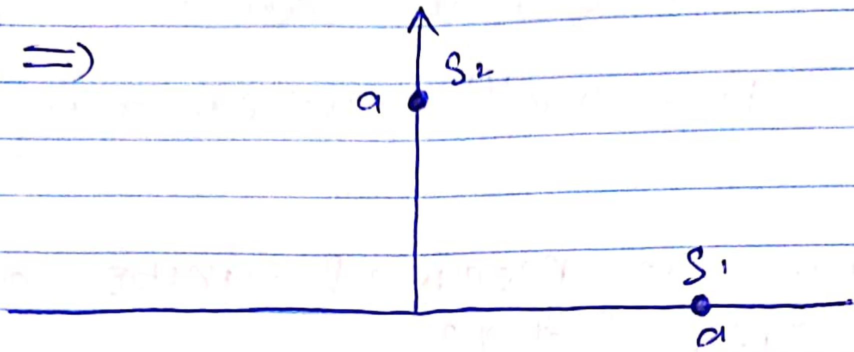
# Communications Project
## Component – II

(a) Given model of the channel & symbols

Symbols ⟹ 




$S_2 \Rightarrow 1$
$S_1 \Rightarrow 0$

Channel ⟹

$$y = hx + n$$

here $x \in \{S_1, S_2\}$; $n$ is additive noise.

$h$ is multiplicative noise.

$h \sim \mathcal{N}_c(0, 1) \rightarrow$ complex gaussian.
$n \sim \mathcal{N}_c(0, \sigma^2) \rightarrow$ gaussian (also complex)

Let us consider an example to understand the problem.

let's say we want to transmit a bit 0 from our channel.
Therefore we will transmit the symbol ~~so~~ $S_1$ through the channel

Hence the received symbol will be

$$y = (a + j0)h + n$$

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now we know that $h$ is a random no. with a complex gaussian distribution.

this random no. can be represented as

$h = re^{j\theta}$. where $r, \theta$ are random values.

now the received symbo can be represented as

$$y = re^{j\theta} \times (a + 0j) + n.$$

as $n, r$ & $\theta$ are random no.s this received symbol can lie any where on the complex plane

if we take $r = 1$, $\theta = \pi/2$ & $n = 0$ then $y$ will become:

$$y = 1e^{j\pi/2}(a) + n = 0 + aj$$

which is infact the symbol $S_2$

so basically the symbols after getting transmitted through the channel end up somewhere, we don't have any control where they will land.

This was more of an intuitive proof, lets prove the above with numbers.

Let us apply our basic ML rule

$H_0 \implies y_0 = (a + 0j)h + n$

$H_1 \implies y_1 = (0 + aj)h + n$

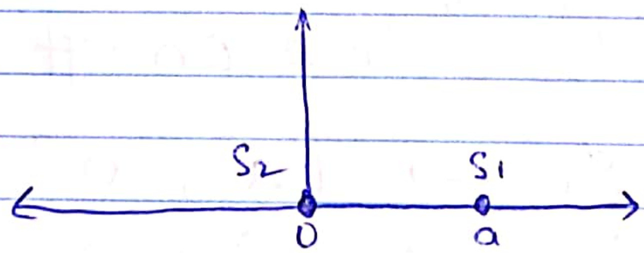
we can clearly see that,
as $h \sim \mathcal{N}_c(0,1)$ & $n \sim \mathcal{N}_c(0,\sigma^2)$

both $y_1$ & $y_0 \sim \mathcal{N}_c(0,a^2) + \mathcal{N}_c(0,\sigma^2)$

if both $y_1$ & $y_0$ will have the same distribu-tion we will never be able to form a decision boundary.

The above statement gives us a idea, How can we make the distributions of $y_1$ & $y_0$ different?

There is one way,
If our symbols would have been like in the case of on-off keying, we indeed will be able to achieve that




Now our ML rule becomes:
$H_0 \implies y_0 = (a + 0j)h + n$; &
$H_1 \implies y_1 = 0 \times h + n$

Hence now,

$$y_0 \sim N_c(0, a^2) + N_c(0, \sigma^2)$$

$$y_1 \sim N(0, \sigma^2)$$

Now we can do something to detect the symbols as the distributions are different.

But the problem is we need to use the symbols given to us in the problem statement;

To solve that problem we can use a combined method.

~~$S_1 \text{ to}$~~ $S_1$ is $a + 0j$ or $(a, 0)$ in complex plane

$S_2$ is $0 + aj$ or $(0, a)$ in complex plane.

* we can interpret $S_1$ as an 'a' followed by a '0' in the on off keying example.

whereas,

* we can interpret $S_2$ as a '0' followed by an 'a' in the on off keying example.

$$\therefore \quad S_1 \Rightarrow [a, 0]$$

$$S_2 \Rightarrow [0, a]$$

So to transmit $S_1$ we will first transmit 'a' through the channel followed by a '0'.

similarly,
we will transmit a '0' through the channel followed by an 'a'. if we want to transmit $S_2$.

after transmitting two symbols separately, we will make a decision rule by taking pairs of received symbols.

let $R, Q$ denote the first symbol received and the second symbol received, respectively.

Then,

$$H_0 \Rightarrow \quad R = ah + n \Rightarrow R \sim \mathcal{N}_c(0, a^2) + \mathcal{N}_c(0, \sigma^2)$$
$$Q = 0h + n \Rightarrow Q \sim \mathcal{N}_c(0, \sigma^2)$$

$$H_1 \Rightarrow \quad R = 0h + n \Rightarrow R \sim \mathcal{N}_c(0, \sigma^2)$$
$$Q = ah + n \Rightarrow Q \sim \mathcal{N}_c(0, \sigma^2) + \mathcal{N}_c(0, a^2)$$

we know $\mathcal{N}_c(0, a^2) + \mathcal{N}_c(0, \sigma^2) = \mathcal{N}_c(0, a^2 + \sigma^2)$
therefore on simplification

$$H_0 \Rightarrow \quad R \sim \mathcal{N}_c(0, a^2 + \sigma^2)$$
$$Q \sim \mathcal{N}_c(0, \sigma^2)$$

$$H_1 \Rightarrow \quad Q \sim \mathcal{N}_c(0, a^2 + \sigma^2)$$
$$R \sim \mathcal{N}_c(0, \sigma^2)$$

if we look closely we will observe that R is nothing but the real part of the original symbols and Q is the imaginary part.

$\therefore$ $y = R + jQ$

Hence $P(y|H_0) = P(R, Q|H_0)$

as we know R & Q are gaussian and independent we can say that

$$P_R(r|H_0) = \frac{1}{\sqrt{2\pi(a^2+\sigma^2)}} \times e^{-\frac{1}{2}\left(\frac{r^2}{\sigma^2+a^2}\right)}$$

$$P_Q(q|H_0) = \frac{1}{\sqrt{2\pi\sigma^2}} \times e^{-\frac{1}{2}q^2/\sigma^2}$$

here $r = x_0 h + n$ (received real part)
$q = x_1 h + n$ (received imaginary part)

$$\therefore P_{RQ}(r, q|H_0) = \frac{1}{2\pi\sigma\sqrt{\sigma^2+a^2}} \times e^{-\frac{1}{2}\left(\frac{r^2}{\sigma^2+a^2}+\frac{q^2}{\sigma^2}\right)}$$

Similarly,

$P(y|H_1) = P(R, Q|H_1)$

$$P_R(r|H_1) = \frac{1}{\sqrt{2\pi\sigma^2}} \times e^{-\frac{1}{2}\left(r^2/\sigma^2\right)}$$

$$P_Q(q|H_1) = \frac{1}{\sqrt{2\pi(\sigma^2+a^2)}} \times e^{-\frac{1}{2}\left(q^2/\sigma^2+a^2\right)}$$

$$\therefore P_{RQ}(r,q|H_1) = \frac{1}{2\pi\sigma\sqrt{\sigma^2+a^2}} e^{-\frac{1}{2}\left(\frac{r^2}{\sigma^2}+\frac{q^2}{\sigma^2+a^2}\right)}$$

Now we can say that the log likelihood ratio can be written as:

$$\log\left(\frac{P(y|H_0)}{P(y|H_1)}\right) = \log\left(\frac{P_{RQ}(r,q|H_0)}{P_{RQ}(r,q|H_1)}\right)$$

$$= \log\left(\cancel{\frac{2\pi\sigma\sqrt{\sigma^2+a^2}}{2\pi\sigma\sqrt{\sigma^2+a^2}}} \times e^{-\frac{1}{2}\left(r^2/\sigma^2+a^2 - \frac{r^2}{\sigma^2} - \frac{q^2}{\sigma^2+a^2} + \frac{q^2}{\sigma^2}\right)}\right)$$

$$= \log\left(e^{-\frac{1}{2}\left(\frac{-r^2a^2+a^2q^2}{(\sigma^2+a^2)\sigma^2}\right)}\right)$$

$$= \frac{a^2(r^2-q^2)}{2\sigma^2(a^2+\sigma^2)}$$

$H_0 \longrightarrow r^2 > q^2$;

$H_1 \longrightarrow q^2 > r^2$;

or in other words, 0 is received if the magnitude of the 1st term in the pair is greater than the 2nd term.

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Code guide:

There are 3 scripts

a.) Main.m — for demo

b.) commSystem.m — simulated system

c.) bitErrRate.m — calculating bit error rate

to get a demo of the project run Main.m, please ensure all the three files are in the same folder and are added to path.