# CSI 4133 Computer Methods in Picture Processing and Analysis

Fall 2024

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# Image enhancement

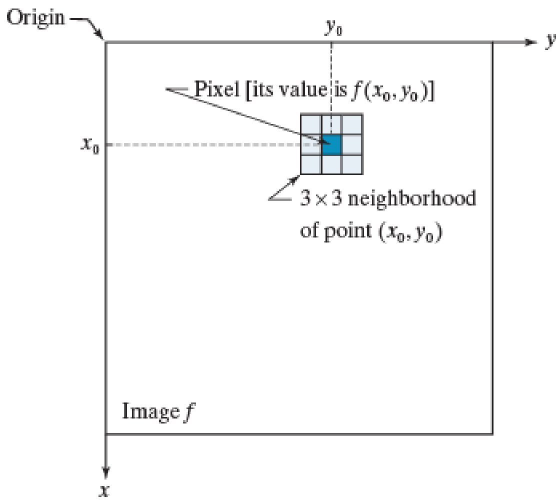
- To process an image so that the result is more suitable than the original image for a specific application
  - Very much problem oriented
  - Different from image restoration
- Two categories
  - **Spatial** domain methods
    - The image plane itself. Direct manipulation of pixels in an image
  - **Frequency** domain
    - Modifying the Fourier transform of an image

# Intensity transformation and spatial filtering

- The spatial domain processes are based on

$$g(x,y) = T\left[f(x,y)\right]$$

**FIGURE 3.1** A $3 \times 3$ neighborhood about a point $(x_0, y_0)$ in an image. The neighborhood is moved from pixel to pixel in the image to generate an output image. Recall from Chapter 2 that the value of a pixel at location $(x_0, y_0)$ is $f(x_0, y_0)$, the value of the image at that location.

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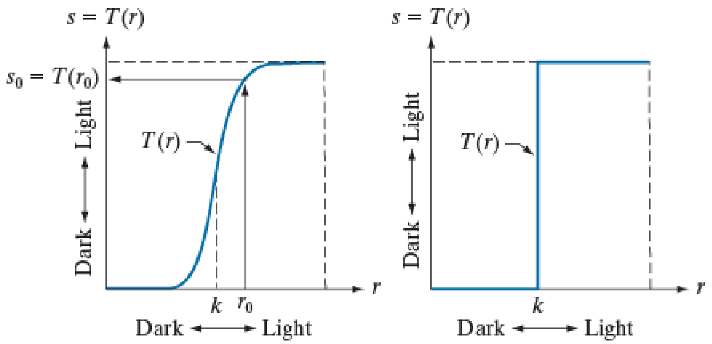
# Intensity transformation and spatial filtering

- At a single point (x,y), it becomes an intensity transformation function $s = T(r)$




a b

**FIGURE 3.2** Intensity transformation functions. (a) Contrast-stretching function. (b) Thresholding function.

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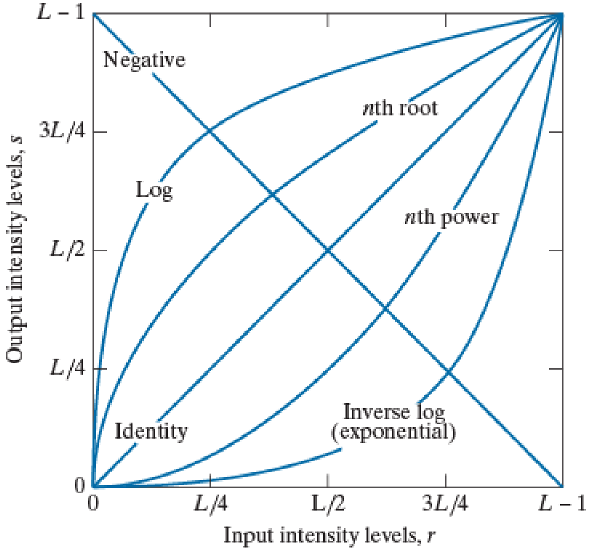
# Intensity transformation functions

**FIGURE 3.3** Some basic intensity transformation functions. Each curve was scaled *independently* so that all curves would fit in the same graph. Our interest here is on the *shapes* of the curves, not on their relative values.

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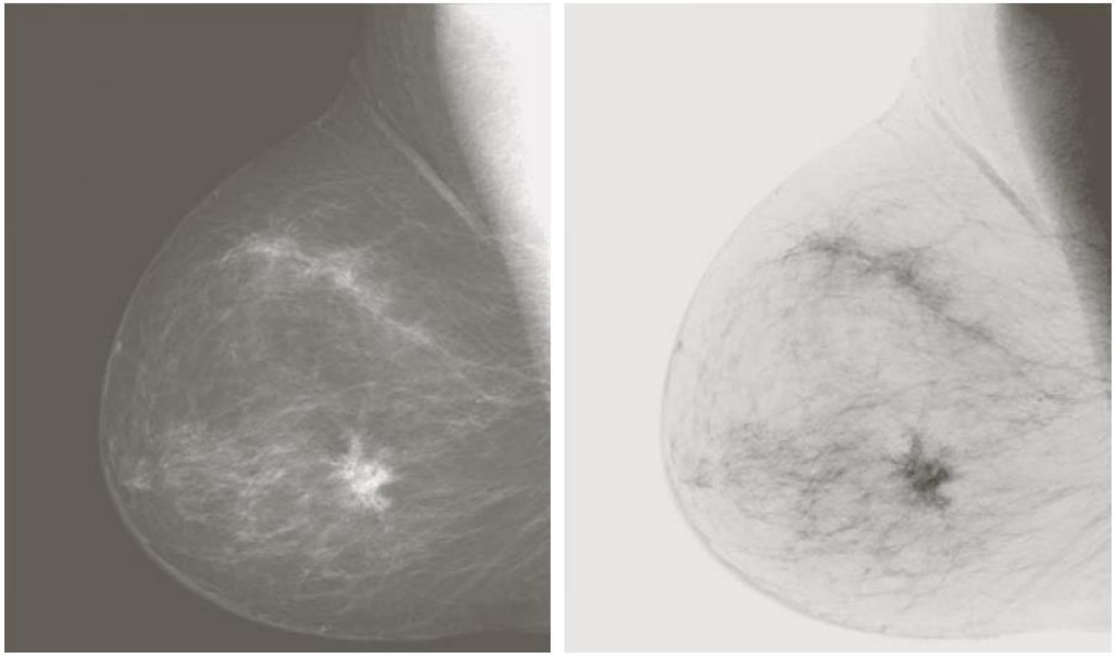
# Gray level transformation

- Image negatives
  - Original image with intensity levels in the range [0, *L*-1]
  - Its negative image is computed: $s = T(r)$ $s = L - 1 - r$



a b

FIGURE 3.4
(a) A digital mammogram. (b) Negative image obtained using Eq. (3-3). (Image (a) Courtesy of General Electric Medical Systems.)

- Digital mammogram showing a small lesion
- Enhancing white or gray detail embedded in dark regions of an image

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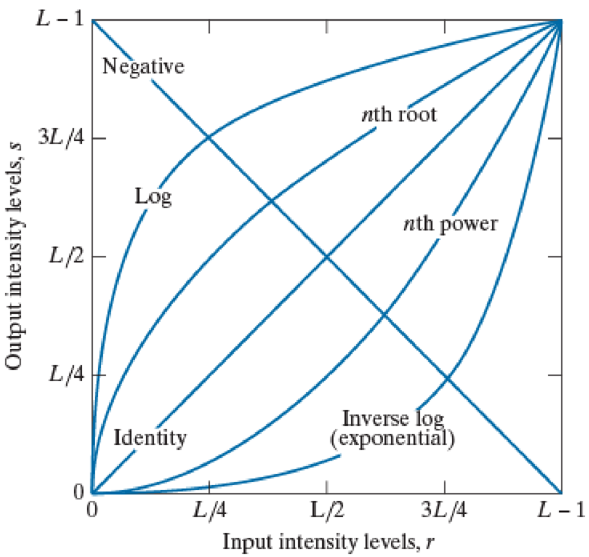
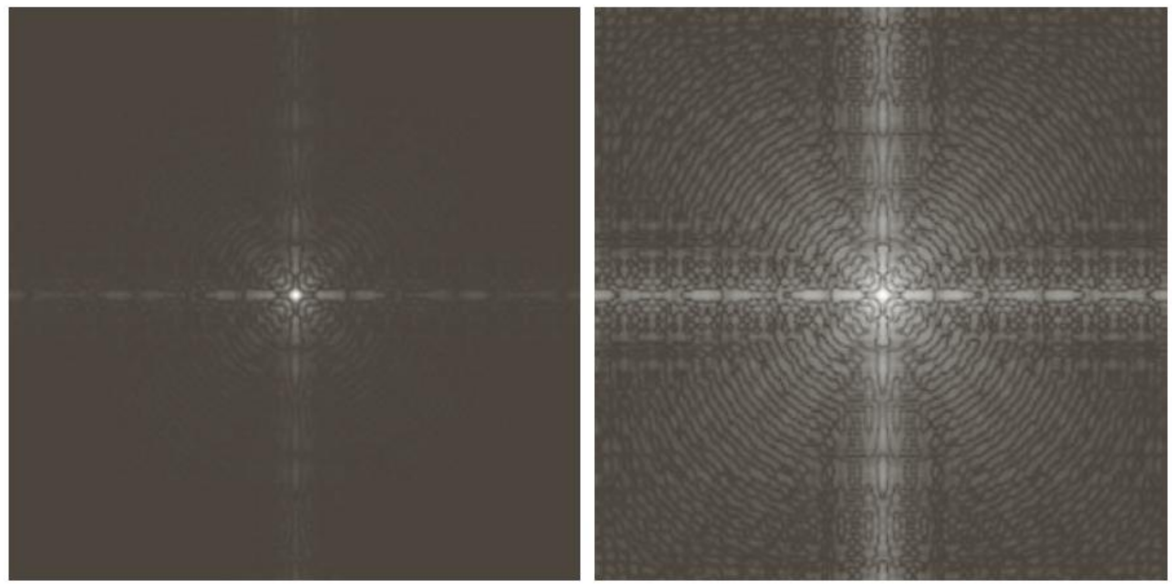
# Log transformations

$$s = T(r) \qquad s = c\log(1 + r)$$

a b

**FIGURE 3.5** (a) Fourier spectrum displayed as a grayscale image. (b) Result of applying the log transformation in Eq. (3-4) with $c = 1$. Both images are scaled to the range [0, 255].



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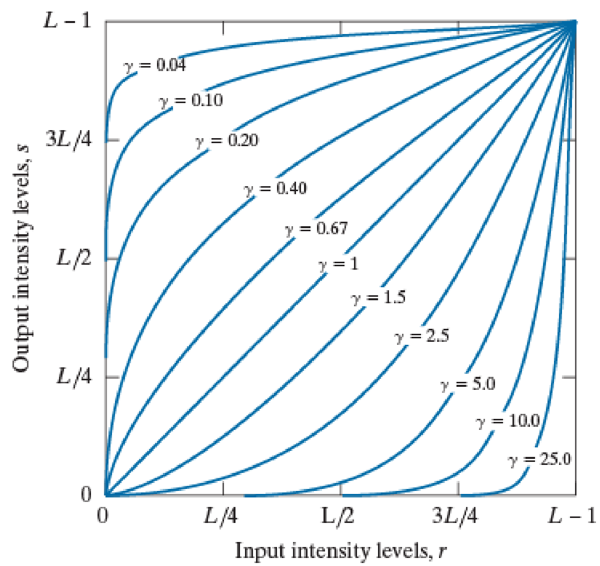
# Power-law (**Gamma**) transformations

$$s = T(r) \qquad s = cr^{\gamma}$$

**FIGURE 3.6** Plots of the gamma equation $s = cr^{\gamma}$ for various values of $\gamma$ ($c = 1$ in all cases). Each curve was scaled *independently* so that all curves would fit in the same graph. Our interest here is on the *shapes* of the curves, not on their relative values.

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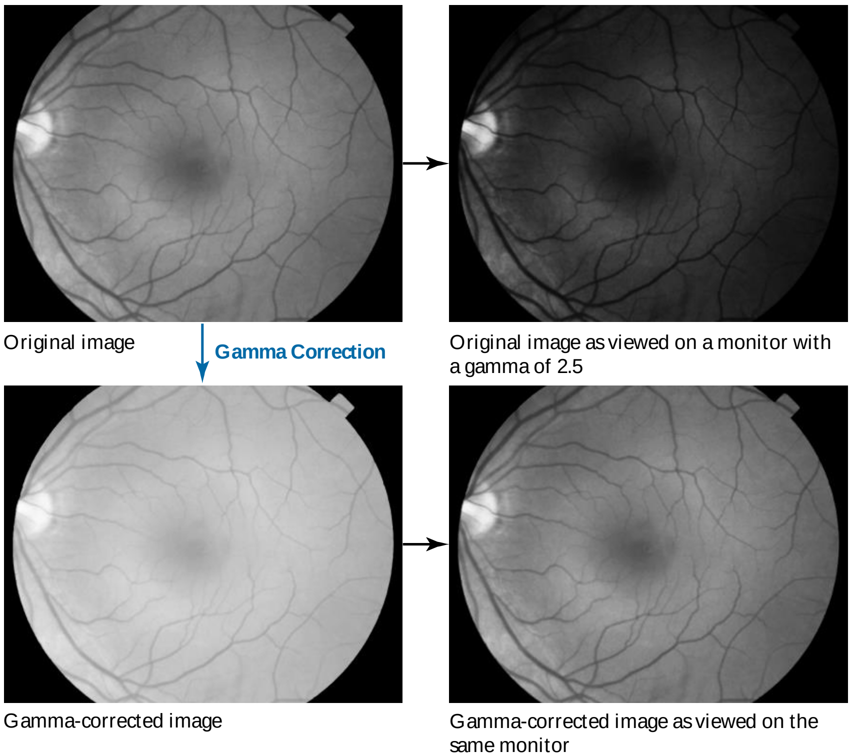
# Gamma correction

- The response of many devices used for image capture, printing, and display obey a power law.
- The exponent in a power-law equation is referred to as **gamma**.
- The process used to correct these power-law response phenomena is called **gamma correction** or gamma encoding.




a b
c d

**FIGURE 3.7** (a) Image of a human retina. (b) Image as as it appears on a monitor with a gamma setting of 2.5 (note the darkness). (c) Gamma-corrected image. (d) Corrected image, as it appears on the same monitor (compare with the original image). (Image (a) courtesy of the National Eye Institute, NIH)

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# Contrast enhancement using power-law intensity transformation

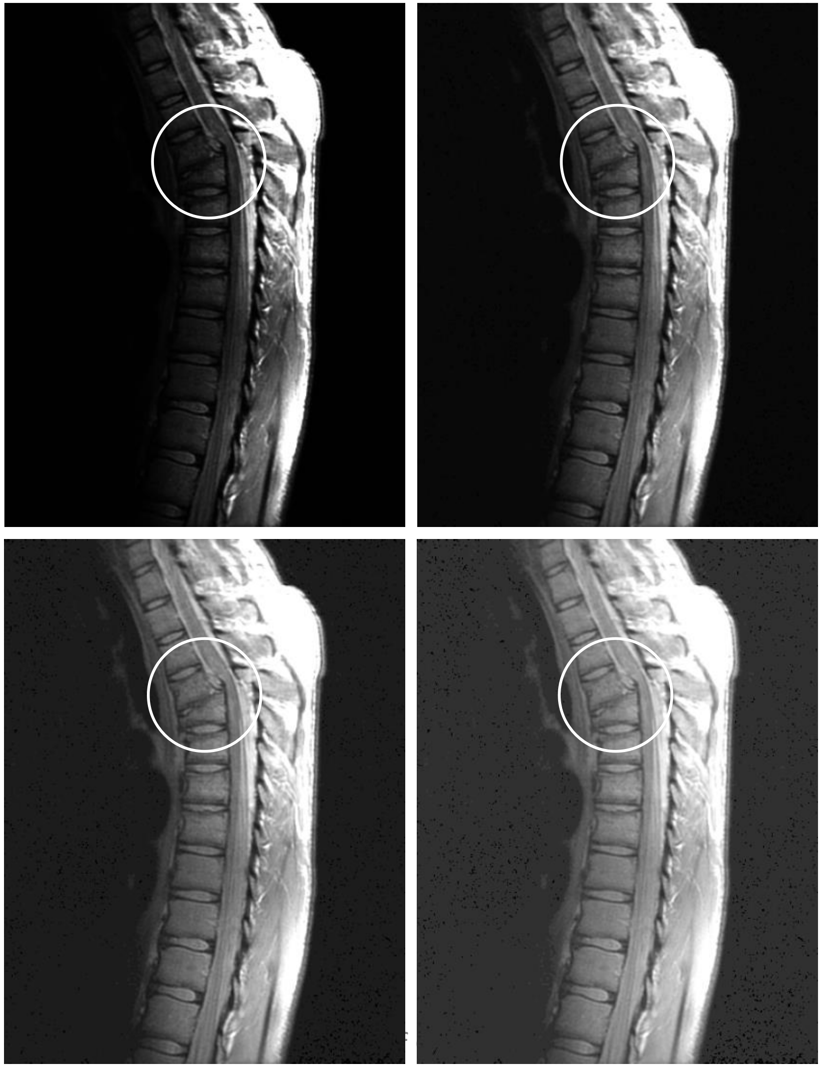


**FIGURE 3.8** (a) Magnetic resonance image (MRI) of a fractured human spine (the region of the fracture is enclosed by the circle). (b)–(d) Results of applying the transformation in Eq. (3-5) with $c = 1$ and $\gamma = 0.6, 0.4,$ and $0.3$, respectively. (Original image courtesy of Dr. David R. Pickens, Department of Radiology and Radiological Sciences, Vanderbilt University Medical Center.)

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# Contrast enhancement using power-law intensity transformation


**FIGURE 3.9** (a) Aerial image. (b)–(d) Results of applying the transformation in Eq. (3-5) with $\gamma = 3.0, 4.0,$ and $5.0$, respectively. ($c = 1$ in all cases.) (Original image courtesy of NASA.)

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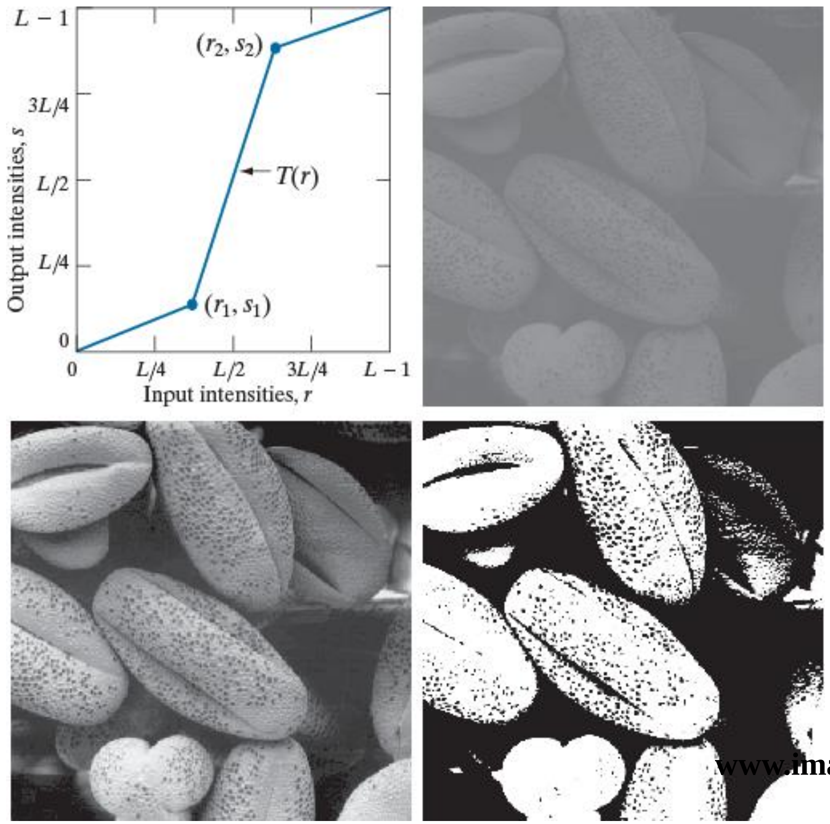
# Contrast stretching

- Piecewise linear transformation



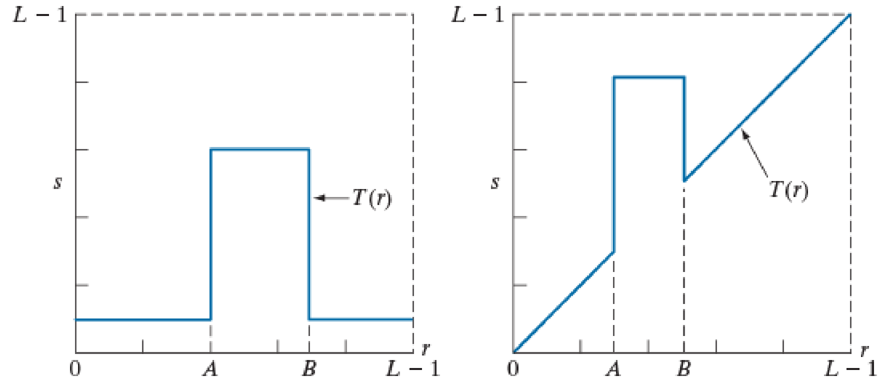

a b
c d

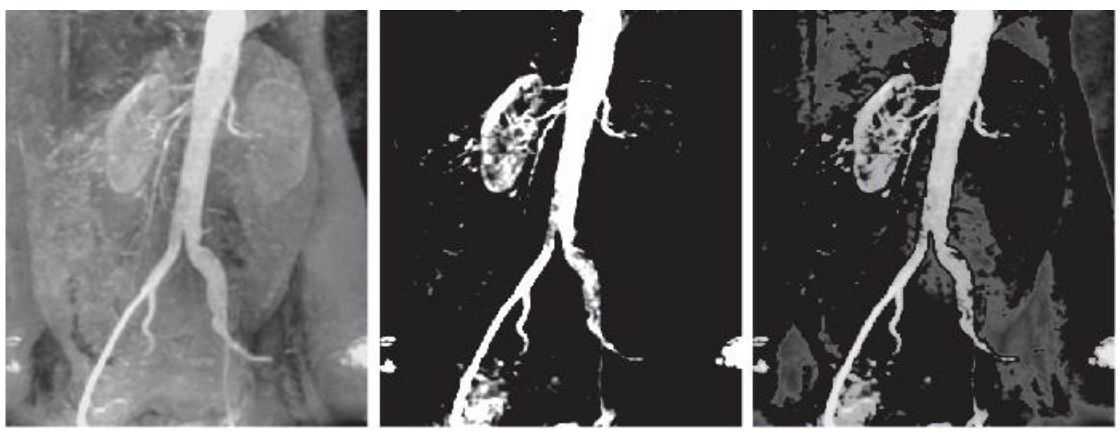
**FIGURE 3.10** Contrast stretching. (a) Piecewise linear transformation function. (b) A low-contrast electron microscope image of pollen, magnified 700 times. (c) Result of contrast stretching. (d) Result of thresholding. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)

# Gray level slicing

a b

**FIGURE 3.11** (a) This transformation function highlights range $[A, B]$ and reduces all other intensities to a lower level. (b) This function highlights range $[A, B]$ and leaves other intensities unchanged.



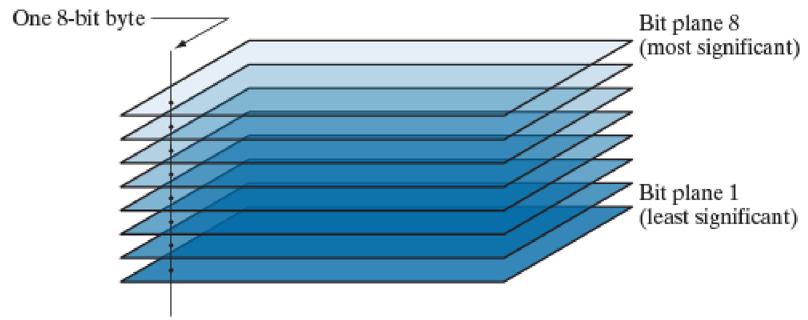
a b c

**FIGURE 3.12** (a) Aortic angiogram. (b) Result of using a slicing transformation of the type illustrated in Fig. 3.11(a), with the range of intensities of interest selected in the upper end of the gray scale. (c) Result of using the transformation in Fig. 3.11(b), with the selected range set near black, so that the grays in the area of the blood vessels and kidneys were preserved. (Original image courtesy of Dr. Thomas R. Gest, University of Michigan Medical School.)

# Bit-plane slicing

- Eight 1-bit planes
  - Ranging from bit-plane 1 for the least significant bit to bit-plane 8 for the most significant bit

**FIGURE 3.13** Bit-planes of an 8-bit image.

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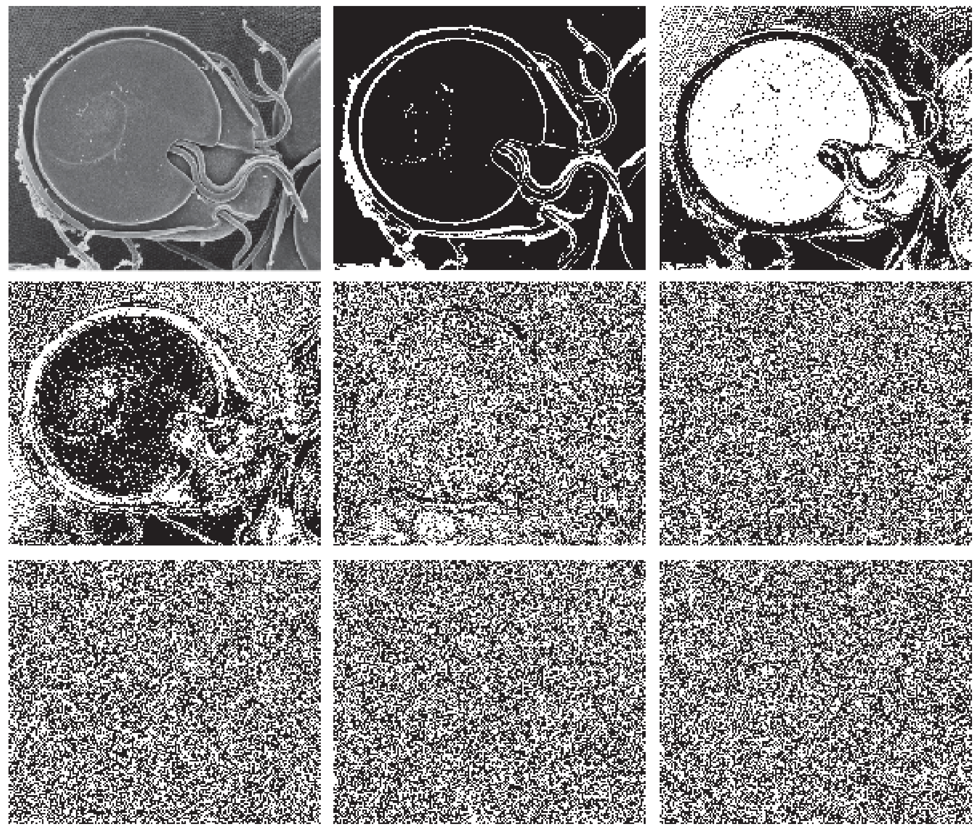
# Bit-plane slicing

a b c
d e f
g h i

**FIGURE 3.14**
(a) An 8-bit gray-scale image of size $837 \times 988$ pixels. (b) through (i) Bit planes 8 through 1, respectively, where plane 1 contains the least significant bit. Each bit plane is a binary image. Figure (a) is an SEM image of a trophozoite that causes a disease called *giardiasis*. (Courtesy of Dr. Stan Erlandsen, U.S. Center for Disease Control and Prevention.)



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# Image reconstruction

a b c

**FIGURE 3.15** Image reconstructed from bit planes: (a) 8 and 7; (b) 8, 7, and 6; (c) 8, 7, 6, and 5.

# Histogram processing

Let $r_k$, for $k = 0,1,2,\ldots,L-1$, denote the intensities of an $L$-level digital image, $f(x,y)$. The *unnormalized histogram* of $f$ is defined as

$$h(r_k) = n_k \quad \text{for } k = 0,1,2,\ldots,L-1 \tag{3-6}$$

where $n_k$ is the number of pixels in $f$ with intensity $r_k$, and the subdivisions of the intensity scale are called *histogram bins*. Similarly, the *normalized histogram* of $f$ is defined as

$$p(r_k) = \frac{h(r_k)}{MN} = \frac{n_k}{MN} \tag{3-7}$$

where, as usual, $M$ and $N$ are the number of image rows and columns, respectively.

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# Image transformation – continuous case

- The intensity of an image may be viewed as a random variable in the interval [0, *L*-1]
- Let $p_r(r)$and $p_s(s)$denote the PDFs (probability density function) of intensity values r and s in two different images
- A transformation function of particular importance:
  - $s = T(r) = (L-1)\int_0^r p_r(w)dw$

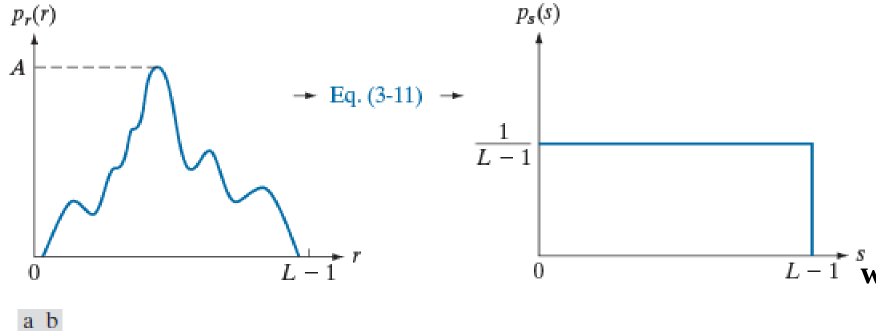


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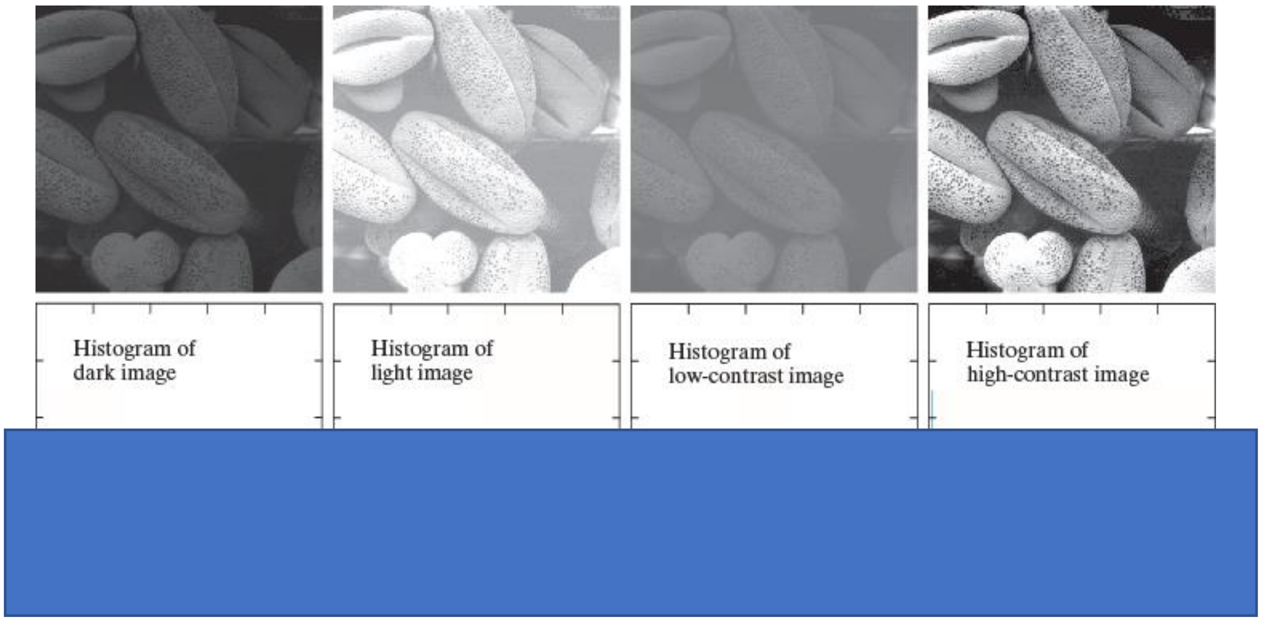
**FIGURE 3.18** (a) An arbitrary PDF. (b) Result of applying Eq. (3-11) to the input PDF. The resulting PDF is always uniform, independently of the shape of the input.

# Histograms




a b c d

**FIGURE 3.16** Four image types and their corresponding histograms. (a) dark; (b) light; (c) low contrast; (d) high contrast. The horizontal axis of the histograms are values of $r_k$ and the vertical axis are values of $p(r_k)$.

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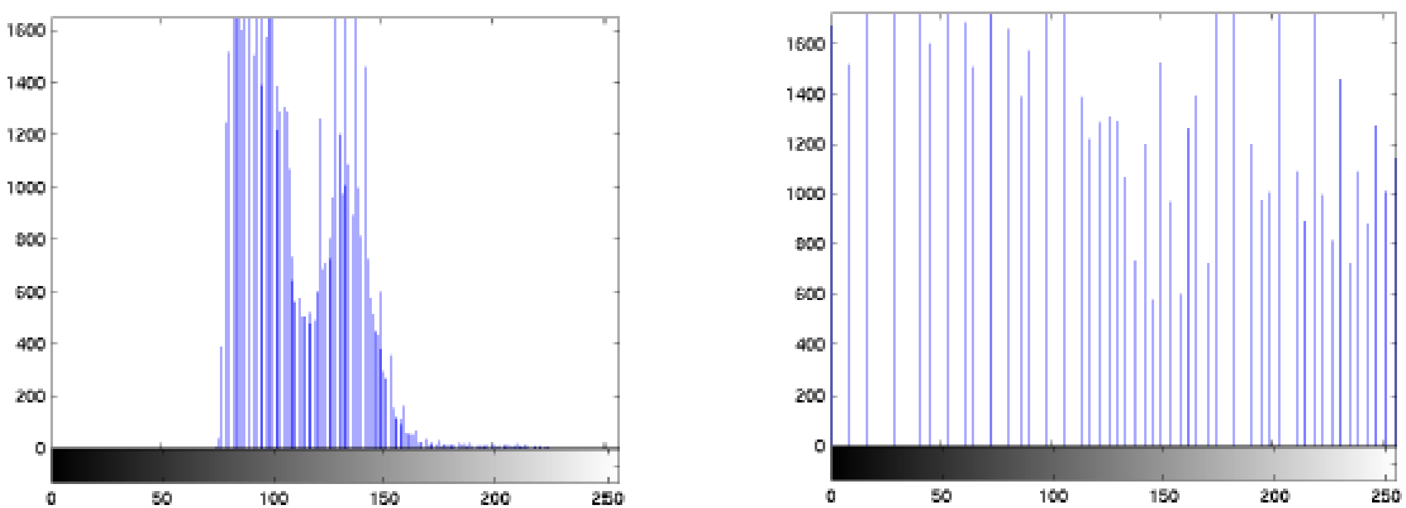
( From Matlab Image Toolbox Guide Fig.10-10 & 10-11 )

# Histogram equalization

- Example



# Image enhancement using the histogram equalization

- **Goal**: map the luminance of each pixel to a new value such that the output image has approximately uniform distribution of gray levels
- By histogram equalization
  - To transform the image intensities in order to obtain a flat histogram

# How to do histogram equalization

- Compute the **probability of occurrence of intensity level** $r_k$ in a digital image by

$$p_r(r_k) = \frac{n_k}{MN}$$

  - Where MN is the number of pixels in the image, $n_k$ is the number of pixels that have intensity $r_k$
- The discrete form of the transformation is:

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j) \qquad k = 0, 1, 2, \ldots, L-1$$

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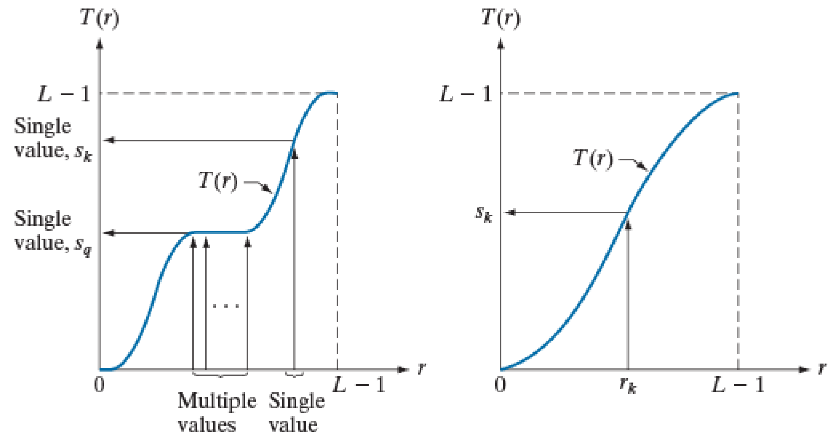
# Monotonic increasing function

a b

**FIGURE 3.17**
(a) Monotonic increasing function, showing how multiple values can map to a single value. (b) Strictly monotonic increasing function. This is a one-to-one mapping, both ways.




Is the transformation function T in the previous slide strictly monotonic?

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# Example of histogram equalization

- A 3-bit image ($L$=8) of size 64x64 pixels ($MN$ = 4096)

**TABLE 3.1** Intensity distribution and histogram values for a 3-bit, $64 \times 64$ digital image.

| $r_k$ | $n_k$ | $p_r(r_k) = n_k/MN$ |
|---|---|---|
| $r_0 = 0$ | 790 | 0.19 |
| $r_1 = 1$ | 1023 | 0.25 |
| $r_2 = 2$ | 850 | 0.21 |
| $r_3 = 3$ | 656 | 0.16 |
| $r_4 = 4$ | 329 | 0.08 |
| $r_5 = 5$ | 245 | 0.06 |
| $r_6 = 6$ | 122 | 0.03 |
| $r_7 = 7$ | 81 | 0.02 |

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# Example of histogram equalization

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| $r_5 = 5$ | 245 | 0.06 |
| $r_6 = 6$ | 122 | 0.03 |
| $r_7 = 7$ | 81 | 0.02 |

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j) \qquad k = 0,1,2,\ldots,L-1$$

$$s_0 = T(r_0) = 7\sum_{j=0}^{0} p_r(r_j) = 7p_r(r_0) = 1.33$$

$$s_1 = T(r_1) = 3.08,\ s_2 = 4.55,\ s_3 = 5.67,\ s_4 = 6.23,\ s_5 = 6.65,\ s_6 = 6.86,\ \text{and}\ s_7 = 7.00.$$

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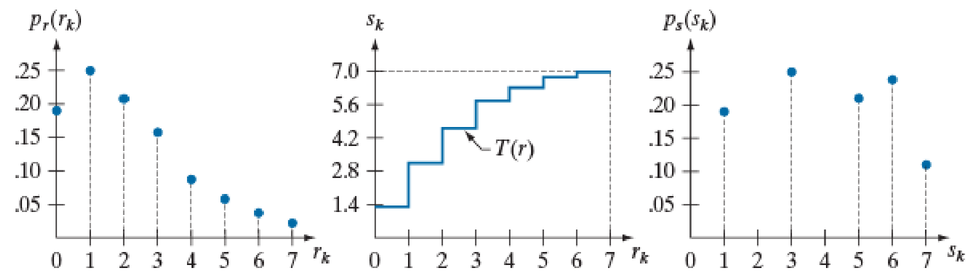
# Example of histogram equalization

- Round values to their nearest integer values in the range of [0,7]

$$s_0 = 1.33 \rightarrow 1 \quad s_2 = 4.55 \rightarrow 5 \quad s_4 = 6.23 \rightarrow 6 \quad s_6 = 6.86 \rightarrow 7$$
$$s_1 = 3.08 \rightarrow 3 \quad s_3 = 5.67 \rightarrow 6 \quad s_5 = 6.65 \rightarrow 7 \quad s_7 = 7.00 \rightarrow 7$$

a b c

**FIGURE 3.19** Histogram equalization. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.




Monotonic increasing

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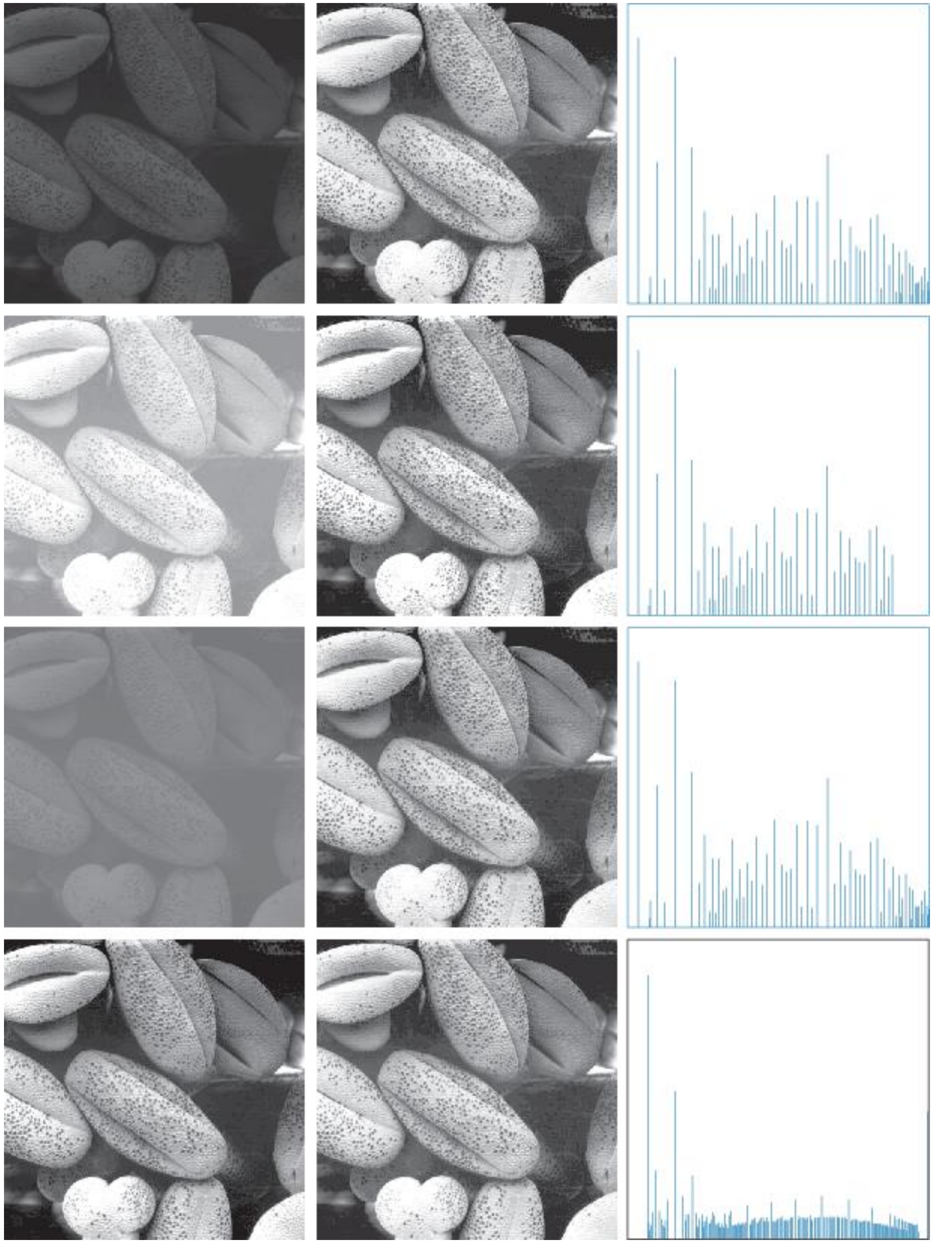
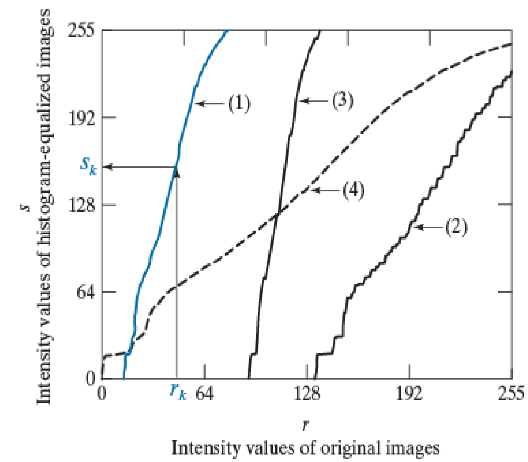


**FIGURE 3.20** Left column: Images from Fig. 3.16. Center column: Corresponding histogram-equalized images. Right column: histograms of the images in the center column (compare with the histograms in Fig. 3.16).

**FIGURE 3.21** Transformation functions for histogram equalization. Transformations (1) through (4) were obtained using Eq. (3-15) and the histograms of the images on the left column of Fig. 3.20. Mapping of one intensity value $r_k$ in image 1 to its corresponding value $s_k$ is shown.

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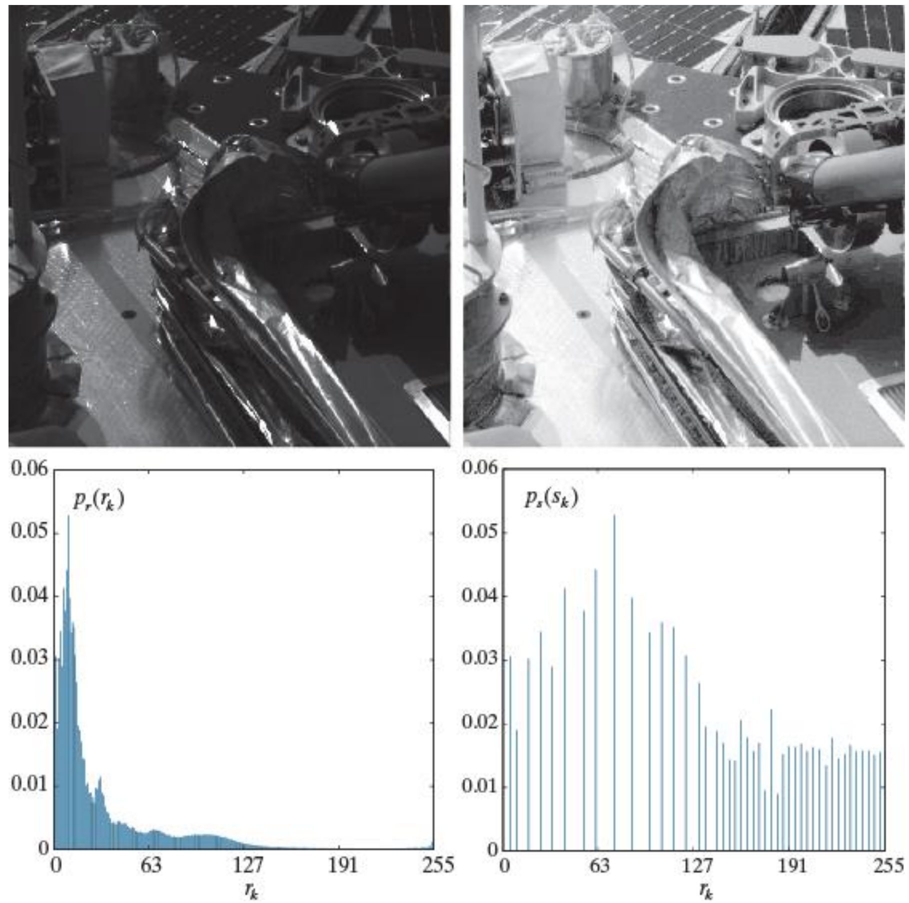


a b
c d

**FIGURE 3.22**
(a) Image from Phoenix Lander. (b) Result of histogram equalization. (c) Histogram of image (a). (d) Histogram of image (b). (Original image courtesy of NASA.)

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# Generalization of histogram equalization

- Histogram matching (**specification**)
  - Useful to be able to specify the shape of the histogram that we wish the processed image to have

# Generalization of histogram equalization

- Steps to achieve histogram specification:
  - Given an input image, we compute histogram equalization transformation:

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j) \qquad k = 0,1,2,\ldots,L-1$$

  - Given a specified histogram of the output image, compute the transformation function

$$G(z_q) = (L-1)\sum_{i=0}^{q} p_z(z_i)$$

    So that $G(z_q) = s_k$

  - Obtain the desired value from the inverse transformation:

$$z_q = G^{-1}(s_k)$$

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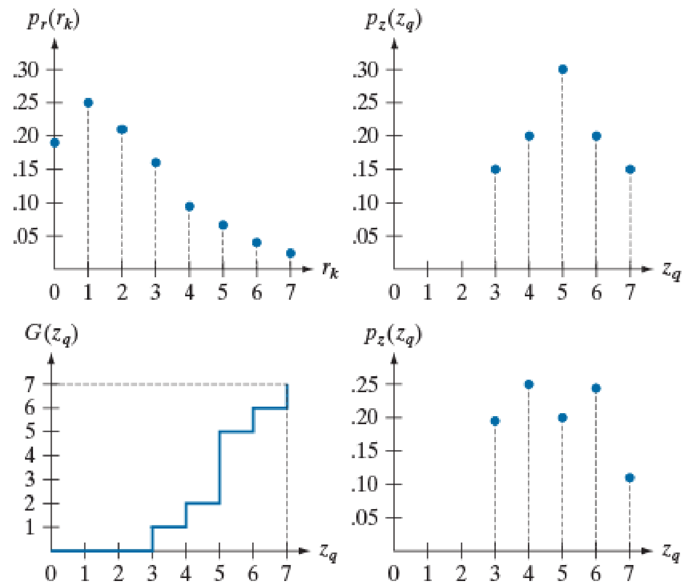


a b
c d

**FIGURE 3.23**
(a) Histogram of a 3-bit image.
(b) Specified histogram.
(c) Transformation function obtained from the specified histogram.
(d) Result of histogram specification. Compare the histograms in (b) and (d).

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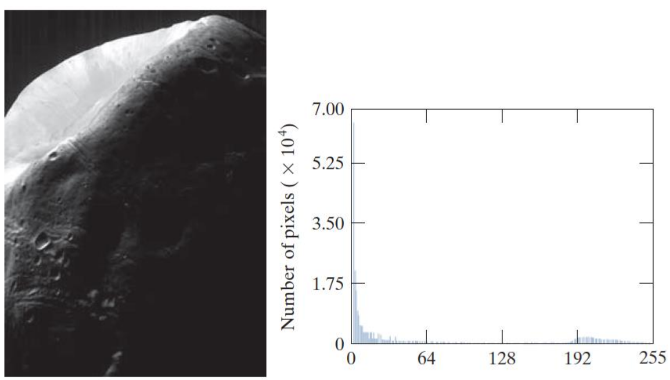
# Histogram equalization

- Equalization not always working, need specification

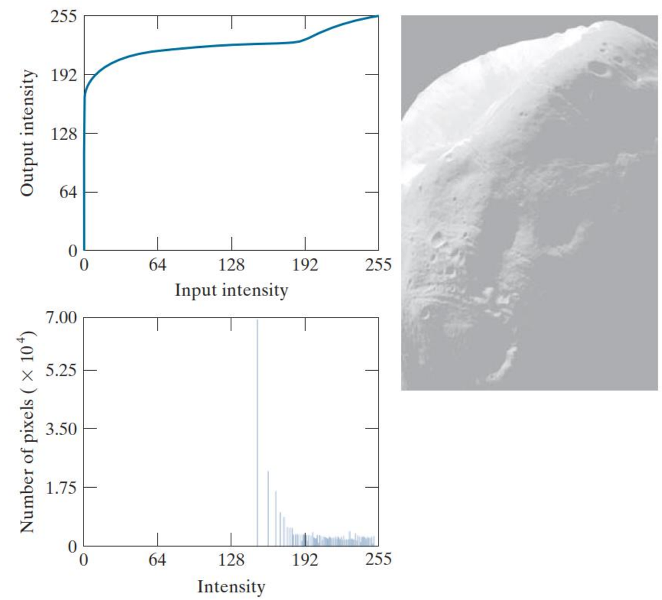
a b

**FIGURE 3.23**
(a) An image, and
(b) its histogram.




a b
c

**FIGURE 3.24**
(a) Histogram equalization transformation obtained using the histogram in Fig. 3.23(b).
(b) Histogram equalized image.
(c) Histogram of equalized image.




Reason: large peak near black in the histogram of the image
A reasonable approach is to modify the histogram of that image so that it does not have that property

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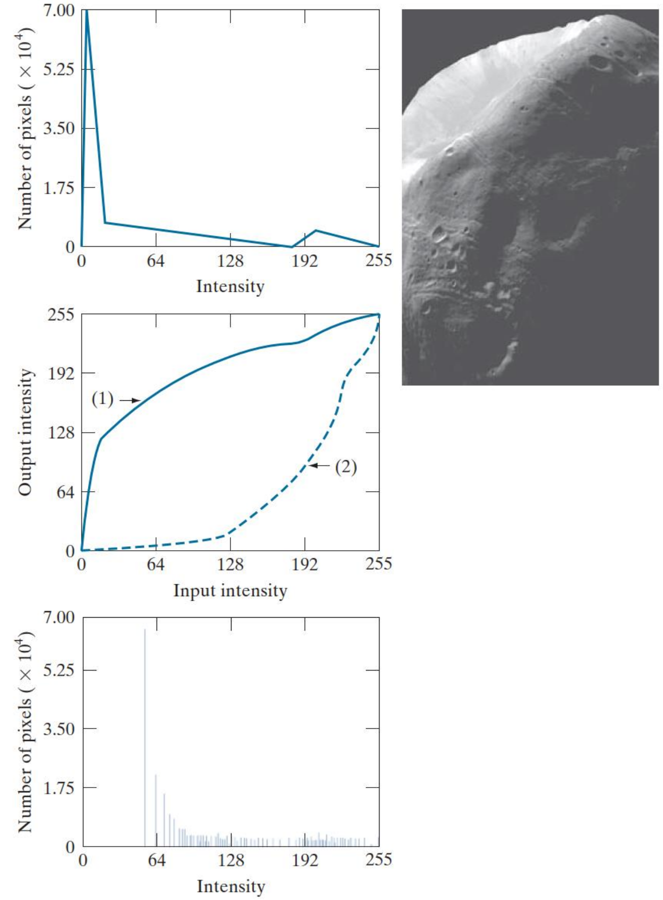
# Specification

- *Modification*: preserves the general shape of the original histogram, but has a smoother transition of levels in the dark region of the intensity scale
- A rather modest change in the original histogram can obtain a significant improvement in appearance




a c
b
d

**FIGURE 3.25** Histogram specification. (a) Specified histogram. (b) Transformation $G(z_q)$, labeled (1), and $G^{-1}(s_k)$, labeled (2). (c) Result of histogram specification. (d) Histogram of image (c).

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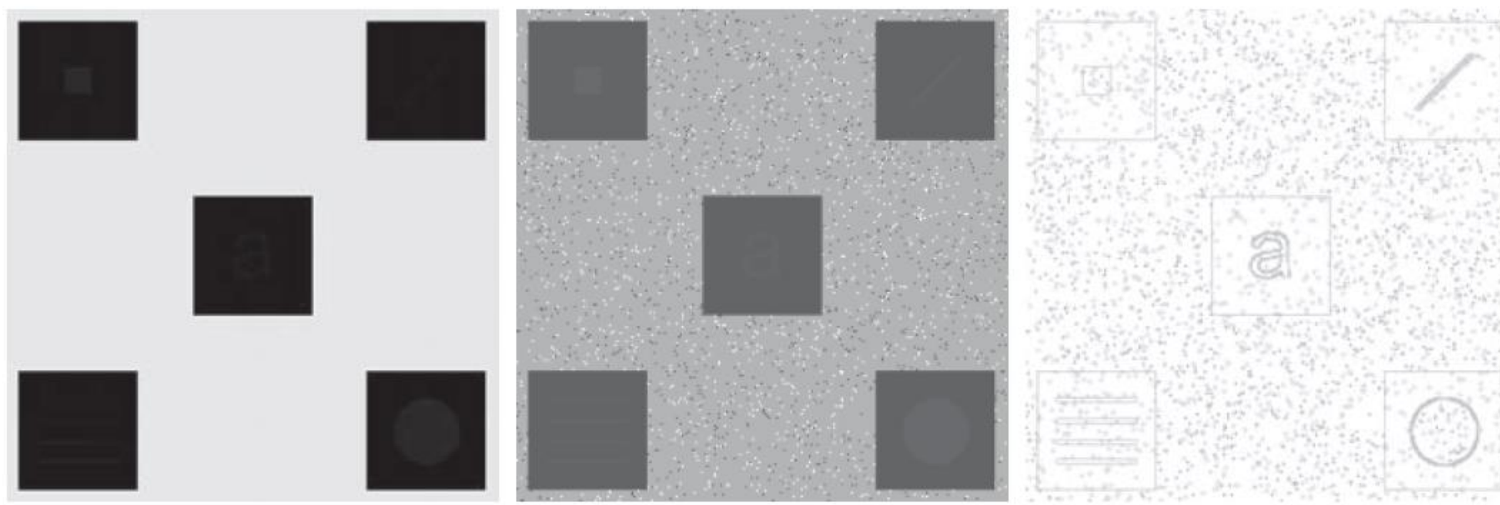
# Local enhancement

- To define a **square** or **rectangular** neighborhood and move the center of this area from pixel to pixel
- At each location, the histogram of the points in the neighborhood is computed and either a histogram equalization or histogram specification is obtained

# Local histogram equalization



a b c

**FIGURE 3.26** (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization.

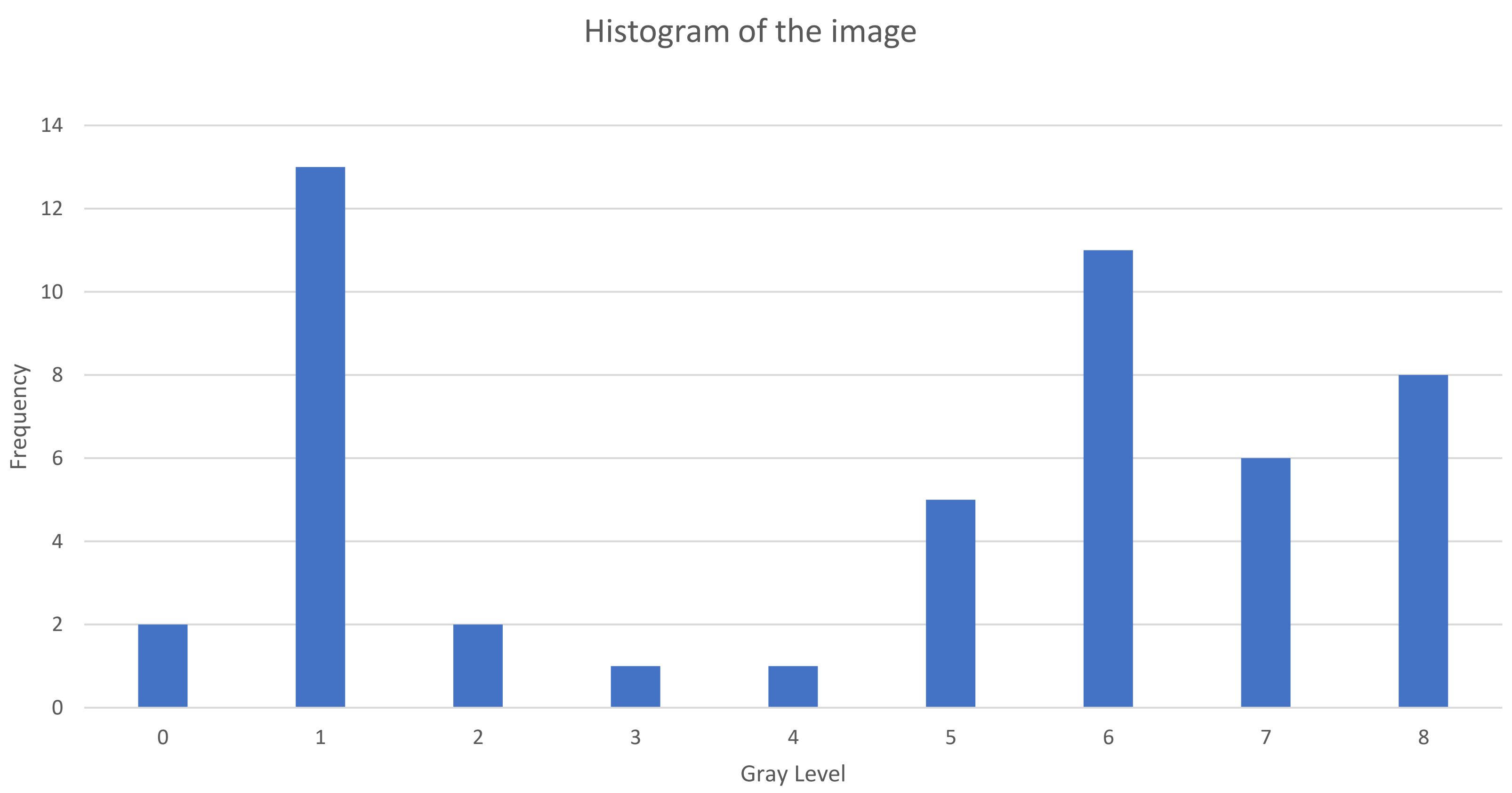
# Histogram

- [question] Consider the following image with 9 gray values

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 8 | 5 | 5 | 5 | 7 | 8 |
| 4 | 7 | 8 | 3 | 2 | 2 | 8 |
| 1 | 7 | 0 | 5 | 6 | 6 | 6 |
| 7 | 1 | 1 | 5 | 6 | 8 | 6 |
| 7 | 1 | 1 | 7 | 1 | 8 | 6 |
| 8 | 1 | 1 | 1 | 6 | 6 | 6 |
| 8 | 1 | 1 | 1 | 1 | 6 | 6 |

- Draw the histogram of this image
- Equalize the histogram of this image and draw the resulting histogram

# Histogram


Histogram of the image
Frequency
14
12
10
8
6
4
2
0
0
1
2
3
4
5
6
7
8
Gray Level

| gray value r_k | histogram = # of pixels | probability of occurrence | transformed value s_k | round of s_k | new histogram |
|---|---|---|---|---|---|
| 0 | 2 | 0.04081633 | 0.326530612 | 0 | 2 |
| 1 | 13 | 0.26530612 | 2.448979592 | 2 | 0 |
| 2 | 2 | 0.04081633 | 2.775510204 | 3 | 13 |
| 3 | 1 | 0.02040816 | 2.93877551 | 3 | 4 |
| 4 | 1 | 0.02040816 | 3.102040816 | 3 | 5 |
| 5 | 5 | 0.10204082 | 3.918367347 | 4 | 0 |
| 6 | 11 | 0.2244898 | 5.714285714 | 6 | 11 |
| 7 | 6 | 0.12244898 | 6.693877551 | 7 | 6 |
| 8 | 8 | 0.16326531 | 8 | 8 | 8 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 8 | 5 | 5 | 5 | 7 | 8 |
| 4 | 7 | 8 | 3 | 2 | 2 | 8 |
| 1 | 7 | 0 | 5 | 6 | 6 | 6 |
| 7 | 1 | 1 | 5 | 6 | 8 | 6 |
| 7 | 1 | 1 | 7 | 1 | 8 | 6 |
| 8 | 1 | 1 | 1 | 6 | 6 | 6 |
| 8 | 1 | 1 | 1 | 1 | 6 | 6 |

# Histogram of new image