USN

10MAT11

First/Second Semester B.E. Degree Examination, June/July 2013

Engineering Mathematics – I

Time: 3 hrs. Max. Marks: 100

Note: ***1. Answer any FIVE full questions, choosing at least two from each part.***
2. Answer all objective type questions only on OMR sheet page 5 of the answer booklet.
3. Answer to objective type questions on sheets other than OMR will not be valued.

PART – A

1 a. Choose your answers for the following : **(04 Marks)**

i) If $y = 3^{5x}$ then y_n is A) $(3\log 5)^n e^{5x}$ B) $(5\log 3)^n e^{5x}$ C) $(5\log 3)^{-n} e^{5x}$ D) $(5\log 3)^n e^{-5x}$

ii) If $y = \cos^2 x$ then y_n is
A) $2^{n+1}\cos(n\pi/2 + 2x)$ B) $2^{n-1}\cos(n\pi/2 + 2x)$ C) $2^{n-1}\cos(n\pi/2 - 2x)$ D) $2^{n+1}\cos(n\pi/2 - 2x)$

iii) The Lagrange's mean value theorem for the function $f(x) = e^x$ in the interval [0, 1] is
A) $C = 0.5413$ B) $C = 2.3$ C) 0.3 D) None of these

iv) Expansion of $\log(1+e^x)$ in powers of x is ______. A) $\log 2 - x/2 + x^2/8 + x^4/192 + --$
B) $\log 2 + x/2 + x^2/8 - x^4/192 + --$ C) $\log 2 + x/2 + x^2/8 + x^4/192 + --$ D) $\log 2 - \frac{x}{2} - \frac{x^2}{8} - \frac{x^4}{192} + --$

b. If $y^{1/m} + y^{-1/m} = 2x$ prove that $(x^2-1)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$. **(06 Marks)**

c. Verify the Rolle's theorem for the functions : $f(x) = e^x(\sin x - \cos x)$ in $(\pi/4, 5\pi/4)$. **(06 Marks)**

d. By using Maclaarin's theorem expand log sec x up to the term containing x^6. **(04 Marks)**

2 a. Choose your answers for the following : **(04 Marks)**

i) The indeterminate form of $\lim\limits_{x\to 0}\frac{a^x - b^x}{x}$ is A) $\log(b/a)$ B) $\log(a/b)$ C) 1 D) –1

ii) The angle between the radius vector and the tangent for the curves $r = a(1-\cos\theta)$ is
A) $\theta/2$ B) $-\theta/2$ C) $\pi/2 + \theta$ D) $\pi/2 - \theta/2$.

iii) The polar form of a curve is_____ **A)** $r = f(\theta)$ **B)** $\theta = f(y)$ **C)** $r = f(x)$ **D)** None of these

iv) The rate at which the curve is bending called ___. A) Radius of curvature; B) Curvature; C) Circle of curvature; D) Evaluate.

b. Evaluate $\lim\limits_{x\to 0}\left(\frac{\sin x}{x}\right)^{1/x^2}$. **(06 Marks)**

c. Find the angles of intersection of the following pairs of curves, $r = a\theta/(1+\theta)$; $r = a/(1+\theta^2)$. **(06 Marks)**

d. Find the radius of curvature at $(3a/2,\ 3a/2)$ on $x^3 + y^3 = 3axy$. **(04 Marks)**

3 a. Choose your answers for the following : **(04 Marks)**

i) If $u = x^2 + y^2$ then $(\partial^2 u)/(\partial x \partial y)$ is equal to **A)** 2 **B)** 0 **C)** 2x **D)** 2y

ii) If $z = f(x, y)$ where $x = u - v$ and $y = uv$ then $(u+v)(\partial z/\partial x)$ is
A) $u(\partial z/\partial u) - v(\partial z/\partial v)$ B) $u(\partial z/\partial u) + v(\partial z/\partial v)$ C) $\partial z/\partial u + \partial z/\partial v$ D) $\partial z/\partial u - \partial z/\partial v$

iii) If $x = r\cos\theta$, $y = r\sin\theta$ then $[\partial(r,\theta)]/[\partial(x,y)]$ is **A)** r **B)** 1/r **C)** 1 **D)** –1

iv) In errors and approximations $\partial x/x$, $\partial y/y$, $\partial f/f$ are called
A) relative error B) percentage error C) error in x, y and f D) none of these

b. If $x^x y^y z^z = c$, show that $\partial^2 z/\partial x\partial y = -[x \log ex]^{-1}$, when $x = y = z$. **(06 Marks)**

c. Obtain the Jacobian of $\partial(x, y, z)/\partial(r, \theta, \phi)$ for change of coordinate from three dimensional Cartesian coordinates to spherical polar coordinates. **(06 Marks)**

d. In estimating the cost of a pile of bricks measured as 2m×15m×1.2m, the tape is stretched +1% beyond the standard length. If the count is 450 bricks to 1 cu.cm and bricks cost of 530 per 1000, find the approximate error in the cost. **(04 Marks)**

4 a. Choose your answers for the following : **(04 Marks)**

i) If $\vec{R} = xi + yj + zk$ then div $\vec{R}$ **A)** 0 **B)** 3 **C)** –3 **D)** 2

ii) If $\overline{F} = 3x^2 i - xyj + (a-3)xz\,k$ is Solenoidal then a is equal to ____ **A)** 0 **B)** –2 **C)** 2 **D)** 3

iii) If $\overline{F} = (x+y+1)i + j - (x+y)k$ then $\overline{F}.\text{curl}\,\overline{F}$ is ___. A) 0 B) x + y C) x + y + z D) x – y

iv) The scale factors for cylindrical coordinate system (ρ, ϕ, z) are given by
A) $(\rho, 1, 1)$ B) $(1, \rho, 1)$ C) $(1, 1, \rho)$ D) none of these

b. Prove that $\text{curl}\,\overline{A} = \text{g rad}(\text{div}\overline{A}) - \nabla^2 A$. **(06 Marks)**

c. Find the constants a, b, c such that the vector $\overline{F} = (x+y+az)i + (bx+2y-z)j + (x+cy+2z)k$ is irrotational. **(06 Marks)**

d. Derive an expression for $\nabla\cdot\overline{A}$ in orthogonal curvilinear coordinates. Deduce $\nabla\cdot\overline{A}$ is rectangular coordinates.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – B

5 a. Choose your answers for the following : **(04 Marks)**

i) The value of $\int_0^\infty e^{-\alpha x} dx$ is ____ A) $1/e$ B) $-1/e$ C) $1/\alpha$ D) $-1/\alpha$

ii) The value of the integral $\int_0^{\pi/2} \sin^7 x dx$ is A) 35/16 B) 16/35 C) –16/35 D) 18/35

iii) The volume generated by revolving the cardioid $r = a(1 + \cos\theta)$ about the initial line is
A) $(3\pi a^2)/8$ B) $(3\pi a^3)/8$ C) $(2\pi a^2)/9$ D) None

iv) The area of the loop of the curve $r = a \sin 3\theta$ is ___ A) $a^2/12$; B) $\pi/12$; C) $\pi a^2/12$; D) None

b. By applying differential under the integral sign evaluate $\int_0^{\pi/2} \frac{\log(1 + y\sin^2 x)}{\sin^2 x} dx$. **(06 Marks)**

c. Evaluate of $\int_0^{\pi/2} \sin^n x\, dx$ where n is any integer. **(06 Marks)**

d. Find the length of the arch of the cycloid $x = a(\theta - \sin\theta)$; $y = a(1 - \cos\theta)$; $0 < \theta \le 2\pi$. **(04 Marks)**

6 a. Choose your answers for the following : **(04 Marks)**

i) The general solution of the differential equation $(dy/dx) = (y/x) + \tan(y/x)$ is
A) $\sin(y/x) = c$ B) $\sin(y/x) = cx$ C) $\cos(y/x) = cx$ D) $\cos(y/x) = c$

ii) An integrating factor for $ydx - xdy = 0$ is A) x/y B) y/x C) $1/(x^2y^2)$ D) $1/(x^2+y^2)$

iii) The differential equation satisfying the relation $x = A\cos(mt - \alpha)$ is
A) $(dx/dt) = 1 - x^2$ B) $(d^2x/dt^2) = -\alpha^2 x$ C) $(d^2x/dt^2) = -m^2 x$ D) $(dx/dt) = -m^2 x$

iv) The orthogonal trajectories of the system given by $r = a\theta$ is
A) $r^2 = ke^\theta$ B) $r = ke^\theta$ C) $r^2 e^{-\theta^2} = k$ D) $r^2 = k e^{-\theta^2}$

b. Solve $(x\cos(y/x) + y\sin(y/x))y - (y\sin(y/x) - x\cos(y/x))\, x(dy/dx) = 0$. **(06 Marks)**

c. Solve $(1 + y^2) + (x - e^{\tan^{-1} y})dy/dx = 0$. **(06 Marks)**

d. Prove that the system of parabola $y^2 = 4a(x + a)$ is self orthogonal. **(04 Marks)**

7 a. Choose your answers for the following : **(04 Marks)**

i) Find the rank of $\begin{bmatrix} 3 & -1 & 2 \\ -6 & 2 & 4 \\ -3 & 1 & 2 \end{bmatrix}$: A) 3 B) 2 C) 4 D) 1

ii) The exact solution of the system of equation $10x + y + z = 12$, $x + 10y + z = 12$, $x + y + 10z = 12$ by inspection is equal to A) (-1, 1, 1) ; B) (1, 1, 1) ; C) (-1, -1, -1) ; D) None

iii) If the given system of linear equations in 'n' variables is consistent then the number of linearly independent – solution is given by A) n ; B) n – 1 ; C) r – n ; D) n – r

iv) The trivial solution for the given system of equations $9x - y + 4z = 0$, $4x - 2y + 3z = 0$, $5x + y - 6z = 0$ is
A) (1, 2, 0) B) (0, 4, 1) C) (0, 0, 0) D) (1, -5, 0).

b. Using elementary transformation reduce each of following matrices to the normal form, $\begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & -1 & 2 & 5 \\ 3 & 1 & 1 & 8 \\ 2 & -2 & 3 & 7 \end{bmatrix}$. **(06 Marks)**

c. Test for consistency and solve the system, $2x + y + z = 10$, $3x + 2y + 3z = 18$, $x + 4y + 9z = 16$. **(06 Marks)**

d. Apply Gauss-Jordan method to solve the system of equations, $2x + 5y + 7z = 52$, $2x + y - z = 0$, $x + y + z = 9$ **(04 Marks)**

8 a. Choose your answers for the following : **(04 Marks)**

i) A square matrix A is called orthogonal if, A) $A = A^2$ B) $A = A^{-1}$ C) $AA^{-1} = I$ D) None

ii) The eigen values of the matrix, $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ are A) 2, 3, 8 B) 2, 3, 9 C) 2, 2, 8 D) None

iii) The eigen vector X of the matrix A corresponding to eigen value λ and satisfy the equation,
A) $AX = \lambda X$ B) $\lambda(A - X) = 0$ C) $XA - A\lambda = 0$ D) $|A - \lambda I|X = 0$

iv) Two square matrices A and B are similar if, A) $A = B$; B) $B = P^{-1}AP$; C) $A' = B'$; D) $A^{-1} = B^{-1}$

b. Show that the transformation, $y_1 = 2x_1 - 2x_2 - x_3$, $y_2 = -4x_1 + 5x_2 + 3x_3$, $y_3 = x_1 - x_2 - x_3$ is, regular and find the inverse transformations. **(06 Marks)**

c. Diagonalize the matrix, $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$. **(06 Marks)**

d. Reduce the quadratic form, $x_1^2 + 2x_2^2 - 7x_3^2 - 4x_1x_2 + 8x_2x_3$ into sum of squares **(04 Marks)**

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