1. Semi-gloup: (Already des cues in previous lecture) 2. Sub serni-group: Consider a Berni group (A, *) and BCA then the system (B, *) is said to be sub semi-group if the set B is closed under the operation *. for example: - Consider a semi- geoup (N,+), where N is the set of all natural numbers and + is the addition operation The algebric system (£, +) is a semi-subgroup of (N,+), where E is the set of all +ve integers 3. Monoid: - let A non-empty set G togther with a binary composition * is said to be Monoid if it satisfies following properties > defination of i, closure property (2) Associative property semi-group (3) Existence of Edentity. in that time only (1), and (2)

properties are sabisfed

(1) + a, b + G = 2 + b + G (ii) + a, b, c e g => (a x b) * c = a * (b * c) (iii) + a e G, J e e G & t a * e = a = e * a. (4) Sub monoid :- let us consider, a Monoid (H,*), where * is the binary oferestion and M is the set of are in+jors. Then (M, *) is sus monoiet of (M, *), where of is defined M, = gai/i i kom o ion, a tre integer and a EM} (5) Q: In a group G, Plove that (as) = b a + a, b ∈ G under the operation multiplication ., Sol: we know that for the existence of the inverse of any a E q 8. E a * b = e = b * a (under operation a.b = e = b.a (of under) Which is correct. (a*b)= a b > x Then we say that (ab) = b a] (a*b)= a+b1 x |a = b (a*b) = 5 * a x Now ... 1+ +1 - ~ (bb+) a

(à*b) = b a (ab)(b'a') = a (bb')a' €= 65 = e = a e at = a at s.a.e = a graate e <u></u> = e = b (ata) b f; ata=e (Stat) (ab) = b^f (e) b = b (eb) = btb 8:06=b } b + b = e : (ab) (btat) = e = (btat) (ab) Then by the defination of inverse (ab) = blat Ploved Q: let G be the set of two by two invertible matrices [a b] belonging to G if ad-bc\$0 is a group with making multiplication. Then which is three. in a normal subgroup of G

Then which is the.

(1)
$$H = 9 \int_0^{1} a \int_0^{1} q \neq 0$$
 is a suspend of G

(ii) $H = 2 \int_0^{1} a \int_0^{1} q \neq 0$ is a suspend of G

(iii) both (i) and (ii)

(iv) None of these.

Sol: First of all me cheek H is a subgeof
of growth G.
Let
$$H_1 = \begin{bmatrix} a_1 & 0 \\ 0 & a_1 \end{bmatrix}$$
, $H_2 = \begin{bmatrix} a_2 & 0 \\ 0 & a_2 \end{bmatrix}$
Here $a_1 \neq 0$, $a_2 \neq 0$
be two elements of H .

$$H_1 H_2 = \begin{bmatrix} a_1 & 0 \\ 0 & a_1 \end{bmatrix} \begin{bmatrix} a_2 & 0 \\ 0 & a_2 \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & 0 \\ 0 & a_1 & a_2 \end{bmatrix} \in H$$

$$\therefore a_1 \neq 0$$

$$a_2 \neq 0$$
Then $a_1 a_2 \neq 0$

It holds for closure property.

Now we check its inverse peroperty.

For that find out H, T,

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$$Adj H_{1} = \begin{bmatrix} a_{1} & 0 \\ 0 & a_{1} \end{bmatrix} = \begin{bmatrix} a_{1} & 0 \\ 0 & a_{1} \end{bmatrix}$$

$$|H_{1}| = |A_{1}| - |A_{2}| - |A_{2}$$

: H is Wesed under multiplication and each clement of H has multiplicative inverse.

: H is subglorif of george G.

let G, = (x 4) be any element of G, zu-yz to

$$Adj G_{1} = \begin{pmatrix} u & -z \\ -y & z \end{pmatrix} = \begin{pmatrix} u & -y \\ -z & z \end{pmatrix}$$

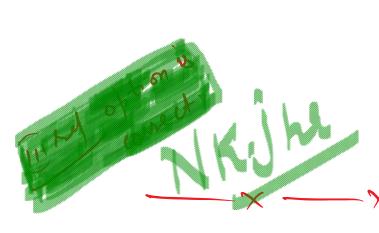
Now
$$G_{1}H_{1}G_{1}^{-1} = \begin{bmatrix} x & y \\ z & u \end{bmatrix}\begin{bmatrix} a_{1} & 0 \\ 0 & a_{1} \end{bmatrix} \begin{bmatrix} u & -y \\ -z & x \end{bmatrix}$$

$$= \frac{1}{xu-y^{2}}\begin{bmatrix} a_{1}(xu-y^{2}) & 0 \\ 0 & a_{1} \end{bmatrix} \begin{bmatrix} a_{1}(xu-y^{2}) & 0 \\ 0 & a_{1} \end{bmatrix} \in H_{1}$$

$$= \frac{1}{xu-y^{2}}\begin{bmatrix} a_{1}(xu-y^{2}) & 0 \\ 0 & a_{1} \end{bmatrix} \in H_{1}$$

.. I G, EG such that G, H, G, EH for H, EH

i. H is normed subgrave, of group of G under the multiplication.



SNote: A susgeont H of a gray q is normal iff

ghat EH for all JEG and hEH.