Problems on Recursion :→

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Q :→ solve $Q(n) - Q(n-1) - 12\,Q(n-2) = (-3)^n + 6(4)^n$

sol :→ Homo soln $Q^h(n)$

Associated Homo eqn $Q(n) - Q(n-1) - 12Q(n-2) = 0$

For char eqn, take $Q(n) = a^n$ in the above eqn

$$a^2 - a - 12 = 0$$
$$\Rightarrow (a-4)(a+3) = 0$$
$$\Rightarrow a = 4, -3$$
$$\therefore Q^h(n) = A(4)^n + B(-3)^n$$

<u>Particular soln $Q^p(n)$</u>

$q(n) = (-3)^n + 6(4)^n = q_1(n) + q_2(n)$

where $q_1(n) = (-3)^n$ and $q_2(n) = 6(4)^n$

i) For $q_1(n) = (-3)^n$ we take $nd_0(-3)^n$ as particular soln ($\because$ -3 is the char root).

ii) For $q_2(n) = 6(4)^n$ we take $nd_1(4)^n$ as particular solution ($\because$ 4 is the char root).

For the particular soln, take $Q(n) = nd_0(-3)^n + nd_1(4)^n$ in the given recc. relation

$$Q(n) - Q(n-1) - 12\,Q(n-2) = (-3)^n + 6(4)^n$$

$\Rightarrow [nd_0(-3)^n + nd_1(4)^n] - [(n-1)d_0(-3)^{n-1} + (n-1)d_1(4)^{n-1}]$
$- 12\,[(n-2)d_0(-3)^{n-2} + (n-2)d_1(4)^{n-2}] = (-3)^n + 6(4)^n$

$\Rightarrow [n(-3)^n - (n-1)(-3)^{n-1} - 12(n-2)(-3)^{n-2}]\,d_0$
$+ [n(4)^n - (n-1)(4)^{n-1} - 12(n-2)(4)^{n-2}]\,d_1 = (-3)^n + 6(4)^n.$

$\Rightarrow [n(-3)^2(-3)^{n-2} - (n-1)(-3)(-3)^{n-2} - 12(n-2)(-3)^{n-2}]\,d_0$
$+ [n(4)^2(4)^{n-2} - (n-1)(4)(4)^{n-2} - 12(n-2)(4)^{n-2}]\,d_0 = (-3)^2(-3)^{n-2} + 4^2(4)^{n-2}\cdot 6$

$\Rightarrow [9n+3(n-1)-12(n-2)]d_0(-3)^{n-2} + [16n-4(n-1)-12(n-2)]d_1(4)^{n-2} = 9(-3)^{n-2} + 16(4)^{n-2}$

$\Rightarrow 21d_0(-3)^{n-2} + 28d_1(4)^{n-2} = 9(-3)^{n-2} + 96(4)^{n-2}$

equate the coeff of $(-3)^{n-2}$ and $(4)^{n-2}$

$21d_0 = 9$ and $28d_1 = 96$

$d_0 = \frac{3}{7}$, $d_1 = \frac{24}{7}$

$$Q^p(n) = nd_0(-3)^n + nd_1(4)^n$$
$$= n\left(\frac{3}{7}\right)(-3)^n + n\left(\frac{24}{7}\right)(4)^n$$
$$= \frac{n}{7}\left[3\cdot(-3)^n + 24\cdot(4)^n\right]$$

Complete soln

$$Q(n) = Q^h(n) + Q^p(n)$$
$$= A(4)^n + B(-3)^n + \frac{n}{7}\left[3(-3)^n + 24(4)^n\right]$$ Ans

Q:→ Solve $y(n) - 4y(n-1) + 4y(n-2) = 3n + 2^n$.

sol:→ Homo soln $y^h(n)$:

Associated Homo eqn $y(n) - 4y(n-1) + 4y(n-2) = 0$

For char eqn, take $y(n) = a^n$

$$a^2 - 4a + 4 = 0$$
$$(a-2)^2 = 0$$
$$a = 2, 2$$
$$y^h(n) = (A + nB)(2)^n$$

Particular soln $y^p(n)$:

$q(n) = 3n + 2^n = q_1(n) + q_2(n)$

where $q_1(n) = 3n$ and $q_2(n) = 2^n$

For $q_1(n) = 3n$ which is a linear polyn, we take particular soln $d_0 + d_1 n$

For $g_2(n) = 2^n$, we take particular soln $n^2 d_2 (2)^n$

($\because$ 2 is the char root which is repeated two times)

Take $y(n) = d_0 + d_1 n + d_2 n^2 (2)^n$ in the given recc relation

$$y(n) - 4y(n-1) + 4y(n-2) = 3n + 2^n.$$

$$\Rightarrow [d_0 + d_1 n + d_2 n^2 (2)^n] - 4[d_0 + d_1(n-1) + d_2(n-1)^2(2)^{n-1}]$$
$$+ 4[d_0 + d_1(n-2) + d_2(n-2)^2(2)^{n-2}] = 3n + 2^n$$

$$\Rightarrow [1-4+4]d_0 + [n - 4(n-1) + 4(n-2)]d_1 + [n^2(2)^n - 4(n-1)^2(2)^{n-1} + 4(n-2)^2(2)^{n-2}]d_2$$
$$= 3n + 2^n$$

$$\Rightarrow d_0 + (n-4)d_1 + [n^2(2)^2 - 4(n-1)^2(2) + 4(n-2)^2]d_2(2)^{n-2} = 3n + 2^2 \cdot 2^{n-2}$$

$$\Rightarrow d_0 + (n-4)d_1 + [4n^2 - 8(n^2-2n+1) + 4(n^2-4n+4)]d_2(2)^{n-2} = 3n + 4(2)^{n-2}$$

$$\Rightarrow d_0 + (n-4)d_1 + 8d_2(2)^{n-2} = 3n + 4(2)^{n-2}$$

$$\Rightarrow (d_0 - 4d_1) + d_1 n + 8d_2(2)^{n-2} = 3n + 4(2)^{n-2}$$

equate the coeff of n^0, n and $(2)^{n-2}$

$$d_0 - 4d_1 = 0 \quad \Rightarrow d_0 = 12$$
$$d_1 = 3$$
$$8d_2 = 4 \quad \Rightarrow d_2 = \frac{1}{2}$$

$$\therefore y^p(n) = d_0 + d_1 n + d_2 n^2 (2)^n$$
$$= 12 + 3n + \frac{1}{2}n^2(2)^n = 12 + 3n + n^2(2)^{n-1}$$

Complete soln

$$y(n) = y^h(n) + y^p(n)$$
$$= (A + nB)(2)^n + 12 + 3n + n^2(2)^{n-1} \quad \underline{\text{Ans}}$$

Q:- Solve

$$y(n+1) + 2y(n) = 3+4^n \quad ; \quad y(0) = 2$$

sol:— Homo soln $y^h(n)$:

Associated Homo eqn $\quad y(n+1) + 2y(n) = 0$

For char eqn, $y(n) = a^n$

$$\Rightarrow \quad a + 2 = 0$$

$$\Rightarrow \quad a = -2$$

$$\therefore \quad y^h(n) = A(-2)^n$$

Particular soln $y^p(n)$:

$q(n) = 3+4^n = q_1(n) + q_2(n)$

where $q_1(n) = 3$ and $q_2(n) = 4^n$

For $q_1(n) = 3$ which is a const., we take d_0 as particular soln

For $q_2(n) = 4^n$, we take $d_1(4)^n$ as particular soln

$\because$ 4 is not char root.

Take $y(n) = d_0 + d_1(4)^n$ in the given recc relation

$$y(n+1) + 2y(n) = 3 + 4^n$$

$$\Rightarrow [d_0 + d_1(4)^{n+1}] + 2[d_0 + d_1(4)^n] = 3 + 4^n$$

$$\Rightarrow [d_0 + 4d_1(4)^n] + [2d_0 + 2d_1(4)^n] = 3 + 4^n$$

$$\Rightarrow 3d_0 + 6d_1(4)^n = 3 + 4^n$$

equate const term and coeff of $(4)^n$

$3d_0 = 3 \quad \Rightarrow d_0 = 1$

$6d_1 = 1 \quad \Rightarrow d_1 = \frac{1}{6}$

$$\therefore \quad y^p(n) = d_0 + d_1(4)^n = 1 + \frac{(4)^n}{6}$$

Complete soln

$$y(n) = y^h(n) + y^p(n) = A(-2)^n + 1 + \frac{(4)^n}{6}$$

As $y(0) = 2$

$$A + 1 + \frac{1}{6} = 2$$

$$A = \frac{5}{6}$$

$$\therefore \quad y(n) = \frac{5}{6}(-2)^n + 1 + \frac{(4)^n}{6} \quad \underline{\text{Ans}}$$

Q:→ Solve

$$f(n+2) - 6f(n+1) + 9f(n) = 3(2)^n + 7(3)^n$$

$$f(0) = 1, \quad f(1) = 4.$$

Sol:→ Homo. Soln $f^h(n)$:-

Associated Homo eqn $\quad f(n+2) - 6f(n+1) + 9f(n) = 0$

For char eqn, $\quad f(n) = a^n$

$$a^2 - 6a + 9 = 0$$

$$\Rightarrow a = 3, 3$$

$$\therefore \quad f(n) = (A + Bn)(3)^n$$

Particular soln $f^p(n)$:-

As $q(n) = 3(2)^n + 7(3)^n = q_1(n) + q_2(n)$

Where $q_1(n) = 3(2)^n$ and $q_2(n) = 7(3)^n$

For $q_1(n) = 3(2)^n$, take $d_0(2)^n$ as particular soln

For $q_2(n) = 7(3)^n$, take $n^2 d_1(3)^n$ as particular soln

Take $f(n) = d_0(2)^n + n^2 d_1(3)^n$ in the given recc. relation

$$f(n+2) - 6f(n+1) + 9f(n) = 3(2)^n + 7(3)^n$$

$$\Rightarrow [d_0(2)^{n+2} + (n+2)^2 d_1(3)^{n+2}] - 6[d_0(2)^{n+1} + (n+1)^2 d_1(3)^{n+1}]$$

$$+ 9[d_0(2)^n + n^2 d_1(3)^n] = 3(2)^n + 7(3)^n$$

$$\Rightarrow [4d_0(2)^n + 9(n+2)^2 d_1(3)^n] - 6[2d_0(2)^n + 3(n+1)^2 d_1(3)^n]$$
$$+ 9[d_0(2)^n + n^2 d_1(3)^n] = 3(2)^n + 7(3)^n$$

$$\Rightarrow [4-12+9]d_0(2)^n + [9(n+2)^2 - 18(n+1)^2 + 9n^2]d_1(3)^n = 3(2)^n + 7(3)^n$$

$$\Rightarrow d_0(2)^n + 18 d_1(3)^n = 3(2)^n + 7(3)^n$$

equate the coeff of $(2)^n$ and $(3)^n$

$d_0 = 3$ and $18d_1 = 7$

$d_1 = \frac{7}{18}$

$$\therefore f^p(n) = d_0(2)^n + n^2 d_1(3)^n$$
$$= 3(2)^n + n^2\left(\frac{7}{18}\right)(3)^n$$
$$= 3\cdot(2)^n + \frac{7}{18}n^2(3)^n$$

Complete soln

$$f(n) = f^h(n) + f^p(n)$$
$$= (A+Bn)(3)^n + 3\cdot(2)^n + \frac{7}{18}n^2(3)^n$$

As $f(0) = 1$ and $f(1) = 4$

$A + 3 = 1$

$A = -2$

$(A+B)\cdot 3 + 3\cdot(2) + \frac{7}{18}(1)^2(3) = 4$

$3A + 3B + 6 + \frac{7}{6} = 4$

$\cancel{3(-2)} + 3B + \cancel{6} + \frac{7}{6} = 4$

$3B = 4 - \frac{7}{6}$

$B = \frac{17}{18}$

$$\therefore f(n) = \left(-2 + \frac{17}{18}n\right)(3)^n + 3\cdot(2)^n + \frac{7}{18}n^2(3)^n$$