USN

10CS42

Fourth Semester B.E. Degree Examination, June/July 2013
Graph Theory and Combinatorics

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. i) Define connected graph. Give an example of a connected graph G where removing any edge of G results in a disconnected graph.

ii) Define complement of a graph. Find an example of a self-complementary graph on four vertices and one on five vertices. **(06 Marks)**

b. Find all (loop-free) non-isomorphic undirected graphs with four vertices. How many of these graphs are connected? **(05 Marks)**

c. Show that the following graphs in Fig. Q1 (c) are isomorphic: **(05 Marks)**

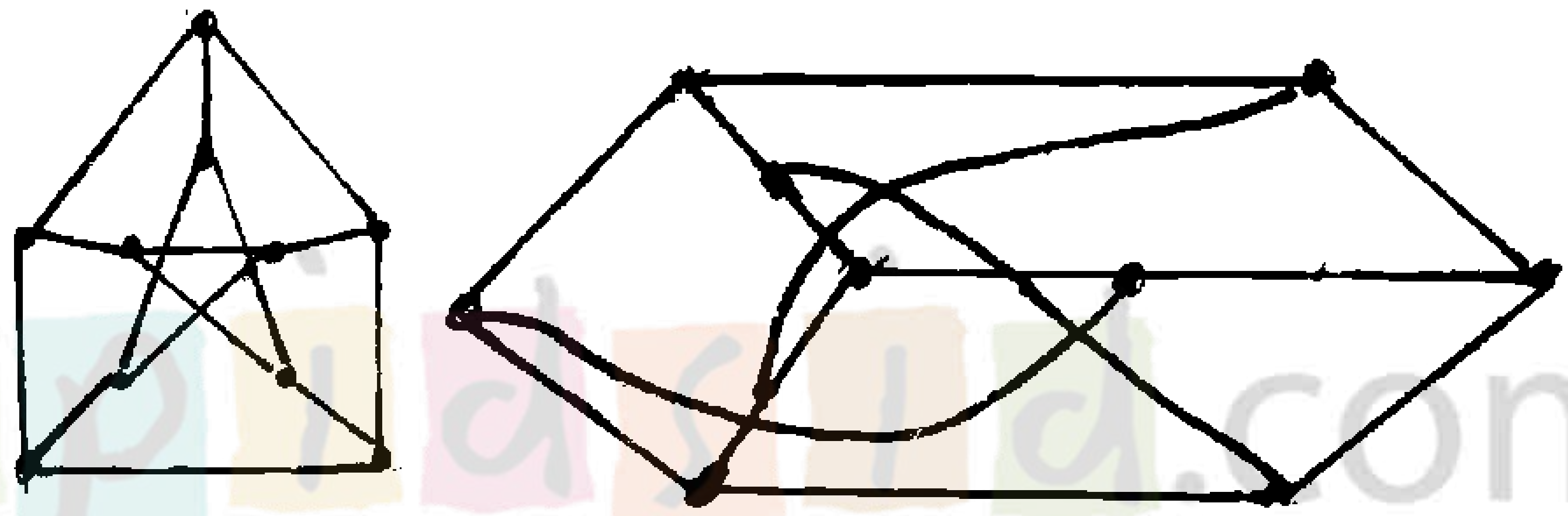

Fig. Q1 (c)

d. How many different paths of length 2 are there in the undirected graph G in Fig. Q1 (d)? **(04 Marks)**

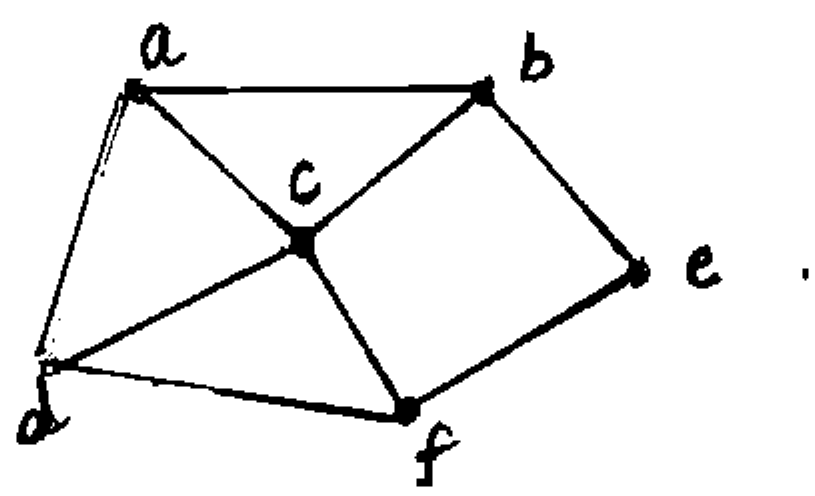

Fig. Q1 (d)

2 a. Define Hamilton cycle. How many edge-disjoint Hamilton cycles exist in the complete graph with seven vertices? Also, draw the graph to show these Hamilton cycles. **(06 Marks)**

b. Define Planar graph. Let G = (V, E) be a connected planar graph or multigraph with $|V| = V$ and $|E| = e$. Let r be the number of regions in the plane determined by a planar embedding 0+G. Then prove that $v - e + r = 2$. **(07 Marks)**

c. i) Find the chromatic number of the complete bipartite graph $K_{m,n}$ and a cycle, C_n on n vertices, $n \geq 3$.

ii) Determine the chromatic polynomial for the graph G in Fig. Q2 (c). **(07 Marks)**

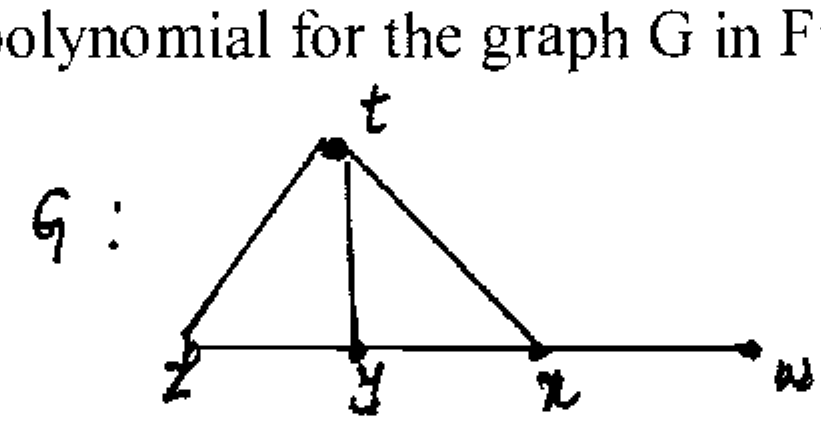

Fig. Q2 (c)

3 a. i) Prove that in every tree $T = (V, E)$, $|E| = |V| - 1$.

ii) Let $F_1 = (V_1, E_1)$ be a forest of seven trees, where $|E_1| = 40$. What is $|V_1|$? **(07 Marks)**

b. Define : i) Spanning tree ii) Binary rooted tree. Find all the nonisomorphic spanning trees of the graph. Fig. Q3 (b). **(06 Marks)**

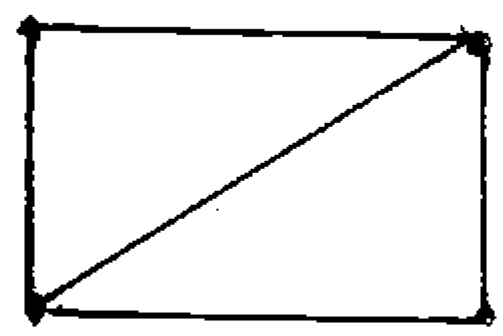

Fig. Q3 (b)

c. Define prefix code. Obtain an optimal prefix code for the message ROAD IS GOOD. Indicate the code. **(07 Marks)**

4 a. Apply Dijkstra's algorithm to the digraph shown in Fig. Q4 (a) and determine the shortest distance from vertex a to each of the other vertices in the graph. **(07 Marks)**

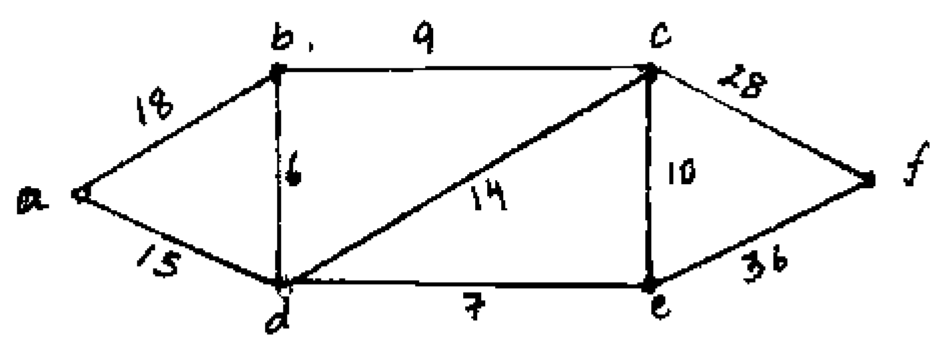

Fig. Q4 (a)

b. Define the following with respect to a graph: i) matching ii) a cut-set. Show that the graph in Fig. Q4 (b) has a complete matching from V_1 to V_2. Obtain two complete matching. **(07 Marks)**

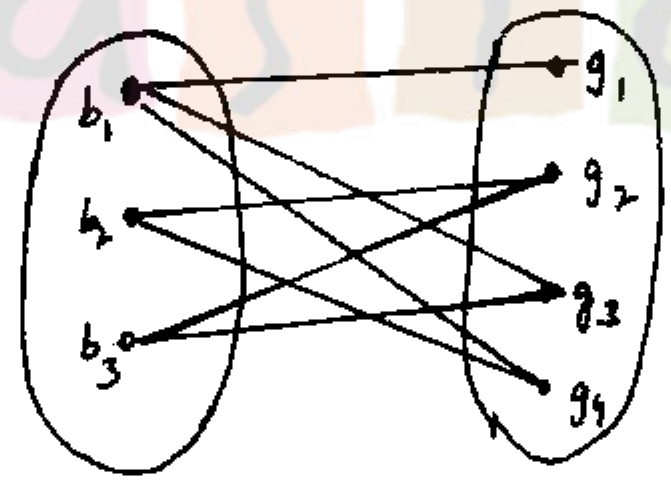

Fig. Q4 (b)

c. For the network shown in Fig. Q4 (c), find the capacities of all the cutsets between A and D, and hence determine the maximum flow between A and D. **(06 Marks)**

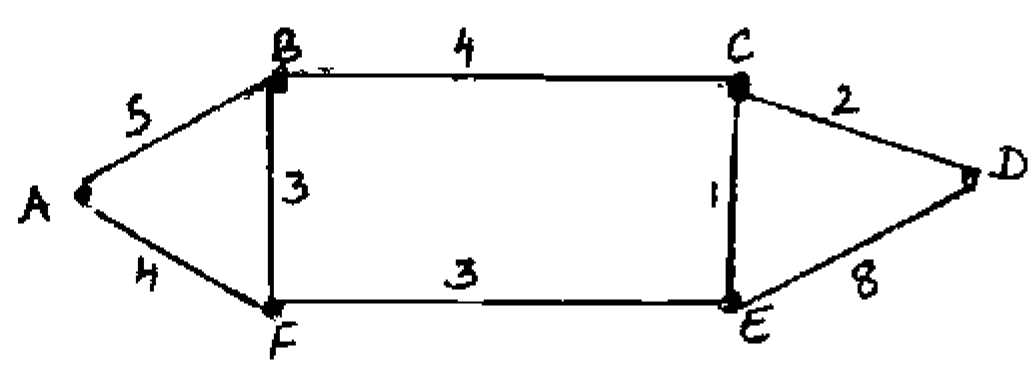

Fig. Q4 (c)

PART – B

5 a. How many arrangements of the letters in MISSISSIPPI have no consecutive S's? **(05 Marks)**

b. i) Find the coefficient of v^2w^4xz in the expansion of $(3v + 2w + x + y + z)^8$.

ii) How many distinct terms arise in the expansion in part (i)? **(05 Marks)**

c. How many positive integers n can we form using the digits 3, 4, 4, 5, 5, 6, 7 if we want n to exceed 5000000? **(05 Marks)**

d. A message is made up of 12 different symbols and is to be transmitted through a communication channel. In addition to the 12 symbols, the transmitter will also send a total of 45 blank spaces between the symbols, with at least three spaces between each pair of consecutive symbols. In how many ways the transmitter sends such a message? **(05 Marks)**

6 a. In how many ways can the 26 letters of the alphabet be permuted so that none of the patterns spin, game, path or net occurs? **(07 Marks)**

b. Define derangement. In how many ways can each of 10 people select a left glove and a right glove out of a total of 10 pairs of gloves so that no person selects a matching pair of gloves? **(06 Marks)**

c. Five teachers T_1, T_2, T_3, T_4, T_5 are to be made class teachers for five classes C_1, C_2, C_3, C_4, C_5, one teacher for each class. T_1 and T_2 do not wish to become the class teachers for C_1 or C_2, T_3 and T_4 for C_4 or C_5 and T_5 for C_3 or C_4 or C_5. In how many ways can the teachers be assigned the work? **(07 Marks)**

7 a. Find the generating function for the following sequences:
i) $1^2, 2^2, 3^2, 4^2, \ldots$ ii) $0^2, 1^2, 2^2, 3^2, \ldots$ iii) $0, 2, 6, 12, 30, \ldots$ **(06 Marks)**

b. Use generating function to determine how many four element subsets of $S = \{1, 2, 3, \ldots 15\}$ contain no consecutive integers? **(07 Marks)**

c. Using exponential generating function, find the number of ways in which 4 of the letters in the words given below be arranged: i) ENGINE ii) HAWAII **(07 Marks)**

8 a. The number of virus affected files in a system is 1000 (to start with) and this number increases 250% every two hours. Use a recurrence relation to determine the number of virus affected files in the system after one day. **(05 Marks)**

b. Solve the recurrence relation:
$a_{n+2} - 10a_{n+1} + 21a_n = 3n^2 - 2, \; n \geq 0$ **(07 Marks)**

c. Using the generating function method, solve the recurrence relation, **(08 Marks)**
$a_n - 3a_{n-1} = n, \; n \geq 1$ given $a_0 = 1$

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