Communications Lab
Experiment 4
Lab Report

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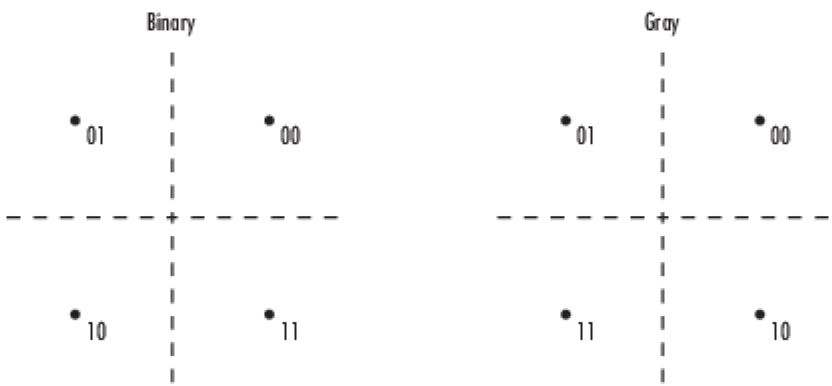
QPSK Modulation & Demodulation:

QPSK stands for Quadrature Phase Shift Keying. QPSK has 4 symbols, one corresponds to 45 phase, second to 135 phase, third to -135 phase and the fourth corresponds to -45 phase.



QPSK simulation:

The simulation contains 8 scripts Main.m, qpskSystem.m, bitRep.m, errorCheck.m, bits2symbol.m, symbol2bits, symbol_detector and repRemove.m

The working of each script is explained in the comments of the code.

An overview of the working process of the simulation**:**

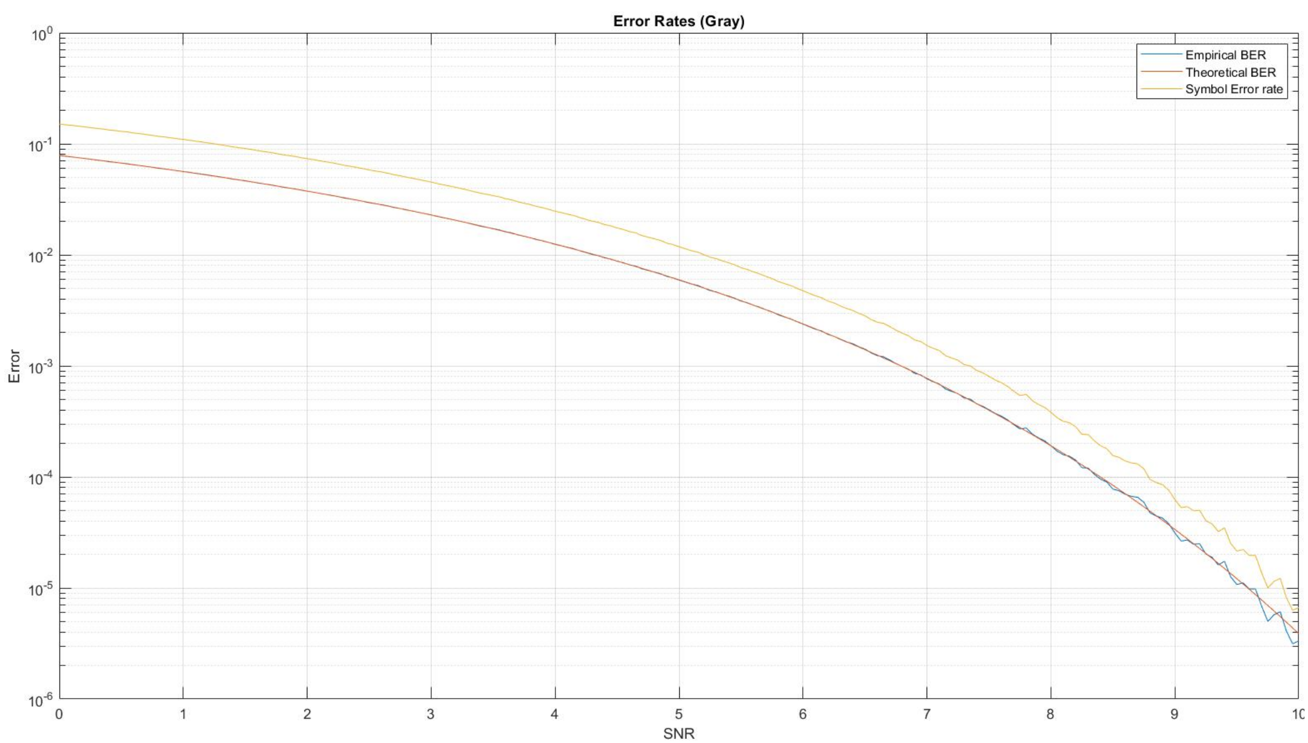
- We take a message of 50 bits.
- This message undergoes preprocessing. First we define if the message has an odd number of bits or even number of bits.
  even:
  We add a set of initialization bits [0 0].
  odd:
  We add a 0 at the end of the sequence to make it even length (this is required because qpsk symbols are formed using 2 bits). After this we add a set of initialization bits [1 1].

  Let's take an example: 1001 → **00**1001
  101 → **11**101**0**

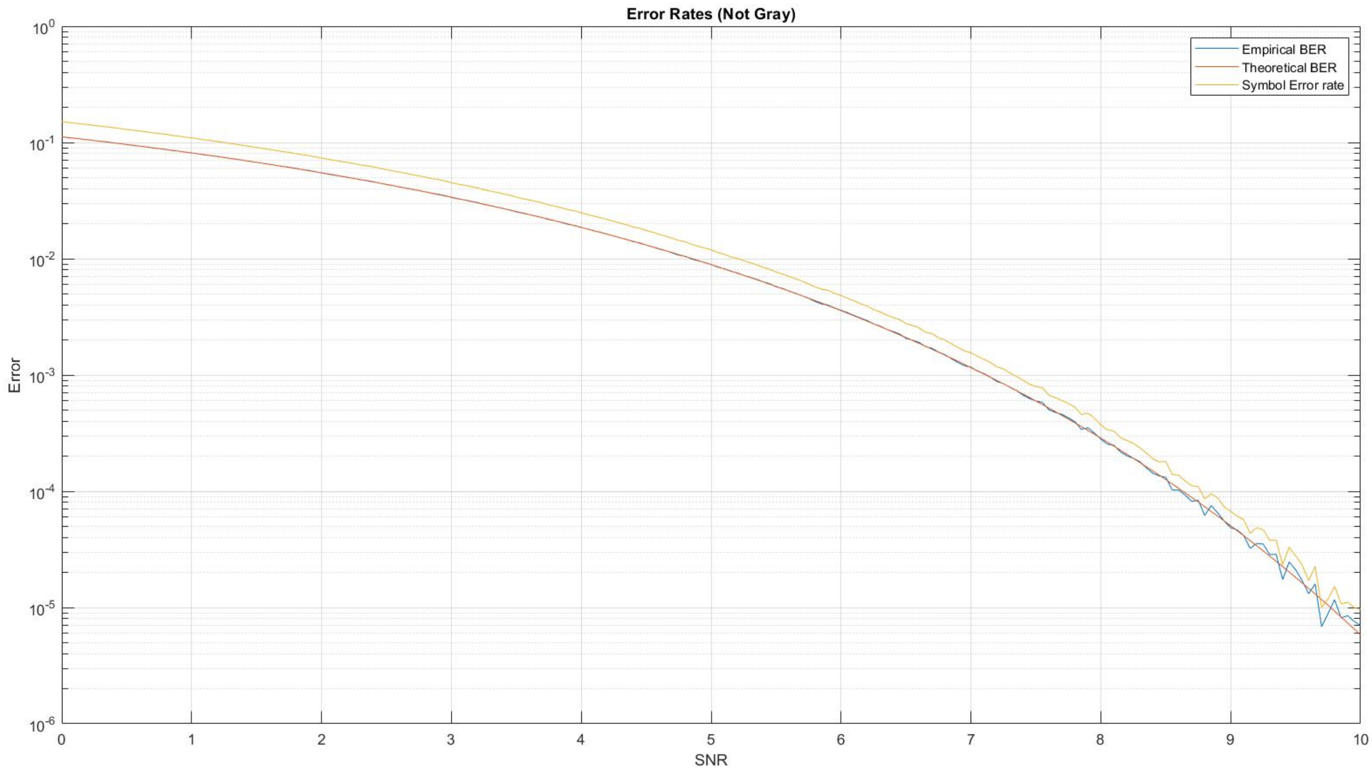
- The set of two bits are repeated ‘reps’ number of times.
  Example if reps = 4 then 001001 will become:
  0 0   0 0   0 0   0 0   1 0   1 0   1 0   1 0   0 1   0 1   0 1   0 1
  Basically we are repeating the qpsk symbols many times.
- Now we convert the set of two bits into QPSK symbols according to the Labelling format mentioned.
  In the above example(for gray labelling):
  1+j   1+j   1+j   1+j   1-j   1-j   1-j   1-j   -1+j   -1+j   -1+j   -1+j
- These symbols are then sent into the channel where noise is added.
- The Noisy symbols are then taken 1 by 1 for detection.
  The detection is done by checking the quadrant in which the symbol lies.
- We take a subset of ‘reps’ length and check which is the most frequent symbol in this rep. Following this to the end, we get the final symbol list.
- These symbols are converted to bits.
  The first 2 bits are checked, if they are 1 1 then we remove these two bits along with the last bit of the sequence (the length of the signal was odd).
  If the first 2 bits are 0 0 then we just remove these two bits.
  This is the final demodulated output.

Error Rates:

1) For Gray labelling

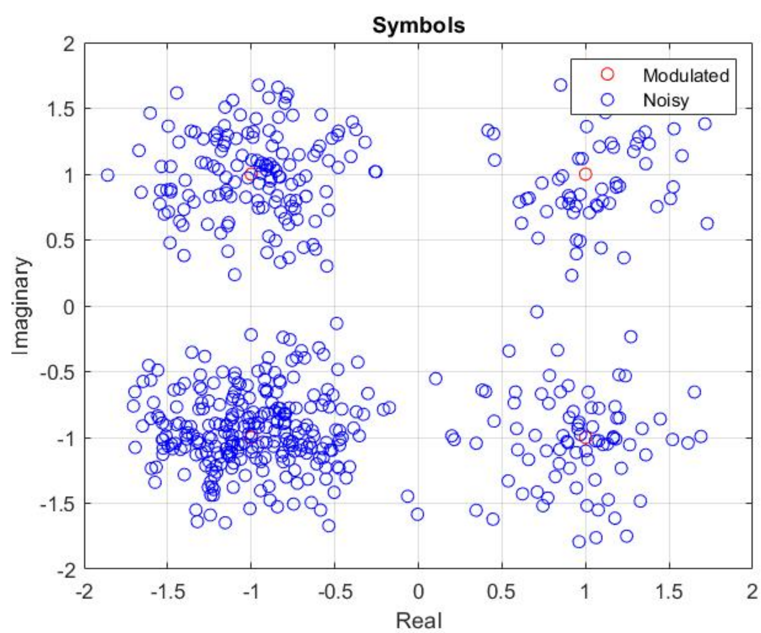


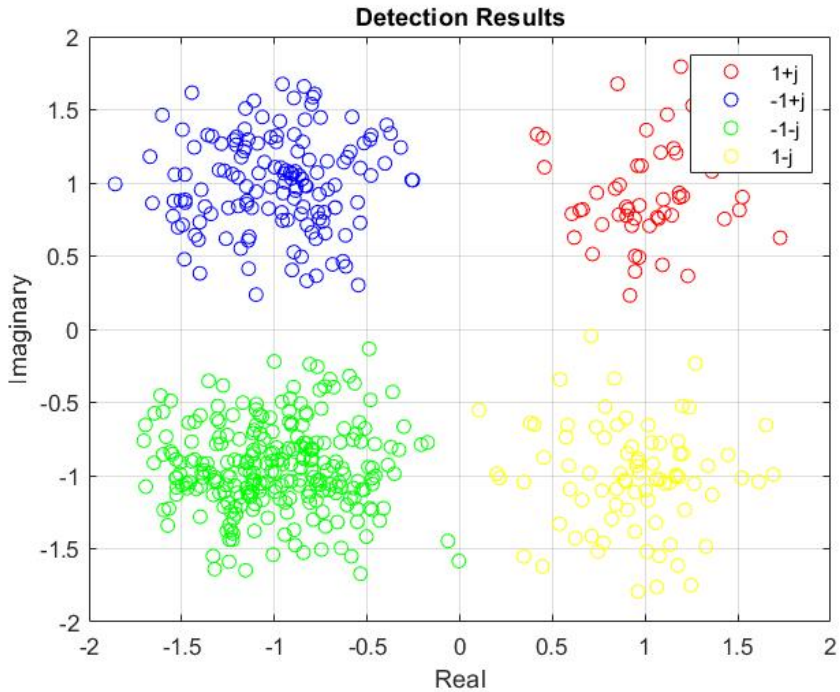
2) Without Gray Labelling:



These plots are attached with the submission.

Constellation Plots for Demo:

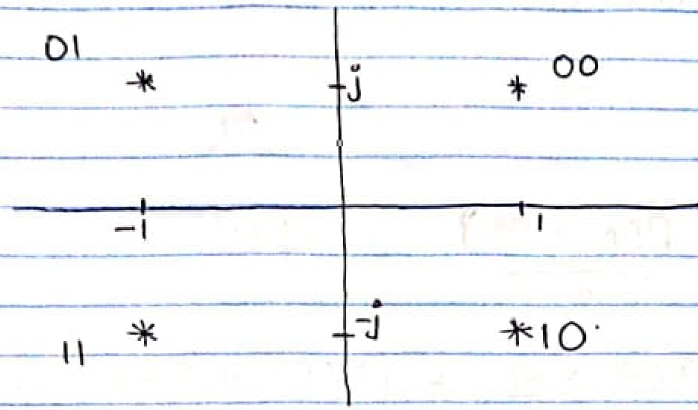



Detection Results
1+j
-1+j
-1-j
1-j
Imaginary
Real
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2

Calculating the Theoretical BER for Gray Labelling and Without Gray labelling:

Calculation of theoretical BER

Case 1 : Gray labelling



let us assume we have a total of 'N' symbols.

we are adding noise to all the 'N' symbols.

$$\text{recvd symbol} = S_i + \underbrace{\frac{1}{\sqrt{2SNR}}\left(N(0,1) + j\,N(0,1)\right)}_{\text{Noise}}$$

we have 3 possible scenarios in which we will get symbol errors.

1.) if real(noise) > 1 & imaginary(noise) < 1

2.) if imaginary(noise) > 1 & real(noise) < 1

3.) if imaginary(noise) > 1 & real(noise) > 1 as we have labelled our constellation with gray labelling,

the bit errors in scenario 1 & 2 will be 1.
But the bit error is 2 in scenario 3.

let us define 3 probabilities

P_1 = Probability of scenario 1

P_2 = Probability of scenario 2

P_3 = Probability of scenario 3

lets calculate these probabilities.

$$P1 = P(\text{real}(\text{noise})>1) \times P(\text{imag}(\text{noise})<1)$$
$$= \cancel{Q(1)}\ P\left(N\left(0, \tfrac{1}{2SNR}\right)>1\right) \times P\left(N\left(0, \tfrac{1}{2SNR}\right)<1\right)$$
$$= Q(\sqrt{2SNR}) \times (1 - Q(\sqrt{2SNR}))$$

similarly

$$P2 = P(\text{imag}(\text{noise})>1) \times P(\text{real}(\text{noise})<1)$$
$$= Q(\sqrt{2SNR}) \times (1 - Q\sqrt{2SNR})$$

and,

$$P3 = P(\text{imag}(\text{noise})>1) \times P(\text{real}(\text{noise})>1)$$
$$= Q^2(\sqrt{2SNR})$$

Scenario 1 & 2 correspond to 1 bit error and scenario 3 correspods to 2 bit error

if no. of symbols = 'N' then no. of bits is = '2N'

no. of erraneous symbols with

a.) scenario 1 = $N \times P1 = N(\Phi(\sqrt{2SNR})(1-\Phi(\sqrt{2SNR}))$

b.) scenario 2 = $N(\Phi(\sqrt{2SNR})(1-\Phi(\sqrt{2SNR}))$

c.) scenario 3 = $N(\Phi^2(\sqrt{2SNR}))$

$$\therefore BER = \frac{1 \times NP1 + 1 \times NP2 + 2 \times NP3}{2N}$$

$$\boxed{BER = \frac{P_1 + P_2}{2} + P_3}$$ for gray labelling

$$\therefore BER = \Phi(\sqrt{2SNR}) - \Phi^2(\sqrt{2SNR}) + \Phi^2(\sqrt{2SNR})$$

$$\therefore \boxed{BER = \Phi(\sqrt{2SNR})}$$

Case 2: without gray labelling

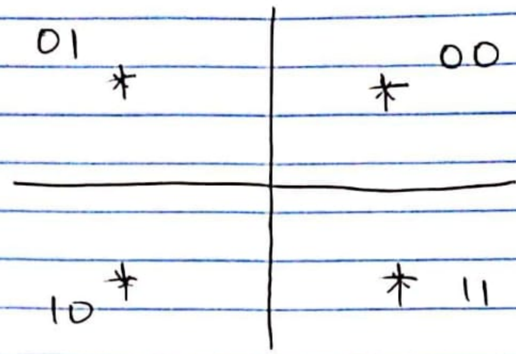
Probability of scenarios is the same

$\therefore P_1 = \Phi(\sqrt{2SNR})(1-\Phi(\sqrt{2SNR}))$

$P_2 = \Phi(\sqrt{2SNR})(1-\Phi(\sqrt{2SNR}))$

$P_3 = \Phi^2(\sqrt{2SNR})$

but now the constellation is as follows:



we have 2 bit error in scenario 2 and 1 bit in the rest.

hence $BER = \dfrac{1 \times P_1 N + 2 \times P_2 N + 1 \times P_3 N}{2N}$

$$\boxed{BER = \frac{P_1 + P_3}{2} + P_2}$$

$$BER = \frac{Q(\sqrt{2SNR})}{2} + Q(\sqrt{2SNR})\left(1 - Q(\sqrt{2SNR})\right)$$

$$\boxed{\therefore BER = 1.5\,Q(\sqrt{2SNR}) - Q^2(\sqrt{2SNR})}$$

without gray labelling