Some Applications of Trigonometry

CHAPTER 9

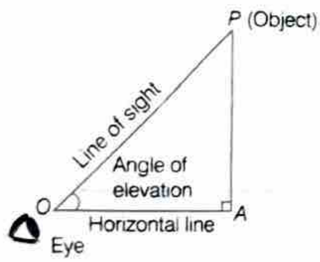
Measuring height and distances is an important application of trigonometry. In this chapter, we shall study the use of trigonometry in measuring the height and distances of towers, buildings and other objects. In solving problems of height and distances of different objects, we need to define few terms as discuss below.

Line of Sight

The line of sight is the line drawn from the eye of an observer to the point in the object viewed by the observer.

Angle of Elevation

The angle of elevation of an object viewed, is the angle formed by the line of sight with the horizontal, when it is above the horizontal level, i.e. the case when we raise our head to look at object.



Let P be the position of the object above the horizontal line OA and O be the eye of the observer. Then, OP is the line of sight and $\angle AOP$ is called **angle of elevation**, because the observer has to elevate (raise) his/her line of sight from the horizontal OA to see the object P.

Some Important Points

(i) A plane level parallel to Earth's surface is called the horizontal plane level and a line drawn parallel to horizontal plane is called a horizontal line.

(ii) If the observer moves towards the perpendicular line (tower/building), then angle of elevation increases and if the observer moves away from the perpendicular line (tower/building), then angle of elevation decreases.

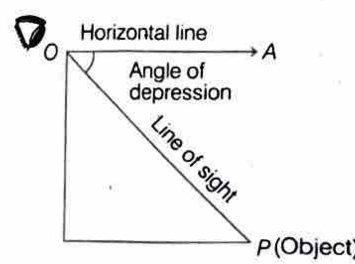
(iii) If the height of tower is doubled and the distance between the observer and foot of the tower is also doubled, then the angle of elevation remains same.

(iv) If the angle of elevation of Sun, above a tower decreases, then the length of shadow of a tower increases and *vice-versa*.

Angle of Depression

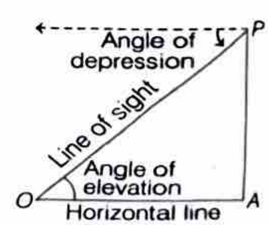
The angle of depression of an object viewed, is the angle formed by the line of sight with the horizontal, when it is below the horizontal level i.e. the case when we lower our head to look at the object.

Let P be the position of the object below the horizontal level OA and O be the eye of the observer. Then, OP is the line of sight and $\angle AOP$ is called an **angle of depression**, because the observer has to depress (lower) his/her line of sight from the horizontal OA to see the object P.



Some Important Points

(i) The angle of elevation of a point P as seen from a point O is always equal to the angle of depression of O as seen from P.



(ii) The angles of elevation and depression are always acute angles.

(iii) In solving problems, observer is represented by a point and object is represented by a line segment or a point.

Example 1. **A tower stands vertically on the ground. From a point on the ground which is $15\sqrt{3}$ m away from the foot of the tower, the angle of elevation of the top of the tower is found to be 30°. Find the height of the tower.**

Sol. Let the height of the tower be H m.

So, $AB = H$ m

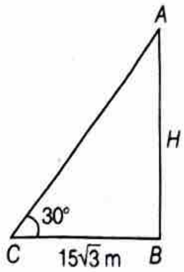
Distance of the point from the foot of the tower $= 15\sqrt{3}$ m

$\Rightarrow \quad CB = 15\sqrt{3}$ m

$\therefore \quad \tan 30° = \dfrac{AB}{BC}$

$\Rightarrow \quad \dfrac{1}{\sqrt{3}} = \dfrac{AB}{15\sqrt{3}}$

$\Rightarrow \quad AB = 15$ m

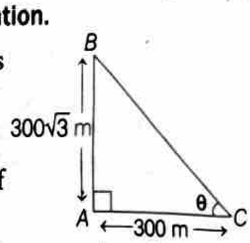


Hence, the height of the tower is 15 m.

Example 2. **If $300\sqrt{3}$ m high tower makes angle of elevation at a point on ground which is 300 m away from its foot, then find the angle of elevation.**

Sol. Let AB be the tower whose height is $300\sqrt{3}$ m, i.e. $AB = 300\sqrt{3}$ m.

Again, let C be the point at a distance of 300 m from the foot of the tower, i.e. $AC = 300$ m.



Here, the angle of elevation is unknown, so let it be θ.

Since, here base and perpendicular are given.

So, in right angled ΔBAC,

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{AB}{AC} = \frac{300\sqrt{3}}{300}$$

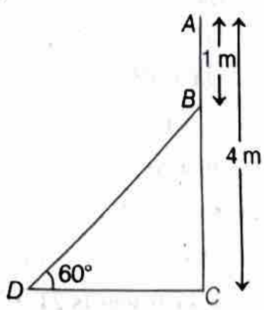
$\Rightarrow \quad \tan\theta = \sqrt{3} = \tan 60°$

$\therefore \quad \theta = 60°$

Hence, the required angle of elevation is 60°.

Example 3. **An electrician has to repair an electric fault on a pole of height 4 m. He needs to reach a point 1 m below the top of the pole to undertake the repair work. What should be the length of the ladder that he should use, when inclined at an angle of 60° to the horizontal, which would enable him to reach the required position? [Take $\sqrt{3} = 1.73$]**

Sol. Let AC be the pole and BD be the ladder.



We have, $AC = 4$ m, $AB = 1$ m and $\angle BDC = 60°$

and $BC = AC - AB = 4 - 1 = 3$ m

In ΔBDC, $\sin 60° = \dfrac{BC}{BD}$

$\Rightarrow \quad \dfrac{\sqrt{3}}{2} = \dfrac{3}{BD} \qquad \left[\because \sin 60° = \dfrac{\sqrt{3}}{2}\right]$

$\Rightarrow \quad BD = \dfrac{3 \times 2}{\sqrt{3}}$

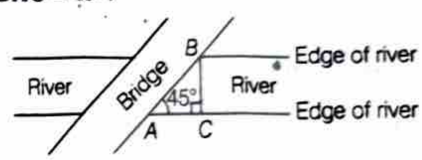
$\Rightarrow \quad BD = 2 \times 1.73$

$\therefore \quad BD = 3.46$ m

Hence, the length of the ladder should be 3.46 m.

Example 4. **A bridge on a river makes an angle of 45° with its edge. If the length along the bridge from one edge to the other is 150 m, then find the width of the river.**

Sol. Let BC be the width of the river and A, B be the ends of river such that $AB = 150$ m = Length of the bridge [given] and $\angle BAC = 45°$.



In right angled ΔACB,

$\sin 45° = \dfrac{P}{H}$

$\Rightarrow \quad \dfrac{1}{\sqrt{2}} = \dfrac{BC}{AB} = \dfrac{BC}{150} \qquad \left[\because \sin 45° = \dfrac{1}{\sqrt{2}}\right]$

$\therefore \quad BC = \dfrac{150}{\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}}$

[by rationalising denominator]

$= \dfrac{150}{2}\sqrt{2} = 75\sqrt{2}$

$= 75 \times 1.414 \qquad [\because \sqrt{2} = 1.414]$

$= 106.05$ m (approx.)

Hence, the width of the river is 106.05 m.

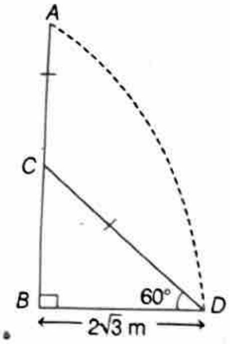
Example 5. **A straight tree is broken due to thunderstorm. The broken part is bent in such a way that the peak of the tree touches the ground at an angle of 60° at a distance of $2\sqrt{3}$ m. Find the whole height of the tree.**

Here, angle of elevation and base of triangle is given and we have to find the whole height of the tree i.e. AB, for this we have to find AC (or CD) and BC.

Sol. Let AB be the tree whose part AC breaks and touches the ground at D.

Then, $BD = 2\sqrt{3}$ m [given]

and $AC = CD$



In right angled ΔCBD, $\cos 60° = \frac{B}{H} = \frac{BD}{CD}$

$\Rightarrow \quad \frac{1}{2} = \frac{2\sqrt{3}}{CD} \quad \left[\because \cos 60° = \frac{1}{2} \text{ and } BD = 2\sqrt{3}\text{ m}\right]$

$\Rightarrow \quad CD = 2 \times 2\sqrt{3} = 4\sqrt{3}$ m

$= 4 \times 1.732 = 6.928$ m $\quad [\because \sqrt{3} = 1.732]$

$\therefore \quad AC = CD = 6.928$ m

Again, in right angled ΔCBD,

$\tan 60° = \frac{P}{B} = \frac{BC}{BD}$

$\Rightarrow \quad \sqrt{3} = \frac{BC}{2\sqrt{3}} \quad [\because \tan 60° = \sqrt{3} \text{ and } BD = 2\sqrt{3}\text{ m}]$

$\Rightarrow \quad BC = \sqrt{3} \times 2\sqrt{3} = 6$ m

Now, $\quad AB = AC + BC$

$= 6.928 + 6$

$= 12.928$ m (approx.)

Hence, the height of the tree is 12.928 m.

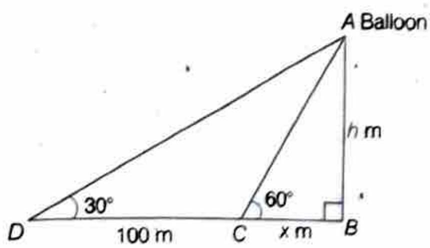
Example 6. One observer estimates the angle of elevation to the basket of a hot air balloon to be 60°, while another observer 100 m away estimates the angle of elevation to be 30°. Find

(i) the height of the basket from the ground.

(ii) the distance of the basket from the first observer's eye.

(iii) the horizontal distance of the second observer from the basket. [take $\sqrt{3} = 1.732$] CBSE 2023 (Standard)

Sol. (i)



In ΔABC,

$\because \tan 60° = \frac{AB}{BC} \Rightarrow \sqrt{3} = \frac{h}{x}$

$\Rightarrow \quad h = \sqrt{3}x$ ...(i)

In ΔABD,

$\tan 30° = \frac{AB}{BD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{100 + x}$

$\Rightarrow \quad 100 + x = \sqrt{3}h$

$\Rightarrow \quad 100 + x = \sqrt{3}(\sqrt{3}x)$ [from Eq. (i)]

$\Rightarrow \quad 100 = 2x \Rightarrow x = 50$ m

Height of basket $= h = \sqrt{3}x$

$= \sqrt{3} \times 50 = 50 \times 1.732 = \mathbf{86.60}$ **m**

(ii) In ΔABC,

$\cos 60° = \frac{BC}{AC}$

$\Rightarrow \quad \frac{1}{2} = \frac{50}{AC}$

$\Rightarrow \quad AC = 100$ m

$\therefore$ Distance of the basket from first observer's eye

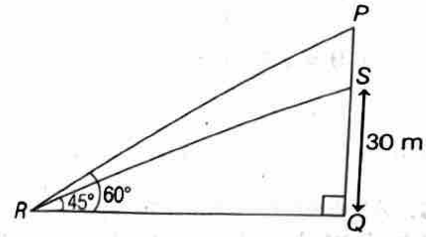
$= 100$ m

(iii) Horizontal distance of the second observer from the basket $= BD = BC + CD$

$= 50 + 100 = 150$ m

Example 7. From a point on the ground, the angle of elevation of the bottom and top of a transmission tower fixed at the top of a 30 m high building are 45° and 60°, respectively. Find the height of the tower. [take $\sqrt{3} = 1.73$]

Sol.



Let QS be the building of height 30 m and PS be the tower.

In right angled ΔSQR,

$\tan 45° = \frac{QS}{QR} \Rightarrow QR = QS = 30$ m

In right angled ΔPQR,

$\tan 60° = \frac{QP}{QR}$

$\Rightarrow \quad \sqrt{3} = \frac{QS + SP}{30} \quad [\because QP = QS + SP]$

$\Rightarrow \quad 30\sqrt{3} = 30 + SP$

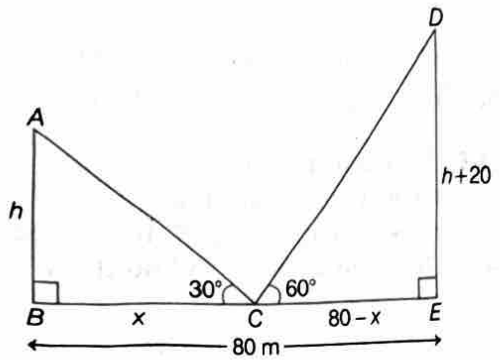
$\Rightarrow \quad SP = 30(\sqrt{3} - 1) = 30(1.73 - 1)$

$= 30 \times 0.73 = 21.9$ m

Hence, the height of the tower is 21.9 m.

Example 8. Two pillars are standing on either side of a 80 m wide road. Height of one pillar is 20 m more than the height of the other pillar. From a point on the road between the pillars, the angle of elevation of the higher pillar is 60°, whereas that of the other pillar is 30°. Find the position of the point between the pillars and the height of each pillar. [take $\sqrt{3}=1.73$] CBSE 2023 (Standard)

Sol.



In ΔABC, $\tan 30° = \dfrac{h}{x}$

$\Rightarrow \quad \dfrac{1}{\sqrt{3}} = \dfrac{h}{x}$

$\Rightarrow \quad x = \sqrt{3}h \quad \ldots\text{(i)}$

In ΔCDE, $\tan 60° = \dfrac{h+20}{80-x}$

$\Rightarrow \quad \sqrt{3} = \dfrac{h+20}{80-x}$

$\Rightarrow \quad \sqrt{3}(80-\sqrt{3}h) = h+20$ [from Eq. (i)]

$\Rightarrow \quad 80\sqrt{3} - 3h = h + 20 \Rightarrow 80\sqrt{3} - 20 = 4h$

$\Rightarrow \quad h = 20\sqrt{3} - 5 \Rightarrow h = 20(1.73) - 5 = 29.6$ m

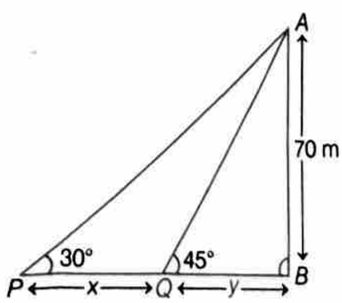
From Eq. (i), $x = 1.73 \times 29.6 = 51.208$ m

Hence, the height of both pillars are 29.6 m and $(29.6 + 20)$ m $= 49.6$ m

Position of point between the pillars are 51.208 m and $(80 - 51.208)$ m $= 28.792$ m.

Example 9. Two persons standing on the same side of a tower in a straight line with it, measure the angles of elevation of the top of the tower as 30° and 45°, respectively. If the height of the tower is 70 m, then find the distance between the two persons.

Sol. Let P and Q be the position of two persons and AB be the height of tower.



Let $PQ = x$ m and $QB = y$ m

In ΔAPB,

$$\tan 30° = \frac{AB}{PB}$$

$\Rightarrow \quad \dfrac{1}{\sqrt{3}} = \dfrac{70}{x+y}$

$\Rightarrow \quad x + y = 70\sqrt{3} \quad \ldots\text{(i)}$

and in ΔAQB,

$$\tan 45° = \frac{AB}{QB}$$

$\Rightarrow \quad 1 = \dfrac{70}{y}$

$\Rightarrow \quad y = 70$ m

Now, put the value of y in Eq. (i), we get

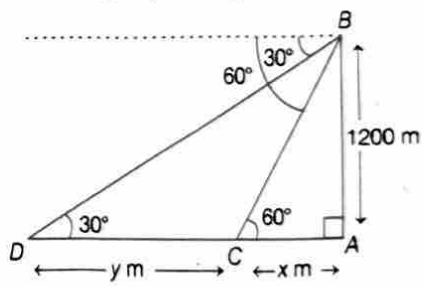
$$x + 70 = 70\sqrt{3}$$

$\Rightarrow \quad x = 70(\sqrt{3}-1)$ m

Hence, the distance between the two persons is $70(\sqrt{3}-1)$ m.

Example 10. An aeroplane is at an altitude of 1200 m find that two ships are sailing towards it in the same direction. The angles of depression of the ships as observed from the aeroplane are 60° and 30°, respectively. Find the distance between both ships.

Sol. Let the aeroplane be at B and two ships be at C and D such that their angles of depression from B are 60° and 30°, respectively. Then, the angles of elevation of B from D and C are 30° and 60°, respectively.



We have, $AB = 1200$ m

Let $AC = x$ m and $CD = y$ m

In right angled ΔBAC, we have

$$\tan 60° = \frac{P}{B} = \frac{AB}{AC}$$

$\Rightarrow \quad \sqrt{3} = \dfrac{1200}{x} \quad [\because \tan 60° = \sqrt{3}]$

$\Rightarrow \quad x = \dfrac{1200}{\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}}$ [by rationalising denominator]

$\Rightarrow \quad x = \dfrac{1200\sqrt{3}}{3}$

$= 400\sqrt{3}$ m $\quad \ldots\text{(i)}$

In right angled ΔBAD, we have

$$\tan 30° = \frac{AB}{AD} = \frac{AB}{DC+CA} \quad [\because AD = DC + CA]$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{1200}{x+y}$$

$$\Rightarrow \quad x + y = 1200\sqrt{3} \quad \left[\because \tan 30° = \frac{1}{\sqrt{3}}\right]$$

$$\Rightarrow \quad y = 1200\sqrt{3} - x \quad \text{...(ii)}$$

On putting the value of x from Eq. (i) in Eq. (ii), we get

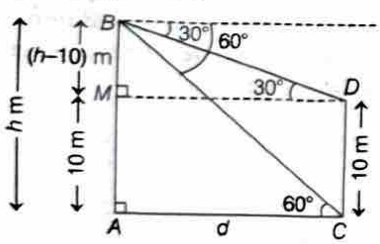
$$y = 1200\sqrt{3} - 400\sqrt{3}$$
$$= 800\sqrt{3}$$
$$= 800 \times 1.732 \quad [\because \sqrt{3} = 1.732]$$
$$= 1385.6 \text{ m}$$

Hence, the distance between both ships is 1385.6 m.

Example 11. **The angle of depression of the top and bottom of 10 m tall building from the top of a multistoried building are 30° and 60°, respectively. Find the height of the multistoried building and the distance between the two buildings.**

Sol. Let the multistoried building AB be h m and the distance between the two buildings be d m and CD be the tall building of 10 m, then

According to the question,



In ΔBMD,

$$\tan 30° = \frac{BM}{MD} = \frac{h-10}{d} = \frac{1}{\sqrt{3}} \quad \left[\because \tan 30° = \frac{1}{\sqrt{3}}\right]$$

$$\Rightarrow \quad \sqrt{3}(h-10) = d \quad \text{...(i)}$$

Now, in ΔBAC,

$$\tan 60° = \frac{AB}{AC} = \frac{h}{d} = \sqrt{3} \quad [\because \tan 60° = \sqrt{3}]$$

$$\Rightarrow \quad h = d\sqrt{3} \quad \text{...(ii)}$$

On putting the value of h from Eq. (ii) in Eq. (i), we get

$$\sqrt{3}(d\sqrt{3} - 10) = d$$

$$\Rightarrow \quad 3d - 10\sqrt{3} = d$$

$$\Rightarrow \quad 3d - d = 10\sqrt{3}$$

$$\Rightarrow \quad d = \frac{10\sqrt{3}}{2} = 5\sqrt{3} \text{ m}$$

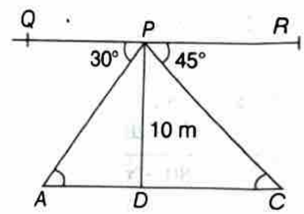
$$\Rightarrow \quad h = 5\sqrt{3} \times \sqrt{3} = 15 \text{ m}$$

So, the height of the multistoried building is 15 m and the distance between the two buildings is $5\sqrt{3}$ m.

Example 12. **From a point of a bridge across a river, the angle of depression of the banks on opposite sides of the river are 30° and 45°, respectively. If the bridge is at a height of 10 m from the banks, then find the width of the river. [use $\sqrt{3} = 1.73$]**

Sol. Let AC represents the width of the river. P is a point on the bridge at a height of 10 m i.e. $PD = 10$ m, then

According to the question,



In ΔAPD,

$$\tan 30° = \frac{PD}{AD}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{10}{AD}$$

$$\Rightarrow \quad AD = 10\sqrt{3} \text{ m} \quad \text{...(i)}$$

In ΔCPD,

$$\tan 45° = \frac{PD}{DC}$$

$$\Rightarrow \quad 1 = \frac{10}{DC}$$

$$\Rightarrow \quad DC = 10 \text{ m} \quad \text{...(ii)}$$

Width of the river

$$AC = AD + DC$$
$$= 10\sqrt{3} + 10 = 10(\sqrt{3} + 1) \text{ m}$$

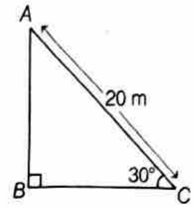
Hence, the width of the river is $10(\sqrt{3} + 1)$ m.

NCERT FOLDER

Complete Solutions of All Exercises of NCERT Textbook

EXERCISE 9.1

Q1. A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with ground level is 30° (see the figure).



Sol. In the given figure, AB is the height of the pole and $AC = 20$ m is the length of rope, which is tied from the top of the pole to the ground at point C.

In right angled ΔABC,

$$\sin 30° = \frac{P}{H} = \frac{AB}{AC} = \frac{AB}{20}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{AB}{20} \qquad \left[\because \sin 30° = \frac{1}{2}\right]$$

$$\therefore \quad AB = \frac{20}{2} = 10 \text{ m}$$

Hence, the height of the pole is 10 m.

Q2. A tree breaks due to storm and the broken part bends, so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point, where the top touches the ground is 8 m. Find the height of the tree.

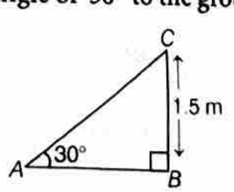
Sol. Do same as Example 5.

Ans. $8\sqrt{3}$ m

Q3. A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she prefers to have a slide whose top is at a height of 1.5 m and is inclined at an angle of 30° to the ground, whereas for elder children, she wants to have a steep slide at a height of 3 m and inclined at an angle of 60° to the ground. What should be the length of the slide in each case?

Sol. ***Case* I Slide for the children below age of 5 yr.**

Let $BC = 1.5$ m be the height of the slide and let slide AC be inclined at an angle of 30° to the ground.



In right angled ΔABC,

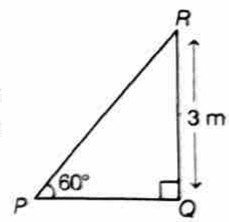
$$\sin 30° = \frac{BC}{AC} \qquad \left[\because \sin\theta = \frac{P}{H}\right]$$

$$\Rightarrow \quad \frac{1}{2} = \frac{1.5}{AC} \qquad \left[\because \sin 30° = \frac{1}{2}\right]$$

$$\Rightarrow \quad AC = 3 \text{ m}$$

***Case* II Slide for the elder children.**

Let $RQ = 3$ m be the height of the slide and let slide PR be inclined at an angle of 60° to the ground.



In right angled ΔPQR,

$$\sin 60° = \frac{RQ}{PR}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{3}{PR} \qquad \left[\because \sin 60° = \frac{\sqrt{3}}{2}\right]$$

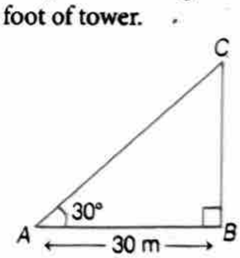
$$\Rightarrow \quad PR = \frac{3 \times 2}{\sqrt{3}} = \frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 2\sqrt{3} \text{ m}$$

[by rationalising denominator]

Hence, the length of slides in each case are 3m and $2\sqrt{3}$ m.

Q4. The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower is 30°. Find the height of the tower.

Sol. Let BC be the height of the tower which is standing on the ground. Let A be a point on the ground which is 30 m away from the foot of tower.



Then, $AB = 30$ m and $\angle BAC = 30°$

In right angled ΔABC,

$$\tan 30° = \frac{BC}{AB} \qquad \left[\because \tan\theta = \frac{P}{B}\right]$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{BC}{30} \qquad \left[\because \tan 30° = \frac{1}{\sqrt{3}}\right]$$

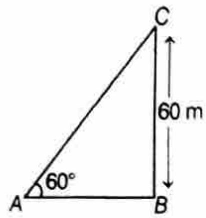
$$\Rightarrow \quad BC = \frac{30}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 10\sqrt{3} \text{ m}$$

[by rationalising denominator]

Hence, the height of the tower is $10\sqrt{3}$ m.

Q5. **A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.**

Sol. Let C be the position of the kite and AC be the length of the string which makes an angle of 60° on the ground. The height of the kite from the ground is $BC = 60$ m.



In right angled ΔABC,

$$\sin 60° = \frac{P}{H} = \frac{BC}{AC}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{60}{AC} \qquad \left[\because \sin 60° = \frac{\sqrt{3}}{2}\right]$$

$$\therefore \quad AC = \frac{60 \times 2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

[by rationalising denominator]

$$= \frac{120\sqrt{3}}{3} = 40\sqrt{3} \text{ m}$$

Hence, the length of the string is $40\sqrt{3}$ m.

Q6. **A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.**

Sol. Let $AB = 30$ m be the height of the building and $DC = 1.5$ m be the height of the boy. The point D be the boy's eyes.

Draw the line $DF \parallel CA$.

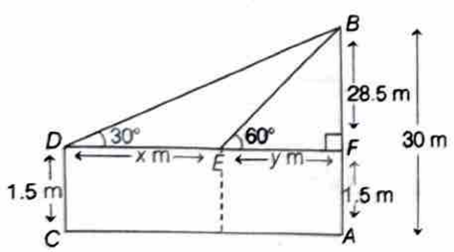
Then, $CD = AF = 1.5$ m

The angle of elevation is $\angle BDF = 30°$.

Let he walked $DE = x$ m towards the building. Then, the angle of elevation is $\angle BEF = 60°$.

Let $EF = y$ m

Now, $BF = AB - AF = 30 - 1.5 = 28.5$ m.



In right angled ΔBFD,

$$\tan 30° = \frac{P}{B} = \frac{BF}{DF} = \frac{BF}{DE + EF} \qquad [\because DF = DE + EF]$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{28.5}{x+y} \qquad \left[\because \tan 30° = \frac{1}{\sqrt{3}}\right]$$

$$\Rightarrow \quad x + y = 28.5\sqrt{3} \text{ m} \qquad \ldots\text{(i)}$$

and in right angled ΔBFE,

$$\tan 60° = \frac{BF}{EF}$$

$$\Rightarrow \quad \sqrt{3} = \frac{28.5}{y} \qquad [\because \tan 60° = \sqrt{3}]$$

$$\Rightarrow \quad y = \frac{28.5}{\sqrt{3}} \text{ m}$$

On putting $y = \frac{28.5}{\sqrt{3}}$ in Eq. (i), we get

$$x + \frac{28.5}{\sqrt{3}} = 28.5\sqrt{3}$$

$$\Rightarrow \quad x = 28.5\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) = 28.5\left(\frac{3-1}{\sqrt{3}}\right)$$

$$= \frac{28.5 \times 2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

[by rationalising denominator]

$$= \frac{57\sqrt{3}}{3} = 19\sqrt{3} \text{ m}$$

Hence, the distance he walked towards the building is $19\sqrt{3}$ m.

Q7. **From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.**

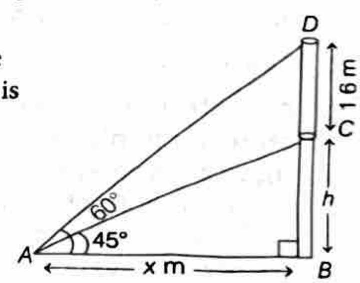
CBSE 2024, 20 (Standard)

Sol. Do same as Example 7. **Ans.** 14.64 m

Q8. **A statue, 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point, the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal.**

Sol. Let $BC = h$ m be the height of the pedestal and $CD = 1.6$m be the length of the statue, which is standing on the pedestal.

Again, let point A be a fixed point on the ground such that the angles of elevation of the top of the statue and bottom of the statue (i.e. top of the pedestal) are $\angle DAB = 60°$ and $\angle CAB = 45°$.



Also, let $AB = x$ m.

In right angled ΔABD,

$$\tan 60° = \frac{P}{B} = \frac{BD}{AB}$$

$$\Rightarrow \quad \sqrt{3} = \frac{BC + CD}{x} \qquad [\because \tan 60° = \sqrt{3}]$$

$$\Rightarrow \quad \sqrt{3} = \frac{h + 1.6}{x} \Rightarrow h = \sqrt{3}x - 1.6 \qquad \ldots\text{(i)}$$

In right angled ΔCBA,

$$\tan 45° = \frac{BC}{AB}$$

$$\Rightarrow \quad 1 = \frac{h}{x} \qquad [\because \tan 45° = 1]$$

$$\Rightarrow \quad x = h$$

On putting $x = h$ in Eq. (i), we get

$$h = \sqrt{3}h - 1.6 \Rightarrow h(\sqrt{3}-1) = 1.6$$

$$\Rightarrow \quad h = \frac{1.6}{(\sqrt{3}-1)} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} \qquad \text{[rationalising]}$$

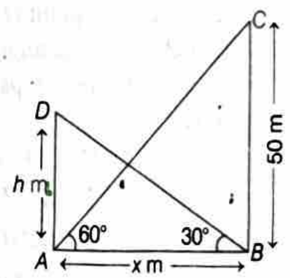
$$= \frac{1.6(\sqrt{3}+1)}{(\sqrt{3})^2 - (1)^2} \qquad [\because (a+b)(a-b) = a^2 - b^2]$$

$$= \frac{1.6}{2}(\sqrt{3}+1) = 0.8(\sqrt{3}+1)\text{ m}$$

Hence, the height of the pedestal is $0.8(\sqrt{3}+1)$ m.

Q9. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building.

Sol. Let $BC = 50$ m be the height of the tower and $AD = h$ m be the height of the building. Angle of elevation of the top of the building from the foot of the tower is $\angle DBA = 30°$ and angle of elevation of the top of the tower from the foot of the building $\angle CAB = 60°$.



Also, let $AB = x$ m be the distance between foots of the tower and the building.

In right angled ΔBAD,

$$\tan 30° = \frac{P}{B} = \frac{AD}{AB}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x} \qquad \left[\because \tan 30° = \frac{1}{\sqrt{3}}\right]$$

$$\Rightarrow \quad h = \frac{x}{\sqrt{3}} \qquad \ldots\text{(i)}$$

and in right angled ΔCBA,

$$\tan 60° = \frac{BC}{AB}$$

$$\Rightarrow \quad \sqrt{3} = \frac{50}{x} \qquad [\because \tan 60° = \sqrt{3}]$$

$$\Rightarrow \quad x = \frac{50}{\sqrt{3}}\text{ m}$$

On putting $x = \frac{50}{\sqrt{3}}$ in Eq. (i), we get

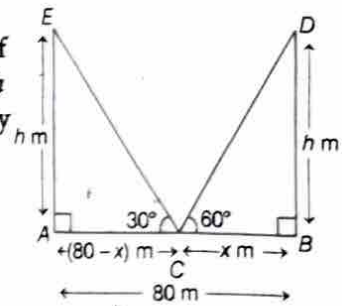
$$h = \frac{50}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{50}{3} = 16\frac{2}{3}\text{ m}$$

Hence, the height of the building is $16\frac{2}{3}$ m.

Q10. Two poles of equal heights are standing opposite to each other on either side of the road, which is 80m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30°, respectively. Find the height of the poles and the distances of the point from the poles.

CBSE 2015, 2019 CBSE Sample paper 2020 (Standard)

Sol. Let $AB = 80$ m be the width of the road. On both sides of the road, poles $AE = BD = h$ m are standing. Let C be any point on AB such that from point C, angles of elevation are $\angle BCD = 60°$ and $\angle ACE = 30°$.



Let $BC = x$ m.

Then, $AC = AB - BC = (80 - x)$ m

In right angled ΔCAE, $\tan 30° = \frac{P}{B} = \frac{AE}{AC}$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{80-x} \qquad \left[\because \tan 30° = \frac{1}{\sqrt{3}}\right]$$

$$\Rightarrow \quad 80 - x = h\sqrt{3} \Rightarrow h\sqrt{3} + x = 80 \qquad \ldots\text{(i)}$$

and in right angled ΔCBD,

$$\tan 60° = \frac{BD}{BC} \Rightarrow \sqrt{3} = \frac{h}{x} \qquad [\because \tan 60° = \sqrt{3}]$$

$$\Rightarrow \quad h = \sqrt{3}x \qquad \ldots\text{(ii)}$$

On putting $h = \sqrt{3}x$ in Eq. (i), we get

$$\sqrt{3}x(\sqrt{3}) + x = 80 \Rightarrow 3x + x = 80$$

$$\Rightarrow \quad 4x = 80 \Rightarrow x = 20\text{ m}$$

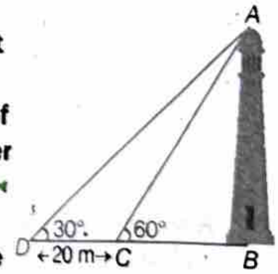
On putting $x = 20$ m in Eq. (ii), we get

$$h = 20\sqrt{3}\text{ m}$$

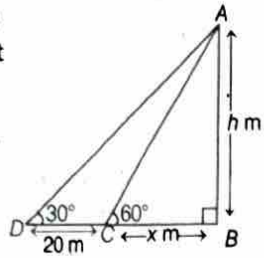
Now, $AC = 80 - x = 80 - 20 = 60$ m

Hence, the height of the poles is $20\sqrt{3}$ m and the distances of the point C from the poles are 60 m and 20 m.

Q11. A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60°. From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30° (see the figure). Find the height of the tower and the width of the canal.



Sol. Let $BC = x$ m be the width of the canal and $AB = h$ m be the height of the tower.



Given, $\angle ACB = 60°$ and $\angle ADB = 30°$

In right angled ΔABD,

$$\tan 30° = \frac{P}{B} = \frac{AB}{DB}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{DC + CB}$$

$$\left[\because \tan 30° = \frac{1}{\sqrt{3}} \text{ and } DB = DC + CB\right]$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{20 + x} \Rightarrow 20 + x = \sqrt{3}h \quad \text{...(i)}$$

and in right angled ΔABC,

$$\tan 60° = \frac{AB}{BC} \Rightarrow \sqrt{3} = \frac{h}{x} \quad [\because \tan 60° = \sqrt{3}]$$

$$\Rightarrow \quad h = \sqrt{3}x \quad \text{...(ii)}$$

On putting $h = \sqrt{3}x$ in Eq. (i), we get

$$20 + x = \sqrt{3}(\sqrt{3}x)$$

$$\Rightarrow \quad 20 + x = 3x \Rightarrow 2x = 20 \Rightarrow x = 10 \text{ m}$$

On putting $x = 10$ m in Eq. (ii), we get

$$h = \sqrt{3}(10) \Rightarrow h = 10\sqrt{3} \text{ m}$$

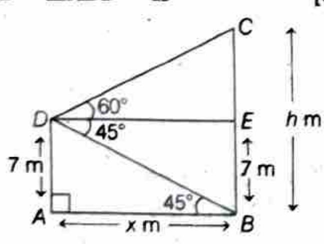
Hence, the height of the tower is $10\sqrt{3}$ m and width of the canal is 10 m.

Q 12. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower. CBSE 2020 (Standard)

Sol. Let $AD = 7$ m be the height of the building and $BC = h$ m be the height of the cable tower. From the top of the building D, the angles of elevation and depression are $\angle CDE = 60°$ and $\angle EDB = 45°$.

From the point D, draw a line $DE \parallel AB$.

Then, $\angle EDB = \angle ABD = 45°$ [alternate angles]



Also, let $AB = DE = x$ m be the distance between building and tower.

In right angled ΔBAD,

$$\tan 45° = \frac{P}{B} = \frac{AD}{AB}$$

$$\Rightarrow \quad 1 = \frac{7}{x} \quad [\because \tan 45° = 1]$$

$$\Rightarrow \quad x = 7 \text{ m} \quad \text{...(i)}$$

and in right angled ΔCED,

$$\tan 60° = \frac{CE}{DE} = \frac{CB - BE}{AB} \quad [\because CE = CB - BE]$$

$$\Rightarrow \quad \sqrt{3} = \frac{h - 7}{x} \quad [\because \tan 60° = \sqrt{3}]$$

$$\Rightarrow \quad h - 7 = x\sqrt{3}$$

$$\Rightarrow \quad h = x\sqrt{3} + 7$$

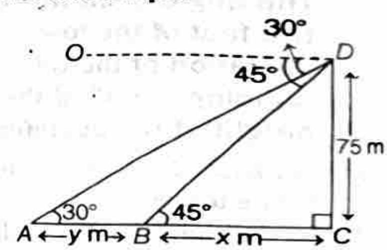
$$\Rightarrow \quad h = 7\sqrt{3} + 7 \quad \text{[from Eq. (i)]}$$

$$\Rightarrow \quad h = 7(\sqrt{3} + 1) \text{ m}$$

Hence, the height of the tower is $7(\sqrt{3} + 1)$ m.

Q 13. As observed from the top of a 75 m high lighthouse from the sea level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, then find the distance between the two ships.

Sol. Let $CD = 75$ m be the height of the lighthouse from the sea level. Let A and B be the positions of two ships on the sea level. From point D of the lighthouse, the angles of depression of two ships A and B are



$$\angle ODA = 30° \text{ and } \angle ODB = 45°$$

$$\therefore \quad \angle CAD = \angle ODA = 30° \quad \text{[alternate angles]}$$

$$\text{and} \quad \angle CBD = \angle ODB = 45° \quad \text{[alternate angles]}$$

Let distance between two ships be $AB = y$ m and $BC = x$ m.

In right angled ΔACD,

$$\tan 30° = \frac{P}{B} = \frac{CD}{AC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{75}{AB + BC}$$

$$\left[\because \tan 30° = \frac{1}{\sqrt{3}} \text{ and } AC = AB + BC\right]$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{75}{y + x} \Rightarrow x + y = 75\sqrt{3} \quad \text{...(i)}$$

and in right angled ΔDCB,

$$\tan 45° = \frac{CD}{BC}$$

$$\Rightarrow \quad 1 = \frac{75}{x} \quad [\because \tan 45° = 1]$$

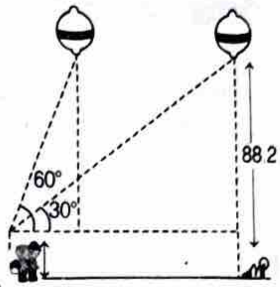
$$\Rightarrow \quad x = 75 \text{ m}$$

On putting $x = 75$ m in Eq. (i), we get

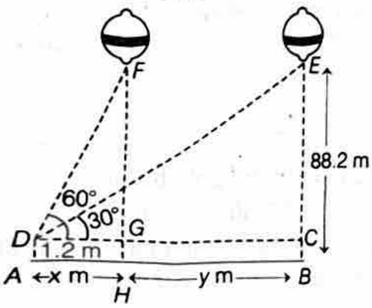
$$75 + y = 75\sqrt{3} \Rightarrow y = 75(\sqrt{3} - 1) \text{ m}$$

Hence, the distance between two ships is $75(\sqrt{3} - 1)$ m.

Q 14. A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After sometime, the angle of elevation reduces to 30° (see the figure). Find the distance travelled by the balloon during the interval. CBSE 2009, CBSE Sample Paper 2020 (Basic)



Sol. Let $AD = 1.2$ m be the height of girl standing on the horizontal line AB and let $FH = EB = 88.2$ m be the height of balloon from the line AB. At the eyes of the girl, the angles of elevation are $\angle FDC = 60°$ and $\angle EDC = 30°$.



Now, $FG = EC = 88.2 - 1.2 = 87$ m

Let the distance travelled by the balloon, $HB = y$ m and $AH = x$ m.

$\therefore \; DG = AH = x$ m and $GC = HB = y$ m

In right angled ΔFGD,

$$\tan 60° = \frac{P}{B} = \frac{FG}{DG} \Rightarrow \sqrt{3} = \frac{87}{x} \quad [\because \tan 60° = \sqrt{3}]$$

$$\Rightarrow \quad x = \frac{87}{\sqrt{3}} \quad \ldots(i)$$

In right angled ΔECD, $\tan 30° = \dfrac{EC}{DC} \Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{87}{DG + GC}$

$$\left[\because \tan 30° = \frac{1}{\sqrt{3}} \text{ and } DC = DG + GC\right]$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{87}{x+y} \Rightarrow x + y = 87\sqrt{3} \quad \ldots(ii)$$

On putting $x = \dfrac{87}{\sqrt{3}}$ from Eq. (i) in Eq. (ii), we get

$$\frac{87}{\sqrt{3}} + y = 87\sqrt{3} \Rightarrow y = 87\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)$$

$$\Rightarrow \quad y = \frac{87\,(3-1)}{\sqrt{3}} = \frac{87 \times 2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{87 \times 2\sqrt{3}}{3}$$

[rationalising]

$$= 29 \times 2\sqrt{3} = 58\sqrt{3} \text{ m}$$

Hence, the distance travelled by the balloon during the interval is $58\sqrt{3}$ m.

Q 15. A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point.

Sol. Let $CD = h$ m be the height of the tower. At point D of the tower, a man is standing and observes the car at an angle of depression of 30°. After six seconds, the angle of depression of the car is 60°.

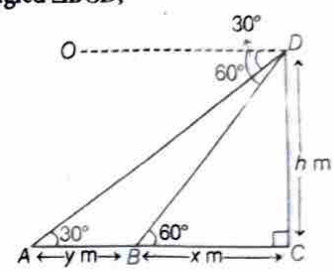
i.e. $\quad \angle ODA = 30°$ and $\angle ODB = 60°$

$\Rightarrow \quad \angle DAC = \angle ODA = 30° \quad$ [alternate angles]

and $\quad \angle DBC = \angle ODB = 60° \quad$ [alternate angles]

Let $AB = y$ m and $BC = x$ m

In right angled ΔBCD,



$$\tan 60° = \frac{P}{B} = \frac{CD}{BC}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{x} \quad [\because \tan 60° = \sqrt{3}]$$

$$\Rightarrow \quad h = \sqrt{3}x \quad \ldots(i)$$

In right angled ΔACD,

$$\tan 30° = \frac{CD}{AC} = \frac{CD}{AB + BC} \quad [\because AC = AB + BC]$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x+y} \quad \left[\because \tan 30° = \frac{1}{\sqrt{3}}\right]$$

$$\Rightarrow \quad x + y = h\sqrt{3}$$

$$\Rightarrow \quad x + y = \sqrt{3}x\,(\sqrt{3}) \quad \text{[from Eq. (i)]}$$

$$\Rightarrow \quad x + y = 3x \quad \ldots(ii)$$

It is given that a car moves from point A to B in six seconds. Let its speed be k km/s.

$$\therefore \quad \text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

$$\Rightarrow \quad 6 = \frac{y}{k} \Rightarrow y = 6k$$

On putting $y = 6k$ in Eq. (ii), we get

$$x + 6k = 3x \Rightarrow 6k = 2x \Rightarrow x = 3k$$

$$\therefore \quad \text{Time} = \frac{\text{Distance}}{\text{Speed}} = \frac{x}{k} = \frac{3k}{k} = 3\text{s}$$

Hence, the car moves from point B to point C in 3 s.

REVIEW EXERCISE

Including Competency Based Questions

Part I

Multiple Choice Questions

1. If the height of the tower is equal to the length of its shadow, then the angle of elevation of the Sun is **CBSE 2023 (Basic)**

(a) 30° (b) 45° (c) 60° (d) 90°

2. A pole 6 m high cast a shadow $2\sqrt{3}$m long on the ground, then the Sun's elevation is

(a) 60° (b) 45° (c) 30° (d) 90°

3. A circus artist is climbing a 30 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground, then the height of pole, if the angle made by the rope with the ground level is 30°, is **Competency Based Question**

(a) 5 m (b) 10 m (c) 15 m (d) 20 m

4. The length of a string between a kite and a point on the ground is 85 m. If the string makes an angle θ with level ground such that $\tan\theta = \frac{15}{8}$, then the height of kite is **Competency Based Question**

(a) 75 m (b) 78.05 m (c) 226 m (d) None of these

5. The top of two poles of height 20 m and 14 m are connected by a wire. If the wire makes an angle of 30° with the horizontal, then the length of the wire is

(a) 12 m (b) 10 m (c) 8 m (d) 6 m

6. An observer 1.5 m tall is 20.5 away from a tower 22 m high, then the angle of elevation of the top of the tower from the eye of the observer is **NCERT Exemplar**

(a) 30° (b) 45° (c) 60° (d) 90°

7. A tree 6 m tall cast a 4m long shadow. At the same time, a flag pole cast a shadow 50 m long. How long is the flag pole? **Competency Based Question**

(a) 75 m (b) 100 m (c) 150 m (d) 50 m

8. A kite is flying at a height of 80 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with ground is 60°, then the length of the string is **Competency Based Question**

(a) 62.37 m (b) 92.37 m (c) 52.57 m (d) 72.57 m

9. From a point on the ground which is 30 m away from the foot of a vertical tower, the angle of elevation of the top of the tower is found to be 60°. The height (in metres) of the tower is **CBSE 2024 (Standard)**

(a) $10\sqrt{3}$ (b) $30\sqrt{3}$ (c) 60 (d) 30

10. From the top of a 8 m high building the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°, then the height of the tower is

(a) 14.124 m (b) 17.124 m (c) 21.856 m (d) 15.124 m

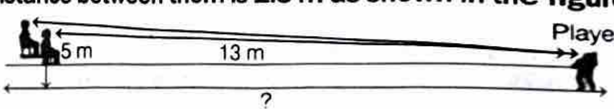
11. Seaweed is found under 80 m deep seafloor. To reach it, a diver makes a 45° dive from a boat. What is the distance travelled by the diver to reach the seafloor? **Competency Based Question**

(a) 80 m (b) 80.2 m (c) $80\sqrt{2}$ m (d) $80\sqrt{3}$ m

12. A 9 m high street-light pole is broken during a storm. The top end of the pole touches the ground at 30°. At what height did the pole break?

(a) 3 m (b) 3.75 m (c) 4.5 m (d) 9 m

13. Two persons are watching a game in a stadium. The distance between them is 1.5 m as shown in the figure.



Based on the above information, what is the distance of the second person from the player?

(a) 10 m (b) 12 m (c) 13.5 m (d) 14.5 m

Case Study Based Questions

14. A group of students of Class X visited India Gate on an education trip. The teacher and students had interest in history as well. The teacher narrated that India Gate, official name Delhi Memorial, originally called All India War Memorial, monumental sandstone arch in New Delhi, dedicated to the troops of British India who died in wars fought between 1914 and 1919. The teacher also said that India Gate, which is located at the eastern end of the Rajpath (formerly called the Kingsway), is about 138 ft (42 m) in height.

(i) What is the angle of elevation if they are standing at a distance of 42 m away from the monument?

(a) 30° (b) 45° (c) 60° (d) 0°

(ii) They want to see the tower at an angle of 60°. So, they want to know the distance where they should stand and hence find the distance.

(a) 24.25 m (b) 20.12 m (c) 42 m (d) 24.64 m

(iii) If the altitude of the Sun is at 60°, then the height of the vertical tower that will cast a shadow of length 20 m is

(a) $20\sqrt{3}$ m (b) $\frac{20}{\sqrt{3}}$ m (c) $\frac{15}{\sqrt{3}}$ m (d) $15\sqrt{3}$ m

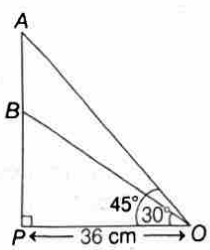
(iv) The ratio of the length of a rod and its shadow is 1 : 1. The angle of elevation of the Sun is

(a) 30° (b) 45° (c) 60° (d) 90°

(v) The angle formed by the line of sight with the horizontal when the object viewed is below the horizontal level is

(a) corresponding angle (b) angle of elevation
(c) angle of depression (d) complete angle

15. Radio towers are used for transmitting a range of communication services including radio and television. The tower will either act as an antenna itself or support one or more antennas on its structure. On a similar concept, a radio station tower was built in two Sections *A* and *B*. Tower is supported by wires from a point *O*.



Distance between the base of the tower and point *O* is 36 cm. From point *O*, the angle of elevation of the top of the Section *B* is 30° and the angle of elevation of the top of Section *A* is 45°.

Based on the above information, answer the following questions :

(i) Find the length of the wire from the point *O* to the top of Section *B*.

(ii) Find the distance *AB*.

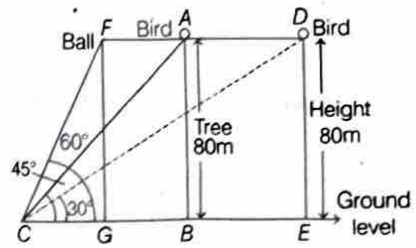
(iii) Find the height of the Section *A* from the base of the tower.

Or Find the area of ΔOPB. **CBSE 2023 (Standard)**

16. One evening, Kaushik was in a park. Children were playing cricket. Birds were singing on a nearby tree of height 80m. He observed a bird on the tree at an angle of elevation of 45°.

When a sixer was hit, a ball flew through the tree frightening the bird to fly away. In 2 s, he observed the bird flying at the same height at an angle of elevation of 30° and the ball flying towards him at the same height at an angle of elevation of 60°.

CBSE Sample paper 2023 (Standard)



(i) At what distance from the foot of the tree was he observing the bird sitting on the tree?

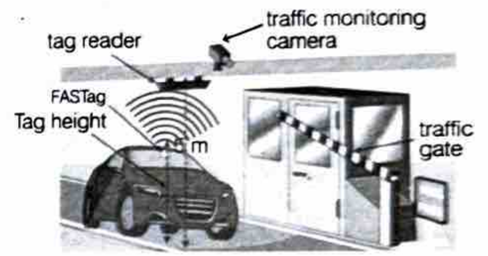
(ii) How far did the bird fly in the mentioned time?

Or After hitting the tree, how far did the ball travel in the sky when Kaushik saw the ball?

(iii) What is the speed of the bird in m/min, if it had flown $20(\sqrt{3}+1)$ m?

17. At a toll plaza, an electronic toll collection system has been installed. FASTag can be used to pay the fare. The tag can be pasted on the windscreen of a car.

At the toll plaza a tag scanner is placed at a height of 6 m from the ground. The scanner reads the information on the tag of the vehicle and debits the desired toll amount from a linked bank account.



For the tag scanner to function properly the speed of a car needs to be less than 30 km per hour.

A car with a tag installed at a height of 1.5 m from the ground enters the scanner zone.

Based on the above information, answer the following questions.

(i) The scanner gets activated when the car's tag is at a distance of 5 m from it. Give one trigonometric ratio for the angle between the horizontal and the line between the car tag and the scanner?

(ii) The scanner reads the complete information on the car's tag while the angle between chip and scanner changes from 30° to 60° due to car movement. What is the distance moved by the car?

(a) $\sqrt{3}$ m (b) $3\sqrt{3}$ m
(c) 4.5 m (d) $4.5\sqrt{3}$ m

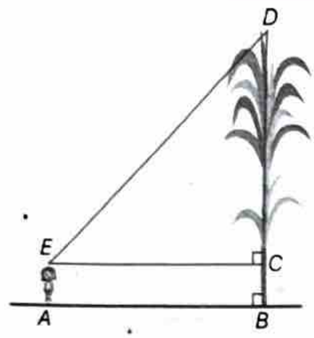
(iii) A vehicle with a tag pasted at a height of 2 m from the ground stops in the scanner zone. The scanner reads the data at an angle of 45°. What is the distance between the tag and the scanner?

(a) 2 m (b) 4 m
(c) $4\sqrt{2}$ m (d) 8 m

Part II

Very Short Answer Type Questions

1. In the given figure, the height of the girl is 1.5 m and the height of the tree is 13.5 m. **Competency Based Question**



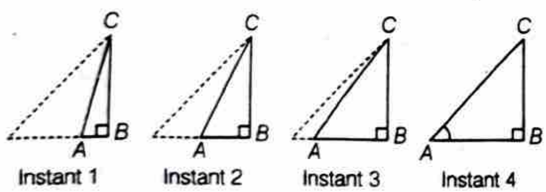
(**Note** The figure is not to scale.)

If $AB = 12\sqrt{3}$ m, what is the angle of elevation of the top of the tree from her eyes?

Show your steps.

2. At the toll plaza, a traffic monitoring camera is installed at a height of 6.2 m. It takes pictures of moving vehicles at regular intervals.

The diagram below shows the position of the camera and a car moving away from it after paying the toll in four instances. The speed of the car is 5 m/s.



Based on the above information, answer the following questions.

The angle made by the camera to the car in instance 1 is $30°$ and changes to $60°$ in instance 4. What is the distance moved by the car? ($\sqrt{3} = 1.73$)

Competency Based Question

3. Find the length of the shadow on the ground on a pole of height 18 m when angle of elevation θ of the Sun is such that $\tan\theta = \frac{6}{7}$.

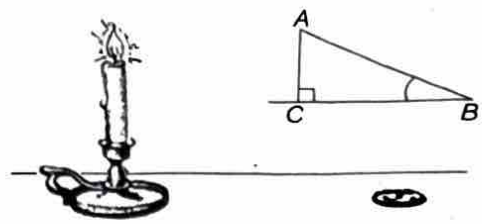
CBSE 2023 (Standard)

4. If the length of the shadow of a tower is increasing, then the angle of elevation of the Sun is also increasing. Is it true? Justify your answer.

NCERT Exemplar

5. If the angles of elevation of the top of the candle from two coins distant 'a' cm and 'b' cm ($a > b$) from its base and in the same straight line from it are $30°$ and $60°$, then find the height of the candle.

CBSE Sample Paper 2020 (Standard)



6. Find the angle of elevation of the Sun when the shadow of a pole h m high is $\sqrt{3}h$ m long.

NCERT Exemplar

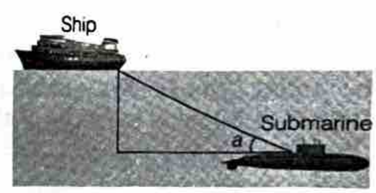
7. A tower stands vertically on the ground. From a point on the ground which is 20 m away from of the foot of the tower, the angle of elevation of the top of the tower is found to be $60°$. Find the height of the tower.

8. A boy 1.7 m tall is standing on a horizontal ground, 50 m away from a building.

The angle of elevation of the top of the building from his eye is $60°$. Calculate the height of the building. (take $\sqrt{3} = 1.73$)

9. The angle of elevation of the top of a building 150 m high, from a point on the ground is $45°$. Find the distance of the point from foot of the building.

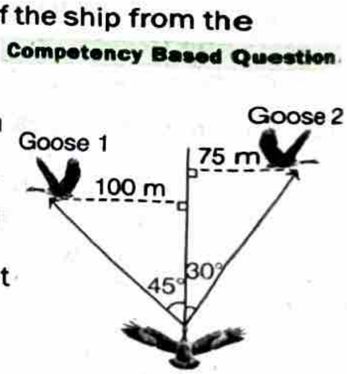
10. Shown below is a submarine scouting an enemy ship in the ocean using a sonar device. Sonar devices send out a sound pulse from a transducer, and then precisely measure the time it takes for the sound pulses to be reflected back to the transducer.



A sonar wave sent by the submarine hits the ship and returns back in 2 sec. The speed of a sonar wave underwater is 1500 m/s and the submarine is diving at a depth of 750 m below sea level.

Find the angle of elevation a of the ship from the submarine. Show your steps. **Competency Based Question**

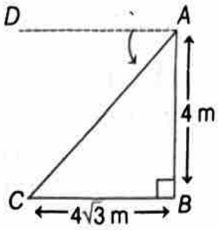
11. The position of an Eagle and two identical geese are shown in the figure below. All the birds are at the same height from the ground. Assume that the Eagle can fly at the same speed in all directions and that the geese are unaware of the Eagle's intention and will not move from their positions.



(**Note** The figure is not to scale.)

If the Eagle wants to attack the goose that is nearer to it, which one should it attack?
Show your steps. (**Note** Use $\sqrt{2}=1.41, \sqrt{3}=1.73$)

12. The figure shows the observation of point C from point A. Find the angle of depression from A.



13. The angle of depression of car parked on the road from the top of a 150 m high tower is 30°. Find the distance of the car from the tower.

Short Answer Type Questions

14. If two towers of heights x m and y m subtend angles of 30° and 60°, respectively at the centre of a line joining their feet, then find the ratio of $x:y$.

15. Two vertical poles of different heights are standing 20m away from each other on the level ground. The angle of elevation of the top of the first pole from the foot of the second pole is 60° and angle of elevation of the top of the second pole from the foot of the first pole is 30°. Find the difference between the heights of two poles. [take $\sqrt{3}=1.732$]

16. A tree is broken due to the storm in such a way that the top of the tree touches the ground and makes an angle of 30° with the ground. Length of the broken upper part of the tree is 8 m. Find the height of the tree before it was broken.

17. A person standing on the bank of a river observes that the angle of elevation of the top of tree standing on the opposite bank is 60°. When he moves 40 m away from the bank, he finds the angle of elevation to be 30°. Find the height of the tree and the width of the river. [take $\sqrt{3}=1.732$]

18. Sachin is a fireman worker. When he throws water on the fire of burning house, he notice that one child is crying for getting help. At once Sachin tied a rope at the top of a pole near the burning house and its other end tied at the ground. He climbed the rope and from the top of the pole picked the child and save his life. Suppose the height of the pole is 20 m and the angle made by the rope with ground is 60°.

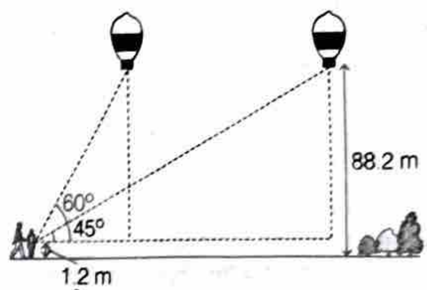
(i) Calculate the distance covered by the fireman to reach the top of the pole.

(ii) Find the distance between the foot of the pole and where he tied the rope at the ground.

Competency Based Question

19. A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After sometime, the angle of elevation reduces 45°. Find the distance travelled by the balloon during the interval.

Competency Based Question



20. A boy standing on a horizontal plane finds a bird flying at a distance of 100 m from him at an elevation of 30°. A girl standing on the roof of a 20 m high building, finds the elevation of the same bird to be 45°. The boy and the girl are on the opposite sides of the bird. Find the distance of the bird from the girl. [given $\sqrt{2}=1.414$]

CBSE 2019

21. The shadow of a flag staff is three times as long as the shadow of the flag staff, when the Sun rays meet the ground at an angle of 60°. Find the angle between the Sun rays and the ground at the time of longer shadow.

22. The angle of elevation of a jet plane from a point A, on the ground is 60°. After a flight of 30 s, the angle of elevation changes to 30°. If the jet plane is flying at a constant height of $3600\sqrt{3}$ m, find the speed of the jet plane.

CBSE 2024 (Standard)

23. There is a small island in the middle of a 100 m wide river and a tall tree stands on the island. P and Q are points directly opposite to each other on two banks and in line with the tree. Suppose the angles of elevation of the top of the tree from P and Q are respectively 30° and 45°.

(i) Find the height of the tree. [take $\sqrt{3} = 1.732$]

(ii) Determine the distance between two trees.

(iii) Which point is farthest from the island?

Competency Based Question

24. The angle of elevation of the top Q of a vertical tower PQ from a point X on the ground is 60°. At a point R, 40 m vertically above X, the angle of elevation of the top Q of tower is 45°. Find the height of the tower PQ and the distance PX. [take, $\sqrt{3}=1.732$]

25. Sunita is an electrician and she has to repair an electric fault on a pole of height 8 m. She needs to reach to a point on the pole 3 m below the top of the pole to undertake the repair work.

(i) What should be the length of the ladder that she should use which when inclined at an angle of 30° from the horizontal, would enable her to reach the required position?

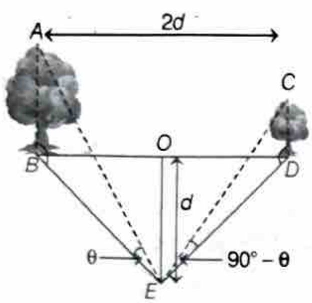
(ii) How far from the foot of the pole should she place the foot of the ladder.

(iii) The space between the pole and ladder is in the triangular shape, find its area.

26. A vertical tower stands on a horizontal plane and is surmounted by a vertical flag staff of height h. At a point on the plane, the angles of elevation of the bottom and the top of the flag staff are α and β, respectively. Prove that the height of the tower is

$$\left(\frac{h\tan\alpha}{\tan\beta-\tan\alpha}\right).$$

27. Two trees are $2d$ m apart. Ajay stood at a point midway between them and started walking in a direction perpendicular to the line connecting the two trees. After walking d metres, he observed the angle of elevations to the tops of the two trees and found them to be complementary.



(**Note** The figure is not to scale.)

If one of the trees is thrice as tall as the other, find the height of the shorter tree, in terms of d. Show your work.

Competency Based Question

28. A player sitting on the top of a tower of height 20 m observes the angle of depression of a ball lying on the ground as $60°$. Find the distance between the foot of the tower and the ball.

29. A peacock is sitting on the top of a tree. It observes a serpent on the ground making an angle of depression of $30°$. The peacock catches the serpent in 12 s with the speed of 300 m/min. What is the height of the tree?

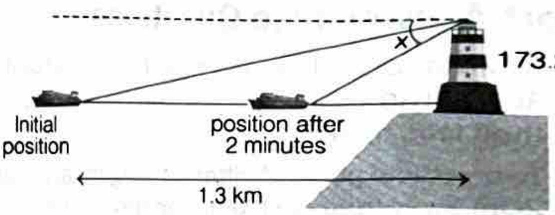
30. From the top of a hill, the angles of depression of two consecutive kilometre stones due East are found to be $30°$ and $45°$. Find the height of the hill.

31. As observed from the top of a lighthouse, 100 m high above sea level, the angle of depression of a ship sailing directly towards it, changes from $30°$ to $60°$. Determine the distance travelled by the ship during the period of observation. [take, $\sqrt{3}=1.732$] **CBSE 2022 (Basic)**

32. Two ships are there in the sea on either side of a lighthouse in such away that the ships and the base of the lighthouse are in the same straight line. The angle of depression of two ships as observed from the top of the lighthouse are $60°$ and $45°$. If the height of the lighthouse is 200 m, then find the distance between the two ships.

33. From the top of a building 50 m high, the angles of depression of the top and bottom of a tower are observed to be $30°$ and $60°$. Find the height of the tower and distance between the building and the tower. [take $\sqrt{3}=1.73$] **CBSE 2023 (Basic)**

34. A ship was moving towards the shore at a uniform speed of 36 km/h. Initially, the ship was 1.3 km away from the foot of a lighthouse which is 173.2 m in height.



(**Note** The figure is not to scale.)

Find the angle of depression x of the top of the lighthouse from the ship after the ship had been moving for 2 min. Show your steps and give reasons.

(**Note** Take $\sqrt{3}=1.732$ and $\sqrt{2}=1.414$)

Competency Based Question

35. A window in a building is at a height of 10 m from the ground. The angle of depression of a point P on the ground from the window is $30°$. The angle of elevation of the top of the building from the point P is $60°$. Find the height of the building.

36. A man standing on the deck of a ship, which is 10 m above the water level. He observes the angle of elevation of the top of a hill is $60°$ and the angle of depression of the base of the hill is $30°$. Calculate the distance of the hill from the ship and height of the hill. **Competency Based Question**

37. A window of a house is h m above the ground. From the window, the angles of elevation and depression of the top and the bottom of another house situated on the opposite side of the lane are found to be α and β, respectively. Prove that the height of the other house is $h(1+\tan\alpha\cot\beta)$ m.

Long Answer Type Questions

38. The angle of elevation θ of the top of a lighthouse as seen by a person on the ground is such that $\tan\theta = \frac{5}{12}$. When the person moves a distance of 240 m towards the lighthouse, the angle of elevation becomes ϕ, such that $\tan\phi = \frac{3}{4}$. Find the height of the lighthouse.

39. The length of the shadow of a tower standing on level ground is found to be $2x$ m longer when the Sun's altitude is $30°$ than when it was $45°$. Prove that the height of tower is $\left(\frac{2x}{\sqrt{3}-1}\right)$ m.

40. The angle of elevation of the top of a tower at a distance of 120 m from a point A on the ground is $45°$. If the angle of elevation of the top of a flag staff fixed at the top of the tower at A is $60°$, then find the height of the flag staff. [use $\sqrt{3} = 1.732$]

41. A balloon is connected to an electric pole. It is inclined at $60°$ to the horizontal by a cable of length 215 m. Determine the height of the balloon from the ground. Also, find the height of the balloon, if the angle of inclination is changed from $60°$ to $30°$.

42. A man in a boat rowing away from a lighthouse 100 m high takes 2 min to change the angle of elevation of the lighthouse from $60°$ to $45°$. Find the speed of boat.

43. The angle of elevation of the top of a building from the foot of the tower is $30°$ and the angle of elevation of the top of the tower from the foot of the building is $60°$. If the tower is 60 m high, then find the height of the building. **CBSE 2020 (Basic), 2013**

44. At the foot of mountain, the elevation of its summit is $45°$. After ascending 1000 m towards to mountain up a slope of $30°$ inclination, the elevation is found to be $60°$. Find the height of the mountain.

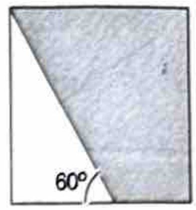
45. A ladder leaning against a vertical wall at an inclination α to the horizontal. Its foot is pulled away from the wall through a distance p, so that its upper end slides a distance q down the wall and then the ladder makes an angle β to the horizontal.

(i) Show that $\frac{p}{q} = \frac{\cos\beta - \cos\alpha}{\sin\alpha - \sin\beta}$.

(ii) Check whether the above result is true for $\alpha = \beta$.

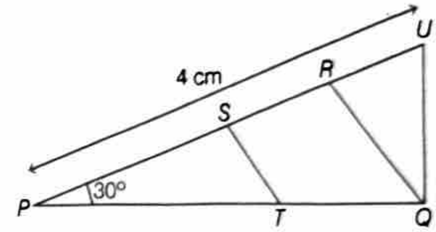
46. A triangle is drawn inside a square of side length 6 cm as shown in figure given below.

Competency Based Question



What is the area of the shaded region?

47. In the figure below, PQU is a right triangle with $\angle PQU = 90°$. ST is parallel to RQ, $PS = RQ$ and $ST = RU$. Calculate the length of SR. **Competency Based Question**



What is the length of SR?

48. Two ships are sailing in the sea on the either side of the lighthouse. The angles of depression of two ships as observed from the top of the lighthouse are $60°$ and $45°$, respectively. If the distance between the ships is $100\left(\frac{\sqrt{3}+1}{\sqrt{3}}\right)$ m, then find the height of the lighthouse.

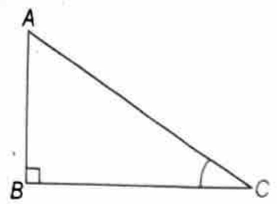
49. A moving boat is observed from the top of a 150 m high cliff moving away from the cliff. The angle of depression of the boat changes from $60°$ to $45°$ in 2 min. Find the speed of the boat in (m/h).

50. A man on a cliff observes a boat at an angle of depression of $30°$ which is approaching the shore to the point immediately beneath the observer with a uniform speed. Six minutes later, the angle of depressions of the boat is found to be $60°$. Find the time taken by the boat from here to reach the shore. **CBSE 2024 (Standard)**

HINTS & SOLUTIONS

Part I

1. (b) Let AB be the tower and BC be its shadow.



Given, $AB = BC$

In right angled ΔABC,

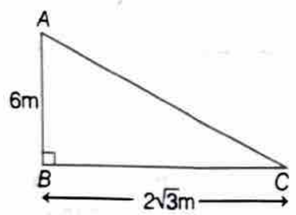
$$\tan C = \frac{\text{Perpendicular}}{\text{Base}}$$

$$= \frac{AB}{BC} = 1$$

$\Rightarrow \tan C = \tan 45°$ $\quad [\because \tan 45° = 1]$

$\Rightarrow \quad C = 45°$

2. (a)



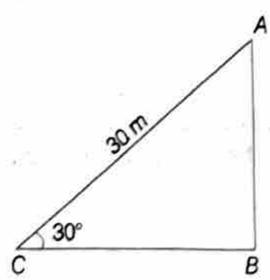
Let $AB = 6$ m be the height of the pole and length of the shadow $BC = 2\sqrt{3}$ m.

In right angled ΔABC,

$$\tan C = \frac{AB}{BC} = \frac{6}{2\sqrt{3}} = \sqrt{3}$$

$\Rightarrow \quad \angle C = 60°$

3. (c) Let AB be the vertical pole and CA be the 30 m long rope such that its one end A is tied from the top of the vertical pole AB and the other end C is tied to a point C on the ground.



In ΔABC, we have

$$\sin 30° = \frac{AB}{AC} \Rightarrow \frac{1}{2} = \frac{AB}{AC}$$

$\Rightarrow \quad AB = 15$ m

Hence, the height of the pole is 15 m.

4. (a) Given, length of the string of the kite,

$$AB = 85 \text{ m}$$

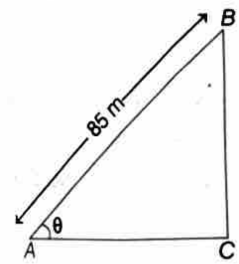
and $\quad \tan\theta = \frac{15}{8} \Rightarrow \cot\theta = \frac{8}{15}$

$\Rightarrow \quad \operatorname{cosec}^2\theta - 1 = \frac{64}{225}$

$\Rightarrow \quad \operatorname{cosec}^2\theta = 1 + \frac{64}{225} = \frac{289}{225}$

$\Rightarrow \quad \operatorname{cosec}\theta = \sqrt{\frac{289}{225}} = \frac{17}{15}$

$\Rightarrow \quad \sin\theta = \frac{15}{17}$



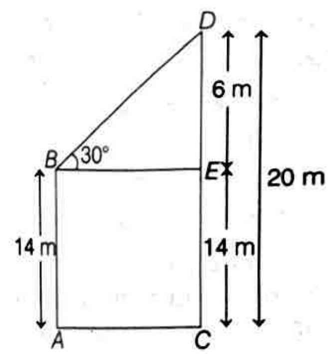
In ΔABC, $\sin\theta = \frac{BC}{AB}$

$\Rightarrow \quad \frac{15}{17} = \frac{BC}{85} \Rightarrow BC = 75$ m

$\therefore$ Height of kite = 75 m

5. (a) Here, $CD = 20$ m $\quad$ **[height of big pole]**

$AB = 14$ m $\quad$ **[height of small pole]**



$\therefore \quad DE = CD - CE$

$\Rightarrow \quad DE = CD - AB \quad [\because AB = CE]$

$\Rightarrow \quad DE = 20 - 14 = 6$ m

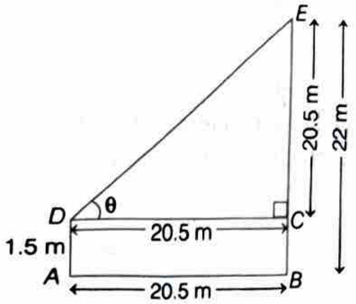
In ΔBDE,

$$\sin 30° = \frac{DE}{BD}$$

$\Rightarrow \quad \frac{1}{2} = \frac{6}{BD} \Rightarrow BD = 12$ m

$\therefore$ Length of wire = 12 m

6. (b) Let $BE = 22$ m be the height of the tower and $AD = 1.5$ m be the height of the observer. The point D be the observer's eye. Draw $DC \parallel AB$.



Then, $AB = 20.5 \text{ m} = DC$

and $EC = BE - BC = BE - AD$

$= 22 - 1.5 = 20.5$ m $\quad [\because BC = AD]$

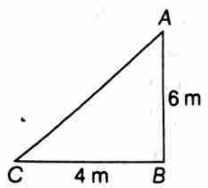
Let θ be the angle of elevation make by observer's eye to the top of the tower i.e. $\angle EDC = \theta$.

In right angled ΔDCE,

$$\tan\theta = \frac{P}{B} = \frac{CE}{DC} = \frac{20.5}{20.5}$$

$\Rightarrow \quad \tan\theta = 1 \Rightarrow \tan\theta = \tan 45° \Rightarrow \theta = 45°$

7. (a) Let AB be height of tree and BC be its shadow.



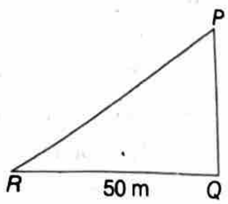
Again, let PQ be height of pole and QR be its shadow.

At the same time, the angle of elevation of tree and poles are equal.

$\therefore \qquad \Delta ABC \sim \Delta PQR$

$\Rightarrow \qquad \dfrac{AB}{BC} = \dfrac{PQ}{QR}$

$\Rightarrow \qquad \dfrac{6}{4} = \dfrac{PQ}{50}$

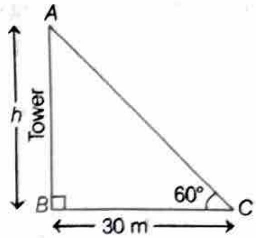


$\Rightarrow \qquad PQ = \dfrac{50 \times 6}{4}$

$\Rightarrow \qquad PQ = 75$ m

8. (b) Do same as Question 5 of NCERT Folder Exercise 9.1.

9. (d) Let the height of the tower AB be h m.



In ΔABC, right angled at B,

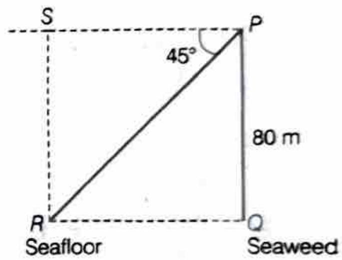
$$\tan 60° = \frac{P}{B} = \frac{AB}{BC}$$

$\Rightarrow \qquad \sqrt{3} = \dfrac{h}{30} \Rightarrow h = 30\sqrt{3}$

$\therefore$ The height of the tower is $30\sqrt{3}$ m.

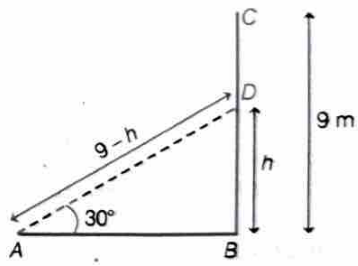
10. (c) **Hint** Do same as Question 12 of NCERT Folder Exercise 9.1.

11. (c) **Hint** Since, diver makes an angle of 45° from the boat, it is an angle of depression.



Find PR using suitable trigonometric ratio.

12. (a) Let us consider the height, $DB = h$ m



Use suitable trigonometric ratio to find the height DB.

$$\sin 30° = \frac{h}{9-h}$$

$\Rightarrow \qquad \dfrac{1}{2} = \dfrac{h}{9-h} \Rightarrow h = 3$ m

13. (c) Find the distance between first person and the player by Pythagoras theorem, which is equal to 12 m.

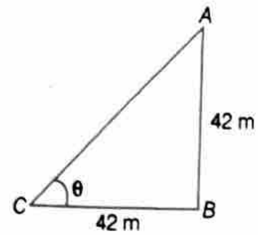
Now, according to the question,

Distance between these two persons is 1.5 m.

$\therefore$ Distance from second person to the player $= 12 + 1.5$ m

$= 13.5$ m

14. (i) (b) Let AB be the monument of height 42 m and C is the point where they are standing such that $BC = 42$ m.

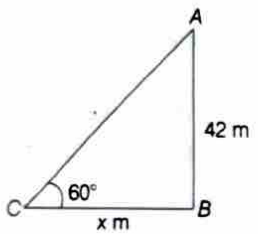


Now, in ΔABC,

$$\tan\theta = \frac{AB}{BC} \Rightarrow \tan\theta = \frac{42}{42} = 1$$

$$\Rightarrow \quad \tan\theta = 1 \Rightarrow \theta = 45°$$

(ii) (a) In ΔABC,



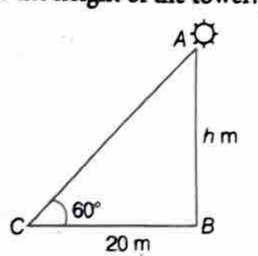
$$\tan 60° = \frac{AB}{BC}$$

$$\Rightarrow \quad \sqrt{3} = \frac{42}{BC}$$

$$\Rightarrow \quad BC = \frac{42}{\sqrt{3}} = \frac{42\sqrt{3}}{3} = 14\sqrt{3}$$

$$= 14 \times 1.732 = 24.248 = 24.25 \text{ m}$$

(iii) (a) Let AB be the height of the tower.

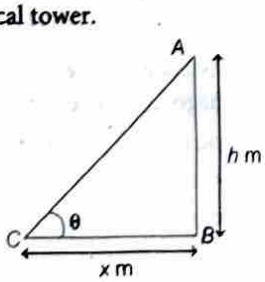


Then, in ΔABC,

$$\tan 60° = \frac{AB}{BC} \Rightarrow \sqrt{3} = \frac{h}{20}$$

$$\Rightarrow \quad h = 20\sqrt{3} \text{ m}$$

(iv) (b) Let h and x be the height and length of shadow of the vertical tower.



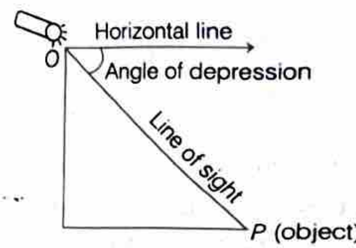
Then, in ΔABC,

$$\tan\theta = \frac{AB}{BC} \Rightarrow \tan\theta = \frac{h}{x}$$

$$\Rightarrow \quad \tan\theta = 1 \qquad [\because h : x = 1 : 1]$$

$$\Rightarrow \quad \theta = 45°$$

(v) (c) The angle of depression of an object viewed, is the angle formed by the line of sight with the horizontal, when it is below the horizontal level.



15. (i) In right angled ΔOPB,

$$\cos 30° = \frac{OP}{OB} = \frac{36}{OB}$$

$$\Rightarrow \quad OB = \frac{36}{\cos 30°} = \frac{36}{\frac{\sqrt{3}}{2}} = \frac{72}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$\Rightarrow \quad OB = 24\sqrt{3} \text{ cm}$$

(ii) In right angled ΔOPB,

$$\tan 30° = \frac{BP}{OP}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{BP}{36}$$

$$\Rightarrow \quad BP = \frac{36}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$\Rightarrow \quad BP = 12\sqrt{3} \text{ cm} \qquad \ldots(i)$$

In right angled ΔAPO,

$$\tan 45° = \frac{AP}{OP}$$

$$\Rightarrow \quad OP = AP \qquad [\because \tan 45° = 1]$$

$$\Rightarrow \quad AP = 36 \text{ cm} \qquad \ldots(ii)$$

$\therefore$ Distance $AB = AP - BP$

$$= 36 - 12\sqrt{3}$$

$$= 12(3 - \sqrt{3}) \text{ cm}$$

(iii) Height of the Section A from the base of the tower

$$= AP = 36 \text{ cm} \qquad \text{[using Eq (ii)]}$$

Or

$$BP = 12\sqrt{3} \text{ cm} \qquad \text{[using Eq. (i)]}$$

$$\therefore \text{ Area of } \Delta OPB = \frac{1}{2} \times OP \times BP$$

$$= \frac{1}{2} \times 36 \times 12\sqrt{3}$$

$$= 216\sqrt{3} \text{ cm}^2$$

16. (i) In right angled ΔABC,

$$\tan 45° = \frac{AB}{BC}$$

$$\Rightarrow \quad BC = AB = 80 \text{ m} \quad \text{... (i)}$$

$$[\because AB = 80 \text{ m and } \tan 45° = 1]$$

$\therefore$ Required distance $= 80$ m

(ii) In right angled ΔDEC,

$$\tan 30° = \frac{DE}{EC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{80}{EC}$$

$$\left[\because DE = 80 \text{ m and } \tan 30° = \frac{1}{\sqrt{3}}\right]$$

$$\Rightarrow \quad EC = 80\sqrt{3} \text{ m}$$

$$\therefore \quad BE = EC - BC$$

$$= 80\sqrt{3} - 80 \quad \text{[using Eq. (i)]}$$

$$= 80(\sqrt{3} - 1) \text{ m}$$

Hence, distance the bird flew

$$= AD = BE$$

$$= 80(\sqrt{3} - 1) \text{ m}$$

Or

In right angled ΔCGF,

$$\tan 60° = \frac{GF}{GC}$$

$$\Rightarrow \quad \sqrt{3} = \frac{80}{GC}$$

$$[\because GF = ED = 80 \text{ m and } \tan 60° = \sqrt{3}]$$

$$\Rightarrow \quad GC = \frac{80\sqrt{3}}{3} \text{ m} \quad \text{...(ii)}$$

Distance travelled by the ball after hitting the tree

$$= AF = BG$$

and
$$BG = BC - GC$$

$$= \left(80 - \frac{80\sqrt{3}}{3}\right)$$

$$= 80\left(\frac{3 - \sqrt{3}}{3}\right) \text{m}$$

[using Eqs. (i) and (ii)]

(iii) Speed of the bird $= \dfrac{\text{Distance}}{\text{Time}}$

$$= \frac{20(\sqrt{3} + 1)}{2} \text{ m/s}$$

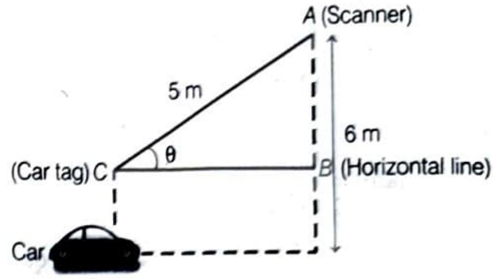
$$= 10(\sqrt{3} + 1) \text{ m/s}$$

$$= 600(\sqrt{3} + 1) \text{ m/min}$$

$$\left[\because 1 \text{ s} = \frac{1}{60} \text{ min}\right]$$

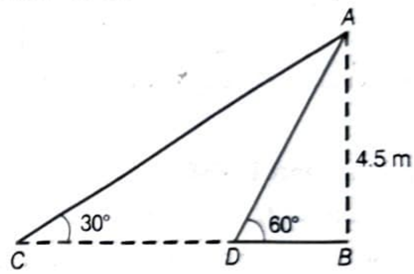
17. (i) For angle θ between car's tag and scanner and horizontal line,

$$\sin\theta = \frac{6 - 1.5}{5} \Rightarrow \sin\theta = \frac{4.5}{5}$$



Ans. $\sin\theta = \dfrac{4.5}{5}$

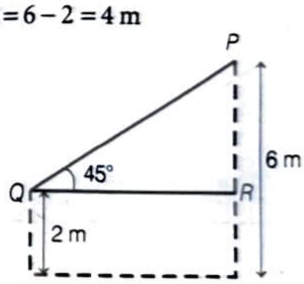
(ii) (b) Let CD be the distance moved by the car.



Use suitable trigonometric ratio to find CB in ΔACB and DB in ΔADB.

Find $CD = CB - DB$

(iii) (c) $PR = 6 - 2 = 4$ m



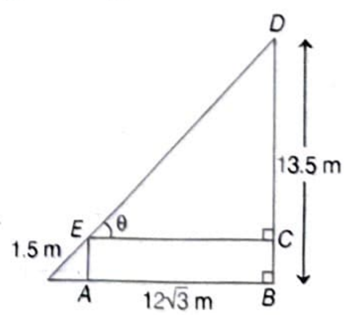
To find PQ, use $\sin 45° = \dfrac{PR}{PQ}$

$$\frac{1}{\sqrt{2}} = \frac{4}{PQ}$$

$$\therefore \quad PQ = 4\sqrt{2} \text{ m}$$

Part II

1. According to the figure,



$$AB = EC = 12\sqrt{3}\text{ m} \quad \ldots(i)$$
$$AE = BC = 1.5\text{ m}$$
$$BD = 13.5\text{ m}$$
$$\Rightarrow DC + BC = 13.5\text{ m}$$
$$DC = 13.5 - 1.5\text{ m}$$
$$= 12\text{ m} \quad \ldots(ii)$$

In ΔDEC,

$$\tan\theta = \frac{DC}{EC}$$
$$= \frac{12}{12\sqrt{3}} \quad \text{[from Eqs. (i) and (ii)]}$$
$$\tan\theta = \frac{1}{\sqrt{3}}$$
$$\therefore \quad \theta = 30° \quad \left[\because \tan 30° = \frac{1}{\sqrt{3}}\right]$$

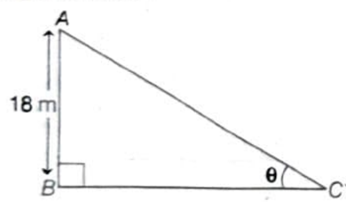
Hence, angle of elevation of top of the tree from her eyes is 30°.

2. **Hint** In instance 1, $\angle ACB = 30°$ and in instance 4, $\angle ACB = 60°$

Then, with the help of suitable trigonometric ratio, find the distance.

Ans. 7.16 m

3. **Hint** Let AB be the pole of height 18 m and BC be the length of the shadow.



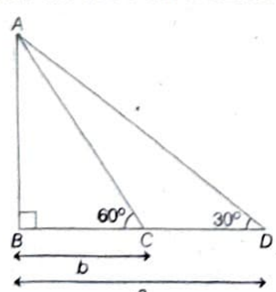
In right angled ΔABC,

$$\tan\theta = \frac{AB}{BC} = \frac{18}{BC} = \frac{6}{7} \quad \left[\text{given}, \tan\theta = \frac{6}{7}\right]$$

Ans. 21 m

4. **False**, because when length of shadow increases, the angle of elevation of the Sun decreases and *vice-versa*.

5. Given, the angle of elevation of the top of candle from two coins distant a cm and b cm from its base.



In ΔABC, $\tan 60° = \frac{AB}{BC} \Rightarrow \sqrt{3} = \frac{AB}{b}$ ...(i)

In ΔABD, $\tan 30° = \frac{AB}{BD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{a}$ ... (ii)

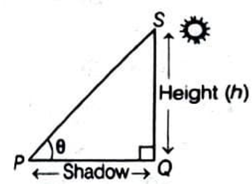
Multiplying Eqs. (i) and (ii), we get

$$\Rightarrow \quad \sqrt{3} \times \frac{1}{\sqrt{3}} = \frac{AB \times AB}{a \times b}$$
$$\Rightarrow \quad AB^2 = ab \Rightarrow AB = \sqrt{ab}$$

6. **Hint** Let SQ be the pole of height h and PQ be the shadow of a pole.

According to the question,

$$SQ = h \text{ and } PQ = \sqrt{3}h$$



Again, let the angle of elevation of the Sun be θ.

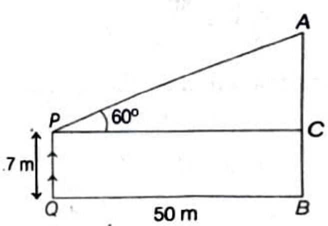
In right angled ΔPQS,

$$\tan\theta = \frac{P}{B} = \frac{QS}{PQ}$$
$$\Rightarrow \quad \tan\theta = \frac{h}{\sqrt{3}h} = \frac{1}{\sqrt{3}} \quad \textbf{Ans. } 30°$$

7. **Hint** Do same as Question 4. of NCERT Folder Exercise 9.1. **Ans.** $20\sqrt{3}$ m

8. Given that boy is standing 50 m away from the building

$\therefore QB = 50$ m



and boy is 1.7 m tall.

$$\therefore \quad PQ = 1.7\text{ m}$$
$$\angle APC = 60°, PC = QB = 50\text{ m}$$
$$\text{and} \quad PQ = CB = 1.7\text{ m}$$

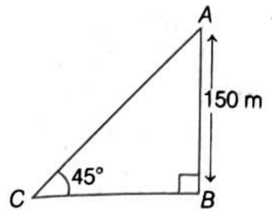
In ΔAPC,

$$\tan 60° = \frac{AC}{CP} = \frac{AC}{50}$$
$$\Rightarrow \quad AC = 50\sqrt{3} \Rightarrow AB - BC = 50\sqrt{3}$$
$$\Rightarrow \quad AB = 50\sqrt{3} + 1.7 = 50 \times 1.73 + 1.7$$
$$\Rightarrow \quad AB = 88.2\text{ m}$$

$\therefore$ Height of building is 88.2 m.

9. **Hint** Let $AB = 150$ m be the height of building and C be a point on the ground such that

$$\angle ACB = 45°$$



In right angled ΔABC,

$$\tan 45° = \frac{P}{B} = \frac{AB}{BC}$$ **Ans.** 150 m

10. Time taken from both side = 2sec

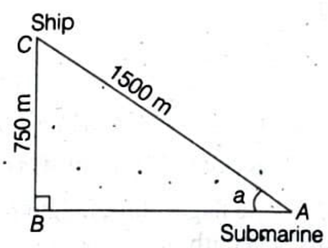
Speed underwater = 1500 m/s

Distance travelled = $1500 \times 2 = 3000$ m

Since, distance travelled by sonar wave to ship and back

$= 3000$ m.

So, distance between ship and sonar $\frac{3000}{2} = 1500$ m



In ΔABC, $\sin a = \frac{BC}{AC} = \frac{750}{1500} = \frac{1}{2}$

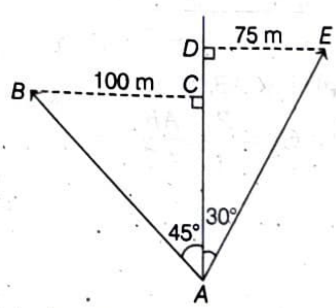
$\therefore \quad \sin a = \sin 30°$ $\quad \left[\because \sin 30° = \frac{1}{2}\right]$

Thus, $a = 30°$

11. Let Eagle is at position A

Goose 1 at B

Goose 2 at E



Now, in right ΔABC,

$$\sin 45° = \frac{BC}{AB}$$

$\Rightarrow \quad \frac{1}{\sqrt{2}} = \frac{100}{AB}$ $\quad \left[\because \sin 45° = \frac{1}{\sqrt{2}}\right]$

$$= 100 \times 1.41$$

$$AB = 141 \text{ m} \quad \ldots\text{(i)}$$

in right ΔADE,

$$\sin 30° = \frac{DE}{AE}$$

$$\frac{1}{2} = \frac{75}{AE} \quad \left[\because \sin 30° = \frac{1}{2}\right]$$

$$AE = 150 \text{ m} \quad \ldots\text{(ii)}$$

Thus, as Goose 1 is closer, so the Eagle would attack it.

12. Hint In right angled $\Delta ABC, \angle B = 90°$

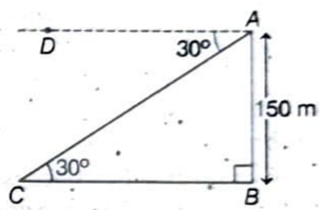
Let $\quad \angle DAC = \theta$

Then, $\quad \angle ACB = \angle DAC = \theta$ $\quad$ [alternate angles]

Now, $\quad \tan\theta = \frac{P}{B} = \frac{AB}{BC} = \frac{4}{4\sqrt{3}} = \frac{1}{\sqrt{3}} = \tan 30°$

Ans. 30°

13. Hint Let AB=150 m be the height of the tower and angle of depression is $\angle DAC$= 30°.



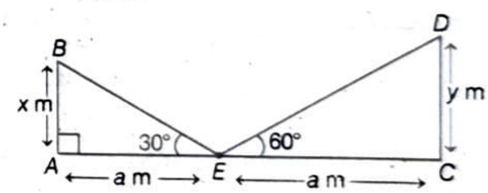
Then, $\angle ACB = \angle DAC = 30°$ $\quad$ [alternate angles]

Now, in right angled ΔABC,

$$\tan 30° = \frac{P}{B} = \frac{AB}{BC}$$

Ans. $150\sqrt{3}$ m

14. Hint Let AB be the tower of height x m and CD be the tower of height y m.



Let E be the mid-point of the line AC.

Then, $\angle AEB = 30°$ and $\angle CED = 60°$.

Also, $AE = EC = a$ m $\quad$ [let]

In right angled ΔBAE,

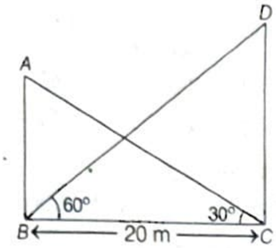
$$\tan 30° = \frac{P}{B} = \frac{AB}{AE} = \frac{x}{a}$$

and in right angled ΔDCE,

$$\tan 60° = \frac{DC}{CE} = \frac{y}{a}$$

Ans. 1 : 3

15. Let two poles be AB and CD.
Given, length of the road = 20 m
So, $BC = 20$ m



Also, $\angle ACB = 30°$ and $\angle DBC = 60°$
In right angled $\Delta ABC, \angle ABC = 90°$

$$\tan 30° = \frac{AB}{BC} \Rightarrow AB = \frac{20}{\sqrt{3}} \text{ m} \quad \ldots \text{(i)}$$

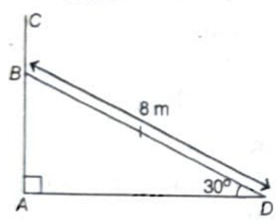
Now, in right angled ΔDBC,

$$\tan 60° = \frac{DC}{BC} \Rightarrow DC = 20\sqrt{3} \text{ m}$$

Distance between heights $= CD - AB$

$$= 20\sqrt{3} - \frac{20}{\sqrt{3}} = \left(\frac{40}{3}\sqrt{3}\right) \text{m} = 23.09 \text{ m}$$

16. Let the initial height of the tree be AC, when the storm come, the tree broke from point B.
The broken part of the tree BC touches the ground at point D, making an angle 30° on the ground.
Also, given $BD = 8$ m

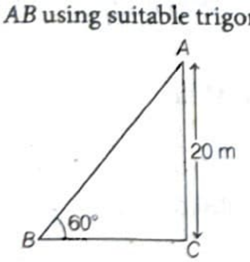


In right ΔABD, $\sin 30° = \frac{AB}{BD} \Rightarrow AB = \frac{8}{2} = 4$ m

$$\therefore \quad AC = AB + BC = AB + BD \; [\because BC = BD]$$
$$= 4 + 8 = 12 \text{ m}$$

17. **Hint** Do same as Question 11 of NCERT Folder Exercise 9.1
Ans. Height of tree = 34.64 m and width of river = 20 m

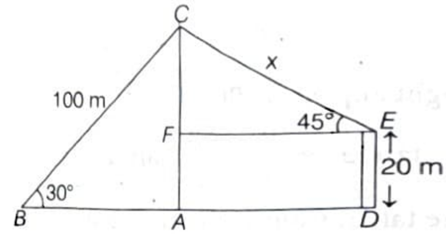
18. **Hint** (i) Find AB using suitable trigonometric ratio,



(ii) Find BC
Ans. (i) $\frac{40}{\sqrt{3}}$ m (ii) $\frac{20}{\sqrt{3}}$ m

19. **Hint** Do same as Question 14 of NCERT Folder Exercise 9.1.
Ans. $29(3 - \sqrt{3})$ m

20. Let C be the position of bird, and B and E be the position of boy and girl respectively.
Let the height of building be $DE = 20$ m
Let the distance of girl from bird be $EC = x$ m



In right ΔBAC,

$$\sin 30° = \frac{AC}{BC}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{AC}{100} \Rightarrow AC = \frac{100}{2} = 50 \text{ m}$$

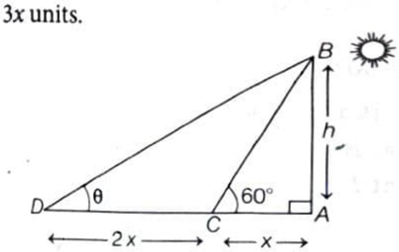
Now, $\quad CF = AC - AF = 50 - 20 = 30$ m $\quad [\because AF = DE]$
In right ΔEFC,

$$\sin 45° = \frac{CF}{CE} \Rightarrow \frac{1}{\sqrt{2}} = \frac{30}{x}$$

$$\Rightarrow \quad x = 30 \times \sqrt{2} = 30 \times 1.414 = 42.42 \text{ m}$$

Hence, the distance of girl from bird is 42.42 m.

21. **Hint** Let AB be the flag staff of height h units and $AC = x$ units be length of its shadow, when the Sun rays meet the ground at an angle of 60°.
Also, let θ be the angle between the Sun rays and the ground, when the length of the shadow of the flag staff is $AD = 3x$ units.



In right angled ΔCAB,

$$\tan 60° = \frac{P}{B} = \frac{AB}{AC} \quad \left[\because \tan\theta = \frac{P}{B}\right]$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{x} \quad [\because \tan 60° = \sqrt{3}]$$

$$\Rightarrow \quad h = \sqrt{3}x \quad \ldots \text{(i)}$$

Now, in right angled ΔDAB,

$$\tan\theta = \frac{AB}{AD} = \frac{AB}{DC + CA} \quad [\because AD = DC + CA]$$

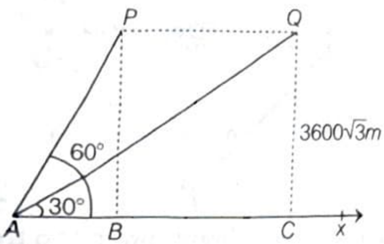
$$\Rightarrow \quad \tan\theta = \frac{h}{2x + x} = \frac{h}{3x}$$

Ans. 30°

22. Let the point on ground be A.

Since, jet plane is flying at a height of $3600\sqrt{3}$ m.

$$BP = CQ = 3600\sqrt{3}\text{ m}$$



We need to find speed of jet plane.

To find speed, we need to find distance travelled by jet plane in 30 sec i.e. BC

Let $\quad BC = x$

In ΔABP, $\tan A = \dfrac{BP}{AB}$ $\quad\left[\because \tan\theta = \dfrac{\text{perpendicular}}{\text{base}}\right]$

$\Rightarrow \quad \tan 60° = \dfrac{3600\sqrt{3}}{AB}$

$\Rightarrow \quad \sqrt{3} = \dfrac{3600\sqrt{3}}{AB}$ $\quad[\because \tan 60° = \sqrt{3}]$

$\Rightarrow \quad AB = 3600$ m

In ΔACQ, $\tan A = \dfrac{CQ}{AC}$

$\tan 30° = \dfrac{3600\sqrt{3}}{AB+BC}$ $\quad[\because AC = AB + BC]$

$\Rightarrow \quad \dfrac{1}{\sqrt{3}} = \dfrac{3600\sqrt{3}}{3600+x}$ $\quad\left[\because \tan 30° = \dfrac{1}{\sqrt{3}}\right]$

$\Rightarrow \quad 3600 + x = 3600 \times 3 \Rightarrow 3600 + x = 10800$

$\Rightarrow \quad x = 10800 - 3600 = 7200$

$\Rightarrow \quad BC = 7200$ m

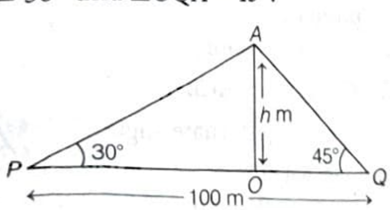
Hence, the distance travelled by jet plane in 30 sec $= 7200$ m

Now, speed of jet plane $= \dfrac{\text{Distance}}{\text{Time}} = \dfrac{7200\text{ m}}{30\text{ sec}} = 240$ m/sec

Thus, the speed of jet plane is 240 m/sec.

23. (i) **Hint** Let OA be the tree of height h m.

Given, $PQ = 100$ m and angles of elevation are $\angle APO = 30°$ and $\angle OQA = 45°$.



In right angled ΔPOA,

$$\tan 30° = \frac{P}{B} = \frac{OA}{OP}$$

$\Rightarrow \quad \dfrac{1}{\sqrt{3}} = \dfrac{h}{OP}$ $\quad\left[\because \tan 30° = \dfrac{1}{\sqrt{3}}\right]$

$\Rightarrow \quad OP = \sqrt{3}h$ $\quad$ ...(i)

Now, in right angled ΔQOA,

$$\tan 45° = \frac{OA}{OQ}$$

$\Rightarrow \quad 1 = \dfrac{h}{OQ}$ $\quad[\because \tan 45° = 1]$

$\Rightarrow \quad OQ = h$ $\quad$ ...(ii)

On adding Eqs. (i) and (ii), we get

$$OP + OQ = \sqrt{3}h + h$$

$\Rightarrow \quad PQ = (\sqrt{3}+1)h$ $\quad[\because OP + OQ = PQ]$

$\Rightarrow \quad 100 = (\sqrt{3}+1)h$ $\quad[\because PQ = 100\text{ m, given}]$

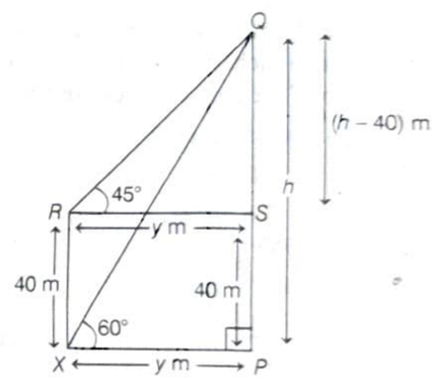
Ans. (i) 36.6 m, (ii) 99.99 m, (iii) Point P

24. Hint Let h be the height of tower,

i.e. $\quad PQ = h$ m and let $PX = y$ m

Now, draw $RS \parallel XP$.

Then, we have $RX = 40\text{ m} = SP$, $\angle QXP = 60°$ and $\angle QRS = 45°$



In right angled ΔXPQ,

$$\tan 60° = \frac{P}{B} = \frac{PQ}{XP}$$

$\Rightarrow \quad \dfrac{\sqrt{3}}{1} = \dfrac{h}{y}$ $\quad[\because \tan 60° = \sqrt{3}]$

$\Rightarrow \quad y = \dfrac{h}{\sqrt{3}}$ $\quad$ ...(i)

In right angled ΔRSQ,

$$\tan 45° = \frac{QS}{RS} \Rightarrow \tan 45° = \frac{PQ - SP}{XP}$$

$[\because QS = PQ - SP \text{ and } RS = XP]$

$\Rightarrow \quad 1 = \dfrac{h-40}{y}$ $\quad[\because \tan 45° = 1]$

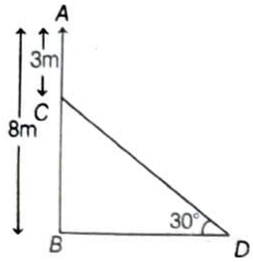
$\Rightarrow \quad y = h - 40$ $\quad$ ...(ii)

Now, solve Eqs. (i) and (ii), to find h and y.

Ans. $PQ = 94.64$ m and $PX = 54.64$ m

25. (i) Let AB be the pole and C be the point 3 m below the top A of the pole. Also, let D be the point on the ground, where ladder CD be placed.

So, $\angle CDB = 30°$ and $BC = (8-3) = 5$ m.



In ΔBCD,

$$\sin 30° = \frac{BC}{CD} \Rightarrow \frac{1}{2} = \frac{5}{CD} \Rightarrow CD = 10 \text{ m}$$

(ii) $\tan 30° = \frac{BC}{BD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{5}{BD} \Rightarrow BD = 5\sqrt{3}$ m

(iii) $\therefore$ Area of $\Delta BCD = \frac{1}{2} \times BD \times BC$

$$= \frac{1}{2} \times 5\sqrt{3} \times 5 = \frac{25\sqrt{3}}{2} \text{ m}$$

26. Hint Let the height of the tower OP be H and R be a point on the plane such that the angle of elevation of the bottom and top of the flag staff are α and β respectively.

Given, height of flag staff, say $PF = h$

Now, $\angle PRO = \alpha$, $\angle FRO = \beta$

Clearly, in right angled ΔPOR,

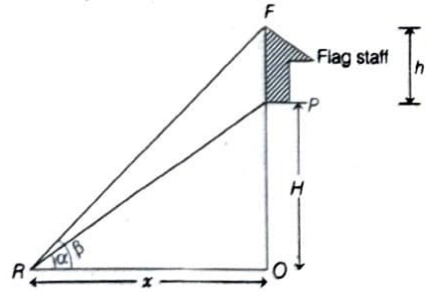
$$\tan\alpha = \frac{PO}{RO} = \frac{H}{x}$$

$$\Rightarrow \quad x = \frac{H}{\tan\alpha} \quad \text{...(i)}$$

and in right angled ΔFOR,

$$\tan\beta = \frac{FO}{RO} = \frac{FP + PO}{RO} = \frac{h+H}{x}$$

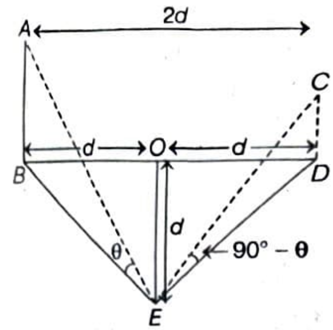
$$\Rightarrow \quad x = \frac{h+H}{\tan\beta} \quad \text{...(ii)}$$



From Eqs. (i) and (ii), we get

$$\frac{H}{\tan\alpha} = \frac{h+H}{\tan\beta}$$

27.



Since, Ajay stands midway between two trees

$\therefore$ $BO = OD = d$ m.

Now, in ΔBOE right angle at O

$$BE^2 = BO^2 + OE^2 \quad \text{[by Pythagoras theorem]}$$

$$BE^2 = d^2 + d^2 = 2d^2$$

$$BE = \sqrt{2}d \quad \text{...(i)}$$

Similarly, $DE = \sqrt{2}d$ ...(ii)

Let $DC = h$ m, ...(iii)

then $AB = 3h$ m ...(iv)

In ΔEBA, $\tan\theta = \frac{AB}{BE}$

$$\tan\theta = \frac{3h}{\sqrt{2}d} \quad \text{[from Eq. (i) and (iv)]}$$

Similarly, in right ΔEDC,

$$\tan(90° - \theta) = \frac{CD}{ED} \quad [\because \tan(90° - \theta) = \cot\theta]$$

$$\Rightarrow \quad \cot\theta = \frac{h}{\sqrt{2}d} \quad \text{[from Eqs. (ii) and (iii)]}$$

On multiplying $\tan\theta$ and $\cot\theta$, we get

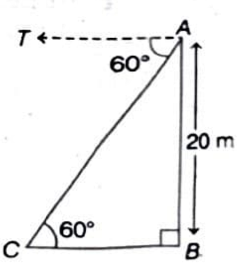
$$\tan\theta \times \cot\theta = \frac{3h}{\sqrt{2}d} \times \frac{h}{\sqrt{2}d}$$

$$1 = \frac{3h^2}{2d^2} \quad [\because \tan\theta \times \cot\theta = 1]$$

$$\Rightarrow \quad \frac{2}{3}d^2 = h^2$$

Thus, $h = d \times \sqrt{\frac{2}{3}}$

28. Hint Let $AB = 20$ m be the height of tower and let the ball lying on the ground at point C.



Given, angle of depression,

$$\angle TAC = 60° = \angle ACB$$

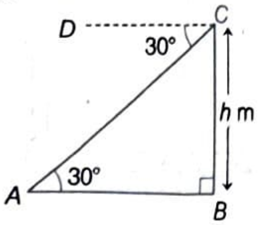
[alternate angles]

In right angled ΔABC,

$$\tan 60° = \frac{P}{B} = \frac{AB}{BC}$$

Ans. 11.55 m

29. Hint Let C be the position of peacock and A be the position of serpent.



Given, $\angle DCA = 30°$

$\Rightarrow \quad \angle BAC = \angle DCA = 30°$ [alternate angles]

$\because$ Distance = Speed × Time

$\therefore \quad AC = 300 \times \frac{12}{60}$ $\left[\because 12\,s = \frac{12}{60}\text{ min}\right]$

$\Rightarrow \quad AC = 60$ m

In right angled ΔABC,

$$\sin 30° = \frac{P}{H} = \frac{BC}{AC} = \frac{h}{60}$$

Ans. 30 m

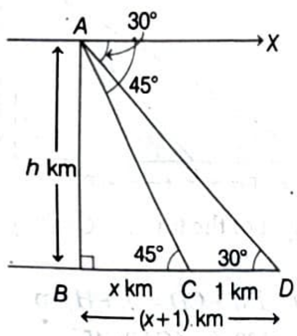
30. Hint Let $AB = h$ km be the height of the hill and C, D be two consecutive stones such that $CD = 1$ km

Let BC be x km, then

$$BD = BC + CD = (x+1) \text{ km}$$

Now, $\angle ADB = \angle XAD = 30°$ [alternate angles]

and $\angle ACB = \angle XAC = 45°$ [alternate angles]



In right angled ΔABC,

$$\tan 45° = \frac{P}{B} = \frac{AB}{BC} \Rightarrow 1 = \frac{h}{x} \Rightarrow x = h$$

Now, in right angled ΔABD,

$$\tan 30° = \frac{AB}{BD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x+1} \quad \left[\because \tan 30° = \frac{1}{\sqrt{3}}\right]$$

Ans. $\left(\frac{\sqrt{3}+1}{2}\right)$ km

31. Hint Do same as Question 13 of NCERT Folder Exercise 9.1. **Ans.** 115.47m

32. Hint Do same as Example 12.
Ans. 315.48 m.

33. Hint Do same as Example 11.
Ans. 33.33 m, 28.9 m

34. Speed of ship = 36 km/h

Time taken = 2 min

$\therefore$ Distance covered by ship in 2 min $= 36 \times \frac{2}{60} = 1.2$ km

Distance of ship from foot of lighthouse $= 1.3 - 1.2$
$= 0.1$ km
$= 100$ m

Let angle of elevation of the ship to the top of the lighthouse $= y°$

Then, $\tan y = \frac{173.2}{100} = 1.732$

$\Rightarrow \quad \tan y = \sqrt{3}$ $[\because 1.732 = \sqrt{3}]$

$\Rightarrow \quad y = 60°$ $[\because \tan\sqrt{3} = \tan 60°]$

Now, $y = x°$ [$\because$ alternate interior angles are equal]

Thus, $x° = 60°$

35. Hint Let QS be the building and R be the position of window.

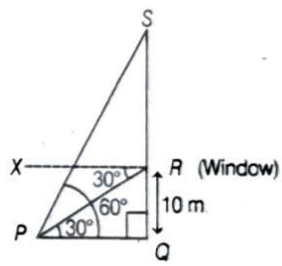
Given, height of the window, $QR = 10$ m

$\angle QPR = \angle XRP = 30°$ [alternate angles]

and $\angle SPQ = 60°$

In right angled ΔPQR,

$$\tan 30° = \frac{P}{B} = \frac{QR}{PQ}$$



$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{10}{PQ}$ $\left[\because \tan 30° = \frac{1}{\sqrt{3}}\right]$

$\Rightarrow \quad PQ = 10\sqrt{3}$ ...(i)

In right angled ΔPQS,

$$\tan 60° = \frac{QS}{PQ} \Rightarrow \sqrt{3} = \frac{QS}{10\sqrt{3}}$$

[$\because \tan 60° = \sqrt{3}$ and from Eq. (i)]

Ans. 30 m

36. Hint Let a man is standing on the deck of a ship at point A such that $AB = 10$ m and let CD be the hill.

Then, $\angle EAD = 60°$

and $\angle CAE = \angle BCA = 30°$ [alternate angles]

Let $BC = x$ m $= AE$ and $DE = h$ m

In right angled ΔAED,

$$\tan 60° = \frac{P}{B} = \frac{DE}{EA} = \frac{h}{x}$$

In right angled ΔABC,

$$\tan 30° = \frac{AB}{BC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{10}{x} \qquad \left[\because \tan 30° = \frac{1}{\sqrt{3}}\right]$$

Ans. Distance of the hill from the ship is $10\sqrt{3}$ m and height of the hill is 40 m.

37. Hint Let W denotes the window and OQ denotes the height of the other house.

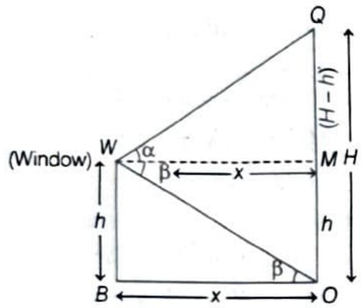
Now, draw $WM \parallel OB$.

Given, height of the window $= WB = h$ m

Then, $OM = BW = h$ m,

$\angle QWM = \alpha$ and $\angle OWM = \beta = \angle WOB$

[alternate angles]



Now, in right angled ΔWBO,

$$\tan\beta = \frac{WB}{OB} = \frac{h}{x}$$

$$\Rightarrow \quad x = \frac{h}{\tan\beta} \qquad \text{...(i)}$$

and in right angled ΔQMW,

$$\tan\alpha = \frac{QM}{WM} = \frac{OQ - MO}{WM}$$

$$\Rightarrow \quad \tan\alpha = \frac{H-h}{x}$$

$$\Rightarrow \quad x = \frac{H-h}{\tan\alpha} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

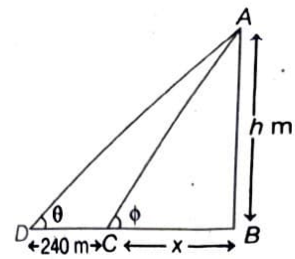
$$\frac{h}{\tan\beta} = \frac{H-h}{\tan\alpha}$$

$$\therefore \quad H = h\left(1 + \tan\alpha \cdot \frac{1}{\tan\beta}\right)$$

$$= h\,(1 + \tan\alpha \cdot \cot\beta) \qquad \left[\because \frac{1}{\tan\theta} = \cot\theta\right]$$

38. Hint Let the height of lighthouse AB be h m and C, D be two points of observer, such that

$\angle ADB = \theta, \angle ACB = \phi$ and $DC = 240$ m.



Let $BC = x$ m.

In right angled ΔABC,

$$\tan\phi = \frac{AB}{BC} \Rightarrow \frac{3}{4} = \frac{AB}{BC} \qquad \left[\because \tan\phi = \frac{3}{4}, \text{ given}\right]$$

$$\Rightarrow \quad h = \frac{3}{4} \times x \qquad \text{...(i)}$$

and in right angled ΔABD,

$$\tan\theta = \frac{AB}{BD} \Rightarrow \frac{5}{12} = \frac{AB}{DC + CB}$$

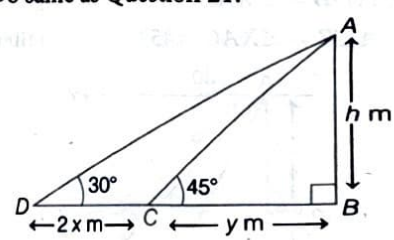
$$\left[\because \tan\theta = \frac{5}{12}, \text{ given and } BD = DC + CB\right]$$

$$\Rightarrow \quad \frac{5}{12} = \frac{h}{240 + x} \qquad \text{...(ii)}$$

Now, solve Eqs. (i) and (ii) to find h.

Ans. 225 m

39. Hint Do same as Question 21.

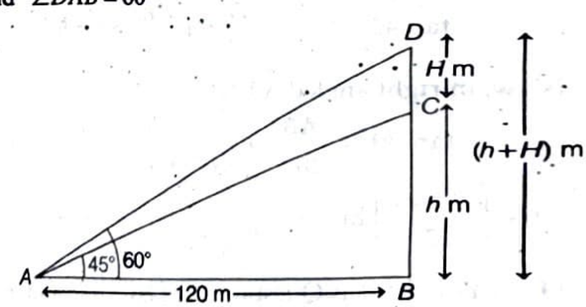


40. Hint Let height of the tower, $BC = h$ m and height of the flag staff $CD = H$ m.

$$\therefore \quad BD = BC + CD = (h + H) \text{ m} \qquad \text{...(i)}$$

Given, $AB = 120$ m, $\angle CAB = 45°$

and $\angle DAB = 60°$



In right angled ΔABC, we get

$$\tan 45° = \frac{BC}{AB} \qquad \left[\because \tan\theta = \frac{P}{B}\right]$$

$$\Rightarrow \quad h = 120 \text{ m} \qquad \text{...(ii)}$$

Now, in right angled ΔABD, we get

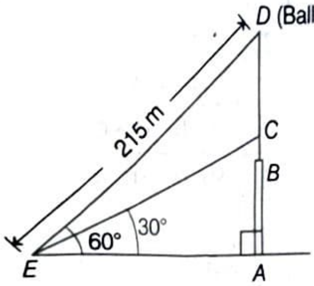
$$\tan 60^\circ = \frac{BD}{AB}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h+H}{120} \quad \text{...(iii)}$$

$[\because \tan 60^\circ = \sqrt{3}$ and from Eq. (i)$]$

Now, solve Eq. (iii) using Eq. (ii). **Ans.** 87.84 m

41. Hint Let D be the position of the balloon, when it is inclined at angle of 60° and AB be the height of the pole.



Given, length of cable,

$$DE = 215 \text{ m}$$

In right angled ΔEAD,

$$\sin 60^\circ = \frac{P}{H} = \frac{AD}{ED} \Rightarrow AD = \frac{215\sqrt{3}}{2} \text{ m}$$

Hence, height of the balloon from the ground is $\frac{215\sqrt{3}}{2}$ m.

Again, in right angled ΔEAD,

$$\cos 60^\circ = \frac{B}{H} = \frac{AE}{DE} = \frac{AE}{215}$$

$$\Rightarrow \quad AE = \frac{215}{2} \text{ m} \quad \text{...(i)}$$

Now, the angle of inclination is changed, say $\angle CEA = 30^\circ$.

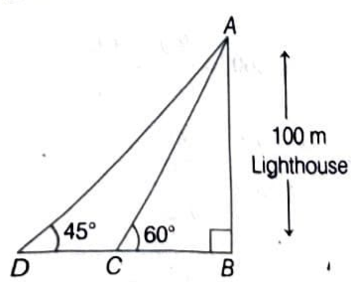
In right angled ΔEAC,

$$\tan 30^\circ = \frac{P}{B} = \frac{AC}{EA} \quad \textbf{Ans. } \frac{215}{2\sqrt{3}} \text{ m}$$

42. Hint Let the height of the lighthouse AB be 100 m and C, D be the positions of man when angle of elevation changes from 60° to 45°, respectively. The man has covered a distance CD in 2 min.

$$\because \quad \text{Speed} = \frac{\text{Distance}}{\text{Time}} \Rightarrow \text{Speed} = \frac{CD}{2} \quad \text{...(i)}$$

In right angled ΔABC,



$$\tan 60^\circ = \frac{P}{B} = \frac{AB}{BC}$$

$$\Rightarrow \quad BC = \frac{100\sqrt{3}}{3} \text{ m} \quad \text{...(ii)}$$

In right angled ΔABD,

$$\tan 45^\circ = \frac{AB}{BD} \Rightarrow BD = 100 \text{ m}$$

Now, $\quad CD = BD - BC$

and $\quad \text{speed} = \frac{CD}{2} = \frac{100\left(\frac{3-\sqrt{3}}{3}\right)}{2}$

Ans. $\frac{50}{3}(3-\sqrt{3})$ m/min

43. Hint Do same as Question 9 of NCERT folder Exercise 9.1. **Ans** 20 m

44. Hint Let F be the foot and S be the summit of the mountain FOS. Also, let the height of mountain be h km.

Then, $\angle OFS = 45^\circ$ and so $\angle OSF = 45^\circ$

$$\Rightarrow \quad OF = OS = h \text{ km}$$

[$\because$ sides opposite to equal angles are equal]

Let $FP = 1000$ m $= 1$ km be the slope, so that

$$\angle OFP = 30^\circ$$

Draw $PM \perp OS$ and $PL \perp OF$

Join PS.

Clearly, $\angle MPS = 60^\circ$ [given]

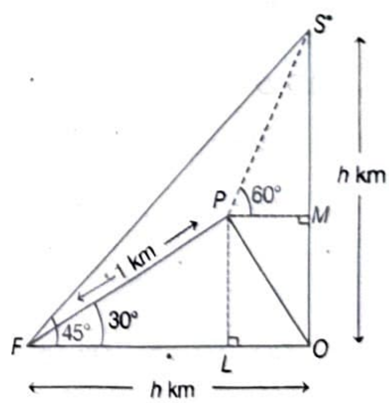
Now, in right angled ΔFLP,

$$\sin 30^\circ = \frac{PL}{PF}$$

$$\Rightarrow \quad PL = PF \sin 30^\circ = 1 \times \frac{1}{2} = \frac{1}{2} \text{ km} \quad \left[\because \sin 30^\circ = \frac{1}{2}\right]$$

$$\Rightarrow \quad OM = PL = \frac{1}{2} \text{ km}$$

Now, $SM = OS - OM = \left(h - \frac{1}{2}\right)$ km $\quad$ ...(i)



Also, in right angled ΔFLP,

$$\cos 30° = \frac{FL}{PF}$$

$$\Rightarrow \quad FL = PF\cos 30° = \left(1 \times \frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{2} \text{ km}$$

Now, $\quad OL = OF - FL \qquad \left[\because LF = \frac{\sqrt{3}}{2}\right]$

$$\Rightarrow \quad OL = \left(h - \frac{\sqrt{3}}{2}\right)$$

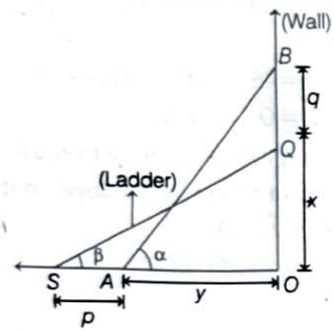
$$\Rightarrow \quad PM = h - \frac{\sqrt{3}}{2} \qquad [\because OL = PM] \ldots\text{(ii)}$$

In right angled ΔSMP,

$$\tan 60° = \frac{SM}{PM}$$

Now, put value of SM and PM from Eqs. (i) and (ii) and hence find h. **Ans.** 1.366 km

45. (i) Let AB and SQ be two positions of the ladder making inclination α and β with the horizontal respectively.



Let $OQ = x$ and $OA = y$

Given that $BQ = q$ and $SA = p$

Clearly, now in ΔBAO,

$$\cos\alpha = \frac{OA}{AB}$$

$$\Rightarrow \quad \cos\alpha = \frac{y}{AB}$$

$$\Rightarrow \quad y = AB\cos\alpha = OA \qquad \ldots\text{(i)}$$

and $\quad \sin\alpha = \dfrac{OB}{AB}$

$$\Rightarrow \quad OB = BA\sin\alpha \qquad \ldots\text{(ii)}$$

Now, in ΔQSO,

$$\cos\beta = \frac{OS}{SQ}$$

$$\Rightarrow \quad OS = SQ\cos\beta = AB\cos\beta$$

$$[\because AB = SQ] \ldots\text{(iii)}$$

and $\quad \sin\beta = \dfrac{OQ}{SQ}$

$$\Rightarrow \quad OQ = SQ\sin\beta = AB\sin\beta$$

$$[\because AB = SQ] \ldots\text{(iv)}$$

Now, $\quad SA = OS - AO$

$$p = AB\cos\beta - AB\cos\alpha$$

[from Eqs. (i) and (iii)]

$$\Rightarrow \quad p = AB(\cos\beta - \cos\alpha) \qquad \ldots\text{(v)}$$

and $\quad BQ = BO - QO$

$$\Rightarrow \quad q = BA\sin\alpha - AB\sin\beta$$

[from Eqs. (ii) and (iv)]

$$\Rightarrow \quad q = AB(\sin\alpha - \sin\beta) \qquad \ldots\text{(vi)}$$

On dividing Eq. (v) by Eq. (vi), we get

$$\frac{p}{q} = \frac{AB(\cos\beta - \cos\alpha)}{AB(\sin\alpha - \sin\beta)} = \frac{\cos\beta - \cos\alpha}{\sin\alpha - \sin\beta}$$

$$\Rightarrow \quad \frac{p}{q} = \frac{\cos\beta - \cos\alpha}{\sin\alpha - \sin\beta}$$

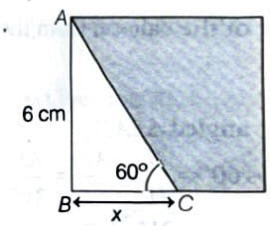
Hence proved.

(ii) No

46. In ΔABC, $\tan 60° = \dfrac{AB}{BC}$

$$\Rightarrow \quad \sqrt{3} = \frac{6}{x}$$

$$\therefore \quad x = \frac{6}{\sqrt{3}} = 2\sqrt{3} \text{ cm}$$



So, area of $\Delta ABC = \frac{1}{2} \times 6 \times 2\sqrt{3} = 6\sqrt{3} \text{ cm}^2$

Hence, area of shaded region

$= $ Area of square $-$ Area of ΔABC

$= 36 - 6\sqrt{3} \text{ cm}^2$

$= 6(6 - \sqrt{3}) \text{ cm}^2$

47. In ΔPQU, $\sin 30° = \dfrac{QU}{4}$

$$\Rightarrow \quad QU = 2 \text{ cm}$$

In ΔPQU,

$$\angle PQU + \angle UPQ + \angle PUQ = 180°$$

$$\Rightarrow \quad 90° + 30° + \angle PUQ = 180°$$

$$\Rightarrow \quad \angle PUQ = 60°$$

In ΔQRU,

$$\cos 60° = \frac{RU}{QU} = \frac{RU}{2}$$

$$\Rightarrow \quad RU = 1 \text{ cm}$$

and $\quad \tan 60° = \dfrac{RQ}{RU}$

$$\Rightarrow \quad \sqrt{3} = \frac{RQ}{1}$$

$\Rightarrow \quad RQ = \sqrt{3}\text{ cm} = PS$

$\therefore \quad PS + SR + RU = 4\text{ cm}$

$\Rightarrow \quad SR = 4 - PS - RU$

$= 4 - \sqrt{3} - 1$

$\Rightarrow \quad SR = (3 - \sqrt{3})\text{ cm}$

Ans. $(3 - \sqrt{3})$ cm

48. **Hint** Let $CD = h$ m be the height of the lighthouse and A, B be two ships on either side of lighthouse, such that their angles of depression are 60° and 45°, respectively.

i.e. $\quad \angle XCB = \angle CBD = 45^\circ \quad$ [alternate angles]

and $\quad \angle YCA = \angle CAD = 60^\circ \quad$ [alternate angles]

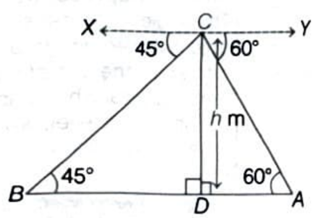
Given, the distance between the ships A and B is

$$AB = \frac{100(\sqrt{3}+1)}{\sqrt{3}}\text{ m}$$

In right angled ΔBDC,

$$\tan 45^\circ = \frac{CD}{BD} \qquad \left[\because \tan\theta = \frac{P}{B}\right]$$

$$\Rightarrow \quad BD = CD \qquad \text{...(i)}$$



and in right angled ΔADC

$$\tan 60^\circ = \frac{CD}{AD}$$

$$\Rightarrow \quad CD = \sqrt{3}\,AD$$

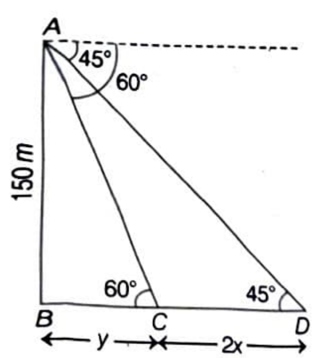
$$\Rightarrow \quad AD = \frac{CD}{\sqrt{3}} \qquad \text{...(ii)}$$

Ans. 100 m

49. Let the speed of boat be x m/min.

$\therefore CD = 2x$

In ΔABC,



$$\frac{150}{y} = \tan 60^\circ$$

$$\Rightarrow \quad y = \frac{150}{\sqrt{3}} = 50\sqrt{3}\text{ m}$$

In ΔABD,

$$\frac{150}{y + 2x} = \tan 45^\circ$$

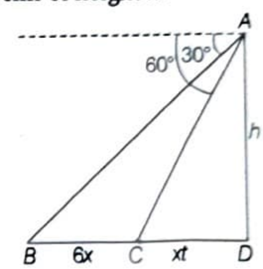
$$\Rightarrow \quad 150 = 50\sqrt{3} + 2x$$

$$\Rightarrow \quad x = 25(3 - \sqrt{3})\text{ m/min}$$

$$= 25 \times 60(3 - \sqrt{3})\text{ m/h}$$

$$= 1500(3 - \sqrt{3})\text{ m/h}$$

50. Let AD be a cliff of height h.



Let the speed of the boat is x m/min.

Let time taken by the boat to reach the shore be t.

In ΔADC,

$$\tan 30^\circ = \frac{xt}{h}$$

[$\because$ distance covered by the boat at a uniform speed of x m/min in t time is xt]

$$\frac{1}{\sqrt{3}} = \frac{xt}{h}$$

$$h = \sqrt{3}xt \qquad \text{...(i)}$$

In ΔABD,

$$\tan 60^\circ = \frac{x(t+6)}{h} \qquad \left[\therefore \tan\theta = \frac{P}{B}\right]$$

[$\therefore$ distance covered by the boat at a uniform speed of x m/min in 6 min is $6x$]

$$\sqrt{3} = \frac{x(t+6)}{h}$$

$$h = \frac{x(t+6)}{\sqrt{3}} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\sqrt{3}xt = \frac{xt + 6x}{\sqrt{3}}$$

$$3xt = xt + 6x$$

$$3t = t + 6$$

$$2t = 6$$

$$t = 3\text{ sec}$$

Therefore, total time taken by the boat to reach the shore is $6 + 3 = 9$ min.

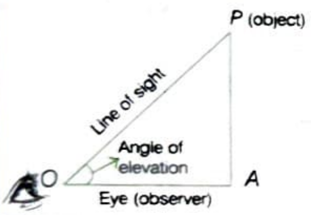
Mind Map

SOME APPLICATIONS OF TRIGONOMETRY

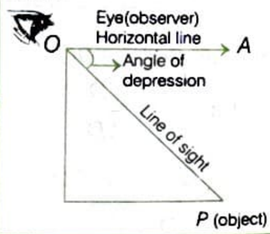
Measuring height or length of an object or the distance between two distant objects is an important application of trigonometry. It can be determined with the help of trigonometric ratios.

Line of Sight
The line of sight is the line drawn from eye of an observer to the point in the object viewed by observer.

Angle of Elevation
The angle of elevation of an object viewed, is the angle formed by the line of sight with the horizontal when it is above the horizontal level,
i.e. the case when we raise our head to look at the object.



Angle of Depression
The angle of depression of an object viewed, is the angle formed by the line of sight with the horizontal when it is below the horizontal level, when we lower our head to look at the object.



CHAPTER 10

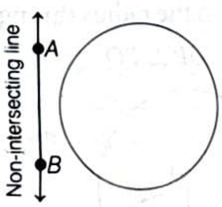
Circles

In previous classes, we have studied about circle which is a collection of all point in a plane and which are at a constant distance (radius) from a fixed point (centre) and various terms related to it like chord, segment, sector, arc, etc. In this chapter, we will discuss different situations that can arise when a circle and a line are given in a plane.

TOPIC 01 Tangent to a Circle

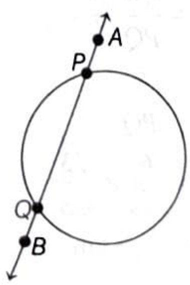
Consider, a line AB and a circle. Now, there can be three possible cases which can arise according to the position of line AB with respect to circle, which are given below.

Case I When there is no common point between a circle and a line AB.



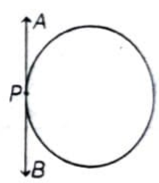
In this case, the line AB is said to be **non-intersecting line** with respect to a circle.

Case II When there are two common points P and Q between a circle and a line AB.



In this case, line AB intersects the circle at two points, so this line is called **secant** of the circle.

Case III When there is only one common point between a circle and a line AB.



In this case, the line AB is said to be the **tangent** to the circle at point P.

Tangent to a Circle

A tangent to a circle is a line that intersects or touches the circle at only one point. The common point of the tangent and the circle is called the **point of contact**.

Also, we can say that the tangent to a circle is a special case of the secant, when the two end points of its corresponding chord coincide. There is only one tangent at a point of the circle.