

Fourth Semester B.E. Degree Examination, Dec.2014/Jan.2015
Graph Theory and Combinatorics

Time: 3 hrs.

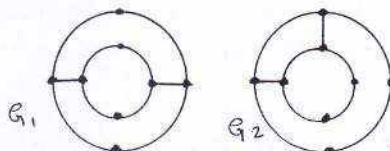
Max. Marks:100

Note: Answer any FIVE full questions, selecting atleast TWO questions from each part.

PART – A

- 1 a. Define graph isomorphism and isomorphic graphs. Determine whether the following graphs are isomorphic or not: (05 Marks)

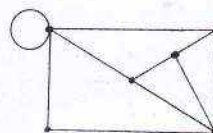
Fig.Q.1(a)



- b. Define complement of a simple graph. Let G be a simple graph of order n . If the size of G is 56 and the size of \bar{G} is 80. What is n ? (05 Marks)
- c. Let $G = (V, E)$ be a connected undirected graph. What is the largest possible value for $|V|$ if $|E| = 19$ and $\deg(v) \geq 4$ for all $v \in V$? (04 Marks)
- d. Write a note on “Konigsberg bridge problem and its solution”. (06 Marks)

- 2 a. Define planar graph. Prove that the Peterson graph is nonplanar. (05 Marks)
- b. Define Hamilton cycle. How many edge disjoint Hamilton cycles exist in the complete graph with seven vertices? Also draw the graph to show these Hamilton cycles. (05 Marks)
- c. Define dual of a planar graph. Construct the dual of the planar graph given in Fig.Q.2(c). (04 Marks)

Fig.Q.2(c)



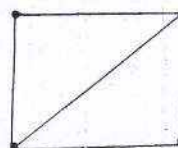
- d. Define chromatic number and chromatic polynomial. Determine the chromatic polynomial for the graph show in Fig.Q.2(d). (06 Marks)

Fig.Q.2(d)



- 3 a. A class room contains 10 micro computer that are to be connected to a wall socket that has 2 outlets. Connections are made by using extension cords that have 2 outlets each. Find the least number of cords needed to get these computer set up for use. (04 Marks)
- b. Apply merge sort to the list -1, 0, 2, -2, 3, 6, -3, 5, 1, 4. (04 Marks)
- c. Find all the spanning trees of the graph shown in Fig.Q.3(c). Also find all the non isomorphic spanning trees. (06 Marks)

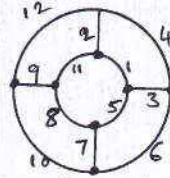
Fig.Q.3(c)



- d. Obtain an optimal prefix code for the message MISSION SUCCESSFUL. Indicate the code for the message. (06 Marks)

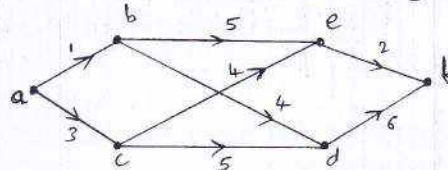
- 4 a. State Krushkal's algorithm. Using Krushkal's algorithm find a minimal spanning tree for the weighted graph shown in Fig.Q.4(a). (08 Marks)

Fig.Q.4(a)



- b. Apply Dijkstra's algorithm the diagram shown in Fig.Q.4(b) and determine the shortest distance from vertex a to each of the other vertices in the directed graph. (06 Marks)

Fig.Q.4(b)



- c. Define the following with one example for each: i) Cut set; ii) Edge connectivity; iii) Vertex connectivity. (06 Marks)

PART - B

- 5 a. A bit is either 0 or 1. A byte is a sequence of 8 bits. Find: i) The number of bytes; ii) The number of bytes that begin with 11 and end 11; iii) the number of bytes that begin with 11 and do not end with 11 and iv) the number of bytes that begin 11 or end with 11. (06 Marks)
- b. How many arrangements of the letters in MISSISSIPPI have no consecutive S's? (05 Marks)
- c. Find the coefficient of x^0 in the expansion of $3\left(x^2 - \frac{2}{x}\right)^{15}$. (05 Marks)
- d. In how many ways can we distribute 7 apples and 6 oranges among 4 children so that each child gets at least 1 apple? (04 Marks)
- 6 a. How many integers between 1 and 300 (inclusive) are
i) Divisible by at least one of 5, 6, 8?
ii) Divisible by none of 5, 6, 8? (06 Marks)
- b. Define derangement. Find the number of derangements of 1, 2, 3, 4. List all the derangements. (06 Marks)
- c. Five teachers T_1, T_2, T_3, T_4, T_5 are to be made class teachers for five classes, C_1, C_2, C_3, C_4, C_5 one teacher for each class. T_1 and T_2 do not wish to become the class teachers for C_1 or C_2 , T_3 and T_4 for C_4 or C_5 , and T_5 for C_3 or C_4 or C_5 . In how many ways can the teachers be assigned the work? (08 Marks)
- 7 a. Find the generating function for the sequence 8, 26, 54, 92.... (06 Marks)
- b. Using generating function, find the number of i) non negative and ii) positive integer solutions of the equation $x_1 + x_2 + x_3 + x_4 = 25$. (08 Marks)
- c. Define exponential generating functions using exponential generating function find the number of ways in which 5 of the letters in the word CALCULUS be arranged. (06 Marks)
- 8 a. The number of bacteria in a culture is 1000 (approximately) and this number increases 250% every two hours. Use a recurrence relation to determine the number of bacteria present after one day. (05 Marks)
- b. Solve the recurrence relation $a_{n+2} - 4a_{n+1} + 3a_n = -200$, $n \geq 0$ and $a_0 = 3000$, $a_1 = 3300$. (07 Marks)
- c. Find the generating function for the recurrence relation $a_{n+1} - a_n = n^2$, $n \geq 0$ and $a_0 = 1$. (08 Marks)
