USN | | | | | | | | | | 10MAT11

First Semester B.E. Degree Examination, January 2013

Engineering Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: 1. *Answer any FIVE full questions, choosing at least two from each part.*
2. Answer all objective type questions only in OMR sheet page 5 of the answer booklet.
3. Answer to objective type questions on sheets other than OMR will not be valued.

PART – A

1 a. Choose the correct answers for the following :

i) The Leibnitz theorem is the formula to find the n^{th} derivative of
A) trigonometric function B) exponential function C) product of two algebraic functions D) product of two functions

ii) The n^{th} derivative of 5^x is : A) $\log 5, 5^x$ B) $(\log 5)^n 5^x$ C) $e^{(\log 5)x}$ D) $(\log 5)^2 e^{(\log 5)x}$

iii) The value of 'c' of the Cauchy mean value theorem for $f(x) = e^x$, $g(x) = e^{-x}$ in (3, 7) is : A) 5 B) 3 C) 0 D) 4

iv) The generalized series of Maclaurin's series expansion is
A) Taylor series B) Exponential series C) Logarithmic series D) Trigonometric series (04 Marks)

b. Verify Rolle's theorem for the function $f(x) = x^2(1-x)^2$ in $0 \le x \le 1$ and also find the value of c. (04 Marks)

c. If $\sin^{-1} y = 2\log(x+1)$, prove that $(x+1)^2 y_{n+2} + (2n+1)(x+1)y_{n+1} + (n^2+4)y_n = 0$. (06 Marks)

d. Expand by using Maclaurin's series, the function $\log(1+\sin x)$ upto fifth degree terms. (06 Marks)

2 a. Choose the correct answers for the following :

i) The curve $r = \frac{a}{1+\cos\theta}$ intersect orthogonally with the following curve : A) $r = \frac{b}{1-\cos\theta}$ B) $r = \frac{c}{1+\sin\theta}$ C) $r = \frac{b}{1-\sin\theta}$ D) $r = \frac{d}{\cos\theta}$

ii) If ϕ be the angle between the tangent and radius vector at any point on the curve $r = f(\theta)$, then $\sin\phi$ equals to
A) $\frac{dr}{ds}$ B) $r\frac{d\theta}{ds}$ C) $r\frac{d\theta}{dr}$ D) $r\frac{dr}{d\theta}$

iii) L Hospital's Rule can be applied to the limits of the form : A) 0/0 B) $0\times\infty$ C) $\infty-\infty$ D) ∞^0

iv) $\underset{x\to\infty}{\text{Lt}}(a^{1/x}-1)x$ is of the following form : A) $0\times\infty$ B) $\infty-\infty$ C) ∞^0 D) 0^0 (04 Marks)

b. Evaluate $\lim\limits_{x\to\pi/2}(\tan x)^{\cos x}$. (04 Marks)

c. Find the radius of curvature for the curve $x^2 y = a(x^2+y^2)$ at the point (-2a, 2a). (06 Marks)

d. Find the Pedal equation for the curve $r(1-\cos\theta) = 2a$. (06 Marks)

3 a. Choose the correct answers for the following :

i) If $f(x,y) = \frac{1}{x^3} + \frac{1}{y^3} + \frac{1}{x^3+y^3}$, then $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$ is : A) 0 B) 9 C) 1 D) $-3f$

ii) If $x = \rho\cos\theta$, $y = \rho\sin\theta$, $z = z$ then $\frac{\partial(x,y,z)}{\partial(\rho,\theta,z)}$: A) ρ B) 1 C) 0 D) θ

iii) If an error of 1% is made in measuring its base and height, the percentage error in the area of a triangle is
A) 0.2% B) 0.02% C) 1% D) 2%

iv) One of the necessary and sufficient condition for a function to have a maximum value is
A) $AC-B^2>0,\ A<0$ B) $AC-B^2=0,\ A=0$ C) $AC-B^2<0,\ A>0$ D) $AC-B^2>0,\ A>0$ (04 Marks)

b. If $V = e^{a\theta}\cos(a\log r)$, prove that $\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} + \frac{1}{r^2}\frac{\partial^2 v}{\partial\theta^2} = 0$. (06 Marks)

c. Examine the function $f(x,y) = 1 + \sin(x^2+y^2)$ for extremum values. (05 Marks)

d. In calculating the volume of right circular cone, errors of 2% and 1% are made in height and radius of the base respectively. Find the percentage error in the volume. (05 Marks)

4 a. Choose the correct answers for the following :

i) If $\vec{F} = \nabla\phi$, then the curl $\vec{F}$: A) solenoidal B) irrotational C) rotational D) none of these

ii) If $V = x^2+y^2+3$ then grad V is : A) $2xi+2yj$ B) $2x+2y$ C) $2xi+2yj+k$ D) $xi+yj$

iii) The value of 'a' of the vector $\vec{F} = (x+3y)i + (x-2z)j + (x+az)k$, which is solenoidal : A) -2 B) -1 C) 0 D) 3

iv) If $R = x^2y + y^2z + z^2x$, then Laplacian of R is : A) $x+y+z$ B) $x-y-z$ C) $2(x+y+z)$ D) $2(x-y+z)$ (04 Marks)

b. Find div $\vec{F}$ and curl $\vec{F}$, where $\vec{F} = \nabla(x^3+y^3+z^3-3xyz)$. (06 Marks)

c. Prove that $\text{curl}(\phi\,\vec{u}) = \phi\,\text{curl}\,\vec{u} + \text{grad}\,\phi\times\vec{u}$. (06 Marks)

d. Show that the cylindrical system is orthogonal. (04 Marks)

PART – B

5 a. Choose the correct answers for the following :

i) The value of $\int_0^{\pi/2}\cos x\sin^{99}x\,dx$ is A) 1/99 B) 1/100 C) $\pi/100$ D) 99/100

ii) The curve $y^2(a^2 + x^2) = x^2(a^2 - x^2)$ is
A) symmetric about the x-axis B) symmetric about the x & y axis C) symmetric about the y-axis D) none of these

iii) The length of the arc $y = f(x)$ from $x = a$ to $x = b$ is
A) $\int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$ B) $\int_a^b \sqrt{1-\left(\frac{dx}{dy}\right)^2}\,dx$ C) $\int_a^b \sqrt{1+\left(\frac{dx}{dy}\right)^2+\left(\frac{dy}{dx}\right)^2}\,dx$ D) none of these

iv) The value of $\int_0^{\pi/2} \sin^4 x\,dx$ is equal to : A) $3\pi/8$ B) $3/8$ C) $\pi/16$ D) $\pi/4$ **(04 Marks)**

b. Obtain the reduction formula for $\int \sin^n x\,dx$. **(04 Marks)**

c. Evaluate $\int_0^a x\sqrt{ax - x^2}\,dx$. **(06 Marks)**

d. Find the area of an arch of the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$. **(06 Marks)**

6 a. Choose the correct answers for the following :

i) The order and degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2} = c\frac{d^2y}{dx^2}$ respectively is
A) one, two B) one, one C) two, one D) three, two

ii) The differential equation $\left[1+e^{x/y}\right]dx + e^{x/y}\left[1-\frac{x}{y}\right]dy = 0$ is
A) homogeneous and linear B) homogeneous and exact C) non-homogeneous and exact D) none of these

iii) The solution of the differential equation $\frac{dy}{dx} = e^{x+y}$: A) $e^x + e^y = c$ B) $e^x + e^{-y} = c$ C) $e^x - e^{-y} = c$ D) $e^{x+y} = c$

iv) Replacing dy/dx by –dx/dy in the differential equation of (x, y, dy/dx) = 0, we get the differential equation of
A) polar trajectory B) orthogonal trajectory C) trajectory D) none of these **(04 Marks)**

b. Solve $\frac{dy}{dx} = \frac{2x - y + 1}{x + 2y - 3}$. **(06 Marks)**

c. Solve $dr + (2r\cot\theta + \sin 2\theta)d\theta = 0$. **(06 Marks)**

d. Find the orthogonal trajectory of the family of coaxial circles $\frac{x^2}{a^2} + \frac{y^2}{b^2+\lambda} = 1$. **(04 Marks)**

7 a. Choose the correct answers for the following :

i) The normal form of the matrix are A) $[I_r, 0]$ B) $\begin{bmatrix} I_r \\ 0 \end{bmatrix}$ C) $\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$ D) all of these

ii) The solution of the simultaneous equations $x + y = 3$, $x - y = 3$ is
A) only trivial B) only unique C) unique and trivial D) none of these

iii) In Gauss Jordan method, the coefficient matrix reduces to matrix
A) diagonal B) unit matrix C) triangular matrix D) none of these

iv) If r is the rank of the matrix [A] of order $m \times n$ then r is : A) $r \le m$ B) $r \le n$ C) $r \ge n$ D) $r \ge m$ **(04 Marks)**

b. Find the rank of the following matrix by elementary transform : $A = \begin{bmatrix} 0 & 2 & 3 & 4 \\ 2 & 3 & 5 & 4 \\ 4 & 8 & 13 & 12 \end{bmatrix}$ **(04 Marks)**

c. Find for what value of k the system of equations $x + y + z = 1$, $x + 2y + 4z = k$, $x + 4y + 6z = k^2$, posses a solution. Solve completely in each case. **(06 Marks)**

d. Solve the following system of equations by Gauss elimination method: $x + y + z = 9$; $x - 2y + 3z = 8$; $2x + y - z = 3$ **(06 Marks)**

8 a. Choose the correct answers for the following :

i) If the determinant of the coefficient matrix is zero, then there exist
A) trivial solution B) non-trivial solution C) unique solution D) no solution

ii) If P is the modal matrix of an orthogonal matrix, then its inverse matrix is equal to
A) P^{-1} B) P C) diagonal matrix D) none of these

iii) The quadratic form for the matrix $A = \begin{bmatrix} a & h \\ h & b \end{bmatrix}$ is : A) $ax^2 + 2hxy + by^2$ B) $ax^2 + by^2$ C) $ax^2 + 2bxy + 2by^2$ D) none of these

iii) The nature of the quadratic function of the matrix having the eigen values [0, 2, 4] is
A) positive definite B) positive semi-definite C) negative definite D) negative semi-definite **(04 Marks)**

b. Reduce the matrix $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$ to the diagonal form and hence find A^4. **(06 Marks)**

c. Find all the eigen values of the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$. **(04 Marks)**

d. Reduce the quadratic form $3x^2 + 3y^2 + 3z^2 + 2xy - 2yz + 2zx$ into canonical form. **(06 Marks)**

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First Semester B.E. Degree Examination, January 2013
Engineering Mathematics – I

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Note: ***1. Answer any FIVE full questions, choosing at least two from each part.***
2. Answer all objective type questions only on OMR sheet page 5 of the answer booklet.
3. Answer to objective type questions on sheets other than OMR will not be valued.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg. 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Choose correct answers for the following : **(04 Marks)**

i) If $y = 3^{2x}$ then y_n = ____ : A) $2^{3x}(2\log 3)^n$ B) $3^{2x}(\log 3)^n$ C) $3^{2\log x}$ D) $3^{2x}(2\log 3)^n$

ii) If $y = \log(1-x)$ the y_n = ____ : A) $\frac{(-1)^{n-1}n!}{(1-x)n}$ B) $\frac{(-1)^{2n-1}(n-1)!}{(1+x)^n}$ C) $\frac{(-1)^{2n-1}(n-1)!}{(1-x)^n}$ D) $\frac{(-1)^{2n+1}(n-1)!}{(1-x)^{n+1}}$

iii) By Rolle's theorem the number C = ____ when $f(x) = x^2 - 4x + 8$ in [1, 3]: A) 1 B) 2 C) 3 D) 4

iv) By Maclaurins series, the expansion $x - \frac{x^3}{3!} + \frac{x^5}{5!}$ ------- is equal to ____ : A) e^x B) cosx C) sinx D) x cosx

b. Find the n^{th} derivative of $x^2 \sin 3x$. **(04 Marks)**

c. Show that $\frac{b-a}{1+b^2} < \tan^{-1}b - \tan^{-1}a < \frac{b-a}{1+a^2}$, if $0 < a < b$ and deduce that $\frac{\pi}{4} + \frac{3}{25} < \tan^{-1}\frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}$. **(06 Marks)**

d. Expand $\tan^{-1}x$ in powers of x -1 upto the term containing fourth degree. **(06 Marks)**

2 a. Choose correct answers for the following : **(04 Marks)**

i) $\underset{x \to 0}{\text{Limit}}\left[\frac{\log \sin ax}{\log \sin bx}\right]$ = ____ : A) 1 B) a/b C) b/a D) ab

ii) The angle between the radius vector and the tangent of the curve $r = \sin\theta + \cos\theta$ is ____
A) $\pi/2 + \theta$ B) $\pi/4 + \theta$ C) $\pi/3 + \theta$ D) $\pi/6 + \theta$

iii) Derivative of arc length for polar curve, the value $ds/d\theta$ = ____
A) $\sqrt{r^2 + \frac{d^2r}{d\theta^2}}$ B) $\sqrt{r + \left(\frac{dr}{d\theta}\right)^2}$ C) $\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}$ D) $\sqrt{r^2 + \left(\frac{d\theta}{dr}\right)^2}$

iv) Radius of curvature of $y = x^2$ at x = 1 is = ____ : A) $5\sqrt{5}$ B) $\frac{4\sqrt{5}}{2}$ C) $\frac{3\sqrt{5}}{2}$ D) $\frac{5\sqrt{5}}{2}$

b. Evaluate $\underset{x \to 0}{\text{Limit}}\left[\frac{a^x + b^x + c^x + d^x}{4}\right]^{1/x}$. **(04 Marks)**

c. Find the angle of intersection between the curves $r^2\sin 2\theta = 4$ and $r^2 = 16\sin 2\theta$. **(06 Marks)**

d. Find the radius of curvature at any point t of the curve $x = a(\cos t + \log\tan t/2)$, $y = a\sin t$. **(06 Marks)**

3 a. Choose correct answers for the following : **(04 Marks)**

i) If $F(u) = \sin u = \frac{x^2y^2}{x^2+y^2}$ the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ = ____ :A) cot u B) tan u C) 2 tanu D) 3 tan u

ii) Jacobian for $x = r\cos\theta$, $y = r\sin\theta$ is ____ : A) r B) $1/r^2$ C) 1/r D) r^2

iii) The necessary condition for u = f(x, y) have maxima or minima is
A) $\partial u/\partial x \neq 0$, $\partial u/\partial y \neq 0$ B) $\partial u/\partial x = 0$, $\partial u/\partial y = 0$ C) $\partial u/\partial x > 0$, $\partial u/\partial y > 0$ D) $\partial u/\partial x < 0$, $\partial u/\partial y < 0$

iv) The percentage error in the area of the rectangle when an error of 1.0% is made in measuring the sides x and y is ____ : A) 4 B) 3 C) 2 D) 1

b. If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$, find the Jacobian of (x, y, z) with respect to r, θ, ϕ. **(04 Marks)**

c. Find the percentage error in computing resistance r of two resistances r_1 and r_2 connected in parallel of both r_1 and r_2 are in error by 2%. **(06 Marks)**

d. Find the extreme values of the function $f(x, y) = x^3y^2(1 - x - y)$. **(06 Marks)**

4 a. Choose correct answers for the following : **(04 Marks)**

i) If $\vec{R}$ is a position vector of any point P(x, y, z) then $\nabla \cdot \vec{R}$ is ____ : A) 0 B) 1 C) 2 D) 3

ii) Any motion in which the curl of the velocity vector is zero, then the vector $\vec{V}$ is said to be
A) solenoidal B) Vector C) Constant D) Irrotational

iii) If ϕ is the scalar point function then the value of curl (grad ϕ) = ____ : A) > 0 B) < 0 C) 0 D) ∞

iv) In orthogonal curvilinear coordinates the value of $\frac{\partial(x,y,z)}{\partial(u,v,w)}$ is = ____
A) $h_1 h_2 h_3$ B) $1/h_1h_2h_3$ C) h_1/h_2h_3 D) h_1h_2/h_3

b. Show that the vector field $F = (x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$ is irrotational and find its scalar potential. **(04 Marks)**

c. Prove that $\nabla \cdot (\phi\vec{A}) = (\nabla\phi)\cdot\vec{A} + \phi(\nabla\cdot\vec{A})$ where ϕ is a scalar field. **(06 Marks)**

d. If $\vec{F}(u,v,w)$ be the vector point function given interms of orthogonal curvilinear coordinates as $F = F_1e_1 + F_2e_2 + F_3e_3$, find curl $\vec{F}$. **(06 Marks)**

PART – B

5 a. Choose correct answers for the following : **(04 Marks)**

i) If $I(\alpha) = \int_0^1 \left[\frac{x^\alpha - 1}{\log x}\right] dx$ then $\frac{dI(\alpha)}{d\alpha} =$ ____ : A) $4/(1+\alpha)$ B) $3/(1+\alpha)$ C) $2/(1+\alpha)$ D) $1/(1+\alpha)$

ii) The value of $\int_0^\pi \sin^4 x\, dx$ is = ____ : A) $3\pi/8$ B) $3\pi/16$ C) $3\pi^2/8$ D) Zero

iii) A curve $r = a(1 + \cos\theta)$ has the length on x-axis (the initial line) ____ : A) a B) 2a C) -2a D) 3a

iv) Special points on x and y-axis for the asteroid $x^{2/3} + y^{2/3} = a^{2/3}$ are ____ : A) $\pm a$ B) $\pm 2a$ C) $\pm 3a$ D) $\pm 4a$

b. Differentiate under the integral sign and hence evaluate the integration $\int_0^\infty \frac{\tan^{-1}(ax)}{x(1+x^2)} dx$. **(04 Marks)**

c. Evaluate $\int_0^{2a} x^2\left(\sqrt{2ax - x^2}\right) dx$. **(06 Marks)**

d. Trace the curve $r = a(1 + \cos\theta)$ and hence find the total length. **(06 Marks)**

6 a. Choose correct answers for the following : **(04 Marks)**

i) The solution of the differential equation $dy/dx = e^{x-y}$ is ____
A) $e^x/e^y = c$ B) $e^y/e^x = c$ C) $e^x + e^y = c$ D) $e^{xy} = c$

ii) If $M(x, y)dx + N(x, y)\, dy = 0$ is said to be exact then the condition is
A) $\partial M/\partial y \neq \partial N/\partial x$ B) $\partial M/\partial y = \partial N/\partial x$ C) $\partial M/\partial y > \partial N/\partial x$ D) $M = N$

iii) The integrating factor for $(x + 2y^3)\, dy/dx = y$ is I.F = ____ : A) log y B) e^y C) $1/y$ D) $y + 1$

iv) For $r = f(\theta)$, the replacement of $dr/d\theta$ to find the orthogonal trajectory is ____
A) $-r\frac{dr}{d\theta}$ B) $-r^2\frac{dr}{d\theta}$ C) $-r^2\frac{d\theta}{dr}$ D) $-r\frac{d\theta}{dr}$

b. Solve $(4x + 6y + 5)\, dy = (3y + 2x + 4)\, dx$. **(04 Marks)**

c. Solve $dy/dx + x\sin 2y = x^3\cos^2 y$. **(06 Marks)**

d. Find the orthogonal trajectory of the system of confocal conics $x^2/(a^2+\lambda) + y^2/(b^2+\lambda) = 1$ where λ is the parameter. **(06 Marks)**

7 a. Choose correct answers for the following : **(04 Marks)**

i) The system of linear equations is said to be consistent then the relation between R(A) and R(A:B) in AX = B is ____ : A) R(A) > R(A:B) B) R(A) < R(A:B) C) R(A) ≠ R(A:B) D) R(A) = R(A:B)

ii) The rank of the matrix $A = \begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix}$ is ____ : A) 0 B) 1 C) 2 D) 3

iii) A square matrix is said to be symmetric matrix is ____ : A) $a_{ij} = a_{ji}$ B) $a_{ij} > a_{ji}$ C) $a_{ij} < a_{ji}$ D) $a_{ij} = -a_{ji}$

iv) In Gauss elimination method the system of equations is transformed into an ____
A) Row matrix B) Column matrix C) Null matrix D) Upper triangular matrix

b. Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & -2 & 3 \\ 2 & 5 & -4 & 6 \\ -1 & -3 & 2 & -2 \\ 2 & 4 & -1 & 6 \end{bmatrix}$. **(04 Marks)**

c. Investigate the value of λ and μ, so that the equations $2x + 3y + 5z = 9$, $7x + 3y - 2z = 8$, $2x + 3y + \lambda z = \mu$ have i) Unique solution; ii) No solution; iii) An infinite number of solutions. **(06 Marks)**

d. Solve the system of equations by Gauss Jordon method: $2x+5y+7z = 52$, $2x+y-z = 0$, $x+y+z = 9$. **(06 Marks)**

8 a. Choose correct answers for the following : **(04 Marks)**

i) Vectors x_1, x_2, x_3 ----- are said to be ----- in a relation $k_1x_1 + x_2k_2 + k_3x_3 + \text{-----} + k_rx_r$ with k_1, k_2 ----- k_r are the scalars not all zero: A) Linearly independent B) Linearly dependent C) Consistent D) Inconsistent

ii) A matrix A is called orthogonal if: A) A = A' B) A/A' = I C) AA' = I D) A'/A = I

iii) Eigen values of the matrix $A = \begin{bmatrix} 2 & 4 \\ 1 & 5 \end{bmatrix}$ are ____ : A) 1, 3 B) 1, 4 C) 1, 5 D) 1, 6

iv) A homogeneous polynomial of second degree in n variables x_1, x_2 ----- is called a ____
A) Canonical form B) Linear form C) Exponential form D) Quadratic form

b. Show that the transformation $y_1 = 2x_1 + x_2 + x_3$, $y_2 = x_1 + x_2 + 2x_3$, $y_3 = x_1 - 2x_3$ is regular, write down the inverse transformation. **(04 Marks)**

c. Find the Eigen values and the corresponding Eigen vectors of the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$. **(06 Marks)**

d. Reduce the quadratic form $x^2 + 5y^2 + z^2 + 2yz + 6xz + 2xy$ to the canonical form and specify the matrix of transformation. **(06 Marks)**