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10MAT11

First Semester B.E. Degree Examination, Dec.2013/Jan.2014
Engineering Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: 1. ***Answer any FIVE full questions, choosing at least two from each part.***
2. ***Answer all objective type questions only in OMR sheet page 5 of the answer booklet.***
3. ***Answer to objective type questions on sheets other than OMR will not be valued.***

PART – A

1 a. Choose the correct answers for the following :

i) If $y = \frac{x+2}{x+1}$, then y_n is

A) $\frac{(-1)^n(n+1)!}{(x+1)^{n+1}}$ B) $\frac{(-1)^n n!}{(x+1)^{n+1}}$ C) $\frac{(-1)^n n!}{(x+1)^n}$ D) $\frac{(-1)^{n-1} n!}{(x+1)^{n+1}}$

ii) If $y = (ax+b)^m$ with $m = n$, then y_n is

A) $n!\, a^n$ B) 0 C) $n!\, b^n$ D) $n!$

iii) The geometrical interpretation of Lagrange's mean value theorem is

A) $f'(c) = \frac{f(b)-f(a)}{b-a}$ B) $f'(c) = \frac{f(b)+f(a)}{b-a}$ C) $\frac{f'(c)}{g'(c)} = \frac{f(b)-f(a)}{g(b)-g(a)}$ D) none of these

iv) The Maclaurin's series expansion of e^{-x} is

A) $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} - - - -$ B) $1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + - - - -$

C) $x - \frac{x^2}{2!} + \frac{x^3}{3!} - - - -$ D) $x + \frac{x^2}{2!} + \frac{x^3}{3!} + - - - -$ **(04 Marks)**

b. If $y = \sin\log(x^2+2x+1)$, prove that $(x+1)^2 y_{n+2} + (2n+1)(x+1)y_n + (n^2+4)y_n = 0$. **(04 Marks)**

c. If x is positive, show that $x > \log(1+x) > x - \frac{1}{2}x^2$. **(06 Marks)**

d. Using Maclourin's series, expand $\log(1+e^x)$ upto the terms containing x^4. **(06 Marks)**

2 a. Choose the correct answers for the following :

i) $\lim_{x\to\frac{\pi}{4}} \left(\frac{1-\tan x}{\frac{\pi}{4} - x}\right)$ is equal to

A) 2 B) – 2 C) 1 D) – 1

ii) If ϕ be the angle between the tangent and radius vector at any point on the curve $r = f(\theta)$, then $\sin\phi$ is equal to

A) dr/ds B) $r\frac{d\theta}{ds}$ C) $r\frac{d\theta}{dr}$ D) ds/dr

iii) The rate at which the curve is bending called

A) radius of curvature B) curvature C) circle of curvature D) evolute

iv) The radius of curvature for polar curve $r = f(\theta)$ is given by

A) $\frac{(r^2+r_1^2)^{3/2}}{r^2+r_1^2-rr_2}$ B) $\frac{(r^2+r_1^2)^{3/2}}{r_1^2+2r^2-rr_2}$ C) $\frac{(r^2+r_1^2)^{3/2}}{r^2+2r_1^2-rr_2}$ D) $\frac{(r^2-r_1^2)^{3/2}}{r^2+2r_1^2-rr_2}$. **(04 Marks)**

b. Find the Pedal equation of the curve $r^m = a^m \cos m\theta$. **(04 Marks)**

c. Find the radius of curvature for the curve $y^2 = \frac{a^2(a-x)}{x}$, where the curve meets the x – axis. **(06 Marks)**

d. Evaluate $\lim_{x\to\infty}\left(\frac{ax+1}{ax-1}\right)^x$. **(06 Marks)**

3 a. Choose the correct answers for the following :

i) If $u = \log(x^2 + y^2 + z^2)$, then $\frac{\partial u}{\partial z}$ is

A) $\frac{2x}{x^2+y^2+z^2}$ B) $\frac{2y}{x^2+y^2+z^2}$ C) $\frac{2z}{x^2+y^2+z^2}$ D) $\frac{2z}{x^2+y^2-z^2}$

ii) If $u = f(x, y)$ and y is a function x, then

A) $\frac{du}{dx} = \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}\frac{dy}{dx}$ B) $\frac{\partial u}{\partial x} = \frac{du}{dx} + \frac{\partial u}{\partial y}\frac{dy}{dx}$

C) $\frac{du}{dx} = \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial x}$ D) $\frac{\partial u}{\partial x} = \frac{du}{dx} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial x}$

iii) If $r = \frac{\partial^2 f}{\partial x^2}, S = \frac{\partial^2 f}{\partial x \partial y}$ & $t = \frac{\partial^2 f}{\partial y^2}$, then the condition for the saddle point is

A) $rt - s^2 < 0$ B) $rt - s^2 = 0$ C) $rt - s^2 > 0$ D) $rt - s^2 \neq 0$

iv) If $u = x + y + z$, $v = y + z$, $z = z$, then $J\left(\frac{u, v, z}{x, y, z}\right)$ is equal to

A) 1 B) −1 C) 0 D) none of these **(04 Marks)**

b. The focal length of a mirror is given by the formula $\frac{1}{v} - \frac{1}{u} = \frac{2}{f}$. If equal errors, 'e' are made in the determination of u and v. show that the resulting error in f is $e\left(\frac{1}{u} + \frac{1}{v}\right)$. **(04 Marks)**

c. If $u = f(2x - 3y, 3y - 4z, 4z - 2x)$, prove that $\frac{1}{2}\frac{\partial u}{\partial x} + \frac{1}{3}\frac{\partial u}{\partial y} + \frac{1}{4}\frac{\partial u}{\partial z} = 0$. **(06 Marks)**

d. If $x = u(1 - v)$, $y = uv$. Prove that $JJ' = 1$. **(06 Marks)**

4 a. Choose the correct answers for the following :

i) Directional derivative is maximum along

A) tangent to the surface B) normal to the surface

C) any unit vector D) coordinate axes

ii) If $r = |x_i + y_j + 2_k|$, then ∇r^n is

A) nr^{n-1} B) r^{n-1} C) $\nabla.\nabla r^n$ D) none of these

iii) If $f = 3x^2 - 3y^2 + 4z^2$, then curl (grad f) is

A) $4x - 6y + 8z$ B) $4x_i - 6y_j + 8z\,k$ C) $\vec{0}$ D) 3

iv) If the base vectors e_1 and e_2 are orthogonal then $|e_1 \times e_2|$ is

A) 0 B) − 1 C) + 1 D) none of these **(04 Marks)**

b. If $\vec{F} = (x + y + 1)i + j - (x + y)k$, show that $\vec{F} \cdot \text{crul}\,\vec{F} = 0$. **(04 Marks)**

c. Find constants 'a' and 'b' such that $\vec{F} = (axy + z^3)i + (3x^2 - z)j + (bxz^2 - y)k$ is irrotational. Also find a scalar function ϕ such that $\vec{F} = \nabla\phi$. **(06 Marks)**

d. Prove that a spherical coordinate system is orthogonal. **(06 Marks)**

PART – B

5 a. Choose the correct answers for the following :

i) $\int_0^{\pi} \sin^7 x \, dx$ is equal to

A) zero B) $\frac{32\pi}{35}$ C) $\frac{32}{35}$ D) $=\frac{35\pi}{32}$

ii) The asymptote of $(2-x)y^2 = x^3$ is
A) $x = 2$ B) y – axis C) x – axis D) none of these

iii) The area of the cordioid $r = a(1 - \cos\theta)$ is

A) $\frac{3\pi a^2}{2}$ B) $\frac{3\pi}{2}$ C) $\frac{a^2}{2}$ D) $\frac{3a^2}{2}$

iv) The entire length f the asteroid $x^{2/3} + y^{2/3} = a^{2/3}$ is
A) 6a B) 3a C) 2a D) a. **(04 Marks)**

b. Evaluate $\int_0^{\pi} \log(1 + a\cos x)dx$ by differentiating under the integral sign. **(04 Marks)**

c. Evaluate $\int_0^{2a} x^2\sqrt{2ax - x^2}\,dx$, using reduction formula. **(06 Marks)**

d. Find the volume of generated by the revolution of the curve $r = a(1 + \cos\theta)$ about the initial line. **(06 Marks)**

6 a. Choose the correct answers for the following :

i) The general solution of the differential equation $dy/dx = (y/x) + \tan(y/x)$ is
A) $\sin(y/x) = c$ B) $\sin(y/x) = cx$ C) $\cos(y/x) = cx$ D) $\cos(y/x) = c$

ii) The family of straight lines passing through the origin is represented by the differential equation :
A) $ydx + xdy = 0$ B) $xdy - ydx = 0$ C) $xdx + ydy = 0$ D) $ydy - xdx = 0$

iii) The homogeneous differential equation $Mdx + Ndy = 0$ can be reduced to a differential equation, in which the variables are separated by the substitution
A) $y = vx$ B) $x + y = v$ C) $xy = v$ D) $x - y = v$

iv) The equation $y - 2x = c$ represents the orthogonal trajectories of the family
A) $y = ae^{-2x}$ B) $x^2 + 2y^2 = a$ C) $xy = a$ D) $x + 2y = a$
(04 Marks)

b. Solve $(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2$. **(04 Marks)**

c. Solve $(1 + xy)\,ydx + (1 - xy)\,xdy = 0$. **(06 Marks)**

d. Find the orthogonal trajectory of the cordioids $r = a(1 - \cos\theta)$. **(06 Marks)**

7 a. Choose the correct answers for the following :

i) If every minor of order 'r' of a matrix A is zero, then rank of A is

A) greater than r B) equal r C) less than or equal to r D) less than r.

ii) The trivial solution for the given system of equations $x + 2y + 3z = 0$, $3x + 4y + 4z = 0$, $7x + 10y + 12z = 0$ is

A) (1, 1, 1) B) (1, 0, 0) C) (0, 1, 0) D) (0, 0, 0)

iii) Matrix has a value. This statement

A) is always true B) depends upon the matrices C) is false D) none of these

iv) If A is singular and $\rho(A) = \rho(A : B)$ then the system has

A) unique solution B) infinitely many solution C) trivial solution D) no solution. **(04 Marks)**

b. Using elementary transformations, find the rank of the matrix

$$\begin{bmatrix} 1 & 2 & 0 & -1 \\ 3 & 4 & 1 & 2 \\ -2 & 3 & 2 & 5 \end{bmatrix}.$$ **(04 Marks)**

c. Show that the system $x + y + z = 4$; $2x + y - z = 1$; $x - y + 2z = 2$ is consistent, solve the system. **(06 Marks)**

d. Apply Gauss – Jordan method to solve the system of equation :
$2x + 5y + 7z = 52$; $2x + y - z = 0$; $x + y + z = 9$. **(06 Marks)**

8 a. Choose the correct answers for the following :

i) A square matrix A is called orthogonal, if

A) $A = A^L$ B) $A^T = A^{-1}$ C) $AA^{-1} = I$ D) none of these

ii) The eigen values of the matrix $\begin{bmatrix} 2 & \sqrt{2} \\ \sqrt{2} & 2 \end{bmatrix}$ are

A) $1 \pm \sqrt{6}$ B) $1 \pm \sqrt{5}$ C) $\sqrt{5}$ D) 1

iii) The index and signature of the quadratic form $x_1^2 + 2X_2^2 - 3X_3^2$ are respectively

A) 2, 1 B) 1, 2 C) 1, 1 D) none of these

iv) Two square matrices A and B are similar, if

A) A = B B) $B = P^{-1}AP$ C) $A^T = B^T$ D) $A^{-1} = B^{-1}$. **(04 Marks)**

b. Reduce the quadratic form $8x^2 + 7y^2 + 3z^2 - 12yz + 4zx - 8xy$ to the canonical form **(04 Marks)**

c. Determine the characteristic roots and eigen vectors of

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}.$$ **(06 Marks)**

d. Reduce the quadratic form $x_1^2 + 2x_2^2 - 7x_3^2 - 4x_1x_2 + 8x_2x_3$ into sum of squares. **(06 Marks)**

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