1. Question 1

   Given $f(x,y)= 8xy$, when $0<=x<=1$, $0<=y<=x$ and $f(x,y)=0$ elsewhere, find

   (i) Marginal density of x

   (ii) Marginal density of y

   (iii) Conditional density of x

   (iv) Conditional density of y

Answer:

$f(x,y) = 8xy$ when $0 \leq x \leq 1$
and $0 \leq y \leq x$

(i) Marginal Density of x

$$f(x) = \int_0^x 8xy\, dy = 8x \int_0^x y\, dy$$

$$= \overset{4}{\cancel{8}}x \left[\frac{y^2}{\cancel{2}}\right]_0^x$$

$$\Rightarrow 4x \times x^2 = 4x^3, \quad 0 \leq x \leq 1$$

(ii) Marginal Density of y

$$f(y) = \int_0^1 8xy\, dx = 8y \int_0^1 x\, dx$$

$$= \overset{4}{\cancel{8}}y \left[\frac{x^2}{\cancel{2}}\right]_0^1$$

$$\Rightarrow 4y, \quad 0 \leq y \leq 1$$

(iii) Conditional Density of x:

$$f_{x|y}(x|y) = \frac{\overset{2}{\cancel{8}}x\cancel{y}}{\cancel{4}\cancel{y}} = 2x \quad 0 \leq y \leq x \leq 1$$

(iv) Conditional Density of y:

$$f_{y|x}(y|x) = \frac{\overset{2}{\cancel{8}}\cancel{x}y}{\cancel{4}x^{\cancel{3}\,2}} = \frac{2y}{x^2}, \quad 0 \leq y \leq x \leq 1$$

Q2.Calculate the regression coefficients from the following :

x:	1	2	3	4	5	6	7	8
y:	3	7	10	12	14	17	20	24

Answer:

X	Y	$X \cdot Y$	$X \cdot X$	$Y \cdot Y$
1	3	3	1	9
2	7	14	4	49
3	10	30	9	100
4	12	48	16	144
5	14	70	25	196
6	17	102	36	289
7	20	140	49	400
8	24	192	64	576

Step 2: Find the sum of every column to get:

$$\sum X = 36\,,\ \sum Y = 107\,,\ \sum X \cdot Y = 599\,,\ \sum X^2 = 204\,,\ \sum Y^2 = 1763$$

Step 3: Use the following formula to work out the correlation coefficient.

$$r = \frac{n \cdot \sum XY - \sum X \cdot \sum Y}{\sqrt{\left[n \sum X^2 - \left(\sum X\right)^2\right] \cdot \left[n \sum Y^2 - \left(\sum Y\right)^2\right]}}$$

$$r = \frac{8 \cdot 599 - 36 \cdot 107}{\sqrt{\left[8 \cdot 204 - 36^2\right] \cdot \left[8 \cdot 1763 - 107^2\right]}} \approx 0.9952$$

Q3.Compute the rank correlation co-efficient for the following data:

Sr. no.	1	2	3	4	5	6	7	8	9
X:	10	15	12	17	13	16	24	14	22

Y:	30	42	45	46	33	34	40	35	39

Answer:

Solution:

x	y	Rx	Ry	$d = Rx - Ry$	d^2
10	30	9	9	0	0
15	42	5	3	2	4
12	45	8	2	6	36
17	46	3	1	2	4
13	33	7	8	-1	1
16	34	4	7	-3	9
24	40	1	4	-3	9
14	35	6	6	0	0
22	39	2	5	-3	9
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$$r = 1 - \frac{6 \cdot \sum d^2}{n\left(n^2 - 1\right)}$$

$$= 1 - \frac{6 \cdot 72}{9 \cdot \left(9^2 - 1\right)}$$

$$= 1 - \frac{6 \cdot 72}{9 \cdot (81 - 1)}$$

$$= 1 - \frac{432}{720}$$

$$= 1 - 0.6$$

$$= 0.4$$

Q4.12 entries in a painting competition were ranked by two judges , Judge I : 5, 2, 3, 4, 1, 6, 8, 7, 10, 9, 12, 11 and by Judge II: 4, 5, 2, 1, 6, 7, 10, 9, 11, 12, 3, 8 . Calculate Spearman's rank correlation coefficient.

Answer:

Solution:

x	y	Rx	Ry	$d = Rx - Ry$	d^2
5	4	8	9	-1	1
2	5	11	8	3	9
3	2	10	11	-1	1
4	1	9	12	-3	9
1	6	12	7	5	25
6	7	7	6	1	1
8	10	5	3	2	4
7	9	6	4	2	4
10	11	3	2	1	1
9	12	4	1	3	9
12	3	1	10	-9	81
11	8	2	5	-3	9
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$$r = 1 - \frac{6 \cdot \sum d^2}{n\left(n^2 - 1\right)}$$

$$= 1 - \frac{6 \cdot 154}{12 \cdot \left(12^2 - 1\right)}$$

$$= 1 - \frac{6 \cdot 154}{12 \cdot (144 - 1)}$$

Q5. A manufacturer claims that only 4% of his products supplied by him are defective. A random sample of 600 products contains 36 defectives. Test the claim of the manufacturer.

Answer:

H_0 : p (proportion of defective apples) $= 0.04$ is true

H_1: $P \neq 0.04$ (population)

$P = 0.04$; $Q = 0.96$; $n = 600$; $x = 36$

p' (sample proportion) $= 36/600 = 0.06$

$$|z| = \frac{0.06 - 0.04}{\sqrt{\frac{0.04 \times 0.96}{600}}} = 2.5$$

Since, $|z| = 2.5 > 1.96$ at 5% significant level.

Hence, Ho is rejected i.e claim is not acceptable.