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USN | | | | | | | | | | **10MAT41**

Fourth Semester B.E. Degree Examination, Dec.2013/Jan.2014

Engineering Mathematics – IV

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Employ Taylor's series method to obtain the value of y at x = 0.1 and 0.2 for the differential equation $\frac{dy}{dx} = 2y + 3e^x$, $y(0) = 0$ considering upto fourth degree term. **(06 Marks)**

b. Determine the value of y when x = 0.1, given that $y(0) = 1$ and $y'' = x^2 + y^2$ using modified Euler's formula. Take h = 0.05. **(07 Marks)**

c. Apply Adams-Bashforth method to solve the equation $\frac{dy}{dx} = x^2(1+y)$, given $y(1) = 1$, $y(1.1) = 1.233$, $y(1.2) = 1.548$, $y(1.3) = 1.979$. Evaluate $y(1.4)$. **(07 Marks)**

2 a. Solve $\frac{dy}{dx} = 1 + zx$, $\frac{dz}{dx} = -xy$, $y(0) = 0$, $z(0) = 1$ at x = 0.3 by taking h = 0.3. Applying Runge-Kutta method of fourth order. **(06 Marks)**

b. Applying Picard's method to compute y(1.1) from the second approximation to the solution of the differential equation $y'' + y^2y' = x^3$. Given that $y(1) = 1$, $y'(1) = 1$. **(07 Marks)**

c. Using the Milne's method obtain an approximate solution at the point x = 0.8 of the problem $\frac{d^2y}{dx^2} = 1 - 2y\frac{dy}{dx}$, give that $y(0) = 0$, $y'(0) = 0$, $y(0.2) = 0.02$, $y'(0.2) = 0.1996$, $y(0.4) = 0.0795$, $y'(0.4) = 0.3937$, $y(0.6) = 0.1762$, $y'(0.6) = 0.5689$. **(07 Marks)**

3 a. Derive Cauchy-Riemann equations in Cartesian form. **(06 Marks)**

b. Give $u - v = (x - y)(x^2 + 4xy + y^2)$ find the analytic function $f(z) = u + iv$. **(07 Marks)**

c. If $f(z) = u + iv$ is an analytic function then prove that $\left(\frac{\partial}{\partial x}|f(z)|\right)^2 + \left(\frac{\partial}{\partial y}|f(z)|\right)^2 = |f'(z)|^2$ **(07 Marks)**

4 a. Find the image of the straight lines parallel to coordinate axes in z-plane under the transformation $w = z^2$. **(06 Marks)**

b. Find the bilinear transformation which maps the points $z = 1, i, -1$ on to the points $w = 0, 1, \infty$. **(07 Marks)**

c. Evaluate $\int_c \frac{e^{2z}}{(z+1)(z+2)}$, where c is the circle $|z| = 3$. **(07 Marks)**

PART – B

5 a. Find the solution of the Laplace equation in cylindrical system leading to Besseis differential equation. **(06 Marks)**

b. If α and β are two distinct roots of $J_n(x) = 0$, then prove that $\int_0^1 x\, J_n(\alpha x)\, J_m(\beta x)\, dx = 0$, $m \neq n$. **(07 Marks)**

c. Express $f(x) = x^4 - 2x^3 + 3x^2 - 4x + 5$ in terms of legendre polynomial. **(07 Marks)**

6 a. A committee consists of 9 students, 2 from first year, 3 from second year and 4 from third year. 3 students are to be removed at random. What is the probability that (i) 3 students belongs to different class (ii) 2 belongs to the same class and third belongs to different class. (iii) All the 3 belongs to the same class. **(06 Marks)**

b. State and prove Baye's theorem. **(07 Marks)**

c. The chance that a doctor will diagnose a disease correctly is 60%. The chance that a patient will die after correct diagnose is 40% and the chance of death after wrong diagnose is 70%. If a patient dies, what is the chance that disease was correctly diagnosed. **(07 Marks)**

7 a. The probability distribution of finite random variable x is given by the following table:

x :	0	1	2	3	4	5	6	7
p(x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

Find k, $p(x < 6)$, $p(x \geq 6)$, $p(3 < x \leq 6)$ **(06 Marks)**

b. Obtain the mean and variance of Poisson distribution. **(07 Marks)**

c. The life of an electric bulb is normally distributed with average life of 2000 hours and standard deviation of 60 hours. Out of 2500 bulbs, find the number of bulbs that are likely to last between 1900 and 2100 hours. Given that $p(0 < z < 1.67) = 0.4525$. **(07 Marks)**

8 a. Explain the following terms:
i) Null hypothesis (ii) Type I and Type II error (iii) Confidence limits. **(06 Marks)**

b. The weight of workers in a large factory are normally distributed with mean 68 kgs. and standard deviation 3 kgs. If 80 samples consisting of 35 workers each are chosen, how many of 80 samples will have the mean between 67 and 68.25 kgs. Given $p(0 < z < 2) = 0.4772$ and $p(0 \leq z \leq 0.5) = 0.1915$. **(07 Marks)**

c. Eleven students were given a test in statistics. They were provided additional coaching and then a second test of equal difficulty was held at the end of coaching. Marks scored by then in the two tests are given below.

Test I	23	20	19	21	18	20	18	17	23	16	19
Test II	24	19	22	18	20	22	20	20	23	20	17

Do the marks give evidence that the student have benefited by extra coaching? Given $t_{0.05}(10) = 2.228$. Test the hypothesis at 5% level of significance. **(07 Marks)**

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