## Generating values from normal distribution

In computer simulations, especially in applications of the Monte-Carlo method, it is often desirable to generate values that are normally distributed. The algorithms listed below all generate the standard normal deviates, since a  $N(\mu, \sigma_2)$  can be generated as  $X = \mu + \sigma Z$ , where Z is standard normal. All these algorithms rely on the availability of a random number generator U capable of producing uniform random variates.

 The Box–Muller method uses two independent random numbers U and V distributed uniformly on (0,1). Then the two random variables X and Y

$$X = \sqrt{-2 \ln U} \cos(2\pi V),$$

$$Y = \sqrt{-2 \ln U} \sin(2\pi V).$$

will both have the standard normal distribution, and will be independent. This formulation arises because for a bivariate normal random vector (X Y) the squared norm  $X^2 + Y^2$  will have the chi-square distribution with two degrees of freedom, which is an easily generated exponential random variable corresponding to the quantity  $-2\ln(U)$  in these equations; and the angle is distributed uniformly around the circle, chosen by the random variable V.

Marsaglia polar method is a modification of the Box–Muller method algorithm, which does not require computation of functions sin() and cos(). In this method U and V are drawn from the uniform (-1,1) distribution, and then  $S = U^2 + V^2$  is computed. If S is greater or equal to one then the method starts over, otherwise two quantities

$$X = U\sqrt{\frac{-2\ln S}{S}}, \qquad Y = V\sqrt{\frac{-2\ln S}{S}}$$

are returned. Again, X and Y will be independent and standard normally distributed.