Communications Lab
Experiment 5
Lab Report

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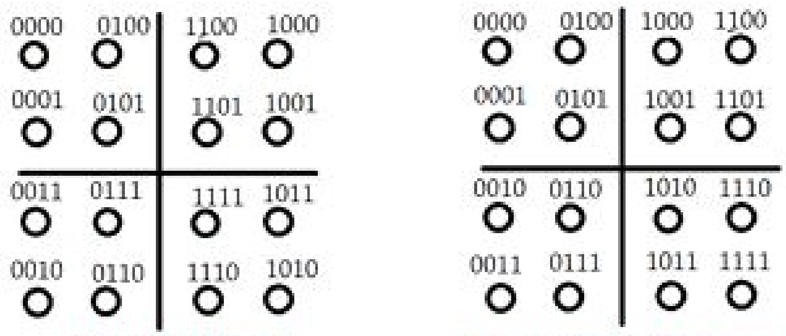
16 QAM Modulation & Demodulation:
QAM stands for Quadrature amplitude modulation. 16 QAM has 16 symbols as shown in the figure below:



16QAM (gray code)   16QAM (natural numbering)

16 QAM simulation:
The simulation contains 8 scripts Main.m, qamSystem.m, bitRep.m, errorCheck.m, bits2symbol.m, symbol2bits, symbol_detector and repRemove.m
The working of each script is explained in the comments of the code.

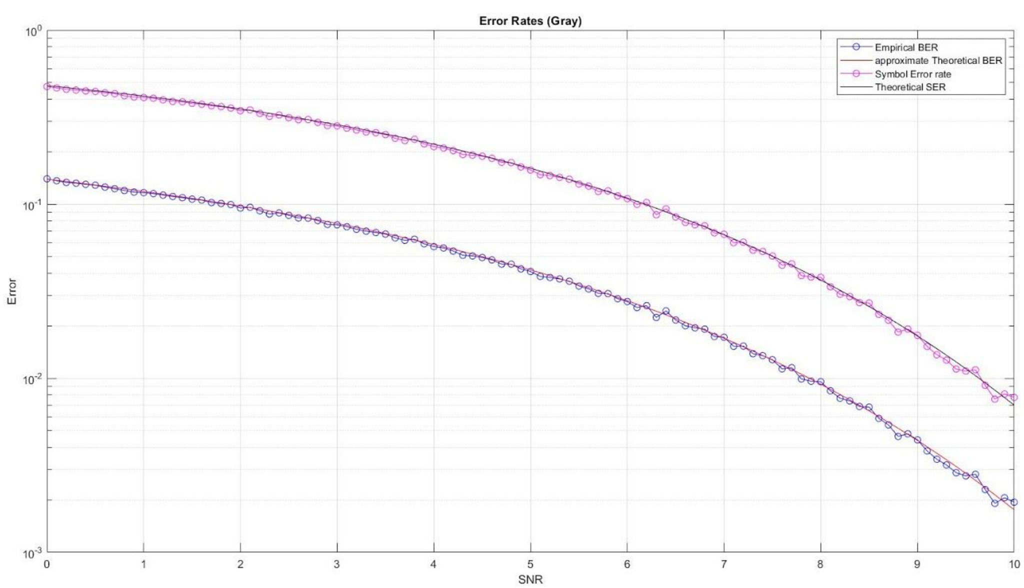
An overview of the working process of the simulation**:**

- We take a message of 64 bits.
- This message undergoes preprocessing. First we define if the message length is divisible by 4, if not we do the following:
  4n+1 Type:
  We add a set of initialization bits [0 0 0 1] and three 0s at the end.
  4n+2 Type:
  We add a set of initialization bits [0 0 1 1] and two 0s at the end.
  4n+3 Type:
  We add a set of initialization bits [0 1 1 1] and one 0 at the end.
  Let's take an example: 1001 → **00**1001
  101 → **0111**101**0**

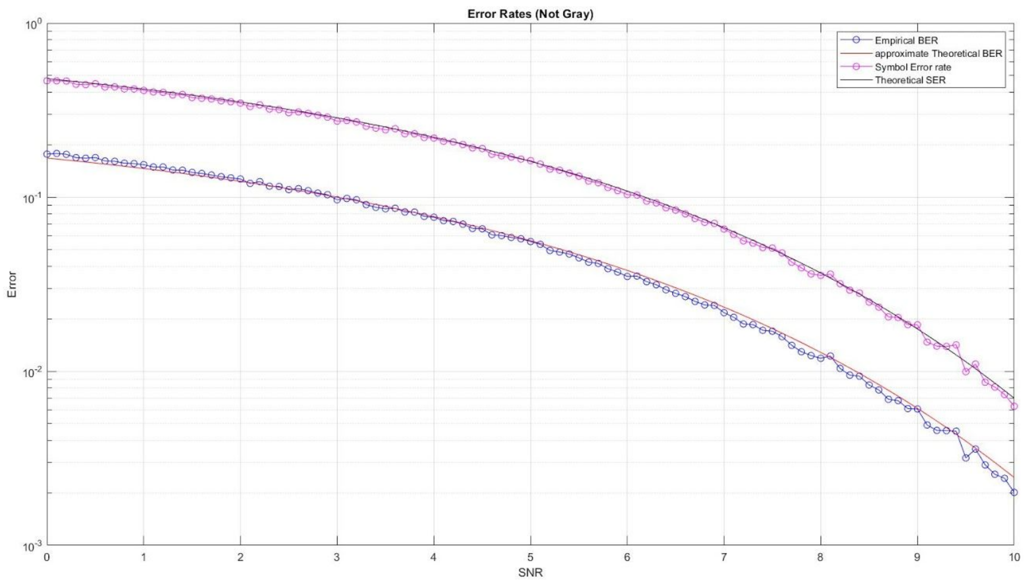
- The set of two bits are repeated 'reps' number of times.
  Example if reps = 4 then 01111010 will become:
  $0\,1\,1\,1\;\;0\,1\,1\,1\;\;0\,1\,1\,1\;\;0\,1\,1\,1\;\;1\,0\,1\,0\;\;1\,0\,1\,0\;\;1\,0\,1\,0\;\;1\,0\,1\,0$
  Basically we are repeating the 16 QAM symbols many times.
- Now we convert the set of four bits into QAM symbols according to the Labelling format mentioned.
  In the above example(for gray labelling):
  $-1-j \quad -1-j \quad -1-j \quad -1-j \quad 3-3j \quad 3-3j \quad 3-3j \quad 3-3j$
- These symbols are then sent into the channel where noise is added.
- The Noisy symbols are then taken 1 by 1 for detection.
  The detection is done by checking the quadrant in which the symbol lies.
- We take a subset of 'reps' length and check which is the most frequent symbol in this rep. Following this to the end, we get the final symbol list.
- These symbols are converted to bits.
  The first 4 bits are checked, if they are:
  0000: only these 4 bits are removed from the received bits.
  0001: these 4 bits along with the last bit are removed from the received bits.
  0011: these 4 bits along with the last two bits are removed from the received bits.
  0111: these 4 bits along with the last three bits are removed.

Error Rates:

1) For Gray labelling

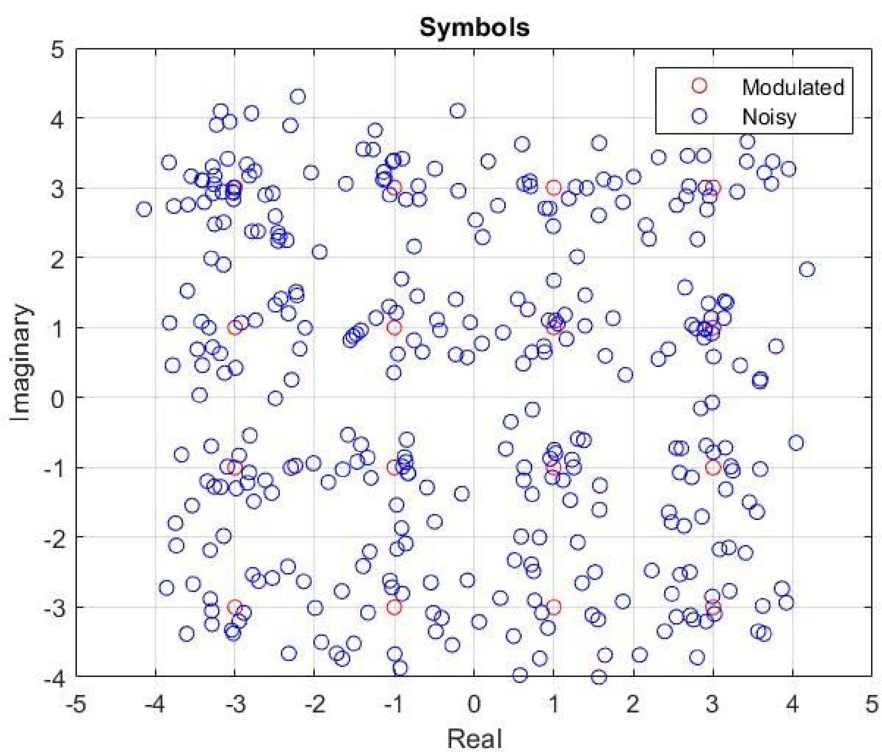


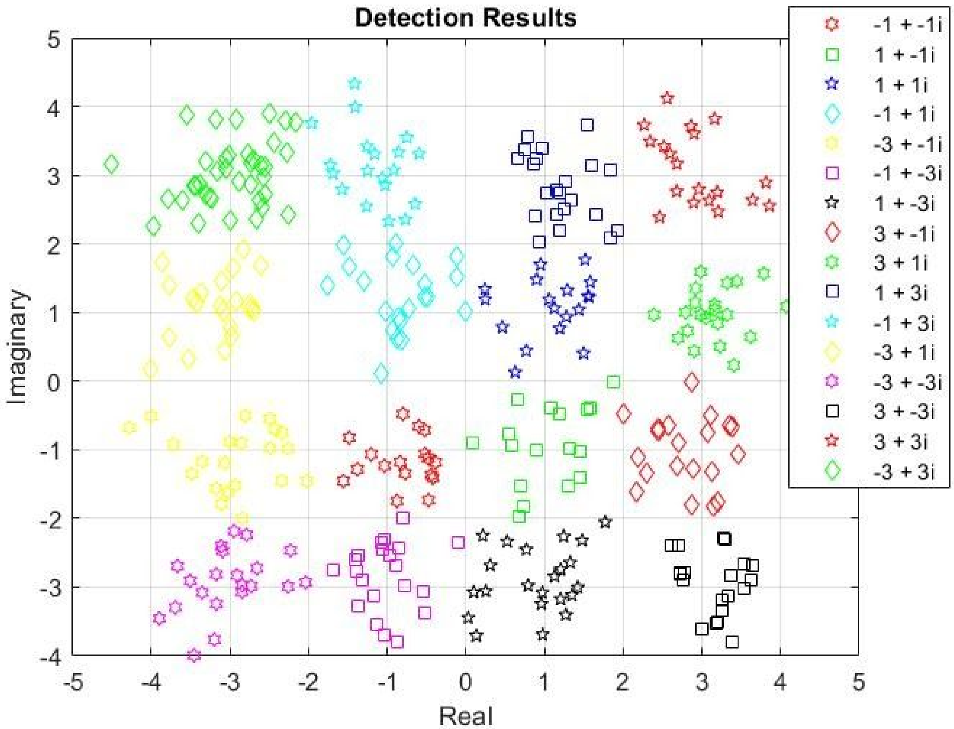
2) Without Gray Labelling:



These plots are attached with the submission.

Constellation Plots for Demo:





BER for Gray labelling is taken from: [link]
BER for Not Gray Labelling is estimated using trial and error.

Calculating the Theoretical SER for Gray Labelling and Without Gray labelling

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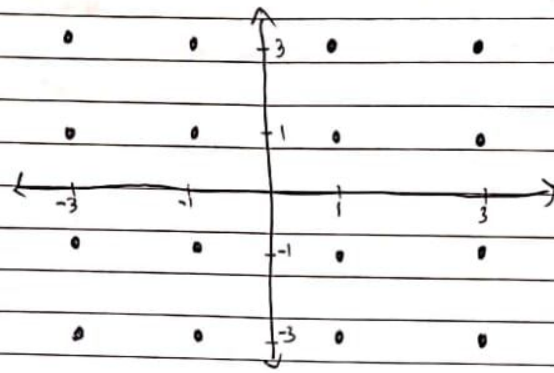
Calculation of Theoretical SER.



As we are concerned only about calculating the theoretical symbol error rate we don't need to know which symbol contains which bits.

Let us divide the 16-QAM symbols into 3 parts:

1.) Middle: $(1+j), (1-j), (-1+j), (-1-j)$
2.) Edges: $(3+j), (3-j), (1+3j), (1-3j), (-3+j), (-3-j), (-1-3j), (-1+3j)$
3) corners: $(3+3j), (3-3j), (-3+3j), (-3-3j)$

Case I (Middle symbols)
In this case there are two possibilities of error
a.) if $|Noise(real)| > 1$
b) if $|Noise(imaginary)| > 1$

$$P_{s,e} = P(a \cup b) = P_a + P_b - P(a \cap b)$$

for middle

$$P_a = 2Q\left(\sqrt{\frac{4snr}{5}}\right), \quad P_b = 2Q\left(\sqrt{\frac{4snr}{5}}\right)$$

$P_{e,s}$ for middle $= 2Q\left(\sqrt{\frac{4snr}{5}}\right) + 2Q\left(\sqrt{\frac{4snr}{5}}\right) - 4Q^2\left(\sqrt{\frac{4snr}{5}}\right)$

$$= 4Q\left(\sqrt{\frac{4snr}{5}}\right) - 4Q^2\left(\sqrt{\frac{4snr}{5}}\right)$$

Case II edges.

in this case there are two possibilities of error.

a) noise(real) > 1

b) |noise (imaginary)| > 1

these conditions are different for each of the 8 edges but as the noise added has same distribution along real and imaginary we can calculate any one and the probability ~~to~~ values will be same for all.

$$P_a = Q\left(\sqrt{\frac{4snr}{5}}\right); \quad P_b = 2Q\left(\sqrt{\frac{4snr}{5}}\right)$$

$$P_{e,s} = P_a + P_b - P(a \cap b)$$

$P_{e,s}$ for edges. $= 3Q\left(\sqrt{\frac{4snr}{5}}\right) - 2Q^2\left(\sqrt{\frac{4snr}{5}}\right)$

Case III

in this case there are two possibilities of errors.

a) noise (real) > 1
b) noise (imaginary) > 1

These conditions vary for each corner but they are symmetric hence the value of probability for any one can be calculated and others will be the same.

$$P_a = Q\left(\sqrt{\frac{4snr}{5}}\right) \; ; \; P_b = Q\left(\sqrt{\frac{4snr}{5}}\right)$$

$$P_{e,s\,\text{corner}} = P_a + P_b - P(a \cap b)$$

$$P_{e,s\,\text{corner}} = 2Q\left(\sqrt{\frac{4snR}{5}}\right) - Q^2\left(\sqrt{\frac{4snR}{5}}\right)$$

finally,

$$SER = \frac{4 \times P_{e,s\,\text{middle}} + 8 \times P_{e,s\,\text{edge}} + 4 \times P_{e,s\,\text{corner}}}{16}$$

Substituting values from above we get:

$$\boxed{SER = 3Q\left(\sqrt{\frac{4snr}{5}}\right) - \frac{9}{4} Q^2\left(\sqrt{\frac{4snr}{5}}\right)}$$

Symbol error rate is independent of labelling format.