USN

10MAT41

Fourth Semester B.E. Degree Examination, December 2012

Engineering Mathematics – IV

Time: 3 hrs. Max. Marks: 100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Using the Taylor's series method, solve the initial value problem $\frac{dy}{dx} = x^2y - 1$, $y(0) = 1$ at the point $x = 0.1$ **(06 Marks)**

b. Employ the fourth order Runge-Kutta method to solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$, $y(0) = 1$ at the points $x = 0.2$ and $x = 0.4$. Take $h = 0.2$. **(07 Marks)**

c. Given $\frac{dy}{dx} = xy + y^2$, $y(0) = 1$, $y(0.1) = 1.1169$, $y(0.2) = 1.2773$, $y(0.3) = 1.5049$. Find $y(0.4)$ using the Milne's predictor-corrector method. Apply the corrector formula twice. **(07 Marks)**

2 a. Employing the Picard's method, obtain the second order approximate solution of the following problem at $x = 0.2$.
$$\frac{dy}{dx} = x + yz,\quad \frac{dz}{dx} = y + zx,\quad y(0) = 1,\quad z(0) = -1.$$ **(06 Marks)**

b. Using the Runge-Kutta method, find the solution at $x = 0.1$ of the differential equation $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} - 2xy = 1$ under the conditions $y(0) = 1$, $y'(0) = 0$. Take step length $h = 0.1$. **(07 Marks)**

c. Using the Milne's method, obtain an approximate solution at the point $x = 0.4$ of the problem $\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} - 6y = 0$, $y(0) = 1$, $y'(0) = 0.1$. Given that $y(0.1) = 1.03995$, $y(0.2) = 1.138036$, $y(0.3) = 1.29865$, $y'(0.1) = 0.6955$, $y'(0.2) = 1.258$, $y'(0.3) = 1.873$. **(07 Marks)**

3 a. If $f(z) = u + iv$ is an analytic function, then prove that $\left(\frac{\partial}{\partial x}|f(z)|\right)^2 + \left(\frac{\partial}{\partial y}|f(z)|\right)^2 = |f'(z)|^2$. **(06 Marks)**

b. Find an analytic function whose imaginary part is $v = e^x\{(x^2 - y^2)\cos y - 2xy\sin y\}$. **(07 Marks)**

c. If $f(z) = u(r, \theta) + iv(r, \theta)$ is an analytic function, show that u and v satisfy the equation $\frac{\partial^2\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial\varphi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\varphi}{\partial\theta^2} = 0$. **(07 Marks)**

4 a. Find the bilinear transformation that maps the points 1, i, –1 onto the points i, 0, –i respectively. **(06 Marks)**

b. Discuss the transformation $W = e^z$. **(07 Marks)**

c. Evaluate $\int_C \frac{\sin\pi z^2 + \cos\pi z^2}{(z-1)^2(z-2)}dz$, where C is the circle $|z| = 3$. **(07 Marks)**

PART – B

5 a. Express the polynomial $2x^3 - x^2 - 3x + 2$ in terms of Legendre polynomials. (06 Marks)

b. Obtain the series solution of Bessel's differential equation $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - n^2)y = 0$ in the form $y = AJ_n(x) + BJ_{-n}(x)$. (07 Marks)

c. Derive Rodrique's formula $P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2 - 1)^n$. (07 Marks)

6 a. State the axioms of probability. For any two events A and B, prove that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. (06 Marks)

b. A bag contains 10 white balls and 3 red balls while another bag contains 3 white balls and 5 red balls. Two balls are drawn at ransom from the first bag and put in the second bag and then a ball is drawn at random from the second bag. What is the probability that it is a white ball? (07 Marks)

c. In a bolt factory there are four machines A, B, C, D manufacturing respectively 20%, 15%, 25% 40% of the total production. Out of these 5%, 4%, 3% and 2% respectively are defective. A bolt is drawn at random from the production and is found to be defective. Find the probability that it was manufactured by A or D. (07 Marks)

7 a. The probability distribution of a finite random variable X is given by the following table:

x_i	-2	-1	0	1	2	3
$p(x_i)$	0.1	k	0.2	2k	0.3	k

Determine the value of k and find the mean, variance and standard deviation. (06 Marks)

b. The probability that a pen manufactured by a company will be defective is 0.1. If 12 such pens are selected, find the probability that (i) exactly 2 will be defective, (ii) at least 2 will be defective, (iii) none will be defective. (07 Marks)

c. In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation, given that A(0.5) = 0.19 and A(1.4) = 0.42, where A(z) is the area under the standard normal curve from 0 to z >0. (07 Marks)

8 a. A biased coin is tossed 400 times and head turns up 120 times. Find the 95% confidence limits for the proportion of heads turning up in infinitely many tosses. (Given that $z_c = 1.96$) (06 Marks)

b. A certain stimulus administered to each of 12 patients resulted in the following change in blood pressure:

5, 2, 8, -1, 3, 0, 6, -2, 1, 5, 0, 4 (in appropriate unit)

Can it be concluded that, on the whole, the stimulus will change the blood pressure. Use $t_{0.05}(11) = 2.201$. (07 Marks)

c. A die is thrown 60 times and the frequency distribution for the number appearing on the face x is given in the following table:

x	1	2	3	4	5	6
Frequency	15	6	4	7	11	17

Test the hypothesis that the die is unbiased.
(Given that $\chi^2_{0.05}(5) = 11.07$ and $\chi^2_{0.01}(5) = 15.09$) (07 Marks)

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