USN

10CS42

Fourth Semester B.E. Degree Examination, June/July 2015
Graph Theory and Combinatorics

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting atleast TWO questions from each part.***

PART – A

1 a. For the following graph determine,
   i) A walk from b to d that is not a trail
   ii) A b–d trail that is not a path
   iii) A path from b to d
   iv) A closed walk from b to b that is not a circuit
   v) A circuit from b to b that is not a cycle
   vi) A cycle from b to b. **(06 Marks)**

Fig.Q1(a)

b. Define subgraph, spanning subgraph, induced subgraph and complete graph with example. **(07 Marks)**

c. Prove that the undirected graph G = (V, E) has an Euler circuit if and only if G is connected and every vertex in G has even degree. **(07 Marks)**

2 a. Define planar graph and prove that the following Petersen graph is nonplanar using Kuratowski's theorem. **(06 Marks)**

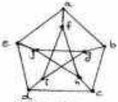

Fig.Q2(a)

b. Prove that in a complete graph with n–vertices, where n is an odd number ≥ 3, there are (n – 1)/2 edge – disjoint Hamiltonian cycles. **(07 Marks)**

c. Find the chromatic polynomial for the following graph. **(07 Marks)**

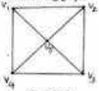

Fig.2Q(c)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

3 a. Prove that in every tree T = (V, E) |V| = |E| + 1. **(06 Marks)**

b. i) If $T_1 = (V_1, E_1)$ and $T_2 = (V_2, E_2)$ be two trees where $|E_1| = 17$ and $|V_2| = 2|V_1|$, then find $|V_1|$, $|V_2|$ and $|E_2|$

ii) Let $F_2 = (V_2, E_2)$ is a forest with $|V_2| = 62$ and $|E_2| = 51$, how many trees determine F_2.

iii) Let $F_1 = (V_1, E_1)$ be a forest of 7 trees where $|E_1| = 40$ what is $|V_1|$? **(07 Marks)**

c. Construct an optimal prefix code for the symbols a, o, q, u, y, z that occur with frequencies 20, 28, 4, 17, 12, 7 respectively. **(07 Marks)**

4 a. Using the Kruskal's algorithm, find a minimal spanning tree of the following weighted graphs. **(06 Marks)**

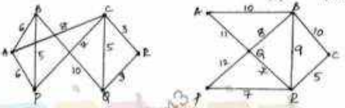

Fig.Q4(a)

b. Using the Dijkstra's algorithm obtain the shortest path from vertex 1 to each of the other vertices in the following graph. **(07 Marks)**

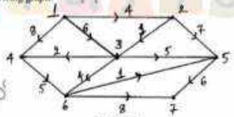

Fig.Q4(b)

c. Prove that in a bipartite graph $G(V_1, V_2, E)$ if there is a positive integer M such that the degree of every vertex in $V_1 \geq M \geq$ the degree of every vertex in V_2, then there exists a complete matching from V_1 to V_2. **(07 Marks)**

PART – B

5 a. i) How many arrangements all there for all letters in the word SOCIOLOGICAL?

ii) In how many of these arrangements, A and G are adjacent?

iii) In how many of these arrangements, all the vowels are adjacent? **(06 Marks)**

b. Determine the co-efficient of :

i) x^9y^3 in the expansion of $(2x - 3y)^{12}$

ii) xyz^2 in the expansion of $(2x - y - z)^4$

iii) $x^2y^2z^3$ in the expansion of $(3x - 2y - 4z)^7$. **(07 Marks)**

c. Determine the number of integer solutions for : $x_1 + x_2 + x_3 + x_4 + x_5 < 40$,

Where :

i) $x_i \geq 0$, $1 \leq i \leq 5$

ii) $x_i \geq -3$, $1 \leq i \leq 5$. **(07 Marks)**

6 a. Find the number of integers between 1 to 10,000 inclusive, which are divisible by none of 5, 6 or 8. **(06 Marks)**

b. Determine in how many ways can the letters in the word ARRANGEMENT be arranged so that there are exactly two pairs of consecutive identical letters. **(07 Marks)**

c. i) Find the rook polynomial for the shaded chessboard

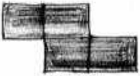

Fig. Q6(c)(i)

ii) Let $A = \{1, 2, 3, 4\}$ and $B = \{u, v, w, x, y, z\}$. How many one to one functions $f : A \to B$ satisfy none of the following conditions :

$C_1 : f(1) = u$ or v ; $C_2 : f(2) = w$; $C_3 : f(3) = w$ or x ; $C_4 : f(4) = x, y$ or z. **(07 Marks)**

7 a. Find the coefficient of x^{12} in $\dfrac{(1+x)^4}{(1-x)^4}$. **(06 Marks)**

b. A ship carries 48 flags, 12 each of the colors red, white, blue and black. Twelve of these flags are placed on a vertical pole inorder to communicate a signal to other ships. Determine, how many of these signals have atleast three white flags or no white flags at all. **(07 Marks)**

c. Find the formula to express : $0^2 + 1^2 + 2^2 + \text{-------} + n^2$ as a function of n using summation on operator. **(07 Marks)**

8 a. Solve the recurrence relation $F_{n+2} = F_{n+1} + F_n$ where $n \geq 0$ and $F_0 = 0$ and $F_1 = 1$. **(06 Marks)**

b. i) A bank pays 6% interest compounded quarterly. If Laura invests $ 100 then how many months must she wait for her money to double?

ii) The number of bacteria in a culture is 1000 and this number increases 250% every 2 hours. Use a recurrence relation to determine the number of bacteria present after one day. **(07 Marks)**

c. Solve the recurrence relation : $a_{n+2} - 5a_{n+1} + 6a_n = 2$, $n \geq 0$, $a_0 = 3$, $a_1 = 7$ using method of generating functions. **(07 Marks)**

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