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14MAT11

First Semester B.E. Degree Examination, Dec.2014/Jan.2015
Engineering Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting ONE full question from each part.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – 1

1 a. If $Y = \cos(m\log x)$, prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (m^2+n^2)y_n = 0$. **(07 Marks)**

b. Find the angle of intersection between the curves $r = a\log\theta$ and $r = \dfrac{a}{\log\theta}$. **(06 Marks)**

c. Derive an expression to find radius of curvature in Cartesian form. **(07 Marks)**

2 a. If $\sin^{-1} y = 2\log(x+1)$ prove that $(x^2+1)y_{n+2} + (2n+1)(x+1)y_{n+1} + (n^2+4)y_n = 0$. **(07 Marks)**

b. Find the pedal equation $r^n = \sec hn\theta$. **(06 Marks)**

c. Show that the radius of curvature of the curve $x^3 + y^3 = 3xy$ at $\left(\dfrac{3}{2}, \dfrac{3}{2}\right)$ is $\dfrac{-3}{8\sqrt{2}}$. **(07 Marks)**

PART – 2

3 a. Find the first four non zero terms in the expansion of $f(x) = \dfrac{x}{e^{x-1}}$. **(07 Marks)**

b. If $\cos u = \dfrac{x+y}{\sqrt{x}+\sqrt{y}}$ show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = -\dfrac{\cot u}{2}$. **(06 Marks)**

c. Find $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$ where $u = x^2+y^2+z^2$, $v = xy+yz+zx$ and $w = x+y+z$. Hence interpret the result. **(07 Marks)**

4 a. If $w = f(x,y)$, $x = r\cos\theta$, $y = r\sin\theta$ show that

$$\left(\frac{\partial t}{\partial x}\right)^2 + \left(\frac{\partial t}{\partial y}\right)^2 - \left(\frac{\partial w}{\partial r}\right)^2 = \frac{1}{r^2}\left(\frac{\partial w}{\partial \theta}\right)^2.$$ **(07 Marks)**

b. Evaluate $\lim\limits_{x\to 0}\left(\dfrac{\sin x}{x}\right)^{\frac{1}{x}}$. **(06 Marks)**

c. Examine the function $f(x, y) = 1+\sin(x^2+y^2)$ for extremum. **(07 Marks)**

PART – 3

5 a. A particle moves along the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$. Find the components of velocity and acceleration at $t = 1$ in the direction $\hat{i} - 2\hat{j} + 2\hat{k}$. **(07 Marks)**

b. Using differentiation under integral sign, evaluate $\displaystyle\int_0^\infty \frac{e^{-\alpha x}\sin x}{x}dx$. **(07 Marks)**

c. Use general rules to trace the curve $y^2(a-x) = x^3$, $a > 0$ **(06 Marks)**

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6 a. If $\vec{v} = \vec{w} \times \vec{r}$, prove that $\text{curl}\,\vec{v} = 2\vec{w}$ where $\vec{w}$ is a constant vector. **(07 Marks)**

b. Show that $\text{div}(\text{curl}\,\vec{A}) = 0$. **(06 Marks)**

c. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $\left|\vec{r}\right| = r$. Find grad div$\left(\frac{\vec{r}}{r}\right)$. **(07 Marks)**

PART – 4

7 a. Obtain the reduction formula for $\int_0^{\frac{\pi}{2}} \cos^n x\,dx$. **(07 Marks)**

b. Solve $(xy^3 + y)dx + 2(x^2y^2 + x + y^4)dy = 0$. **(06 Marks)**

c. Show that the orthogonal trajectories of the family of cardioids $r = a\cos^2\left(\frac{\theta}{2}\right)$ is another family of cardioids $r = b\sin^2\left(\frac{\theta}{2}\right)$. **(07 Marks)**

8 a. Evaluate $\int_0^{\pi} x\sin^2 x\cos^4 x\,dx$. **(07 Marks)**

b. Solve $\frac{dy}{dx} - y\tan x = y^2 \sec x$. **(06 Marks)**

c. If the temperature of the air is 30°C and the substance cools from 100°C to 70°C in 15 minutes, find when the temperature will be 40°C. **(07 Marks)**

PART – 5

9 a. Solve $3x - y + 2z = 12$, $x + 2y + 3z = 11$, $2x - 2y - z = 2$ by Gauss elimination method. **(06 Marks)**

b. Diagonalize the matrix, $A = \begin{bmatrix} -1 & 1 & 2 \\ 0 & -2 & -1 \\ 0 & 0 & -3 \end{bmatrix}$. **(07 Marks)**

c. Determine the largest eigen value and the corresponding eigen vector of $A = \begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}$.

Starting with $[0, \quad 0, \quad 1]^T$ as the initial eigenvector. Perform 5 iterations. **(07 Marks)**

10 a. Show that the transformation $y_1 = x_1 + 2x_2 + 5x_3$, $y_2 = 2x_1 + 4x_2 + 11x_3$, $y_3 = -x_2 + 2x_3$ is regular and find the inverse transformation. **(06 Marks)**

b. Solve by LU decomposition method $2x + y + 4z = 12$, $8x - 3y + 2z = 20$, $4x + 11y - z = 33$. **(07 Marks)**

c. Reduce the quadratic form $2x^2 + 2y^2 - 2xy - 2yz - 2zx$ into canonical form. Hence indicate its nature, rank, index and signature. **(07 Marks)**

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