CSI 4133 Computer Methods in Picture Processing and Analysis

Fall 2024

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Outline

- Pixel neighbor, adjacency, set, structuring element
- Erosion and dilation
- Opening and closing
- Hole filling
- Convex hull
- Thinning, thickening, skeleton
- Grayscale mathematical morphology

Neighbours of a pixel

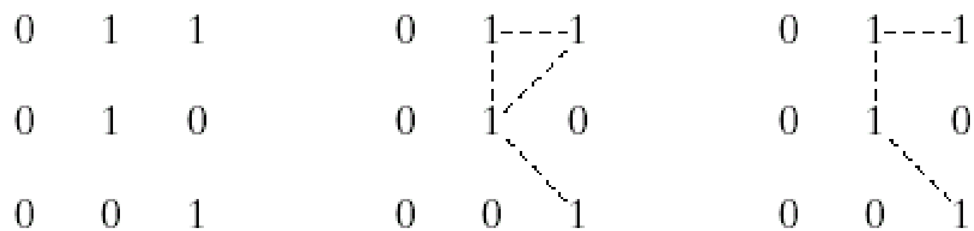
- A pixel p at coordinates (x, y) has
  - Four horizontal and vertical neighbours
    - (x + 1, y), (x – 1, y), (x, y + 1), (x, y – 1)
    - 4-neighbors of p
    - $N_4(p)$
  - Four diagonal neighbours
    - (x + 1, y + 1), (x + 1, y - 1), (x - 1, y + 1), (x - 1, y - 1)
    - $N_D(p)$
    - $N_8(p)$: 8-neighbors of p together with the 4-neighbors

Neighbours of a pixel

- V - the set of gray-level values used to define adjacency.
- 4-adjacency
  - Two pixels p and q with values from V are 4-adjacent if q is in the set $N_4(p)$
- 8-adjacency
  - Two pixels p and q with values from V are 8-adjacent if q is in the set $N_8(p)$
- m-adjacency (mixed adjacency)
  - Two pixels p and q with values from V are m-adjacent if
  - q is in the set $N_4(p)$, or
  - q is in the set $N_D(p)$ and the set $N_4(p) \cap N_4(q)$ has no pixels whose values are from V

Neighbours of a pixel

- 8-adjacency vs. m-adjacency



a b c

FIGURE 2.26 (a) Arrangement of pixels; (b) pixels that are 8-adjacent (shown dashed) to the center pixel; (c) *m*-adjacency.

Connectivity, region, boundary

- S : a subset of pixels in an image.
- Two pixels p and q are said to be connected in S
  - if there exists a path between them consisting entirely of pixels in S
- R : a subset of pixels in an image.
- R : a region of the image
  - if R is a connected set.
- The boundary (i.e. border or contour) of a region R is the set of pixels in the region that have one or more neighbours that are not in R.

Mathematical morphology

- A tool for extracting image components that are useful in the representation and description of region shape
  - Such as boundaries, skeletons, and convex hull
- The language of mathematical morphology is set theory
- Morphological expressions are written in terms of structuring elements and a set, A, of foreground pixels, or in terms of structuring elements and an image, I, that contains A.

Set

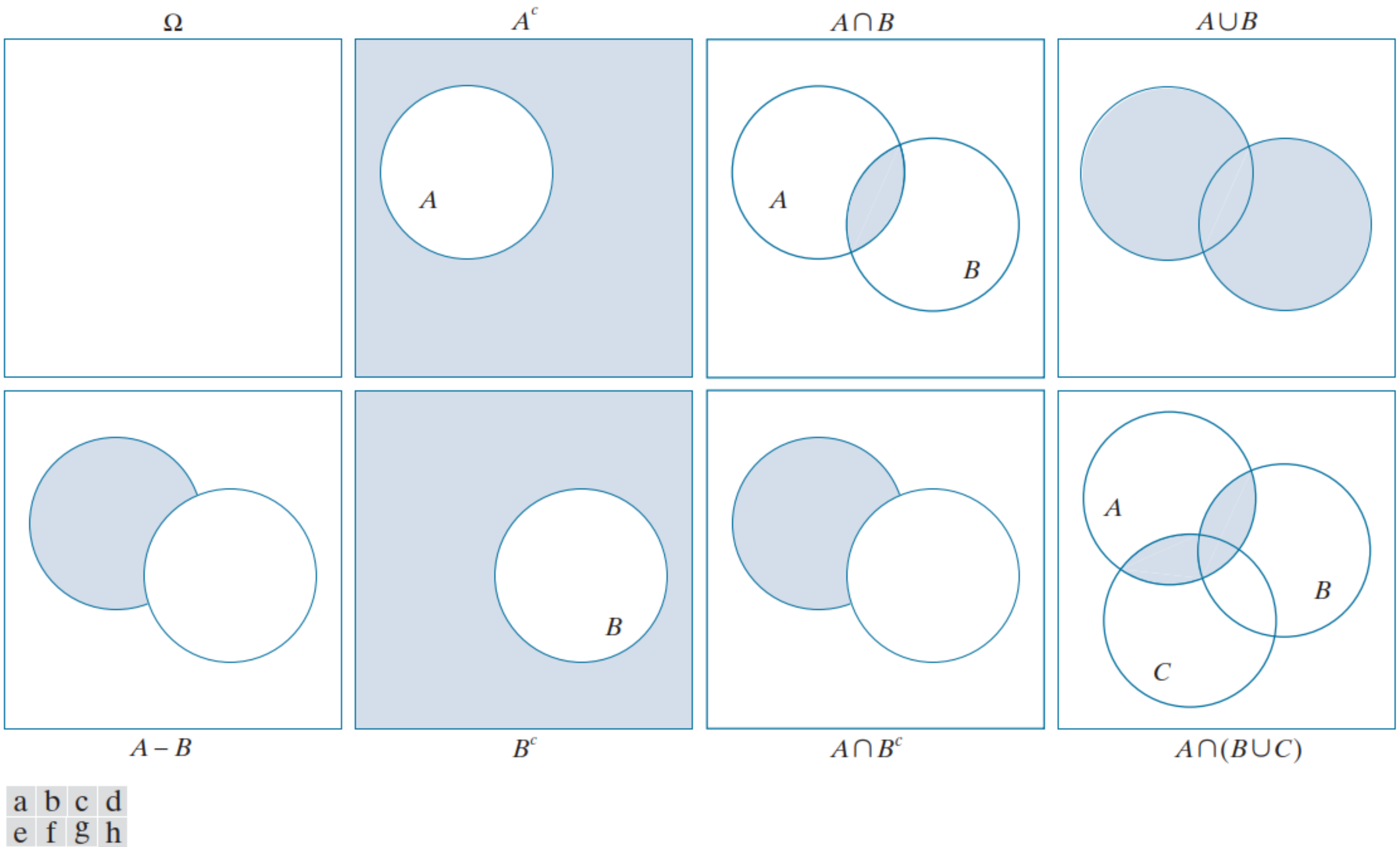


FIGURE 2.35 Venn diagrams corresponding to some of the set operations in Table 2.1. The results of the operations, such as A^c, are shown shaded. Figures (e) and (g) are the same, proving via Venn diagrams that $A - B = A \cap B^c$ [see Eq. (2-40)].

Set

- In image processing, we use morphology with two types of sets of pixels: *objects* and *structuring elements* (SE)
- The *reflection* of a set (structuring element) B about its origin, denoted by $\hat{B}$, is defined as

$$\hat{B} = \left\{ w \mid w = -b, \text{ for } b \in B \right\}$$

- The *translation* of a set B by point $z=(z_1,z_2)$, denoted $(B)_z$, is defined as

$$(B)_z = \left\{ c \mid c = b + z, \text{ for } b \in B \right\}$$

Set

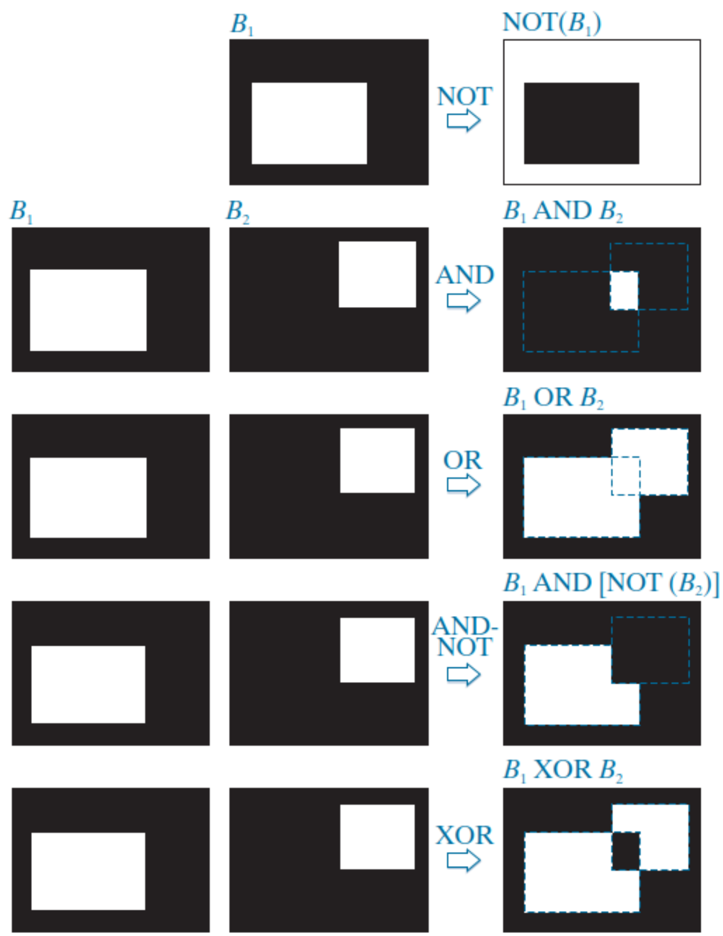
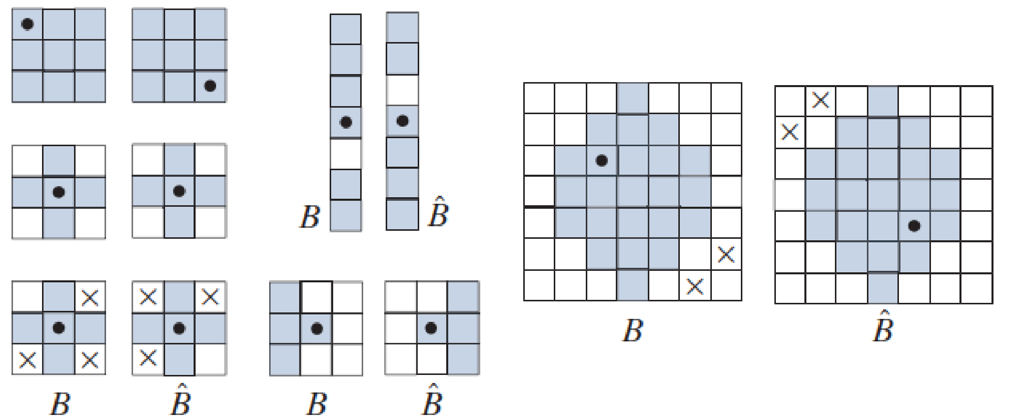


FIGURE 2.37 Illustration of logical operations involving foreground (white) pixels. Black represents binary 0's and white binary 1's. The dashed lines are shown for reference only. They are not part of the result.

Structuring elements (SE)

FIGURE 9.2
Structuring elements and their reflections about the origin (the ×'s are don't care elements, and the dots denote the origin). Reflection is rotation by 180° of an SE about its origin.

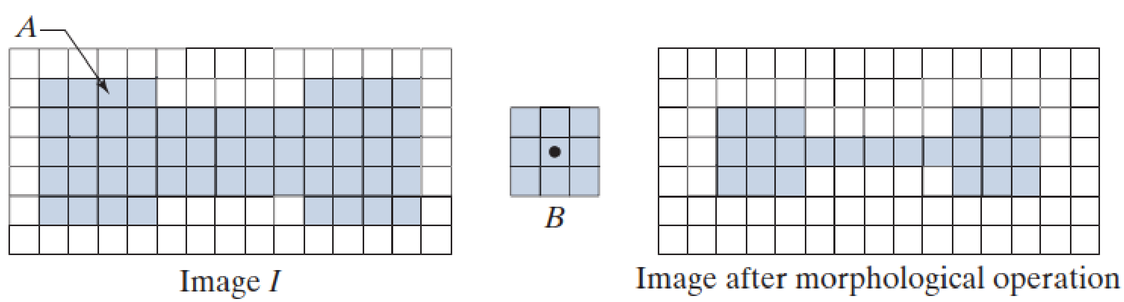


Morphological operation

- Transform B over image I, and at each increment of translation, if B is completely contained in A, mark the location of the origin of B as a foreground pixel in the new image

a b c

FIGURE 9.3
(a) A binary image containing one object (set), *A*. (b) A structuring element, *B*. (c) Image resulting from a morphological operation (see text).

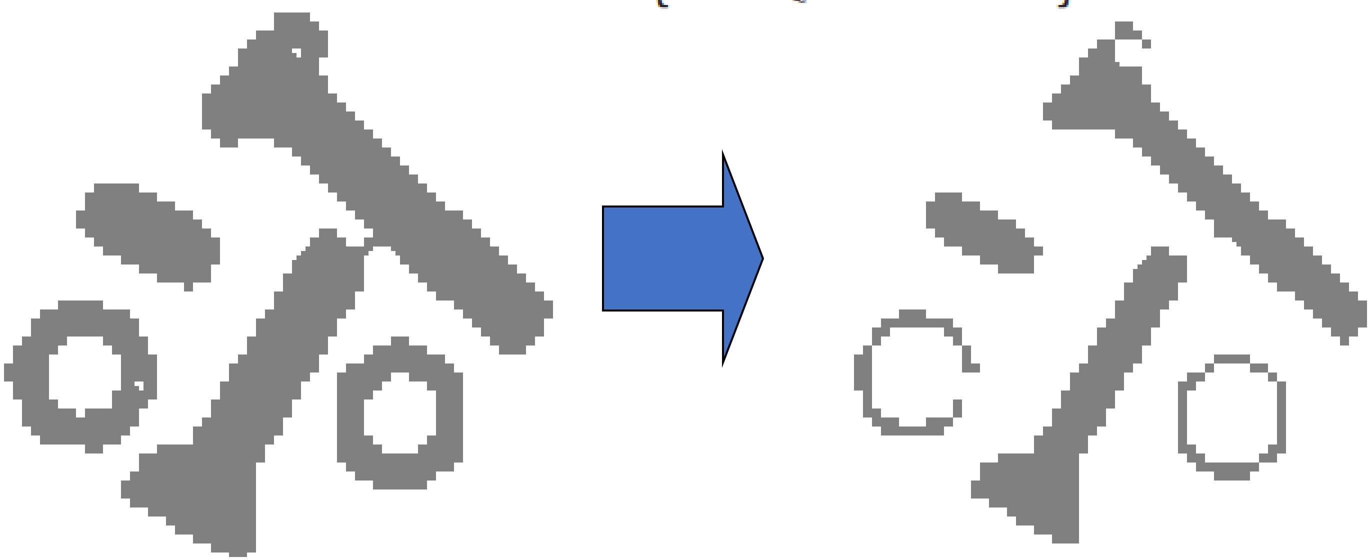


Erosion

- Where the structuring element 'fit' the image

$$A \ominus B = \left\{ z \middle| (B)_z \subseteq A \right\}$$

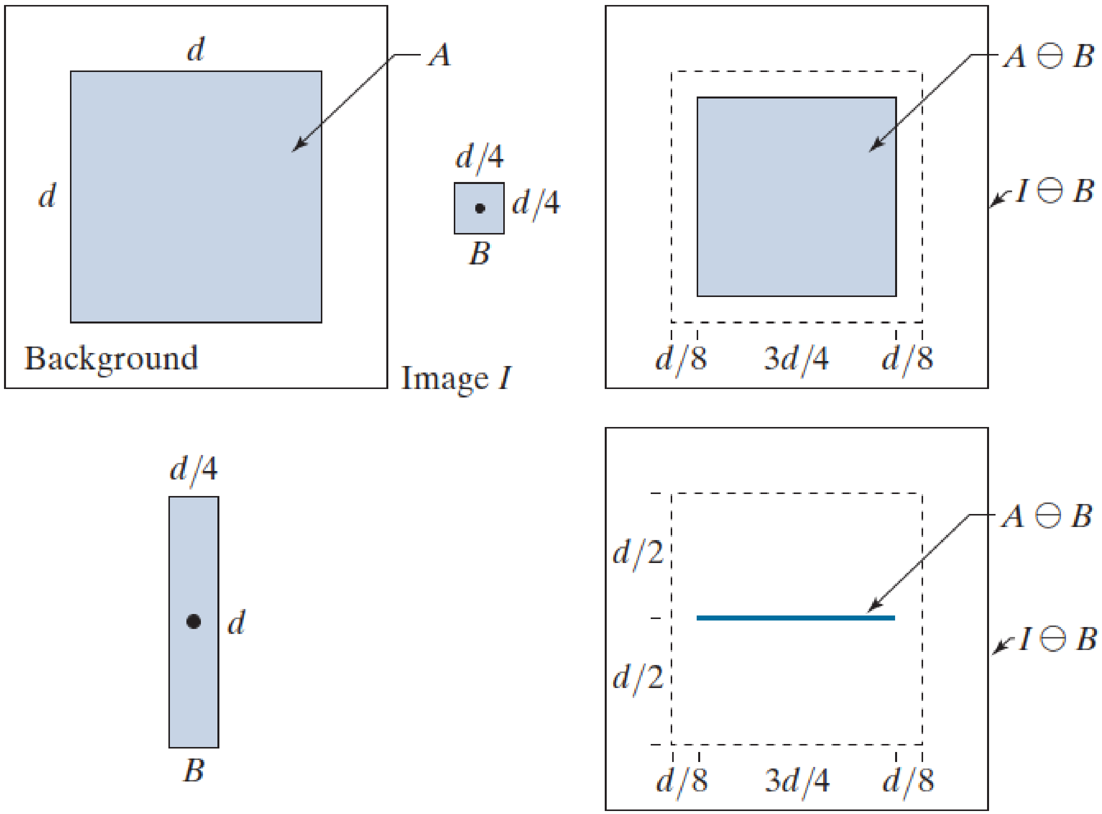
$$A \ominus B = \left\{ z \middle| (B)_z \cap A^c = \varnothing \right\}$$



Erosion

a b c
d e

FIGURE 9.4
(a) Image I, consisting of a set (object) A, and background.
(b) Square SE, B (the dot is the origin).
(c) Erosion of A by B (shown shaded in the resulting image).
(d) Elongated SE.
(e) Erosion of A by B. (The erosion is a line.) The dotted border in (c) and (e) is the boundary of A, shown for reference.



Erosion

a	b
c	d

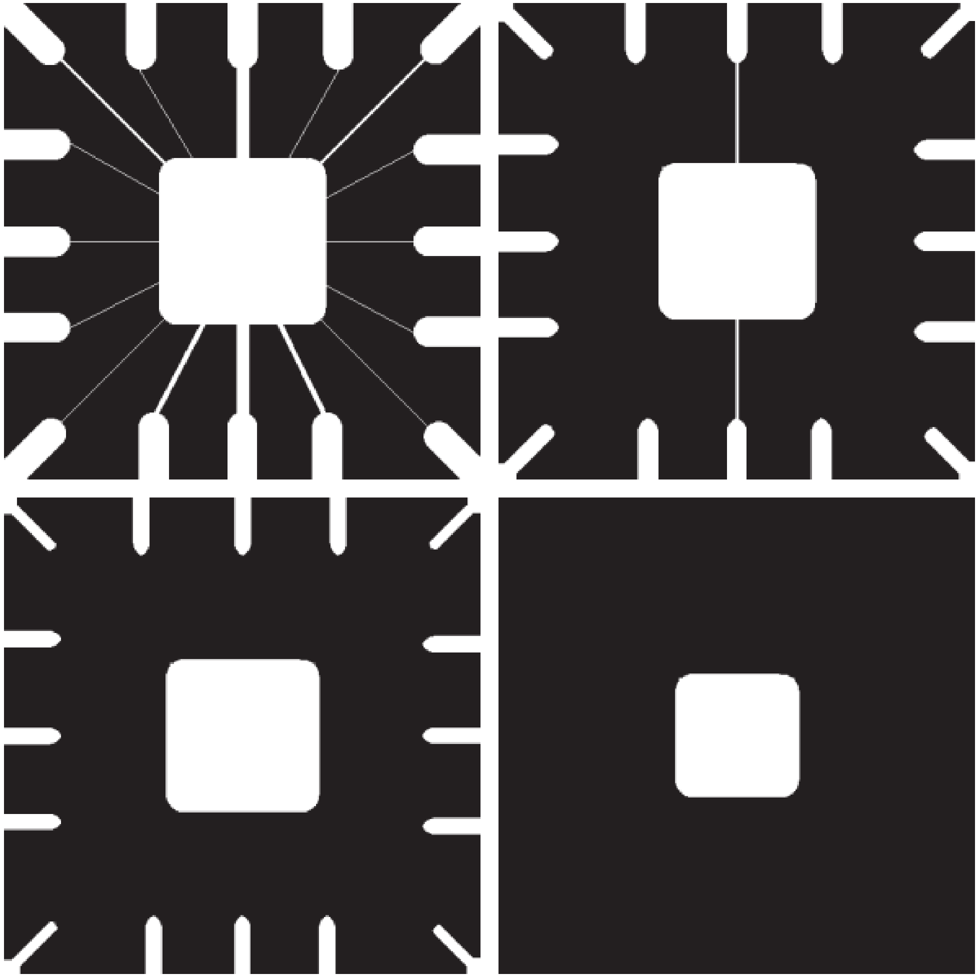
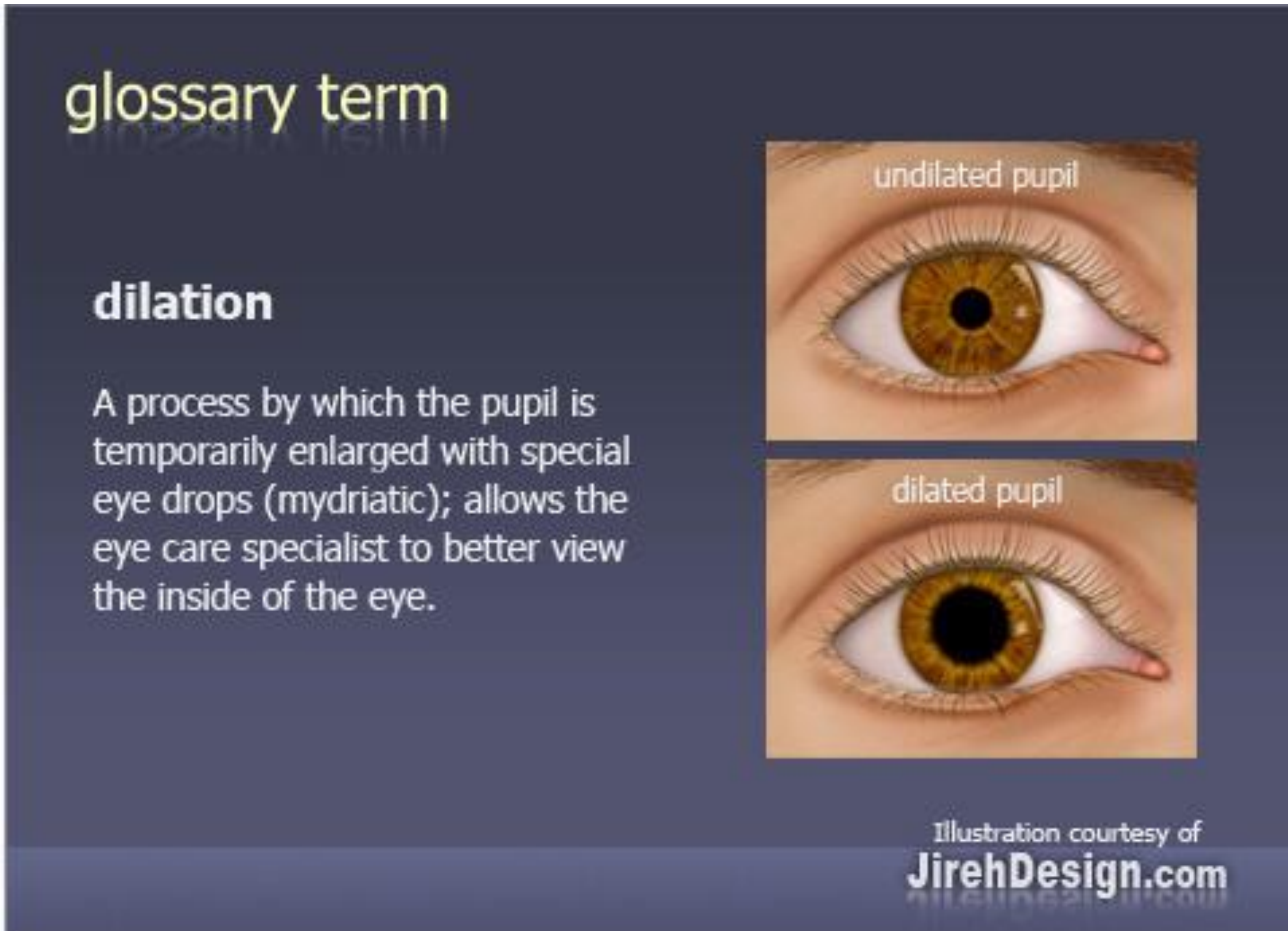


FIGURE 9.5 Using erosion to remove image components. (a) A 486×486 binary image of a wire-bond mask in which foreground pixels are shown in white. (b)–(d) Image eroded using square structuring elements of sizes 11×11, 15×15, and 45×45 elements, respectively, all valued 1.

Erosion

- Erosion shrinks or thins objects in a binary image.
  - Erosion as a morphological filtering operation, where image details smaller than the structuring element are filtered (removed) from the image
- One use is for eliminating irrelevant details (in terms of size) from a binary image

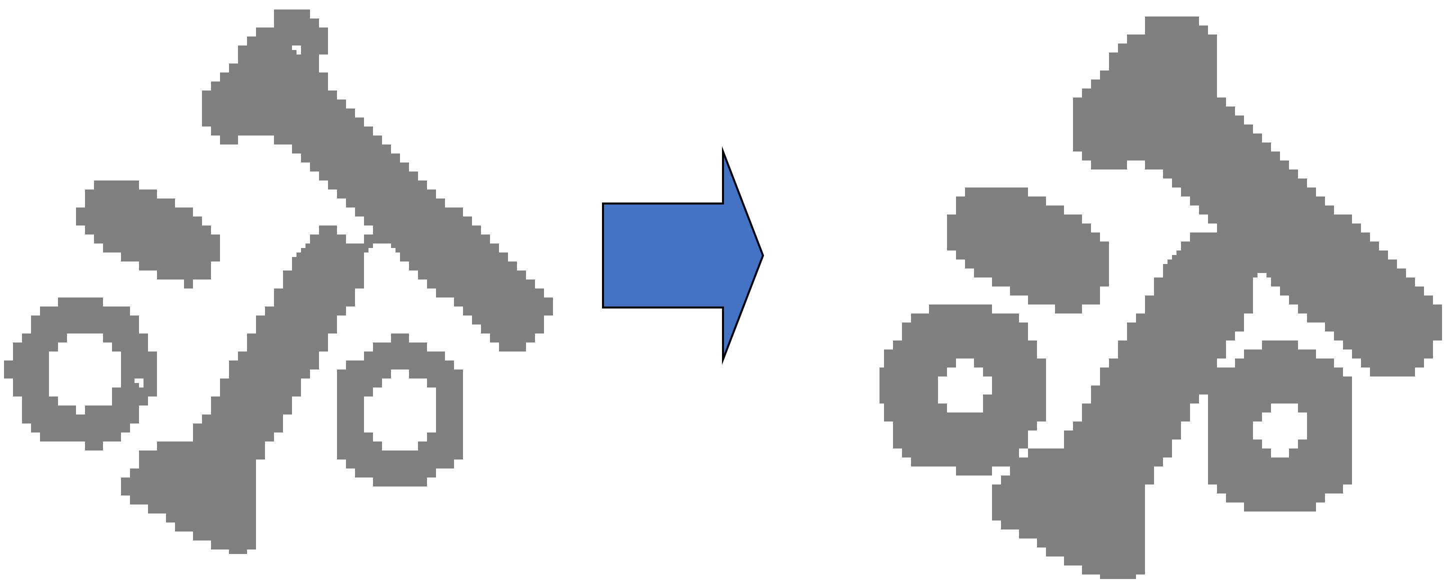
Dilation


glossary term
dilation
A process by which the pupil is temporarily enlarged with special eye drops (mydriatic); allows the eye care specialist to better view the inside of the eye.
undilated pupil
dilated pupil
Illustration courtesy of
JirehDesign.com

Dilation

- Where the structuring element 'hit' the image

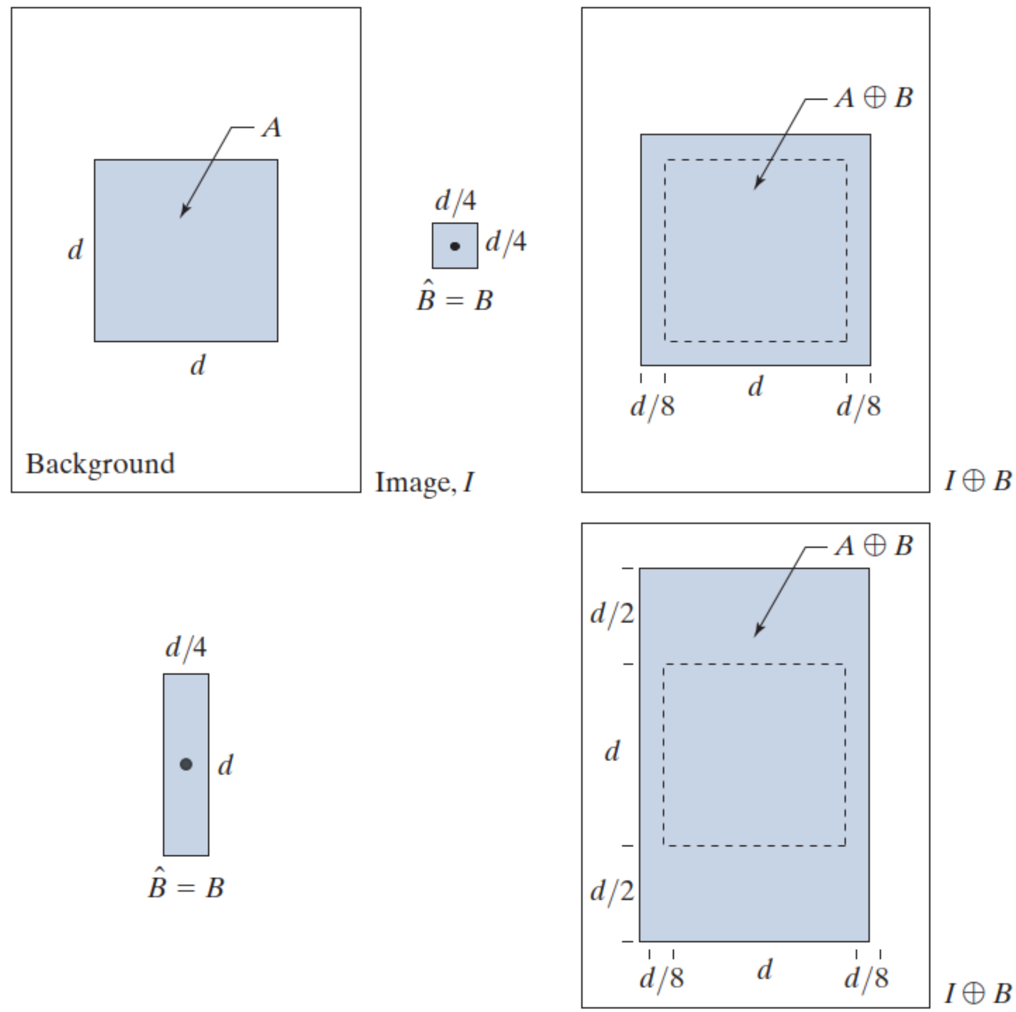
$$A \oplus B = \left\{ z \,\middle|\, (\hat{B})_z \cap A \neq \varnothing \right\}$$



Dilation

a b c
d e

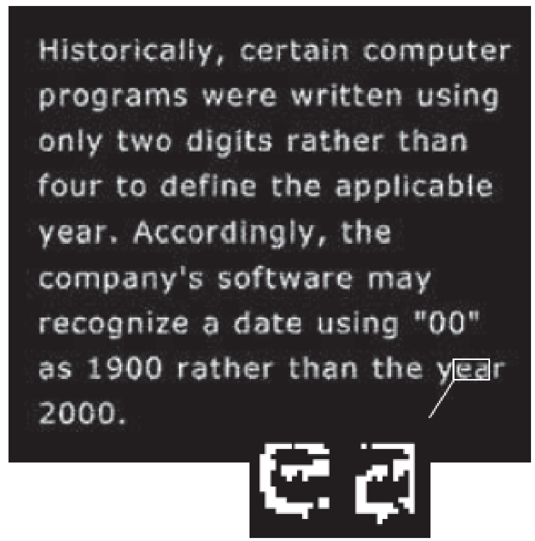
FIGURE 9.6
(a) Image I, composed of set (object) A and background. (b) Square SE (the dot is the origin). (c) Dilation of A by B (shown shaded). (d) Elongated SE. (e) Dilation of A by this element. The dotted line in (c) and (e) is the boundary of A, shown for reference.



Dilation

a c
b

FIGURE 9.7
(a) Low-resolution text showing broken characters (see magnified view).
(b) Structuring element.
(c) Dilation of (a) by (b). Broken segments were joined.

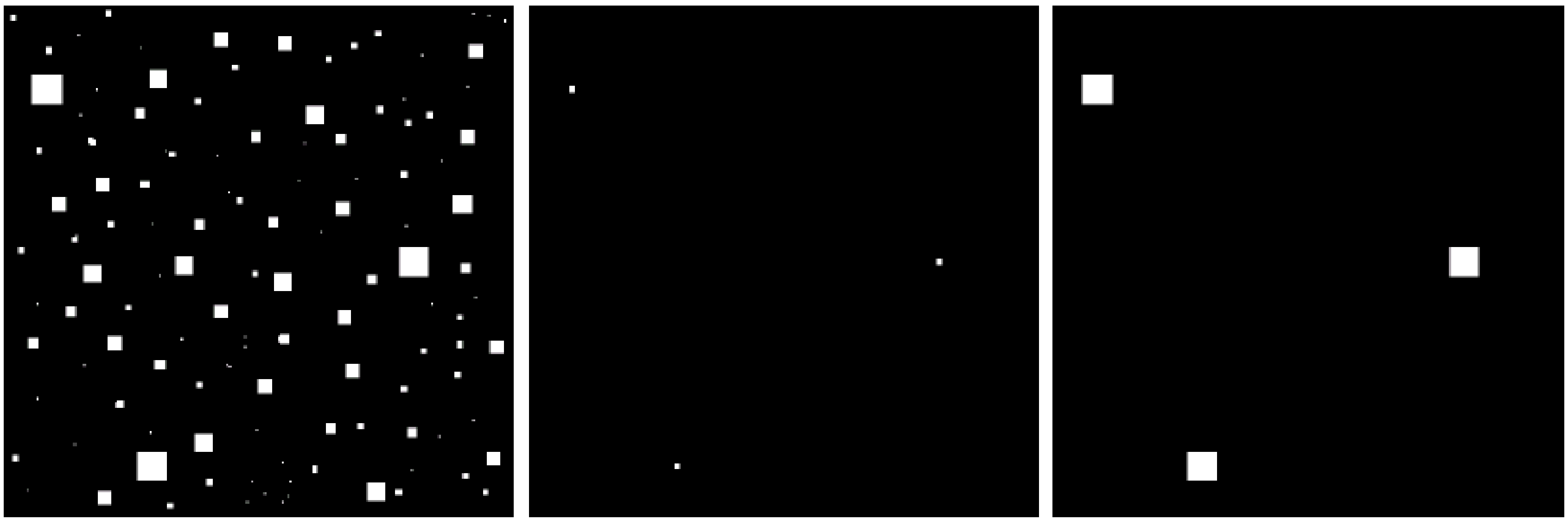


1	1	1
1	1	1
1	1	1

Dilation

- Dilation “grows” or “thickens” objects in a binary image
- The manner and extent of this thickening is controlled by the shape and size of the structuring element used

Erosion and dilation



a b c

FIGURE 9.7 (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

Duality

Erosion and dilation are *duals* of each other with respect to set complementation and reflection. That is,

$$(A \ominus B)^c = A^c \oplus \hat{B} \tag{9-8}$$

and

$$(A \oplus B)^c = A^c \ominus \hat{B} \tag{9-9}$$

- The duality property is useful when the SE values are symmetric with respect to its origin; then, we can obtain the erosion of A simply by dilating its background with the same structuring element and complementing the result.

Opening and closing

- Opening generally smooths the contour of an object, breaks narrow isthmuses, and eliminates thin protrusions

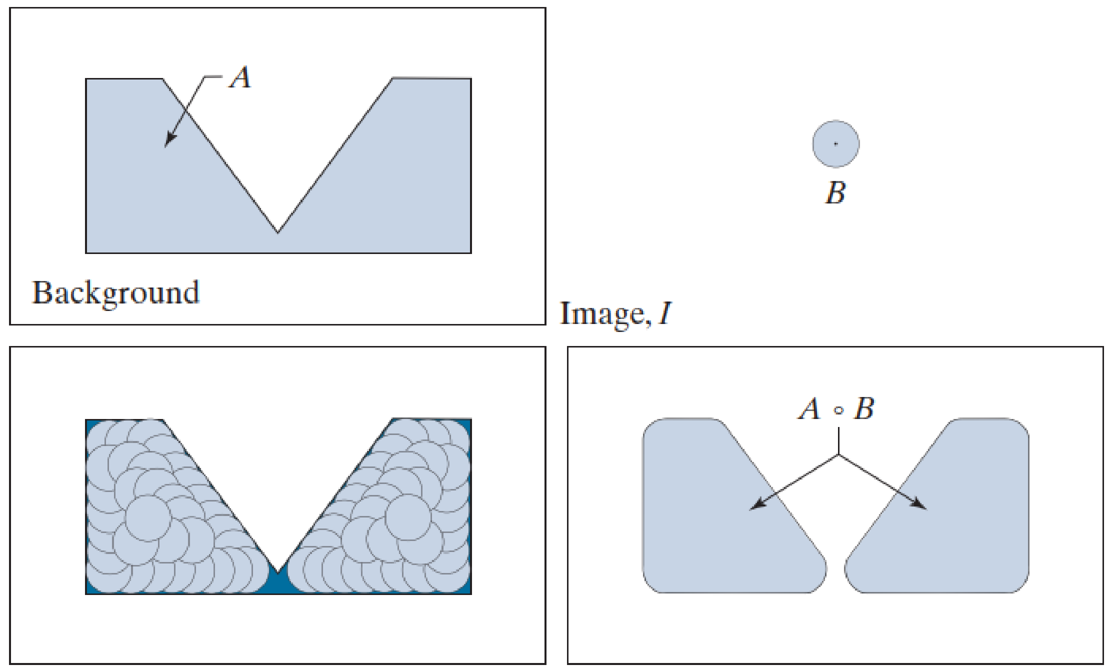
$$A \circ B = (A \ominus B) \oplus B$$

- Closing also tends to smooth sections of contours, but as opposed to opening, it generally fuses narrow breaks and long thin gulfs, eliminates small holes, and fills gaps in the contour

$$A \bullet B = (A \oplus B) \ominus B$$

Opening

$$A \circ B = (A \ominus B) \oplus B$$



a b
c d

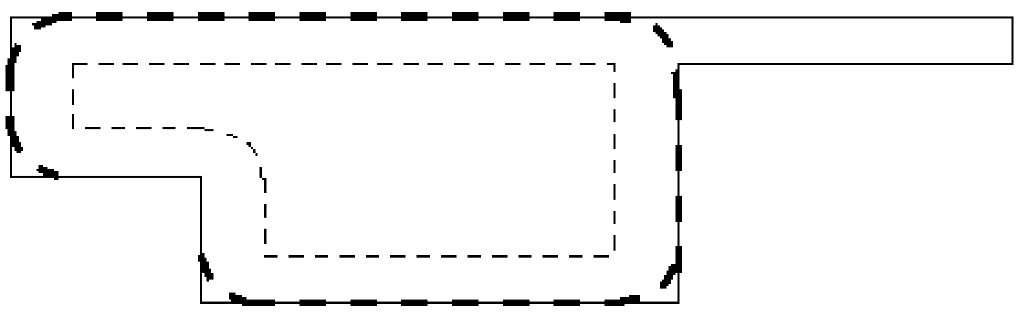
FIGURE 9.8
(a) Image I, composed of set (object) A and background.
(b) Structuring element, B.
(c) Translations of B while being contained in A. (A is shown dark for clarity.)
(d) Opening of A by B.

- Geometric interpretation: the opening of A by B is the union of all the translations of B so that B fits entirely in A.

$$A \circ B = \bigcup \left\{ (B)_z \,\middle|\, (B)_z \subseteq A \right\}$$

Opening

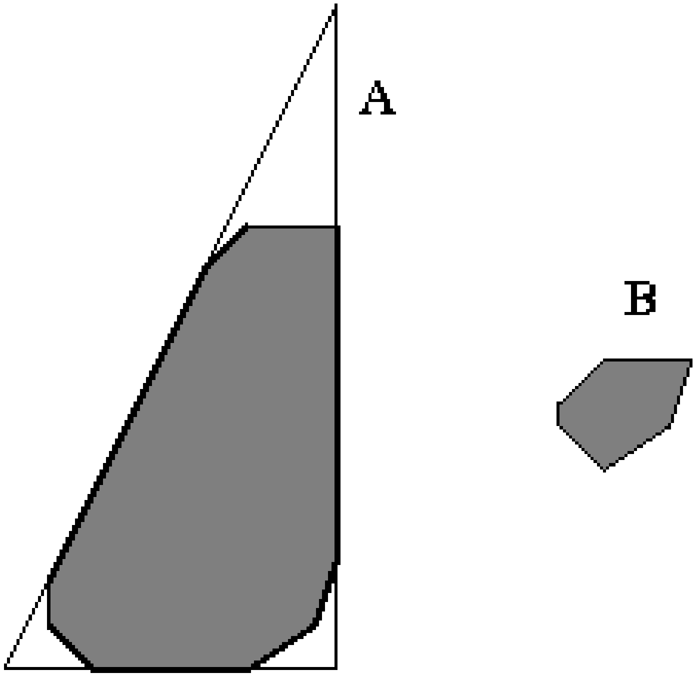
- ‘rounding from the inside’
- The opening (given by the dark dashed lines) of A (given by the solid lines). The structuring element B is a disc. The internal dashed structure is A eroded by B.



http://homepages.inf.ed.ac.uk/rbf/CVonline/LOCAL_COPIES/OWENS/LECT3/node3.html

Opening

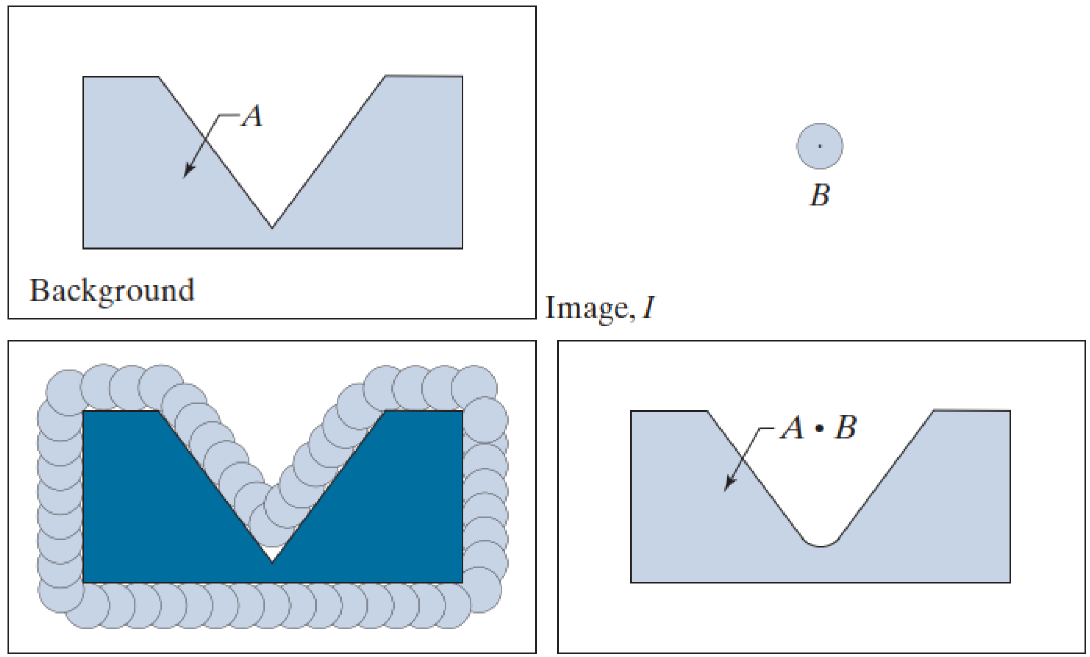
- The opening of A by structuring element B



http://homepages.inf.ed.ac.uk/rbf/CVonline/LOCAL_COPIES/OWENS/LECT3/node3.html

Closing

$$A \bullet B = (A \oplus B) \ominus B$$



a	b
c	d

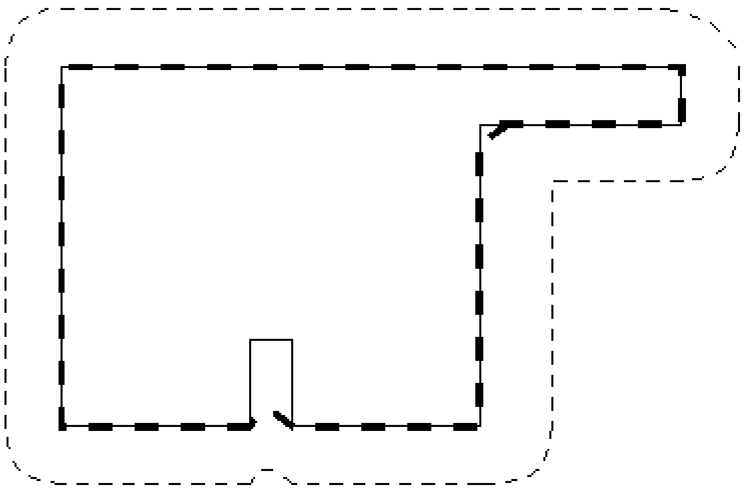
FIGURE 9.9 (a) Image I, composed of set (object) A, and background. (b) Structuring element B. (c) Translations of B such that B does not overlap any part of A. (A is shown dark for clarity.) (d) Closing of A by B.

- The closing is the complement of the union of all translations of B that do not overlap A.

$$A \bullet B = \left[\bigcup \left\{ (B)_z \mid (B)_z \cap A = \varnothing \right\} \right]^c$$

Closing

- 'smoothing from the outside'
- The closing of A by the structuring element B



http://homepages.inf.ed.ac.uk/rbf/CVonline/LOCAL_COPIES/OWENS/LECT3/node3.html

Duality and properties

$$(A \circ B)^c = (A^c \bullet \hat{B}) \qquad (A \bullet B)^c = (A^c \circ \hat{B})$$

Morphological opening has the following properties:

(a) $A \circ B$ is a subset of A.
(b) If C is a subset of D, then $C \circ B$ is a subset of $D \circ B$.
(c) $(A \circ B) \circ B = A \circ B$.

Similarly, closing satisfies the following properties:

(a) A is a subset of $A \bullet B$.
(b) If C is a subset of D, then $C \bullet B$ is a subset of $D \bullet B$.
(c) $(A \bullet B) \bullet B = A \bullet B$.

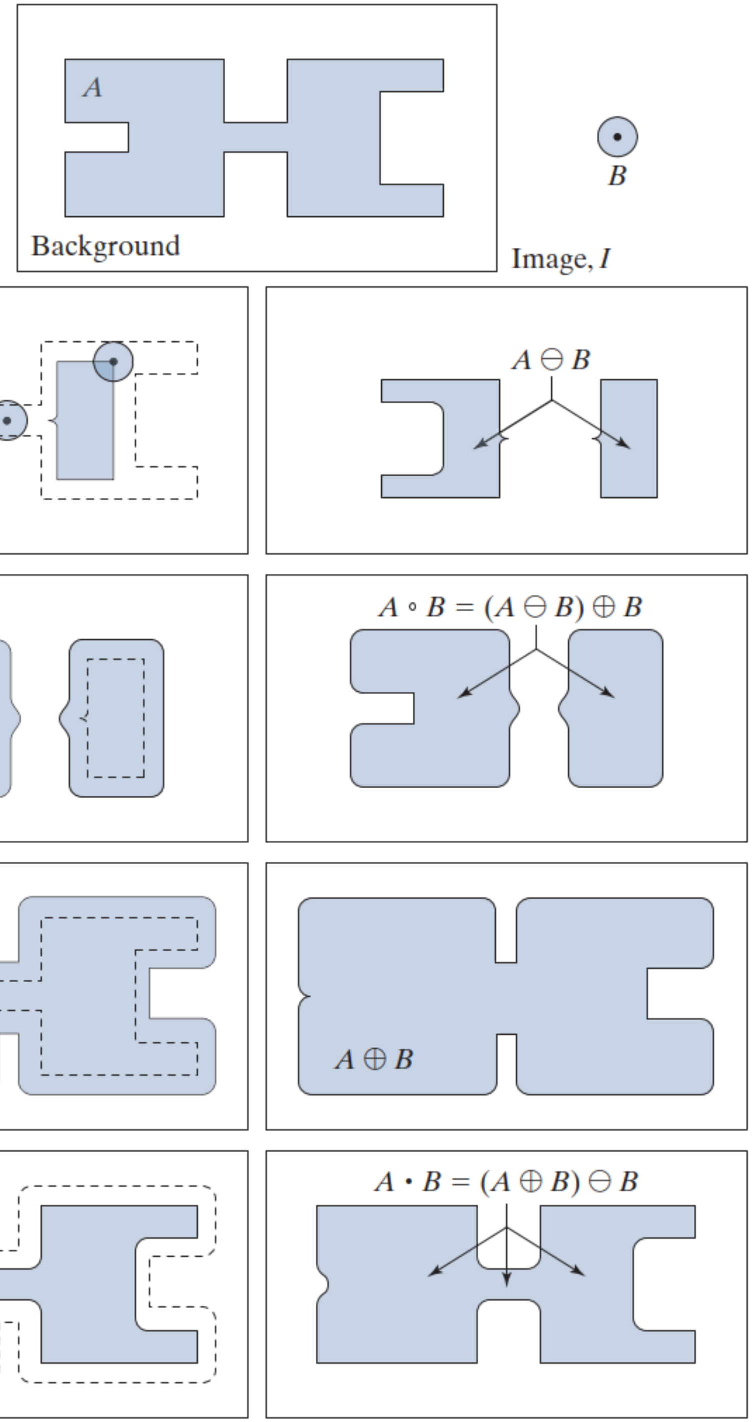


FIGURE 9.10 Morphological opening and closing. (a) Image I, composed of a set (object) A and background; a solid, circular structuring element is shown also. (The dot is the origin.) (b) Structuring element in various positions. (c)-(i) The morphological operations used to obtain the opening and closing.

Hit-or-miss transform (HMT)

$$
\begin{aligned}
I \circledast B_{1,2} &= \left\{ z \middle| \left(B_1\right)_z \subseteq A \text{ and } \left(B_2\right)_z \subseteq A^c \right\} \\
&= \left(A \ominus B_1\right) \cap \left(A^c \ominus B_2\right)
\end{aligned}
$$

a b
c d
e f

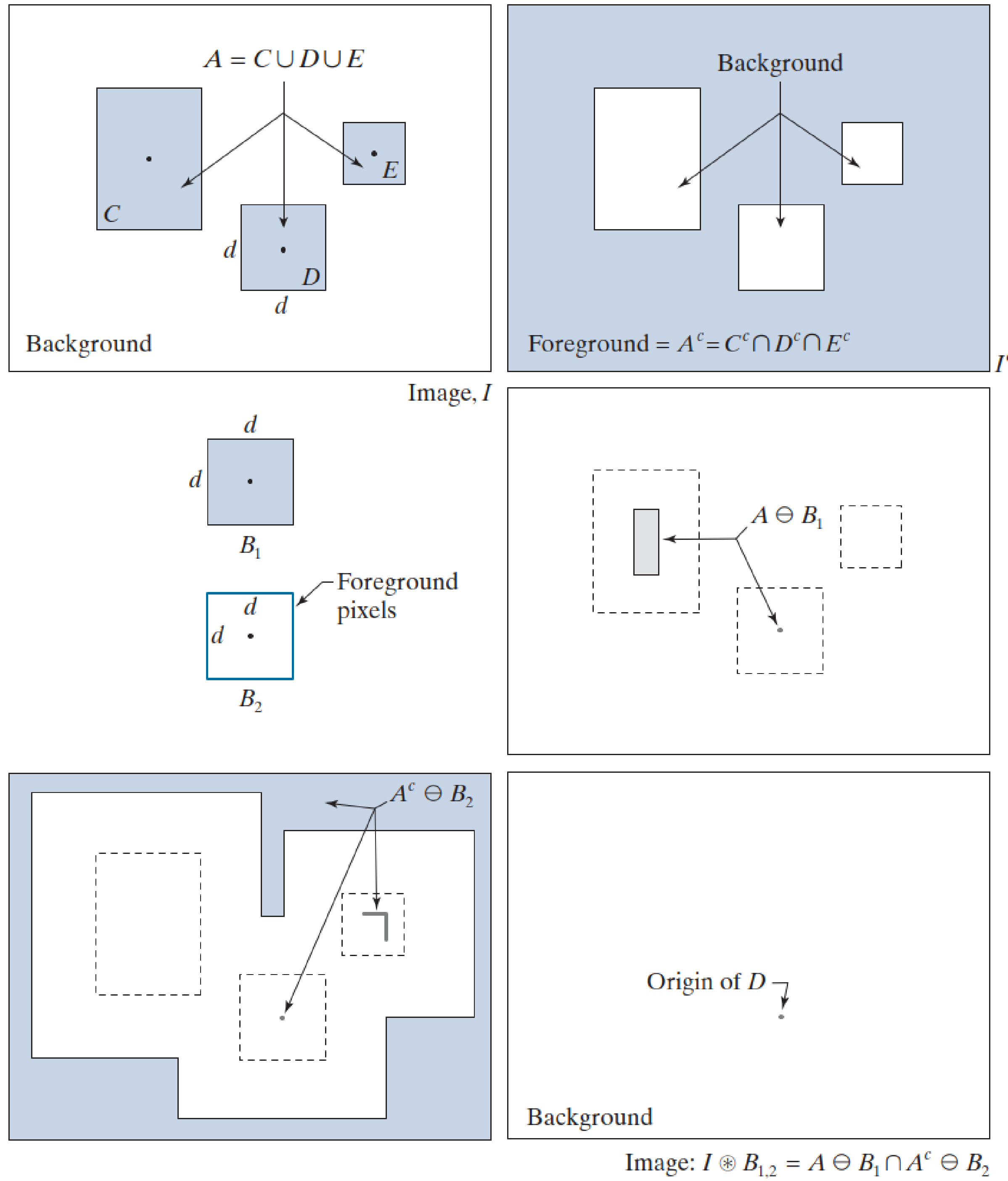


FIGURE 9.12
(a) Image consisting of a foreground (1's) equal to the union, A, of set of objects, and a background of 0's.
(b) Image with its foreground defined as A^c.
(c) Structuring elements designed to detect object D.
(d) Erosion of A by B_1.
(e) Erosion of A^c by B_2.
(f) Intersection of (d) and (e), showing the location of the origin of D, as desired. The dots indicate the origin of their respective components. Each dot is a single pixel.

HMT

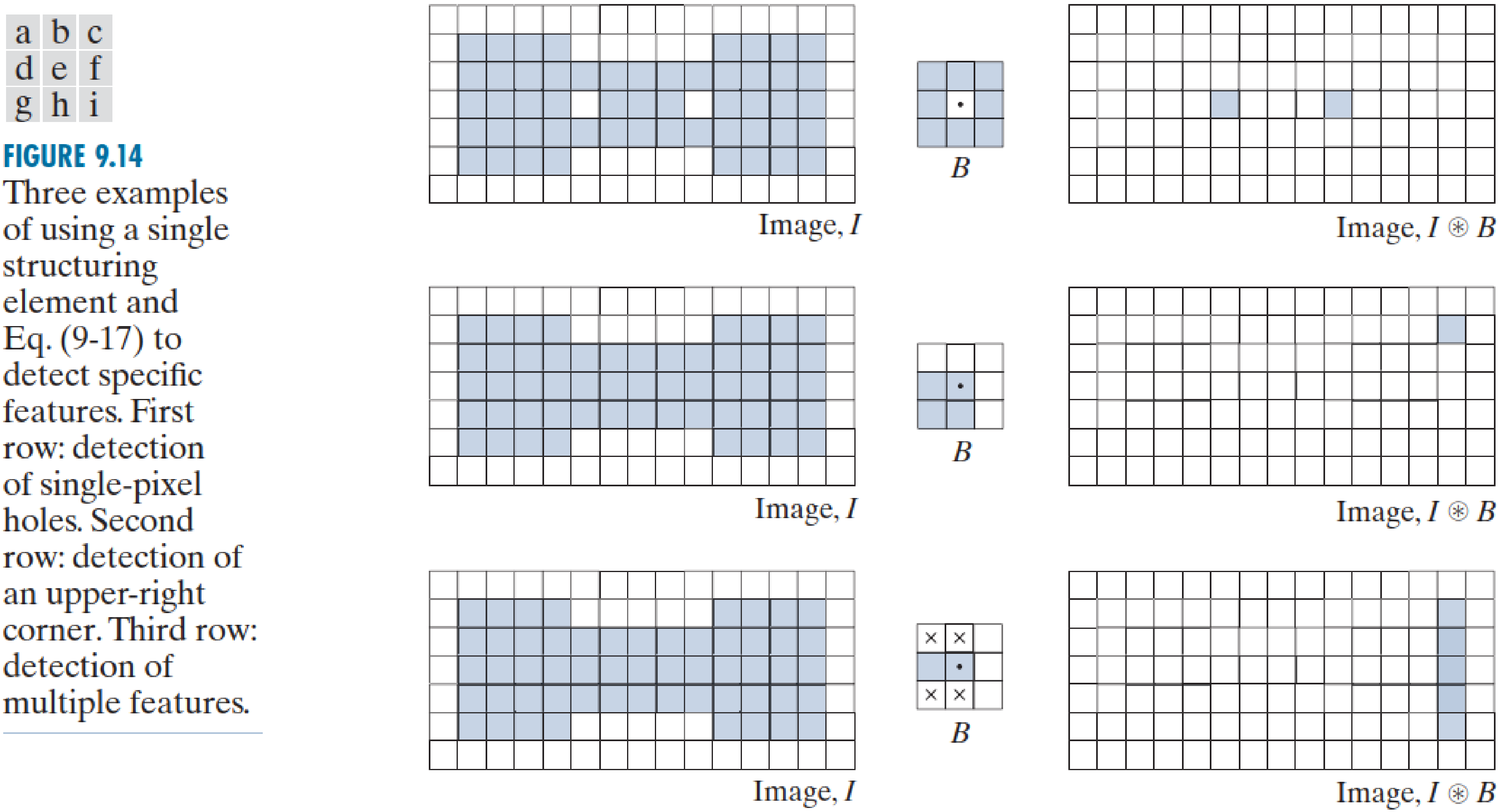


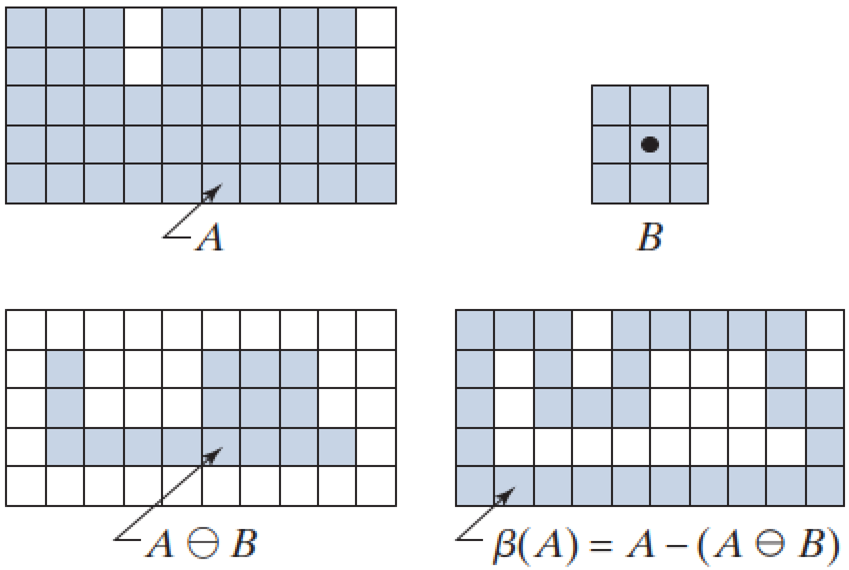
FIGURE 9.14 Three examples of using a single structuring element and Eq. (9-17) to detect specific features. First row: detection of single-pixel holes. Second row: detection of an upper-right corner. Third row: detection of multiple features.

Boundary extraction

$$\beta(A) = A - (A \ominus B)$$

a	b
c	d

FIGURE 9.15
(a) Set, A, of foreground pixels.
(b) Structuring element.
(c) A eroded by B.
(d) Boundary of A.



Effects

- The morphological filter $(A \circ B) \bullet B$ can be used to eliminate ‘salt and pepper’ noise
- Hole filling can be accomplished iteratively using dilations, complementation, and intersections

$$X_k = \left(X_{k-1} \oplus B\right) \cap I^c \qquad k = 1, 2, 3, \ldots$$

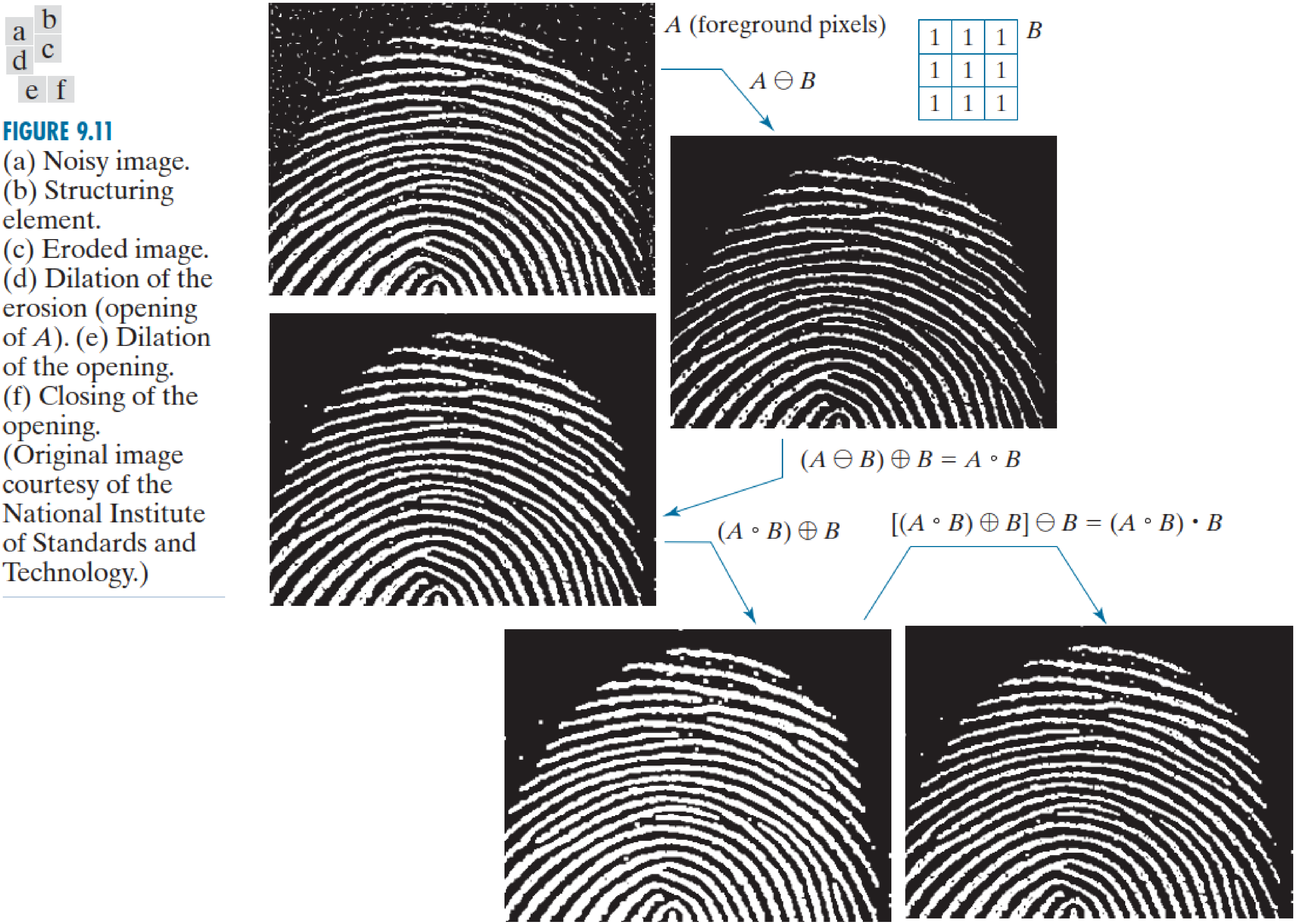


FIGURE 9.11
(a) Noisy image. (b) Structuring element. (c) Eroded image. (d) Dilation of the erosion (opening of A). (e) Dilation of the opening. (f) Closing of the opening. (Original image courtesy of the National Institute of Standards and Technology.)

Hole filling

a	b	c
d	e	f
g	h	i

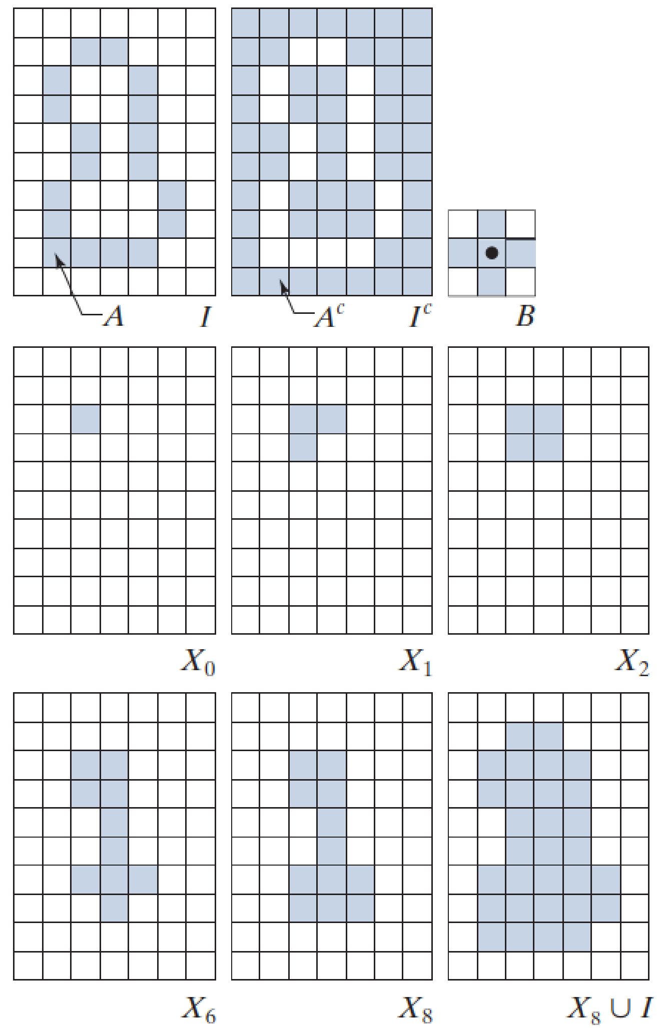


FIGURE 9.17
Hole filling.
(a) Set A (shown shaded) contained in image I.
(b) Complement of I.
(c) Structuring element B. *Only* the foreground elements are used in computations
(d) Initial point inside hole, set to 1.
(e)–(h) Various steps of Eq. (9-19).
(i) Final result [union of (a) and (h)].

Extraction of connected components

$$X_k = \left(X_{k-1} \oplus B\right) \cap I \qquad k = 1, 2, 3, \ldots$$

	a	
b	c	d
e	f	g

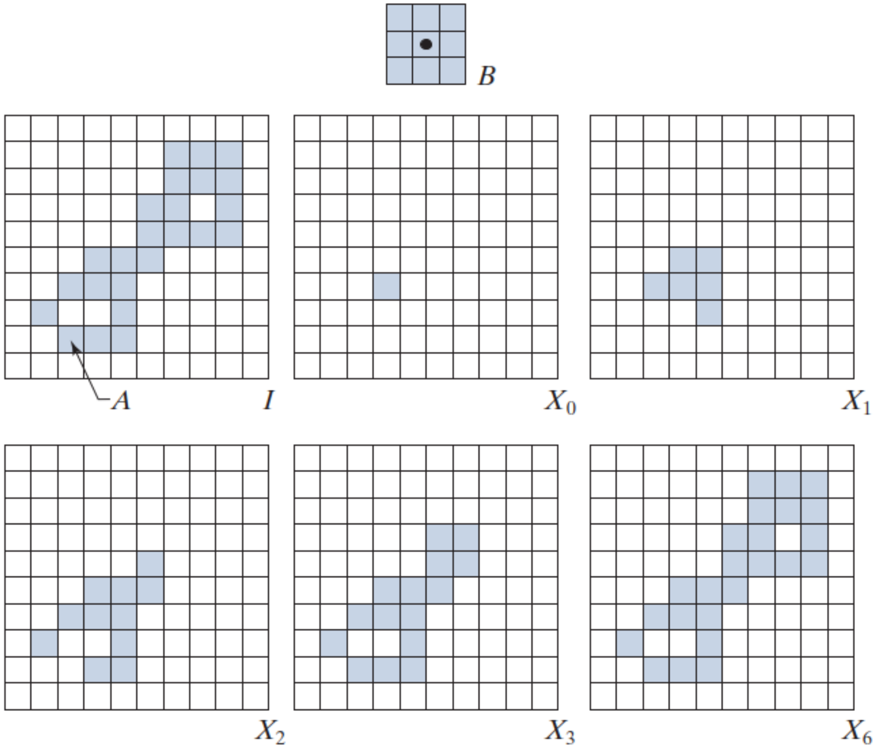


FIGURE 9.19 (a) Structuring element. (b) Image containing a set with one connected component. (c) Initial array containing a 1 in the region of the connected component. (d)–(g) Various steps in the iteration of Eq. (9-20)

Convex hull

$$X_k^i = \left(X_{k-1}^i \circledast B^i\right) \cup X_{k-1}^i \quad i = 1, 2, 3, 4 \quad \text{and} \quad k = 1, 2, 3, \ldots$$

with $X_0^i = I$. When the procedure converges using the ith structuring element (i.e., when $X_k^i = X_{k-1}^i$), we let $D^i = X_k^i$. Then, the convex hull of A is the union of the four results:

$$C(A) = \bigcup_{i=1}^{4} D^i \qquad (9\text{-}22)$$

a
b c d
e f g
h

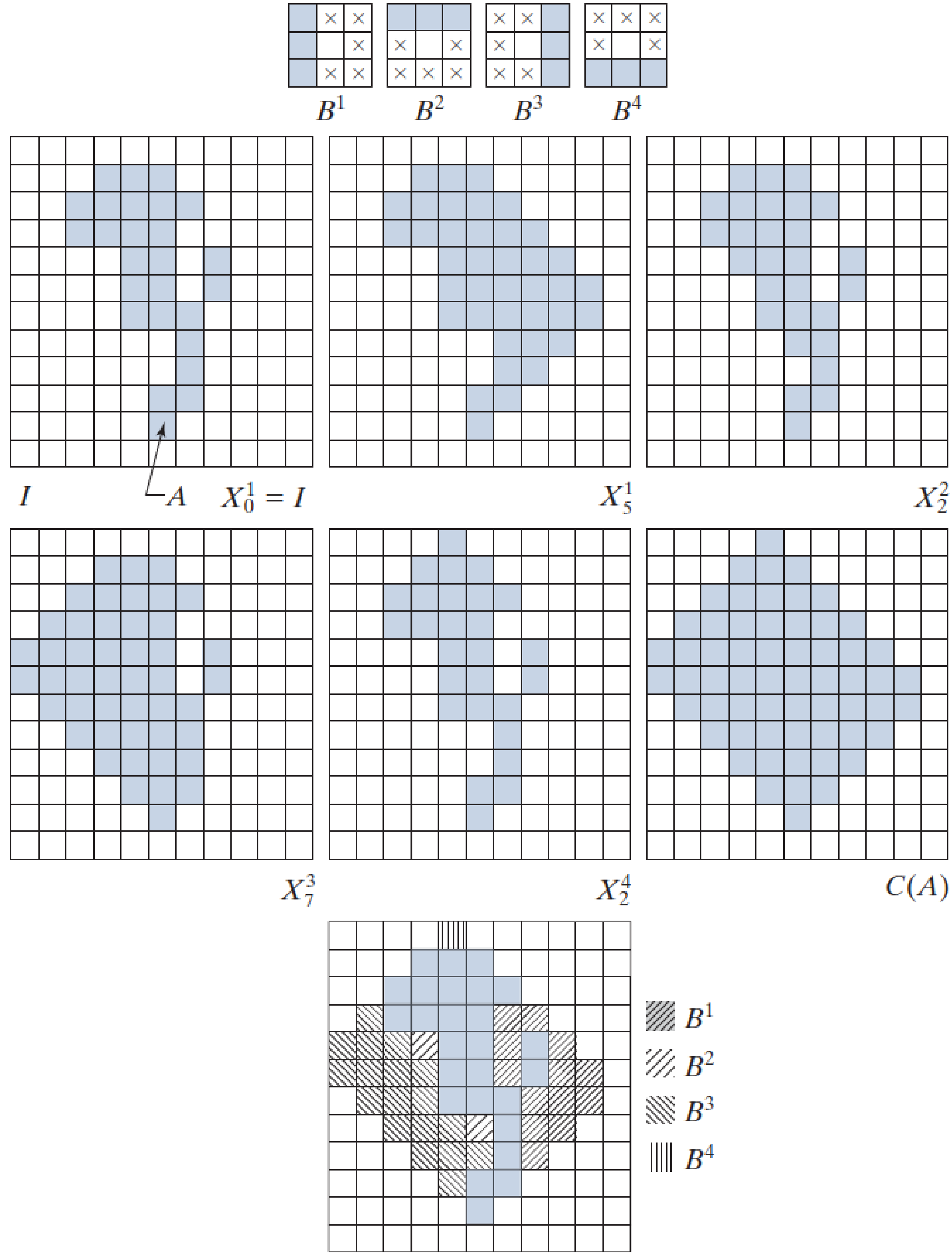


FIGURE 9.21
(a) Structuring elements.
(b) Set A.
(c)–(f) Results of convergence with the structuring elements shown in (a).
(g) Convex hull.
(h) Convex hull showing the contribution of each structuring element.