# EFFECT OF RESIDUAL STRESS ON THE PULL-IN BEHAVIOUR IN A ELECTROSTATICALLY ACTUATED CLAMPED-CLAMPED MICROBEAM

Summer Internship Report

Submitted to

**Indian Institute of Technology, Kharagpur**

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**(23ME63R26)**

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**June-August, 2024**

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# ACKNOWLEDGEMENT

I would like to express my sincere gratitude to my supervisor, Professor Goutam Chakraborty, Mechanical Engineering Department, IIT Kharagpur, for providing me an opportunity to work on a project under him and for his support and guidance.

Furthermore, I would like to thank Gyana Ranjan Saw sir, PhD Research Scholar for providing me continuous assistance in completing my Internship work.

August, 2024

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# ABSTRACT

In the current work, the effect of residual stress on the pull-in behaviour for a Clamped-clamped microbeam has been studied. Residual stress can be induced for many reasons, during fabrication material inhomogeneity, Cold working, differential thermal expansion or quenching. It is important to have accurate estimation of the pull-in voltage when beam is subjected to both residual stress and electrostatic force. In this regard, an energy based approach Raleigh-Ritz method is adopted to find out the pull-in voltage. The deflection can be approximated by assuming a shape function satisfying all the boundary conditions. In this work the factors such as fringing fields, mid-plane stretching and influence of axial residual stresses were taken into consideration.

MATHEMATICA symbolic-mathematical software is adopted to obtain the results. The values of pull-in voltages for different stress values were compared with the results of FEA simulation using COMSOL and showed good agreement.

# INTRODUCTION

Electrostatically actuated Micro-beams are commonly used as the core component in MEMS sensors that measure things by detecting changes in electrical charge. A MEMS capacitive sensor essentially converts physical forces like pressure or vibration into electrical signals. It does this by using a steady electrical charge or voltage to monitor changes in electrical capacitance caused by the physical force affecting the sensor.

The electrostatic force used to operate the sensor isn't constant, it is nonlinear in nature associated with the bias voltage and can cause the beam to collapse on the base if the voltage exceeds a limit. This phenomenon is called Pull-in effect and corresponding voltage as Pull-in voltage. The bias voltage has to be properly controlled to avoid this effect. This is a critical issue because it affects how well the sensor works, how fast it responds, and how much input it can handle without breaking.

A simplified method is Lumped-mass model for calculating the voltage at which the beam collapses assumes the beam moves like a spring and ignores the complex electric field at the edges of the beam. This method predicts that the beam will fail when it's about one-third of the way to the base. However, this is inaccurate because the electric field at the beam edges significantly increases the electrical force (fringe-field effect), often doubling or tripling it.

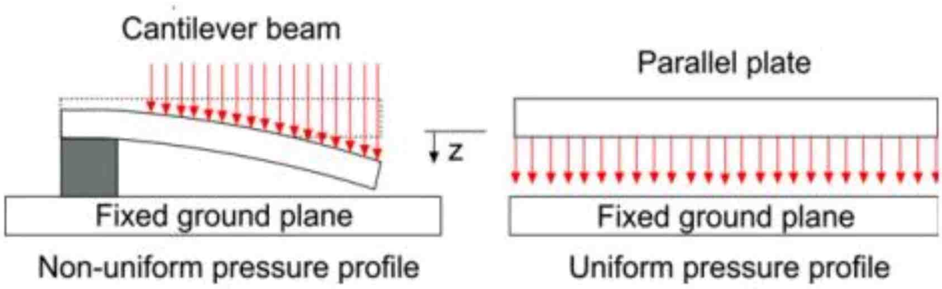



Fig 01: Non-uniformity of pressure profile in 2D beam model unlike Lumped-mass model (DOI: 10.1088/0960-1317/15/4/012)

There are other factors also that affect the Pull-in behaviour such as Squeeze-film damping effect and Residual stress effect. Squeeze-film damping arises from the viscous forces generated when a fluid (usually air) is trapped between the moving micro-beam and a fixed surface. As the micro-beam oscillates, the fluid is squeezed and released, creating a damping force that opposes the motion. However, by increasing the gap height can reduce the damping.

In reality, MEMS Micro-beams are generally curled because of residual stresses aroused during the time of fabrication. This can also arise because of thermomechanical mismatches in multi-layered beams. These can cause difficulties in estimating Pull-in voltages accurately. So Pull-in characteristics of the curled micro-beams need to be addressed.

## LITERATURE REVIEW

**[5] Title:** A closed-form model for the pull-in voltage of electrostatically actuated cantilever beams

(**DOI:** 10.1088/0960-1317/15/4/012)

In this particular work, a closed-form analytical model has been developed that determines the pull-in voltage of a cantilever beam actuated by electrostatic force. This model takes into account the effects of the fringing field capacitance. For large beamwidth–airgap ratio, the fringing fields can be neglected.

For lumped parameter model of a parallel plate configuration, any fringing field capacitance associated with the capacitor electrodes is neglected and the capacitor electrodes and the contacts are assumed to be perfect.

But for cantilever beam geometry, the electrostatic force becomes increasingly non-uniform due to a redistribution of the charges as the beam deforms. The tip experiences higher attractive force compared to region closer to the fixed end. While considering the fringing field effects the capacitance has been expressed as:

$$C_{\mathrm{CB}} = \varepsilon_0 l\left[\left(\frac{w}{d_0}\right) + 0.77 + 1.06\left(\frac{w}{d_0}\right)^{0.25} + 1.06\left(\frac{h}{d_0}\right)^{0.5}\right] + 1.06\varepsilon_0 w\left(\frac{l}{d_0}\right)^{0.25}$$

where $\varepsilon_0$ is the permittivity of free space, $\varepsilon_r$ is the dielectric constant, C is the capacitance and $d_0$ is the initial thickness of the airgap.

Because the expression of capacitance has been derived after considering fringe field effect, so the modified expression of spring constant will also be different for Cantilever beam because of spring softening effect.

A uniform Linearized model of the electrostatic force has been developed by linearizing the electrostatic force about the zero deflection point ($z_0 = 0$). After reaching the final closed-form expression for the pull-in voltage $V_{PI}$, they validated the results with FEA results and found good agreement.

However, they have neglected the effects due to squeezed film damping in the airgap due to static case at equilibrium.

## [2] Title: Pull-in voltage analysis of electrostatically actuated beam structures with fixed–fixed and fixed–free end conditions

(**DOI:** 10.1088/0960-1317/12/4/319)

This model takes into account effects of partial electrodes, axial stress, non-linear stiffening, charge-redistribution and fringing fields and thus eliminates the deficiencies of the other models.

In this paper, a closed-form expression of the pull-in voltage of electrostatically actuated beams with fixed–fixed end conditions as well as with fixed–free end conditions is derived based on the lumped model.

Here an approximate expression for the pull-in voltage $V_{PI}$ is obtained by substituting the expressions for the effective stiffness $K_{eff}$ from equation (2), effective area $A_{eff}$ and the gap spacing $d_0$.

The effective stiffness $K_{eff}$ is expressed by considering the stiffness due to bending, membrane stretching of the beam which includes built-in stress as well as induced non-linear stress due to bending. Whereas the effective area $A_{eff}$ accounts for two effects, namely the fringing field effects and the charge-redistribution effects. Fringing field effects begin to influence for very small widths ($w < d_0$) of the beam and charge-redistribution because the gap is not the same throughout the length of the beam.

## [9] Title: Dynamic pull-in and snap-through behavior in micro/nano mechanical memories considering squeeze film damping

(**DOI:** 10.1007/s00542-016-3026-9)

In this paper, behavior of micro/nano mechanical memories was studied. Here they considered effect of squeeze-film damping assuming general form of Reynolds equation and the results were compared for various ambient pressures.

Electrostatic force, squeeze film damping and mid-plane stretching, which all are nonlinear, are important effects in this investigation.

In their research, they considered the squeeze film damping because of ambient pressure and neglected the effect of intermolecular forces i.e. material damping since the characteristic dimension taken of larger than 500 nm.

Their results showed that, by increasing the length of a curved-beam, effect of ambient pressure is more considerable and by increase in ambient pressure voltage increases slightly. But in their work, they have not included the fringe field effect on total capacitance.

**[8] Title:** Analytical model of electrostatic fixed-fixed micro beam for pull-in voltage

(**DOI:** 10.1109/AIM.2008.4601763)

A closed-form expression of the pull-in voltage of electrostatically actuated beams is derived by Rayleigh–Ritz method. While describing the deflection profile of a beam of uniform cross section by the following non-linear differential equation of equilibrium:

$$\hat{E}I\frac{\partial^4 w(x)}{\partial x^4} - \left[\frac{\hat{E}A}{2L}\int_0^L \left(\frac{\partial w(x)}{\partial x}\right)^2 dx + \hat{N}\right]\frac{\partial^2 w(x)}{\partial x^2} = 0$$

From the given expression the analytical solution is cumbersome because of non-linear electrostatic force and stretch stress gradient non-linear stiffening. Thus Rayleigh–Ritz method is used an approximate solution expressed in the form of admissible trial functions containing undetermined parameters.

This model takes into account effects of large deflection, non-linear stiffening, residual stress, and fringing fields. Analytical results are to be compared with FEM results and the other published results available in the literature. The numerical results were compared with FEA simulation and found out good agreement.

**[9] Title:** A Novel Semianalytical Approach for Finding Pull-In Voltages of Micro Cantilever Beams Subjected to Electrostatic Loads and Residual Stress Gradients

(**DOI:** 10.1109/JMEMS.2011.2105246)

Their work has pointed out an analytical formulation for deducing the pull-in voltage of a curled cantilever beam due to residual stress gradients. These residual gradients arise during the time of fabrication process. As a result, the mechanical strain energies of the structures (i.e., an initially curved beam due to internal stress gradients and an initially flat and stress-free beam) are not the same.

Their proposed method was energy based modelling, by assuming an admissible deformation shape and using the energy method they determined the coefficients of the shape functions. The pull-in voltage was obtained by partial derivative of total energy with coefficients (where, Total energy ($U_T$) = Strain energy (Us) + electrostatics work (Ue)).

The results show that beams with a higher stress gradient have a larger pull-in voltage. the results of FEM and the proposed method agree quite well.

# SCOPE OF WORK

The analysis can be further extended by exploring the effect residual axial stress taken constant along beam axis. In addition, taking into account the mid-plane stretching makes the governing equation non-linear, which makes the problem more challenging.

As mentioned, the resultant governing equation includes non-linear terms, the analytical approach to reach to the solution is quite cumbersome because of the electrostatic force and mid-plane stretching. Instead it can be modelled by the help of Raleigh-Ritz method which assumes an approximate function in the form of admissible trial functions containing undefined parameters. Those undefined parameters can be found by the principle of variational minimization of total potential energy.

# METHODOLOGY

Assuming an approximated displacement function satisfying the all the boundary conditions

Determining Strain energy of the microbeam and work potential due to electrostatic energy produced by the charges

Calculating total Potential Energy U of the microbeam

By performing partial derivative of U wrt unknown coefficient states the condition of Equilibrium and second derivative equals to zero shows the transition from stable to unstable Equilibrium.

Solving these two equations simultaneously yields the pull-in voltage $V_P$ of the fixed-fixed microbeam.

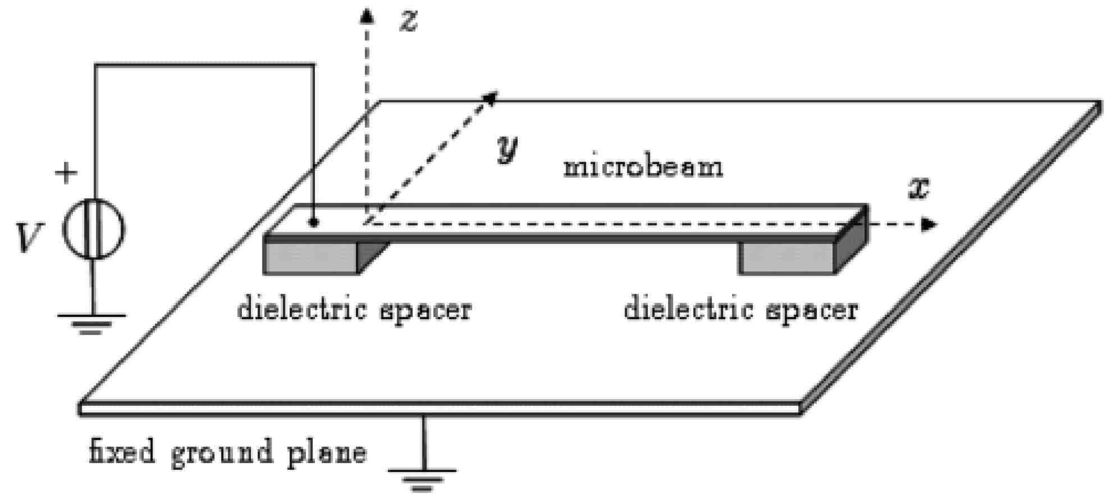



Fig 01: Schematic of an electrostatically actuated Clamped-Clamped microbeam

DOI: 10.1109/JMEMS.2006.880204

The governing non-linear differential equation according to large deflection beam theory is given as:

$$EI\frac{d^4w}{dx^4} - N(w)\frac{d^2w}{dx^2} = \tilde{F}_e(w) \qquad -(1)$$

Where N is the axial force constant along the beam axis. For Clamped-Clamped beam it is given as:

$$N(w) = \frac{EA}{2\ell}\int_0^\ell \left(\frac{dw}{dx}\right)^2 dx + N_0$$

Here $N_0$ is the residual axial load which is $N_0 = \sigma * A$, σ is the effective residual axial stress equals to $\sigma_0*(1-v)$, where $\sigma_0$ is the initial biaxial uniform stress and v is the poisons ratio for the material of the beam. The non-linear term arises by considering the mid-plane stretching effect.

Here $\tilde{F}_e(w)$ is the deflection dependent electrostatic force term per unit length.

The given equation involves non-linear terms which is cumbersome to solve analytically. Therefore, an alternative approach of Energy method can be better option.

Here an approximated displacement function is considered that satisfies all the boundary conditions. The displacement function can be: $W = a_1(1 - \cos(2\pi X))$

The boundary conditions for Clamped-Clamped beam:

$$\begin{aligned} w(0) &= 0 \\ \frac{dw}{dx}(0) &= 0 \\ w(\ell) &= 0 \\ \frac{dw}{dx}(\ell) &= 0 \end{aligned}$$

We know strains for large deflections of a beam $\varepsilon_{xx} = \frac{\partial u_x}{\partial x} - \frac{\partial^2 w}{\partial x^2} y + \frac{1}{2}(\frac{\partial w}{\partial x})^2$

Now Strain energy of the microbeam

$U_{se} = \frac{1}{2}\int_V \quad \varepsilon_{xx}(\sigma_{xx} + \sigma_x)dV = \frac{1}{2}\int_V \hat{E}\, \varepsilon_{xx}^2 dV + \frac{1}{2}\int_V \quad \varepsilon_{xx}\sigma_x dV$

After simplification,

$$\frac{\hat{E}A}{2}\int_0^L \quad (\frac{\partial u_x}{\partial x})^2 dx + \frac{\hat{E}A}{2}\int_0^L (\frac{\partial u_x}{\partial x})\,(\frac{\partial w}{\partial x})^2 dx$$
$$+\frac{\hat{E}I}{2}\int_0^L \quad (\frac{\partial^2 w}{\partial x^2})^2 dx + \frac{A}{2}\int_0^L \quad \sigma_x(\frac{\partial u_x}{\partial x})dx + \frac{A}{4}\int_0^L \quad \sigma_x(\frac{\partial w}{\partial x})^2 dx$$

The first term is the strain energy due to axial residual force,

The third term is the strain energy due to bending because of electrostatic force,

And the rest of the terms arise because of the nonlinear components of strain.

The work potential/ electrostatic potential energy is given as:

$$U_{el} = -\int_0^L \quad \frac{\varepsilon_0 b_{eff} V^2}{2g_{eff}}dx = -\int_0^L \quad \frac{0.5\varepsilon_0 b V^2}{d_0 - w(x)}(1 + 0.65\frac{d_0 - w(x)}{b})dx$$

Here, $g_{eff}$ = $d_0$-w(x) ; electrode gap at defection w(x) of mid-point of beam

$b_{eff}$ = $b_{eff} = b(1 + 0.65\frac{g_{eff}}{b})$ ; width after considering fringe-field effect

So The Total Potential Energy of the Microbeam: U = $U_{se}$- $U_{el}$

As we know Raleigh-Ritz method talks about the minimum potential energy,

The first derivative wrt unknown coefficient equals to zero tells the system is in equilibrium. When the beam comes across pull-in stage, the system goes from stable to unstable equilibrium. So for critical case, the second derivative should be zero. So we will end up with two equations, upon solving them we reached to the expression of Pull-in voltage $V_p$.

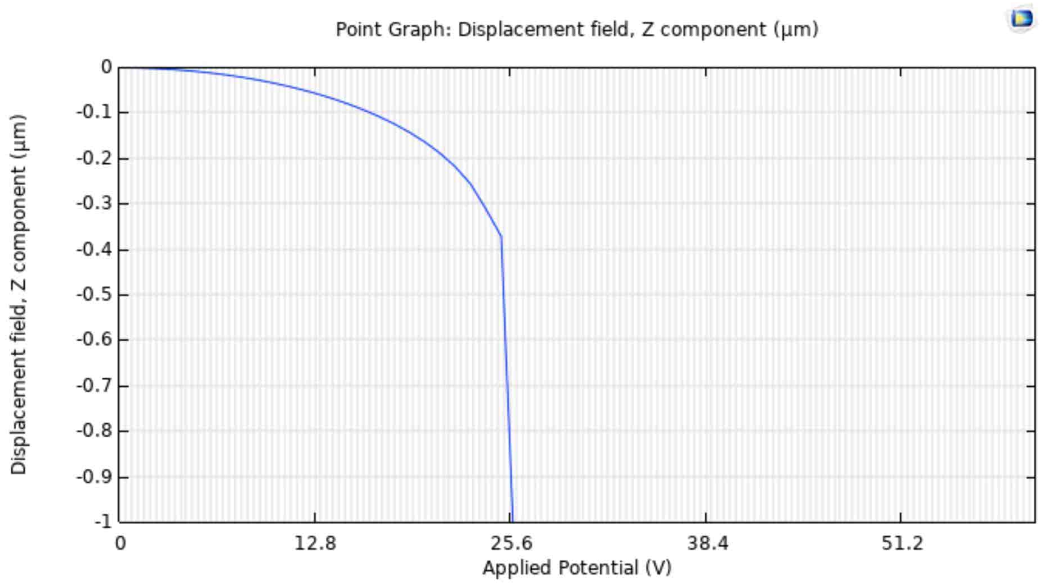
EFFECT OF RESIDUAL STRESS ON THE PULL- IN BEHAVIOUR IN A ELECTROSTATICALLY ACTUATED MICROBEAM

```
In[•]:= Clear["Global *"]

(*******************************************************************************)

(*Geometric data*)
l = 300 * 10^-6; (*Beam length in m*)
b = 50 * 10^-6; (*Beam width in m*)
h = 3 * 10^-6; (*Beam thickness in m*)
d0 = 1.5 * 10^-6; (*Dielectric initial gap in m*)

(*Material properties of the Beam*)
Eb = 169 * 10^9; (*Elastic Modulus in Pa*)
ρ = 2329; (*Mass density in kg/m^3*)
ν = 0.28; (*Poisson's ratio*)

(*Material properties of Dielectric*)
ϵ0 = 8.8541878128 * 10^-12; (*Permittivity of free space in m^-3 kg^-1 s^4 A^2*)
ϵr = 1; (*Relative permittivity of Air*)

M = l * b * h; (*mass of the Beam*)
Ab = b * h; (*cross-sectional area of the beam*)
Ib = (b * h^3) / 12; (*Area moment of inertia of the Beam*)
K = (3 * Eb * Ib) / l^3 ;(*flextural modulus of the Beam*)

(*For b>5*h Plane-strain condition prevails,
thus effective modulus to be considered to account width effects*)
If[b ≥ 5 * h, Eb = Eb / (1 - ν^2), Eb = Eb];



(*******************Performing Raleigh-Ritz method***************************)

(************For clamped-clamped beam************)

(*Approximate Displacement function*)
w = a * (1 - Cos[2 Pi x1 / l]);

(*film gap when beam deflects by w(x)*)
d = d0 - w;
(*Considering fringing-field effects*)
beff = b * (1 + 0.65 * (d0 - w) / b);

σ0 = 50 * 10^6; (* Original biaxial stress in Pa*)
σx = σ0 * (1 - ν); (*Uniaxial stress in Pa*)
```

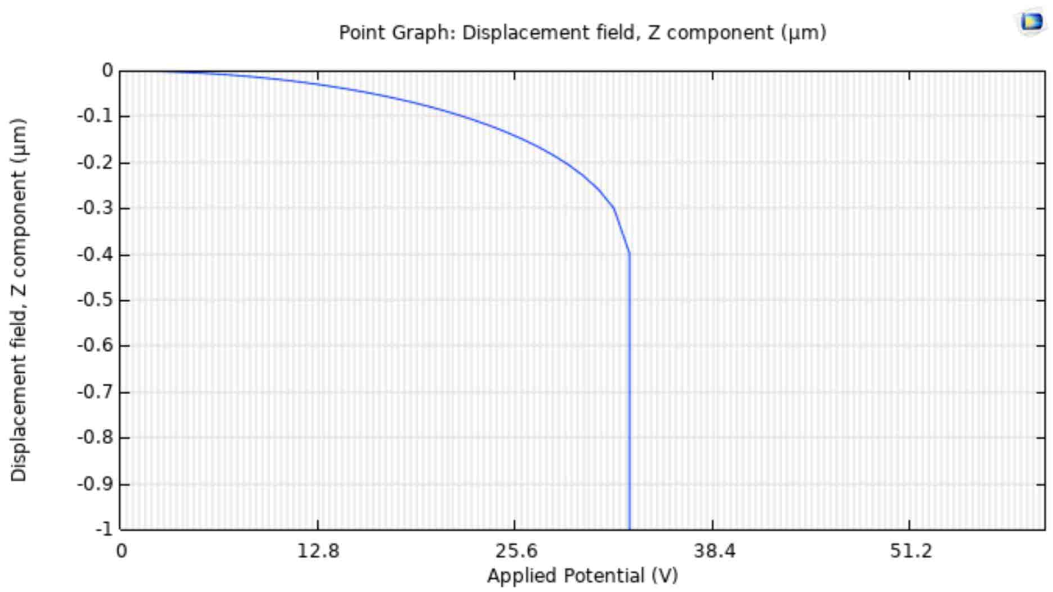
```
(*Deducing Total Potential Energy*)
(*strain Energy*)
U1 =
  Integrate[(0.125 * Eb * Ab / l^2) * (Integrate[D[w, x1]^2, {x1, 0, l}])^2, {x1, 0, l}];
U2 = Integrate[
   (0.25 * Eb * Ab / l) * (Integrate[D[w, x1]^2, {x1, 0, l}]) * D[w, x1]^2, {x1, 0, l}];
U3 = Integrate[(0.5 * Eb * Ib) * (D[w, {x1, 2}]^2), {x1, 0, l}];
U4 = Integrate[(0.25 * Ab * σx / l) * (Integrate[D[w, x1]^2, {x1, 0, l}]), {x1, 0, l}];
U5 = Integrate[(0.25 * Ab * σx) * (D[w, x1]^2), {x1, 0, l}];
Use = U1 + U2 + U3 + U4 + U5;

(*Work Potential*)
Wp = Integrate[(0.5 * ϵ0 * V^2) * (beff / (d0 - w)), {x1, 0, l}];

U = Use - Wp;

(*Conditions for stable equillibrium*)
cond1 = D[U, a]; (*Condition for equillibrium*)
cond2 = D[U, {a, 2}];
(* at the transition between stable to an unstable equillibrium*)

soln = Solve[{cond1 == 0, cond2 == 0}, {a, V}];
Vp = soln[[2]] (*Required Pull-in Voltage*)
```

# Pull-in voltage graph from COMSOL

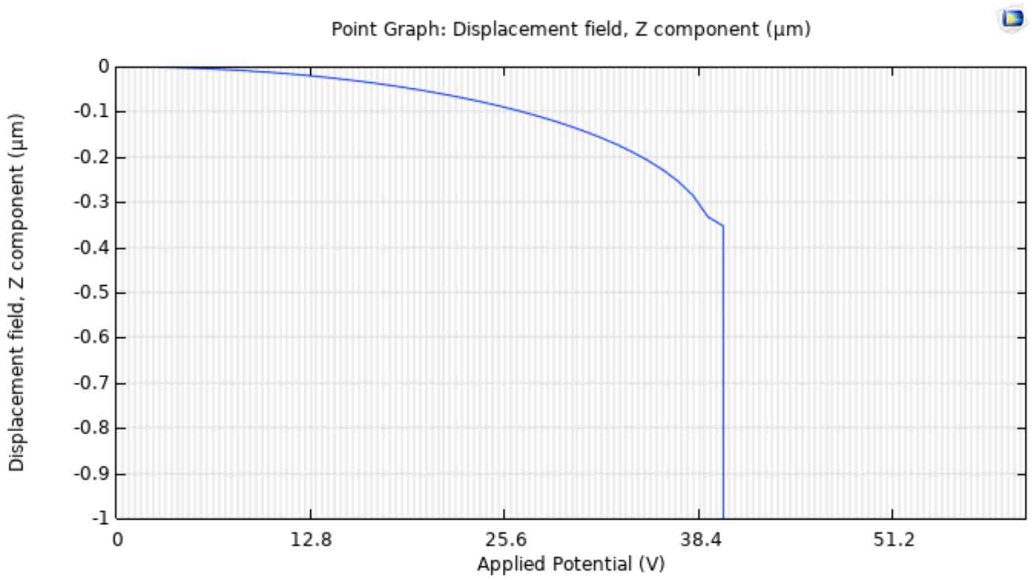
For -50 MPa



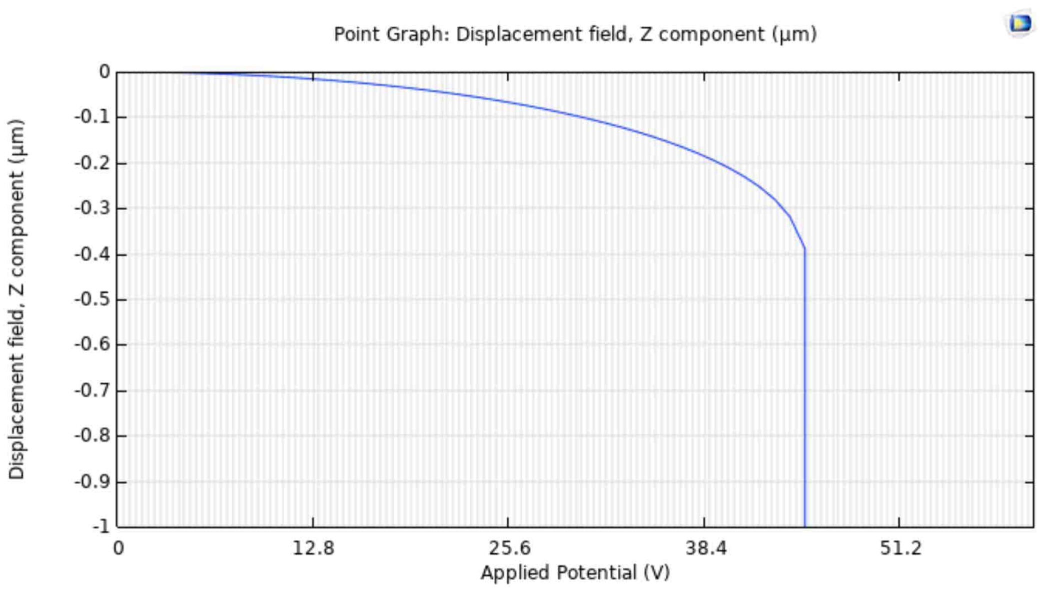

For -25 MPa



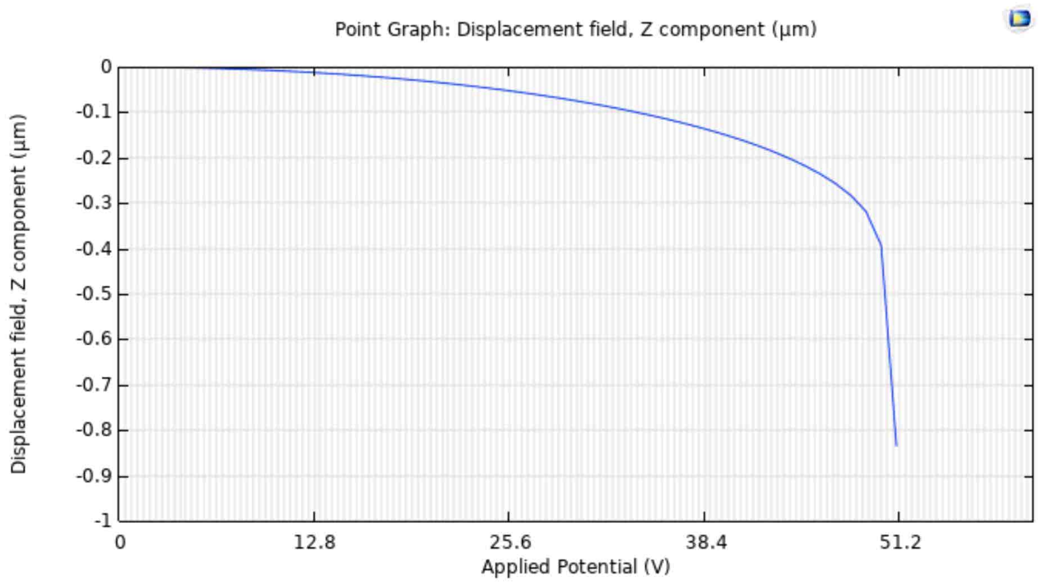
For 0 MPa




For 25 MPa



For 50 MPa


Point Graph: Displacement field, Z component (µm)
Displacement field, Z component (µm)
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
-0.9
-1
0
12.8
25.6
38.4
51.2
Applied Potential (V)

# RESULTS

**<u>The input data:</u>**

length L = 300 µm

breadth b = 50 µm

Thickness h = 3 µm

Initial gap between electrodes $d_0$ = 1.5 3 µm

Mass density of material of beam ρ = 2329 kg/$m^3$

Elastic Modulus of material of beam E (Silicon (110)) = 169 GPa

Poisson's ratio ρ (Silicon (110)) = 0.06

Relative Permittivity of electrode = 11.7

Relative Permittivity of Dielectric (Air) = 1

<u>COMPARISON OF PULL-IN VOLTAGE</u>

| Residual stress $\sigma_x$ (in MPa) | Via MATHEMATICA (in V) | Via COMSOL Multiphysics (in V) | %age difference |
|---|---|---|---|
| -50 | 26.2 | 25.8 | 1.528 |
| -25 | 33.6 | 32.8 | 2.381 |
| 0 | 40.1 | 40.0 | 0.250 |
| 25 | 45.5 | 45.2 | 0.659 |
| 50 | 50.6 | 49.6 | 1.976 |

# CONCLUSION

A closed-form expression for the pull-in voltage has been derived for a clamped-clamped beam based on the Raleigh-Ritz method. The approximated displacement function was taken trigonometric whose admissible function satisfies all the boundary conditions. The results fits well with FEA results of COMSOL.

It was found that with increase residual axial stress the pull-in voltage also increases. One reason can be residual tensile stress in the beam acts like a pre-stretched spring, increasing the beam's stiffness. To overcome this increased stiffness and cause pull-in, a higher electrostatic force (and thus, a higher voltage) is required.

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