

Pair of Linear Equations in Two Variables

In earlier classes, we initiated the study of linear equations, where we studied about linear equations in one and two variable(s) and their graphs. In this chapter, we shall study about pair of linear equations in two variables, various methods of solving them and related word problems.

TOPIC 01 Pair of Linear Equations in Two Variables

Linear Equation in Two Variables

A linear equation in two variables (unknowns), say x and y is usually represented in the following form:

$ax + by + c = 0$, where $a \neq 0, b \neq 0$; a, b and c are real constants.

e.g. $3x - y + 7 = 0$ and $7x + y = 3$
are linear equations in two variables x and y .

Pair of Linear Equations in Two Variables

Two linear equations in the same two variables, say x and y , are called pair of linear equations in two variables.

The general form of pair of linear equations in two variables x and y is $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, where $a_1, b_1, c_1, a_2, b_2, c_2$ are all real numbers such that $a_1, b_1 \neq 0$ and $a_2, b_2 \neq 0$.

Sometimes we call a pair of linear equations in two variables as system of linear equations in two variables.

Solution of a Pair of Linear Equations in Two Variables

Any pair of values of x and y which satisfies both the equations, $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, is called a solution of a given pair of linear equations.

A pair of linear equations in two variables can be solved by two methods namely graphical method and algebraic method.

Solution of a pair of linear equations



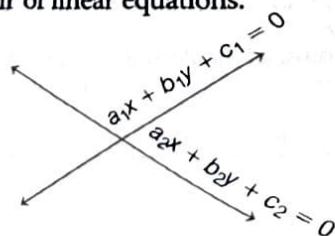
Graphical Method

Let us consider a pair of linear equations in two variables, $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$.

To find the solution graphically, first draw the graph of both equations on the same graph paper with same scale of representation.

Then, proceed as follows:

- If the graph of system of linear equations will represent two intersecting lines, then coordinates of point of intersection say (a, b) is the solution of the pair of linear equations. In this case, there will be only one point of intersection. So, the given system of equations will have unique solution and such pair of linear equations, is called consistent pair of linear equations.



(ii) If the graph of system of linear equations will represent two parallel lines, there is no point of intersection and consequently there is no pair of values of x and y which satisfy both equations. Thus, given system of equations have no solution. Such pair of linear equations, is called inconsistent pair of linear equations.

$$\begin{array}{c} \xleftarrow{a_1x + b_1y + c_1 = 0} \\ \xleftarrow{a_2x + b_2y + c_2 = 0} \end{array}$$

(iii) If the graph of system of linear equations will represent coincident or overlapping lines, there are infinitely many common points. Thus, given system of equations have infinitely many solutions. Such pair of linear equations is called dependent pair of linear equations. Dependent pair of linear equations is always consistent.

Example 1. Solve graphically

$$2x - 3y + 13 = 0, 3x - 2y + 12 = 0$$

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Sol. We have, $2x - 3y + 13 = 0$

...(i)

and $3x - 2y + 12 = 0$... (ii)

$$\text{Now, } 2x = 3y - 13 \Rightarrow x = \frac{3y - 13}{2}$$

When $y = 1$, we have

$$x = \frac{3 \times 1 - 13}{2} = -5$$

When $y = 3$, we have

$$x = \frac{3 \times 3 - 13}{2} = \frac{-4}{2} = -2$$

Table for $2x - 3y + 13 = 0$ is

| | | |
|--------|------------|------------|
| x | -5 | -2 |
| y | 1 | 3 |
| Points | $A(-5, 1)$ | $B(-2, 3)$ |

$$\text{Now, } 3x - 2y + 12 = 0 \Rightarrow 3x = 2y - 12 \Rightarrow x = \frac{2y - 12}{3}$$

When $y = 0$, we have

$$x = \frac{2 \times 0 - 12}{3} = \frac{-12}{3} = -4$$

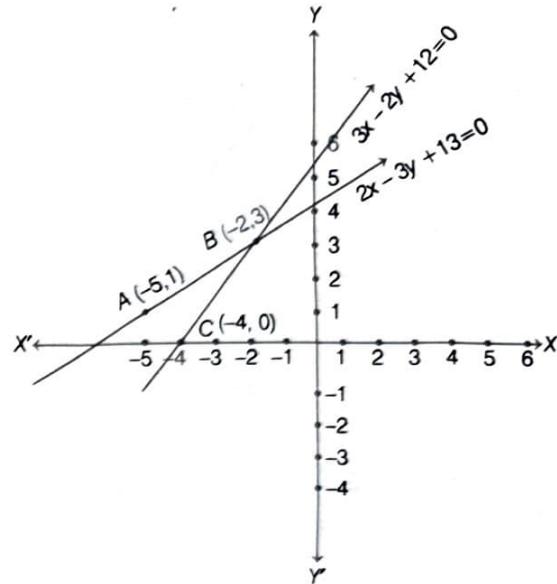
When $y = 3$, we have

$$x = \frac{2 \times 3 - 12}{3} = \frac{-6}{3} = -2$$

Table for $3x - 2y + 12 = 0$ is

| | | |
|--------|------------|------------|
| x | -4 | -2 |
| y | 0 | 3 |
| Points | $C(-4, 0)$ | $B(-2, 3)$ |

Clearly, lines (i) and (ii) intersect each other at point $B(-2, 3)$.



Hence, $x = -2, y = 3$ is the solution of the given system of equations.

Example 2. Write an equation of a line passing through the point representing solution of the pair of linear equations $x + y = 2$ and $2x - y = 1$. How many such lines can we find?

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Sol. Given, pair of linear equations is

$$x + y - 2 = 0 \quad \dots(i)$$

$$\text{and} \quad 2x - y - 1 = 0 \quad \dots(ii)$$

Now, table for $x + y - 2 = 0$ or $y = 2 - x$ is

| | | |
|-------------|-----------|-----------|
| x | 0 | 2 |
| $y = 2 - x$ | 2 | 0 |
| Points | $A(0, 2)$ | $B(2, 0)$ |

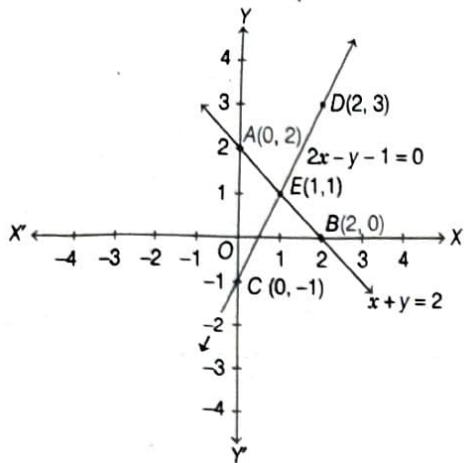
Plot the points $A(0, 2)$ and $B(2, 0)$ and join them to get the straight line AB .

Table for $2x - y - 1 = 0$ or $y = 2x - 1$ is

| | | |
|--------------|------------|-----------|
| x | 0 | 2 |
| $y = 2x - 1$ | -1 | 3 |
| Points | $C(0, -1)$ | $D(2, 3)$ |

Plot the points $C(0, -1)$ and $D(2, 3)$ and join them to get the straight line CD .

The lines AB and CD intersect at $E(1, 1)$. So, the solution of the given pair of linear equations is $(1, 1)$.



It is clear from the graph that infinite lines can pass through the intersection point of linear equations

$$x + y = 2 \text{ and } 2x - y = 1$$

i.e. point $E(1, 1)$ satisfies many linear equations such as $y = x$, $2x + y = 3$, $x + 2y = 3$ and so on.

Example 3. Solve graphically the pair of linear equations $3x - 4y + 3 = 0$ and $3x + 4y - 21 = 0$. Find the coordinates of the vertices of triangular region formed by these lines and X -axis. Also, calculate the area of this triangle.

Sol. Given, pair of linear equations is

$$3x - 4y + 3 = 0 \quad \dots(i)$$

$$\text{and} \quad 3x + 4y - 21 = 0 \quad \dots(ii)$$

Table for $3x - 4y + 3 = 0$ or $y = \frac{3x+3}{4}$ is

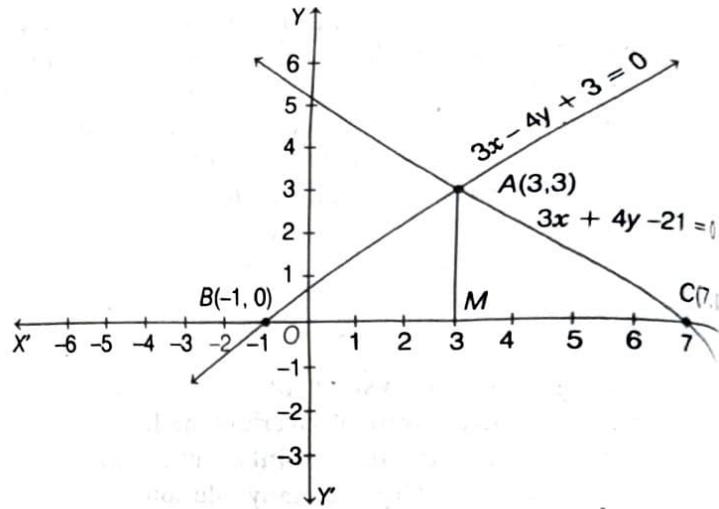
| | | |
|----------------------|---------|----------|
| x | 3 | -1 |
| $y = \frac{3x+3}{4}$ | 3 | 0 |
| Points | A(3, 3) | B(-1, 0) |

Plot the points $A(3, 3)$ and $B(-1, 0)$ on the graph paper and join them to get the straight line AB .

Now, table for $3x + 4y - 21 = 0$ or $y = \frac{21-3x}{4}$ is

| | | |
|-----------------------|---------|---------|
| x | 3 | 7 |
| $y = \frac{21-3x}{4}$ | 3 | 0 |
| Points | A(3, 3) | C(7, 0) |

Plot the points $A(3, 3)$ and $C(7, 0)$ on same graph paper and join them to get the straight line AC .



The lines AB and AC intersect each other at point $A(3, 3)$. So, the solution of the given pair of linear equations is $(3, 3)$. The coordinates of the vertices of ΔABC are $A(3, 3)$, $B(-1, 0)$ and $C(7, 0)$.

$$\therefore \text{Area of } \Delta ABC = \frac{1}{2} \times AM \times BC = \frac{1}{2} \times 3 \times 8 = 12 \text{ sq units}$$

Example 4. Show graphically that the following pair of equations is inconsistent (i.e. has no solution).

$$3x - 4y - 1 = 0 \text{ and } 2x - \frac{8}{3}y + 5 = 0$$

Sol. Given, pair of equations is $3x - 4y - 1 = 0$

$$\text{and} \quad 2x - \frac{8}{3}y + 5 = 0$$

Now, let us find atleast two solutions of each of the above equations, as shown in the following tables.

Table for equation $3x - 4y - 1 = 0$ or $y = \frac{3x-1}{4}$ is

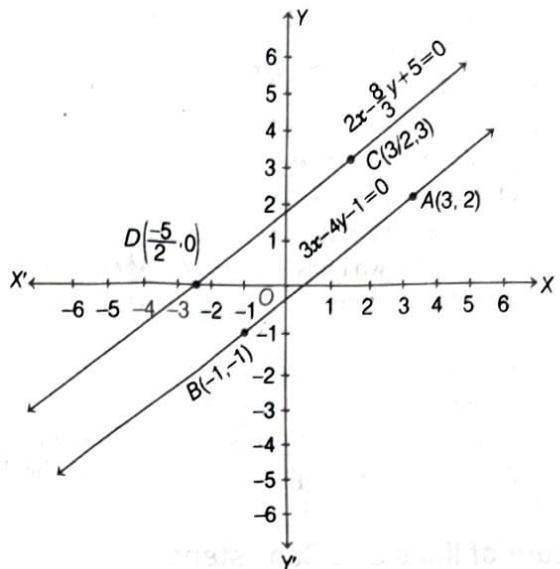
| | | |
|----------------------|---------|-----------|
| x | 3 | -1 |
| $y = \frac{3x-1}{4}$ | 2 | -1 |
| Points | A(3, 2) | B(-1, -1) |

Plot the points $A(3, 2)$ and $B(-1, -1)$ on a graph paper and join them to get the line AB .

Table for equation $2x - \frac{8}{3}y + 5 = 0$ or $y = \frac{6x+15}{8}$ is

| | | |
|-----------------------|--------------------------------|---------------------------------|
| x | $\frac{3}{2}$ | $-\frac{5}{2}$ |
| $y = \frac{6x+15}{8}$ | 3 | 0 |
| Points | $C\left(\frac{3}{2}, 3\right)$ | $D\left(-\frac{5}{2}, 0\right)$ |

Plot the points $C\left(\frac{3}{2}, 3\right)$ and $D\left(-\frac{5}{2}, 0\right)$ on same graph paper and join them to get the line CD .



From the graph, it is clear that lines represented by the equations $3x - 4y - 1 = 0$ and $2x - \frac{8}{3}y + 5 = 0$ are parallel.

So, the two lines have no common point. Hence, the given pair of equations is inconsistent. **Hence proved.**

Example 5. Show graphically that system of equations $x + 2y = 5$, $3x + 6y = 15$ has infinitely many solutions.

System of linear equations has infinitely many solutions, if the graph of the equations represents coincident lines.

Sol. Given, pair of equations is $x + 2y = 5$ and $3x + 6y = 15$

Now, let us find atleast two solutions of each of the above equations, as shown in the following tables.

Table for $x + 2y = 5$ or $y = \frac{5-x}{2}$ is

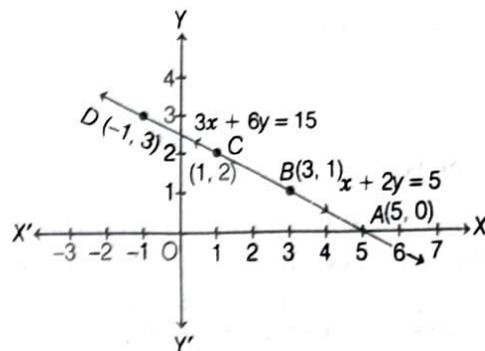
| | | |
|---------------------|---------|---------|
| x | 5 | 3 |
| $y = \frac{5-x}{2}$ | 0 | 1 |
| Points | A(5, 0) | B(3, 1) |

Plot the points $A(5, 0)$ and $B(3, 1)$ on graph paper and join them to get the line AB .

Table for $3x + 6y = 15$ or $y = \frac{15-3x}{6}$ is

| | | |
|-----------------------|---------|----------|
| x | 1 | -1 |
| $y = \frac{15-3x}{6}$ | 2 | 3 |
| Points | C(1, 2) | D(-1, 3) |

Plot the points $C(1, 2)$ and $D(-1, 3)$ on same graph paper and join them to get the line CD .



From the graph it is clear that lines represented by the equations $x + 2y = 5$ and $3x + 6y = 15$ are coincident to each other.

So, the system of equations have infinitely many solutions.

Example 6. Form a pair of linear equations in two variables using the following information and solve it graphically. 5 yr ago, Sagar was twice as old as Tiru. 10 yr later, Sagar's age will be 10 yr more than Tiru's age. Find their present ages.

Sol. Let the present age of Sagar be x yr and the age of Tiru be y yr.

$$\begin{aligned} 5 \text{ yr ago, Sagar's age} &= (x - 5) \text{ yr} \\ \text{and Tiru's age} &= (y - 5) \text{ yr} \end{aligned}$$

According to the given condition,

$$\begin{aligned} (x - 5) &= 2(y - 5) \\ \Rightarrow x - 5 &= 2y - 10 \\ \Rightarrow x - 2y + 5 &= 0 \end{aligned}$$

$$\begin{aligned} \text{After 10 yr, Sagar's age} &= (x + 10) \text{ yr} \\ \text{and Tiru's age} &= (y + 10) \text{ yr} \end{aligned}$$

According to the given condition,

$$\begin{aligned} x + 10 &= (y + 10) + 10 \\ \Rightarrow x + 10 &= y + 20 \\ \Rightarrow x - y - 10 &= 0 \end{aligned}$$

Thus, we get the following pair of linear equations

$$\begin{aligned} x - 2y + 5 &= 0 & \dots(i) \\ \text{and } x - y - 10 &= 0 & \dots(ii) \end{aligned}$$

Now, let us draw the graphs of Eqs. (i) and (ii), by finding at least two solutions of each of the above equations.

The solutions of the equations are given in the following tables.

Table for $x - 2y + 5 = 0$ or $y = \frac{x+5}{2}$ is

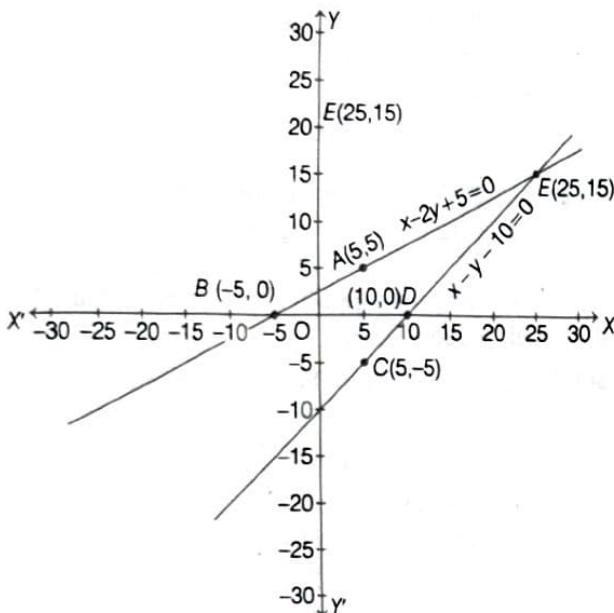
| | | |
|---------------------|---------|----------|
| x | 5 | -5 |
| $y = \frac{x+5}{2}$ | 5 | 0 |
| Points | A(5, 5) | B(-5, 0) |

Table for $x - y - 10 = 0$ or $y = x - 10$ is

| | | |
|--------------|----------|----------|
| x | 5 | 10 |
| $y = x - 10$ | -5 | 0 |
| Points | C(5, -5) | D(10, 0) |

Plot the points A(5, 5) and B(-5, 0) on a graph paper and join them to get the line AB.

Similarly, plot the points C(5, -5) and D(10, 0) on the same graph paper and join them to get the line CD.



It is clear from the graph that, lines AB and CD intersect each other at point E(25, 15).

So, $x=25$ and $y=15$ is the required solution.

Hence, Sagar's present age = 25 yr
and Tiru's present age = 15 yr

Example 7. Champa went to a 'Sale' to purchase some pants and skirts. When her friends asked her, how many of each she had bought, she answered, 'The number of skirts is two less than twice the number of pants purchased. Also, the number of skirts is four less than four times the number of pants purchased'. Help her friends to find out how many pants and skirts, had Champa bought?

Sol. Let number of pants be x and number of skirts be y . Then,

$$\text{Condition I} \quad \text{Number of skirts} = 2 \times \text{Number of pants} - 2 \\ \Rightarrow y = 2x - 2 \quad \dots(i)$$

$$\text{Condition II} \quad \text{Number of skirts} = 4 \times \text{Number of pants} - 4 \\ \Rightarrow y = 4x - 4 \quad \dots(ii)$$

Now, let us find atleast two solutions of each of the above equations, as shown in the following tables.

Table for $y = 2x - 2$

| | | |
|--------------|---------|----------|
| x | 2 | 0 |
| $y = 2x - 2$ | 2 | -2 |
| Points | A(2, 2) | B(0, -2) |

Table for $y = 4x - 4$

| | | |
|--------------|----------|---------|
| x | 0 | 1 |
| $y = 4x - 4$ | -4 | 0 |
| Points | P(0, -4) | Q(1, 0) |

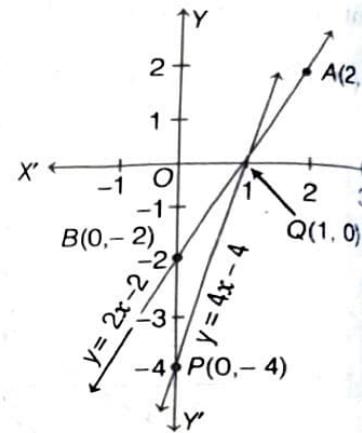
Now, plot the points A(2, 2) and B(0, -2) on a graph paper and join them to get the line AB.

Similarly, plot the points P(0, -4) and Q(1, 0) on same graph paper and join them to get the line PQ.

From the graph, it is clear that the two lines intersect at the point (1, 0).

$\therefore x=1$ and $y=0$ is the solution of the pair of linear equations.

Hence, the number of pants, she purchased is one number of skirts, she purchased is zero i.e. she did any skirt.



Nature of lines and Consistency

The nature of lines and consistency corresponding to equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, is shown in the table given below:

| Compare the ratios | Graphical representation | Algebraic interpretation | Consis... |
|--|--------------------------|-------------------------------|--------------------|
| $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ | Intersecting lines | Exactly one solution (unique) | System consists... |
| $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ | Coincident lines | Infinitely many solutions | System consists... |
| $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ | Parallel lines | No solution | System inconsis... |

Each of the cases in the table can be understood with the help of following example.

Example 8. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}, \frac{c_1}{c_2}$ and

without drawing them, find out whether the lines representing the following pairs of linear equations intersect at a point or are parallel or coincide.

$$(i) 3x - 5y + 8 = 0, 7x + 6y - 9 = 0$$

$$(ii) 4x + 3y - 7 = 0, 12x + 9y = 21$$

$$(iii) x - 2y + 5 = 0, 8y - 4x + 20 = 0$$

Sol. (i) The given pair of linear equations is

$$3x - 5y + 8 = 0$$

$$\text{and} \quad 7x + 6y - 9 = 0$$

On comparing the given equations with standard form of pair of linear equations i.e. $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we get

$$a_1 = 3, b_1 = -5, c_1 = 8$$

$$\text{and} \quad a_2 = 7, b_2 = 6, c_2 = -9$$

Here, $\frac{a_1}{a_2} = \frac{3}{7}$ and $\frac{b_1}{b_2} = \frac{-5}{6}$

$$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

\therefore The lines representing the given pair of linear equations will intersect at a point.

(ii) The given pair of linear equations is

$$4x + 3y - 7 = 0 \quad \dots(i)$$

$$\text{and} \quad 12x + 9y - 21 = 0 \quad \dots(ii)$$

On comparing the above equations with standard form of pair of linear equations, we get

$$a_1 = 4, b_1 = 3, c_1 = -7$$

$$\text{and} \quad a_2 = 12, b_2 = 9, c_2 = -21$$

$$\text{Now, } \frac{a_1}{a_2} = \frac{4}{12} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{3}{9} = \frac{1}{3}$$

$$\text{and} \quad \frac{c_1}{c_2} = \frac{-7}{-21} = \frac{1}{3}$$

$$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

\therefore The lines representing the given pair of linear equations will coincide.

(iii) The given pair of linear equations is

$$x - 2y + 5 = 0 \text{ and } -4x + 8y + 20 = 0$$

On comparing the above equations with standard form of pair of linear equations, we get

$$a_1 = 1, b_1 = -2, c_1 = 5$$

$$\text{and} \quad a_2 = -4, b_2 = 8, c_2 = 20$$

$$\text{Now, } \frac{a_1}{a_2} = -\frac{1}{4}, \frac{b_1}{b_2} = \frac{-2}{8} = -\frac{1}{4} \text{ and } \frac{c_1}{c_2} = \frac{5}{20} = \frac{1}{4}$$

$$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

\therefore The lines representing the given pair of linear equations are parallel.

Example 9. Find out which pair of linear equations are consistent/inconsistent.

(i) $4x - 5y - 12 = 0, 10y + 20 = 8x$

(ii) $3x + 2y = 8, 6x - 4y = 9$ [CBSE Sample Paper 2023 (Basic)]

Sol. (i) The given pair of linear equations is

$$4x - 5y - 12 = 0 \text{ and } -8x + 10y + 20 = 0$$

On comparing with standard form of pair of linear equations, we get

$$a_1 = 4, b_1 = -5, c_1 = -12$$

$$\text{and} \quad a_2 = -8, b_2 = 10, c_2 = 20$$

$$\text{Here, } \frac{a_1}{a_2} = \frac{4}{-8} = -\frac{1}{2}, \frac{b_1}{b_2} = \frac{-5}{10} = -\frac{1}{2}$$

$$\text{and} \quad \frac{c_1}{c_2} = -\frac{12}{20} = -\frac{3}{5}$$

$$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

\therefore The given pair of linear equations is inconsistent.

(ii) Given, pair of linear equations is

$$3x + 2y = 8$$

$$\Rightarrow 3x + 2y - 8 = 0$$

$$\text{and} \quad 6x - 4y = 9$$

$$\Rightarrow 6x - 4y - 9 = 0$$

On comparing with standard form of pair of linear equations, we get

$$a_1 = 3, b_1 = 2, c_1 = -8$$

$$\text{and} \quad a_2 = 6, b_2 = -4, c_2 = -9$$

$$\text{Here, } \frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2}$$

$$\text{and} \quad \frac{b_1}{b_2} = \frac{2}{-4} = \frac{-1}{2}$$

$$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

\therefore The given pair of linear equations is consistent.

Example 10. If the lines given by $2x + ky = 1$ and $3x - 5y = 7$ has unique solution, then find the value of k .

Sol. The given equations can be rewritten as $2x + ky - 1 = 0$ and $3x - 5y - 7 = 0$.

On comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we get

$$a_1 = 2, b_1 = k, c_1 = -1 \text{ and } a_2 = 3, b_2 = -5, c_2 = -7$$

$$\text{For unique solution, } \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow \frac{2}{3} \neq \frac{k}{-5} \Rightarrow k \neq \frac{-10}{3}$$

Thus, given lines have a unique solution for all real values of k , except $\frac{-10}{3}$.

Example 11. Find the value of k for which the system of equations $kx - 5y = 2; 6x + 2y = 7$ has no solution.

Sol. Given, pair of linear equations is

$$kx - 5y - 2 = 0 \text{ and } 6x + 2y - 7 = 0$$

On comparing with standard form of pair of linear equations, we get

$$a_1 = k, b_1 = -5, c_1 = -2 \text{ and } a_2 = 6, b_2 = 2, c_2 = -7$$

$$\text{For no solution, } \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\Rightarrow \frac{k}{6} = \frac{-5}{2} \neq \frac{-2}{-7}$$

$$\Rightarrow \frac{k}{6} = -\frac{5}{2}$$

$$\Rightarrow k = -15$$

Example 12. For what value of k , does the system of linear equations

$$2x + 3y = 7 \text{ and } (k-1)x + (k+2)y = 3k$$

have infinite number of solutions?

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Sol. Given, system of linear equations

$$2x + 3y = 7 \text{ and } (k-1)x + (k+2)y = 3k$$

On comparing the above equations with standard form of pair of linear equations, we get

$$a_1 = 2, b_1 = 3, c_1 = -7$$

$$\text{and } a_2 = k-1, b_2 = k+2, c_2 = -3k$$

For infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\therefore \frac{2}{k-1} = \frac{3}{k+2} = \frac{-7}{-3k}$$

$$\text{Consider } \frac{2}{k-1} = \frac{7}{3k} \text{ and } \frac{3}{k+2} = \frac{7}{3k}$$

$$\Rightarrow 6k = 7k - 7 \text{ and } 9k = 7k + 14$$

$$\Rightarrow k = 7 \text{ and } 2k = 14$$

$$\Rightarrow k = 7 \text{ and } k = 7$$

$$\therefore k = 7$$

Example 13. For what value of p , will the following system of linear equations represent parallel lines?

$$-x + py = 1 \text{ and } px - y = 1$$

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Sol. Given, pair of equations is

$$-x + py - 1 = 0 \quad \dots(i)$$

$$\text{and } px - y - 1 = 0 \quad \dots(ii)$$

On comparing the given equations with standard form i.e. $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we get

$$a_1 = -1, b_1 = p, c_1 = -1$$

$$\text{and } a_2 = p, b_2 = -1, c_2 = -1$$

$$\text{For parallel lines, } \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\Rightarrow \frac{-1}{p} = \frac{p}{-1} \neq \frac{-1}{-1} \quad \dots(iii)$$

On taking I and II terms, we get

$$\frac{-1}{p} = \frac{p}{-1}$$

$$\Rightarrow p^2 = 1$$

$$\Rightarrow p = \pm 1$$

Since, $p = -1$ does not satisfy the last two terms of Eq. (iii).
 $\therefore p = 1$ is the required value.

Hence, for $p = 1$, the given system of equations will represent parallel lines.

Try These 3.1

1. Solve the following system of linear equations graphically

$$x - y + 1 = 0$$

$$x + y = 5$$

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2. Graphically, find whether the following pair of equations has no solution, unique solution or infinitely many solutions.

$$15x - 30y + 1 = 0, 3x - \frac{24}{4}y + \frac{1}{5} = 0$$

3. Check graphically, whether the following pair of linear equations is consistent. If yes, solve it graphically.

$$2x - y = 0, x + y = 0$$

4. Solve graphically, the pair of linear equations $x - y = 1$ and $2x + y - 10 = 0$. Also, find the vertices of the triangle formed by these lines and X-axis and shade the triangular region.

5. Solve graphically the pair of linear equations $3x + y - 11 = 0, x - y - 1 = 0$. Also, find the vertices of the triangle formed by these lines and Y-axis.

6. Solve the following pairs of linear equations graphically and find the vertices of the triangle formed by these lines and Y-axis.

$$x - y + 1 = 0, 3x + 2y - 12 = 0$$

7. Check graphically, whether the pair of linear equations $4x - y - 8 = 0$ and $2x - 3y + 6 = 0$ is consistent. Also, find the vertices of the triangle formed by these lines with the X-axis.

8. Two straight paths are represented by the lines $7x - 5y = 3$ and $21x - 15y = 5$. Check whether they cross each other.

9. Write a pair of linear equations which has the unique solution at $x = -1$ and $y = 3$. How many such pairs can you write?

Directions (Q.Nos. 10-13) Form the pair of linear equations in the following problems and find their solutions graphically.

10. 4 chairs and 3 tables cost ₹ 210 and 5 chairs and 2 tables cost ₹ 175. Find the cost of one chair and one table separately.

11. The cost of 4 pens and 4 pencil boxes is ₹ 100. Three times the cost of a pen is ₹ 15 more than the cost of a pencil box. Find the cost of a pen and a pencil box.

12. 2 years ago, Salim was thrice as old as his daughter. 6 years later, he will be 4 yr older than twice her age. How old are they now?

13. Two numbers are in the ratio 5 : 6. If 8 is subtracted from each of the numbers, the ratio becomes 4 : 5. Find the numbers.

- 4.** On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the following pairs of linear equations are consistent or inconsistent?
- $\frac{4}{3}x + 2y = 8$ and $2x + 3y = 12$
 - $4x - y = 4$ and $3x + 2y = 14$
 - $3x - 5y = 11$ and $6x - 10y = 7$
 - $6x - 3y = 12$ and $2x - y = 4$
 - $6x + 5y = 11$ and $9x + \frac{15}{2}y = 21$
- 5.** On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines representing the following pairs of linear equations intersect at a point or are parallel or coincide?
- $2x + 3y = 3$ and $x - 2y = 2$
 - $4x - 2y = 10$ and $6x - 3y = 12$
 - $3x - 4y - 1 = 0$ and $2x - \frac{8}{3}y + 5 = 0$
 - $x - 3y - 7 = 0$ and $3x - 9y = 21$
 - $-3x + 4y = 5$ and $\frac{9}{2}x - 6y + \frac{15}{2} = 0$
- 6.** Find the value of k for which the following system of equations has a unique solution.
- $kx + 2y = 5$, $3x + y = 1$
 - $x - 3y = 5$, $5x + ky = 10$
 - $kx + 2y = 4$, $8x + ky = 11$
 - $4x - 5y = k$, $2x - 3y = 12$
 - $2x - 3y = k$, $3x - 2y = 6$
- 7.** Find the value of k for which the following system of equations has no solution.
- $4x + 5y = 17$, $kx + 15y = 33$
 - $2x + ky = 8$, $x + y = 6$
 - $kx + 2y = 5$, $8x + ky = 20$
 - $3x + y = 1$, $(2k - 1)x + (k - 1)y = (2k + 1)$
 - $3x - y - 5 = 0$, $6x - 2y + k = 0$
- 8.** Find the value of k for which the following system of equations has infinitely many solutions.
- $4x + 7y = 10$; $(k + 2)x + 21y = 3k$
 - $x + 2y + 7 = 0$, $2x + ky + 14 = 0$
 - $2x + (k - 2)y = k$; $6x + (2k - 1)y = 2k + 5$
 - $2x + 3y = 2$; $(k + 2)x + (2k + 1)y = 2(k - 1)$
 - $x + (k + 1)y = 5$, $(k + 1)x + 9y = (8k - 1)$
- 9.** Find the values of p and q for which the following system of equations has infinitely many solutions.
- $(2p - 1)x + 3y = 5$; $3x + (q - 1)y = 2$
 - $2x + 3y = 7$; $(p + q + 1)x + (p + 2q + 2)y = 4(p + q) + 1$
 - $2x + 3y = 7$, $(p + q)x + (2p - q)y = 21$
- 10.** Find the value of k , for which system of equations $kx + 3y = 3$ and $12x + ky = 6$ represent parallel lines.
- 21.** For what value of k , the pair of linear equations $x + 2y = 3$, $5x + ky + 7 = 0$ represents
- intersecting lines
 - parallel lines
 - Is there any value of k for which the given equations represents coincident lines?
- 22.** Given the linear equation $5x + 8y - 20 = 0$, write another linear equation in two variables such that the geometrical representation of the pair so formed is
- intersecting lines
 - parallel lines
 - coincident lines

Answers

- Hint Do same as Example 1. Ans. $x = 2$, $y = 3$
- Hint The graph of given equations represents coincident lines. Hence, the given pair of equations has infinitely many solutions.
- Hint The graph of given linear equations intersect each other at origin. So, the given pair of linear equations is consistent.
Ans. $x = 0$, $y = 0$
- Hint Do same as Example 3.
Ans. $x = 3$, $y = 4$; Vertices of triangle are $(3, 4)$, $(-1, 0)$ and $(5, 0)$.
- Hint Do same as Example 3.
Ans. $x = 3$, $y = 2$; Vertices of triangle are $(3, 2)$, $(0, -1)$ and $(0, 11)$
- Hint Do same as Example 3.
Ans. $x = 2$, $y = 3$; The vertices of the triangle formed by these lines and Y -axis are $E(0, 6)$, $A(0, 1)$ and $D(2, 3)$.
- Hint Do same as Example 3.
Ans. Yes, The vertices of triangle formed by these lines with X -axis are $E(-3, 0)$, $B(2, 0)$ and $C(3, 4)$.
- Hint Two straight paths are parallel to each other. Hence, they do not cross each other.
- Ans. $3x + 2y = 3$, $x + y = 2$; Infinitely many pairs are possible.
- Hint Let cost of one chair be ₹ x and cost of one table be ₹ y , then
 $4x + 3y = 210$... (i)
and $5x + 2y = 175$... (ii)
Draw graphs of above equation and find the coordinates of intersecting point.
Ans. Cost of one chair = ₹ 15, Cost of one table = ₹ 50
- Hint Do same as Question 10.
Ans. Cost of one pen = ₹ 10, Cost of one pencil box = ₹ 15
- Hint Do same as Example 6.
Salim's age = 38 yr, Daughter's age = 14 yr
- Hint Let the two numbers be x and y , then

$$\frac{x}{y} = \frac{5}{6} \Rightarrow y = \frac{6x}{5}$$
 ... (i)
Also,

$$\frac{x-8}{y-8} = \frac{4}{5}$$

$$\Rightarrow 5x - 4y = 8$$
 ... (ii)
Draw graphs of Eqs. (i) and (ii) to get the required numbers.
Ans. 40 and 48.

14. (i) Hint On comparing the given equations with standard form, we get

$$a_1 = \frac{4}{3}, b_1 = 2, c_1 = -8 \quad \text{and} \quad a_2 = 2, b_2 = 3, c_2 = -12$$

$$\text{Now, } \frac{a_1}{a_2} = \frac{2}{3}, \frac{b_1}{b_2} = \frac{2}{3} \text{ and } \frac{c_1}{c_2} = \frac{2}{3}; \therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Hence, given equations is consistent.

- (ii) Hint Do same as Part (i). Ans. Consistent.
 (iii) Hint Do same as Part (i). Ans. Inconsistent.
 (iv) Hint Do same as Part (i). Ans. Consistent.
 (v) Hint Do same as Part (i). Ans. Inconsistent.

15. Hint Do same as Example 8.

- Ans. (i) Intersecting lines (ii) Parallel lines
 (iii) Parallel lines (iv) Coincident lines
 (v) Coincident lines

16. (i) Hint For unique solution,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow \frac{k}{3} \neq \frac{2}{1} \Rightarrow k \neq 6$$

- (ii) Hint Do same as Part (i). Ans. $k \neq -15$

- (iii) Hint Do same as Part (i). Ans. $k \neq \pm 4$

$$(iv) \text{ Hint For unique solution, } \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow \frac{4}{2} \neq \frac{-5}{-3}$$

which is true for any real values of k .

- (v) Hint Do same as Part (iv). Ans. For all real values k .

17. (i) Hint For no solution, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$$\Rightarrow \frac{4}{k} = \frac{5}{15} \neq \frac{-17}{-33} \Rightarrow \frac{4}{k} = \frac{5}{15} \Rightarrow k = 12$$

- (ii) Hint Do same as Part (i). Ans. $k = 2$

- (iii) Hint Do same as Part (i). Ans. $k = \pm 4$

- (iv) Hint Do same as Part (i). Ans. $k = 2$

- (v) Hint Do same as Part (i). Ans. $k \neq 10$

18. (i) Hint For infinitely many solutions, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$$\Rightarrow \frac{4}{k+2} = \frac{7}{21} = \frac{-10}{-3k} \Rightarrow \frac{4}{k+2} = \frac{7}{21} \Rightarrow k = 10$$

- (ii) Hint Do same as Part (i). Ans. $k = 4$

- (iii) Hint Do same as Part (i). Ans. $k = 5$

- (iv) Hint Do same as Part (i). Ans. $k = 4$

- (v) Hint Do same as Part (i). Ans. $k = 2$

$$19. (i) \text{ Hint For infinitely many solutions, } \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \frac{2p-1}{3} = \frac{3}{q-1} = \frac{-5}{-2} \quad \text{Ans. } p = \frac{17}{4}, q = \frac{11}{5}$$

- (ii) Hint Do same as Part (i). Ans. $p = 3, q = 2$

- (iii) Hint Do same as Part (i). Ans. $p = 5, q = 1$

$$20. \text{ Hint For parallel lines, } \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \quad \text{Ans. } k = -6$$

$$21. \text{ Hint (i) For intersecting lines, } \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$(ii) \text{ For parallel lines, } \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$(iii) \text{ For coincident lines, } \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

- Ans. (i) $k \neq 10$ (ii) $k = 10$

- (iii) There is no value of k for which given system has many solutions i.e. represent coincident lines.

22. (i) Hint Given linear equation is $5x + 8y - 20 = 0$

$$\text{For intersecting lines, } \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$\text{Here, } a_1 = 5, b_1 = 8$$

- Any line intersecting with Eq. (i) may be taken

$$a_2x + b_2y + c_2 = 0, \text{ where } \frac{5}{a_2} \neq \frac{8}{b_2}$$

$$\text{Ans. } 8x + 5y - 25 = 0 \text{ or } 8x + 5y - 15 = 0$$

$$(ii) \text{ For parallel line, } \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\therefore \frac{5}{a_2} = \frac{8}{b_2} = \frac{-20}{c_2}$$

$$\text{Ans. } 10x + 16y + 15 = 0 \text{ or } 5x + 8y - 15 = 0$$

- (iii) Hint Any line coincident to Eq. (i) may be taken

$$a_2x + b_2y + c_2 = 0$$

$$\text{where, } \frac{5}{a_2} = \frac{8}{b_2} = \frac{-20}{c_2}$$

$$\text{Ans. } 10x + 16y - 40 = 0 \text{ or } 15x + 24y - 60 = 0$$

TOPIC 02 Algebraic Methods for Solving a Pair of Linear Equations

The graphical method is suitable for integer solutions but it cannot be suitable for non-integer solutions, since it may not be as accurate as we need. So, there are several algebraic methods, which can also be used for solving a pair of linear equations. Some of the algebraic methods for solving a pair of linear equations are:

- (i) Substitution Method
- (ii) Elimination Method

Substitution Method

In this method, value of one variable can be found out in terms of other variable from one of the given equation and this value is substituted in other equation, then we get an equation in one variable, which can be solved easily.

To understand the substitution method more clearly, we use following steps :

Step I Consider the linear equations

$$\begin{aligned} a_1x + b_1y + c_1 &= 0 \quad \dots(i) \\ a_2x + b_2y + c_2 &= 0 \quad \dots(ii) \end{aligned}$$

Step II Choose any one of the equation say Eq. (i), find the value of one variable, say y in terms of other variable i.e. x .

Step III Substitute this value of y in the Eq. (ii) and reduce it to an equation in one variable i.e. in terms of x , which can be solved.

Step IV Substitute the value of x obtained in Step III in the equation used in Step II to obtain the value of the other variable i.e. y .

Example 1. Solve the following system of equations $x + 8y = 19$ and $2x + 11y = 28$, by substitution method and verify it.

Sol. Given, equations are

$$x + 8y = 19 \quad \dots(i)$$

and $2x + 11y = 28 \quad \dots(ii)$

From Eq. (i), $x = 19 - 8y \quad \dots(iii)$

On substituting $x = 19 - 8y$ in Eq. (ii), we get

$$\begin{aligned} 2(19 - 8y) + 11y &= 28 \\ \Rightarrow 38 - 16y + 11y &= 28 \\ \Rightarrow 5y &= 38 - 28 = 10 \\ \Rightarrow y &= \frac{10}{5} = 2 \end{aligned}$$

Now, on putting $y = 2$ in Eq. (iii), we get

$$\begin{aligned} x &= 19 - 8 \times 2 \\ \Rightarrow x &= 19 - 16 = 3 \end{aligned}$$

Thus, $x = 3$ and $y = 2$ is the required solution.

Verification

On putting $x = 3$ and $y = 2$ in Eqs. (i) and (ii) respectively, we get

$$\text{From Eq. (i), LHS} = x + 8y = 3 + 8(2) = 3 + 16 = 19 = \text{RHS}$$

$$\begin{aligned} \text{From Eq. (ii), LHS} &= 2x + 11y = 2(3) + 11(2) \\ &= 6 + 22 = 28 = \text{RHS} \end{aligned}$$

Hence, the solution $x = 3, y = 2$ is verified.

Example 2. Solve the following system of linear equations

$$ax + by - a + b = 0 \quad \dots(i)$$

$$\text{and } bx - ay - a - b = 0. \quad \dots(ii)$$

Sol. The given system can be rewritten as

$$ax + by = a - b \quad \dots(i)$$

$$\text{and } bx - ay = a + b \quad \dots(ii)$$

From Eq. (i), we get

$$\begin{aligned} by &= a - b - ax \\ \Rightarrow y &= \frac{a - b - ax}{b} \quad \dots(iii) \end{aligned}$$

On substituting the value of y in Eq. (ii), we get

$$\begin{aligned} bx - a \left[\frac{a - b - ax}{b} \right] &= a + b \\ \Rightarrow b^2x - a(a - b - ax) &= b(a + b) \\ \Rightarrow b^2x - a^2 + ab + a^2x &= ab + b^2 \\ \Rightarrow (b^2 + a^2)x &= ab + b^2 + a^2 - ab \Rightarrow x = \frac{a^2 + b^2}{a^2 + b^2} = 1 \end{aligned}$$

On substituting $x = 1$ in Eq. (iii), we get

$$y = \frac{a - b - a}{b} \Rightarrow y = \frac{-b}{b} = -1$$

Hence, solution of the given system of linear equations is $x = 1$ and $y = -1$.

Example 3. Romila went to a stationary shop and purchased 2 pencils and 3 erasers for ₹ 9. Her friend Sonali saw the new variety of pencils and erasers with Romila and she also bought 4 pencils and 6 erasers of the same kind for ₹ 18. Find the cost of each pencil and each eraser.

Sol. Let cost of one pencil be ₹ x and cost of one eraser be ₹ y .

Then, according to given conditions, we get the following equations $2x + 3y = 9 \quad \dots(i)$

and $4x + 6y = 18 \quad \dots(ii)$

From Eq. (i), we get

$$x = \frac{9 - 3y}{2}$$

On substituting the value of x in Eq. (ii), we get

$$4\left(\frac{9-3y}{2}\right) + 6y = 18$$

$$\Rightarrow 2(9-3y) + 6y = 18 \Rightarrow 18 - 6y + 6y = 18 \Rightarrow 18 = 18$$

This equation has no variable but this is true for all values of y . [∴ LHS=RHS]

Thus, we do not get a specific value of y as a solution and so we cannot obtain a specific value of x .

Hence, Eqs. (i) and (ii) have infinitely many solutions and we cannot find a unique cost of pencil and an eraser.

Note It This situation has arisen, because both the given equations $2x+3y=9$ and $4x+6y=18$ [i.e. $2(2x+3y)=2\times 9$] are the same.

Example 4. A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator. It becomes $\frac{1}{4}$ when 8 is added to the denominator. Find the fraction. CBSE 2023 (Standard)

Sol. Let the numerator be x and the denominator be y .

$$\text{So, the fraction} = \frac{x}{y}$$

According to the question,

Condition I

$$\frac{x-1}{y} = \frac{1}{3} \Rightarrow 3(x-1) = y$$

$$\Rightarrow 3x - 3 = y \Rightarrow 3x - y - 3 = 0 \quad \dots(i)$$

Condition II

$$\frac{x}{y+8} = \frac{1}{4} \Rightarrow 4x = y + 8$$

$$\Rightarrow 4x - y = 8 \Rightarrow y = 4x - 8 \quad \dots(ii)$$

On substituting the value of y in Eq. (i), we get

$$3x - (4x - 8) - 3 = 0$$

$$\Rightarrow 3x - 4x + 8 - 3 = 0 \Rightarrow x = 5$$

Now, on substituting the value of x in Eq. (ii), we get

$$y = 4(5) - 8 = 20 - 8 = 12$$

Hence, the fraction $\frac{x}{y}$ is $\frac{5}{12}$.

Try These 3.2

1. Find the solution of the following system of equations by substitution method.

- (i) $x + y = 8, 2x - 3y = 1$
- (ii) $3x + 2y = 10, 12x + 8y = 30$
- (iii) $2x - 7y = 11, 6x - 21y = 33$
- (iv) $\sqrt{2}x + \sqrt{5}y = 0, \sqrt{6}x + \sqrt{15}y = 0$
- (v) $3x - y = 3, 9x - 3y = 9$

2. Solve the following pair of linear equations by substitution method.

$$(i) \frac{3x - 4y}{2} = 10, \frac{3x + 2y}{4} = 2$$

$$(ii) y = \frac{2}{3}x + 6, 2y - 4x = 20$$

$$(iii) 3x - \frac{y+7}{11} = 8, 2y + \frac{x+11}{7} = 10$$

$$(iv) 1.1x + 1.5y + 2.3 = 0, 0.7x - 0.2y = 2$$

$$(v) 0.2x + 0.3y = 1.3, 0.4x + 0.5y = 2.3$$

$$(vi) \sqrt{7}x + \sqrt{11}y = 0, \sqrt{3}x - \sqrt{5}y = 0$$

$$(vii) 7(y+3) - 2(x+2) = 14, 4(y-2) + 3(x-3) = 2$$

3. Solve for x and y by substitution method

$$(i) x + y = a - b, ax - by = a^2 + b^2$$

$$(ii) \frac{x}{a} + \frac{y}{b} = 2, ax - by = a^2 - b^2$$

$$(iii) \frac{bx}{a} - \frac{ay}{b} + a + b = 0, bx - ay + 2ab = 0$$

$$(iv) 2(ax - by) + a + 4b = 0, 2(bx + ay) + (b - 4a) = 0$$

$$(v) 2\left(\frac{x}{a}\right) + \frac{y}{b} = 2, \frac{x}{a} - \frac{y}{b} = 4$$

4. (i) Solve $2x - 3y = 13$ and $7x - 2y = 20$ and hence find the value of m for which $y = mx + 7$.

(ii) Solve $5x + 4y = 10$ and $3x - 2y + 16 = 0$ and hence find the value of m for which $y = mx + 3$.

Directions (Q. Nos. 5-7) Form the pair of linear equations the following problems and find their solutions by substitution method.

5. A man has ₹ 100 in ₹ 1 coins and 50 paise coins. All 50 paise coins are worth as much as all the ₹ 1 coins. How many coins of each he has?

6. In $\triangle ABC$, $\angle C = 5\angle B = 3(\angle A + \angle B)$, find all angles of $\triangle ABC$.

7. The perimeter of rectangular lawn is 54 m. It is reduced in size, so that length is $\frac{3}{5}$ th and breadth is $\frac{3}{4}$ th of the original dimensions. The perimeter of the reduced rectangle is 36 m. What were the original dimensions of the lawn?

Answers

1. (i) Hint Given, equations are $x + y = 8$

and $2x - 3y = 1$

From Eq. (i), $y = 8 - x$

Then, from Eq. (ii), $2x - 3(8 - x) = 1$

$\Rightarrow 5x = 25 \Rightarrow x = 5$

$\therefore y = 8 - 5 = 3$

Hence, $x = 5, y = 3$

(ii) Hint Given, equations are $3x + 2y = 10$

and $12x + 8y = 30$

From Eq. (i), $y = \frac{10 - 3x}{2}$

Then, from Eq. (ii), $12x + 8\left(\frac{10 - 3x}{2}\right) = 30$

$$\Rightarrow 12x + 40 - 12x = 30 \Rightarrow 40 = 30$$

Since, this equality is false, therefore no solution exist.

(iii) Hint Given equations are $2x - 7y = 11$... (i)

and $6x - 21y = 33$... (ii)

From Eq. (i), $y = \frac{2x - 11}{7}$

Then, from Eq. (ii), $6x - 21\left(\frac{2x - 11}{7}\right) = 33$

$$\Rightarrow 6x - 6x + 33 = 33 \Rightarrow 33 = 33$$

This equality is true for all values of x , therefore given pair of equations have infinitely many solutions.

(iv) Hint Do same as Part (iii).

Ans. Infinitely many solutions.

(v) Hint Do same as Part (iii).

Ans. Infinitely many solutions.

2. (i) Hint Given equations are

$$\frac{3x - 4y}{2} = 10 \Rightarrow 3x - 4y = 20$$

and $\frac{3x + 2y}{4} = 2 \Rightarrow 3x + 2y = 8$

Now, do same as Q.1 (ii).

Ans. $x = 4, y = -2$

(ii) Hint Given equations are $y = \frac{2}{3}x + 6$

$$\Rightarrow -2x + 3y = 18$$

and $-4x + 2y = 20$

Now, do same as Q. 1 (ii).

Ans. $x = -3, y = 4$

(iii) Hint Given equations are

$$3x - \frac{y+7}{11} = 8 \Rightarrow y = 33x - 95$$

and $2y + \frac{x+11}{7} = 10$

$$\Rightarrow 14y + x = 59$$

Now, do same as Q.1 (ii). Ans. $x = 3, y = 4$

(iv) Hint Multiplying the given equations by 10, we get

$$11x + 15y + 23 = 0$$

and $7x - 2y = 20$

Now, do same as Q.1 (ii) Ans. $x = 2, y = -3$

(v) Hint Do same as Part (iv) Ans. $x = 2, y = 3$

(vi) Hint Given equations are $\sqrt{7}x + \sqrt{11}y = 0$... (i)

and $\sqrt{3}x - \sqrt{5}y = 0$... (ii)

From Eqs. (ii), $y = \frac{\sqrt{3}x}{\sqrt{5}}$

Then, from Eq. (i), $\sqrt{7}x + \sqrt{11} \times \frac{\sqrt{3}}{\sqrt{5}}x = 0 \Rightarrow x = 0$

$$\therefore y = \frac{\sqrt{3}}{5} \times 0 = 0$$

Hence, $x = 0, y = 0$

(vii) Hint First rewrite the given equations as

$$2x - 7y - 3 = 0 \text{ and } 3x + 4y - 19 = 0$$

Then, do same as Q.1 (ii) Ans. $x = 5, y = 1$

3. (i) Given equations are

$$x + y = a - b \Rightarrow y = a - b - x \quad \dots(i)$$

and $ax - by = a^2 + b^2 \quad \dots(ii)$

On putting $y = a - b - x$ in Eq. (ii), we get

$$ax - b(a - b - x) = a^2 + b^2$$

$$\Rightarrow (a+b)x = a^2 + b^2 + ab - b^2$$

$$\Rightarrow x = \frac{a(a+b)}{(a+b)} = a$$

$$\therefore y = a - b - a = -b$$

Hence, $x = a, y = -b$

(ii) Hint Do same as Part (i). Ans. $x = a, y = b$

(iii) Hint Do same as Part (i). Ans. $x = -a, y = b$

(iv) Hint Do same as Part (i). Ans. $x = -\frac{1}{2}, y = 2$

(v) Hint Do same as Part (i) Ans. $x = 2a, y = -2b$

4. Hint First, solve the given equations similar to question 1 to get the values of x and y and then put these values in $y = mx + 7$ and $y = mx + 3$ to get required values of m .

(i) Ans. $x = 2, y = -3, m = -5$

(ii) Ans. $x = -2, y = 5, m = -1$

5. Let the number of ₹ 1 and 50 paise coins be x and y ,

respectively. Then, $x + \frac{1}{2}y = 100 \quad \dots(i)$

and $x = \frac{1}{2}y \quad \dots(ii)$

Then, from Eq. (i),

$$\frac{1}{2}y + \frac{1}{2}y = 100 \Rightarrow y = 100$$

$$\therefore \text{From Eq. (ii), } x = \frac{1}{2} \times 100 = 50$$

Hence, ₹ 1 coins = 50, 50 paise coins = 100

6. Hint Let $\angle A$ and $\angle B$ be x and y , respectively.

Then, $\angle C = 5\angle B = 3(\angle A + \angle B)$

$$\Rightarrow \angle C = 5y \text{ and } \angle C = 3(x + y)$$

$$\Rightarrow 5y = 3(x + y)$$

$$\Rightarrow 2y - 3x = 0 \quad \dots(i)$$

Also, $\angle A + \angle B + \angle C = 180^\circ \Rightarrow x + y + 5y = 180^\circ$

$$\Rightarrow x + 6y = 180^\circ \quad \dots(ii)$$

Now, solve Eqs. (i) and (ii) to find $\angle A$ and $\angle B$.

Hence, find $\angle C$.

Ans. $\angle A = 18^\circ, \angle B = 27^\circ, \angle C = 135^\circ$

7. Hint Let the original length and breadth of the lawn be x m and y m, respectively.

Then, $2(x + y) = 54 \Rightarrow x + y = 27$

$$\text{and } 2\left(\frac{3}{5}x + \frac{3}{4}y\right) = 36 \Rightarrow 4x + 5y = 120$$

Now, do same as Q. 1 (i).

Ans. Length = 15 m, Breadth = 12 m

TOPIC 03 Elimination Method

In this method, one variable out of the two variables is eliminated by making the coefficients of that variable equal in both the equations.

After eliminating that variable, the left equation is an equation in one variable, which can be solved easily.

Value of one variable obtained in this way can be substituted in any one of the two given equations to find the value of other variable. This method can be understood with the help of following Steps:

Step I Consider the equation

$$a_1x + b_1y + c_1 = 0 \quad \dots(i)$$

$$a_2x + b_2y + c_2 = 0 \quad \dots(ii)$$

Step II First multiply both the equations by some suitable non-zero constants to make the coefficient of the variable to be numerically equal.

Step III Add or subtract the one equation from the other, so that one variable gets eliminated.

Step IV Solve the equation in one variable x (or y), so obtained to get its value.

Step V Substitute this value x (or y) in either of the original equation to get the value of other variable.

Example 1. Solve the following pair of linear equations

$$41x + 53y = 135 \text{ and } 53x + 41y = 147.$$

Sol. Given, pair of linear equations is

$$41x + 53y = 135 \quad \dots(i)$$

$$\text{and} \quad 53x + 41y = 147 \quad \dots(ii)$$

On adding Eqs. (i) and (ii), we get

$$94x + 94y = 282$$

$$\Rightarrow x + y = 3 \quad [\text{dividing both sides by 94}] \quad \dots(iii)$$

On subtracting Eq. (i) from Eq. (ii), we get

$$12x - 12y = 12$$

$$\Rightarrow x - y = 1 \quad [\text{dividing both sides by 12}] \quad \dots(iv)$$

Now, adding Eqs. (iii) and (iv), we get

$$2x = 4$$

$$\Rightarrow x = 2$$

On substituting $x = 2$ in Eq. (iii), we get

$$y = 3 - 2 = 1$$

Hence, $x = 2$ and $y = 1$ is the required solution.

Note It In the above question, coefficient of x and y look alike, therefore for easy calculation we can use the above process.

Example 2. Use elimination method to find all solutions of the following pair of linear equations

$$(i) 2x + 3y = 8, 4x + 6y = 7$$

$$(ii) ax + by - a + b = 0, bx - ay - a - b = 0$$

Sol. (i) Given, pair of linear equations is

$$2x + 3y = 8$$

$$\text{and} \quad 4x + 6y = 7$$

On multiplying Eq. (i) by 2, to make the coefficient x equal, we get

$$4x + 6y = 16$$

On subtracting Eq. (ii) from Eq. (iii), we get

$$(4x - 4x) + (6y - 6y) = 16 - 7$$

$$\Rightarrow 0 = 9$$

which is a false equation involving no variable.
So, the given pair of linear equations has no solution.
i.e. this pair of linear equations is inconsistent.

(ii) Given, pair of linear equations is

$$ax + by = a - b$$

$$\text{and} \quad bx - ay = a + b$$

On multiplying Eq. (i) by a and Eq. (ii) by b and then, we get

$$a^2x + aby + b^2x - aby = a^2 - ab + ba + b^2$$

$$\Rightarrow (a^2 + b^2)x = (a^2 + b^2) \Rightarrow x = \frac{a^2 + b^2}{a^2 + b^2} = 1$$

On putting $x = 1$ in Eq. (i), we get

$$a + by = a - b \Rightarrow y = -\frac{b}{b} = -1$$

Hence, $x = 1$ and $y = -1$, which is the required solution.

Example 3. Sabina went to a bank ATM to withdraw ₹ 2000. She received ₹ 50 and ₹ 100 notes only. If she got 25 notes in all, how many notes of ₹ 50 and ₹ 100 did she receive?

Sol. Let number of ₹ 50 notes Sabina received be x and number of ₹ 100 notes be y :

According to the question,

Total money withdrawn = ₹ 2000

$$\Rightarrow 50x + 100y = 2000$$

$$\Rightarrow x + 2y = 40$$

and total number of notes = 25

$$\Rightarrow x + y = 25$$

On subtracting Eq. (ii) from Eq. (i), we get

$$y = 15$$

From Eq. (ii), $x = 25 - 15 = 10$

∴ Number of ₹ 50 notes = 10

Number of ₹ 100 notes = 15

Example 4. The sum of the digits of a 2-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.

CBSE 2024 (Basic)

Sol. Let the number be $10x + y$.

$$\text{Given, } x + y = 9 \quad \dots(\text{i})$$

According to the question,

$$9(10x + y) = 2(10y + x)$$

$$\Rightarrow 90x + 9y = 20y + 2x$$

$$\Rightarrow 88x - 11y = 0$$

$$\Rightarrow 8x - y = 0 \quad \dots(\text{ii})$$

On adding Eqs. (i) and (ii), we get

$$9x = 9$$

$$\Rightarrow x = 1$$

On substituting $x = 1$ in Eq. (i), we get

$$1 + y = 9$$

$$y = 8$$

The number is $10x + y = 10 \times 1 + 8 = 18$

Therefore, $x = 1$ and $y = 8$ are the required digits and the number is 18.

Example 5. The ratio of incomes of two persons is 9 : 7 and the ratio of their expenditures is 4 : 3. If each of them manages to save ₹ 2000 per month. To find their monthly incomes, form a pair of linear equations from the above data and solve them by elimination method. Also, verify the solution.

Sol. Given, ratio of incomes = 9 : 7

and ratio of their expenditures = 4 : 3

Saving of each person = ₹ 2000

Let incomes of two persons be $9x$, $7x$ and their expenditures be $4y$, $3y$.

Then, linear equations so formed are

$$9x - 4y = 2000 \quad \dots(\text{i})$$

$$\text{and } 7x - 3y = 2000 \quad \dots(\text{ii})$$

We make the coefficients of x numerically equal in both equations.

On multiplying Eq. (i) by 7 and Eq. (ii) by 9, we get

$$63x - 28y = 14000 \quad \dots(\text{iii})$$

$$\text{and } 63x - 27y = 18000 \quad \dots(\text{iv})$$

On subtracting Eq. (iv) from Eq. (iii), we get

$$-28y + 27y = 14000 - 18000$$

$$\Rightarrow -y = -4000$$

$$\Rightarrow y = 4000$$

On putting $y = 4000$ in Eq. (i), we get

$$9x - 4 \times 4000 = 2000$$

$$\Rightarrow 9x = 2000 + 16000$$

$$\Rightarrow x = \frac{18000}{9} = 2000$$

Thus, monthly income of both persons are 9(2000) and 7(2000) i.e. ₹ 18000 and ₹ 14000, respectively.

Verification

On putting $x = 2000$ and $y = 4000$ in Eqs. (i) and (ii) respectively, we get

$$\text{From Eq. (i), LHS} = 9x - 4y = 9(2000) - 4(4000)$$

$$= 18000 - 16000$$

$$= 2000 = \text{RHS}$$

$$\text{From Eq. (ii), LHS} = 7x - 3y = 7(2000) - 3(4000)$$

$$= 14000 - 12000$$

$$= 2000 = \text{RHS}$$

Hence, the solution is verified.

Example 6. Two people are 16 km apart on a straight road. They start walking at the same time. If they walk towards each other with different speeds, they will meet in 2 h. Had they walked in the same direction with same speeds as before, they would have met in 8 h. Find their walking speeds.

CBSE 2023 (Standard)

Sol. Let the walking speed of 1st person be x km/h and of 2nd person be y km/h.

It is given that if they walk towards each other, they will meet in 2 h.

\Rightarrow Distance covered by 1st person in 2 h

$$+ \text{Distance covered by 2nd person in 2 h} = 16 \text{ km}$$

$$16 = (x + y) \times 2$$

$$\Rightarrow 16 = 2x + 2y \quad [\because \text{Distance} = \text{Speed} \times \text{Time}]$$

$$\Rightarrow x + y = 8 \quad \dots(\text{i})$$

Now, it is also given that if they walk in the same direction, then they will meet in 8 h.

\Rightarrow Distance covered by 1st person in 8 h

$$= \text{Distance covered by 2nd person in 8 h} + 16 \text{ km}$$

$$\Rightarrow 8x = 8y + 16$$

$$\Rightarrow 8(x - y) = 16$$

$$\Rightarrow x - y = 2 \quad \dots(\text{ii})$$

On adding Eqs. (i) and (ii), we get

$$2x = 10$$

$$\Rightarrow x = 5$$

From Eq. (ii),

$$5 - y = 2$$

$$\Rightarrow y = 3$$

Hence, walking speeds of both persons are 5 km/h and 3 km/h respectively.

Try These 3.3

1. Solve the following pair of equations by using elimination method.

- $8x + 5y = 11, x + y = 4$
- $x - y = 3, 3x - 2y = 10$
- $4x - 3y = 8, 6x - y = \frac{29}{3}$
- $11x + 15y + 23 = 0, 7x - 2y - 20 = 0$
- $3x - 2y = 5, 2x - 5y = 4$
- $x - 2y = 3, 3x - 3y = 5$
- $0.4x + 0.3y = 1.7, 0.7x - 0.2y = 0.8$
- $\frac{x}{4} + \frac{y}{3} = -\frac{1}{12}, \frac{x}{2} - \frac{5}{4}y = \frac{7}{4}$
- $\frac{x}{3} + \frac{y}{3} = 14, \frac{5x}{6} - \frac{y}{3} + 7 = 0$
- $\frac{x}{10} + \frac{y}{5} + 1 = 15, \frac{x}{8} + \frac{y}{6} = 15$

2. Solve the following pair of linear equations by elimination method.

- $ax + by = c, a^2x + b^2y = c^2$
- $(a + b)x + (a - b)y = 2ab, (a + b)x - (a - b)y = ab$
- $\frac{bx}{a} + \frac{ay}{b} = a^2 + b^2, x + y = 2ab,$
- $5ax + 6by = 28; 3ax + 4by = 18$
- $\frac{ax}{b} - \frac{by}{a} = a + b; ax - by = 2ab$

3. Solve the following pair of linear equations by elimination method.

- $89x + 91y = 449, 91x + 89y = 451$
- $117x + 231y = 579, 231x + 117y = 465$
- $217x + 131y = 913, 131x + 217y = 827$
- $41x - 17y = 99, 17x - 41y = 75$
- $23x - 29y = 98, 29x - 23y = 110$

4. Find the solution of the pair of equations $\frac{x}{10} + \frac{y}{5} - 1 = 0$

$$\text{and } \frac{x}{8} + \frac{y}{6} = 15 \text{ and find } \lambda, \text{ if } y = \lambda x + 5.$$

NCERT Exemplar

Directions (Q. Nos. 5-8) Form the pair of linear equation in the following problems and find their solutions (if they exist) by the elimination method.

- 5 yr ago, Amit was thrice as old as Baljeet. 10 yr hence, Amit shall be twice as old as Baljeet. What are their present ages? CBSE 2023 (Standard)
- Sheena went to a bank to withdraw ₹ 1000 and asked the cashier to give her ₹ 100 and ₹ 50 notes only. She got 14 notes in all. Find how many notes of ₹ 100 and ₹ 50 she received?

7. The sum of the digits of a two-digit number is 6. 17 times this number is 5 times of the number by reversing the order of digits. Find the number.

8. The area of a rectangle gets reduced by 80 sq units when its length is reduced by 5 units and the breadth is increased by 2 units. If we increase the length by 10 units and decrease the breadth by 5 units, then the area is increased by 50 sq units. Find the length and the breadth of the rectangle.

Answers

1. (i) Given, pair of linear equations is $8x + 5y = 11$

$$\text{and } x + y = 4$$

On multiplying Eq. (ii) by 8 and then subtracting Eq. (i), we get $-3y = -21 \Rightarrow y = 7$

On putting $y = 7$ in Eq. (ii), we get

$$x = 4 - 7 = -3$$

Hence, $x = -3, y = 7$

(ii) Hint Do same as Part (i). Ans. $x = 4, y = 1$

(iii) Hint Do same as Part (i). Ans. $x = \frac{3}{2}, y = -\frac{2}{3}$

(iv) Hint Do same as Part (i). Ans. $x = 2, y = -3$

(v) Hint Do same as Part (i). Ans. $x = \frac{17}{11}, y = -\frac{11}{11}$

(vi) Hint Do same as Part (i). Ans. $x = \frac{1}{3}, y = -\frac{4}{3}$

(vii) Given, pair of linear equations is $0.4x + 0.3y = 1.7$

$$\text{and } 0.7x - 0.2y = 0.8$$

On multiplying Eq. (i) by 20 and Eq. (ii) by 30 and adding both equations, we get

$$29x = 58 \Rightarrow x = 2$$

On putting $x = 2$ in Eq. (ii), we get

$$0.7 \times 2 - 0.8 = 0.2y \Rightarrow y = 3$$

Hence, $x = 2$ and $y = 3$

(viii) Given, pair of linear equations is

$$\frac{x}{4} + \frac{y}{3} = -\frac{1}{12}$$

$$\Rightarrow 3x + 4y = -1$$

$$\text{and } \frac{x}{2} - \frac{5}{4}y = \frac{7}{4} \Rightarrow 2x - 5y = 7$$

Now, do same as Part (i).

$$\text{Ans. } x = 1, y = -1$$

(ix) Hint Do same as Part (viii).

$$\text{Ans. } x = 6, y = 36$$

(x) Hint Do same as Part (viii).

$$\text{Ans. } x = 80, y = 30$$

2. (i) Given pair of linear equation is

$$ax + by = c$$

$$a^2x + b^2y = c^2$$

On multiplying Eq. (i) by a and then subtracting from Eq. (ii), we get

$$b^2y - aby = c^2 - ca \\ \Rightarrow by(b-a) = c(c-a) \Rightarrow y = \frac{c(c-a)}{b(b-a)}$$

On putting this value of y in Eq. (i), we get

$$ax = c - \frac{bc(c-a)}{b(b-a)} = \frac{cb-ac-c^2+ac}{(b-a)} \\ \Rightarrow x = \frac{c(b-c)}{a(b-a)} \\ \text{Hence, } x = \frac{c(b-c)}{a(b-a)}, y = \frac{c(c-a)}{b(b-a)}$$

(ii) Hint Do same as Part (i).

$$\text{Ans. } x = \frac{3ab}{2(a+b)}, y = \frac{ab}{2(a-b)}$$

(iii) Hint Do same as Part (i). Ans. $x = ab$, $y = ab$

(iv) Hint Do same as Part (i). Ans. $x = \frac{2}{a}$, $y = \frac{3}{b}$

(v) Hint Do same as Part (i). Ans. $x = b$, $y = -a$

3. Hint Do same as Example 1.

Ans.

- (i) $x = 3$, $y = 2$
- (ii) $x = 1$, $y = 2$
- (iii) $x = 3$, $y = 2$
- (iv) $x = 2$, $y = -1$
- (v) $x = 3$, $y = -1$

4. Hint By solving both equations as in Question 1 (viii), find the values of x and y and then put these values in $y = \lambda x + 5$ to get required value of λ .

$$\text{Ans. } x = 340, y = -165, \lambda = -\frac{1}{2}$$

5. Let present age of Amit be x yr and present age of Baljeet be y yr.

5 yr ago,

Age of Amit = $x - 5$

Age of Baljeet = $y - 5$

According to the question,

$$x - 5 = 3(y - 5) \\ \Rightarrow x - 5 = 3y - 15 \\ \Rightarrow x - 3y = -10 \quad \dots(i)$$

After 10 yr,

Age of Amit = $x + 10$

Age of Baljeet = $y + 10$

According to the question,

$$x + 10 = 2(y + 10) \\ \Rightarrow x + 10 = 2y + 20 \\ \Rightarrow x - 2y = 10 \quad \dots(ii)$$

On subtracting Eq. (ii) from Eq. (i), we get

$$-y = -20 \Rightarrow y = 20$$

On substituting the value of y in Eq. (ii), we get

$$x - 2(20) = 10 \\ \Rightarrow x - 40 = 10 \Rightarrow x = 50$$

Hence, present age of Amit = 50 yr

and present age of Baljeet = 20 yr

6. Hint Let the number of ₹ 100 and ₹ 50 notes be x and y , respectively. Then, according to the question,

$$x + y = 14 \text{ and } 100x + 50y = 1000$$

Now, do same as Question 1 (i) to solve for x and y .

$$\text{Ans. ₹ 100 notes} = 6 \text{ and ₹ 50 notes} = 8$$

7. Hint Do same as Example 4.

Ans. The required two-digit number is 15.

8. Hint Let x and y be length and breadth of rectangle.

Then, its area = xy

According to the question,

$$(x-5)(y+2) = xy - 80 \Rightarrow 2x - 5y = -70 \\ (x+10)(y-5) = xy + 50 \Rightarrow -5x + 10y = 100$$

Now, do same as Question 1 (i) to solve for x and y .

$$\text{Ans. Length} = 40 \text{ units, breadth} = 30 \text{ units}$$

EXERCISE 3.1

Q1. Form the pair of linear equations in the following problems and find their solutions graphically.

- 10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, then find the number of boys and girls who took part in the quiz.
- 5 pencils and 7 pens together cost ₹ 50, whereas 7 pencils and 5 pens together cost ₹ 46. Find the cost of one pencil and that of one pen.

Sol. (i) Let the number of boys be x and the number of girls be y .

Then, according to the question, $x + y = 10$

Also, number of girls = 4 + Number of boys

$$y = x + 4$$

Thus, we get the following pair of linear equations

$$x + y = 10 \quad \dots(i)$$

$$\text{and} \quad y = x + 4 \quad \dots(ii)$$

∴ Table for linear equation $x + y = 10$ or $y = 10 - x$ is

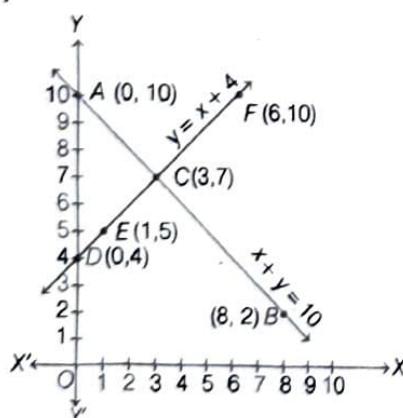
| | | | |
|--------------|---------|--------|--------|
| x | 0 | 8 | 3 |
| $y = 10 - x$ | 10 | 2 | 7 |
| Points | A(0,10) | B(8,2) | C(3,7) |

Plot the points A, B and C and join them to get the line ACB.

Table for linear equation $y = x + 4$ is

| | | | |
|-------------|--------|--------|---------|
| x | 0 | 1 | 6 |
| $y = x + 4$ | 4 | 5 | 10 |
| Points | D(0,4) | E(1,5) | F(6,10) |

Plot the points D, E and F and join them to get the line DEF.



The two lines AB and DF intersect at the point

So, $x = 3$ and $y = 7$ is the required solution of the linear equations.

Hence, the required number of boys is 3 and

(ii) Let the cost of one pencil be ₹ x and the cost of one pen be ₹ y . Then, according to the question,

Cost of 5 pencils and 7 pens = ₹ 50

$$\therefore 5x + 7y = 50$$

and cost of 7 pencils and 5 pens = ₹ 46

$$\therefore 7x + 5y = 46$$

Thus, we get the following pair of linear equations

$$5x + 7y = 50$$

$$\text{and} \quad 7x + 5y = 46$$

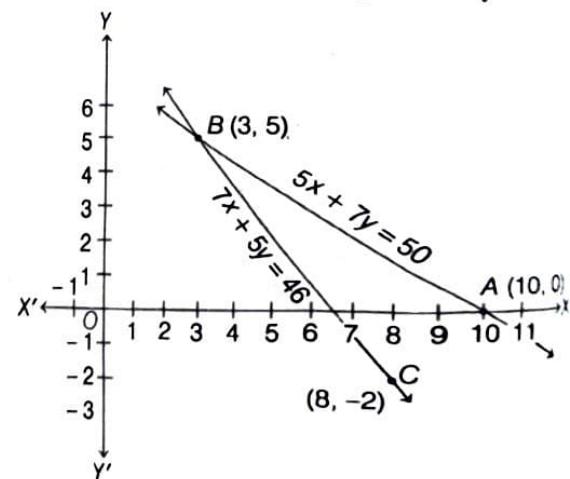
Table for linear equation $5x + 7y = 50$ or $y =$

| | | |
|-------------------------|----------|------|
| x | 10 | |
| $y = \frac{50 - 5x}{7}$ | 0 | |
| Points | A(10, 0) | B() |

Table for linear equation $7x + 5y = 46$ or $y =$

| | | |
|-------------------------|----------|------|
| x | 8 | |
| $y = \frac{46 - 7x}{5}$ | -2 | |
| Points | C(8, -2) | B() |

Plot the points A, B and C on graph paper and to get the lines AB and BC, respectively.



The two lines AB and BC intersect at the point $B(3, 5)$. So, $x = 3$ and $y = 5$ is the required solution. Hence, the cost of one pencil is ₹ 3 and the cost of one pen is ₹ 5.

Q2. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out

whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincident:

- (i) $5x - 4y + 8 = 0; 7x + 6y - 9 = 0$
- (ii) $9x + 3y + 12 = 0; 18x + 6y + 24 = 0$
- (iii) $6x - 3y + 10 = 0; 2x - y + 9 = 0$

Sol. (i) The given pair of linear equations is

$$5x - 4y + 8 = 0 \quad \dots(i)$$

$$\text{and } 7x + 6y - 9 = 0 \quad \dots(ii)$$

On comparing with standard form of pair of linear equations, we get $a_1 = 5, b_1 = -4, c_1 = 8$

and $a_2 = 7, b_2 = 6, c_2 = -9$

Here, $\frac{5}{7} \neq \frac{-4}{6}$ i.e. $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

So, lines (i) and (ii) intersect at a point.

- (ii) Do same as Part (i). Ans. Coincident lines
- (iii) Do same as Part (i). Ans. Parallel lines

Q3. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out

whether the following pairs of linear equations are consistent or inconsistent.

- (i) $3x + 2y = 5; 2x - 3y = 7$
- (ii) $2x - 3y = 8; 4x - 6y = 9$
- (iii) $\frac{3}{2}x + \frac{5}{3}y = 7; 9x - 10y = 14$
- (iv) $5x - 3y = 11; -10x + 6y = -22$
- (v) $\frac{4}{3}x + 2y = 8; 2x + 3y = 12$

Sol. (i) The given equations can be rewritten as

$$3x + 2y - 5 = 0 \text{ and } 2x - 3y - 7 = 0$$

On comparing with standard form of pair of linear equations, we get $a_1 = 3, b_1 = 2, c_1 = -5$

and $a_2 = 2, b_2 = -3, c_2 = -7$

Now, $\frac{a_1}{a_2} = \frac{3}{2}, \frac{b_1}{b_2} = -\frac{2}{3} \text{ and } \frac{c_1}{c_2} = \frac{5}{7}$

Thus, $\frac{3}{2} \neq -\frac{2}{3}$ i.e. $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence, the pair of linear equations is consistent.

- (ii) The given equations can be rewritten as

$$2x - 3y - 8 = 0 \text{ and } 4x - 6y - 9 = 0$$

On comparing with standard form of pair of linear equations, we get

$$a_1 = 2, b_1 = -3, c_1 = -8$$

$$\text{and } a_2 = 4, b_2 = -6, c_2 = -9$$

$$\text{Now, } \frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{-3}{-6} = \frac{1}{2} \text{ and } \frac{c_1}{c_2} = \frac{-8}{-9} = \frac{8}{9}$$

$$\text{Thus, } \frac{1}{2} = \frac{1}{2} \neq \frac{8}{9} \text{ i.e. } \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Hence, the pair of linear equations is inconsistent.

- (iii) Do same as Part (i). Ans. Consistent

- (iv) The given equations can be rewritten as

$$5x - 3y - 11 = 0 \text{ and } -10x + 6y + 22 = 0$$

On comparing with standard form of pair of linear equations, we get $a_1 = 5, b_1 = -3, c_1 = -11$

$$\text{and } a_2 = -10, b_2 = 6, c_2 = 22$$

$$\text{Now, } \frac{a_1}{a_2} = \frac{5}{-10} = -\frac{1}{2}, \frac{b_1}{b_2} = \frac{-3}{6} = -\frac{1}{2}$$

$$\text{and } \frac{c_1}{c_2} = \frac{-11}{22} = -\frac{1}{2}$$

$$\text{Thus, } -\frac{1}{2} = -\frac{1}{2} = -\frac{1}{2}$$

$$\text{i.e. } \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Hence, the pair of linear equations is consistent (or dependent).

- (v) Do same as Part (iv). Ans. Consistent (or dependent)

Q4. Which of the following pairs of linear equations are consistent/inconsistent?

If consistent, obtain the solution graphically:

- (i) $x + y = 5; 2x + 2y = 10$
- (ii) $x - y = 8; 3x - 3y = 16$
- (iii) $2x + y - 6 = 0; 4x - 2y - 4 = 0$
- (iv) $2x - 2y - 2 = 0; 4x - 4y - 5 = 0$

Sol. (i) Given, pair of linear equations is

$$x + y = 5 \Rightarrow x + y - 5 = 0 \quad \dots(i)$$

$$\text{and } 2x + 2y = 10 \Rightarrow 2x + 2y - 10 = 0 \quad \dots(ii)$$

On comparing with standard form of pair of linear equations, we get

$$a_1 = 1, b_1 = 1, c_1 = -5$$

$$\text{and } a_2 = 2, b_2 = 2, c_2 = -10$$

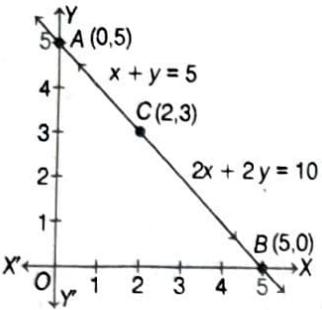
$$\text{Here, } \frac{a_1}{a_2} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{1}{2} \text{ and } \frac{c_1}{c_2} = \frac{-5}{-10} = \frac{1}{2}$$

$$\text{Thus, } \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{2}$$

So, the pair of linear equations is consistent.

Now, table for $x + y = 5$ or $y = 5 - x$ is

| | | | |
|-------------|---------|---------|---------|
| x | 0 | 5 | 2 |
| $y = 5 - x$ | 5 | 0 | 3 |
| Points | A(0, 5) | B(5, 0) | C(2, 3) |



Here, Eq. (ii) is same as Eq. (i) because on dividing Eq (ii) by 2 on both sides, it becomes same as Eq (i). Therefore, table for both equations is same.

On plotting and joining these points on graph paper, we observe that the lines are coincident, so pair of linear equations have infinitely many solutions and hence consistent. All the points lying on the line ACB satisfy the given pair of linear equations.

(ii) Given, pair of linear equations is

$$x - y = 8 \Rightarrow x - y - 8 = 0 \quad \dots(i)$$

$$\text{and } 3x - 3y = 16 \Rightarrow 3x - 3y - 16 = 0 \quad \dots(ii)$$

On comparing with standard form of pair of linear equations, we get

$$a_1 = 1, b_1 = -1, c_1 = -8$$

$$\text{and } a_2 = 3, b_2 = -3, c_2 = -16$$

$$\text{Here, } \frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{1}{3} \text{ and } \frac{c_1}{c_2} = \frac{-8}{-16} = \frac{1}{2}$$

$$\text{Thus, } \frac{1}{3} = \frac{1}{3} \neq \frac{1}{2} \text{ i.e. } \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

So, the pair of linear equations is inconsistent.

(iii) Given, pair of linear equations is

$$2x + y - 6 = 0 \quad \dots(i)$$

$$\text{and } 4x - 2y - 4 = 0 \quad \dots(ii)$$

On comparing with standard form of pair of linear equations, we get $a_1 = 2, b_1 = 1, c_1 = -6$

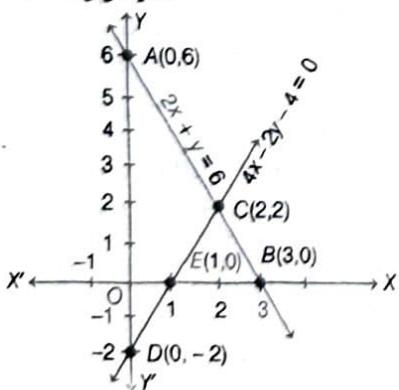
$$\text{and } a_2 = 4, b_2 = -2, c_2 = -4$$

$$\text{Here, } \frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{1}{-2} = -\frac{1}{2} \text{ and } \frac{c_1}{c_2} = \frac{-6}{-4} = \frac{3}{2}$$

$$\text{Thus, } \frac{1}{2} \neq -\frac{1}{2} \text{ i.e. } \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Hence, the given pair of linear equations is consistent.

Now, on plotting the graph similar to Q. 1 (i), we get the following graph.



The lines AB and DE intersect at point C(2, 2). Hence, the unique solution is $x = 2, y = 2$.

(iv) Do same as Part (ii). Ans. Inconsistent

Q5. Half of the perimeter of a rectangular garden whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden.

Perimeter of a rectangular garden = 2 (length + width). Use this relation to make a pair of linear equations and on graph paper, then intersection point gives the required dimensions.

Sol. Let the length of the garden be x m and its width y m. Then, perimeter of the rectangular garden

$$= 2(\text{length} + \text{width}) = 2(x + y)$$

$$\text{Therefore, half of the perimeter} = (x + y) \text{ m}$$

$$\text{But given that half of the perimeter} = 36 \text{ m}$$

$$\therefore (x + y) = 36$$

$$\text{Also, length of garden} = 4 + \text{width of garden}$$

$$x = y + 4 \Rightarrow x - y = 4$$

$$\text{Table for } x + y = 36 \text{ or } y = 36 - x \text{ is}$$

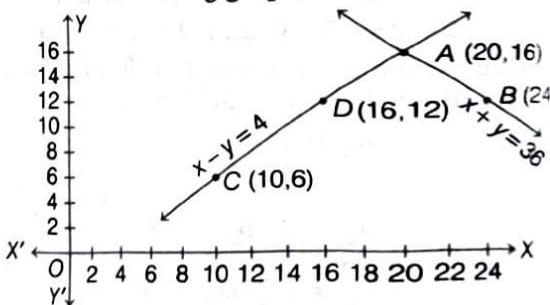
| | | |
|--------------|-----------|-----------|
| x | 20 | 24 |
| $y = 36 - x$ | 16 | 12 |
| Points | A(20, 16) | B(24, 12) |

Plot the points A(20, 16) and B(24, 12) and join them to form straight line AB.

Table for $x - y = 4$ or $y = x - 4$ is

| | | |
|-------------|----------|-----------|
| x | 10 | 16 |
| $y = x - 4$ | 6 | 12 |
| Points | C(10, 6) | D(16, 12) |

Plot the points C(10, 6) and D(16, 12) and join them to form straight line CD. The two lines intersect at point A(20, 16).



Hence, length of rectangular garden is 20 m and width is 16 m.

Q6. Given the linear equation $2x + 3y - 8 = 0$, write another linear equation in two variables such that their geometrical representation of the pair so formed

- (i) intersecting lines
- (ii) parallel lines
- (iii) coincident lines

Q6. Given, linear equation is $2x + 3y - 8 = 0$ (i)

(i) For intersecting lines, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Here, $a_1 = 2, b_1 = 3, c_1 = -8$.

Any line intersecting with Eq. (i) may be taken as

$a_2x + b_2y + c_2 = 0$, where $\frac{2}{a_2} \neq \frac{3}{b_2}$ or $2b_2 \neq 3a_2$

$\therefore 3x + 2y - 9 = 0$ or $3x + 2y - 7 = 0$ is intersect to given line.

(ii) For parallel lines, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$.

Here, $a_1 = 2, b_1 = 3, c_1 = -8$

\therefore Any line parallel to Eq. (i) may be taken as

$a_2x + b_2y + c_2 = 0$, where $\frac{2}{a_2} = \frac{3}{b_2} \neq \frac{-8}{c_2}$

$\therefore 6x + 9y + 7 = 0$ or $2x + 3y - 12 = 0$ is parallel to given line.

(iii) For coincident lines, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Any line coincident to Eq. (i) may be taken as

$a_2x + b_2y + c_2 = 0$, where $\frac{2}{a_2} = \frac{3}{b_2} = \frac{-8}{c_2}$

$\therefore 4x + 6y - 16 = 0$ or $6x + 9y - 24 = 0$ coincides with the given line. There can be several linear equations in each of Eqs. (i), (ii) and (iii).

Q7. Draw the graphs of the equations $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Determine the coordinates of the vertices of the triangle formed by these lines and the X-axis and shade the triangular region.

Sol. Given, pair of linear equations is $x - y + 1 = 0$ and

$3x + 2y - 12 = 0$.

Table for $x - y + 1 = 0$ or $y = x + 1$ is

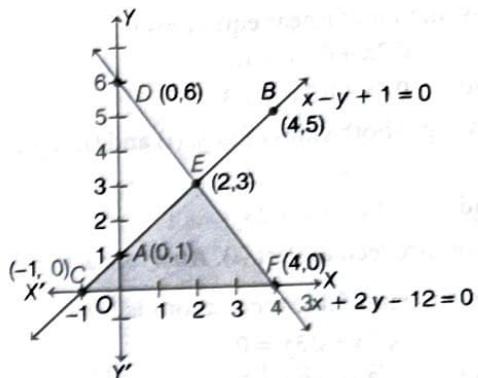
| | | | |
|-------------|---------|---------|----------|
| x | 0 | 4 | -1 |
| $y = x + 1$ | 1 | 5 | 0 |
| Points | A(0, 1) | B(4, 5) | C(-1, 0) |

Plot the points A(0, 1), B(4, 5), C(-1, 0) and join them to get a line CB.

Table for $3x + 2y - 12 = 0$ or $y = \frac{12 - 3x}{2}$ is

| | | | |
|-------------------------|---------|---------|---------|
| x | 0 | 2 | 4 |
| $y = \frac{12 - 3x}{2}$ | 6 | 3 | 0 |
| Points | D(0, 6) | E(2, 3) | F(4, 0) |

Plot the points D(0, 6), E(2, 3), F(4, 0) and join them to get a line DF.



Clearly, the two lines intersect each other at the point E(2, 3). Hence, $x = 2$ and $y = 3$ is the solution of the given pair of equations. The line DE cuts X-axis at the point C(-1, 0). Hence, the coordinates of the vertices of the ΔEFC , so formed are E(2, 3), F(4, 0) and C(-1, 0).

EXERCISE 3.2

Q1. Solve the following pairs of linear equations by the substitution method.

(i) $x + y = 14; x - y = 4$

(ii) $s - t = 3; \frac{s}{3} + \frac{t}{2} = 6$

(iii) $3x - y = 3; 9x - 3y = 9$

(iv) $0.2x + 0.3y = 1.3; 0.4x + 0.5y = 2.3$

(v) $\sqrt{2}x + \sqrt{3}y = 0; \sqrt{3}x - \sqrt{8}y = 0$

(vi) $\frac{3x}{2} - \frac{5y}{3} = -2; \frac{x}{3} + \frac{y}{2} = \frac{13}{6}$

Sol. (i) Given, pair of linear equations is

$x + y = 14$... (i)

and $x - y = 4 \Rightarrow y = x - 4$... (ii)

On substituting the value of y from Eq. (ii) in

Eq. (i), we get $x + x - 4 = 14$

$\Rightarrow 2x = 18$

$\Rightarrow x = 9$

On substituting $x = 9$ in Eq. (ii), we get

$y = 9 - 4 = 5 \Rightarrow y = 5$

Hence, $x = 9$ and $y = 5$.

(ii) Do same as Part (i). Ans. $s = 9, t = 6$

(iii) Given, pair of linear equations is

$3x - y = 3 \Rightarrow y = 3x - 3$... (i)

and $9x - 3y = 9$... (ii)

On substituting the value of y from Eq. (i) in Eq. (ii), we get

$9x - 3(3x - 3) = 9$

$\Rightarrow 9 = 9$

This equation has no variable but it is true for all values of x . So, this pair of linear equations has infinitely many solutions.

(iv) Given, pair of linear equations is

$$0.2x + 0.3y = 1.3 \quad \dots(i)$$

$$\text{and } 0.4x + 0.5y = 2.3 \quad \dots(ii)$$

Multiply both sides of Eqs. (i) and (ii) by 10, we get

$$2x + 3y = 13 \quad \dots(iii)$$

$$\text{and } 4x + 5y = 23 \quad \dots(iv)$$

Now, proceed as Part (i). Ans. $x = 2, y = 3$

(v) Given, pair of linear equations is

$$\sqrt{2}x + \sqrt{3}y = 0 \quad \dots(i)$$

$$\text{and } \sqrt{3}x - \sqrt{8}y = 0$$

$$\Rightarrow y = \frac{\sqrt{3}x}{\sqrt{8}} \quad \dots(ii)$$

On substituting the value of y in Eq. (i), we get

$$\begin{aligned} \sqrt{2}x + \sqrt{3} \times \left(\frac{\sqrt{3}x}{\sqrt{8}} \right) &= 0 \\ \Rightarrow \sqrt{2}x + \frac{3x}{\sqrt{8}} &= 0 \\ \Rightarrow \sqrt{2} \times \sqrt{8}x + 3x &= 0 \\ \Rightarrow 4x + 3x &= 0 \quad [\because \sqrt{2} \times \sqrt{8} = \sqrt{16} = 4] \\ \Rightarrow 7x &= 0 \Rightarrow x = 0 \\ \text{or } -4x + 3x &= 0 \Rightarrow -x = 0 \Rightarrow x = 0 \end{aligned}$$

On substituting $x = 0$ in Eq. (ii), we get

$$y = \frac{\sqrt{3} \times 0}{\sqrt{8}} = 0 \Rightarrow y = 0$$

Hence, $x = 0$ and $y = 0$.

(vi) Given, pair of linear equations is

$$\begin{aligned} \frac{3x - 5y}{2 - 3} &= -2 \\ \Rightarrow \frac{9x - 10y}{6} &= -2 \quad [\text{taking LCM}] \\ \Rightarrow 9x - 10y &= -12 \quad \dots(i) \\ \text{and } \frac{x + y}{3 - 2} &= \frac{13}{6} \\ \Rightarrow \frac{2x + 3y}{6} &= \frac{13}{6} \\ \Rightarrow 2x + 3y &= 13 \quad \dots(ii) \end{aligned}$$

Now, proceed as Part (i). Ans. $x = 2, y = 3$

Q2. Solve $2x + 3y = 11$ and $2x - 4y = -24$ and hence find the value of m for which $y = mx + 3$.

Firstly, find the values of x and y from given equations and then put the values of x and y in $y = mx + 3$, to get the value of m .

Sol. Given, pair of linear equations is

$$2x + 3y = 11 \quad \dots(i)$$

$$\text{and } 2x - 4y = -24 \quad \dots(ii)$$

From Eq. (ii), we have

$$4y = 2x + 24$$

$$\begin{aligned} \Rightarrow y &= \frac{2x + 24}{4} \\ \Rightarrow y &= \frac{x + 12}{2} \end{aligned}$$

On substituting the value of y from Eq. (iii) in Eq. (i), we get

$$2x + 3 \left(\frac{x + 12}{2} \right) = 11$$

$$\Rightarrow 4x + 3(x + 12) = 11 \times 2$$

$$\Rightarrow 4x + 3x + 36 = 22 \Rightarrow 7x = 22 - 36$$

$$\Rightarrow 7x = -14 \Rightarrow x = -2$$

On substituting $x = -2$ in Eq. (iii), we get

$$y = \frac{-2 + 12}{2} \Rightarrow y = \frac{10}{2} \Rightarrow y = 5$$

On substituting $x = -2$ and $y = 5$ in the equation $y = mx + 3$, we get

$$5 = m \times (-2) + 3$$

$$5 = -2m + 3$$

$$2m = 3 - 5 = -2$$

$$m = -1$$

Q3. Form the pair of linear equations for the following problems and find their solutions by substitution method.

(i) The difference between two numbers is 26 and one number is three times the other. Find the numbers.

(ii) The larger of two supplementary angles exceeds the smaller by 18° . Find them.

(iii) The coach of a cricket team buys 7 bats and 6 balls for ₹ 3800. Later, she buys 3 bats and 5 balls for ₹ 1750. Find the cost of each bat and each ball.

(iv) The taxi charges in a city consist of a fixed charge together with the charge for the distance covered. For a distance of 10 km, the charge paid is ₹ 135 and for a journey of 15 km, the charge paid is ₹ 155. What are the fixed charges and the charges per km? How much does a person have to pay for travelling a distance of 25 km?

(v) A fraction becomes $\frac{9}{11}$, if 2 is added to both the numerator and the denominator. If 3 is added to both the numerator and the denominator, it becomes $\frac{5}{6}$. Find the fraction.

(vi) Five years hence, the age of Jacob will be 11 times that of his son. Five years ago, Jacob's age was seven times that of his son. What are their present ages?

Sol. (i) Let the two numbers be x and y ($x > y$). According to the question,

$$x - y = 26$$

and

$$x = 3y$$

On substituting the value of x from Eq. (ii) in Eq. (i),

$$\text{we get } 3y - y = 26$$

$$\Rightarrow 2y = 26$$

$$\Rightarrow y = 13$$

On substituting $y = 13$ in Eq. (ii), we get

$$x = 3 \times 13$$

$$\Rightarrow x = 39$$

Hence, the two numbers are 39 and 13.

(ii) Let the supplementary angles be x and y ($x > y$).

$$\text{Then, } x + y = 180^\circ \quad \dots(\text{i})$$

Now, according to the question,

$$x - y = 18^\circ \quad \dots(\text{ii})$$

From Eq. (ii), we have

$$y = x - 18^\circ \quad \dots(\text{iii})$$

On substituting the value of y from Eq. (iii) in Eq. (i), we get

$$x + x - 18^\circ = 180^\circ$$

$$\Rightarrow 2x = 198^\circ$$

$$\Rightarrow x = 99^\circ$$

On substituting $x = 99^\circ$ in Eq. (iii), we get

$$y = 99^\circ - 18^\circ$$

$$\Rightarrow y = 81^\circ$$

Hence, the required angles are 99° and 81° .

(iii) Let cost of one bat be ₹ x and cost of one ball be ₹ y .

According to the question,

Cost of 7 bats and 6 balls = ₹ 3800

$$\text{i.e. } 7x + 6y = 3800 \quad \dots(\text{i})$$

Cost of 3 bats and 5 balls = ₹ 1750

$$\text{i.e. } 3x + 5y = 1750 \quad \dots(\text{ii})$$

Now, solve the Eqs. (i) and (ii) similar to Question 1(i).

Then, $x = 500$ and $y = 50$.

Hence, cost of one bat is ₹ 500 and cost of one ball is ₹ 50.

(iv) Let fixed charge be ₹ x and charge per kilometre be ₹ y .

According to the question,

$$x + 10y = 105 \quad \dots(\text{i})$$

$$\text{and } x + 15y = 155 \quad \dots(\text{ii})$$

On solving Eqs. (i) and (ii) similar to Question 1 (i), we get

$$x = 5 \text{ and } y = 10$$

Hence, fixed charge is ₹ 5 and charge per kilometre is ₹ 10.

Now, amount to be paid for travelling 25 km

$$= \text{Fixed charge} + ₹ 10 \times 25$$

$$= ₹ 5 + ₹ 250 = ₹ 255$$

Hence, the amount paid by the person for travelling 25 km is ₹ 255.

(v) Let x/y be the fraction.

Condition I When 2 is added to both numerator and denominator, then

$$\text{New fraction} = \frac{x+2}{y+2}$$

According to the question,

$$\frac{x+2}{y+2} = \frac{9}{11}$$

$$\Rightarrow 11 \times (x+2) = 9 \times (y+2)$$

$$\Rightarrow 11x + 22 = 9y + 18$$

$$\Rightarrow 11x - 9y + 4 = 0 \quad \dots(\text{i})$$

Condition II When 3 is added to both numerator and denominator, then

$$\text{New fraction} = \frac{x+3}{y+3}$$

According to the question,

$$\frac{x+3}{y+3} = \frac{5}{6}$$

$$\Rightarrow 6 \times (x+3) = 5 \times (y+3)$$

$$\Rightarrow 6x + 18 = 5y + 15$$

$$\Rightarrow 6x - 5y + 3 = 0 \quad \dots(\text{ii})$$

Now, from Eq. (ii), we have

$$5y = 6x + 3 \Rightarrow y = \frac{6x+3}{5} \quad \dots(\text{iii})$$

On substituting the value of y from Eq. (iii) in Eq. (i), we get

$$11x - 9 \times \left(\frac{6x+3}{5} \right) + 4 = 0$$

$$\Rightarrow 55x - 9 \times (6x+3) + 20 = 0 \quad [\text{multiplying by 5}]$$

$$\Rightarrow 55x - 54x - 27 + 20 = 0$$

$$\Rightarrow x = 7$$

On putting $x = 7$ in Eq. (iii), we get

$$y = \frac{6 \times 7 + 3}{5} \Rightarrow y = \frac{45}{5} = 9$$

Hence, the required fraction is $\frac{7}{9}$.

(vi) Let x (in years) be the present age of Jacob's son and y (in years) be the present age of Jacob.

After 5 yr, Jacob's son age = $(x + 5)$ yr

and Jacob's age = $(y + 5)$ yr

According to the question,

$$(y+5) = 3(x+5) \Rightarrow y+5 = 3x+15$$

$$\Rightarrow 3x - y + 10 = 0 \quad \dots(\text{i})$$

5 yr ago, Jacob's son age = $(x - 5)$ yr

and Jacob's age = $(y - 5)$ yr

According to the question,

$$(y-5) = 7(x-5)$$

$$\Rightarrow 7x - y - 30 = 0 \quad \dots(\text{ii})$$

From Eq. (i), we have

$$y = 3x + 10 \quad \dots(\text{iii})$$

On substituting $y = 3x + 10$ in Eq. (ii), we get

$$7x - (3x + 10) - 30 = 0 \Rightarrow 4x - 40 = 0 \Rightarrow x = 10$$

On putting $x = 10$ in Eq. (iii), we get

$$y = 3 \times 10 + 10 \Rightarrow y = 40$$

Hence, the present ages of Jacob and his son are respectively, 40 yr and 10 yr.

EXERCISE 3.3

Q1. Solve the following pair of linear equations by the elimination method and the substitution method.

$$(i) x+y=5 \text{ and } 2x-3y=4$$

$$(ii) 3x+4y=10 \text{ and } 2x-2y=2$$

$$(iii) 3x-5y-4=0 \text{ and } 9x=2y+7$$

$$(iv) \frac{x}{2} + \frac{2y}{3} = -1 \text{ and } x - \frac{y}{3} = 3$$

Sol. (i) **By elimination method**

$$\text{Given, } x+y=5 \quad \dots(i)$$

$$\text{and } 2x-3y=4 \quad \dots(ii)$$

On multiplying Eq. (i) by 3 and Eq. (ii) by 1 and then adding them, we get

$$3(x+y) + 1(2x-3y) = 3 \times 5 + 1 \times 4$$

$$\Rightarrow 3x+3y+2x-3y = 15+4$$

$$\Rightarrow 5x = 19 \Rightarrow x = \frac{19}{5}$$

On putting $x = \frac{19}{5}$ in Eq. (i), we get

$$\frac{19}{5} + y = 5 \Rightarrow y = 5 - \frac{19}{5}$$

$$\Rightarrow y = \frac{25-19}{5} \Rightarrow y = \frac{6}{5}$$

$$\text{Hence, } x = \frac{19}{5} \text{ and } y = \frac{6}{5}$$

By substitution method

$$\text{Given, } x+y=5 \quad \dots(i)$$

$$\text{and } 2x-3y=4 \quad \dots(ii)$$

$$\text{From Eq. (i), we have } y = 5-x \quad \dots(iii)$$

On putting the value of y from Eq. (iii) in Eq. (ii), we get

$$2x-3(5-x)=4 \Rightarrow 2x-15+3x=4$$

$$\Rightarrow 5x=19$$

$$\Rightarrow x=\frac{19}{5}$$

On putting $x = \frac{19}{5}$ in Eq. (iii), we get

$$y = 5 - \frac{19}{5} \Rightarrow y = \frac{6}{5}$$

$$\text{Hence, } x = \frac{19}{5} \text{ and } y = \frac{6}{5}$$

(ii) Do same as Part (i). Ans. $x = 2$ and $y = 1$

(iii) Do same as Part (i). Ans. $x = \frac{9}{13}$ and $y = -\frac{5}{13}$

(iv) **By elimination method**

$$\text{Given, } \frac{x}{2} + \frac{2y}{3} = -1 \quad \dots(i)$$

$$\text{and } x - \frac{y}{3} = 3 \quad \dots(ii)$$

On multiplying Eq. (i) by 1 and Eq. (ii) by 2, we get

$$\frac{x}{2} + \frac{2y}{3} = -1 \quad \dots(iii)$$

$$\text{and } 2x - \frac{2y}{3} = 6 \quad \dots(iv)$$

On adding Eqs. (iii) and (iv), we get

$$\frac{x}{2} + 2x = -1 + 6 \Rightarrow \frac{5}{2}x = 5 \Rightarrow x = 2$$

On putting $x = 2$ in Eq. (ii), we get

$$2 - \frac{y}{3} = 3 \Rightarrow \frac{-y}{3} = 1$$

$$\Rightarrow y = -3$$

Hence, $x = 2$ and $y = -3$.

By substitution method

$$\text{Given, } \frac{x}{2} + \frac{2y}{3} = -1$$

$$\text{and } x - \frac{y}{3} = 3$$

From Eq. (ii), we have

$$\frac{y}{3} = x - 3 \Rightarrow y = 3(x - 3)$$

On substituting $y = 3(x - 3)$ in Eq. (i), we get

$$\frac{x}{2} + \frac{2}{3} \times 3(x - 3) = -1$$

$$\Rightarrow \frac{x}{2} + 2(x - 3) = -1$$

$$\Rightarrow x + 4(x - 3) = -2 \quad [\text{multiplying both sides}]$$

$$\Rightarrow 5x - 12 = -2$$

$$\Rightarrow 5x = 10 \Rightarrow x = 2$$

On substituting $x = 2$ in Eq. (iii), we get

$$y = 3(2 - 3) = -3$$

Hence, $x = 2$ and $y = -3$.

Q2. Form the pair of linear equations in the following problems and find their solutions (if they exist) by elimination method.

(i) If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes $\frac{1}{2}$ if we only add 1 to the denominator. What is the fraction?

(ii) Five years ago, Nuri was thrice as old as Sonu. Five years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu?

(iii) The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.

(iv) Meena went to a bank to withdraw ₹ 2000. She asked the cashier to give her ₹ 50 and ₹ 100 notes only. Meena got 25 notes in all. Find how many notes of ₹ 50 and ₹ 100 she received?

(v) A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid ₹ 27 for a book kept for seven days, while Susy paid ₹ 21 for the book kept for five days. Find the fixed charge and the charge for each extra day.

- Sol.** (i) Let the fraction be x/y .
 According to the Condition I,

$$\frac{x+1}{y-1} = 1$$

 $\Rightarrow x+1 = y-1 \Rightarrow x-y = -2 \quad \dots(i)$
- According to the Condition II,

$$\frac{x}{y+1} = \frac{1}{2}$$

 $\Rightarrow 2x = y+1 \Rightarrow 2x-y = 1 \quad \dots(ii)$
- On subtracting Eq. (i) from Eq. (ii), we get
 $(2x-y)-(x-y) = 1+2 \Rightarrow x = 3$
- On putting $x = 3$ in Eq. (i), we get
 $3-y = -2 \Rightarrow y = 5$
- Hence, the required fraction is $\frac{3}{5}$.
- (ii) Let the present age of Nuri be x yr and the present age of Sonu be y yr.
 5 yr ago, Nuri's age = $(x-5)$ yr
 and Sonu's age = $(y-5)$ yr
 According to the question, $x-5 = 3(y-5)$
 $\Rightarrow x-3y = -10 \quad \dots(i)$
- After 10 yr, Nuri's age = $(x+10)$ yr
 and Sonu's age = $(y+10)$ yr
 According to the question,
 $x+10 = 2(y+10) \Rightarrow x-2y = 10 \quad \dots(ii)$
- On subtracting Eq. (i) from Eq. (ii), we get
 $(x-2y)-(x-3y) = 10+10 \Rightarrow -2y+3y = 20 \Rightarrow y = 20$
- On substituting $y = 20$ in Eq. (ii), we get
 $x-2\times 20 = 10 \Rightarrow x = 50$
- Therefore, present age of Nuri is 50 yr and present age of Sonu is 20 yr.
- (iii) Let x be the digit at unit place and y be the digit at tens place of the two-digit number.
 Then, two-digit number = $x+10y$
 According to the question, $x+y = 9 \quad \dots(i)$
 When we reverse the order of the digits, new two-digit number = $y+10x$.
 According to the question,
 $9\times(x+10y) = 2\times(y+10x)$
 $\Rightarrow 9x+90y = 2y+20x$
- $\Rightarrow 88y = 11x$
 $\Rightarrow x = 8y$
 or $x-8y = 0 \quad \dots(ii)$
- On subtracting Eq. (ii) from Eq. (i), we get
 $(x+y)-(x-8y) = 9-0 \Rightarrow 8y+y = 9$
 $\Rightarrow 9y = 9 \Rightarrow y = 1$
- On putting $y = 1$ in Eq. (ii), we get
 $x = 8\times 1 \Rightarrow x = 8$
- Hence, the required two-digit number is
 $x+10y = 8+10\times 1 = 18$
- (iv) Let number of ₹ 50 notes be x and number of ₹ 100 notes be y .
 According to the question,
 $x+y = 25 \quad \dots(i)$
 and $50\times x+100\times y = 2000$
 $\Rightarrow x+2y = 40$ [dividing both sides by 50] ... (ii)
 On subtracting Eq. (i) from Eq. (ii), we get
 $(x+2y)-(x+y) = 40-25 \Rightarrow y = 15$
 On putting $y = 15$ in Eq. (i), we get
 $x+15 = 25 \Rightarrow x = 10$
- Hence, the number of ₹ 50 notes is 10 and number of ₹ 100 notes is 15.
- (v) Let the fixed charges for the first three days be ₹ x and the additional charge per day be ₹ y .
 According to the question,
 Amount paid by Saritha for 7 days = ₹ 27
 i.e. $x+4\times y = 27 \Rightarrow x+4y = 27 \quad \dots(i)$
 [since, ₹ 4y are to be paid for 4 extra days]
 Amount paid by Susy for 5 days = ₹ 21
 i.e. $x+2y = 21 \quad \dots(ii)$
 [since, ₹ 2y are to be paid for 2 extra days]
- On subtracting Eq. (ii) from Eq. (i), we get
 $2y = 27-21$
 $\Rightarrow 2y = 6 \Rightarrow y = 3$
- On putting $y = 3$ in Eq. (i), we get
 $x+4\times 3 = 27$
 $\Rightarrow x = 27-12 \Rightarrow x = 15$
- Hence, the fixed charge for first three days is ₹ 15 and additional charge for each extra day is ₹ 3.

REVIEW EXERCISE

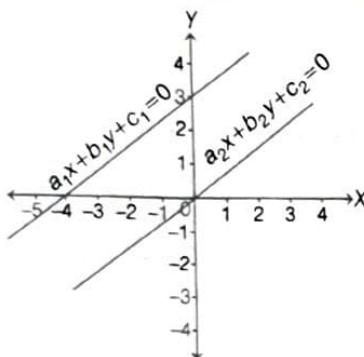
Including Competency Based Questions

Part I

Multiple Choice Questions

1. The given pair of linear equations is non-intersecting. Which of the following statement is true?

CBSE Sample Paper 2023 (Standard)



- (a) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
 (b) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
 (c) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} = \frac{c_1}{c_2}$
 (d) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

2. The lines represented by linear equations $x = a$ and $y = b$ ($a \neq b$) are

CBSE 2024 (Basic)

- (a) intersecting at (a, b) (b) intersecting at (b, a)
 (c) parallel (d) coincident

3. The pair of linear equations $x + 2y + 5 = 0$ and $-3x = 6y - 1$ has

CBSE 2024 (Standard)

- (a) unique solution
 (b) exactly two solutions
 (c) infinitely many solutions
 (d) no solution

4. Which of the following pair of equations are inconsistent?

- (a) $3x - y = 9, x - \frac{y}{3} = 3$
 (b) $4x + 3y = 24, -2x + 3y = 6$
 (c) $5x - y = 10, 10x - 2y = 20$
 (d) $-2x + y = 3, -4x + 2y = 10$

5. The value of k for which the pair of linear equations $3x + 5y = 8$ and $kx + 15y = 24$ has infinitely many solutions, is

CBSE 2022 Term I (Basic)

- (a) 3 (b) 9
 (c) 5 (d) 15

6. The pair of equations $ax + 2y = 9$ and $3x + by = 1$ represent parallel lines, where a, b are integers.

CBSE 2022

- (a) $a = b$ (b) $3a = 2b$ (c) $2a = 3b$ (d) $ab = 1$

7. If the lines represented by equations $3x + 2y = 5$ and $2x + 5y + 1 = 0$ are parallel, then the value of y is

CBSE 2022

- (a) $\frac{2}{5}$ (b) $-\frac{5}{4}$ (c) $\frac{3}{2}$ (d) $\frac{15}{4}$

8. The value of k for which the system of equations $kx + 2y = 5$ and $3x + 4y = 1$ have no solution is

CBSE 2022

- (a) $k = \frac{3}{2}$ (b) $k \neq \frac{3}{2}$ (c) $k \neq \frac{2}{3}$ (d) $k = \frac{2}{3}$

9. The value of a for which the lines $x = 1, y = 2$ and $a^2x + 2y - 20 = 0$ are concurrent, is

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- (a) 1 (b) 8 (c) -4 (d) -2

10. Two lines are given to be parallel. The equation of the first line is $3x - 2y = 5$. The equation of the second line can be

CBSE 2022

- (a) $9x + 8y = 7$ (b) $-12x - 8y = 7$
 (c) $-12x + 8y = 7$ (d) $12x + 8y = 7$

11. The values of x and y satisfying the two equations $32x + 33y = 34$ and $33x + 32y = 31$ respectively are

Competency Based

- (a) -1 and 2 (b) -1 and 4 (c) 1 and -2 (d) -1 and 3

12. The solution of the pair of equations $x + y = 5$ and $ax - by = a^2 - b^2$ is

CBSE 2022

- (a) $x = b, y = a$ (b) $x = -a, y = b$
 (c) $x = a, y = b$ (d) $x = a, y = -b$

13. 3 chairs and 1 table cost ₹ 900; whereas 5 chairs and 3 tables cost ₹ 2100. If the cost of 1 chair is ₹ x and the cost of 1 table is ₹ y , then the situation can be represented algebraically as

Competency Based

- (a) $3x + y = 900, 3x + 5y = 2100$
 (b) $x + 3y = 900, 3x + 5y = 2100$
 (c) $3x + y = 900, 5x + 3y = 2100$
 (d) $x + 3y = 900, 5x + 3y = 2100$

14. The ratio of a two-digit number and the sum of its digits is 7 : 1. How many such two-digit numbers are possible?

Competency Based

- (a) 1 (b) 4 (c) 9 (d) infinite

Assertion-Reason Type Questions

5. Assertion The value of $q = \pm 2$, if $x = 3, y = 1$ is the solution of the line $2x + y - q^2 - 3 = 0$.

Reason The solution of the line will satisfy the equation of the line.

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
- (b) Both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
- (c) Assertion is correct but Reason is incorrect.
- (d) Assertion is incorrect but Reason is correct.

6. Assertion The graphical representation of $x + 2y - 4 = 0$ and $2x + 4y - 12 = 0$ will be a pair of parallel lines.

Reason Let $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ be two linear equations and if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, then the pair of equations represent parallel lines and they have no solution.

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
- (b) Both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
- (c) Assertion is correct but Reason is incorrect.
- (d) Assertion is incorrect but Reason is correct.

7. Assertion The graphical representation of $2x + y = 6$ and $2x - y + 2 = 0$ will be a pair of parallel lines.

Reason When $k = 1$, then linear equations $5x + ky = 4$ and $15x + 3y = 12$ have infinitely many solutions.

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
- (b) Both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
- (c) Assertion is correct but Reason is incorrect.
- (d) Assertion is incorrect but Reason is correct.

8. Assertion The value of k for which the system of equations $kx - y = 2, 6x - 2y = 3$ has a unique solution is 3.

Reason The system of linear equations

$a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

has a unique solution, if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
- (b) Both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
- (c) Assertion is correct but Reason is incorrect.
- (d) Assertion is incorrect but Reason is correct.

19. Assertion Pair of linear equations : $9x + 3y + 12 = 0, 18x + 6y + 24 = 0$ have infinitely many solutions.

Reason Pair of linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ have infinitely many solutions, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
- (b) Both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
- (c) Assertion is correct but Reason is incorrect.
- (d) Assertion is incorrect but Reason is correct.

Case Study Based Questions

20. A book store shopkeeper gives books on rent for reading. He has variety of books in his store related to fiction, stories and quizzes etc. He takes a fixed charge for the first two days and an additional charge for subsequent day. Amruta paid ₹ 22 for a book and kept for 6 days; while Radhika paid ₹ 16 for keeping the book for 4 days.



Assume that the fixed charge be ₹ x and additional charge (per day) be ₹ y . Based on the above information, answer any four of the following questions

CBSE 2022 (Standard)

- (i) The situation of amount paid by Radhika, is algebraically represented by
 - (a) $x - 4y = 16$
 - (b) $x + 4y = 16$
 - (c) $x - 2y = 16$
 - (d) $x + 2y = 16$
- (ii) The situation of amount paid by Amruta, is a algebraically represented by
 - (a) $x - 2y = 11$
 - (b) $x - 2y = 22$
 - (c) $x + 4y = 22$
 - (d) $x - 4y = 11$
- (iii) What are the fixed charges for a book?
 - (a) ₹ 9
 - (b) ₹ 10
 - (c) ₹ 13
 - (d) ₹ 15
- (iv) What are the additional charges for each subsequent day for a book?
 - (a) ₹ 6
 - (b) ₹ 5
 - (c) ₹ 4
 - (d) ₹ 3
- (v) What is the total amount paid by both, if both of them have kept the book for 2 more days?
 - (a) ₹ 35
 - (b) ₹ 52
 - (c) ₹ 50
 - (d) ₹ 58

- 21.** A coaching institute conducts Mathematics classes in two batches I and II and fee for rich and poor children are different. In batch I there are 20 poor and 5 rich children, whereas in batch II, there are 5 poor and 25 rich children. The total monthly collection of fees from batch I is ₹ 9000 and from batch II is ₹ 26000. Assume that each poor child pays ₹ x per month and each rich child pays ₹ y per month. **CBSE 2023 (Standard)**



Based on the above information, answer the following questions.

- (i) Represents the information given above in terms of x and y .
 - (ii) Find the monthly fee paid by a poor child.
Or Find the difference in the monthly fee paid by a poor child and a rich child.
 - (iii) If there are 10 poor and 20 rich children in batch II, what is the total monthly collection of fees from batch II?
- 22.** Lokesh is a production manager in Mumbai, hires a taxi everyday to go to his office. The taxi charges in Mumbai consists of a fixed charges together with the charges or the distance covered. His office is at a distance of 10 km from his home. For a distance of 10 km to his office. Lokesh paid ₹ 105. While coming back home, he took another route. He covered a distance of 15 km and the charges paid by him were ₹ 155.

CBSE 2023 (Basic)

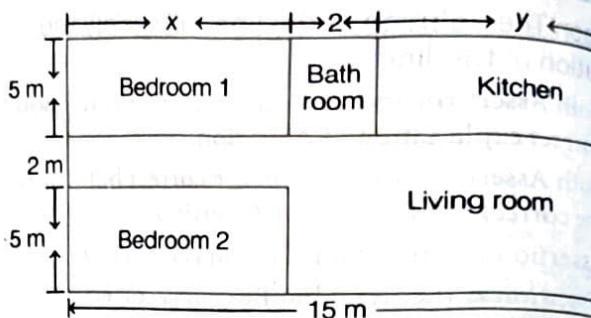


Based on the above information, answer the following questions.

- (i) What are the fixed charges?
- (ii) What are the charges per km?
- (iii) If fixed charges are ₹ 20 and charges per km are ₹ 10, then how much Lokesh have to pay for travelling a distance of 10 km?

Or Find the total amount paid by Lokesh for travelling 10 km from home to office and 25 km from office to home. (Fixed charges and charges per km are same as in parts (i) and (ii)).

- 23.** Amit is planning to buy a house and the layout below. The design and the measurement has made such that areas of two bedrooms and kitchen together is 95 m^2 . **CBSE Qu**



Based on the above information, answer the questions:

- (i) Form the pair of linear equations in two variables from this situation.
- (ii) Find the length of the outer boundary of the layout.
- (iii) Find the area of each bedroom and kitchen in the layout.
- (iv) Find the area of living room in the layout.
- (v) Find the cost of laying tiles in kitchen at ₹ 50 per m^2 .

- 24.** Tickets for a play can be booked online as well as purchased from the theatre. A 10% discount was available on online ticket purchase. Simran likes to watch plays. She purchased the ticket online. The ticket and food cost her ₹ 600. The cost of food was one-third the cost of the ticket.

Based on the above information, answer the questions.

- (i) Represent the relation between the cost of a ticket and the cost of food for Simran algebraically. Also, represent the relation between the cost of a ticket and the cost of food for Simran and food spent algebraically.
- (ii) Simran purchased the ticket for a musical play from the theatre. How much can she spend on food if the ticket cost ₹ 600?
- (iii) A group of friends went to watch a play. Some of them purchased tickets online and some bought them at the theatre. If two more had purchased online tickets the total ticket price would be ₹ 100 less. Is this true for any group greater than 2? Why?
- (iv) In the theatre canteen, two packets of popcorn and a mango drink cost ₹ 330. One packet of popcorn and two mango drinks cost ₹ 300. What is the cost of the packet of popcorn?
 - (a) 100
 - (b) 120
 - (c) 150
 - (d) 200

Part II

Very Short Answer Type Questions

Directions (Q.Nos 1-3) Without drawing them, find out whether the line, representing the following pairs of linear equations intersect at a point or parallel or coincide.

1. $5x - 4y + 8 = 0; 7x + 6y - 9 = 0$

2. $6x - 3y + 10 = 0; 2x - y + 9 = 0$

3. $9x + 3y + 12 = 0; 18x + 6y + 24 = 0$

Directions (Q. Nos. 4-5) Given below is a pair of linear equations.

$4x + y = 8$ and $4x - 2y = 16$

Based on the above information, answer the following questions.

Competency Based Question

4. Is the given pair of equations consistent? Justify your answer.

5. Does $x = 3$ and $y = -3$ satisfies the pair of linear equations? Justify your answer.

6. Find the number of solutions of the pair of equations $x + 2y + 5 = 0, -3x - 6y + 1 = 0$

7. For what values of p does the pair of equations

$4x + py + 8 = 0$ and $2x + 2y + 2 = 0$ has unique solutions?

CBSE Sample Paper 2022 Term I (Basic)

8. What should be the value of λ , for the given equations to have infinitely many solutions?

$5x + \lambda y = 4$ and $15x + 3y = 12$

9. For what value of k , the pair of linear equations

$3x + y = 3$ and $6x + ky = 8$ does not have a solution.

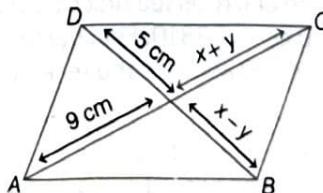
10. If $2x + y = 13$ and $4x - y = 17$, then find the value of $(x - y)$.

CBSE 2024 (Standard)

11. If $x = a, y = b$ is the solution of the equations $x - y = 2$ and $x + y = 4$, then find the values of a and b .

NCERT Exemplar

12. In the given figure, ABCD is parallelogram. Find the values of x and y .



Short Answer Type Questions

13. Draw the graph of lines $x = -2$ and $y = 3$. Write the vertices of the figure formed by these lines, X-axis and Y-axis. Also, find the area of the figure. **NCERT Exemplar**

14. Draw the graph of the pair of linear equations

$x - y + 2 = 0$ and $4x - y - 4 = 0$. Calculate the area of the triangle formed by the lines so drawn and the X-axis.

NCERT Exemplar

15. Solve graphically, the pair of equations $2x + y = 6$ and $2x - y + 2 = 0$. Find the ratio of the areas of the two triangles formed by the lines representing these equations with X-axis and the lines with Y-axis.

16. Equation $2x = 5y + 4$ is given. Write another linear equation, so that the lines represented by the pair are

- (i) intersecting
- (ii) coincident
- (iii) parallel

Competency Based Question

17. Two straight paths are represented by the lines

$7x - 5y = 3$ and $14x - 10y = 5$. Check whether the paths cross each other.

Competency Based Question

18. Determine the values of a and b , for which the following pairs of linear equations has infinitely many solutions

$$3x - (a+1)y = 2b-1 \text{ and } 5x + (1-2a)y = 3b$$

19. For which value(s) of λ , does the pair of linear equations $\lambda x + y = \lambda^2$ and $x + \lambda y = 1$ have

- (i) no solution?
- (ii) infinitely many solutions?
- (iii) a unique solution?

NCERT Exemplar

Directions (Q.Nos 20-25) Solve the following pair of equations by substitution method.

20. $\frac{3}{2}x - y = \frac{1}{4}; x + \frac{1}{2}y = 1$

21. $3x + y = 4; 2(y - 5) = -5x$

22. $1.4x + 3.9y = 6.4; 0.2x - 1.3y = 1.2$

23. $0.1x - 0.2y = 2; x + y = 17$

24. $\frac{x+y}{a} = a+b; \frac{x}{a^2} + \frac{y}{b^2} = 2, a, b \neq 0$

25. $ax + by = 1; bx + ay = \frac{2ab}{a^2 + b^2}$

NCERT Exemplar

Directions (Q.Nos 26-30) Solve the following pair of equations by elimination method.

26. $3x - 4y = 11; 7x - 5y = 4$

27. $2x + 3y - 5 = 0; 3x - 2y - 14 = 0$

28. $3x + 2y = 7; 2x - 5y + 8 = 0$

29. $0.5x + 0.2y = 1.6; 0.9x - 0.3y = 0.9$

30. $\frac{x}{7} + \frac{y}{3} = 5; \frac{x}{7} - \frac{y}{9} = 1$

31. Show that the following system of equations has unique solution.

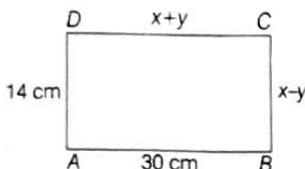
$$3x+5y=12, 5x+3y=4$$

Also, find the solution of the given system of equations.

NCERT Exemplar

32. In figure, ABCD is a rectangle. Find the value of x and y.

CBSE 2018



33. Sum of two numbers is 105 and their difference is 45. Find the numbers.

CBSE 2024 (Standard)

34. The combined ages of two people is 34. If one person is 6 yr younger than the other, then find their ages.

35. Father's age is 3 times the sum of ages of his two children. After 5 yr, his age will be twice the sum of ages of the two children. Find the age of father.

Competency Based Question

36. Find a, if the line $3x+ay=8$ passes through the intersection of lines represented by equations $3x-2y=10$ and $5x+y=8$.

Competency Based Question

37. Given below are two lines such that $l_1 \parallel l_2$.

$$l_1 : 2x+2y+2=0$$

$$l_2 : 3x-3y+3=0$$

- (i) Using comparison of ratios of coefficients, write the equation of a line l_3 in two variables, such that it intersects l_1 at exactly one point.

- (ii) Find the point of intersection of l_2 and l_3 .

Show your steps.

Competency Based Question

38. There are some students in the two examination halls A and B. To make the number of students equal in each hall, 10 students are sent from A to B.

But if 20 students are sent from B to A, the number of students in A becomes double the number of students in B. Find the number of students in the two halls.

NCERT Exemplar

39. If the angles of a triangle are x, y and 40° and the difference between the two angles x and y is 30° , then find the values of x and y .

NCERT Exemplar

40. A company has a locker in which valuable documents are kept. The passcode is a four-digit number of the form $xyyx$. The Chief Executive Officer (CEO) and the Vice President (VP) of the company have each been given one clue. On solving both clues, the passcode that opens the locker can be found.

CEO's clue : When twice the ones digit is subtracted from the tens digit, the result is 1.

VP's clue : Three more than the tens digit is three times the ones digit.

Find the passcode that opens the locker. Show your work.

Competency Based

41. Determine algebraically, the vertices of the triangle formed by the lines

$$(i) 3x - y = 3, 2x - 3y = 2 \text{ and } x + 2y = 8.$$

NCERT

$$(ii) 5x - y = 5, 6x + y = 17 \text{ and } x + 2y = 1$$

42. Two chairs and three tables cost ₹ 5650 whereas three chairs and two tables cost ₹ 7100. Find the cost of a chair and a table separately.

NCERT

43. Ten students of Class X took part in Value Based Education activities. If the number of boys is 4 less than the number of girls. Represent the situation algebraically and graphically.

Long Answer Type Questions

44. Determine graphically, the vertices of the triangle formed by the lines

$$y = x, 3y = x, x + y = 8.$$

NCERT

45. The sum of a two-digit number and number obtained by reversing the order of digits is 99. If the digits of the number differ by 3, then find the numbers.

46. A man, when asked how many hens and buffaloes he has, told that his animals have 120 eyes and 100 legs. How many hens and buffaloes has he?

47. In a flight of 2800 km, an aircraft was slowed down due to bad weather. Its average speed is reduced by 100 km/h and by doing so, the time of flight increased by 30 min. Find the original duration of the flight.

CBSE 2024

48. A railway half ticket cost half the full fare but the reservation charges are the same on a half ticket as on a full ticket. One reserved first class ticket from stations A to B costs ₹ 2530. Also, one reserved first class ticket and one reserved first class half ticket from stations A to B costs ₹ 3810. Find the full first class ticket from stations A to B and also the reservation charge for a ticket.

NCERT

49. Ananya had red, blue and yellow marbles in the ratio 4:5:3. She gave all her red marbles and some blue marbles to Neha. The ratio of the number of blue marbles and yellow marbles left with Ananya is 2:1. If Ananya gave 20 marbles to Neha, then how many of them are red marbles? Show your work.

Competency Based

HINTS & SOLUTIONS

Part I

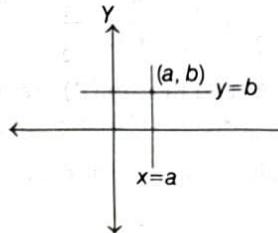
1. (b) Since, the given pair of linear equations is non-intersecting i.e. parallel.

$$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

2. (a) Given equations are $x = a$ and $y = b$ ($a \neq b$).

The graph of $x = a$ is a line parallel to Y -axis and $y = b$ is a line parallel to X -axis.

\therefore The angle between them is 90° i.e. they intersect each other and the solution is (a, b) .



Hence, the point of intersection is (a, b) .

3. (d) Given equations are

$$x + 2y + 5 = 0$$

and $-3x + 6y - 1 \Rightarrow -3x - 6y + 1 = 0$

Here, $a_1 = 1, b_1 = 2, c_1 = 5$

and $a_2 = -3, b_2 = -6, c_2 = 1$

$$\therefore \frac{a_1}{a_2} = \frac{1}{-3}, \frac{b_1}{b_2} = \frac{2}{-6} = \frac{-1}{3}, \frac{c_1}{c_2} = \frac{5}{1}$$

Since, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, the given pair of equations have no solution.

4. (d) On comparing the above equations with standard form of pair of linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we get

- (a) Consistent (b) Consistent
 (c) Consistent (d) Inconsistent

5. (b) \because Given, pair of linear equations has infinitely many solutions.

$$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \frac{3}{k} = \frac{5}{15} = \frac{8}{24} \Rightarrow k = 9$$

6. (d) Given equations are to be parallel, if

$$\frac{a}{3} = \frac{2}{b} \neq \frac{9}{18}$$

$$\text{Taking } \frac{a}{3} = \frac{2}{b} \Rightarrow ab = 6$$

7. (d) Given equations are to be parallel, if

$$\frac{3}{2} = \frac{2m}{5} \neq \frac{2}{-1}$$

$$\text{On taking } \frac{3}{2} = \frac{2m}{5} \Rightarrow 4m = 15 \Rightarrow m = \frac{15}{4}$$

8. (a) \because The given system of equations have no solution.

$$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\Rightarrow \frac{k}{3} = \frac{2}{4} \neq \frac{-5}{-1} \Rightarrow k = \frac{3}{2}$$

9. (c) Given lines are $x = 1, y = 2$ and $a^2x + 2y - 20 = 0$.

Since, $x = 1, y = 2$ and $a^2x + 2y - 20 = 0$ are concurrent, i.e. $x = 1, y = 2$ and $a^2x + 2y - 20 = 0$ having a common solution.

So, $x = 1, y = 2$ is a solution of given equation

$$\therefore a^2 \cdot 1 + 2 \cdot 2 - 20 = 0$$

$$\Rightarrow a^2 - 16 = 0$$

$$\Rightarrow a^2 = 16 \Rightarrow a = -4, 4$$

10. (c) Hint Condition for two lines to be parallel,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

11. (a) Given equations are

$$32x + 33y = 34 \quad \dots(i)$$

$$33x + 32y = 31 \quad \dots(ii)$$

On solving Eqs. (i) and (ii), we get

$$x = -1 \text{ and } y = 2$$

12. (c) Given, equations

$$x + y = a + b \quad \dots(i)$$

$$ax - by = a^2 - b^2 \quad \dots(ii)$$

On multiplying Eq. (i) by b , we get

$$bx + by = ab + b^2 \quad \dots(iii)$$

On adding Eqs. (ii) and (iii), we get

$$ax + bx = a^2 + ab$$

$$\Rightarrow (a + b)x = a(a + b) \Rightarrow x = a$$

$$\text{From Eq. (i), } x + y = x + b \Rightarrow y = b$$

13. (c) Given, cost of 1 chair = ₹ x

$$\therefore \text{Cost of 3 chairs} = ₹ 3x$$

$$\text{and cost of 5 chairs} = ₹ 5x$$

$$\text{Also, given cost of 1 table} = ₹ y$$

$$\therefore \text{Cost of 3 tables} = ₹ 3y$$

So, the given situation can be represented as

$$3x + y = 900 \text{ and } 5x + 3y = 2100$$

14. (b) Let ones digit of the number be x and tens digit of the number be y .

$$\therefore \text{Two-digit number} = 10y + x$$

According to the question,

$$\begin{aligned}\frac{10y+x}{x+y} &= \frac{7}{1} \\ \Rightarrow \quad \frac{x}{y} &= \frac{1}{2}\end{aligned}$$

\therefore Possible two-digit numbers are 21, 42, 63, 84.

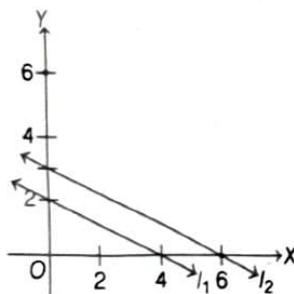
15. (a) As, $x = 3, y = 1$ is the solution of

$$\begin{aligned}2x + y - q^2 - 3 &= 0 \\ \Rightarrow 2 \times 3 + 1 - q^2 - 3 &= 0 \\ \Rightarrow 4 - q^2 &= 0 \\ \Rightarrow q^2 - 4 &= 0 \Rightarrow q = \pm 2\end{aligned}$$

Both Assertion (A) and Reason (R) are correct and Reason (R) is the correct explanation of Assertion (A).

16. (a) $l_1 : x + 2y - 4 = 0$ and $l_2 : 2x + 4y - 12 = 0$

| | | |
|-----|---|---|
| x | 4 | 0 |
| y | 0 | 2 |
| x | 6 | 0 |
| y | 0 | 3 |



So, the lines l_1 and l_2 are parallel to each other.

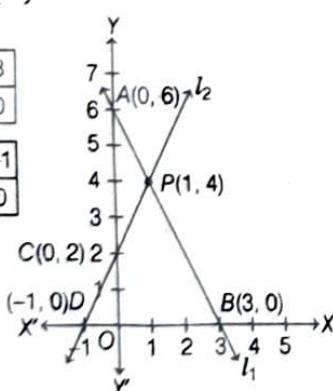
Both Assertion (A) and Reason (R) are correct and Reason (R) is the correct explanation of Assertion (A).

17. (d) **Assertion** We have,

$$l_1 : 2x + y - 6 = 0; l_2 : 2x - y + 2 = 0$$

$$l_1 : y = 6 - 2x; l_2 : y = 2x + 2$$

| | | |
|-----|---|----|
| x | 0 | 3 |
| y | 6 | 0 |
| x | 0 | -1 |
| y | 2 | 0 |



From graph, it is clear that the given lines are not parallel lines.

Reason The linear equations $5x + ky - 4 = 0$ and $15x + 3y - 12 = 0$ have infinitely many solutions.

$$\therefore \frac{5}{15} = \frac{k}{3} = \frac{-4}{-12} \Rightarrow \frac{1}{3} = \frac{k}{3} = \frac{1}{3} \Rightarrow k = 1$$

\therefore Assertion (A) is incorrect but Reason (R) is correct.

18. (d) Given, system of linear equations has a unique solution, if

$$\begin{aligned}\frac{k}{6} &\neq \frac{-1}{-2} \\ \Rightarrow \quad \frac{k}{6} &\neq \frac{1}{2} \Rightarrow k \neq 3\end{aligned}$$

\therefore So, Assertion (A) is incorrect and Reason (R) is correct.

19. (a) From the given equations, we have

$$\begin{aligned}\frac{9}{18} &= \frac{3}{6} = \frac{12}{24} \\ \frac{1}{2} &= \frac{1}{2} = \frac{1}{2} \text{ i.e. } \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}\end{aligned}$$

Both Assertion (A) and Reason (R) are correct and Reason (R) is the correct explanation of Assertion (A).

20. (i) Hint Fixed charge + (Additional charge per unit consumption) \times Number of units

$$\text{Ans. (d)} \quad x + 2y = 16$$

$$\text{(ii) Hint Do same as Q.20. Ans. (c)} \quad x + 4y = 22$$

$$\text{(iii) Hint } x + 2y = 16$$

$$x + 4y = 22$$

Solve Eqs. (i) and (ii) to find the value of x

$$\text{Ans. (b)} \quad ₹ 10$$

$$\text{(iv) Hint Substitute the value of } x = 10 \text{ in Eq. (i)}$$

$$\text{Ans. (d)} \quad ₹ 3$$

$$\text{(v) Hint Required amount}$$

$$= (x + 2y) + (x + 4y) + 2y + 10$$

$$\text{Put } x = 10, y = 3 \text{ in above. Ans. (c)} \quad ₹ 50$$

21. (i) For batch I, $20x + 5y = 9000$

$$\text{For batch II, } 5x + 25y = 26000$$

- (ii) Hint Solve the above Eqs. (i) and (ii). Ans.

Or

On substituting $x = 200$ in Eq. (i), we get
 $y = 1000$

\therefore Difference in the monthly fee paid by a poor child and a rich child = $y - x = ₹ 800$

- (iii) Hint Total monthly collection of fees, if there are 10 poor and 20 rich children = $10x + 20y$

Put the value of $x = 200$ and $y = 1000$.

$$\text{Ans. } ₹ 22000$$

22. Hint Do same as Question 3 (iv) of NCERT Exercise 3.2.

$$\text{Ans. (i)} \quad ₹ 5, \quad \text{(ii)} \quad ₹ 10, \quad \text{(iii)} \quad ₹ 120$$

Or

$$\text{Fixed charges} = ₹ 5$$

$$\text{Charges per km} = ₹ 10$$

$$\text{Total distance travelled} = 10 + 25 = 35 \text{ km}$$

$$\text{Total amount paid} = 5 + 10(35) = ₹ 355$$

23. (i) From the given figure, we see that area of two bedrooms
 $= 2(5x) = 10x \text{ m}^2$

$$\therefore \text{Area of kitchen} = 5 \times y = 5y \text{ m}^2$$

According to the question,

$$\text{Area of the two bedrooms and area of kitchen} = 95 \text{ m}^2$$

$$\therefore 10x + 5y = 95$$

$$\Rightarrow 2x + y = 19 \quad [\text{dividing both sides by 5}] \dots (\text{i})$$

$$\text{Also, length of the home} = 15 \text{ m}$$

$$\therefore x + 2 + y = 15$$

$$\Rightarrow x + y = 13 \dots (\text{ii})$$

Hence, pair of linear equations is

$$2x + y = 19 \text{ and } x + y = 13$$

- (ii) The length of the outer boundary of the layout
 $= 2(l + b) = 2(15 + 12) = 2(27) = 54 \text{ m}$

- (iii) On solving Eqs. (i) and (ii), we get

$$x = 6 \text{ and } y = 7$$

$$\therefore \text{Area of each bedroom} = 5 \times x = 5 \times 6 = 30 \text{ m}^2$$

$$\text{and area of kitchen} = 5 \times y = 5 \times 7 = 35 \text{ m}^2$$

- (iv) \therefore Area of living room

$$= 15 \times (5 + 2) - \text{Area of bedroom 2}$$

$$= 15 \times 7 - 5 \times 6$$

$$= 105 - 30 = 75 \text{ m}^2$$

- (v) Since, area of kitchen $= 5 \times y = 5 \times 7 = 35 \text{ m}^2$
But it is also given, the cost of laying tiles in kitchen
at the rate of ₹ 50 per m^2 .
 \therefore Total cost of laying tiles in the kitchen $= 35 \times 50$
 $= ₹ 1750$

24. (i) Let the cost of a ticket be ₹ x and the cost of food be ₹ y .

Given, the cost of food is one-third of the cost of the ticket.

$$\Rightarrow y = \frac{1}{3}x \Rightarrow x = 3y \dots (\text{i})$$

Also, the ticket and food cost her ₹ 600

$$\Rightarrow x + y = 600 \dots (\text{ii})$$

- (ii) Substituting $x = 3y$ from Eq. (i) in Eq. (ii),

$$3y + y = 600$$

$$\Rightarrow 4y = 600 \Rightarrow y = 150$$

$$\text{From Eq. (i), } x = 3y \Rightarrow x = 3 \times 150 = 450$$

Now, Simran purchased the ticket from the theatre so the ticket cost increases by 10%.

$$\therefore \text{Cost of ticket} = 450 + 450 \times \frac{10}{100} = 495$$

So, she spend ₹ $(600 - 495) = ₹ 105$ on food in ₹ 600.

- (iii) It is true for any group because the difference of ₹ 100 depends on online and offline tickets and not on the group size.

- (iv) (b) Hint Do same as Example 3 of Topic 2.

Part II

1. Hint Do same as Example 8 of Topic 1.

Ans. Intersect at a point.

2. Hint Do same as Example 8 of Topic 1.

Ans. Parallel lines

3. Hint Do same as Example 8 of Topic 1.

Ans. Coincident lines

4. Hint Do same as Example 9 of Topic 2.

Ans. Consistent.

5. Putting $x = 3$ and $y = -3$ in Eqs. (i) and (ii).

LHS of Eq. (i), $4(3) - 3 = 12 - 3 = 9 \neq 8$ (RHS)

LHS of Eq. (ii), $4(3) - 2(-3) = 12 + 6 = 18 \neq 16$ (RHS)

So, the points $x = 3$ and $y = -4$ do not satisfy the given equations.

6. Hint Here, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

So, the pair of equations have no solution. Ans. 0

7. Hint Do same as Example 10 of Topic 1. Ans. $p \neq 4$

8. Hint Here, $\frac{a_1}{a_2} = \frac{5}{15} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{\lambda}{3}$ and $\frac{c_1}{c_2} = \frac{-4}{-12} = \frac{1}{3}$

For infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{1}{3} = \frac{\lambda}{3} = \frac{1}{3} \quad \text{Ans. } \lambda = 1$$

9. Hint Do same as Example 11 of Topic 1. Ans. $k = 2$

10. Hint Add both the given equations.

$$\text{Ans. } x - y = 2$$

11. Hint The values a and b will satisfy given equations.

Thus, we have

$$a - b = 2 \dots (\text{i})$$

$$\text{and} \quad a + b = 4 \dots (\text{ii})$$

Now, solving Eqs. (i) and (ii) to find a and b .

$$\text{Ans. } a = 3, b = 1$$

12. Hint We know that diagonals of a parallelogram bisect each other.

$$\therefore x + y = 9 \dots (\text{i})$$

$$\text{and} \quad x - y = 5 \dots (\text{ii})$$

Now, solving Eqs. (i) and (ii) to find x and y .

$$\text{Ans. } x = 7, y = 2$$

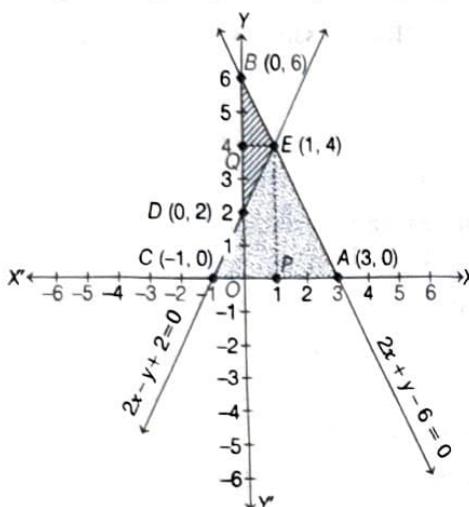
13. Hint Do same as Example 3 of Topic 1.

Ans. $(0, 0), (0, 3), (-2, 3), (-2, 0)$ and area = 6 sq units.

14. Hint Do same as Example 3 of Topic 1.

Ans. 6 sq units

15. Hint We plot the given pair of equations as follows:



$$\text{Then, area of } \triangle ACE = \frac{1}{2} \times AC \times PE$$

$$= \frac{1}{2} \times 4 \times 4 = 8 \text{ sq units}$$

$$\text{and area of } \triangle BDE = \frac{1}{2} \times BD \times QE = \frac{1}{2} \times 4 \times 1 \\ = 2 \text{ sq units Ans. } 4 : 1$$

16. Hint Do same as Question 6 of NCERT Folder Exercise 3.1. **Ans.** (i) $5x - 2y + 8 = 0$

$$\text{(ii)} 4x - 10y - 8 = 0 \text{ (iii)} 2x - 5y + 10 = 0$$

17. Hint Here, $\frac{a_1}{a_2} = \frac{7}{14} = \frac{1}{2}$, $\frac{b_1}{b_2} = \frac{-5}{-10} = \frac{1}{2}$ and $\frac{c_1}{c_2} = \frac{3}{5}$

Thus, for the given equations, we have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

So, we conclude that the two paths are parallel to each other. **Ans.** No

18. Hint Here, $a_1 = 3$, $b_1 = -(a+1)$, $c_1 = -(2b-1)$

$$\text{and } a_2 = 5, b_2 = 1-2a, c_2 = -3b$$

For infinitely many solutions, we must have

$$\frac{3}{5} = \frac{-(a+1)}{1-2a} = \frac{2b-1}{3b}$$

Now, take two terms at a time i.e. I and II, I and III and solve them. **Ans.** $a = 8, b = 5$

19. Hint Here, $a_1 = \lambda$, $b_1 = 1$, $c_1 = -\lambda^2$

$$\text{and } a_2 = 1, b_2 = \lambda, c_2 = -1$$

(i) For no solution, we must have $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$$\Rightarrow \frac{\lambda}{1} = \frac{1}{\lambda} \neq \frac{\lambda^2}{1} \text{ Ans. } \lambda = -1$$

(ii) For infinitely many solutions, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{\lambda}{1} = \frac{1}{\lambda} = \frac{\lambda^2}{1} \text{ Ans. } \lambda = 1$$

(iii) For a unique solution, we must have

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow \frac{\lambda}{1} \neq \frac{1}{\lambda} \Rightarrow \lambda^2 \neq 1 \text{ Ans. } \lambda \neq \pm 1$$

20. Hint Do same as Question 1 of NCERT Folder Exercise 3.2. **Ans.** $x = \frac{9}{14}, y = \frac{5}{7}$

21. Hint Do same as Question 1 of NCERT Folder Exercise 3.2. **Ans.** $x = -2, y = 10$

22. Hint Do same as Question 1 of NCERT Folder Exercise 3.2. **Ans.** $x = 5, y = -\frac{2}{13}$

23. Hint Do same as Question 1 of NCERT Folder Exercise 3.2. **Ans.** $x = 18, y = -1$

24. Hint Do same as Question 3 (i) of Topic 2 Exercise 3.2. **Ans.** $x = a^2, y = b^2$

25. Hint Do same as Question 3 (i) of Topic 2 Exercise 3.2. **Ans.** $x = \frac{a}{a^2 + b^2}, y = \frac{b}{a^2 + b^2}$

26. Hint Do same as Question 1 of NCERT Folder Exercise 3.3. **Ans.** $x = -3, y = -5$

27. Hint Do same as Question 1 of NCERT Folder Exercise 3.3. **Ans.** $x = 4, y = -1$

28. Hint Do same as Question 1 of NCERT Folder Exercise 3.3. **Ans.** $x = 1, y = 2$

29. Hint First multiply the pair of equations on both sides by 10 and then do same as Question 1 of NCERT Folder Exercise 3.3. **Ans.** $x = 2, y = 3$

30. Hint Do same as Question 1 of NCERT Folder Exercise 3.3. **Ans.** $x = 14, y = 9$

31. Hint Compare the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}, \frac{c_1}{c_2}$ and then solve by algebraic method. **Ans.** $x = -1, y = 1$

32. Hint We know that opposite sides of a rectangle are equal.

$$\therefore x + y = 30$$

$$\text{and } x - y = 14$$

On solving Eqs. (i) and (ii), we get

$$\text{Ans. } x = 22 \text{ cm and } y = 8 \text{ cm.}$$

33. Given, sum of two numbers is 105 and their difference is 45.

Let the two numbers be x and y . ($x > y$)

$$\begin{aligned} \therefore x + y &= 105 & \dots(i) \\ \text{and } x - y &= 45 & \dots(ii) \end{aligned}$$

On adding Eqs. (i) and (ii), we get

$$2x = 150 \Rightarrow x = 75$$

On putting the value of x in Eq. (i), we get

$$75 + y = 105 \Rightarrow y = 30$$

Hence, the numbers are 75 and 30.

34. Hint Let the ages of two people be x yr and y yr respectively (assume $x > y$)

According to the question,

$$\begin{aligned} x + y &= 34 & \dots(i) \\ x - y &= 6 & \dots(ii) \end{aligned}$$

Now, solve Eqs. (i) and (ii) to find x and y .

Ans. 20 yr, 14 yr

35. Hint Let father's age be x yr and sum of ages of his two children be y yr.

For condition I $x = 3y$...(i)

For condition II After 5 yr, age of father = $(x+5)$ yr

After 5 yr, sum of ages of two children = $(y+10)$ yr
 $\Rightarrow x+5 = 2(y+10)$...(ii)

Now, solve Eqs. (i) and (ii) to find x . **Ans.** 45 yr

36. Hint Points of intersection of lines represented by

$$3x - 2y = 10 \quad \dots(i)$$

and $5x + y = 8 \quad \dots(ii)$

is the common solution of this system of equations.

On solving this system of equations, we get the point of intersection of the lines (i) and (ii) is $(2, -2)$.

Now, the line $3x + ay = 8$ passes through the point $(2, -2)$.

So, this point will satisfy the equation $3x + ay = 8$.

$$\therefore (3 \times 2) + [(-2) \times a] = 8 \quad \text{Ans. } a = -1$$

37. (i) Let the equation of a line l_3 is $x + 2y - 5 = 0$

it intersect l_1 : $2x + 2y + 2 = 0$ at exactly one point, if

$$\frac{2}{1} \neq \frac{2}{2}$$

$\therefore x + 2y - 5 = 0$ is intersect to given line l_1 .

(ii) We have, l_2 : $3x - 3y + 3 = 0$...(i)

and l_3 : $x + 2y - 5 = 0$...(ii)

Multiply Eq. (i) by 2 and Eq. (ii) by 3 after that add both to find the values of x and y .

Ans. $(1, 2)$.

8. Hint Let the number of students in halls A and B be x and y , respectively.

According to the question,

$$x - 10 = y + 10 \Rightarrow x - y = 20 \quad \dots(i)$$

$$\text{and } (x + 20) = 2(y - 20) \Rightarrow x - 2y = -60 \quad \dots(ii)$$

Solve Eqs. (i) and (ii) to find x and y . **Ans.** 100, 80

39. Hint We have, $x + y + 40^\circ = 180^\circ$

[\because sum of all the angles of a triangle is 180°]

$$\Rightarrow x + y = 140^\circ \quad \dots(i)$$

$$\text{Also, } x - y = 30^\circ \quad \dots(ii)$$

Solve Eqs. (i) and (ii) to find x and y . **Ans.** $85^\circ, 55^\circ$

40. Hint Given, the ones digit is x and the tens digit is y .

According to CEO's clue,

$$y - 2x = 1 \quad \dots(i)$$

According to VP's clue,

$$y + 3 = 3x \quad \dots(ii)$$

Now, solve Eqs. (i) and (ii).

Ans. Required passcode is 4994.

41. (i) Hint Given, equation of lines are

$$3x - y = 3 \quad \dots(i)$$

$$2x - 3y = 2 \quad \dots(ii)$$

and $x + 2y = 8 \quad \dots(iii)$

Let lines (i), (ii) and (iii) represent the sides of ΔABC , say AB , BC and CA , respectively.

Now, solve Eqs. (i) and (ii) and then Eqs. (ii) and (iii) to find coordinates of points A, B, C .

Ans. $A(2, 3), B(1, 0)$ and $C(4, 2)$

(ii) Do same as Part (i). **Ans.** $(1, 0), (3, -1)$ and $(2, 5)$

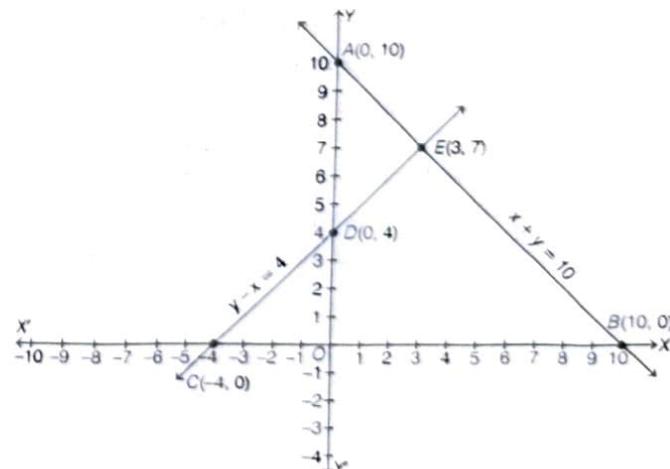
42. Hint Do same as Example 3 of Topic 2.

Ans. Cost of one chair = ₹ 2000 and cost of one table = ₹ 550

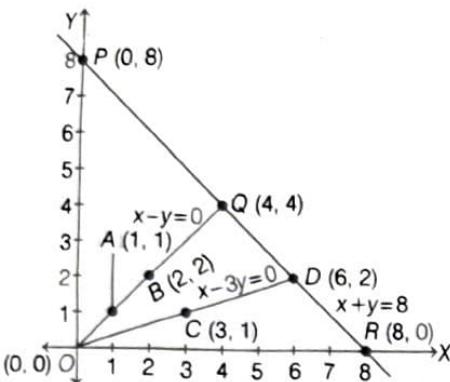
43. Hint Do same as Question 1 (i) of NCERT folder

Exercise 3.1. **Ans.** $x = 3, y = 7$

Graphical representation



- 44. Hint** Plot the lines as follows $(0, 0)$, $(4, 4)$, $(6, 2)$.



- 45. Hint** Let the unit place digit be x and tens place digit be y .

$$\therefore \text{Original number} = 10y + x$$

Number obtained by reversing the order of digits

$$= 10x + y$$

$$\text{Sum of both numbers} = 99$$

[given]

$$(10y + x) + (10x + y) = 99$$

$$\Rightarrow 11x + 11y = 99 \Rightarrow x + y = 9 \quad \dots(i)$$

[dividing both sides by 11]

Also, given that the digits of the numbers differ by 3.

$$\text{If } y > x, \text{ then } y - x = 3 \quad \dots(ii)$$

$$\text{and if } y < x, \text{ then } x - y = 3 \quad \dots(iii)$$

Now, solve Eqs. (i) and (ii) and then Eqs. (i) and (iii) to find x and y .

Ans. 63, 36

- 46. Hint** Let number of hens be x and number of buffaloes be y . Then, according to given information, we get the following equations.

$$2x + 2y = 120 \text{ and } 2x + 4y = 180$$

$$\text{i.e. } x + y = 60 \text{ and } x + 2y = 90$$

Ans. Number of buffaloes = 30

and Number of hens = 30

- 47. Total distance to be travelled by aircraft**

$$= 2800 \text{ km}$$

Let the original average speed be x km/h.

$$\text{Original duration of the flight} = \frac{2800}{x} \text{ h}$$

It is given that the average speed is reduced by 100 km/h.

So, the reduced average speed = $(x - 100)$ km/h

Time taken to cover the distance at the reduced

$$\text{speed} = \frac{2800}{x-100} \text{ h}$$

According to the given condition,

$$\frac{2800}{x-100} - \frac{2800}{x} = \frac{30}{60}$$

$$\Rightarrow \frac{2800x - 2800(x-100)}{x(x-100)} = \frac{1}{2}$$

$$\Rightarrow x(x-100) = 560000$$

$$\Rightarrow x^2 - 100x - 560000 = 0$$

$$\Rightarrow x^2 - 800x + 700x - 560000 = 0$$

$$\Rightarrow x(x-800) + 700(x-800) = 0$$

$$\Rightarrow (x-800)(x+700) = 0$$

$$\Rightarrow x = 800, x \neq -700 \quad [\text{speed cannot be negative}]$$

$$\Rightarrow x = 800$$

$$\therefore \text{Original duration of flight} = \frac{2800}{800} \text{ h} = \frac{7}{2} \text{ h}$$

- 48. Hint** Let the cost of full first class fare be $\text{₹ } x$

Then, the cost of half first class fare be $\text{₹ } \frac{x}{2}$. And the reservation charges be $\text{₹ } y$ per ticket.

Case I The cost of one reserved first class ticket from stations A to B = $\text{₹ } 2530 \Rightarrow x + y = 2530$

Case II The cost of one reserved first class ticket from stations A to B and one reserved first class half ticket from station A to B = $\text{₹ } 3810$

$$\Rightarrow x + y + \frac{x}{2} + y = 3810$$

$$\Rightarrow \frac{3x}{2} + 2y = 3810$$

$$\Rightarrow 3x + 4y = 7620$$

Solve Eqs. (i) and (ii) to find value of x and y .

Ans. Full first class fare from stations A to B = $\text{₹ } 2500$ and the reservation charges for a ticket = $\text{₹ } 250$

- 49. Ananya had $4x$ red, $5x$ blue and $3x$ yellow marbles.**

Let number of blue marbles given to Neha be y .

According to question,

$$4x + y = 20$$

$$\text{and } \frac{5x - y}{3x} = \frac{7}{9}$$

Ans. Number of red marbles = 12

Mind Map

PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

Two linear equations in the same two variables, say x and y , are called pair of linear equations in two variables. The general form of pair of linear equations in two variables x and y is $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, where $a_1, b_1, c_1, a_2, b_2, c_2$ are all real numbers.

Solution of Pair of Linear Equations in Two Variables

Any pair of values of x and y which satisfies both the equations, $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ is called a solution of a given pair of linear equation.
e.g. $x = 2$ and $y = 3$ is the solution of pair of linear equations $2x + 3y - 13 = 0$ and $4x - 5y + 7 = 0$.

Graphical Method

Let us consider a pair of linear equations in two variables, $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$. To find the solution graphically, first draw the graph of both equations on the same graph paper with same scale of representation.

Then,

- (i) If the lines intersect at a point, the pair of equations is consistent and the point of intersection gives the unique solution of the equations.
- (ii) If the lines are parallel, then there is no solution. The pair of linear equations is inconsistent.
- (iii) If the lines coincide, then there are infinitely many solutions. The pair of equations is consistent. Each point on the line is a solution.

Algebraic Methods

Algebraic method is also suitable for non-integer solutions. Different algebraic methods for solving pair of linear equations are given below.

(I) Substitution Method

In this method, value of one variable can be found out in terms of other variable from one of the given equations and this value is substituted in other equation, then we get an equation in one variable, which can be solved easily.
e.g. Solve $x + y = 3$ and $x + 2y = 5$

$$\text{Sol. } x + y = 3 \quad \dots \text{(i)}$$

$$\text{and } x + 2y = 5 \quad \dots \text{(ii)}$$

On substituting $x = 3 - y$ from Eq. (i) in Eq. (ii), we get

$$3 - y + 2y = 5 \Rightarrow y = 5 - 3 = 2$$

$$\therefore x = 1$$

(II) Elimination Method

In this method, one variable out of the two variables is eliminated by making the coefficients of that variable equal in both the equations.

After eliminating that variable, the left equation is an equation in one variable, which can be solved easily.
e.g. Solve $x + 2y = 6$ and $x + y = 5$

$$\text{Sol. } x + 2y = 6 \quad \dots \text{(i)}$$

$$\text{and } x + y = 5$$

On multiplying by 2 in Eq. (ii) on both sides, we get
 $2x + 2y = 10 \quad \dots \text{(ii)}$

On subtracting Eq. (i) from Eq. (ii), we get $x = 4 \quad \dots \text{(iii)}$
Now, on putting $x = 4$, we get $y = 1$

Nature of lines and Consistency

The nature of lines and consistency to linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, is shown in the table given below:

| Compare the ratios | Graphical representation | Algebraic interpretation | Consistency |
|--|--------------------------|------------------------------|----------------------------------|
| $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ | intersecting lines | Exactly one solution(unique) | System is consistent |
| $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ | Coincident lines | Infinity many solution | System is consistent (dependent) |
| $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ | Parallel lines | No solution | System is inconsistent |