USN | | | | | | | | | | | **10MAT21**

Second Semester B.E. Degree Examination, Dec.2013/Jan.2014

Engineering Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: 1. ***Answer any FIVE full questions, choosing at least two from each part.***
2. ***Answer all objective type questions only on OMR sheet page 5 of the answer booklet.***
3. ***Answer to objective type questions on sheets other than OMR will not be valued.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Choose the correct answers for the following : **(04 Marks)**

i) Suppose the equation to be solved is of the form, $y = f(x, p)$ then differentiating x we get equation of the form,

A) $\phi\left(x, p, \frac{dp}{dy}\right) = 0$ B) $\phi\left(y, p, \frac{dp}{dx}\right) = 0$ C) $\phi(x, y, p) = 0$ D) $\phi(x, y, 0) = 0$

ii) The general solution of the equation, $p^2 - 3p + 2 = 0$ is

A) $(y + x - c)(y + 2x - c)$ B) $(y - x - c)(y - 2x - c) = 0$
C) $(-y - x - c)(y - 2x - c) = 0$ D) $(y - [illegible] - c)(y + x - c) = 0$

iii) Clairaut's equation is of the form,

A) $x = py + f(p)$ B) $y = p^2 + f(p)$ C) $y = px + f(p)$ D) None of these

iv) Singular solution of $y = px + 2p^2$ is,

A) $y^2 + 8y = 0$ B) $x^2 - 8y = 0$ C) $x^2 + 8y - c = 0$ D) $x^2 + 8y = 0$

b. Solve $p^2 + 2p\cosh x + 1 = 0$. **(04 Marks)**

c. Find singular solution of $p = \sin(y - xp)$. **(06 Marks)**

d. Solve the equation, $y^2(y - xp) = x^4p^2$ using substitution $X = \frac{1}{x}$ and $Y = \frac{1}{y}$. **(06 Marks)**

2 a. Choose the correct answers for the following : **(04 Marks)**

i) A second order linear differential equation has,

A) two arbitrary solution B) One arbitrary solution
C) no arbitrary solution D) None of these

ii) If 2, 4i and -4i are the roots of A.E of a homogeneous linear differential equation then its solution is,

A) [illegible] $+ e^{[illegible]}(\cos 4x + \sin 4x)$ B) $C_1e^{2x} + C_2\cos 4x + C_3\sin 4x$
C) $C_1e^{2x} + C_2e^x\cos 4x + C_3e^x\sin 4x$ D) $C_1e^{2x}\cos 4x + C_2e^{2x}\sin 4x$

iii) P.I. of $(D + 1)^2 y = e^{-x+3}$

A) $\frac{x^2}{[illegible]}$ B) x^3e^x C) $\frac{x^3}{3}e^{-x+3}$ D) $\frac{x^2}{2}e^{-x+3}$

iv) Particular integral of $f(D)y = e^{ax}V(x)$ is,

A) $\frac{e^{ax}V(x)}{f(D)}$ B) $e^{ax}\frac{1}{f(D)}[V(x)]$ C) $e^{ax}\frac{1}{f(D+a)}[V(x)]$ D) $\frac{1}{f(D+a)}\left[e^{ax}V(x)\right]$

b. Solve $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - y = 0$. **(04 Marks)**

c. Solve $y'' - 3y' + 2y = 2\sin x\cos x$ **(06 Marks)**

d. Solve the system of equation, $\frac{dx}{dt} - 2y = \cos 2t$, $\frac{dy}{dt} + 2x = \sin 2t$. **(06 Marks)**

3 a. Choose the correct answers for the following : **(04 Marks)**

i) In $x^2y'' + xy' - y = 0$ if $e^t = x$ then we get x^2y'' as,

A) (D-1)y B) (D+1)y C) D(D+1)y D) None of these

ii) In second order homogeneous differential equation, $P_0(x)y'' + P_1(x)y' + P_2(x)y = 0$, $x = a$ is a singular point if,

A) $P_0(a) > 0$ B) $P_0(a) \neq 0$ C) $P_0(a) = 0$ D) $P_0(a) < 0$

iii) The general solution of $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$ is,

A) $y = C_1x - C_2\frac{1}{x}$ B) $C_1x + C_2\frac{1}{x}$ C) $C_1x + C_2x$ D) $C_1x - C_2x$

iv) Frobenius series solution of second order linear differential equation is of the form,

A) $x^m\sum_{r=0}^{\infty} a_r x^r$ B) $\sum_{i=0}^{\infty} a_i x^i$ C) $\sum_{r=0}^{\infty} a_r x^r$ D) None of these

b. Solve $y'' + a^2y = \sec ax$ by the method of variation of parameters. **(04 Marks)**

c. Solve $x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = e^x$. **(06 Marks)**

d. Obtain the series solution of $\frac{dy}{dx} - 2xy = 0$. **(06 Marks)**

4 a. Choose the correct answers for the following : **(04 Marks)**

i) PDE of $az + b = a^2x + y$ is,

A) $\frac{\partial z}{\partial x}\cdot\frac{\partial z}{\partial y} = 1$ B) $\frac{\partial z}{\partial x}\cdot\frac{\partial z}{\partial y} = 0$ C) $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$ D) $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 1$

ii) The solution of PDE $Z_{xx} = \ldots$ is,

A) $z = x^2 + xf(y) + g(y)$ B) $z = x^2y^2 + xf(y) + g(y)$

C) $z = x^2y^2 + f(x) + g(x)$ D) $z = y^2 + xf(y) + g(y)$

iii) The subsidiary equations of $(y^2 + z^2)p + x(yq - z) = 0$ are,

A) $\frac{dx}{P} = \frac{dy}{q} = \frac{dz}{R}$ B) $\frac{dx}{y^2+z^2} = \frac{dy}{x} = \frac{dz}{xz}$

C) $\frac{dx}{y^2+z^2} = \frac{dy}{xy} = \frac{dz}{xz}$ D) None of these

iv) In the method of separation of variables to solve $xz_n + z_t = 0$ the assumed solution is of the form

A) $X(x)Y(x)$ B) $X(y)Y(y)$ C) $X(t)Y(t)$ D) $X(x)T(t)$

b. Solve $\frac{\partial^3 z}{\partial x^2 \partial y} = \cos(2x + 3y)$. **(04 Marks)**

c. Solve $xp - yq = y^2 - x^2$ **(06 Marks)**

d. Solve $3u_x + 2u_y = 0$ by the separation of variable method given that $u = 4e^{-x}$ when $y = 0$. **(06 Marks)**

PART – B

5 a. Choose the correct answers for the following : **(04 Marks)**

i) $\int_0^1 \int_0^{x^2} e^{y/x} dydx =$ ________
A) 1 B) - 1/2 C) 1/2 D) None of these

ii) The integral $\iint_R f(x,y)dxdy$ by changing to polar form becomes.
A) $\iint_R \phi(r,\theta)drd\theta$ B) $\iint_R f(r,\theta)drd\theta$ C) $\iint_R f(r,\theta)rdrd\theta$ D) $\iint_R \phi(r,\theta)rdrd\theta$

iii) For a real positive number n, the Gamma function $\Gamma(n) =$ ________
A) $\int_0^\infty x^{n-1}e^{-x}dx$ B) $\int_0^1 x^{n-1}e^{-x}dx$ C) $\int_0^\infty x^n e^{-x}dx$ D) $\int_0^1 x^n e^{-x}dx$

iv) The Beta and Gamma functions relation for $B(m,n) =$ ________
A) $\frac{\Gamma(m)\Gamma(n)}{\Gamma(m-n)}$ B) $\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ C) $\Gamma(m)\Gamma(n)$ D) $\Gamma(m n)$

b. By changing the order of integration evaluate, $\int_0^a \int_{x/a}^{\sqrt{x/a}} (x^2+y^2)dydx$, $a > 0$. **(04 Marks)**

c. Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} dzdydx$. **(06 Marks)**

d. Express the integral $\int_0^1 \frac{dx}{\sqrt{1-x^n}}$ in terms of the Gamma function. Hence evaluate $\int_0^1 \frac{dx}{\sqrt{1-x^{1/4}}}$. **(06 Marks)**

6 a. Choose the correct answers for the following : **(04 Marks)**

i) The scalar surface integral of $\vec{f}$ over s, where s is a surface in a three-dimensional region R is given by, $\int_s \vec{f}.nds =$ ________ by Gauss divergence theorem.
A) $\iiint_V \nabla \cdot \vec{f}\, dV$ B) $\iint_S \vec{f}\, dxdy$ C) $\iiint_V \nabla \cdot \vec{F}\, dV$ D) None of these

ii) If all the surface are closed in a region containing volume V then the following theorem is applicable.
A) Stoke's theorem B) Green's theorem C) Gauss divergence theorem D) None of these

iii) The value of $\oint_c \{(3xy - x^2)dx + (x^2+y^2)dy\}$ by using Green's theorem is.
A) Zero B) One C) Two D) Three

iv) $\iint_s \vec{f}.nds =$ ________, where $\vec{f} = xi+yj+2k$ and S is the surface of the sphere $x^2+y^2+z^2 = a^2$
A) $4\pi a$ B) $4\pi a^2$ C) $4\pi a^3$ D) 4π

b. Find the work done by a force $\vec{f} = (2y - x^2)i + 6yzj - 8xz^2k$ from the point (0, 0, 0) to the point (1, 1, 1) along the straight-line joining these points. **(04 Marks)**

c. If C is a simple closed curve in the xy-plane, prove by using Green's theorem that the integral $\oint_c \frac{1}{2}(xdy - ydx)$ represents the area A enclosed by C. Hence evaluate $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. **(06 Marks)**

d. Verify Stoke's theorem for $\vec{f} = (2x-y)i - yz^2j - y^2zk$ for the upper half of the sphere $x^2+y^2+z^2 = 1$. **(06 Marks)**

7 a. Choose the correct answers for the following : (04 Marks)

i) $L[t^n] =$ ________

A) $\frac{n}{s^{n+1}}$ B) $\frac{n}{s^{n-1}}$ C) $\frac{n!}{s^{n-1}}$ D) $\frac{n!}{s^{n+1}}$

ii) $L[e^{-3t}] =$ ________

A) $\frac{3}{s-3}$ B) $\frac{3}{s+3}$ C) $\frac{1}{s+3}$ D) $\frac{1}{s-3}$

iii) $L\{f(t-a)H(t-a)\}$ is equal to,

A) $\frac{3!}{(s+2)^4}$ B) $\frac{3!}{(s-2)^4}$ C) $\frac{3}{(s-2)^4}$ D) $\frac{3}{(s-2)}$

iv) $L\{\delta(t-1)\} =$ ________

A) e^{-s} B) e^{s} C) e^{as} D) e^{-as}

b. Evaluate $L\{\sin^3 2t\}$. (06 Marks)

c. Find $L\{f(t)\}$, given that $f(t) = \begin{cases} 2, & 0 < t < 3 \\ 1, & t > 3 \end{cases}$ (06 Marks)

d. Express $f(t) = \begin{cases} t^2 & 0 < t < 2 \\ 4t & 2 < t \le 4 \\ 8 & t > 4 \end{cases}$ in terms of unit step function and hence find their Laplace transform. (04 Marks)

8 a. Choose the correct answers for the following : (04 Marks)

i) $L^{-1}\{\cos at\} =$ ________

A) $\frac{s}{s^2+a^2}$ B) $\frac{s}{s^2-a^2}$ C) $\frac{1}{s^2+a^2}$ D) $\frac{1}{s^2-a^2}$

ii) $L^{-1}\{\bar{f}(s-a)\} =$ ________

A) $e^{t}f(t)$ B) $e^{at}f(t)$ C) $e^{-at}f(t)$ D) None of these

iii) $L^{-1}\left\{\cot^{-1}\left(\frac{2}{s^2}\right)\right\} =$ ________

A) $\frac{\sin t}{t}$ B) $\frac{\sinh at}{t}$ C) $\frac{\sin at}{t}$ D) $\frac{\sinh t}{t}$

iv) For the function $f(t) = 1$, convolution theorem condition.

A) Not satisfied B) Satisfied with some condition
C) Satisfied D) None of these

b. Find the inverse Laplace transform of $\frac{2s^2-6s+5}{(s-1)(s-2)(s-3)}$. (04 Marks)

c. Find $L^{-1}\left\{\frac{s}{(s-1)(s^2+4)}\right\}$ using convolution theorem. (06 Marks)

d. Solve differential equation $y''(t) + y = F(t)$, where $F(t) = \begin{cases} 0 & 0 < t < 1 \\ 2 & t > 1 \end{cases}$. Given that $y(0) = 0 = y'(0)$. (06 Marks)

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