First Semester B.E. Degree Examination, June / July 2014
Engineering Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: 1. *Answer any FIVE full questions, choosing at least two from each part.*
2. *Answer all objective type questions only on OMR sheet page 5 of the answer booklet.*
3. *Answer to objective type questions on sheets other than OMR will not be valued.*

PART – A

1 a. Choose the correct answers for the following : **(04 Marks)**

i) If $y_n = (\sqrt{17})^n e^{4x} \cos\left(x + n\tan^{-1}\frac{1}{4}\right)$ then $y =$ ______

A) $e^{4x}\cos x$ B) $e^{2x}\sin 3x$ C) $e^x \cos x$ D) None of these

ii) $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$.........is,

A) Taylor's series B) Exponential series C) Meclaurin's series D) None of these

iii) In the Rolle's theorem if $F'(c) = 0$ then the tangent at the point $x = c$ is,

A) parallel to y-axis B) parallel to x-axis C) parallel to both axes D) None of these

iv) If $y = 3^x$ then $y_n =$ ______

A) $(\log x)3^n$ B) $3(\log x)^n$ C) $3^n \log 3^x$ D) $3^x(\log_e 3)^n$

b. If $x = \sin t$, $y = \sin pt$ prove that, $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (p^2 - n^2)y_n = 0$. **(04 Marks)**

c. State and prove Cauchy's mean value theorem in [0, 16]. **(06 Marks)**

d. Expand $\sqrt{1+\sin 2x}$ by using Meclaurin's expansion. **(06 Marks)**

2 a. Choose the correct answers for the following : **(04 Marks)**

i) The value of $\underset{x\to\infty}{\text{lt}} (1+x)^{1/x}$ is,

A) e B) 1 C) $\frac{1}{e}$ D) ∞

ii) The angle between two curves $r = ae^{\theta}$ and $re^{\theta} = b$ is,

A) $\frac{\pi}{2}$ B) $\frac{\pi}{4}$ C) 0 D) π

iii) $\frac{ds}{dt} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$

A) Polar form B) Parametric form C) Cartesian form D) None of these

iv) $\underset{x\to\infty}{\text{lt}} \frac{\log x}{\cot x} =$ ______

A) 1 B) 0 C) 2 D) -2

b. Find a & b, if $\underset{x\to 0}{\text{lt}} \frac{x(1+a\cos x) - b\sin x}{x^3} = 1$. **(04 Marks)**

c. Find the pedal equation of the curve $r^2 = a^2\cos 2\theta$ **(06 Marks)**

d. Find the radius of curvature at any point t of the curve $x = a(t+\sin t)$ and $y = a(1-\cos t)$. **(06 Marks)**

3 a. Choose the correct answers for the following : **(04 Marks)**

i) If $u = (x-y)^2 + (y-z)^2 + (z-x)^2$ then $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$ is,

A) 1 B) 24 C) $2(x+y+z)$ D) 0

ii) $e^x \cos y = \frac{e}{\sqrt{2}}\left[1 + (x-1) - \left(y - \frac{\pi}{4}\right) + \frac{(x-1)^2}{2} - (x-1)\left(y - \frac{\pi}{4}\right) - \frac{1}{2}\left(y - \frac{\pi}{4}\right)^2\right] + \ldots\ldots$

A) $\left(1, \frac{\pi}{4}\right)$ B) $(0, 0)$ C) $(1, 1)$ D) $\left(\frac{\pi}{4}, 1\right)$

iii) At (a, b) $\frac{\partial^2 u}{\partial x^2} = A$, $\frac{\partial^2 u}{\partial y^2} = B$ and $\frac{\partial^2 u}{\partial x \partial y} = H$ and if $AB - H^2 < 0$ then such a point is called,

A) Maximum B) Minimum C) Saddle D) Extremum

iv) If $J = \frac{\partial(u,v)}{\partial(x,y)}$, $J' = \frac{\partial(x,y)}{\partial(u,v)}$, then JJ' is,

A) 0 B) 2 C) ∞ D) 1

b. If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$ then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$. **(04 Marks)**

c. If $u = \frac{xy}{z}$, $v = \frac{yz}{x}$, $w = \frac{zx}{y}$ then show that $J\left(\frac{u,v,w}{x,y,z}\right) = 4$ verify $JJ' = 1$. **(06 Marks)**

d. For the kinetic energy $E = \frac{1}{2}mv^2$ find approximately the change in E as the mass m changes from 49 to 49.5 and the velocity 'v' changes from 1600 to 1590. **(06 Marks)**

4 a. Choose the correct answers for the following : **(04 Marks)**

i) The value of $\nabla \times \nabla\varphi$ is,

A) 0 B) $\vec{R}$ C) φ D) 3

ii) Any motion in which the curl of the velocity vector is zero, then the vector $\vec{v}$ is said to be,

A) Constant B) Solenoidal C) Vector D) Irrotational

iii) In orthogonal curvilinear co-ordinates the Jacobian $J = \frac{\partial(x,y,z)}{\partial(u,v,w)}$ is,

A) $\frac{h_1}{h_2 h_3}$ B) $\frac{1}{h_1 h_2 h_3}$ C) $h_1 h_2 h_3$ D) $\frac{h_3}{h_1 h_2}$

iv) A gradient of the scalar point function φ, $\nabla\varphi$ is,

A) Scalar function B) Vector function C) φ D) zero

b. Find the value of the constant a such that the vector field, $\vec{F} = (axy - z^3)i + (a-2)x^2 j + (1-a)xz^2 k$ is irrotational and hence find a scalar function φ such that $\vec{F} = \nabla\varphi$. **(04 Marks)**

c. Prove that $\text{curl}(\text{curl}\,\vec{A}) = \nabla\left(\nabla.\vec{A}\right) - \nabla^2\vec{A}$. **(06 Marks)**

d. Express $\nabla^2\psi$ in orthogonal curvilinear co-ordinates. **(06 Marks)**

PART – B

5 a. Choose the correct answers for the following : **(04 Marks)**

i) The value of $\int_0^{\pi/8} \cos^3(4x)dx$ is,

A) $\frac{1}{3}$ B) $\frac{1}{6}$ C) $\frac{\pi}{3}$ D) $\frac{1}{2}$

ii) If the equation of the curve remains unchange after changing θ to $-\theta$ the curve $r = f(\theta)$ is symmetrical about,

A) A line perpendicular to initial line through pole
B) Radially symmetric about the point pole.
C) Symmetry does not exist
D) Initial line

iii) The volume of the curve $r = a(1+\cos\theta)$ about the initial line is,

A) $\frac{4\pi a^3}{3}$ B) $\frac{2\pi a^3}{3}$ C) $\frac{8\pi a^3}{3}$ D) $\frac{\pi a^3}{3}$

iv) The assymptote for the curve $x^3 + y^3 = 3axy$ is equal to,

A) $x+y+a=0$ B) $x-y-a=0$ C) No Assymptote D) $x+y-a=0$

b. Evaluate $\int_0^{\pi} \frac{\log(1+\sin\alpha\cos x)}{\cos x}dx$. **(04 Marks)**

c. Evaluate $\int_0^{2a} x^2\sqrt{2ax - x^2}\,dx$. **(06 Marks)**

d. Find the area of surface of revolution about x-axis of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$. **(06 Marks)**

6 a. Choose the correct answers for the following : **(04 Marks)**

i) In the homogeneous differential equation, $\frac{dy}{dx} = \frac{f_1(xy)}{f_2(xy)}$ the degree of the function, $f_1(xy)$ and $f_2(xy)$ are,

A) Different B) Relatively prime C) Same D) None of these

ii) The integrating factor of the differential equation, $\frac{dy}{dx} + \cot xy = \cos x$ is,

A) $\cos x$ B) $\sin x$ C) $-\sin x$ D) $\cot x$

iii) Replacing $\frac{dy}{dx}$ by $\left(-\frac{dy}{dx}\right)$ in the differential equation $f\left(x, y, \frac{dy}{dx}\right) = 0$ we get the differential equation of,

A) Polar trajectory B) Orthogonal trajectory
C) Parametric trajectory D) Parallel trajectory.

iv) Two families of curves are said to be orthogonal if every member of either family cuts each member of the other family at,

A) Zero angle B) Right angle C) $\frac{\pi}{6}$ D) $\frac{2\pi}{3}$

b. Solve $(1 + e^{x/y})dx + e^{x/y}\left(1 - \frac{x}{y}\right)dy = 0$. **(04 Marks)**

c. Solve $\frac{dy}{dx} + x\sin 2y = x^3\cos^2 y$. **(06 Marks)**

d. Find the orthogonal trajectories of $r^2 = a^2\cos^2\theta$. **(06 Marks)**

7 a. Choose the correct answers for the following : **(04 Marks)**

i) $A = \begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix}$ is called,

A) Scalar matrix B) Diagonal matrix C) Identity matrix D) None of these

ii) If r = n and x = y = z = 0. The equations have only ________ solution.

A) Non trivial B) Trivial C) Unique D) Infinite

iii) In Gauss Jordan method, the coefficient matrix can be reduced to,

A) Echelon form B) Unit matrix C) Triangular form D) Diagonal matrix

iv) The inverse square matrix A is given by,

A) $|A|$ B) $\dfrac{\text{adj}A}{|A|}$ C) adjA D) $\dfrac{|A|}{\text{adj}A}$

b. Find the Rank of the matrix, $\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$. **(05 Marks)**

c. Investigate the values of λ and μ such that the system of equations, $x+y+z=6$, $x+2y+3z=10$, $x+2y+\lambda z=\mu$ may be i) Unique solution ii) Infinite solution iii) No solution. **(06 Marks)**

d. Using Gauss elimination method solve,
$2x_1 - x_2 + 3x_3 = 1$, $-3x_1 + 4x_2 - 5x_3 = 0$, $x_1 + 3x_2 - 6x_3 = 0$ **(05 Marks)**

8 a. Choose the correct answers for the following : **(04 Marks)**

i) A square matrix A of order 3 has 3 linearly independent eigen vectors then a matrix P can be found such that $P^{-1}AP$ is a,

A) Diagonal matrix B) Unit matrix
C) Singular matrix D) Symmetric matrix

ii) The eigen values of matrix $\begin{bmatrix} 2 & \sqrt{2} \\ \sqrt{2} & 2 \end{bmatrix}$ are,

A) $2 \pm \sqrt{6}$ B) $2 \pm \sqrt{2}$ C) $1 - \sqrt{6}$ D) None of these

iii) Solving the equations $x+2y+3z=0$, $3x+4y+4z=0$, $7x+10y+12z=0$. x, y and z values are,

A) $x=y=z=0$ B) $x=y=z=1$ C) $x \neq y \neq z \neq 1$ D) None of these

iv) The index and significance of the quadratic form, $x_1^2 + 2x_2^2 - 3x_3^2$ are respectively ________ and ________.

A) Index = 1, Signature = 1 B) Index = 1, Signature = 2
C) Index = 2, Signature = 1 D) None of these.

b. Find all the eigen values and the corresponding eigen vectors of the matrix,
$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$. **(04 Marks)**

c. Reduce the matrix $A = \begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix}$ into a diagonal matrix. **(06 Marks)**

d. Reduce the quadratic form $3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy$ to the canonical form. **(06 Marks)**

* * * * *