Rules of Inferences

① Modus ponen or Rule of detachment
Premises $p, p \to q$

Conclusion q

i.e $(p \wedge (p \to q)) \to q$ tautology

$$\frac{\begin{array}{c} p \\ p \to q \end{array}}{\therefore q}$$

② Modus tollens
Premises $\sim q, p \to q$

Conclusion $\sim q$

i.e $(\sim q \wedge (p \to q)) \to \sim q$ tautology

$$\frac{\begin{array}{c} \sim q \\ p \to q \end{array}}{\therefore \sim q}$$

③ Hypothetical Syllogism or Chain rule
Premises $p \to q, q \to r$

Conclusion $p \to r$

i.e $((p \to q) \wedge (q \to r)) \to (p \to r)$ tautology

$$\frac{\begin{array}{c} p \to q \\ q \to r \end{array}}{\therefore p \to r}$$

④ Disjunctive Syllogisms
Premises $\sim p, p \to q$

Conclusion q

i.e $(\sim p \wedge (p \to q)) \to q$ tautology

$$\frac{\begin{array}{c} \sim p \\ p \to q \end{array}}{\therefore q}$$

⑤ Addition
Premise p

Conclusion $p \vee q$

$$\frac{p}{\therefore p \vee q}$$

$$\therefore p \rightarrow (p \lor q) \text{ tautology}$$

⑥ Simplication

Premise $p \land q$

Conclusion p

$$\frac{p \land q}{\therefore p}$$

i·e $p \land q \rightarrow p$ tautology

⑦ Conjunction

Premises p, q

Conclusion $p \land q$

$$\frac{\begin{array}{c} p \\ q \end{array}}{\therefore p \land q}$$

i·e $(p) \land (q) \rightarrow p \land q$ tautology

⑧ Resolution

Premises $p \lor q, \sim p \lor r$

Conclusion $q \lor r$

$$\frac{\begin{array}{c} p \lor q \\ \sim p \lor r \end{array}}{\therefore q \lor r}$$

i·e $(p \lor q) \land (\sim p \lor r) \rightarrow (q \lor r)$ tautology

Q:→ Determine the validity of the following without using truth tables

" Either I will pass the exam or I will not be graduate. If I do not graduate, I will go to USA. I failed. Thus, I will go to USA."

sol:→ Let p: I will pass the exam

q: I will graduate

r: I will go to USA

Premises

$P_1: \quad p \lor \sim q$

$P_2: \quad \sim q \to r$

$P_3: \quad \sim p$

Conclusion $Q: \quad r$

Argument $\qquad p \lor \sim q \,, \sim q \to r \,, \sim p \vdash r$

1) $p \lor \sim q$ Premise

2) $\sim q \to r$ Premise

3) $\sim p$ Premise

4) $\sim q$ Simplification of 1) and 3)

5) r Modus ponens of 2) and 4)

Q:→ Let the following statements

"It is snowing. If it is warm, then it is not snowing. If it is not warm then I cannot go for swimming." Show that the statement " I cannot go for swimming" is a true statement."

sol:- Let $p:$ It is snowing

$q:$ It is warm

$r:$ I can go for swimming

Premises

$P_1: \quad p$

$P_2: \quad q \to \sim p$

$$B : \sim q \rightarrow \sim r$$

Conclusion $\quad Q : \sim r$

1)	p	Premise
2)	$q \rightarrow \sim p$	Premise
3)	$p \rightarrow \sim q$	Contrapositive 2)
4)	$\sim q \rightarrow \sim r$	Premise
5)	$p \rightarrow \sim r$	Hypothetical Syllogism of 3) and 4)
6)	$\sim r$	Modus tollen of 1) and 5)

$$\boxed{p \rightarrow q \equiv \sim q \rightarrow \sim p}$$

$p \rightarrow q$
$q \rightarrow r$
$\therefore p \rightarrow r$

p
$p \rightarrow q$
$\therefore q$