

Question 1

Given $f(x,y) = 8xy$, when $0 \leq x \leq 1$, $0 \leq y \leq x$ and $f(x,y) = 0$ elsewhere, find

- (i) Marginal density of x
- (ii) Marginal density of y
- (iii) Conditional density of x
- (iv) Conditional density of y

Answer:

$$f(x,y) = 8xy \quad \text{when } 0 \leq x \leq 1 \\ \text{and } 0 \leq y \leq x$$

(i) Marginal density of x

$$f(x) = \int_0^x 8xy \, dy = 8x \int_0^x y \, dy \\ = 8x \left[\frac{y^2}{2} \right]_0^x$$

$$\Rightarrow 4x \times x^2 = 4x^3, \quad 0 \leq x \leq 1$$

(ii) Marginal density of y

$$f(y) = \int_0^1 8xy \, dx = 8y \int_0^1 x \, dx \\ = 8y \left[\frac{x^2}{2} \right]_0^1$$

$$\Rightarrow 4y, \quad 0 \leq y \leq 1$$

(iii) Conditional Density of x :

$$f_{x|y}(x|y) = \frac{8xy}{4y} = 2x, \quad 0 \leq y \leq x \leq 1$$

(iv) Conditional Density of y :

$$f_{y|x}(y|x) = \frac{8xy}{4x^2} = \frac{2y}{x}, \quad 0 \leq y \leq x \leq 1$$