$$J(\theta_0, \theta_1) = \frac{1}{2M} \sum_{i=1}^{m} \left(h_\theta x^{(i)} - y^{(i)}\right)^2$$

hypothesis function

error function

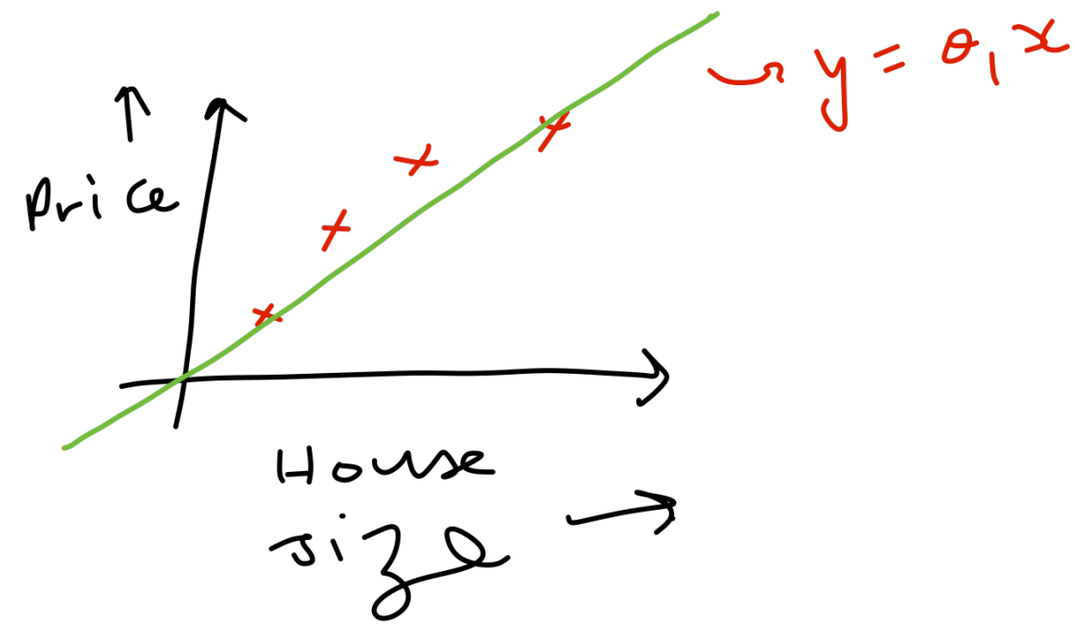
Simplifying for One Dimensional Visualization



$$h_\theta(x) = \theta_0 + \theta_1 x$$

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left[h_\theta(x^{(i)}) - y^{(i)} \right]^2$$

original



$$h_\theta(x) = \cancel{\theta_0}^{\,0} + \theta_1 x$$

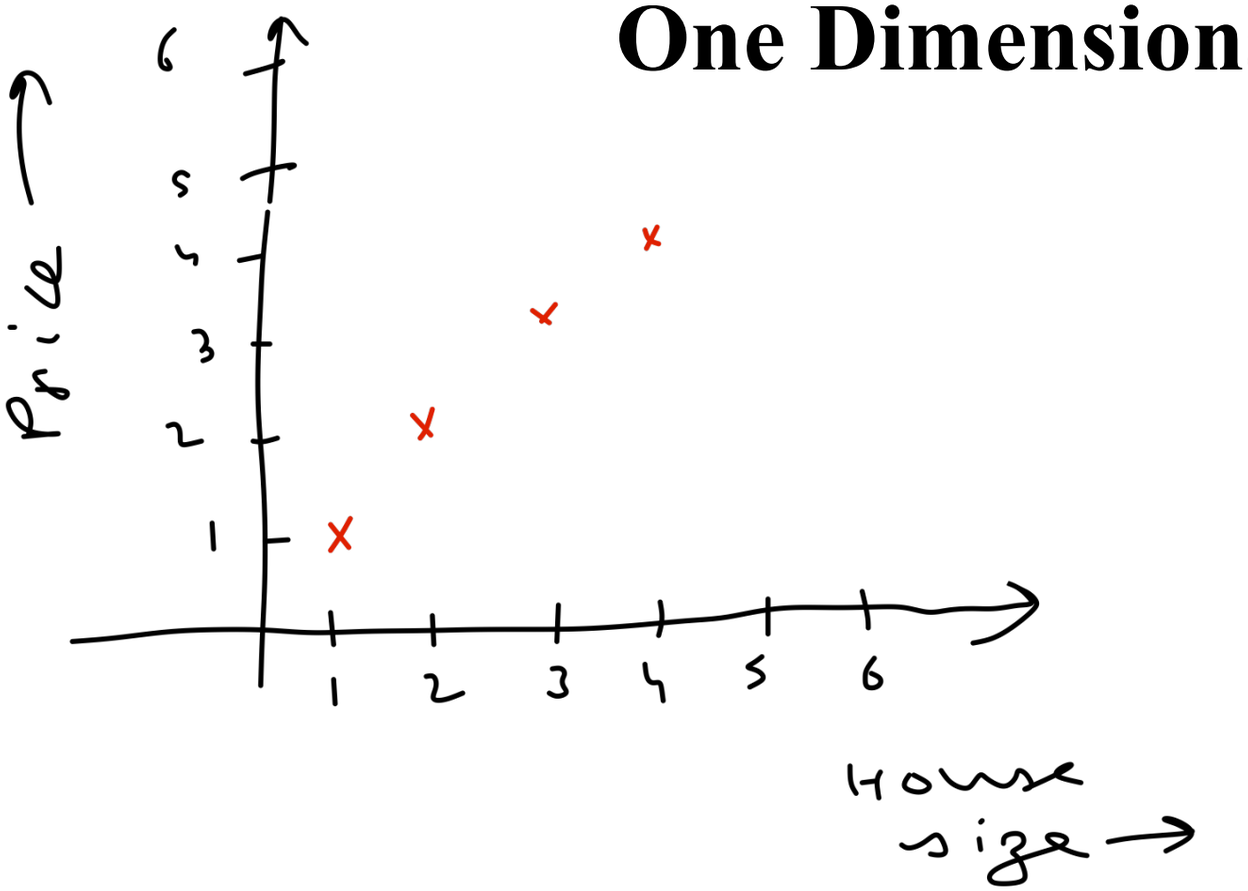
$$h_\theta(x) = \theta_1 x$$

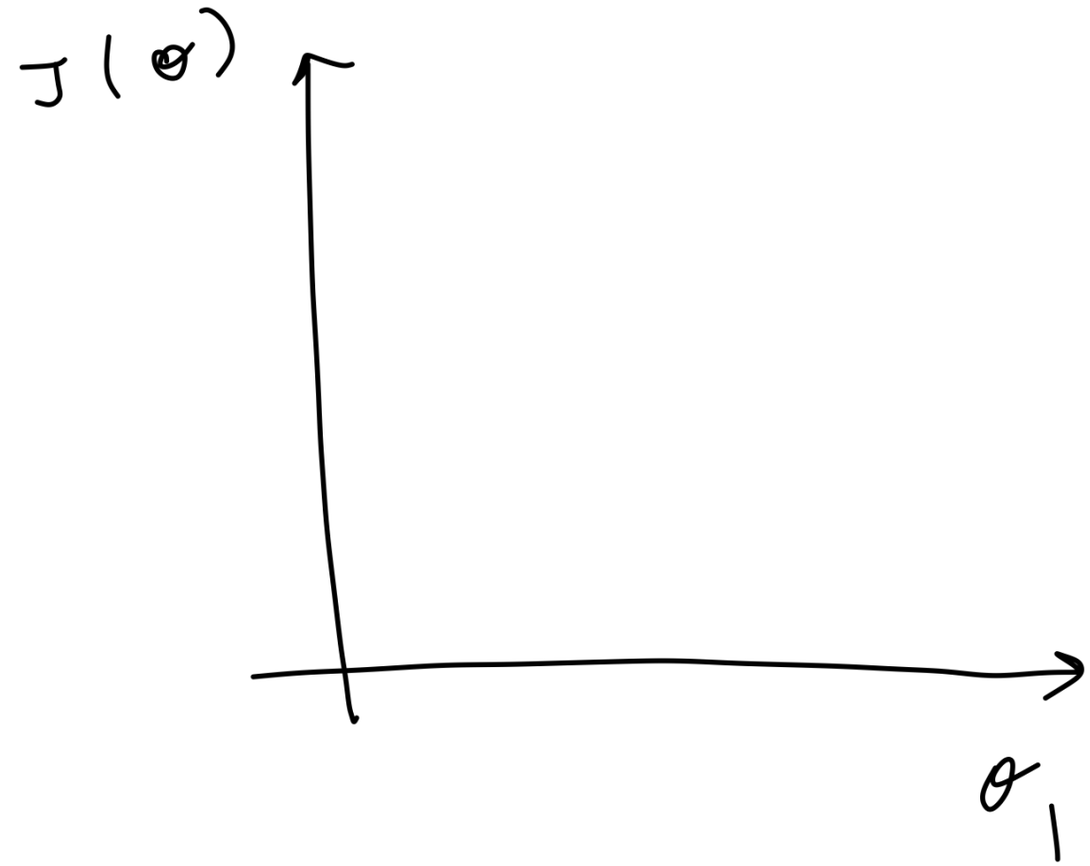
$$J(\theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left[h_\theta(x^{(i)}) - y^{(i)} \right]^2$$

simplified

Rajat Modi

One Dimensional Visualization

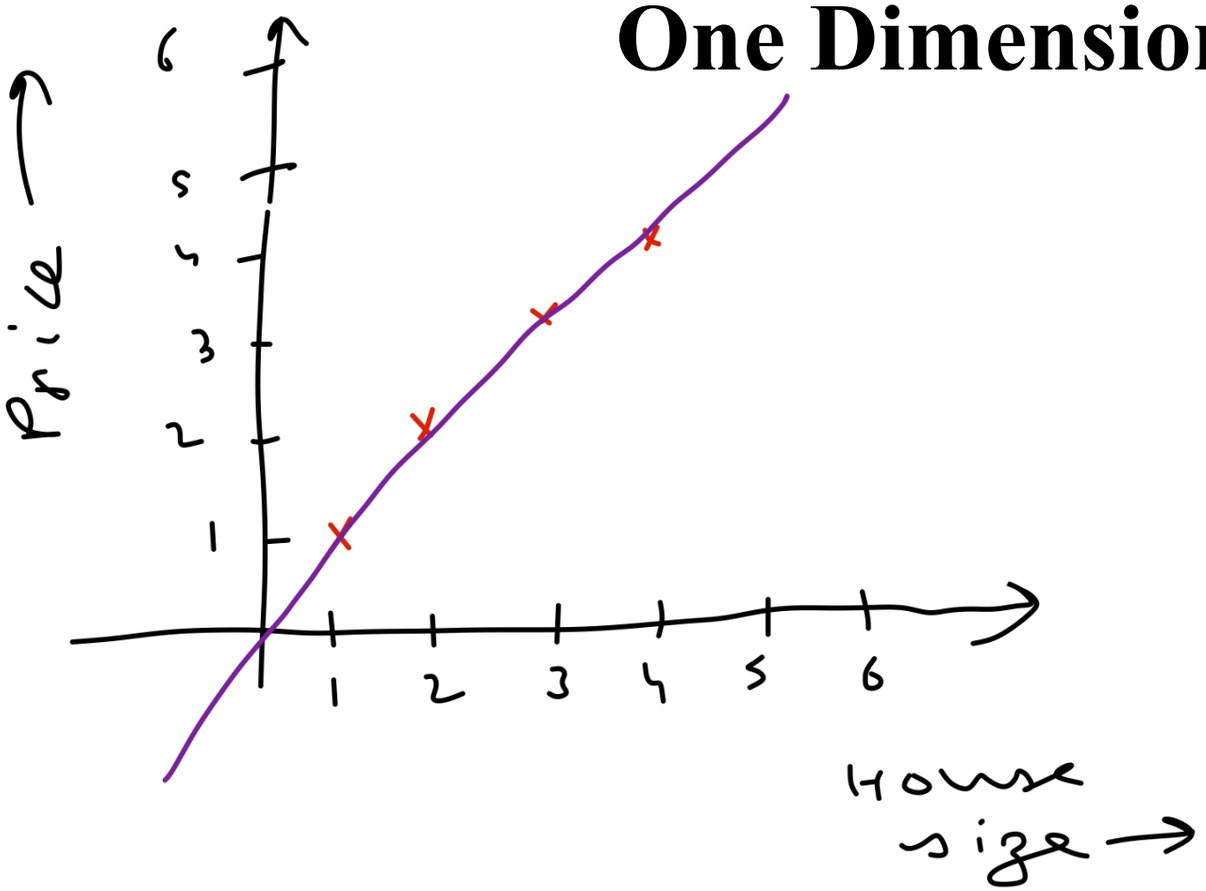


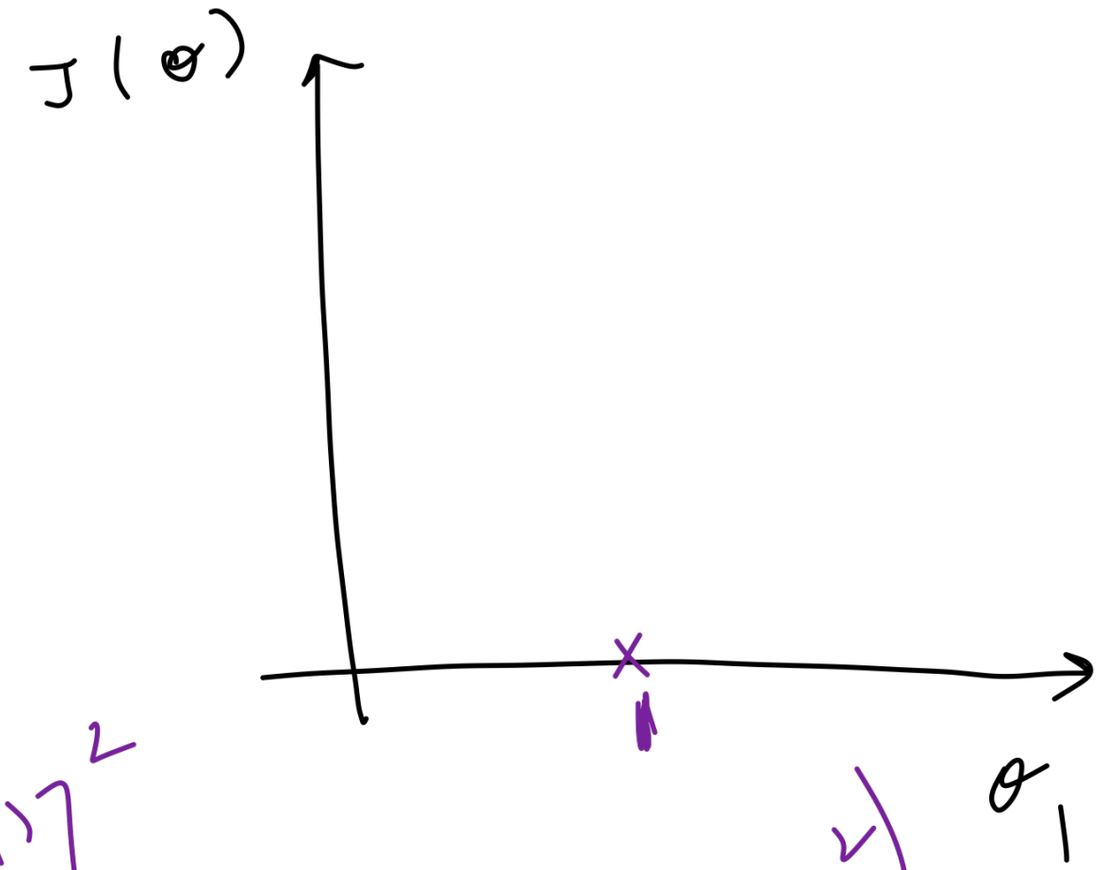


$$h_\theta = \theta_1 x$$

$$J(\theta_1, x) = \frac{1}{2m} \sum_{i=1}^{m} \left(\theta_1 x^{(i)} - y^{(i)}\right)^2$$

One Dimensional Visualization





$\theta_1 = 1$

$h_\theta(x) = x$

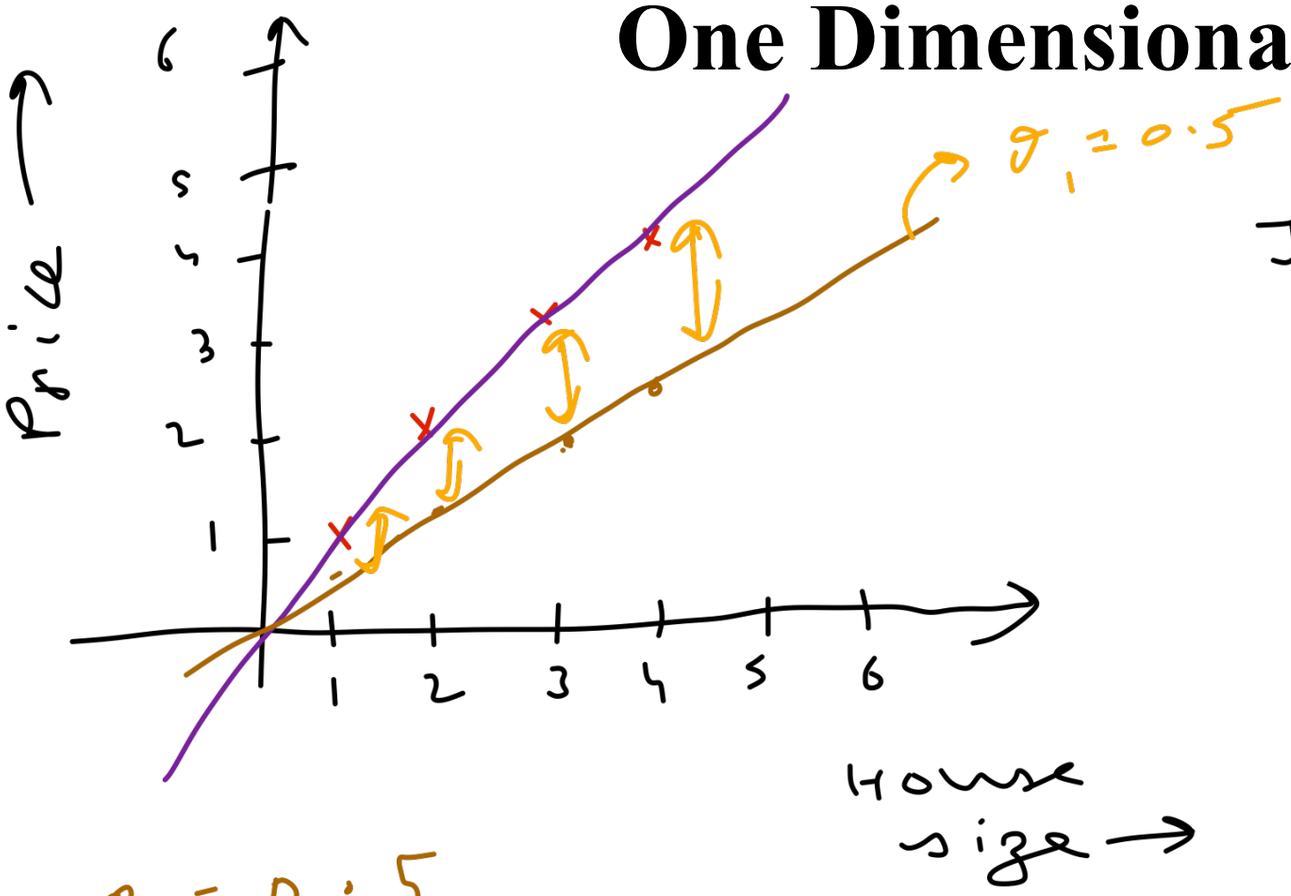
$$J(\theta_1) = \frac{1}{2m}\sum_{i=1}^{m}\left[\theta_1 x^{(i)} - y^{(i)}\right]^2$$

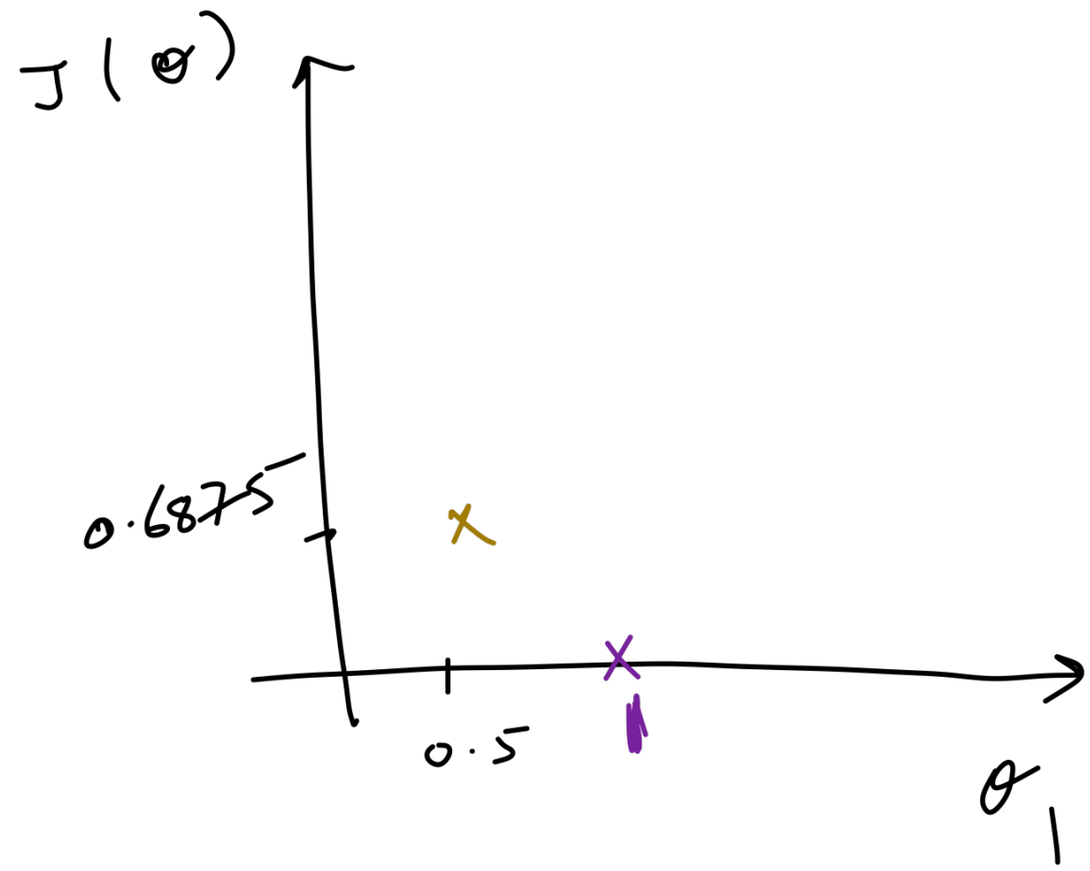
$$J(1) = \frac{1}{2\times 4}\left((1-1)^2 + (2-2)^2 + (3-3)^2 + (4-4)^2\right)$$

$J(1) = 0$

Rajat Modi

One Dimensional Visualization





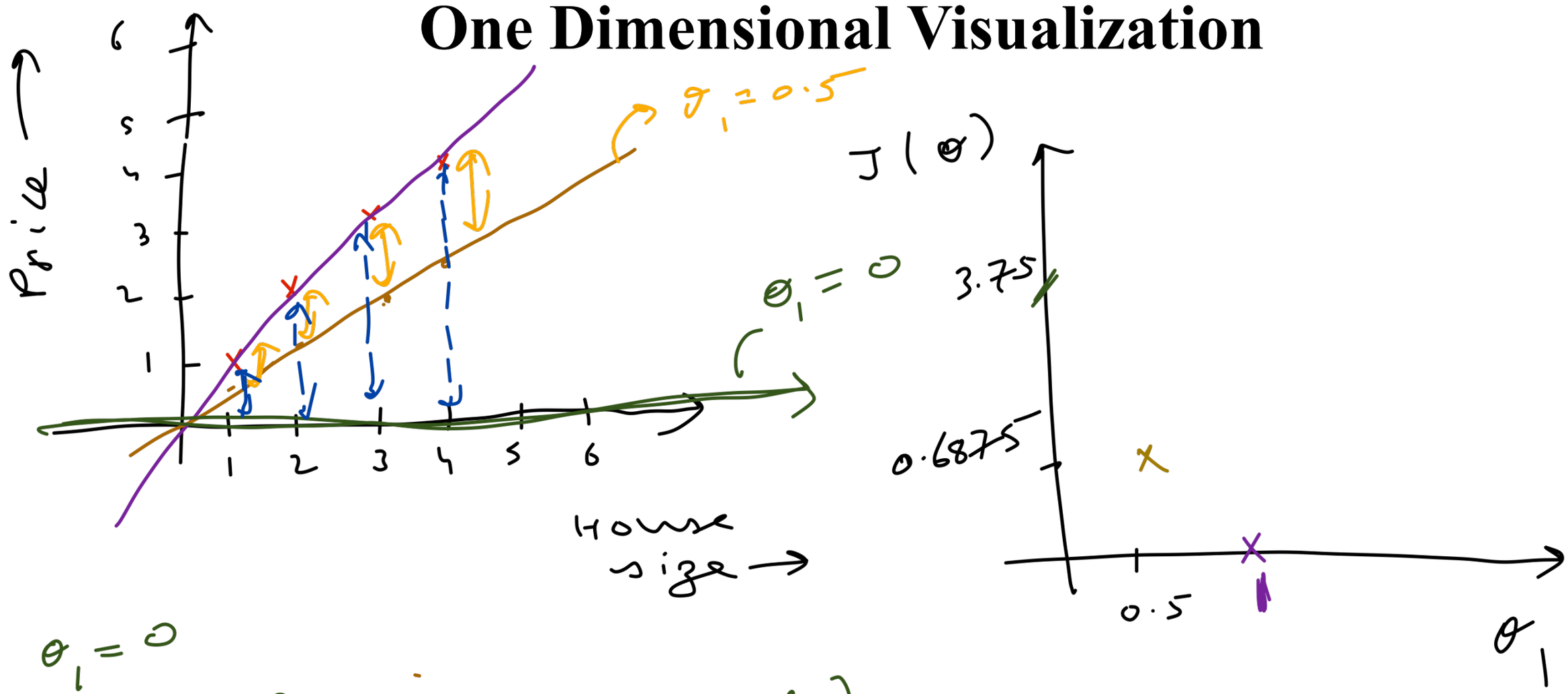
$\theta_1 = 0.5$

$h_\theta(x) = 0.5x$

$$J(0.5) = \frac{1}{2\times 4}\left[\left(1-\frac{1}{2}\right)^2 + (2-1)^2 + \left(3-\frac{3}{2}\right)^2 + (4-2)^2\right]$$

$$= \frac{1}{8}\left[\frac{1}{4} + 1 + \frac{1}{4} + 4\right] = \frac{11}{16} = \underline{0.6875}$$

One Dimensional Visualization



$\theta_1 = 0$

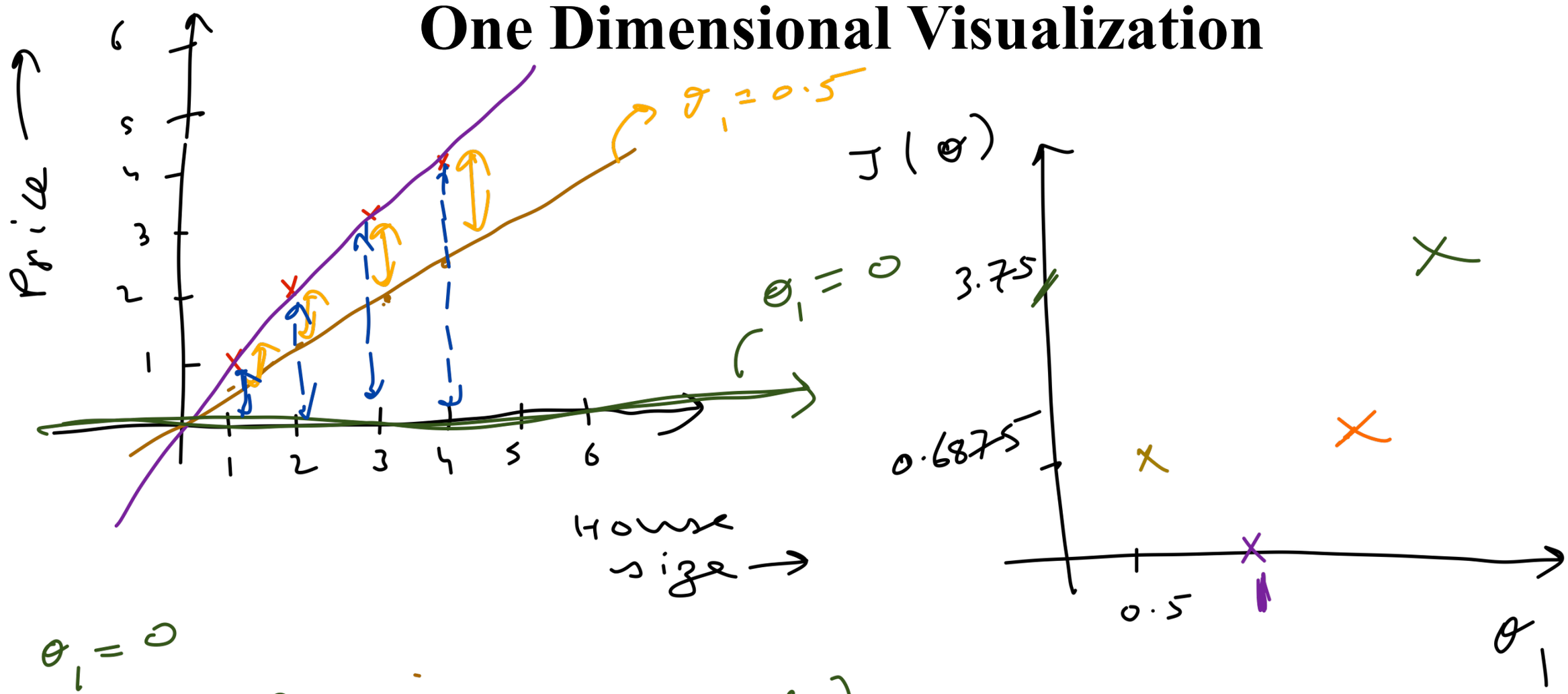
$h_\theta(x) = 0$

$$J(\theta) = \frac{1}{2\times 4}\left(1^2 + 2^2 + 3^2 + 4^2\right)$$

$$= \frac{1}{8}\left(1 + 4 + 9 + 16\right) = \frac{\cancel{30}^{15}}{\cancel{8}_{4}} = 3.75$$

Rajat Modi

One Dimensional Visualization



$\theta_1 = 0$

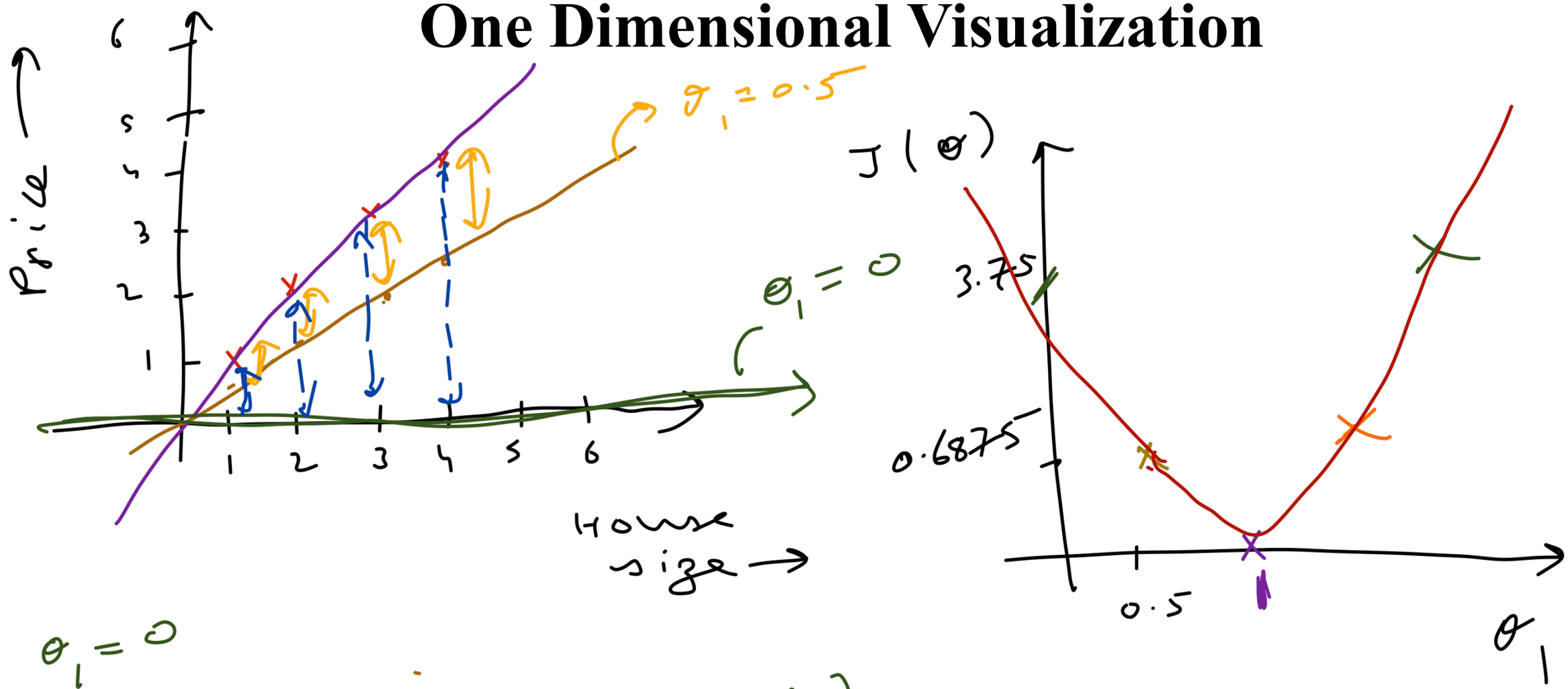
$h_\theta(x) = 0$

$$J(\theta) = \frac{1}{2\times 4}\left(1^2 + 2^2 + 3^2 + 4^2\right)$$

$$= \frac{1}{8}\left(1 + 4 + 9 + 16\right) = \frac{\cancel{30}^{\,15}}{\cancel{8}_{\,4}} = 3.75$$

Rajat Modi

One Dimensional Visualization



$\theta_1 = 0$

$h_\theta(x) = 0$

$$J(0) = \frac{1}{2\times 4}\left(1^2 + 2^2 + 3^2 + 4^2\right)$$

$$= \frac{1}{8}\left(1 + 4 + 9 + 16\right) = \frac{\cancel{30}^{15}}{\cancel{8}_{4}} = 3.75$$

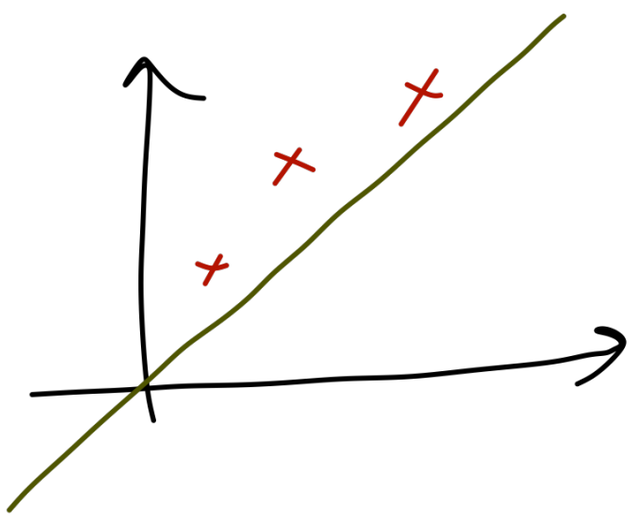
Rajat Modi

CONCLUSION

$h_\theta(x) = \theta_0 + \theta_1 x$

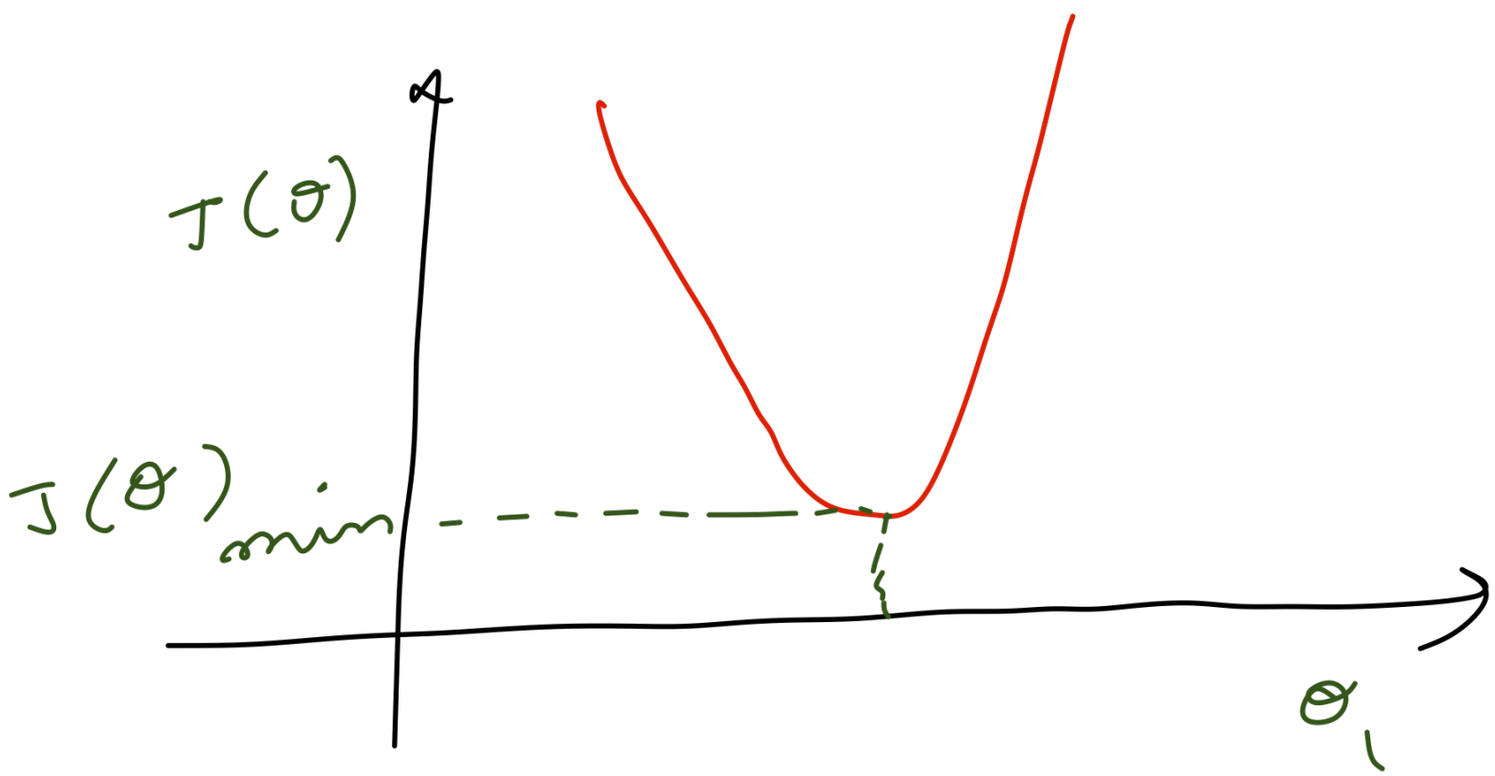
for $\theta_0 = 0$

$h_\theta(x)$ is a line passing through origin



CONCLUSION

The graph of loss looks like



$\theta_0 = 0$.

THANK YOU :-)

next

visualization of $J(\theta_0, \theta_1)$ in 3 dimensions

upvote, subscribe & comment

Rajat Modi