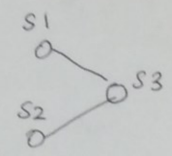
Implementing N. BP



$$p(\underline{s}) = \frac{1}{Z}\, \phi_1(s_1)\, \phi_2(s_2)\, \phi_3(s_3)\, \psi_{13}(s_1, s_3)\, \psi_{23}(s_2, s_3)$$

$$p(s_3) = \frac{1}{Z}\, \phi_3(s_3)\, m_{13}(s_3)\, m_{23}(s_3)$$

$$m_{13}(s_3) = \int \phi_1(s_1)\, \psi_{13}(s_1, s_3)\, ds_1$$

Representation:

$$\phi_i : r_i \rightarrow e^{-\frac{1}{2}(c_i \cdot r_i) s_i^2 + (d_i \cdot r_i) s_i}$$

$i = 1, 2, 3$

$\underline{c_i}$, $\underline{d_i}$ are determined by the parameters of the tuning curves of $\underline{r_i}$.

$$\psi_{13} : R_{13} \rightarrow e^{-\frac{1}{2}(a_{11} \cdot R_{13}) s_1^2 - \frac{1}{2}(a_{33} \cdot R_{13}) s_3^2 + (a_{13} \cdot R_{13}) s_1 s_3 + (a_1 \cdot R_{13}) s_1 + (a_3 \cdot R_{13}) s_3}$$

All the $\underline{a}$ vectors are determined by the P of tuning curves of R_{13}

Similarly, $\psi_{23} : R_{23}$ and a set of $\underline{b}$ vectors

$\underline{b}_{22}$, $\underline{b}_{33}$, $\underline{b}_{23}$, $\underline{b}_2$, and $\underline{b}_3$.
determined by params of tuning curves of R_{23}

Populations

$$M_{13} : \text{represents } m_{13} \rightarrow e^{-\frac{1}{2}(\eta_{13} \cdot M_{13}) s_3^2 + (\gamma_{13} \cdot M_{13}) s_3}$$

Similarly,

$$M_{23} : m_{23} \rightarrow e^{-\frac{1}{2}(\eta_{23} \cdot M_{23}) s_3^2 + (\gamma_{23} \cdot M_{23}) s_3}$$

$$r_{final} : \text{represents the marginal } p(s_3) \rightarrow e^{-\frac{1}{2}(A_{final} \cdot r_{final}) s_3^2 + (B_{final} \cdot r_{final}) s_3}$$

Obtaining the messages.

$$\eta_{13} \cdot M_{13} = a_{33} \cdot R_{13} - \frac{(a_{13} \cdot R_{13})^2}{c_1 \cdot r_1 + a_{11} \cdot R_{13}}$$

$$\gamma_{13} \cdot M_{13} = a_3 \cdot R_{13} + \frac{(a_{13} \cdot R_{13})(a_1 \cdot R_{13} + d_1 \cdot r_1)}{c_1 \cdot r_1 + a_{11} \cdot R_{13}}$$

$$\underline{M}_{13} = (\eta_{13} \cdot M_{13})\tilde{\eta}_{13} + (\gamma_{13} \cdot M_{13})\tilde{\gamma}_{13}$$

where $\tilde{\eta}_{13}$ and $\tilde{\gamma}_{13}$ are chosen such that they are orthogonal to each other.

$$\tilde{\eta}_{13} = \frac{\eta_{13}}{K} \rightarrow \text{normalizing constant / scale param.}$$

Similarly $\underline{M}_{23}$ can be constructed.

$$\eta_{23} \cdot M_{23} = b_{33} \cdot R_{23} - \frac{(b_{23} \cdot R_{23})^2}{(c_2 \cdot r_2 + b_{22} \cdot R_{23})}$$

$$\gamma_{23} \cdot M_{23} = b_3 \cdot R_{13} + \frac{(b_{23} \cdot R_{23})(b_2 \cdot R_{23} + d_2 \cdot r_2)}{c_2 \cdot r_2 + b_{22} \cdot R_{23}}$$

$$\underline{M}_{23} = (\eta_{23} \cdot M_{23})\tilde{\eta}_{23} + (\gamma_{23} \cdot M_{23})\tilde{\gamma}_{23}$$

The final population.

$$A_{final} \cdot r_{final} = \eta_{13} \cdot M_{13} + \eta_{23} \cdot M_{23} + c_3 \cdot r_3 = \alpha$$

$$B_{final} \cdot r_{final} = \gamma_{13} \cdot M_{13} + \gamma_{23} \cdot M_{23} + d_3 \cdot r_3 = \beta$$

$$\boxed{\underline{r_{final}} = \alpha \tilde{A}_{final} + \beta \tilde{B}_{final}}$$