USN

10MAT21

Second Semester B.E. Degree Examination, June/July 2013

Engineering Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: 1. ***Answer any FIVE full questions, choosing at least two from each part.***
2. ***Answer all objective type questions only in OMR sheet page 5 of the answer booklet.***
3. ***Answer to objective type questions on sheets other than OMR will not be valued.***

PART – A

1 a. Choose the correct answers for the following : **(04 Marks)**

i) A differential equation of the first order but of higher degree, solvable for y, has the solution as
A) $F(x, y, c) = 0$ B) $F(x, c_1, c_2) = 0$
C) $F(x, p, c) = 0$ D) $F_1(x, y, c).F_2(x, y\ c) = 0$

ii) If $c^2x^2 + 1 = 2cy$ is the general solution of a differential equation then its singular solution is
A) $y = x$ B) $y = -x$ C) both (A) and (B) D) none of these

iii) The general solution of the differential equation $p = \log(px - y)$ is
A) $y = px + e^p$ B) $y = px - e^p$ C) $y = px - e^c$ D) $y = cx - e^c$

iv) The differential equation $xp^2 + x = 2yp$ can be solvable for
A) p B) y C) x D) all of these

b. Solve $xyp^2 + p(3x^2 - 2y^2) - 6xy = 0$. **(05 Marks)**

c. Solve $y = p\sin p + \cos p$. **(05 Marks)**

d. Solve $y^2 \log y = xyp + p^2$. **(06 Marks)**

2 a. Choose the correct answers for the following : **(04 Marks)**

i) $\dfrac{1}{f(D)}(e^{3x}x^2) =$

A) $e^{3x}\dfrac{1}{f(D-3)}x^2$ B) $e^{3x}\dfrac{1}{f(D+3)}x^2$ C) $x^2\dfrac{1}{f(D-3)}e^{3x}$ D) $x^2\dfrac{1}{f(D+3)}e^{3x}$

ii) The roots of auxillary equation of $(D^4 + 2D^3 - 5D^2 - 6D)y = 0$ are
A) $-1, -1, 2, -3$ B) $0, -1, 2, -3$ C) $0, 1, -2, 3$ D) $0, -1, 2, 3$

iii) The particular integral of $(-D + 2)^3y = 3e^{2x}$ is

A) $\dfrac{x^3e^{2x}}{3}$ B) $\dfrac{x^3e^{2x}}{2}$ C) $-\dfrac{x^3e^{2x}}{2}$ D) $-\dfrac{x^3e^{2x}}{6}$

iv) If $\dfrac{dx}{dt} - 2y = 0$, $\dfrac{dy}{dt} - 2x = 0$ then y is a function of
A) e^{2t} and e^{-2t} B) e^{2it} and e^{-2it} C) e^t and e^{-2t} D) none of these

b. Solve $(D^3 - 6D^2 + 11D - 6)y = 2^x + \cos 2x$. **(05 Marks)**

c. Solve $(D^2 - 4D + 4)y = 8x^2e^{2x}\sin 2x$. **(05 Marks)**

d. Solve $\dfrac{dx}{dt} + \dfrac{dy}{dt} + 2x + y = 0$, $\dfrac{dy}{dt} + 5x + 3y = 0$. **(06 Marks)**

3 a. Choose the correct answers for the following : (04 Marks)

i) The complementary function of $x^2y'' + 4xy' + 2y = e^x$ is

A) $c_1e^{-x} + c_2e^{-2x}$ B) $c_1(-x) + c_2(-2x)$ C) $c_1e^{-2} + c_2e^{2z}$ D) $\frac{c_1}{x} + \frac{c_2}{x^2}$

ii) If $y = u(x)\cdot 1 + v(x)\cdot e^{2x}$ is a particular integral of $y'' + y = \operatorname{cosec} x$ in the method of variation of parameters then $v(x) =$

A) e^{-x} B) e^{-2x} C) e^{2x} D) $-e^{-x}$

iii) The roots of the auxillary equation of the transformed equation of :

$(2x+1)^2y'' - 2(2x+1)y' - 12y = 6x + 5$ are

A) 3, –1 B) –3, 1 C) 12, –4 D) none of these

iv) Indicial equation is related to

A) singular point B) regular singular point
C) ordinary point D) none of these

b. Solve $(D^2 + 1)y = \operatorname{Tan} x$ by method of variation of parameters. (05 Marks)

c. Solve $x^2y'' - xy' + 2y = x\sin(\log x)$. (05 Marks)

d. Solve $(1 + x^2)y'' + xy' - y = 0$ in series solution. (06 Marks)

4 a. Choose the correct answers for the following : (04 Marks)

i) $z = (x-a)^2 + (y-b)^2$, a and b are arbitrary constants, is a solution of

A) $z = 2p^2 + 2q^2$ B) $4z = p^2 + q^2$ C) $p = 2(x-a)$ D) $q = 2(y-b)$

ii) For z = (x + a) (x + b), z = 0 is a

A) singular solution B) general solution
C) particular solution D) complete solution

iii) Suitable set of multipliers to solve $(y^2 + z^2)p + xyq = zx$.

A) 0, 1, 1 B) x, –y, –z C) $1, -\frac{y}{x}, -\frac{z}{x}$ D) all of these

iv) Taking $Z = X(x)\cdot Y(y)$ is a solution of a partial differential equation then this procedure is called

A) separation of derivatives B) Lagrange's method
C) separation of variables D) Partial separation of variables

b. Form a partial differential equation by eliminating arbitrary function from the relation $z = f\left(\frac{xy}{z}\right)$. (05 Marks)

c. Solve $xp - yq = y^2 - x^2$. (05 Marks)

d. Solve $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$ by the method of separation of variables. (06 Marks)

PART – B

5 a. Choose the correct answers for the following : (04 Marks)

i) $\int_0^1 \int_0^{1-y} (x^2 - y^2)\,dx\,dy =$

A) 0 B) $\frac{1}{12}$ C) $\frac{1}{6}$ D) none of these

5 a. ii) $\int_0^1 \int_0^2 \int_0^{2-x-y} dzdydx =$

A) 3 B) 2 C) 1 D) none of these

iii) $\int_0^1 \left[\log\left(\frac{1}{x}\right)\right]^{\frac{1}{2}} dx =$

A) $\Gamma\left(\frac{1}{2}\right)$ B) $\Gamma\left(\frac{3}{2}\right)$ C) $\Gamma\left(\frac{5}{2}\right)$ D) none of these

iv) $\int_0^{\pi/2} \cos^m x \ dx =$

A) $\frac{1}{2}\beta\left(\frac{m-1}{2}, \frac{1}{2}\right)$ B) $\beta\left(\frac{m+1}{2}, \frac{1}{2}\right)$ C) $\frac{1}{2}\beta\left(\frac{m+1}{2}, \frac{1}{2}\right)$ D) $2\beta\left(\frac{m+1}{2}, \frac{1}{2}\right)$

b. Change into polar coordinates and evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dydx$. **(05 Marks)**

c. Evaluate $\int_{-c}^{c} \int_{-b}^{b} \int_{-a}^{a} (x^2 + y^2 + z^2) dzdydx$. **(05 Marks)**

d. Prove that $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. **(06 Marks)**

6 a. Choose the correct answers for the following : **(04 Marks)**

i) Which theorem gives a relation between surface integral and volume integral?

A) Green's B) Stoke's C) Divergence D) None of these

ii) If c is x + y = 1 from (0, 1) to (1, 1) then $\int_C (y^2 dx + x^2 dy) =$

A) 0 B) 1 C) 2 D) 3

iii) The work done by the force $\overline{F} = yI + xJ + zK$ moves a particle from (0, 0, 0) to (2, 1, 1) along the curve $x = t^2$, $y = t$, $z = 0$ is

A) $3t^2$ B) 0 C) 1 D) none of these

iv) If S is any closed surface enclosing the volume, V then by Divergence theorem, the value of $\int_S \overline{R} \cdot d\overline{S}$ is

A) V B) 2V C) 3V D) none of these

b. Use Green's theorem to evaluate $\int_C [(y - \sin x)dx + \cos x dy]$ where c is enclosed by $y = 0$, $x = \frac{\pi}{2}$, $y = \frac{2}{\pi}x$. **(05 Marks)**

c. Use Stoke's theorem to evaluate $\int_S \text{curl}\,\overline{F} \cdot d\overline{S}$ where $\overline{F} = yI + (x - 2xz)J - xyK$ and S is the surface of the sphere $x^2 y^2 + z^2 = a^2$ above the xy-plane. **(05 Marks)**

d. By transforming to a triple integral, evaluate $\int_S \{x^3 dydz + x^2 ydzdx + x^2 zdxdy\}$ where S is the closed surface bounded by the planes $z = 0$, $z = b$ and the cylinder $x^2 + y^2 = a^2$. **(06 Marks)**

7 a. Choose the correct answers for the following : **(04 Marks)**

i) $L(2\cosh 2t) =$

A) $\frac{4}{s^2-4}$ B) $\frac{4s}{s^2-4}$ C) $\frac{2s}{s^2-4}$ D) none of these

ii) $L\left(\frac{\sin t}{t}\right) =$

A) $\cot^{-1} s$ B) $\frac{1}{s^2+1}$ C) $\tan^{-1} s$ D) $\cot^{-1}(s-1)$

iii) $L(f'(t)) =$

A) $s\,f(t) - f(0)$ B) $s\,f'(s) - f(0)$ C) $f(s) - f(0)$ D) none of these

iv) $L(\sin 2t \cdot \delta(t-2)) =$

A) $e^{2s}\sin 4$ B) $e^{-2s}\sin 2$ C) $e^{-4s}\sin 2$ D) $e^{-2s}\sin 4$

b. Prove that $L(t^n) = \frac{n!}{s^{n+1}}$ if n is a positive integer. **(05 Marks)**

c. Find $L\left(\frac{e^{-t}\sin t}{t}\right)$ and hence find $\int_0^\infty \frac{e^{-t}\sin t}{t}\,dt$. **(05 Marks)**

d. Express : $f(t) = t-1, \quad 1 < t < 2$

$= -t-3, \quad 2 < t < 3$

$= 0, \quad$ otherwise

in terms of unit step function and hence find $L(f(t))$. **(06 Marks)**

8 a. Choose the correct answers for the following : **(04 Marks)**

i) $L^{-1}(s^{-5/2}) =$

A) $\frac{2t^{3/2}}{\sqrt{\pi}}$ B) $\frac{4t^{3/2}}{3\sqrt{\pi}}$ C) $\frac{8t^{3/2}}{15\sqrt{\pi}}$ D) none of these

ii) $L^{-1}(\bar{f}(s)\cdot\bar{g}(s)) =$

A) $f(t)\cdot g(t)$ B) $\int_0^t f(u)g(t-u)\,du$ C) $\int_0^t f(t-u)g(u)\,du$ D) either (B) or (C)

iii) $L^{-1}\left(\frac{1}{s^2+5}\right) =$

A) $\frac{1}{5}\sin\sqrt{t}$ B) $\frac{1}{\sqrt{5}}\sin\sqrt{5t}$ C) $\frac{1}{\sqrt{5}}\sin\sqrt{5}\,t$ D) $\sin\sqrt{5}\,t$

iv) $L^{-1}\left(\int_s^\infty F(s)\,ds\right) =$

A) $t\,f(t)$ B) $\frac{f(t)}{t}$ C) $\frac{f(s)}{s}$ D) none of these

b. Find $L^{-1}\left\{\log\frac{s+1}{s-1}\right\}$. **(05 Marks)**

c. Find $L^{-1}\left[\frac{1}{4s^2-9}\right]$ by using convolution theorem. **(05 Marks)**

d. Solve by using Laplace transformation $y''' + 2y'' - y' - 2y = 0$ where $y = 1$, $\frac{dy}{dt} = 2 = \frac{d^2y}{dt^2}$ at $t = 0$. **(06 Marks)**