USN

10MAT41

Fourth Semester B.E. Degree Examination, June/July 2014

Engineering Mathematics – IV

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Obtain a solution upto the third approximation of y for x = 0.2 by Picard's method, given that $\frac{dy}{dx} + y = e^x$; $y(0) = 1$. **(06 Marks)**

b. Apply Runge-Kutta method of order 4, to find an approximate value of y for x = 0.2 in steps of 0.1, if $\frac{dy}{dx} = x + y^2$ given that y = 1 when x = 0. **(07 Marks)**

c. Using Adams-Bashforth formulae, determine y(0.4) given the differential equation $\frac{dy}{dx} = \frac{1}{2}xy$ and the data, y(0) = 1, y(0.1) = 1.0025, y(0.2) = 1.0101, y(0.3) = 1.0228. Apply the corrector formula twice. **(07 Marks)**

2 a. Apply Picard's method to find the second approximation to the values of 'y' and 'z' given that $\frac{dy}{dx} = z$, $\frac{dz}{dx} = x^3(y+z)$, given y = 1, $z = \frac{1}{2}$ when x = 0. **(06 Marks)**

b. Using Runge-Kutta method, solve $\frac{d^2y}{dx^2} - x\left(\frac{dy}{dx}\right)^2 + y^2 = 0$ for x = 0.2 correct to four decimal places. Initial conditions are x = 0, y = 1, y′ = 0. **(07 Marks)**

c. Obtain the solution of the equation $\frac{2d^2y}{dx^2} = 4x + \frac{dy}{dx}$ at the point x = 1.4 by applying Milne's method given that y(1) = 2, y(1.1) = 2.2156, y(1.2) = 2.4649, y(1.3) = 2.7514, y′(1) = 2, y′(1.1) = 2.3178, y′(1.2) = 2.6725 and y′(1.3) = 3.0657. **(07 Marks)**

3 a. Define an analytic function in a region R and show that f(z) is constant, if f(z) is an analytic function with constant modulus. **(06 Marks)**

b. Prove that $u = x^2 - y^2$ and $v = \frac{y}{x^2 + y^2}$ are harmonic functions of (x, y) but are not harmonic conjugate. **(07 Marks)**

c. Determine the analytic function f(z) = u + iv, if $u - v = \frac{\cos x + \sin x - e^{-y}}{2(\cos x - \cosh y)}$ and f(π/2) = 0. **(07 Marks)**

4 a. Find the images of the circles |z| = 1 and |z| = 2 under the conformal transformation $w = z + \frac{1}{z}$ and sketch the region. **(06 Marks)**

b. Find the bilinear transformation that transforms the points 0, i, ∞ onto the points 1, –i, –1 respectively. **(07 Marks)**

c. State and prove Cauchy's integral formula and hence generalized Cauchy's integral formula. **(07 Marks)**

PART – B

5 a. Obtain the solution of the equation $x^2\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}+\left(x^2-\dfrac{1}{4}\right)y=0$. (06 Marks)

b. Obtain the series solution of Legendre's differential equation.

$$(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+n(n+1)y=0$$ (07 Marks)

c. State Rodrigue's formula for Legendre polynomials and obtain the expression for $P_4(x)$ from it. Verify the property of Legendre polynomials in respect of $P_4(x)$ and also find $\int_{-1}^{1} x^3 P_4(x)dx$. (07 Marks)

6 a. Two fair dice are rolled. If the sum of the numbers obtained is 4, find the probability that the numbers obtained on both the dice are even. (06 Marks)

b. Given that $P(\overline{A}\cup\overline{B})=\dfrac{7}{12}$, $P(A\cap\overline{B})=\dfrac{1}{6}=P(\overline{A}\cap B)$. Prove that A and B are neither independent nor mutually disjoint. Also compute $P(A/B)+P(B/A)$ and $P(\overline{A}/\overline{B})+P(\overline{B}/\overline{A})$. (07 Marks)

c. Three machines M_1, M_2 and M_3 produces identical items. Of their respective outputs 5%, 4% and 3% of items are faulty. On a certain day, M_1 has produced 25% of the total output, M_2 has produced 30% and M_3 the remainder. An item selected at random is found to be faulty. What are the chances that it was produced by the machine with the highest output? (07 Marks)

7 a. In a quiz contest of answering 'Yes' or 'No', what is the probability of guessing atleast 6 answers correctly out of 10 questions asked? Also find the probability of the same if there are 4 options for a correct answer. (07 Marks)

b. Define exponential distribution and obtain the mean and standard deviation of the exponential distribution. (07 Marks)

c. If X is a normal variate with mean 30 and standard deviation 5, find the probabilities that (i) $26 \le X \le 40$, (ii) $X \ge 45$, (iii) $|X-30|>5$. [Give that $\phi(0.8)=0.2881$, $\phi(2.0)=0.4772$, $\phi(3.0)=0.4987$, $\phi(1.0)=0.3413$] (06 Marks)

8 a. Certain tubes manufactured by a company have mean life time of 800 hrs and standard deviation of 60 hrs. Find the probability that a random sample of 16 tubes taken from the group will have a mean life time (i) between 790 hrs and 810 hrs, (ii) less than 785 hrs, (iii) more than 820 hrs. [$\phi(0.67)=0.2486$, $\phi(1)=0.3413$, $\phi(1.33)=0.4082$]. (06 Marks)

b. A set of five similar coins is tossed 320 times and the result is:

No. of heads:	0	1	2	3	4	5
Frequency:	6	27	72	112	71	32

Test the hypothesis that the data follow a binomial distribution. [Given that $\psi^2_{0.05}(5)=11.07$] (07 Marks)

c. It is required to test whether the proportion of smokers among students is less than that among the lectures. Among 60 randomly picked students, 2 were smokers. Among 17 randomly picked lecturers, 5 were smokers. What would be your conclusion? (07 Marks)

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