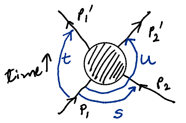
A1) We have all particles of mass m i.e. for the two body scattering;



visualization of Mandelstam variables

So, the Mandelstam variables are; $S = (P_1+P_2)^2 = (P_1'+P_2')^2$

$t = (P_1'-P_1)^2 = (P_2'-P_2)^2$ and $u = (P_2'-P_1)^2 = (P_1'-P_2)^2$

The kinematical constraints are Energy-Momentum conservation.

$\therefore S+t+u = 3P_1^2 + P_2^2 + P_2'^2 + P_1'^2 + 2P_1P_2 - 2P_1'P_1 - 2P_2'P_1$

$= 3P_1^2 + P_2^2 + P_2'^2 + P_1'^2 + 2P_1(P_2 - P_1' - P_2')$

and by conservation of momentum, $P_1+P_2 = P_1'+P_2' = 0$ at constant mass $=m$ and at COM frame; $\vec{P}_1+\vec{P}_2 = \vec{P}_1'+\vec{P}_2' = 0$.

$\Rightarrow P_2 - P_1 - P_2 = -P_1$

$\therefore S+t+u = \cancel{3}P_1^2 + P_2^2 + P_2'^2 + P_1'^2 - \cancel{2P_1^2} = P_1^2 + P_2^2 + P_1'^2 + P_2'^2$

$= 4m^2$

$\Rightarrow \boxed{S+t+u = 4m^2}$

Now, we have; $S = (P_1+P_2)^2 = P_1^2 + P_2^2 + 2P_1P_2$ where $P_1 = (E_1 = \sqrt{\vec{P}_1^2+m^2}, \vec{P}_1)$

$P_2 = (E_2 = \sqrt{\vec{P}_2^2+m^2}, \vec{P}_2)$ at COM frame; $\vec{P}_1+\vec{P}_2 = 0$.

$= 2m^2 + 2\sqrt{(\vec{P}_1^2+m^2)(\vec{P}_2^2+m^2)} - 2\vec{P}_1\cdot\vec{P}_2$

$= 2m^2 + 2m^2\left[\sqrt{\left(1+\frac{\vec{P}_1^2}{m^2}\right)\left(1+\frac{\vec{P}_2^2}{m^2}\right)} - \frac{\vec{P}_1\cdot\vec{P}_2}{m^2}\right] = 2m^2 + 2m^2\left[\sqrt{1+\frac{\vec{P}_1^2+\vec{P}_2^2}{m^2}+\frac{(\vec{P}_1\cdot\vec{P}_2)^2}{m^4}} - \frac{\vec{P}_1\cdot\vec{P}_2}{m^2}\right]$

By comparison, we have; $\sqrt{1+\frac{\vec{P}_1^2+\vec{P}_2^2}{m^2}+\frac{(\vec{P}_1\cdot\vec{P}_2)^2}{m^4}} \geq 1 + \frac{\vec{P}_1\cdot\vec{P}_2}{m^2}$ i.e. $\sqrt{1+\frac{\vec{P}_1^2+\vec{P}_2^2}{m^2}+\frac{(\vec{P}_1\cdot\vec{P}_2)^2}{m^4}} - \frac{\vec{P}_1\cdot\vec{P}_2}{m^2} \geq 1$

$\therefore S = 2m^2 + 2m^2\left[\sqrt{1+\frac{\vec{P}_1^2+\vec{P}_2^2}{m^2}+\frac{(\vec{P}_1\cdot\vec{P}_2)^2}{m^4}} - \frac{\vec{P}_1\cdot\vec{P}_2}{m^2}\right] \geq 2m^2+2m^2 = 4m^2$

$\Rightarrow \boxed{S \geq 4m^2}$

Now; $t = (P_1'-P_1)^2 = 2m^2 + 2\vec{P}_1'\cdot\vec{P}_1 - 2\sqrt{(\vec{P}_1^2+m^2)(\vec{P}_1'^2+m^2)} = 2m^2 - 2m^2\left[\sqrt{1+\frac{\vec{P}_1^2+\vec{P}_1'^2}{m^2}+\frac{(\vec{P}_1\cdot\vec{P}_1')^2}{m^4}} - \frac{\vec{P}_1\cdot\vec{P}_1'}{m^2}\right]$

Similarly, we have; $\left[\sqrt{1+\frac{\vec{P}_1^2+\vec{P}_1'^2}{m^2}+\frac{(\vec{P}_1\cdot\vec{P}_1')^2}{m^4}} - \frac{\vec{P}_1\cdot\vec{P}_1'}{m^2}\right] \geq 1$ i.e. $t = 2m^2 - 2m^2\left[\sqrt{1+\frac{\vec{P}_1^2+\vec{P}_1'^2}{m^2}+\frac{(\vec{P}_1\cdot\vec{P}_1')^2}{m^4}} - \frac{\vec{P}_1\cdot\vec{P}_1'}{m^2}\right]$

$\leq 2m^2 - 2m^2 = 0$

$\Rightarrow \boxed{t \leq 0}$

Similarly; $u = (P_2'-P_1)^2 = P_2'^2 + P_1^2 - 2P_1P_2'$

$= 2m^2 - 2\sqrt{(m^2+\vec{P}_2'^2)(m^2+\vec{P}_1^2)} + 2\vec{P}_1\cdot\vec{P}_2' = 2m^2 - 2m^2\underbrace{\left[\sqrt{1+\frac{(\vec{P}_2'\cdot\vec{P}_1)^2}{m^4}+\frac{\vec{P}_1^2+\vec{P}_2'^2}{m^2}} - \frac{\vec{P}_1\cdot\vec{P}_2'}{m^2}\right]}_{\geq 1}$

$\therefore u \leq 2m^2 - 2m^2 \Rightarrow \boxed{u \leq 0}$

A2) We have $dLIPS = (2\pi)^4\delta^4(P_f - P_i)\prod_{j=1}^{n}\frac{d^3\vec{P}_j}{(2\pi)^3 2E_j}$ $\longrightarrow$ Lorentz Invariant Phase space.

where P_f = Total momenta of final particles

P_i = Total momenta of Initial particles.

In the COM frame of 2 body final state, we have;

$\vec{P}_f = \vec{P}_i = 0$ i.e. 3-momentum of all final (& initial) particles $=0$.

we have $\underbrace{1+2}_{\text{initial}} \longrightarrow \underbrace{3+4}_{\text{final}}$ $\Rightarrow LIPS = \int dLIPS = \int_f (2\pi)^4\delta^4(P_f-P_i)\prod_{j=1}^{n}\frac{d^3\vec{P}_j}{(2\pi)^3 2E_j}$

over all final momenta

$f \to$ all final momenta

individual final particles.

$\therefore$ here $P_i = P_1+P_2$ and

$P_f = P_3+P_4$ $\quad \vec{P}_1+\vec{P}_2 = \vec{P}_3+\vec{P}_4 = 0$

$\Rightarrow \vec{P}_1 = -\vec{P}_2$ and $\vec{P}_3 = -\vec{P}_4 \Rightarrow |\vec{P}_3| = |-\vec{P}_4| = p$ (let) and we have $E_3 = \sqrt{p^2+m_3^2}$; $E_4 = \sqrt{p^2+m_4^2}$

$\therefore LIPS = \int_{f=3,4} \cancel{(2\pi)^4}\,\delta(E_3+E_4-E)\,\delta^3(\vec{P}_3+\vec{P}_4-\vec{P}_1-\vec{P}_2)\frac{d^3\vec{P}_3}{\cancel{(2\pi)^3}2E_3}\frac{d^3\vec{P}_4}{(2\pi)^{\cancel{3}2}2E_4}$ where $j=f=3,4 \to$ final particles.

and E = initial particles energy in COM frame.

$= \frac{1}{4\pi^2}\int_{f=3,4}\frac{d^3\vec{P}_3\,d^3\vec{P}_4}{2E_3\,2E_4}\,\delta(\underbrace{E_3+E_4}_{=E_f}-\underbrace{E}_{=E_i})\,\delta^3(0)$ since $\vec{P}_1+\vec{P}_2 = \vec{P}_3+\vec{P}_4 = 0$.

$= \frac{1}{16\pi^2}\int_{f=3}\frac{d^3\vec{P}_3}{E_3E_4}\delta(E_3+E_4-E) = \frac{1}{16\pi^2}\int\frac{|\vec{P}_3|^2 d|\vec{P}_3|\,\overbrace{\sin\theta\, d\theta\, d\phi}^{=d\Omega}}{E_3E_4}\delta(E_3+E_4-E)$

and we have $|\vec{P}_3| = |-\vec{P}_4| = p$ i.e. $\frac{1}{16\pi^2}\int\frac{p^2dp\,d\Omega}{E_3E_4}\delta(E_3+E_4-E)$

and we have $E_3 = \sqrt{p^2+m_3^2}$ and $E_4 = \sqrt{p^2+m_4^2}$ i.e. $\Rightarrow \frac{1}{16\pi^2}\int\frac{p^2dp\,d\Omega}{E_3E_4}\delta\left(\sqrt{p^2+m_3^2}+\sqrt{p^2+m_4^2}-E\right)$

For $E = \sqrt{p^2+m_3^2} + \sqrt{p^2+m_4^2}$ we have;

$E^2 = 2p^2 + m_3^2 + m_4^2 + 2\sqrt{p^4 + m_3^2m_4^2 + p^2(m_4^2+m_3^2)}$

$\Rightarrow (E^2 - 2p^2 - m_3^2 - m_4^2)^2 = 4(p^4 + m_3^2m_4^2 + p^2(m_4^2+m_3^2))$ $\Rightarrow (E^2-m_3^2-m_4^2)^2 + 4p^4 - 4p^2(E^2-\cancel{m_3^2}-\cancel{m_4^2}) = 4p^4 + 4m_3^2m_4^2 + \cancel{4p^2m_4^2} + \cancel{4p^2m_3^2}$

$\Rightarrow (E^2-m_3^2-m_4^2)^2 - 4m_3^2m_4^2 = \frac{4p^2E^2}{4E^2}$

$4E^2$

$\Rightarrow p = |\vec{P}_3| = \frac{1}{\ldots}\left[(E^2-m_3^2-m_4^2)^2 - 4m_3^2m_4^2\right]^{1/2}$

$$\Rightarrow \frac{(E^2-m_3^2-m_4^2)^2 - 4m_3^2m_4^2}{4E^2} = \frac{4p^2\cancel{E^2}}{4\cancel{E^2}}$$

$$= 4p^4 + 4m_3^2m_4^2 + 4p^2m_4^2 + 4p^2m_3^2$$

$$\Rightarrow p^2 = \frac{1}{4E^2}\left[(E^2-m_3^2-m_4^2)^2 - 4m_3^2m_4^2\right] \Rightarrow \underbrace{p=|p_3|}_{\text{always +ve}} = \underbrace{\frac{1}{2E}\left[(E^2-m_3^2-m_4^2)^2-4m_3^2m_4^2\right]^{1/2}}_{=p'}$$

Now, we have;

$$\frac{1}{16\pi^2}\int \frac{p^2\,dp\,d\Omega}{E_3E_4}\,\delta\left(\sqrt{p^2+m_3^2}+\sqrt{p^2+m_4^2}-E\right)$$ and by changing the dirac-delta function from $\delta\left(\sqrt{p^2+m_3^2}+\sqrt{p^2+m_4^2}-E\right)$ to $\delta(p-p')$ we use the property $\delta(E'-E) = \dfrac{\delta(p-p')}{\frac{d}{dp}(E'-E)}$ i.e.

$$\Rightarrow \text{LIPS} = \frac{1}{16\pi^2}\int\frac{p^2\,dp\,d\Omega}{E_3E_4}\;\frac{\delta(p-p')}{\left.\frac{d}{dp}\left(\sqrt{p^2+m_3^2}+\sqrt{p^2+m_4^2}-E\right)\right|_{p=p'}}$$

$$= \frac{1}{16\pi^2}\int\frac{p^2\,dp\,d\Omega}{\cancel{E_3E_4}}\;\frac{\delta(p-p')\,\cancel{E_3E_4}}{E\,p'} = \boxed{\frac{1}{16\pi^2}\frac{p'\,d\Omega}{E}}$$

where $p' = \dfrac{1}{2E}\left[(E^2-m_3^2-m_4^2)^2-4m_3^2m_4^2\right]^{1/2}$

Now, we have the differential scattering cross section $\frac{d\sigma}{d\Omega}$ as;

$$d\sigma = \int \frac{S}{4I}\underbrace{(2\pi)^4\,\delta^4(P_f-P_i)\prod_{j=1}^{n}\frac{d^3\vec{p}_j}{(2\pi)^3\,2E_j}}_{dLIPS}\,|M_{fi}|^2$$ where f = final particles, i = initial particles.

$$\Rightarrow d\sigma = \underbrace{\frac{S}{4I}|M_{fi}|^2}_{\text{assuming no spatial dependence}}\int dLIPS = \frac{S}{4I}|M_{fi}|^2\frac{1}{16\pi^2}\frac{p'\,d\Omega}{E} = \frac{S\,p'\,|M_{fi}|^2\,d\Omega}{64\pi^2 I\,E}$$

∴ differential scattering cross section; $\boxed{\dfrac{d\sigma}{d\Omega} = \dfrac{S\,p'\,|M_{fi}|^2}{64\pi^2 E I}}$

For $\frac{\lambda\phi^4}{4!}$ theory, we have; $M_{fi} = -\lambda$ since $\langle f|\,S-\mathbb{1}\,|i\rangle = -\lambda i(2\pi)^4\delta^4(P_1+P_2-P_3-P_4)$ (upto first order in λ)

and $S=1$ i.e. $\dfrac{d\sigma}{d\Omega} = \dfrac{\lambda^2 p'}{64\pi^2 E I}$

∴ Total crossection; $\boxed{\sigma = \dfrac{\lambda^2 p'}{16\pi E I}}$ → for scalar particles the $d\Omega$ is integrated over to get 4π.

A3> we have a particle of mass M decaying into two particles of mass m_1 and m_2 as;

$\underset{M}{1} \longrightarrow \underset{m_1}{2} + \underset{m_2}{3}$ and we have the differential decay rate as;

$d\Gamma = \dfrac{1}{2M}|M_{fi}|^2\,dLIPS$ and for our case, we have

$$dLIPS = \int\frac{(2\pi)^4\,\delta^4(P_2+P_3-P_1)}{(2\pi)^6\,2E_2\,2E_3}\,d^3\vec{p}_2\,d^3\vec{p}_3\,d\Omega$$

we consider the rest frame of the decaying particle i.e. in $1. \longrightarrow \vec{p}_1 = \vec{p}_2+\vec{p}_3 = 0$ or $|\vec{p}_2| = |-\vec{p}_3| = p$ (let)

$$\therefore dLIPS = \int\frac{1}{2E_2\,2E_3}\times\frac{1}{(2\pi)^2}\,\underbrace{\delta^3(\vec{p}_2+\vec{p}_3-\vec{p}_1)}_{=\delta^3(0)}\,\delta(E_2+E_3-\underbrace{M}_{=E_1})\,d^3\vec{p}_2\,d^3\vec{p}_3\,d\Omega \quad = (M,0,0,0)$$

$$= \int\frac{p^2\,dp}{(2\pi)^2}\,\frac{\delta\left(\sqrt{p^2+m_1^2}+\sqrt{p^2+m_2^2}-M\right)}{2E_2\,2E_3}\,d\Omega$$ and from previous calculation, we have; $p=p'$ and evaluate the integral at $p=p'$ where;

$$= \frac{d\Omega}{16\pi^2}\int\frac{p^2\,dp}{\cancel{E_2E_3}}\,\frac{\delta(p-p')\,\cancel{E_2E_3}}{M\,p'} \qquad p' = \frac{1}{2M}\left[(M^2-m_1^2-m_2^2)^2-4m_1^2m_2^2\right]^{1/2}$$

$$= \boxed{\frac{p'\,d\Omega}{16\pi^2 M}}$$ where p'

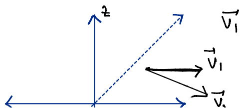
we have the differential decay rate as; $d\Gamma = \dfrac{1}{2M}|M_{fi}|^2\dfrac{p'\,d\Omega}{16\pi^2 M} = \boxed{\dfrac{|M_{fi}|^2\,p'\,d\Omega}{32\pi^2M^2}}$

For decay of a scalar particle, we have $|M_{fi}|^2$ independent of angular terms i.e. we can integrate over $d\Omega$.

$$\Rightarrow \text{Decay rate, } \Gamma = \int d\Gamma = \frac{p'\,|M_{fi}|^2}{8\pi M^2}$$

$$= \frac{|M_{fi}|^2}{8\pi M^2}\,\frac{1}{2M}\left[(M^2-m_1^2-m_2^2)^2-4m_1^2m_2^2\right]^{1/2}$$

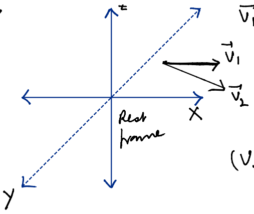
A4>



$\vec{v}_1$ and $\vec{v}_2$ are not necessarily collinear.

Let the $\vec{v}_1$ be moving along the X-axis and $\vec{v}_2$ moving in a different direction wrt $\vec{v}_1$.

A4)



$\vec{v}_1$ and $\vec{v}_2$ are not necessarily collinear.

Let the $\vec{v}_1$ be moving along the X-axis and $\vec{v}_2$ moving in a different direction wrt $\vec{v}_1$.

By Einstein's velocity addition, we have the components of relative velocity as;

$$(V_{rel})_x = \frac{(V_2)_x - V_1}{1-(V_2)_x V_1} \; ; \; (V_{rel})_y = \frac{(V_2)_y\sqrt{1-v_1^2}}{1-(V_2)_x V_1} \; ; \; (V_{rel})_z = \frac{(V_2)_z\sqrt{1-v_1^2}}{1-(V_2)_x V_1}$$

∴ we have;

$$|V_{rel}|^2 = \frac{1}{[1-(V_2)_x V_1]^2}\left[((V_2)_x - V_1)^2 + (V_2)_y^2(1-v_1^2) + (V_2)_z^2(1-v_1^2)\right]$$

$$= \frac{\underbrace{(v_1^2 + v_2^2 - 2V_1(V_2)_x)}_{(\vec{v}_1-\vec{v}_2)^2} - V_1^2(V_2)_y^2 - V_1^2(V_2)_z^2}{[1-(V_2)_x V_1]^2}$$

where $(V_2)_m$ is the m-th component of $\vec{V}_2$.

Now, we have $(V_2)_x V_1 = V_1 (V_2)_x = \vec{v}_1\cdot\vec{v}_2$ and $V_1\hat{n} \times V_2\hat{n}$ where $V_2\hat{n} = (V_2)_x\hat{x} + (V_2)_y\hat{y} + (V_2)_z\hat{z}$

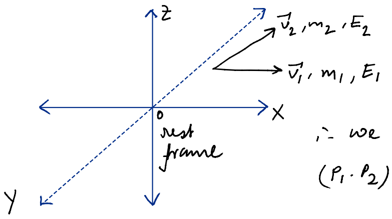
from here $\rightarrow = V_1(V_2)_y\hat{z} - V_1(V_2)_z\hat{y}$

$$\therefore |V_{rel}|^2 = \frac{(\vec{v}_1-\vec{v}_2)^2 - (\vec{v}_1\times\vec{v}_2)^2}{(1-\vec{v}_1\cdot\vec{v}_2)^2}$$

$$\Rightarrow \text{Relative velocity}; \; |V_{rel}| = \frac{\sqrt{(\vec{v}_1-\vec{v}_2)^2 - (\vec{v}_1\times\vec{v}_2)^2}}{(1-\vec{v}_1\cdot\vec{v}_2)}$$

→ this factor is coming which is mentioned in Landau – Lifshitz.

A5) Two particles of masses $(m_1 \,\&\, m_2)$ with Energies E_1, E_2 moving with velocities $\vec{v}_1$ & $\vec{v}_2$ in arbitrary direction.



we have defined the scalar $I = (p_1\cdot p_2)^2 - m_1^2 m_2^2$

and in relativistic frame, we have;

$$p_1 = \left(\underbrace{E_1}_{=\frac{m_1}{\sqrt{1-\vec{v}_1^2}}}, \frac{m_1\vec{v}_1}{\sqrt{1-\vec{v}_1^2}}\right) ; \; p_2 = \left(\underbrace{E_2}_{=\frac{m_2}{\sqrt{1-\vec{v}_2^2}}}, \frac{m_2\vec{v}_2}{\sqrt{1-\vec{v}_2^2}}\right)$$

∴ we have;

$$(p_1\cdot p_2)^2 = \left(\frac{m_1 m_2}{\sqrt{(1-\vec{v}_1^2)(1-\vec{v}_2^2)}} - \frac{m_1 m_2\,\vec{v}_1\cdot\vec{v}_2}{\sqrt{(1-\vec{v}_1^2)(1-\vec{v}_2^2)}}\right)^2$$

$$\Rightarrow (p_1\cdot p_2)^2 - m_1^2 m_2^2 = \frac{m_1^2 m_2^2}{(1-\vec{v}_1^2)(1-\vec{v}_2^2)}\left[1 + (\vec{v}_1\cdot\vec{v}_2)^2 - 2\vec{v}_1\cdot\vec{v}_2\right] - m_1^2 m_2^2$$

$$= \frac{m_1^2 m_2^2}{(1-\vec{v}_1^2)(1-\vec{v}_2^2)}\left[\cancel{1} + (\vec{v}_1\cdot\vec{v}_2)^2 - 2\vec{v}_1\cdot\vec{v}_2 - \cancel{1} + \vec{v}_2^2 + \vec{v}_1^2 - \vec{v}_1^2\vec{v}_2^2\right]$$

$$= \frac{m_1^2 m_2^2}{(1-\vec{v}_1^2)(1-\vec{v}_2^2)}\left[(\vec{v}_1-\vec{v}_2)^2 - v_1^2 v_2^2\underbrace{(1-\cos^2\theta)}_{=\sin^2\theta}\right] = \frac{m_1^2 m_2^2}{\underbrace{(1-\vec{v}_1^2)}_{=E_1^2}\underbrace{(1-\vec{v}_2^2)}_{=E_2^2}}\left[(\vec{v}_1-\vec{v}_2)^2 - (\vec{v}_1\times\vec{v}_2)^2\right]$$

$$= E_1^2 E_2^2\left[(\vec{v}_1-\vec{v}_2)^2 - (\vec{v}_1\times\vec{v}_2)^2\right]$$

$$\therefore \text{we have} \quad \boxed{I = E_1^2 E_2^2\left[(\vec{v}_1-\vec{v}_2)^2 - (\vec{v}_1\times\vec{v}_2)^2\right]}$$

Now, in COM frame, we have $\vec{p}_1 + \vec{p}_2 = 0$ i.e. $|\vec{p}_1| = |-\vec{p}_2| = p$ and $\vec{p}_1\cdot\vec{p}_2 = -p^2$

$$\therefore I = (p_1\cdot p_2)^2 - m_1^2 m_2^2 = (E_1E_2 + p^2)^2 - (E_1^2 - p^2)(E_2^2 - p^2)$$

$$\Rightarrow I = \cancel{E_1^2E_2^2} + 2p^2E_1E_2 + \cancel{p^4} - \cancel{E_1^2E_2^2} - \cancel{p^4} + p^2(E_1^2 + E_2^2)$$

$$= p^2(E_1^2 + E_2^2 + 2E_1E_2) = p^2(E_1+E_2)^2$$

$$\therefore \boxed{I = (E_1+E_2)^2 p^2} \quad \text{or} \quad \boxed{\sqrt{I} = (E_1+E_2)|p|}$$

A6) we have, $\mathcal{L} = \frac{1}{2}(\partial_\mu\varphi)^2 - \frac{1}{2}m^2\varphi^2 - \frac{g}{3!}\varphi^3(x) \Rightarrow \mathcal{H}_{int} = \frac{g}{3!}\varphi^3(x)$ i.e. we have;

This theory is used to show internal lines which are not present in $\frac{\lambda\varphi^4}{4!}$ theory.

we have, the S-matrix as:

$$S = T\left(e^{-i\int \mathcal{H}_{int}(x)\,d^4x}\right) = T\left(e^{-i\int \frac{g}{3!}\varphi^3(x)\,d^4x}\right) = \sum_{n=0}^{\infty}\int \frac{(-i)^n}{n!}T(\mathcal{H}_{int}(x_1), \mathcal{H}_{int}(x_n))\,d^4x_1 \ldots d^4x_n$$

$$\Rightarrow S = \mathbb{1} - \frac{ig}{3!}\int d^4x\, T(\varphi^3(x)) + \frac{(-ig)^2}{(3!)^2}\int d^4x_1\, d^4x_2\, T(\varphi^3(x_1), \varphi^3(x_2)) + \ldots$$

$$\Rightarrow S = \mathbb{1} - \frac{ig}{3!}\int d^4x\, T(:\varphi^3(x):) + \frac{(-ig)^2}{(3!)^2}\int d^4x_1\, d^4x_2\, T(:\varphi^3(x_1):, :\varphi^3(x_2):)$$

where I have normal ordered the interaction Hamiltonian.

Now, we have; $\varphi(x) = \int \frac{d^3p}{}\,\frac{1}{}\,(a_p^\dagger e^{ipx} + a_p e^{-ipx}) = \varphi + \varphi_+$

where I have normal ordered the interaction Hamiltonian.

Now, we have; $\varphi(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left(a_p^\dagger e^{ipx} + a_p e^{-ipx}\right) = \underset{\text{for } a^+}{\varphi_-} + \underset{\text{for } a}{\varphi_+}$

$$\therefore \; S = \mathbb{1} - \frac{ig}{3!}\int d^4x \; \left(\varphi_-^3 + \varphi_+^3 + 3\varphi_-\varphi_+^2 + 3\varphi_-^2\varphi_+\right) + \Big[\frac{(-ig)^2}{(3!)^2}\int d^4x_1\, d^4x_2 \,(:\varphi^3(x_1)\varphi^3(x_2):$$

$$+\, 3^2 \times \varphi(x_1)\varphi(x_2) \; :\varphi^2(x_1)\,\varphi^2(x_2):$$

$$\boxed{+\, \frac{3^2\times 2^2}{2!}\left(\varphi(x_1)\varphi(x_2)\right)^2 :\varphi(x_1)\,\varphi(x_2): + \frac{3^2\times 2^2}{3!}\left(\varphi(x_1)\,\varphi(x_2)\right)^3}\;\Big]$$

and we have; 2 particle scattering process i.e. $\underbrace{p_1 + p_2}_{|i\rangle} \longrightarrow \underbrace{p_1' + p_2'}_{|f\rangle}$

Now, when we have the scattering, we have; $\langle p_1', p_2' | T | p_1, p_2\rangle$ where T-matrix is $T = S - \mathbb{1}$

i.e. we neglect the no-scattering part (i.e. $\mathbb{1}$) and we have;

$$\langle p_1', p_2' | T | p_1, p_2\rangle = \int d^4a_1\, d^4a_2\, d^4a_3\, d^4a_4 \; e^{i(p_1a_1 + p_2a_2)}\, e^{-i(p_1'a_3 + p_2'a_4)}$$

$$\times \langle 0 | T\{\varphi_1\varphi_2\varphi_3\varphi_4\, e^{-ig/3!\int d^4x\, \varphi^3(x)}\} |0\rangle$$

and when we expand the above form like before, we have; for $O(g)$ only the $3\varphi_-\varphi_+^2$ term will remain which reduces the 2 incoming particles to 1 and then again generates 2 particles i.e.;

$O(g)$: $-\frac{ig}{3!}\int d^4x \,(3\varphi_-\varphi_+^2)$ and similarly $O(g^2)$: $\frac{(-ig)^2}{(3!)^2}\int d^4x_1\, d^4x_2\; 3^2\times \underbrace{\varphi(x_1)\varphi(x_2)}_{=D_F(x_1-x_2)}$

$\underbrace{:\varphi^2(x_1)\;\varphi^2(x_2):}$

this contributes 2 creation and 2 annihilation operators.

$\Rightarrow O(g)$: $-\frac{ig}{3!}\int d^4x\,(3\varphi_-\varphi_+^2)$ and;

$$O(g^2) = \frac{(-ig)^2}{(3!)^2}\int d^4x_1\, d^4x_2\; 9\, D_F(x_1-x_2)\; :\left(\varphi_-(x_1) + \varphi_+(x_1)\right)^2\left(\varphi_-(x_2) + \varphi_+(x_2)\right)^2:$$

$$= \frac{(-ig)^2}{(3!)^2}\int d^4x_1\, d^4x_2\; 9\, D_F(x_1-x_2)\left(\varphi_-(x_1)\,\varphi_+(x_2)\right)^2$$

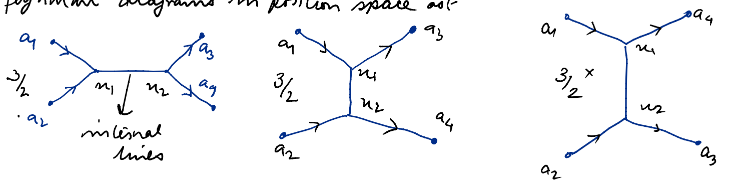
$\therefore$ we have; the scattering as; $\langle p_1', p_2' | T | p_1, p_2\rangle = -\frac{ig}{3!}\int d^4a_1\, d^4a_2\, d^4a_3\, d^4a_4\, e^{i(p_1a_1+p_2a_2)}\, e^{-i(p_1'a_3 + p_2'a_4)}$

$$3\int d^4x\, \langle 0| a_{p_1'} a_{p_2'}\, \varphi_-\varphi_+^2\, a_{p_1}^\dagger a_{p_2}^\dagger |0\rangle$$

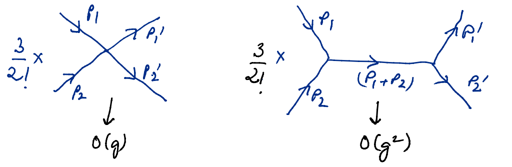
$$+ \frac{(-ig)^2}{(3!)^2}\int d^4a_1\, d^4a_2\, d^4a_3\, d^4a_4\, e^{i(p_1a_1 + p_2a_2)}\, e^{-i(p_1'a_3 + p_2'a_4)}$$

$$9\int d^4x_1\, d^4x_2\; D_F(x_1-x_2)\; \langle 0| a_{p_1'} a_{p_2'}\left(\varphi_-(x_1) + \varphi_+(x_2)\right)^2 a_{p_1}^\dagger a_{p_2}^\dagger |0\rangle$$

By observing the expressions, we have a familiar form for the $\frac{\lambda\varphi^4}{4!}$ theory & we have the feynman diagrams in position space as:



In momentum space, we have the propagators change to $D_F(p_1+p_2)$ from $D_F(x_1-x_2)$, as we did, which indicates the internal line;



We have the Feynmann rules as:

* For $O(g)$: we have $\langle p_1', p_2' | T | p_1, p_2\rangle = -\frac{ig}{\cancel{3!}_{\,2}}\int d^4a_1\, d^4a_2\, d^4a_3\, d^4a_4\, e^{i(p_1a_1 + p_2a_2)}\, e^{-i(p_1'a_3 + p_2'a_4)}$

$$\cancel{3}\int d^4x\, \langle 0| a_{p_1'} a_{p_2'}\, \varphi_-\varphi_+^2\, a_{p_1}^\dagger a_{p_2}^\dagger |0\rangle$$

$$= -\frac{ig}{2}\int d^4x \left[e^{-i(p_1' + p_2' - p_1 - p_2)x}\right]$$

$$= -ig\, \underline{(2\pi)^4\, \delta(p_1' + p_2' - p_1 - p_2)}$$

Calculation similar to φ^4-theory.

* For $O(g^2)$: we have; $\int d^4a_1\, d^4a_2\, d^4a_3\, d^4a_4\, e^{i(p_1a_1 + p_2a_2)}\, e^{-i(p_1'a_3 + p_2'a_4)} \times$

$$\int d^4x_1 \int d^4x_2 \ \frac{(-ig)^2}{(3!)^2} D_F(a_1-x_1) D_F(a_2-x_1) D_F(x_1-x_2) D_F(x_2-a_3) D_F(x_2-a_4)$$

$$= \frac{(-ig)^2}{(3!)^2}(2\pi)^4 \delta^4(p_1'+p_2'-p_1-p_2) D_F(p_1) D_F(p_2) \underbrace{D_F(p_1+p_2)}_{\text{internal line.}} D_F(p_1') D_F(p_2')$$

* Every $(-ig)$ represents a 3 point vertex and D_F represents an internal line.

* To each internal line, we associate a propagator with the value of the four-momentum given by Energy-momentum conservation.

A7) We have the Lagrangian density;

$\mathcal{L} = \frac{1}{2}(\partial_\mu \Phi)^2 - \frac{1}{2}M^2\Phi^2 + \frac{1}{2}(\partial_\mu\varphi)^2 - \frac{1}{2}m^2\varphi^2 - \mu\Phi\varphi^2$ where a Φ particle decays into two φ particles.

Now; if we mode expand both the scalar fields, we have;

$\Phi = \int \frac{d^3p}{(2\pi)^3}(a_p^+ e^{ipx} + a_p e^{-ipx})$ & $\varphi = \int \frac{d^3q}{(2\pi)^3}(b_q^+ e^{iqx} + b_q e^{-iqx})$ and $\mathcal{H}_{int} = \mu\Phi\varphi^2$

$= \Phi_- + \Phi_+$ $\qquad = \varphi_- + \varphi_+$

and we have $1 \to 2+3$ decay as; $|i\rangle = \sqrt{2E_p}\, a_p^+|0\rangle$; $|f\rangle = \sqrt{4E_{q_1}E_{q_2}}\, b_{q_1}^+ b_{q_2}^+|0\rangle$

$\therefore$ we have ; $S = T(e^{-i\int \mathcal{H}_{int} d^4x}) = T(e^{-i\int :\mu\Phi\varphi^2: d^4x})$ where we have normal ordered the interacting hamiltonian.

$\approx \mathbb{1} - i\mu \int d^4x \, T(:\Phi\varphi^2:) + \dots$

keeping upto $O(\mu)$, we have; $S = \mathbb{1} - i\mu\int d^4x \, T(:(\Phi_- + \Phi_+)(\varphi_- + \varphi_+)^2:)$

If we observe, we notice that the term with $\varphi_-^2\Phi_+$ will only contribute for the decay.

$\therefore$ we have;

$\langle f|\underbrace{S-\mathbb{1}}_{=T}|i\rangle = (-i\mu)\int d^4x \langle 0|\sqrt{4E_{q_1}E_{q_2}}\, b_{q_1} b_{q_2}\, \varphi_-^2\Phi_+ \sqrt{2E_p}\, a_p^+|0\rangle$

$= (-i\mu)\int d^4x \sqrt{8E_pE_{q_1}E_{q_2}} \langle 0| b_{q_1} b_{q_2} \iint \frac{d^3l_1}{(2\pi)^3}\frac{d^3l_2}{(2\pi)^3}\frac{b_{l_1}^+ b_{l_2}^+}{\sqrt{4E_{l_1}E_{l_2}}} e^{i(l_1-l_2)x}$

$\int \frac{d^3l_3}{(2\pi)^3}\frac{a_{l_3}e^{-il_3x}}{\sqrt{2E_{l_3}}} a_p^+|0\rangle$

$= \frac{(-i\mu)\sqrt{8E_pE_{q_1}E_{q_2}}}{(2\pi)^6}(2\pi)^4 \int \frac{d^3l_1\, d^3l_2\, d^3l_3}{\sqrt{8E_{l_1}E_{l_2}E_{l_3}}}(2\pi)^3\delta(l_3-l_1-l_2)\,\delta(\vec{l_3}-\vec{p})\,[\langle 0|b_{q_1} b_{l_1}^+ b_{q_2} b_{l_2}^+|0\rangle + (2\pi)^3\delta(\vec{l_1}-\vec{l_2})\langle 0|b_{q_1}b_{l_2}^+|0\rangle]$

By reducing, we have;

$= \underline{(2\pi)^4(-2i\mu)\,\delta(q_1+q_2-p)}$ where $|\mathcal{M}_{fi}| = 2\mu$

So, we have; decay rate ; $\Gamma = \frac{1}{8\pi M^2}\frac{|\mathcal{M}_{fi}|^2}{2M}\sqrt{(M^2-(m_1+m_2)^2)(M^2-(m_1-m_2)^2)}$

and we have $m_1 = m_2 = m$ and $M > 2m$ i.e.

$\Gamma = \frac{1}{\cancel{16}8\pi M^2}\frac{\cancel{4}\mu^2}{\cancel{2M}}\sqrt{(M^2-4m^2)}\times \cancel{M}\times\frac{1}{2} = \frac{\mu^2}{4\pi M^2}\sqrt{M^2-4m^2}\times\frac{1}{2}$

$\therefore$ we have lifetime of Φ as; $\boxed{\frac{1}{\Gamma} = \frac{8\pi M^2}{\mu^2\sqrt{M^2-4m^2}}}$

this factor of $\frac{1}{2}$ is added since the final particles are <u>identical</u>.

A8) we have $\langle 0|T(\varphi_1\varphi_2\varphi_3\varphi_4 e^{-\frac{i\lambda}{4!}\int d^4x\,\varphi^4(x)})|0\rangle$ and keeping upto λ^2 order, we have;

$\langle 0|T(\varphi_1\varphi_2\varphi_3\varphi_4 e^{-\frac{i\lambda}{4!}\int d^4x\,\varphi^4(x)})|0\rangle = \langle 0|T[\varphi_1\varphi_2\varphi_3\varphi_4(\mathbb{1} - \frac{i\lambda}{4!}\int d^4x\,\varphi(x) + \frac{(-i\lambda)^2}{2(4!)^2}\iint d^4x\, d^4y\, \varphi^4(x)\varphi^4(y)]|0\rangle$

$= \underbrace{\langle 0|T(\varphi_1\varphi_2\varphi_3\varphi_4)|0\rangle}_{} - \frac{i\lambda}{4!}\int d^4x \langle 0|T(\varphi_1\varphi_2\varphi_3\varphi_4\varphi^4(x))|0\rangle + \frac{(-i\lambda)^2}{2(4!)^2}\int d^4x\int d^4y \langle 0|T(\varphi_1\varphi_2\varphi_3\varphi_4 \varphi^4(x)\varphi^4(y))|0\rangle$

$= D_F(x_1,x_2)D_F(x_3,x_4) + D_F(x_1,x_3)D_F(x_2,x_4) + D_F(x_1,x_4)D_F(x_2,x_3) \rightarrow$ from wick's theorem

and the other terms give;

$O(\lambda) : \left(-\frac{i\lambda}{4!}\right)\Big\{\int d^4x\,[D_F(x_1,x_2)D_F(x_3,x_4) + D_F(x_1,x_3)D_F(x_2,x_4) + D_F(x_1,x_4)D_F(x_2,x_3)](\overset{D_F(x-x)}{D_F(0)})^2$

$+ \int d^4x\,[D_F(x_1,x_2)D_F(x_3,x)D_F(x_4,x) + D_F(x_3,x_4)D_F(x_2,x)D_F(x_1,x) + D_F(x_2,x_4)D_F(x_1,x)D_F(x_3,x)$

$+ D_F(x_1,x_3)D_F(x_2,x)D_F(x_4,x) + D_F(x_1,x_4)D_F(x_2,x)D_F(x_3,x) + D_F(x_2,x_3)$

$D_F(x_1,x)D_F(x_4,x)]\,\underset{\hookrightarrow D_F(x-x)}{D_F(0)} + \int d^4x\, D_F(x_4,x)D_F(x_3,x)D_F(x_2,x)D_F(x_1,x)\Big\}$

$O(\lambda^2) : \frac{1}{2}\left(-\frac{i\lambda}{4!}\right)^2\Big\{\int d^4x\int d^4y\,[D_F(x_1,x_2)D_F(x_3,x_4) + D_F(x_1,x_3)D_F(x_2,x_4) + D_F(x_1,x_4)D_F(x_2,x_3)]\,\overset{D_F(x-x)}{D_F(0)}\,\overset{D_F(y-y)}{D_F(0)}$

$$\hookrightarrow D_F(x-x) \qquad \cdots D_F(x_2,x)\, D_F(x_1,x)\}$$

$$O(\lambda^2): \frac{1}{2}\left(\frac{-i\lambda}{4!}\right)^2 \Big\{ \int d^4x \int d^4y \left[D_F(x_1,x_2)D_F(x_3,x_4) + D_F(x_1,x_3)D_F(x_2,x_4) + D_F(x_1,x_4)D_F(x_2,x_3)\right] \overset{D_F(x-x)}{D_F(0)}\, \overset{D_F(y-y)}{D_F(0)}$$

$$+ \int dx^4 \int dy^4 \left[D_F(x_1,x_2)D_F(x_3,x_4) + D_F(x_1,x_3)D_F(x_2,x_4) + D_F(x_1,x_4)D_F(x_2,x_3)\right] (D_F(x-x))^2 (D_F(x-y))^2$$

$D_F(y-y)$

$$+ \int dx^4 \int dy^4 \left[D_F(x_1,x_2)D_F(x_3,x_4) + D_F(x_1,x_3)D_F(x_2,x_4) + D_F(x_1,x_4)D_F(x_2,x_3)\right] (D_F(x-y))^2$$

$$+ \int dx^4 \int dy^4 \big[D_F(x_1,x_2)D_F(x_3,x)D_F(x_4,x) + D_F(x_3,x_4)D_F(x_1,x)D_F(x_2,x) + D_F(x_2,x_3)D_F(x_1,x)D_F(x_4,x)$$
$$+ D_F(x_2,x_4)D_F(x_1,x)D_F(x_3,x) + D_F(x_1,x_3)D_F(x_2,x)D_F(x_4,x) + D_F(x_1,x_4)D_F(x_2,x)D_F(x_3,x)\big](D_F(y-y))^2 D_F(x-x)$$

$$+ \int dx^4 \int dy^4 \big[D_F(x_1,x_2)D_F(x_3,x)D_F(x_4,x) + D_F(x_3,x_4)D_F(x_1,x)D_F(x_2,x) + D_F(x_2,x_3)D_F(x_1,x)D_F(x_4,x)$$
$$+ D_F(x_2,x_4)D_F(x_1,x)D_F(x_3,x) + D_F(x_1,x_3)D_F(x_2,x)D_F(x_4,x) + D_F(x_1,x_4)D_F(x_2,x)D_F(x_3,x)\big](D_F(x-y))^2 D_F(y-y)$$

$$+ \int dx^4 \int dy^4 \big[D_F(x_1,x_2)D_F(x_3,x)D_F(x_4,x) + D_F(x_3,x_4)D_F(x_1,x)D_F(x_2,x) + D_F(x_2,x_3)D_F(x_1,x)D_F(x_4,x)$$
$$+ D_F(x_2,x_4)D_F(x_1,x)D_F(x_3,x) + D_F(x_1,x_3)D_F(x_2,x)D_F(x_4,x) + D_F(x_1,x_4)D_F(x_2,x)D_F(x_3,x)\big](D_F(x-x))^2 D_F(y-y)$$

$$+ \int dx^4 \int dy^4 \big[D_F(x_1,x_2)D_F(x_3,x)D_F(x_4,x) + D_F(x_3,x_4)D_F(x_1,x)D_F(x_2,x) + D_F(x_2,x_3)D_F(x_1,x)D_F(x_4,x)$$
$$+ D_F(x_2,x_4)D_F(x_1,x)D_F(x_3,x) + D_F(x_1,x_3)D_F(x_2,x)D_F(x_4,x) + D_F(x_1,x_4)D_F(x_2,x)D_F(x_3,x)\big](D_F(x-y))^2 D_F(x-x)$$

$$+ \int dx^4 D_F(x_1,x)D_F(x_2,x)D_F(x_3,x)D_F(x_4,x)(D_F(y-y))^2 + \int d^4x\, D_F(x_1,y)D_F(x_2,y)D_F(x_3,y)D_F(x_4,y)\,(D_F(x-x))^2 \Big\}$$

$\longrightarrow$ Four point Green's function

Now the momentum space Green's function is given as:-

The first term for λ^0: $\frac{1}{(2\pi)^{16}}\int\left[D_F(x_1,x_2)D_F(x_3,x_4) + D_F(x_1,x_3)D_F(x_2,x_4) + D_F(x_1,x_4)D_F(x_2,x_3)\right] d^4x_1\, d^4x_2\, d^4x_3\, d^4x_4$
and other terms;

$$O(\lambda): \frac{1}{(2\pi)^{16}}\left(\frac{-i\lambda}{4!}\right)\Big\{ \int d^4x \left[D_F(x_1,x_2)D_F(x_3,x_4) + D_F(x_1,x_3)D_F(x_2,x_4) + D_F(x_1,x_4)D_F(x_2,x_3)\right] (\overset{D_F(x-x)}{D_F(0)})^2$$
$$+ \int d^4x \big[D_F(x_1,x_2)D_F(x_3,x)D_F(x_4,x) + D_F(x_3,x_4)D_F(x_2,x)D_F(x_1,x) + D_F(x_2,x_4)D_F(x_1,x)D_F(x_3,x)$$
$$+ D_F(x_1,x_3)D_F(x_2,x)D_F(x_4,x) + D_F(x_1,x_4)D_F(x_2,x)D_F(x_3,x) + D_F(x_2,x_3)D_F(x_1,x)D_F(x_4,x)\big] \underset{\hookrightarrow D_F(x-x)}{D_F(0)} + \int d^4x\, D_F(x_4,x)D_F(x_3,x)D_F(x_2,x)D_F(x_1,x)\Big\} dx_1\, dx_2\, dx_3\, dx_4$$

$$O(\lambda^2): \frac{1}{(2\pi)^{16}}\frac{1}{2}\left(\frac{-i\lambda}{4!}\right)^2 \Big\{ \int d^4x \int d^4y \left[D_F(x_1,x_2)D_F(x_3,x_4) + D_F(x_1,x_3)D_F(x_2,x_4) + D_F(x_1,x_4)D_F(x_2,x_3)\right] \overset{D_F(x-x)}{D_F(0)}\, \overset{D_F(y-y)}{D_F(0)}$$

$$+ \int dx^4 \int dy^4 \left[D_F(x_1,x_2)D_F(x_3,x_4) + D_F(x_1,x_3)D_F(x_2,x_4) + D_F(x_1,x_4)D_F(x_2,x_3)\right] (D_F(x-x))^2 (D_F(x-y))^2$$

$D_F(y-y)$

$$+ \int dx^4 \int dy^4 \left[D_F(x_1,x_2)D_F(x_3,x_4) + D_F(x_1,x_3)D_F(x_2,x_4) + D_F(x_1,x_4)D_F(x_2,x_3)\right] (D_F(x-y))^2$$

$$+ \int dx^4 \int dy^4 \big[D_F(x_1,x_2)D_F(x_3,x)D_F(x_4,x) + D_F(x_3,x_4)D_F(x_1,x)D_F(x_2,x) + D_F(x_2,x_3)D_F(x_1,x)D_F(x_4,x)$$
$$+ D_F(x_2,x_4)D_F(x_1,x)D_F(x_3,x) + D_F(x_1,x_3)D_F(x_2,x)D_F(x_4,x) + D_F(x_1,x_4)D_F(x_2,x)D_F(x_3,x)\big](D_F(y-y))^2 D_F(x-x)$$

$$+ \int dx^4 \int dy^4 \big[D_F(x_1,x_2)D_F(x_3,x)D_F(x_4,x) + D_F(x_3,x_4)D_F(x_1,x)D_F(x_2,x) + D_F(x_2,x_3)D_F(x_1,x)D_F(x_4,x)$$
$$+ D_F(x_2,x_4)D_F(x_1,x)D_F(x_3,x) + D_F(x_1,x_3)D_F(x_2,x)D_F(x_4,x) + D_F(x_1,x_4)D_F(x_2,x)D_F(x_3,x)\big](D_F(x-y))^2 D_F(y-y)$$

$$+ \int dx^4 \int dy^4 \big[D_F(x_1,x_2)D_F(x_3,x)D_F(x_4,x) + D_F(x_3,x_4)D_F(x_1,x)D_F(x_2,x) + D_F(x_2,x_3)D_F(x_1,x)D_F(x_4,x)$$
$$+ D_F(x_2,x_4)D_F(x_1,x)D_F(x_3,x) + D_F(x_1,x_3)D_F(x_2,x)D_F(x_4,x) + D_F(x_1,x_4)D_F(x_2,x)D_F(x_3,x)\big](D_F(x-x))^2 D_F(y-y)$$

$$+ \int dx^4 \int dy^4 \big[D_F(x_1,x_2)D_F(x_3,x)D_F(x_4,x) + D_F(x_3,x_4)D_F(x_1,x)D_F(x_2,x) + D_F(x_2,x_3)D_F(x_1,x)D_F(x_4,x)$$
$$+ D_F(x_2,x_4)D_F(x_1,x)D_F(x_3,x) + D_F(x_1,x_3)D_F(x_2,x)D_F(x_4,x) + D_F(x_1,x_4)D_F(x_2,x)D_F(x_3,x)\big](D_F(x-y))^2 D_F(x-x)$$

$$+ \int dx^4 D_F(x_1,x)D_F(x_2,x)D_F(x_3,x)D_F(x_4,x)(D_F(y-y))^2 + \int d^4x\, D_F(x_1,y)D_F(x_2,y)D_F(x_3,y)D_F(x_4,y)$$

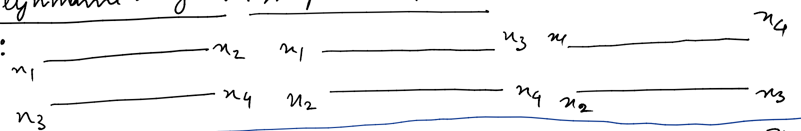
$D_F(x_3,x) \,] (D_F(x-y))\, D_F(x-x)$

$$+ \int dx^4\, D_F(x_1,x)\, D_F(x_2,x)\, D_F(x_3,x)\, D_F(x_4,x)\,(D_F(y-y))^2 + \int d^4x\, D_F(x_1,y)\, D_F(x_2,y)\, D_F(x_3,y)\, D_F(x_4,y)\,(D_F(x-x))^2 \Big\} d^4x_1\, d^4x_2\, d^4x_3\, d^4x_4$$

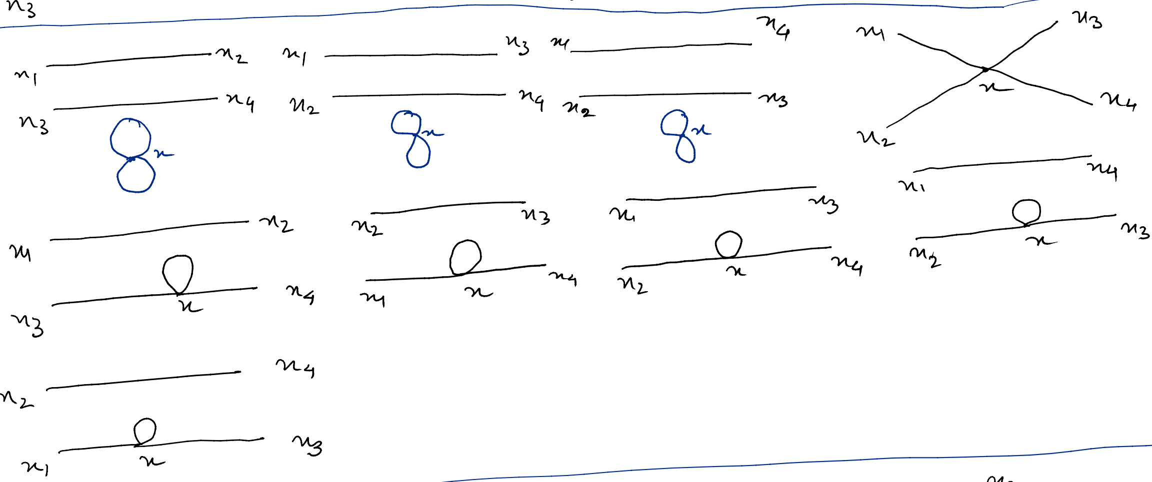
$\longrightarrow$ Momentum space green's function .

The Feynmann diagrams in position space:-

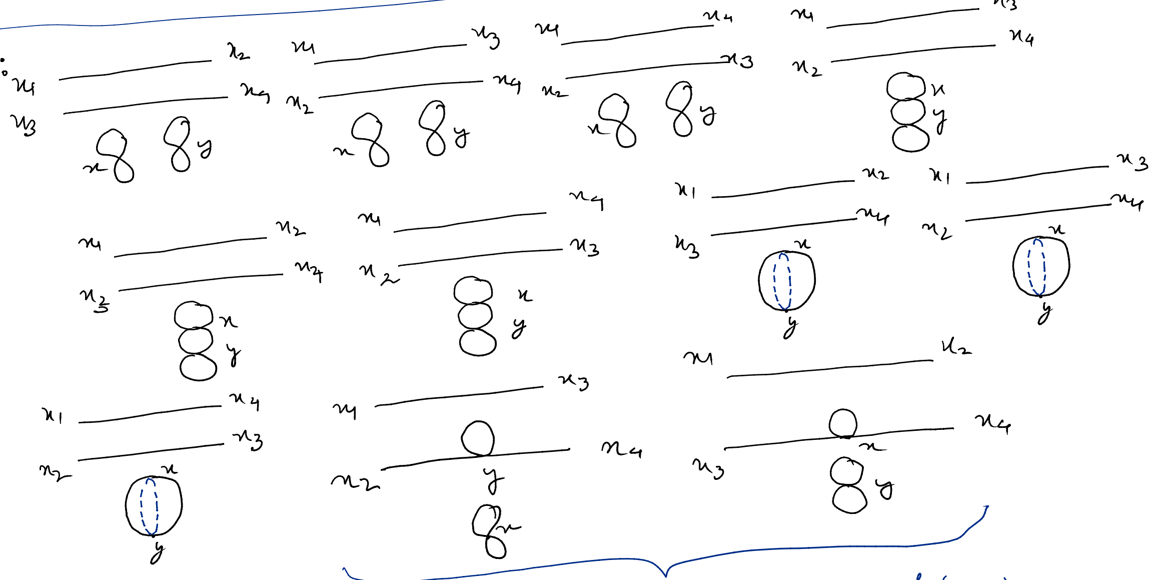
$O(\lambda^0)$:



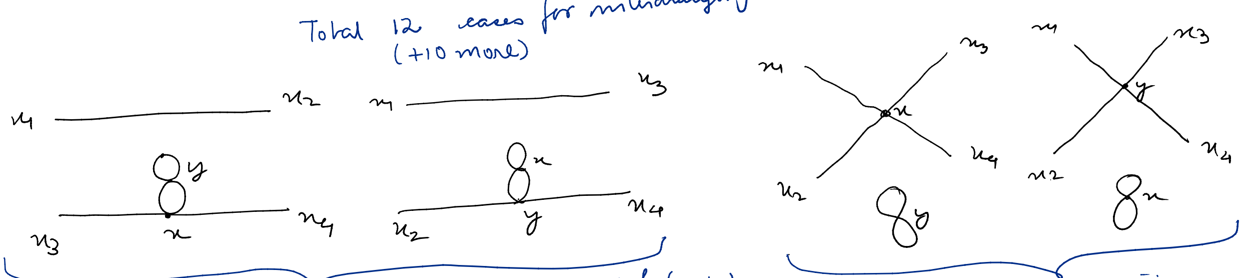
$O(\lambda)$:



$O(\lambda^2)$:



Total 12 cases for interchanging (x_1,x_2,x_3,x_4) & (x,y)
(+10 more)



Total 12 cases for interchanging (x_1,x_2,x_3,x_4) & (x,y)
(+10 more)

2 more combinations by interchanging (x_3,x_4)

—X—