CHAPTER 8

Introduction to Trigonometry

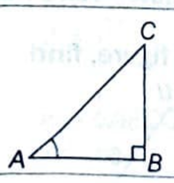
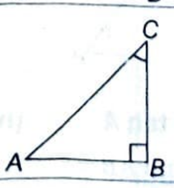
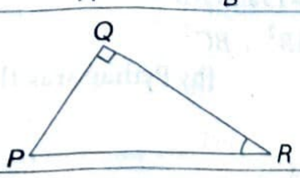
The word **trigonometry** is derived from the Greek words 'tri ' (meaning three), 'gon' (meaning sides) and 'metron' (meaning measure).

Trigonometry is the study of the relationships between the sides and angles of a triangle. In this chapter, we will study ratios of the sides of a right angled triangle with respect to its acute angles. First, we will define the values of trigonometric ratios for angles of measure between 0° and 90°. Then, we will also calculate trigonometric ratios for some specific angles and establish some identities involving these ratios, which are called trigonometric identities.

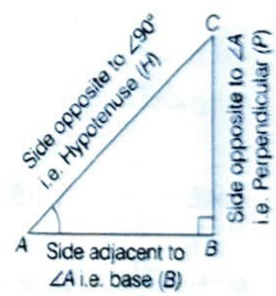
TOPIC 01 Trigonometric Ratios

The ratios of the sides of a right angled triangle with respect to its acute angles, are called **trigonometric ratios.** Trigonometric ratios are also called ***T*-ratios.**

To understand the trigonometric ratios, first of all we will understand the concept of perpendicular, base and hypotenuse in a right angled triangle. For any acute angle (which is also known as the **angle of reference**) in a right angled triangle, the side opposite to the acute angle is called the **perpendicular** (P), the side adjacent to this acute angle is called the **base** (B) and side opposite to the right angle is called the **hypotenuse** (H).

(i)		Angle of reference = $\angle A$ Perpendicular = BC Base = AB Hypotenuse = AC
(ii)		Angle of reference = $\angle C$ Perpendicular = AB Base = BC Hypotenuse = AC
(iii)		Angle of reference = $\angle R$ Perpendicular = PQ Base = QR Hypotenuse = PR

Trigonometric ratios of $\angle A$ in right angled ΔABC are defined as follows:



(i) sine of $\angle A$ or $\sin A = \dfrac{\text{Perpendicular}}{\text{Hypotenuse}} = \dfrac{P}{H} = \dfrac{BC}{AC}$

(ii) cosine of $\angle A$ or $\cos A = \dfrac{\text{Base}}{\text{Hypotenuse}} = \dfrac{B}{H} = \dfrac{AB}{AC}$

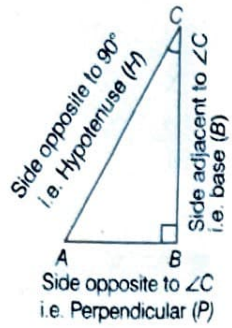
(iii) tangent of $\angle A$ or $\tan A = \dfrac{\text{Perpendicular}}{\text{Base}} = \dfrac{P}{B} = \dfrac{BC}{AB}$

(iv) cosecant of $\angle A$ or $\operatorname{cosec} A = \dfrac{\text{Hypotenuse}}{\text{Perpendicular}} = \dfrac{H}{P} = \dfrac{AC}{BC}$

(v) secant of $\angle A$ or $\sec A = \dfrac{\text{Hypotenuse}}{\text{Base}} = \dfrac{H}{B} = \dfrac{AC}{AB}$

(vi) cotangent of $\angle A$ or $\cot A = \dfrac{\text{Base}}{\text{Perpendicular}} = \dfrac{B}{P} = \dfrac{AB}{BC}$

Similarly, trigonometric ratios of $\angle C$ in right angled ΔABC are as follows :



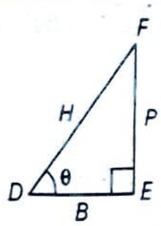
(i) $\sin C = \dfrac{AB}{AC}$ (ii) $\cos C = \dfrac{BC}{AC}$

(iii) $\tan C = \dfrac{AB}{BC}$ (iv) $\operatorname{cosec} C = \dfrac{AC}{AB}$

(v) $\sec C = \dfrac{AC}{BC}$ (vi) $\cot C = \dfrac{BC}{AB}$

A Popular Technique to Remember T-ratios

$\sin\theta$	$\cos\theta$	$\tan\theta$
Pandit (P)	Badari (B)	Prasad (P)
Har (H)	Har (H)	Bholay (B)



Then, $\sin\theta = \dfrac{P}{H}$, $\cos\theta = \dfrac{B}{H}$, $\tan\theta = \dfrac{P}{B}$,

$\operatorname{cosec}\theta = \dfrac{H}{P}$, $\sec\theta = \dfrac{H}{B}$, $\cot\theta = \dfrac{B}{P}$

where, P is perpendicular, B is base and H is hypotenuse.

Some Important Points

(i) The symbol $\sin A$ is used as an abbreviation for 'the sine of $\angle A$.' $\sin A$ is not the product of 'sin' and 'A'. 'sin' separated from 'A' has no meaning. This interpretation follows for other trigonometric ratios also.

(ii) Each trigonometric ratio is a real number and has no unit.

(iii) The value of each of the trigonometric ratio of an angle does not depend on the size of the triangle. It depends only on the angle.

(iv) The values of the trigonometric ratios of an angle do not vary with the lengths of the sides of the triangle, if the angle remains the same.

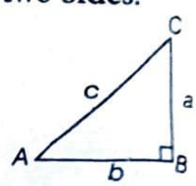
(v) If one of the trigonometric ratios of an acute angle is known, then the remaining trigonometric ratios of that angle can be easily determined.

(vi) In an isosceles right angled ΔABC, right angled at B, the trigonometric ratios obtained by taking either $\angle A$ or $\angle C$, gives the same value.

(vii) As we know that the hypotenuse is the longest side in a right angled triangle, so the value of $\sin A$ or $\cos A$ is always less than 1 (or in particular case equal to 1), whereas the value of $\sec A$ or $\operatorname{cosec} A$ is always greater than or equal to 1.

Pythagoras Theorem

In right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.



In ΔABC, $AC^2 = BC^2 + AB^2$

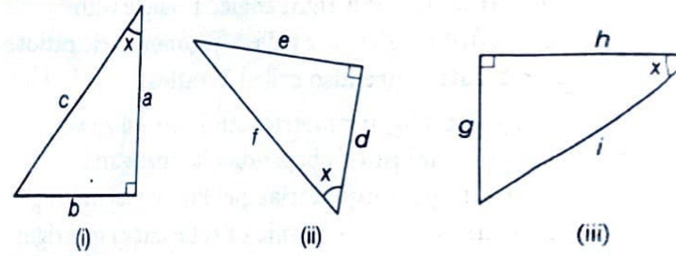
$\Rightarrow$ $c^2 = a^2 + b^2$

or $a^2 = c^2 - b^2$

or $b^2 = c^2 - a^2$

Note It (i) $(\sin A)^2 = \sin^2 A \neq \sin A^2$ (ii) $(\sin A)^{-1} = \dfrac{1}{\sin A} \neq \sin^{-1} A$

Example 1. **Indicate the perpendicular, the hypotenuse and the base (in that order) with respect to the angle marked x.**



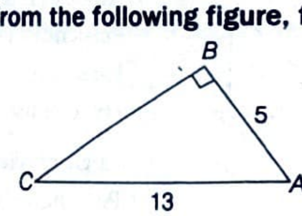
Sol. With reference to angle x,

(i) perpendicular $= b$, hypotenuse $= c$ and base $= a$

(ii) perpendicular $= e$, hypotenuse $= f$ and base $= d$

(iii) perpendicular $= g$, hypotenuse $= i$ and base $= h$

Example 2. **From the following figure, find**



(i) sin A **(ii) cos C** **(iii) tan A** **(iv) cosec C**

Sol. Given, $\angle ABC = 90°, AC = 13$ and $AB = 5$

Now, in ΔABC, $AC^2 = AB^2 + BC^2$ [by Pythagoras theorem]

$\Rightarrow$ $13^2 = 5^2 + BC^2$

$\Rightarrow$ $BC^2 = 169 - 25 = 144$

$\therefore$ $BC = \sqrt{144} = 12$ [taking positive square root]

(i) $\sin A = \dfrac{\text{Perpendicular}}{\text{Hypotenuse}} = \dfrac{BC}{AC} = \dfrac{12}{13}$

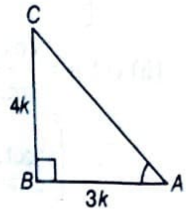
(ii) $\cos C = \dfrac{\text{Base}}{\text{Hypotenuse}} = \dfrac{BC}{AC} = \dfrac{12}{13}$

(iii) $\tan A = \dfrac{\text{Perpendicular}}{\text{Base}} = \dfrac{BC}{AB} = \dfrac{12}{5}$

(iv) $\text{cosec } C = \dfrac{\text{Hypotenuse}}{\text{Perpendicular}} = \dfrac{AC}{AB} = \dfrac{13}{5}$

Example 3. If $\tan A = \dfrac{4}{3}$, find the other trigonometric ratios of $\angle A$.

Sol. First, draw a ΔABC, right angled at B.



Given, $\tan A = \dfrac{4}{3}$

$\Rightarrow \dfrac{\text{Perpendicular}}{\text{Base}} = \dfrac{4}{3}$

$\Rightarrow \dfrac{BC}{AB} = \dfrac{4}{3}$

Let, $BC = 4k$ and $AB = 3k$ where, k is any positive integer.

Using Pythagoras theorem in ΔABC,

$$AC^2 = BC^2 + AB^2$$

$\Rightarrow AC^2 = (4k)^2 + (3k)^2 = 16k^2 + 9k^2$

$\Rightarrow AC^2 = 25k^2$

$\Rightarrow AC = \sqrt{25k^2} = 5k$ [taking positive square root]

Now, $\sin A = \dfrac{BC}{AC} = \dfrac{4k}{5k} = \dfrac{4}{5}$,

$\cos A = \dfrac{AB}{AC} = \dfrac{3k}{5k} = \dfrac{3}{5}$,

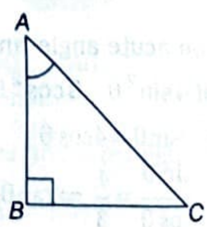
$\cot A = \dfrac{AB}{BC} = \dfrac{3k}{4k} = \dfrac{3}{4}$,

$\text{cosec } A = \dfrac{AC}{BC} = \dfrac{5k}{4k} = \dfrac{5}{4}$,

and $\sec A = \dfrac{AC}{AB} = \dfrac{5k}{3k} = \dfrac{5}{3}$.

Example 4. In a ΔABC, right angled at B, if $\tan A = 1$, verify that $2 \sin A \cos A = 1$.

Sol. Given, a ΔABC in which $\angle B = 90°$.



In ΔABC, $\tan A = \dfrac{\text{Perpendicular}}{\text{Base}} = \dfrac{BC}{AB} = 1$ [given]

$\Rightarrow BC = AB$

Let $AB = BC = k$, where k is a positive number.

Now, $AC = \sqrt{AB^2 + BC^2}$ [by Pythagoras theorem]

$= \sqrt{(k)^2 + (k)^2} = k\sqrt{2}$

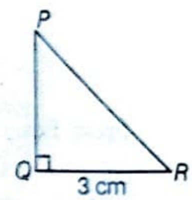
$\therefore \sin A = \dfrac{\text{Perpendicular}}{\text{Hypotenuse}} = \dfrac{BC}{AC} = \dfrac{k}{\sqrt{2}k} = \dfrac{1}{\sqrt{2}}$

and $\cos A = \dfrac{\text{Base}}{\text{Hypotenuse}} = \dfrac{AB}{AC}$

$= \dfrac{k}{\sqrt{2}k} = \dfrac{1}{\sqrt{2}}$

Now, $2 \sin A \cos A = 2\left(\dfrac{1}{\sqrt{2}}\right)\left(\dfrac{1}{\sqrt{2}}\right) = 1$ **Hence proved.**

Example 5. In ΔPQR right angled at Q, $QR = 3$ cm and $PR - PQ = 1$ cm. Determine the values of $\sin R$, $\cos R$ and $\tan R$.



Sol. Given, a ΔPQR in which $\angle Q = 90°$ and $QR = 3$ cm.

Also, $PR - PQ = 1$ ...(i)

On applying Pythagoras theorem in ΔPQR, we get

$$PR^2 = PQ^2 + QR^2$$

$\Rightarrow QR^2 = PR^2 - PQ^2$

$\Rightarrow (3)^2 = PR^2 - PQ^2$ [given $QR = 3$cm]

$\Rightarrow PR^2 - PQ^2 = 9$

$\Rightarrow (PR + PQ)(PR - PQ) = 9$ [$\because a^2 - b^2 = (a+b)(a-b)$]

$\Rightarrow (PR + PQ)(1) = 9$ [from Eq. (i)]

$\Rightarrow PR + PQ = 9$ ...(ii)

On adding Eqs. (i) and (ii), we get

$$PR - PQ + PR + PQ = 1 + 9$$

$\Rightarrow 2PR = 10 \Rightarrow PR = 5$ cm

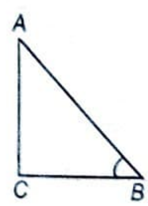
On substituting $PR = 5$ cm in Eq. (i), we get

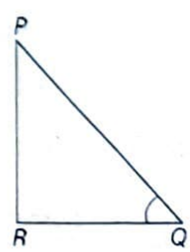
$$5 - PQ = 1 \Rightarrow PQ = 4 \text{ cm}$$

$\therefore PR = 5$ cm and $PQ = 4$ cm

Now, $\sin R = \dfrac{PQ}{PR} = \dfrac{4}{5}$, $\cos R = \dfrac{QR}{PR} = \dfrac{3}{5}$ and $\tan R = \dfrac{PQ}{QR} = \dfrac{4}{3}$

Example 6. If $\angle B$ and $\angle Q$ are acute angles such that $\sin B = \sin Q$, then prove that $\angle B = \angle Q$.





Sol. Let us consider two right angled triangles, ΔABC and ΔPQR, where $\sin B = \sin Q$.

We have, $\sin B = \frac{AC}{AB}$ and $\sin Q = \frac{PR}{PQ}$

Then, $\frac{AC}{AB} = \frac{PR}{PQ}$

Therefore, $\frac{AC}{PR} = \frac{AB}{PQ} = k$, say ...(i)

Now, using Pythagoras theorem,

$$BC = \sqrt{AB^2 - AC^2} \text{ and } QR = \sqrt{PQ^2 - PR^2}$$

So, $$\frac{BC}{QR} = \frac{\sqrt{AB^2 - AC^2}}{\sqrt{PQ^2 - PR^2}} = \frac{\sqrt{k^2PQ^2 - k^2PR^2}}{\sqrt{PQ^2 - PR^2}}$$

$$= \frac{k\sqrt{PQ^2 - PR^2}}{\sqrt{PQ^2 - PR^2}} = k \quad ...(ii)$$

From Eqs. (i) and (ii), we have

$$\frac{AC}{PR} = \frac{AB}{PQ} = \frac{BC}{QR}$$

Then, using SSS criterion $\Delta ACB \sim \Delta PRQ$ and therefore $\angle B = \angle Q$.

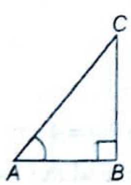
Relations between Trigonometric Ratios

The relations between trigonometric ratios can be studied under the two heads (categories) given below:

Reciprocal Relation

Relations between trigonometric ratios and their reciprocals are given below :

(i) $\sin A = \frac{1}{\operatorname{cosec} A}$ or $\operatorname{cosec} A = \frac{1}{\sin A}$



$$\left[\because \sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{1}{AC/BC} = \frac{1}{\operatorname{cosec} A}\right]$$

(ii) $\cos A = \frac{1}{\sec A}$ or $\sec A = \frac{1}{\cos A}$

$$\left[\because \cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{1}{AC/AB} = \frac{1}{\sec A}\right]$$

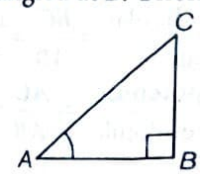
(iii) $\tan A = \frac{1}{\cot A}$ or $\cot A = \frac{1}{\tan A}$

$$\left[\because \tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{BC}{AB} = \frac{1}{AB/BC} = \frac{1}{\cot A}\right]$$

Note It (i) $\sin A \cdot \operatorname{cosec} A = 1$ (ii) $\cos A \cdot \sec A = 1$
(iii) $\tan A \cdot \cot A = 1$

Quotient Relation

Draw a ΔABC right angled at B. Then,



(i) $\tan A = \frac{\sin A}{\cos A}$

$$\left[\because \tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{BC}{AB} = \frac{BC/AC}{AB/AC} = \frac{\sin A}{\cos A}\right]$$

(ii) $\cot A = \frac{\cos A}{\sin A}$

$$\left[\because \cot A = \frac{\text{Base}}{\text{Perpendicular}} = \frac{AB}{BC} = \frac{AB/AC}{BC/AC} = \frac{\cos A}{\sin A}\right]$$

Example 7. If $m \cot A = n$, then find the value of $\frac{m \sin A - n \cos A}{n \cos A + m \sin A}$.

Sol. Given, $m \cot A = n$

$\Rightarrow \quad m \cdot \frac{1}{\tan A} = n \quad \left[\because \cot\theta = \frac{1}{\tan\theta}\right]$

$\Rightarrow \quad \tan A = \frac{m}{n}$ ...(i)

Now, $$\frac{m \sin A - n \cos A}{n \cos A + m \sin A} = \frac{m \cdot \frac{\sin A}{\cos A} - n}{n + m \cdot \frac{\sin A}{\cos A}}$$

[dividing numerator and denominator by $\cos A$]

$$= \frac{m \tan A - n}{n + m \tan A} = \frac{m \cdot \frac{m}{n} - n}{n + m \cdot \frac{m}{n}} \quad \text{[from Eq. (i)]}$$

$$= \frac{\frac{m^2 - n^2}{n}}{\frac{n^2 + m^2}{n}} = \frac{m^2 - n^2}{m^2 + n^2}$$

Example 8. If θ is an acute angle and $3\sin\theta = 4\cos\theta$, then find the value of $4\sin^2\theta - 3\cos^2\theta + 2$.

Sol. Given, equation is $3\sin\theta = 4\cos\theta$

$\Rightarrow \quad \frac{\sin\theta}{\cos\theta} = \frac{4}{3} \Rightarrow \tan\theta = \frac{4}{3}$ ...(i)

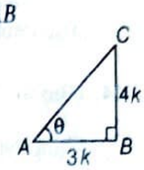
Now, draw a right angled ΔABC such that $\angle B = 90°$ and let $\angle A = \theta$.

From Eq. (i), $\tan\theta = \frac{4}{3} = \frac{\text{Perpendicular}}{\text{Base}} = \frac{BC}{AB}$

Let $BC = 4k$ and $AB = 3k$

where, k is a positive integer.

Using Pythagoras theorem, we have



$$CA^2 = AB^2 + BC^2$$

$\Rightarrow \quad CA^2 = (3k)^2 + (4k)^2 = 9k^2 + 16k^2$

$\Rightarrow \quad CA^2 = 25k^2$

$\Rightarrow \quad CA = 5k$ [taking positive square root both sides]

Now, $\quad \sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{4k}{5k} = \frac{4}{5}$

and $\quad \cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{3k}{5k} = \frac{3}{5}$

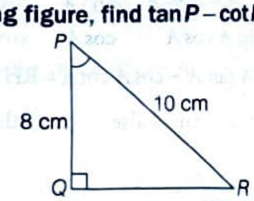
$\therefore \quad 4\sin^2\theta - 3\cos^2\theta + 2 = 4\left(\frac{4}{5}\right)^2 - 3\left(\frac{3}{5}\right)^2 + 2$

$$= 4\left(\frac{16}{25}\right) - 3\left(\frac{9}{25}\right) + 2 = \frac{64}{25} - \frac{27}{25} + 2$$

$$= \frac{64 - 27 + 50}{25} = \frac{87}{25} = 3\frac{12}{25}$$

Try These 8.1

1. In each of the following, one of the six trigonometric ratios is given. Find the values of the other trigonometric ratios.
   (i) $\sin A = \frac{12}{13}$ (ii) $\tan\theta = \frac{a}{b}$ (iii) $\sin A = \frac{3}{5}$ (iv) $\cot A = \frac{8}{15}$
2. If $\tan A = \frac{3}{4}$, then show that $\sin A \cos A = \frac{12}{25}$.
   NCERT Exemplar
3. In the following figure, find $\tan P - \cot R$.



4. If $\sin\theta = \frac{24}{25}$, then find the value of $\sin^2\theta + \cos^2\theta$.
5. If $\sin A = \frac{3}{5}$, then find the value of $\sin A \cos B + \cos A \sin B$, where $\angle C$ is $90°$.
6. In a ΔABC, $\angle B = 90°$. If $AB = 2$ cm and $AC = 3$ cm, then find the value of $\sin A$.
7. If $\sin\alpha = \frac{1}{2}$ and α is an acute angle, then find the value of $(3\cos\alpha - 4\cos^3\alpha)$.
8. If $\cos\theta = \frac{2}{3}$, then find the value of $2\sec^2\theta + 2\tan^2\theta - 9$.
9. In ΔABC, right angled at B, $AC + BC = 49$ cm, $AB = 7$ cm. Determine the values of $\sin A$, $\cos A$, $\tan C$ and $\sec C$.
10. In a ΔPQR, $\angle Q = 90°$. If $PQ = 10$ cm and $PR = 15$ cm, then find the value of $\tan^2 P + \sec^2 P + 1$.
11. If $17\cos A = 8$, then find $15\,\text{cosec}\,A - 8\sec A$.
12. If $\sec\theta = \frac{4}{\sqrt{7}}$, then prove that $\sqrt{\frac{2\tan^2\theta - \text{cosec}^2\theta}{2\cos^2\theta - \cot^2\theta}} = \frac{20}{7}$.
13. In a ΔABC, right angle at B, having $AB = 5$ cm, and $BC = 12$ cm, then find $\sin A$, $\cos C$ and $\tan A$.
14. If $3\tan A = 4$, then prove that
    (i) $\sqrt{\frac{\sec A - \text{cosec}\,A}{\sec A + \text{cosec}\,A}} = \frac{1}{\sqrt{7}}$ (ii) $\sqrt{\frac{1 - \sin A}{1 + \cos A}} = \frac{1}{2\sqrt{2}}$
15. If $\angle A$ and $\angle Q$ are acute angles such that $\tan A = \tan Q$, then show that $\angle A = \angle Q$.
16. If $\tan\theta = \frac{4}{3}$, then find $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta}$.
17. If $5\tan\alpha = 4$, then prove that $\frac{5\sin\alpha - 3\cos\alpha}{5\sin\alpha + 2\cos\alpha} = \frac{1}{6}$.
18. If $3\tan A = 4\sin A$, then find the relation between $\text{cosec}\,A$ and $\cot A$.
19. If $\sqrt{3}\sin\theta = \cos\theta$, then find the value of $\frac{\sin\theta\tan\theta(1 + \cot\theta)}{\sin\theta + \cos\theta}$.
20. If $22\cos A - 3\sin A = 20\sin A$, then find the value of $\tan^2 A + \sin^2 A \cdot \sec^2 A$.
21. Prove that $(1 + \tan A + \cot A)(\sin A - \cos A)$
    $= \sin A \tan A - \cot A \cos A$.
22. State whether the following are true or false. Justify your answer.
    (i) The value of $\tan A$ is always greater than 1.
    (ii) $\sec A = \frac{4}{3}$ for some value of $\angle A$.
    (iii) $\sin A$ is the product of sin and A.
    (iv) $\sin\theta = \frac{12}{5}$ for some $\angle\theta$.

Answers

1. **Hint** Do same as Example 3.

Ans. (i) $\cos A = \frac{5}{13}$, $\tan A = \frac{12}{5}$, $\cot A = \frac{5}{12}$, $\sec A = \frac{13}{5}$, $\text{cosec}A = \frac{13}{12}$

(ii) $\sin\theta = \frac{a}{\sqrt{a^2 + b^2}}$, $\cos\theta = \frac{b}{\sqrt{a^2 + b^2}}$, $\cot\theta = \frac{b}{a}$,

$\sec\theta = \frac{\sqrt{a^2 + b^2}}{b}$, $\text{cosec}\theta = \frac{\sqrt{a^2 + b^2}}{a}$

(iii) $\cos A = \frac{4}{5}$, $\tan A = \frac{3}{4}$, $\cot A = \frac{4}{3}$, $\sec A = \frac{5}{4}$, $\text{cosec}\,A = \frac{5}{3}$

(iv) $\sin A = \frac{15}{17}$, $\cos A = \frac{8}{17}$, $\tan A = \frac{15}{8}$, $\sec A = \frac{17}{8}$, $\text{cosec}\,A = \frac{17}{15}$

2. **Hint** Do same as Example 4.

3. **Hint** Use Pythagoras theorem, to find QR, then find $\tan P$ and $\cot R$. **Ans.** 0

4. **Hint** Use Pythagoras theorem to find the base, after using it, find the value of $\cos\theta$, then put in given expression. **Ans.** 1

5. **Hint** Use Pythagoras theorem, to find base, then on the basis of angle find $\sin B$ and $\cos B$ and then put the values in given expression. **Ans.** 1

6. **Hint** Do same as Example 2 (i). **Ans.** $\frac{\sqrt{5}}{3}$

7. **Hint** As, $\sin\alpha = \frac{1}{2} = \frac{P}{H}$

Using Pythagoras theorem, $(1)^2 + B^2 = (2)^2 \Rightarrow B = \sqrt{3}$

Now, $\cos\alpha = \frac{B}{H} = \frac{\sqrt{3}}{2}$

Put the value in the given expression. **Ans.** 0

8. **Hint** Do same as Question 7. **Ans.** −2

9. **Hint** Do same as Example 5.

Ans. $\sin A = \frac{24}{25}, \tan C = \frac{7}{24}, \cos A = \frac{7}{25}$ and $\sec C = \frac{25}{24}$.

10. **Hint** Use Pythagoras theorem to find QR and then find $\sec P$, $\tan P$ and put in given expression. **Ans.** $\frac{9}{2}$

11. **Hint** As, $\cos A = \frac{8}{17} = \frac{B}{H}$

Using Pythagoras theorem,

$P^2 + 8^2 = 17^2 \Rightarrow P = 15$

Then, $15 \operatorname{cosec} A - 8 \sec A$

$$= 15\left(\frac{H}{P}\right) - 8\left(\frac{H}{B}\right) = 15\left(\frac{17}{15}\right) - 8\left(\frac{17}{8}\right) = 17 - 17 = 0$$

12. **Hint** As, $\sec\theta = \frac{4}{\sqrt{7}} = \frac{H}{B}$

Using Pythagoras theorem, $P^2 = H^2 - B^2 = 16 - 7 = 9 \Rightarrow P = 3$

Now, $\tan\theta = \frac{P}{B} = \frac{3}{\sqrt{7}}, \operatorname{cosec}\theta = \frac{H}{P} = \frac{4}{3}, \cos\theta = \frac{B}{H} = \frac{\sqrt{7}}{4}, \cot\theta = \frac{\sqrt{7}}{3}$

Now, put these values in LHS of given expression.

13. **Hint** Do same as Example 2.

Ans. $\sin A = \frac{12}{13}, \cos C = \frac{12}{13}, \tan A = \frac{12}{5}$

14. **Hint** As, $3 \tan A = 4 \Rightarrow \tan A = \frac{4}{3} = \frac{P}{B}$

Using Pythagoras theorem, $H^2 = 4^2 + 3^2 \Rightarrow H = 5$

$$\sec A = \frac{H}{B} = \frac{5}{3}, \operatorname{cosec} A = \frac{H}{P} = \frac{5}{4}$$

$$\cos A = \frac{B}{H} = \frac{3}{5}, \sin A = \frac{P}{H} = \frac{4}{5}$$

Now, put these values in LHS of given equations in part (i) and (ii).

15. **Hint** Do same as Example 6.

16. **Hint** Do same as Example 7. **Ans.** 7

17. **Hint** Do same as Example 7.

18. **Hint** $3 \operatorname{cosec} A = 4 \cot A$

19. **Hint** As, $\sqrt{3} \sin\theta = \cos\theta \Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$

$$\therefore \frac{\sin\theta \tan\theta (1 + \cot\theta)}{\sin\theta + \cos\theta} = \frac{\tan\theta \tan\theta (1 + \cot\theta)}{\tan\theta + 1}$$ **Ans.** $\frac{1}{\sqrt{3}}$

20. **Hint** Given, $22 \cos A - 3 \sin A = 20 \sin A$

$$\Rightarrow \frac{22}{20} \cot A - \frac{3}{20} = 1 \Rightarrow \cot A = \frac{23}{22} \Rightarrow \tan A = \frac{22}{23}$$

Now, put in given expression and simplify. **Ans.** $2 \times \left(\frac{22}{23}\right)^2$

21. **Hint** LHS $= (1 + \tan A + \cot A)(\sin A - \cos A)$

$$= \left(1 + \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right)(\sin A - \cos A)$$

$$= \left(\frac{\sin A \cos A + \sin^2 A + \cos^2 A}{\sin A \cos A}\right)(\sin A - \cos A)$$

$$= \frac{\sin^3 A - \cos^3 A}{\sin A \cos A} = \frac{\sin^2 A}{\cos A} - \frac{\cos^2 A}{\sin A}$$

$= \sin A \tan A - \cos A \cot A =$ RHS **Hence proved**

22. (i) False (ii) True (iii) False (iv) False

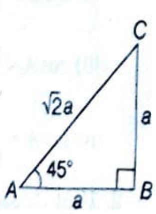
TOPIC 02 Trigonometric Ratios of Some Specific Angles

Determination of trigonometric ratios of some specific angles by geometrical method is discussed below:

Trigonometric Ratios for an Angle of 45°

Let ΔABC be an isosceles right angled triangle in which $\angle B = 90°$ and $\angle A = 45°$, then third angle will also be 45°

i.e. $\angle C = 45°$.



Here, $\angle A = \angle C \Rightarrow BC = AB = a$

[since, sides opposite to equal angles of a triangle are also equal]

Then, $AC = \sqrt{2}a$ units [by Pythagoras theorem]

Now, by the definitions of the trigonometric ratios, we have,

$$\sin 45° = \frac{P}{H} = \frac{BC}{AC} = \frac{a}{\sqrt{2}a} = \frac{1}{\sqrt{2}}$$

Similarly, we can find other trigonometric ratios.

Trigonometric Ratios for Angles of 60° and 30°

Let ΔABC be an equilateral triangle with each side equal to $2a$. Then, each angle of the triangle is 60°

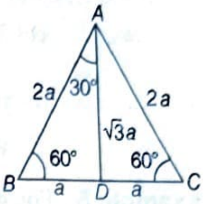
i.e. $\angle A = \angle B = \angle C = 60°$

Now, draw perpendicular AD from A to the side BC.

Here, $\Delta ADB \cong \Delta ADC$ [by RHS congruency rule]

Thus, $BD = DC = a$ units

On applying Pythagoras theorem in right angled ΔADB, we get



$$AD = \sqrt{AB^2 - BD^2}$$

$$= \sqrt{4a^2 - a^2} = \sqrt{3}a$$

For angle 60°

In right angled ΔADB, we have

$$\sin 60° = \frac{P}{H} = \frac{AD}{AB} = \frac{\sqrt{3}a}{2a} = \frac{\sqrt{3}}{2};$$

Similarly, we can find other trigonometric ratios.

For angle 30°

In right angled ΔADB, we have

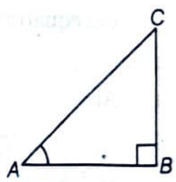
$$\sin 30° = \frac{P}{H} = \frac{BD}{AB} = \frac{a}{2a} = \frac{1}{2}$$

Similarly, we can find other trigonometric ratios.

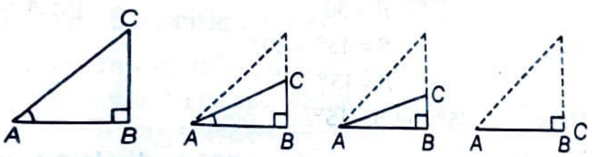
Trigonometric Ratios for Angles of 0° and 90°

For angle 0°

In ΔABC, as $\angle A$ gets smaller and smaller, the length of the side BC decreases and the point C gets closer to point B.



Finally, when $\angle A$ becomes very close to 0°, AC becomes almost the same as AB and BC gets very close to 0, so the value of $\sin A = \frac{BC}{AC}$ is very close to 0 and the value of $\cos A = \frac{AB}{AC}$ is very close to 1.

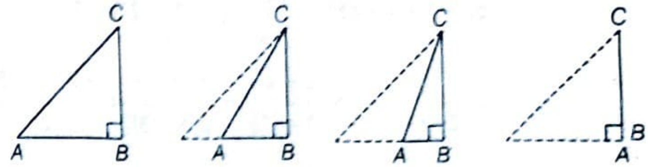


Thus, when $\angle A = 0°$, $\sin 0° = \frac{BC}{AC} = \frac{0}{AC} = 0$

and $\cos 0° = \frac{AB}{AC} = \frac{AB}{AB} = 1.$

For angle 90°

In ΔABC, as $\angle A$ gets larger and larger, $\angle C$ gets smaller and smaller and the length of the side AB goes on decreasing i.e. point A gets closer to point B. Finally, when $\angle A$ is very close to 90°, $\angle C$ becomes very close to 0° and the side AC almost coincides with side BC and the side AB is nearly zero. So, $\sin A = \frac{P}{H} = \frac{BC}{AC}$ is very close to 1 and $\cos A = \frac{B}{H} = \frac{AB}{AC}$ is very close to 0.



Thus, when

$$\angle A = 90°,$$

$$\sin 90° = \frac{BC}{AC} = \frac{BC}{BC} = 1,$$

$$\cos 90° = \frac{AB}{AC} = \frac{0}{AC} = 0$$

Using these, we can find other trigonometric ratios.

Values of Trigonometric Ratios for Some Specific Angles

Angles	0° or 0	30° or $\frac{\pi}{6}$	45° or $\frac{\pi}{4}$	60° or $\frac{\pi}{3}$	90° or $\frac{\pi}{2}$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞
$\text{cosec}\,\theta$	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞
$\cot\theta$	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

Here, '∞' stands for undefined (not defined) value.

Note it

(i) The value of $\sin\theta$ increases from 0 to 1 and $\cos\theta$ decreases from 1 to 0, where $0 \le \theta \le 90°$.

(ii) Division by 0 is not allowed, hence 1/0 is an indeterminate (not defined) value.

(iii) In the case of $\tan\theta$, the values increase from 0 to ∞, where $0 \le \theta \le 90°$.

(iv) In the case of $\cot\theta$, the values decrease from ∞ to 0, where $0 \le \theta \le 90°$.

(v) In the case of $\text{cosec}\,\theta$, the values decrease from ∞ to 1, where $0 \le \theta \le 90°$.

(vi) In the case of $\sec\theta$, the values increase from 1 to ∞, where $0 \le \theta \le 90°$.

Example 1. Evaluate $\dfrac{\sin 30^\circ + \tan 45^\circ}{\sec 30^\circ + \cot 45^\circ}$. CBSE 2023 (Basic)

Sol. We have, $\dfrac{\sin 30^\circ + \tan 45^\circ}{\sec 30^\circ + \cot 45^\circ} = \dfrac{\frac{1}{2}+1}{\frac{2}{\sqrt{3}}+1} = \dfrac{3}{2} \times \dfrac{\sqrt{3}}{2+\sqrt{3}}$

$$= \frac{3\sqrt{3}}{2(2+\sqrt{3})} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{6\sqrt{3}-9}{2(4-3)} = \frac{3(2\sqrt{3}-3)}{2}$$

Example 2. Evaluate $\dfrac{\cos 45^\circ + \sin 60^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$.

CBSE 2024 (Standard)

Sol. $\because$ We know that

$$\cos 45^\circ = \frac{1}{\sqrt{2}}, \sin 60^\circ = \frac{\sqrt{3}}{2}, \sec 30^\circ = \frac{2}{\sqrt{3}} \text{ and } \operatorname{cosec} 30^\circ = 2$$

Now, $\dfrac{\cos 45^\circ + \sin 60^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ} = \dfrac{\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}}{\frac{2}{\sqrt{3}} + 2} = \dfrac{\frac{2+\sqrt{6}}{2\sqrt{2}}}{\frac{2+2\sqrt{3}}{\sqrt{3}}}$

$$= \frac{(2+\sqrt{6})\sqrt{3}}{2\sqrt{2}(2+2\sqrt{3})} = \frac{2\sqrt{3}+\sqrt{18}}{4\sqrt{2}+4\sqrt{6}}$$

$$= \frac{2\sqrt{3}+3\sqrt{2}}{4\sqrt{2}+4\sqrt{6}} = \frac{2\sqrt{3}+3\sqrt{2}}{4\sqrt{2}+4\sqrt{6}} \times \frac{4\sqrt{2}-4\sqrt{6}}{4\sqrt{2}-4\sqrt{6}}$$

$$= \frac{8\sqrt{6}-8\sqrt{18}+12\times 2-12\sqrt{12}}{32-96}$$

$$= \frac{8\sqrt{6}-24\sqrt{2}+24-24\sqrt{3}}{-64} = \frac{-\sqrt{6}+3\sqrt{2}-3+3\sqrt{3}}{8}$$

$$= \frac{+\sqrt{2}(-\sqrt{3}+3)+\sqrt{3}(-\sqrt{3}+3)}{8} = \frac{(-\sqrt{3}+3)(\sqrt{2}+\sqrt{3})}{8}$$

$$\therefore \frac{\cos 45^\circ + \sin 60^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ} = \frac{(-\sqrt{3}+3)(\sqrt{2}+\sqrt{3})}{8}$$

Example 3. Evaluate $8\sqrt{3}\operatorname{cosec}^2 30^\circ \sin 60^\circ \cos 60^\circ \cos^2 45^\circ \sin 45^\circ \tan 30^\circ \operatorname{cosec}^3 45^\circ$.

Sol. We have, $8\sqrt{3}\operatorname{cosec}^2 30^\circ \sin 60^\circ \cos 60^\circ \cos^2 45^\circ \sin 45^\circ \tan 30^\circ \operatorname{cosec}^3 45^\circ$

$$= 8\sqrt{3}(\operatorname{cosec} 30^\circ)^2 \sin 60^\circ \cdot \cos 60^\circ (\cos 45^\circ)^2 \sin 45^\circ \tan 30^\circ (\operatorname{cosec} 45^\circ)^3$$

$$= 8\sqrt{3}(2)^2 \times \frac{\sqrt{3}}{2} \cdot \frac{1}{2} \times \left(\frac{1}{\sqrt{2}}\right)^2 \times \left(\frac{1}{\sqrt{2}}\right) \times \frac{1}{\sqrt{3}} \times (\sqrt{2})^3$$

$$\left[\because \operatorname{cosec} 30^\circ = 2, \sin 60^\circ = \frac{\sqrt{3}}{2}, \cos 60^\circ = \frac{1}{2}, \sin 45^\circ = \cos 45^\circ = \frac{1}{\sqrt{2}}, \tan 30^\circ = \frac{1}{\sqrt{3}} \text{ and } \operatorname{cosec} 45^\circ = \sqrt{2}\right]$$

$$= 8\sqrt{3} \times \sqrt{3} \times \frac{1}{2} \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{3}} \times 2\sqrt{2} = 8\sqrt{3}$$

Example 4. If $2\cos 3\theta = \sqrt{3}$, then find the value of θ.

Sol. We have, $2\cos 3\theta = \sqrt{3}$

$\Rightarrow \quad \cos 3\theta = \dfrac{\sqrt{3}}{2} \Rightarrow \cos 3\theta = \cos 30^\circ \quad \left[\because \cos 30^\circ = \dfrac{\sqrt{3}}{2}\right]$

$\Rightarrow \quad 3\theta = 30^\circ$, as 3θ and 30° are acute angles.

$\therefore \quad \theta = 10^\circ$

Example 5. For $A = 30^\circ$ and $B = 60^\circ$, verify that $\sin(A+B) = \sin A \cos B + \cos A \sin B$. CBSE 2023 (Basic)

Sol. Given, $A = 30^\circ$ and $B = 60^\circ$

$$\text{LHS} = \sin(A+B) = \sin(30^\circ + 60) = \sin 90^\circ = 1$$

$$\text{RHS} = \sin A \cos B + \cos A \sin B$$

$$= \sin 30^\circ \cos 60^\circ + \cos 30^\circ \sin 60^\circ$$

$$= \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} = \frac{1+3}{4} = \frac{4}{4} = 1$$

$\therefore$ LHS = RHS **Hence proved.**

Example 6. If $\sin(A-B) = \dfrac{1}{2}, \cos(A+B) = \dfrac{1}{2}$, $0^\circ < A+B \le 90^\circ$, $A > B$, then find the values of A and B.

Sol. Given, $\quad \sin(A-B) = \dfrac{1}{2}$

$\Rightarrow \quad \sin(A-B) = \sin 30^\circ \quad [\because \sin 30^\circ = \frac{1}{2}]$

On equating both sides, we get

$$A - B = 30^\circ \quad \text{...(i)}$$

Also, $\quad \cos(A+B) = \dfrac{1}{2}$

$\Rightarrow \quad \cos(A+B) = \cos 60^\circ \quad [\because \cos 60^\circ = \frac{1}{2}]$

On equating both sides, we get

$$A + B = 60^\circ \quad \text{...(ii)}$$

On adding Eqs. (i) and (ii), we get

$$2A = 90^\circ \Rightarrow A = 45^\circ$$

From Eq. (i), we get

$$45^\circ - B = 30^\circ \quad [\because A = 45^\circ]$$

$\Rightarrow \quad B = 45^\circ - 30^\circ$

$\therefore \quad B = 15^\circ$

Hence, $A = 45^\circ$ and $B = 15^\circ$.

Example 7. If $\sin x + \cos y = 1$, $x = 30^\circ$ and y is an acute angle, then find the value of y. CBSE 20..

Sol. Given, equation is $\sin x + \cos y = 1$

Put $x = 30^\circ$, we get $\sin 30^\circ + \cos y = 1$

$\Rightarrow \quad \cos y = 1 - \sin 30^\circ \Rightarrow \cos y = 1 - \dfrac{1}{2}$

$\Rightarrow \quad \cos y = \dfrac{1}{2} \Rightarrow \cos y = \cos 60^\circ$

$\Rightarrow y = 60^\circ$, which is acute angle.

Example 8. In a ΔABC, right angled at B, $\angle A = \angle C$, find the value of

(i) $\sin A \cos C + \cos A \sin C$. (ii) $\sin A \sin B + \cos A \cos B$

Sol. Given, $\angle A = \angle C$

Now, $\angle A + \angle B + \angle C = 180°$

[by angle sum property of triangle]

$\Rightarrow \quad 90° + \angle A + \angle C = 180° \quad [\because \angle B = 90°]$

$\Rightarrow \quad \angle A + \angle C = 90°$

$\Rightarrow \quad 2\angle A = 90°$ and $2\angle C = 90°$ $[\because \angle A = \angle C]$

$\Rightarrow \quad \angle A = 45°$ and $\angle C = 45°$

(i) $\sin A \cos C + \cos A \sin C$

$= \sin 45° \cos 45° + \cos 45° \sin 45°$

$= \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{1}{2} + \frac{1}{2} = 1$

$\left[\because \sin 45° = \cos 45° = \frac{1}{\sqrt{2}}\right]$

(ii) $\sin A \sin B + \cos A \cos B$

$= \sin 45° \sin 90° + \cos 45° \cos 90°$

$= \frac{1}{\sqrt{2}} \times 1 + \frac{1}{\sqrt{2}} \times 0 = \frac{1}{\sqrt{2}} + 0 = \frac{1}{\sqrt{2}}$

$[\because \sin 90° = 1 \text{ and } \cos 90° = 0]$

Example 9. **Find acute angles A and B, if $\sin(A + 2B) = \frac{\sqrt{3}}{2}$ and $\cos(A + 4B) = 0°$, $A > B$.**

Sol. We have, $\sin(A + 2B) = \frac{\sqrt{3}}{2} = \sin 60° \quad \left[\because \sin 60° = \frac{\sqrt{3}}{2}\right]$

$\Rightarrow \quad A + 2B = 60° \quad$ ...(i)

Again, $\cos(A + 4B) = 0° = \cos 90° \quad [\because \cos 90° = 0]$

$\Rightarrow \quad A + 4B = 90° \quad$ ...(ii)

On subtracting Eq. (i) from Eq. (ii), we get

$A + 4B - A - 2B = 90° - 60° \Rightarrow 2B = 30° \Rightarrow B = 15°$

On substituting $B = 15°$ in Eq. (i), we get

$A + 2 \times 15° = 60° \Rightarrow A + 30° = 60° \Rightarrow A = 60° - 30° = 30°$

Hence, $A = 30°$ and $B = 15°$

Problems Based on Trigonometric Ratios of Some Specific Angles

There are many different problems in which one or more angles is given to us and we have to find the side or sides of a triangle using these trigonometric ratios of specific angles. For these problems, we have no particular method to solve but we can understand method of solving these type of problems with the help of examples.

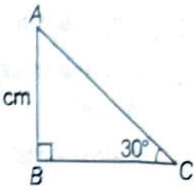
Example 10. **In right angled ΔABC, right angled at B, $AB = 5$ cm and $\angle ACB = 30°$. Determine the length of the sides BC and AC.**

Sol. Given, $\angle C = \angle ACB = 30°$ and $AB = 5$ cm

Now, $\tan 30° = \frac{AB}{BC} = \frac{5}{BC} \quad \left[\because \tan\theta = \frac{\text{Perpendicular}}{\text{Base}}\right]$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{5}{BC} \quad \left[\because \tan 30° = \frac{1}{\sqrt{3}}\right]$

$\Rightarrow \quad BC = 5\sqrt{3}$ cm



By Pythagoras theorem, we have

$AC = \sqrt{AB^2 + BC^2} = \sqrt{5^2 + (5\sqrt{3})^2}$

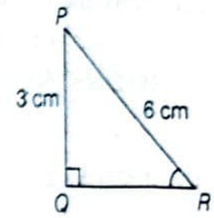
$= \sqrt{25 + 75} = \sqrt{100} = 10$

$\Rightarrow \quad AC = 10$ cm

[taking positive square root]

$\therefore$ Length of side $BC = 5\sqrt{3}$ cm and $AC = 10$ cm.

Example 11. **In the following figure, ΔPQR is right angled at Q, $PQ = 3$ cm and $PR = 6$ cm. Determine $\angle QPR$ and $\angle PRQ$.**



Sol. Given, $PQ = 3$ cm and $PR = 6$ cm

$\therefore \quad \sin R = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{PQ}{PR}$

$\Rightarrow \quad \sin R = \frac{3}{6} \Rightarrow \sin R = \frac{1}{2}$

$\Rightarrow \quad \sin R = \sin 30° \quad \left[\because \sin 30° = \frac{1}{2}\right]$

$\therefore \quad R = 30°$ or $\angle PRQ = 30°$

In ΔPQR, $\angle P + \angle Q + \angle R = 180°$

[by angle sum property of triangle]

$\Rightarrow \quad \angle P + 90° + 30° = 180° \Rightarrow \angle P = 180° - 120°$

$\therefore \quad \angle P = 60°$ or $\angle QPR = 60°$

Try These 8.2

1. **Evaluate $5\sin^2 45° - \sec 60° \cot^2 30°$.** CBSE 2024 (Basic)
2. **Find the value of**
   $4\tan 45° + \sqrt{3}\cot 60° + 3\sin^2 60° + \tan 30° \cot 45°$
3. **Find the value of $(\sin 30° + \cos 30°) - (\sin 60° + \cos 60°)$.**
4. **Find the value of $3\sin 30° - 4\sin^3 60°$.**
5. **If $\tan A + \sin^2 45° - \cos^2 30° - \text{cosec}^2 45° \sec^2 60° = 4$, then find $\tan A$.**
6. **Find the value of x in each of the following**
   (i) $x\tan 45° \cos 60° = \sin 60° \cot 60°$
   (ii) $-x\sin 90° \cos 30° + \sin 30° \cos 90° = \frac{4}{5}$
   (iii) $\cos 2x = \cos 60° \cos 30° + \sin 60° \sin 30°$
7. **Prove that $\sqrt{6} \cdot (\sqrt{2} + \sqrt{3})\left[\frac{\text{cosec } 60°}{2} - \cos 45°\right] = -1$**
8. **Show that $(\sqrt{3} + 1)(3 - \cot 30°) = \tan^3 60° - 2\sin 60°$.** NCERT Exemplar
9. **If ΔABC is right angled at C, then find the value of $\cos(A + B)$.** NCERT Exemplar
10. **State whether the following statements are true or false. Justify your answer.**
    (i) $\cos(A + B) = \cos A + \cos B$
    (ii) $\tan\theta = \cot\theta$ for all values of θ.

11. Find the value of $3\tan^2\theta + 2\sin\theta\cos\theta$, for $\theta = 45°$.

12. Find the value of $\operatorname{cosec}^2 A - 2\sin A + \tan A \cos A + 5$, for $A = 60°$.

13. If $\sin\theta - \cos\theta = 0$, $(0 \le \theta \le 90°)$, then find the value of θ.

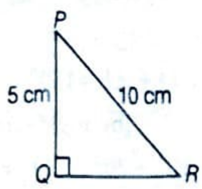
14. Determine the value of x, such that
$$2\operatorname{cosec}^2 30° + x\sin^2 60° - \frac{3}{4}\tan^2 30° = 10.$$

15. Prove the following
(i) $2(\cos^2 45° + \tan^2 60°) - 6(\sin^2 45° - \tan^2 30°) = 6$
(ii) $2(\cos^4 60° + \sin^4 30°) - (\tan^2 60° - \cot^2 45°) + 3\sec^2 30° = 9/4$

16. If $\sin(A+B) = 1$ and $\sin(A-B) = \frac{1}{2}; 0 \le A+B \le 90°$ and $A > B$, then find the value of A and B.

17. In an acute ΔABC, if $\tan(A+B-C) = 1$ and $\sec(B+C-A) = 2$, then find the values of $\angle A$, $\angle B$ and $\angle C$.

18. In the following figure, ΔPQR is right angled at Q, $PQ = 5$ cm, $PR = 10$ cm. Determine $\angle QPR$ and $\angle PRQ$.



Answers

1. We have, $5\sin^2 45° - \sec 60° \cot^2 30°$
$$= \left[5 \times \left(\frac{1}{\sqrt{2}}\right)^2\right] - (2 \times (\sqrt{3})^2)$$
$$= \left(5 \times \frac{1}{2}\right) - (2 \times 3) = \frac{5}{2} - 6 = \frac{5-12}{2} = -\frac{7}{2}$$

2. $\frac{29\sqrt{3}+4}{4\sqrt{3}}$ 3. 0 4. $\frac{3(1-\sqrt{3})}{2}$ 5. $\frac{49}{4}$

6. (i) 1 (ii) $\frac{-8}{5\sqrt{3}}$ (iii) 15

9. Hint $A + B + C = 180° \Rightarrow A + B = 180° - C \Rightarrow A + B = 90°$ Ans.

10. (i) Hint Let $A = 60°$ and $B = 30°$. Then, $\cos(60° + 30°) = \cos 90° = 0$ and $\cos 60° + \cos 30° = \frac{1}{2} + \frac{\sqrt{3}}{2}$ **Ans. False (ii) False**

11. 4 12. $\frac{38-3\sqrt{3}}{6}$

13. $\sin\theta - \cos\theta = 0 \Rightarrow \sin\theta = \cos\theta \Rightarrow \tan\theta = 1 \Rightarrow \theta = 45°$

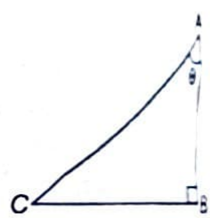
14. $x = 3$

16. Hint Do same as Example 9. **Ans.** $A = 60°, B = 30°$

17. Hint Solve the following equations to find A, B, C.
$A + B - C = 45°, B + C - A = 60°, A + B + C = 180°$
Ans. $A = 60°, B = \frac{105°}{2}, C = \frac{135°}{2}$

18. Hint Do same as Example 11.
Ans. $\angle QPR = 60°$ and $\angle PRQ = 30°$.

TOPIC 03 Trigonometric Identities



We know that, an equation is called **an identity** when it is true for all values of the variables involved. Similarly, an equation involving trigonometric ratios of an angle is called **a trigonometric identity**, if it is true for all values of the angle (s) involved. For any acute angle θ, we have three identities

i.e. (i) $\sin^2\theta + \cos^2\theta = 1$ (ii) $1 + \tan^2\theta = \sec^2\theta$ (iii) $\cot^2\theta + 1 = \operatorname{cosec}^2\theta$

Conversion of Trigonometric Ratios in Terms of Other Trigonometric Ratios

	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$	$\sec\theta$	$\operatorname{cosec}\theta$
$\sin\theta$	$\sin\theta$	$\sqrt{1-\cos^2\theta}$	$\frac{\tan\theta}{\sqrt{1+\tan^2\theta}}$	$\frac{1}{\sqrt{1+\cot^2\theta}}$	$\frac{\sqrt{\sec^2\theta-1}}{\sec\theta}$	$\frac{1}{\operatorname{cosec}\theta}$
$\cos\theta$	$\sqrt{1-\sin^2\theta}$	$\cos\theta$	$\frac{1}{\sqrt{1+\tan^2\theta}}$	$\frac{\cot\theta}{\sqrt{1+\cot^2\theta}}$	$\frac{1}{\sec\theta}$	$\frac{\sqrt{\operatorname{cosec}^2\theta-1}}{\operatorname{cosec}\theta}$
$\tan\theta$	$\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$	$\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$	$\tan\theta$	$\frac{1}{\cot\theta}$	$\sqrt{\sec^2\theta-1}$	$\frac{1}{\sqrt{\operatorname{cosec}^2\theta-1}}$
$\cot\theta$	$\frac{\sqrt{1-\sin^2\theta}}{\sin\theta}$	$\frac{\cos\theta}{\sqrt{1-\cos^2\theta}}$	$\frac{1}{\tan\theta}$	$\cot\theta$	$\frac{1}{\sqrt{\sec^2\theta-1}}$	$\sqrt{\operatorname{cosec}^2\theta-1}$
$\sec\theta$	$\frac{1}{\sqrt{1-\sin^2\theta}}$	$\frac{1}{\cos\theta}$	$\sqrt{1+\tan^2\theta}$	$\frac{\sqrt{1+\cot^2\theta}}{\cot\theta}$	$\sec\theta$	$\frac{\operatorname{cosec}\theta}{\sqrt{\operatorname{cosec}^2\theta-1}}$
$\operatorname{cosec}\theta$	$\frac{1}{\sin\theta}$	$\frac{1}{\sqrt{1-\cos^2\theta}}$	$\frac{\sqrt{1+\tan^2\theta}}{\tan\theta}$	$\sqrt{1+\cot^2\theta}$	$\frac{\sec\theta}{\sqrt{\sec^2\theta-1}}$	$\operatorname{cosec}\theta$

Note it $\sin^2\theta = (\sin\theta)^2$ but $\sin\theta^2 \ne (\sin\theta)^2$. The same is true for all other trigonometric ratios.

Example 1. **Express the ratios cos A, tan A and sec A in terms of sin A.**

Sol. We know that $\cos^2 A + \sin^2 A = 1$,

$\Rightarrow \quad \cos^2 A = 1 - \sin^2 A$

$\Rightarrow \quad \cos A = \sqrt{1 - \sin^2 A}$

Now, $\quad \tan A = \dfrac{\sin A}{\cos A} = \dfrac{\sin A}{\sqrt{1 - \sin^2 A}}$

and $\quad \sec A = \dfrac{1}{\cos A} = \dfrac{1}{\sqrt{1 - \sin^2 A}}$

Example 2. **Prove that** $\dfrac{1 + \sec A}{\sec A} = \dfrac{\sin^2 A}{1 - \cos A}$

CBSE 2023 (Basic)

Sol. $\text{LHS} = \dfrac{1 + \sec A}{\sec A} = \dfrac{1 + \dfrac{1}{\cos A}}{\dfrac{1}{\cos A}}$

$= \dfrac{\cos A + 1}{\cos A} \times \cos A = \cos A + 1$

$\text{RHS} = \dfrac{\sin^2 A}{1 - \cos A} = \dfrac{1 - \cos^2 A}{1 - \cos A} \quad [\because \sin^2\theta + \cos^2\theta = 1]$

$= \dfrac{(1 + \cos A)(1 - \cos A)}{1 - \cos A} = 1 + \cos A$

$\therefore$ LHS = RHS **Hence proved.**

Example 3. **Prove that**

$$\sqrt{\frac{\sec A - 1}{\sec A + 1}} + \sqrt{\frac{\sec A + 1}{\sec A - 1}} = 2\operatorname{cosec} A$$

CBSE 2023 (Standard)

Sol. $\text{LHS} = \sqrt{\dfrac{\sec A - 1}{\sec A + 1}} + \sqrt{\dfrac{\sec A + 1}{\sec A - 1}}$

$= \dfrac{\sec A - 1 + \sec A + 1}{\sqrt{(\sec A + 1)(\sec A - 1)}}$

$= \dfrac{2\sec A}{\sqrt{\sec^2 A - 1}}$

$= \dfrac{2\sec A}{\sqrt{\tan^2 A}} \quad [\because \sec^2\theta - \tan^2\theta = 1]$

$= \dfrac{2\sec A}{\tan A} = 2 \times \dfrac{1}{\cos A} \times \dfrac{\cos A}{\sin A}$

$= 2\operatorname{cosec} A$

= RHS **Hence proved.**

Example 4. **Prove that**

$$\frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\sin A + \cos A} = \frac{2}{2\sin^2 A - 1}.$$

CBSE 2024 (Standard)

Sol. $\text{LHS} = \dfrac{\sin A + \cos A}{\sin A - \cos A} + \dfrac{\sin A - \cos A}{\sin A + \cos A}$

$= \dfrac{(\sin A + \cos A)^2 + (\sin A - \cos A)^2}{(\sin A - \cos A)(\sin A + \cos A)}$

$= \dfrac{\left[\begin{array}{r}\sin^2 A + 2\sin A\cos A + \cos^2 A + \sin^2 A \\ -2\sin A\cos A + \cos^2 A\end{array}\right]}{\sin^2 A - \cos^2 A}$

$[\because (a \pm b)^2 = a^2 + b^2 \pm 2ab]$

$= \dfrac{2\sin^2 A + 2\cos^2 A}{\sin^2 A - \cos^2 A} = \dfrac{2(\sin^2 A + \cos^2 A)}{\sin^2 A - \cos^2 A}$

$= \dfrac{2}{\sin^2 A - \cos^2 A}$

$= \dfrac{2}{\sin^2 A - (1 - \sin^2 A)}$

$= \dfrac{2}{\sin^2 A - 1 + \sin^2 A}$

$= \dfrac{2}{2\sin^2 A - 1}$

= RHS **Hence proved.**

Example 5. **Prove that** $\dfrac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \dfrac{1}{\sec\theta - \tan\theta}$, **using the identity** $\sec^2\theta = 1 + \tan^2\theta$.

Sol. $\text{LHS} = \dfrac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \dfrac{\tan\theta - 1 + \sec\theta}{\tan\theta + 1 - \sec\theta}$

[dividing numerator and denominator by $\cos\theta$]

$= \dfrac{(\tan\theta + \sec\theta) - 1}{(\tan\theta - \sec\theta) + 1} = \dfrac{[(\tan\theta + \sec\theta) - 1][\tan\theta - \sec\theta]}{[(\tan\theta - \sec\theta) + 1][\tan\theta - \sec\theta]}$

[multiplying and dividing by $(\tan\theta - \sec\theta)$]

$= \dfrac{(\tan^2\theta - \sec^2\theta) - (\tan\theta - \sec\theta)}{(\tan\theta - \sec\theta + 1)(\tan\theta - \sec\theta)} \quad \left[\begin{array}{r}\because (a-b)(a+b) \\ = a^2 - b^2\end{array}\right]$

$= \dfrac{-1 - \tan\theta + \sec\theta}{(\tan\theta - \sec\theta + 1)(\tan\theta - \sec\theta)} \quad [\because \tan^2 A - \sec^2 A = -1]$

$= \dfrac{-(\tan\theta - \sec\theta + 1)}{(\tan\theta - \sec\theta + 1)(\tan\theta - \sec\theta)} = \dfrac{-1}{\tan\theta - \sec\theta}$

$= \dfrac{1}{\sec\theta - \tan\theta} = \text{RHS}$ **Hence proved.**

Example 6. **If** $a\cos\theta + b\sin\theta = m$ **and** $a\sin\theta - b\cos\theta = n$, **then prove that** $a^2 + b^2 = m^2 + n^2$

CBSE 2023 (Standard)

Sol. Given, $a\cos\theta + b\sin\theta = m \quad \ldots\text{(i)}$

$a\sin\theta - b\cos\theta = n \quad \ldots\text{(ii)}$

On squaring both sides of Eqs. (i) and (ii), we get

$$a^2\cos^2\theta + b^2\sin^2\theta + 2ab\cos\theta\sin\theta = m^2 \quad \text{...(iii)}$$

$$a^2\sin^2\theta + b^2\cos^2\theta - 2ab\cos\theta\sin\theta = n^2 \quad \text{...(iv)}$$

On adding Eqs. (iii) and (iv), we get

$$a^2(\cos^2\theta + \sin^2\theta) + b^2(\sin^2\theta + \cos^2\theta) = m^2 + n^2$$

$$\Rightarrow \quad a^2 + b^2 = m^2 + n^2 \qquad [\because \cos^2 A + \sin^2 A = 1]$$

Hence proved.

Example 7. If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, then show that $(m^2 - n^2)^2 = 16\,mn$ or $(m^2 - n^2) = 4\sqrt{mn}$.

Sol. Given, $\tan\theta + \sin\theta = m$ ...(i)

and $\tan\theta - \sin\theta = n$ ...(ii)

On adding Eqs. (i) and (ii), we get

$$2\tan\theta = m + n \Rightarrow \tan\theta = \frac{m+n}{2}$$

$$\therefore \quad \cot\theta = \frac{1}{\tan\theta} = \frac{2}{m+n} \quad \text{...(iii)}$$

On subtracting Eq. (ii) from Eq. (i), we get

$$2\sin\theta = m - n \Rightarrow \sin\theta = \frac{m-n}{2}$$

$$\therefore \quad \operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{2}{m-n} \quad \text{...(iv)}$$

We know that $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$$\Rightarrow \quad \left(\frac{2}{m-n}\right)^2 - \left(\frac{2}{m+n}\right)^2 = 1 \quad [\text{from Eqs. (iii) and (iv)}]$$

$$\Rightarrow \quad \frac{4}{(m-n)^2} - \frac{4}{(m+n)^2} = 1$$

$$\Rightarrow \quad 4\left[\frac{1}{(m-n)^2} - \frac{1}{(m+n)^2}\right] = 1$$

$$\Rightarrow \quad 4\left[\frac{(m+n)^2 - (m-n)^2}{(m-n)^2(m+n)^2}\right] = 1$$

$$\Rightarrow \quad 4\left[\frac{(m^2+n^2+2mn) - (m^2+n^2-2mn)}{(m-n)^2(m+n)^2}\right] = 1$$

$$[\because (A \pm B)^2 = A^2 + B^2 \pm 2AB]$$

$$\Rightarrow \quad 4\left[\frac{2mn + 2mn}{(m-n)^2(m+n)^2}\right] = 1$$

$$\Rightarrow \quad \frac{16mn}{[(m-n)(m+n)]^2} = 1$$

$$\Rightarrow \quad \frac{16mn}{(m^2-n^2)^2} = 1$$

$$\Rightarrow \quad (m^2 - n^2)^2 = 16mn$$

$$\therefore \quad (m^2 - n^2) = 4\sqrt{mn}$$

[taking positive square root]

Hence proved.

Try These 8.3

1. Express the trigonometric ratios $\cos A$, $\cot A$ and $\operatorname{cosec} A$ in terms of $\tan A$.
2. Write all the other trigonometric ratios of $\angle A$ in terms of $\operatorname{cosec} A$.

Directions (Q. Nos. 3-7) Show that

3. $\sqrt{\dfrac{1+\cos A}{1-\cos A}} = \operatorname{cosec} A + \cot A$.
4. $(1 - \sin A + \cos A)^2 = 2(1 - \sin A)(1 + \cos A)$
5. $\dfrac{\sin\theta}{1+\cos\theta} + \dfrac{1+\cos\theta}{\sin\theta} = 2\operatorname{cosec}\theta$
6. $\sqrt{(1-\cos^2\theta)\sec^2\theta} = \tan\theta$
7. $\dfrac{1-\cos\theta}{1+\cos\theta} = (\operatorname{cosec}\theta - \cot\theta)^2$
8. If $\dfrac{\cot A}{1+\operatorname{cosec} A} - \dfrac{\cot A}{1-\operatorname{cosec} A} = \dfrac{k}{\cos A}$, then find the value of k.
9. Find the values of the constant a and b for which $\sin x \times \cos x\,(5\tan x + 2\cot x) = a + b\sin^2 x$.

Directions (Q. Nos. 10-26) Prove that

10. $(1+\tan^2\theta) + \left(1 + \dfrac{1}{\tan^2\theta}\right) = \dfrac{1}{\sin^2\theta - \sin^4\theta}$.
11. $(1 + \cot\theta - \operatorname{cosec}\theta)(1 + \tan\theta + \sec\theta) = 2$.
12. $\tan^2\theta + \cot^2\theta + 2 = \sec^2\theta + \operatorname{cosec}^2\theta$.
13. $\tan\theta - \cot\theta = \dfrac{2\sin^2\theta - 1}{\sin\theta \cdot \cos\theta}$.
14. $\dfrac{\tan\theta}{1-\cot\theta} + \dfrac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta\operatorname{cosec}\theta$ CBSE 2023 (Standard)
15. $\dfrac{1-\cos A+\sin A}{\sin A+\cos A-1} = \dfrac{1+\sin A}{\cos A}$.
16. Prove that $(\operatorname{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta) = 1$ CBSE 2024 (Standard)
17. $\sqrt{\dfrac{\sin A+1}{1-\sin A}} + \sqrt{\dfrac{1-\sin A}{\sin A+1}} = 2\sec A$.
18. $\sqrt{\dfrac{\sec A - \tan A}{\sec A + \tan A}}\sqrt{\dfrac{\operatorname{cosec} A - \cot A}{\operatorname{cosec} A + \cot A}}$ $= (\sec A - \tan A)(\operatorname{cosec} A - \cot A)$.
19. $\sec^2\theta - \dfrac{\sin^2\theta - 2\sin^4\theta}{2\cos^4\theta - \cos^2\theta} = 1$.
20. $\dfrac{1+\tan^3\theta}{1+\tan\theta} + \tan\theta - \sec^2\theta = 0$.
21. $(\tan\theta + \sec\theta - 1)(\tan\theta + 1 + \sec\theta) = \dfrac{2\sin\theta}{1-\sin\theta}$.

22. $\dfrac{\cos A}{1-\tan A}+\dfrac{\sin^2 A}{\sin A-\cos A}=\sin A+\cos A.$

23. $\dfrac{\operatorname{cosec}\theta+\cot\theta}{\operatorname{cosec}\theta-\cot\theta}=(\operatorname{cosec}\theta+\cot\theta)^2$

$$=1+2\cot^2\theta+2\operatorname{cosec}\theta\cdot\cot\theta.$$

24. $\dfrac{\sin A+\sin B}{\cos A+\cos B}+\dfrac{\cos A-\cos B}{\sin A-\sin B}=0.$

25. $\dfrac{\sin A}{1+\cos A}+\dfrac{\sin A}{1-\cos A}=\sqrt{\dfrac{1+\cos A}{1-\cos A}}+\sqrt{\dfrac{1-\cos A}{1+\cos A}}$

$$=2\operatorname{cosec} A.$$

26. $\left(\dfrac{\sin A}{1+\cos A}+\dfrac{1+\cos A}{\sin A}\right)\left(\dfrac{\sin A}{1-\cos A}-\dfrac{1-\cos A}{\sin A}\right)$

$$=4\operatorname{cosec} A\cot A.$$

27. If $\sin\theta+\cos\theta=p$ and $\sec\theta+\operatorname{cosec}\theta=q$, then prove that $q(p^2-1)=2p$. CBSE 2023 (Standard)

28. Given that

$$2\sin A\cos A+(\cos A+\sin A)^2-(2\cos A+\sin A)^2=p\sin^2 A+q.$$ Find the value of p and q.

29. If $\sec\theta+\tan\theta=p$, show that $\sec\theta-\tan\theta=\dfrac{1}{p}$.

Hence, find the value of $\cos\theta$ and $\sin\theta$.

30. If $\cos\theta-\sin\theta=\sqrt{2}\sin\theta$, then prove that

$$\cos\theta+\sin\theta=\sqrt{2}\cos\theta.$$

31. Show that

$$2(\sin^6\theta+\cos^6\theta)-3(\sin^4\theta+\cos^4\theta)+1=0.$$

32. If $m=\cos\theta-\sin\theta$ and $n=\cos\theta+\sin\theta$,

show that $\sqrt{\dfrac{m}{n}}+\sqrt{\dfrac{n}{m}}=\dfrac{2}{\sqrt{1-\tan^2\theta}}.$

Answers

1. **Hint** $\cos A=\dfrac{1}{\sec A}=\dfrac{1}{\sqrt{1+\tan^2 A}}$

$[\because \sec^2 A=1+\tan^2 A\Rightarrow \sec A=\sqrt{1+\tan^2 A}]$

$$\cot A=\frac{1}{\tan A}$$

$$\operatorname{cosec} A=\sqrt{1+\cot^2 A}$$

$[\because \operatorname{cosec}^2 A=1+\cot^2 A\Rightarrow \operatorname{cosec} A=\sqrt{1+\cot^2 A}]$

$$=\sqrt{1+\left(\frac{1}{\tan A}\right)^2}=\sqrt{1+\frac{1}{\tan^2 A}}$$

$$=\sqrt{\frac{\tan^2 A+1}{\tan^2 A}}=\frac{\sqrt{1+\tan^2 A}}{\tan A}$$

2. (i) **Hint** We know that $\sin^2 A+\cos^2 A=1$

$\Rightarrow\quad \cos^2 A=1-\sin^2 A$

$\Rightarrow\quad \cos A=\sqrt{1-\sin^2 A}$ [taking positive square root]

$\Rightarrow\quad \cos A=\sqrt{1-\dfrac{1}{\operatorname{cosec}^2 A}}$

$\Rightarrow\quad \cos A=\sqrt{\dfrac{\operatorname{cosec}^2 A-1}{\operatorname{cosec}^2 A}}=\dfrac{\sqrt{\operatorname{cosec}^2 A-1}}{\operatorname{cosec} A}$ $[\because \sqrt{a^2}=a]$

(ii) **Hint** $\sin A=\dfrac{1}{\operatorname{cosec} A}$

(iii) **Hint** We know that $\operatorname{cosec}^2 A-\cot^2 A=1$

$\Rightarrow\quad \cot^2 A=\operatorname{cosec}^2 A-1$ [taking positive square root]

$\Rightarrow\quad \cot A=\sqrt{\operatorname{cosec}^2 A-1}$

(iv) **Hint** $\tan A=\dfrac{1}{\cot A}=\dfrac{1}{\sqrt{\operatorname{cosec}^2 A-1}}$ [using part (iii)]

(v) **Hint** $\sec A=\dfrac{1}{\cos A}=\dfrac{\operatorname{cosec} A}{\sqrt{\operatorname{cosec}^2 A-1}}$ [using part (i)]

3. **Hint** LHS $=\sqrt{\dfrac{1+\cos A}{1-\cos A}\times\dfrac{1+\cos A}{1+\cos A}}$

4. **Hint** LHS $=(1-\sin A+\cos A)^2$

$$=(1)^2+\sin^2 A+\cos^2 A-2\sin A(1)-2\sin A\cdot\cos A+2\cos A$$

5. **Hint** LHS $=\dfrac{\sin^2\theta+\cos^2\theta+2\cos\theta+1}{(1+\cos\theta)\sin\theta}=\dfrac{1+2\cos\theta+1}{(1+\cos\theta)\sin\theta}$

6. **Hint** Use $1-\cos^2\theta=\sin^2\theta$ in LHS, then simplify it.

7. **Hint** LHS $=\dfrac{1-\cos\theta}{1+\cos\theta}\times\dfrac{1-\cos\theta}{1-\cos\theta}=\dfrac{(1-\cos\theta)^2}{1-\cos^2\theta}$

$[\because (a+b)(a-b)=(a^2-b^2)]$

$=\left(\dfrac{1-\cos\theta}{\sin\theta}\right)^2$ $[\because \sin^2 A+\cos^2 A=1]$

8. **Hint** $\dfrac{\cot A}{1+\operatorname{cosec} A}-\dfrac{\cot A}{1-\operatorname{cosec} A}=\dfrac{k}{\cos A}$

Taking LHS $=\dfrac{\cot A}{1+\operatorname{cosec} A}-\dfrac{\cot A}{1-\operatorname{cosec} A}$

$=\dfrac{\frac{\cos A}{\sin A}}{1+\frac{1}{\sin A}}-\dfrac{\frac{\cos A}{\sin A}}{1-\frac{1}{\sin A}}$ $\left[\because \operatorname{cosec} A=\dfrac{1}{\sin A}\text{ and }\cot A=\dfrac{\cos A}{\sin A}\right]$

$=\dfrac{\cos A}{1+\sin A}-\dfrac{\cos A}{\sin A-1}$

$=\dfrac{\cos A[(\sin A-1)-(1+\sin A)]}{(1-\sin^2 A)}$

$=\dfrac{-2\cos A}{\cos^2 A}$ $[\because 1-\sin^2 A=\cos^2 A]$

$=\dfrac{-2}{\cos A}$

Ans. $k=-2$

9. **Hint** $\sin x\cdot\cos x\left(\dfrac{5\sin x}{\cos x}+\dfrac{2\cos x}{\sin x}\right)=a+b\sin^2 x$

Ans. $a=2, b=3$

10. **Hint** LHS $=\sec^2\theta+\dfrac{1+\tan^2\theta}{\tan^2\theta}$ $[\because \sec^2 A-\tan^2 A=1]$

$$=\sec^2\theta+\frac{\sec^2\theta}{\sec^2\theta-1}$$

$$=\frac{\sec^4\theta-\sec^2\theta+\sec^2\theta}{\sec^2\theta-1}$$

$$=\frac{\frac{1}{\cos^4\theta}}{\frac{1}{\cos^2\theta}-1}=\frac{1}{\cos^4\theta\frac{(1-\cos^2\theta)}{\cos^2\theta}}\qquad\left[\because \sec A=\frac{1}{\cos A}\right]$$

$$=\frac{1}{\cos^2\theta\cdot\sin^2\theta}=\frac{1}{(1-\sin^2\theta)\sin^2\theta}$$

$$=\frac{1}{\sin^2\theta-\sin^4\theta}=\text{RHS}$$ **Hence proved.**

11. Hint LHS $=1+\tan\theta+\sec\theta+\cot\theta+\cot\theta\cdot\tan\theta+\cot\theta\cdot\sec\theta-\operatorname{cosec}\theta-\operatorname{cosec}\theta\cdot\tan\theta-\operatorname{cosec}\theta\cdot\sec\theta$

$$=1+\tan\theta+\sec\theta+\cot\theta+1+\operatorname{cosec}\theta-\operatorname{cosec}\theta-\sec\theta-\operatorname{cosec}\theta\cdot\sec\theta$$

$$=2+\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}-\frac{1}{\cos\theta\cdot\sin\theta}$$

$$=\frac{2\cos\theta\cdot\sin\theta+\sin^2\theta+\cos^2\theta-1}{\cos\theta\cdot\sin\theta}=2=\text{RHS}$$

$[\because \cos^2A+\sin^2A=1]$ **Hence proved.**

12. Hint LHS $=(1+\tan^2\theta)+(1+\cot^2\theta)=\sec^2\theta+\operatorname{cosec}^2\theta$

$=\text{RHS}$ **Hence proved.**

13. Hint RHS $=\dfrac{2\sin^2\theta-1}{\sin\theta\cdot\cos\theta}=\dfrac{\sin^2\theta+\sin^2\theta-1}{\sin\theta\cdot\cos\theta}$

14. Hint LHS $=\dfrac{\tan\theta}{1-\cot\theta}+\dfrac{\cot\theta}{1-\tan\theta}$

$$=\frac{\tan\theta}{1-\frac{1}{\tan\theta}}+\frac{\frac{1}{\tan\theta}}{1-\tan\theta}$$

$$=\frac{\tan^2\theta}{\tan\theta-1}+\frac{1}{\tan\theta(1-\tan\theta)}=\frac{1-\tan^3\theta}{\tan\theta(1-\tan\theta)}$$

$$=\frac{(1-\tan\theta)(1+\tan\theta+\tan^2\theta)}{\tan\theta(1-\tan\theta)}$$

$$=\frac{\sec^2\theta+\tan\theta}{\tan\theta}\qquad[\because 1+\tan^2A=\sec^2A]$$

$$=\frac{\sec^2\theta}{\tan\theta}+1=\frac{1}{\cos^2\theta}\times\frac{\cos\theta}{\sin\theta}+1$$

$=\sec\theta\operatorname{cosec}\theta+1=\text{RHS}$ **Hence proved.**

15. Hint LHS $=\dfrac{1-\cos A+\sin A}{\sin A+\cos A-1}=\dfrac{\sec A+\tan A-1}{1-(\sec A-\tan A)}$

[dividing numerator and denominator by $\cos A$]

$$=\frac{(\sec A+\tan A)-1}{1-(\sec A-\tan A)}\times\frac{1+(\sec A-\tan A)}{1+(\sec A-\tan A)}$$

$$=\frac{(\sec A+\tan A)+(\sec^2A-\tan^2A)-1-(\sec A-\tan A)}{1-(\sec^2A+\tan^2A-2\tan A\sec A)}$$

$$=\frac{2\tan A}{2\tan A(\sec A-\tan A)}\qquad[\because \sec^2A-\tan^2A=1]$$

$$=\frac{1}{\sec A-\tan A}=\frac{\sec A+\tan A}{(\sec A-\tan A)(\sec A+\tan A)}$$

$$=\frac{\sec A+\tan A}{1}$$

$$=\frac{1+\sin A}{\cos A}=\text{RHS}$$ **Hence proved.**

16. Hint To prove $(\operatorname{cosec}\theta-\sin\theta)(\sec\theta-\cos\theta)(\tan\theta+\cot\theta)=1$

Proof LHS $=(\operatorname{cosec}\theta-\sin\theta)(\sec\theta-\cos\theta)(\tan\theta+\cot\theta)$

$$=\left(\frac{1}{\sin\theta}-\sin\theta\right)\left(\frac{1}{\cos\theta}-\cos\theta\right)\left(\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}\right)$$

$$\left[\begin{array}{l}\because \operatorname{cosec}A=\dfrac{1}{\sin A},\ \sec A=\dfrac{1}{\cos A}\\ \tan A=\dfrac{\sin A}{\cos A},\ \cot A=\dfrac{\cos A}{\sin A}\end{array}\right]$$

$$=\frac{1-\sin^2\theta}{\sin\theta}\times\frac{1-\cos^2\theta}{\cos\theta}\times\frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}$$

$[\because \sin^2A+\cos^2A=1\Rightarrow 1-\cos^2A=\sin^2A$

and $1-\sin^2A=\cos^2A]$

$$=\frac{\cos^2\theta}{\sin\theta}\times\frac{\sin^2\theta}{\cos\theta}\times\frac{1}{\cos\theta\sin\theta}=1=\text{RHS}$$

Hence proved.

17. Hint LHS $=\sqrt{\dfrac{1+\sin A}{1-\sin A}\times\dfrac{1+\sin A}{1+\sin A}}+\sqrt{\dfrac{1-\sin A}{1+\sin A}\times\dfrac{1-\sin A}{1-\sin A}}$

$$=\frac{1+\sin A}{\cos A}+\frac{1-\sin A}{\cos A}=2\sec A=\text{RHS}$$

Hence proved.

18. Hint Do same as Question 17.

19. Hint LHS $=\sec^2\theta-\dfrac{\dfrac{\sin^2\theta}{\cos^4\theta}-\dfrac{2\sin^4\theta}{\cos^4\theta}}{\dfrac{2\cos^4\theta}{\cos^4\theta}-\dfrac{\cos^2\theta}{\cos^4\theta}}$

$$=\sec^2\theta-\frac{\tan^2\theta\cdot\sec^2\theta-2\tan^4\theta}{2-\sec^2\theta}$$

$$=\sec^2\theta-\frac{\tan^2\theta(\sec^2\theta-2\tan^2\theta)}{2-\sec^2\theta}$$

$$=\sec^2\theta-\frac{\tan^2\theta(2-\sec^2\theta)}{2-\sec^2\theta}$$

$$=\sec^2\theta-\tan^2\theta$$

$=1=\text{RHS}$ **Hence proved.**

20. Hint LHS $=\dfrac{1+\tan(\tan^2\theta)}{1+\tan\theta}+\tan\theta-\sec^2\theta$

21. Hint LHS $=(\sec\theta+\tan\theta-1)(\sec\theta+\tan\theta+1)$

$$=(\sec\theta+\tan\theta)^2-1$$

$$=1+\tan^2\theta+\tan^2\theta+2\sec\theta\tan\theta-1$$

$$=2\tan^2\theta+2\sec\theta\tan\theta$$

$$=2\tan\theta(\tan\theta+\sec\theta)$$

$$=\frac{2\sin\theta}{\cos\theta}\left(\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}\right)$$

22. Hint LHS, write $\tan A = \dfrac{\sin A}{\cos A}$

23. Hint LHS $= \dfrac{\operatorname{cosec}\theta + \cot\theta}{\operatorname{cosec}\theta - \cot\theta} \times \dfrac{\operatorname{cosec}\theta + \cot\theta}{\operatorname{cosec}\theta + \cot\theta}$

$$= \frac{(\operatorname{cosec}\theta + \cot\theta)^2}{\operatorname{cosec}^2\theta - \cot^2\theta} = (\operatorname{cosec}\theta + \cot\theta)^2$$

$$= \operatorname{cosec}^2\theta + \cot^2\theta + 2\operatorname{cosec}\theta \cdot \cot\theta$$

24. Hint LHS $= \dfrac{\sin^2 A - \sin^2 B + \cos^2 A - \cos^2 B}{(\cos A + \cos B)(\sin A - \sin B)}$

$$= \frac{1-1}{(\cos A + \cos B)(\sin A - \sin B)} = 0$$

25. Hint First term $= \sin A\left(\dfrac{1-\cos A + 1 + \cos A}{1-\cos^2 A}\right) = \sin A \cdot \dfrac{2}{\sin^2 A}$

$$= \frac{2}{\sin A} = 2\operatorname{cosec} A \qquad \left[\because \frac{1}{\sin\theta} = \operatorname{cosec}\theta\right]$$

$=$ Third term

Second term $= \sqrt{\dfrac{1+\cos A}{1-\cos A}} + \sqrt{\dfrac{1-\cos A}{1+\cos A}}$

$$= \sqrt{\frac{1+\cos A}{1-\cos A} \times \frac{1+\cos A}{1+\cos A}} + \sqrt{\frac{1-\cos A}{1+\cos A} \times \frac{1-\cos A}{1-\cos A}}$$

$$= \frac{1+\cos A}{\sqrt{\sin^2 A}} + \frac{1-\cos A}{\sqrt{\sin^2 A}} \qquad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$= \frac{1+\cos A + 1 - \cos A}{\sin A} = \frac{2}{\sin A} = 2\operatorname{cosec} A$$

$$= \text{Third term} \qquad \left[\because \frac{1}{\sin\theta} = \operatorname{cosec}\theta\right]$$

Hence proved.

26. Hint Given, $\left(\dfrac{\sin A}{(1+\cos A)} + \dfrac{1+\cos A}{\sin A}\right)\left(\dfrac{\sin A}{1-\cos A} - \dfrac{1-\cos A}{\sin A}\right)$

$$= \left(\frac{\sin^2 A + 1 + \cos^2 A + 2\cos A}{\sin A(1+\cos A)}\right)\left(\frac{\sin^2 A - (1+\cos^2 A - 2\cos A)}{\sin A(1-\cos A)}\right)$$

$$[\because \sin^2 A + \cos^2 A = 1]$$

$$= \left(\frac{2+2\cos A}{\sin A(1+\cos A)}\right)\left(\frac{1-\cos^2 A - 1 - \cos^2 A + 2\cos A}{\sin A(1-\cos A)}\right)$$

$$[\sin^2 A = 1 - \cos^2 A]$$

$$= \frac{2(1+\cos A)}{\sin A(1+\cos A)} \cdot \frac{2\cos A(1-\cos A)}{\sin A(1-\cos A)}$$

$= 4\operatorname{cosec} A \cdot \cot A =$ RHS

27. Hint Given, $\sin\theta + \cos\theta = p$ ...(i)

and $\sec\theta + \operatorname{cosec}\theta = q$

$$\Rightarrow \frac{1}{\cos\theta} + \frac{1}{\sin\theta} = q \Rightarrow \frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta} = q \qquad \text{...(ii)}$$

On squaring Eq. (i), we get

$$(\sin\theta + \cos\theta)^2 = p^2$$

$$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = p^2$$

$$\Rightarrow 1 + 2\sin\theta\cos\theta = p^2 \qquad [\because \sin^2 A + \cos^2 A = 1]$$

On subtracting 1 from both the sides, we get

$$\Rightarrow 2\sin\theta\cos\theta = p^2 - 1 \qquad \text{...(iii)}$$

From Eq. (ii), we get

$$\frac{2(\sin\theta + \cos\theta)}{2\sin\theta\cos\theta} = q$$

$$\Rightarrow 2(\sin\theta + \cos\theta) = 2q\sin\theta\cos\theta$$

$$\Rightarrow 2p = q(p^2 - 1) \qquad [\text{using Eqs. (i) and (iii)}]$$

Hence proved.

28. Hint First, expand the terms in left hand side of the equation and simplify them.

Then, compare the value of LHS with RHS of identity to get required values.

Ans. $p = 3$ and $q = -3$

29. Hint Given, $\sec\theta + \tan\theta = p$ ...(i)

$$\text{LHS} = \sec\theta - \tan\theta \times \frac{\sec\theta + \tan\theta}{\sec\theta + \tan\theta}$$

$$= \frac{1}{\sec\theta + \tan\theta} = \frac{1}{p} \qquad \text{...(ii)}$$

On adding Eqs. (i) and (ii), we get

$$2\sec\theta = p + \frac{1}{p}$$

$$\Rightarrow \cos\theta = \frac{2p}{p^2+1}$$

$$\therefore \sin\theta = \sqrt{1 - \left(\frac{2p}{p^2+1}\right)^2} = \frac{p^2-1}{p^2+1}$$

30. Hint Given, $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$

$$\Rightarrow \cos\theta = \sqrt{2}\sin\theta + \sin\theta = \sin\theta(\sqrt{2}+1)$$

$$\Rightarrow \sin\theta = \frac{\cos\theta}{(\sqrt{2}+1)} \Rightarrow \sin\theta = \frac{\cos\theta}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1}$$

$$\Rightarrow \sin\theta = \frac{(\sqrt{2}-1)\cos\theta}{2-1} = \sqrt{2}\cos\theta - \cos\theta$$

$\therefore \cos\theta + \sin\theta = \sqrt{2}\cos\theta$ **Hence proved.**

31. Hint LHS $= 2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1$

$$= 2[(\sin^2\theta)^3 + (\cos^2\theta)^3] - 3(\sin^4\theta + \cos^4\theta) + 1$$

$$= 2[(\sin^2\theta + \cos^2\theta)(\sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta)] - 3\sin^4\theta - 3\cos^4\theta + 1$$

$$= -\sin^4\theta - \cos^4\theta - 2\sin^2\theta\cos^2\theta + 1$$

$$= 1 - (\sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta)$$

$$= 1 - [(\sin^2\theta + \cos^2\theta)^2]$$

$= 1 - 1 = 0 =$ RHS **Hence proved.**

32. Hint $\sqrt{\dfrac{m}{n}} + \sqrt{\dfrac{n}{m}} = \sqrt{\dfrac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}} + \sqrt{\dfrac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta}}$

$$= \frac{\cos\theta - \sin\theta}{\sqrt{\cos^2\theta - \sin^2\theta}} + \frac{\cos\theta + \sin\theta}{\sqrt{\cos^2\theta - \sin^2\theta}}$$

$$= \frac{2\cos\theta}{\sqrt{\cos^2\theta - \sin^2\theta}}$$

NCERT FOLDER

Complete Solutions of All Exercises of NCERT Textbook

EXERCISE 8.1

Q1. **In ΔABC, right angled at B, $AB = 24$ cm and $BC = 7$ cm. Determine**

(i) $\sin A, \cos A$ (ii) $\sin C, \cos C$.

Sol. In right angled ΔABC,

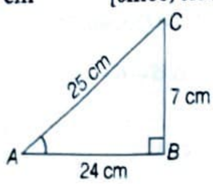
$$AB = 24 \text{ cm}, BC = 7 \text{ cm}$$

Now, $AC^2 = AB^2 + BC^2$ [by Pythagoras theorem]

$\Rightarrow \quad AC^2 = (24)^2 + (7)^2 = 576 + 49 = 625$

$\Rightarrow \quad AC = \sqrt{625} = 25$ [taking positive square root]

$\therefore \quad AC = 25$ cm [since, side cannot be negative]



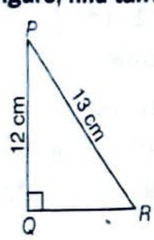
(i) With reference to $\angle A$, we have

Base $= AB = 24$ cm, perpendicular $= BC = 7$ cm and hypotenuse $= AC = 25$ cm

$\therefore \sin A = \frac{P}{H} = \frac{BC}{AC} = \frac{7}{25}$ and $\cos A = \frac{B}{H} = \frac{AB}{AC} = \frac{24}{25}$

(ii) Do same as part (i). **Ans.** $\sin C = \frac{24}{25}, \cos C = \frac{7}{25}$

Q2. **In the following figure, find $\tan P - \cot R$**



Sol. In right angled ΔPQR,

$PQ = 12$ cm, $PR = 13$ cm [given]

Then, $PQ^2 + QR^2 = PR^2$ [by Pythagoras theorem]

$\Rightarrow \quad (12)^2 + QR^2 = (13)^2$

$\Rightarrow \quad 144 + QR^2 = 169 \Rightarrow QR^2 = 169 - 144 = 25$

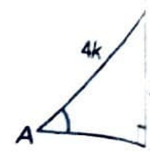
$\Rightarrow \quad QR = 5$ cm [taking positive square root since, side cannot be negative]

Now, $\tan P = \frac{P}{B} = \frac{QR}{PQ} = \frac{5}{12}$ and $\cot R = \frac{B}{P} = \frac{QR}{PQ} = \frac{5}{12}$

$\therefore \quad \tan P - \cot R = \frac{5}{12} - \frac{5}{12} = 0$

Q3. **If $\sin A = \frac{3}{4}$, calculate $\cos A$ and $\tan A$.**

Sol. Given, $\sin A = \frac{3}{4} \Rightarrow \frac{P}{H} = \frac{3}{4} \Rightarrow \frac{BC}{AC} = \frac{3}{4}$



Let $\quad BC = 3k, AC = 4k$

where, k is any positive integer.

In right angled ΔABC, we have

$BC^2 + AB^2 = AC^2$ [by Pythagoras theore

$\Rightarrow \quad (3k)^2 + AB^2 = (4k)^2$

$\Rightarrow \quad 9k^2 + AB^2 = 16k^2$

$\Rightarrow \quad AB^2 = 16k^2 - 9k^2 = 7k^2 \Rightarrow AB = k\sqrt{7}$

[taking positive square root sinc side cannot be negat

$\therefore \quad \cos A = \frac{B}{H} = \frac{k\sqrt{7}}{4k} = \frac{\sqrt{7}}{4}$

and $\quad \tan A = \frac{P}{B} = \frac{3k}{k\sqrt{7}} = \frac{3}{\sqrt{7}}$

Q4. **Given, $15 \cot A = 8$, find $\sin A$ and $\sec A$.**

Sol. Do same as Question 3. **Ans.** $\sin A = \frac{15}{17}, \sec A = \frac{17}{8}$

Q5. **Given, $\sec \theta = \frac{13}{12}$, calculate all other trigonometric ratios.**

Sol. Do same as Example 3 of Topic 1.

Ans.

$\sin\theta = \frac{5}{13}, \cos\theta = \frac{12}{13}, \tan\theta = \frac{5}{12}, \text{cosec } \theta = \frac{13}{5}, \cot\theta = \frac{12}{5}$

Q6. **If $\angle A$ and $\angle B$ are acute angles such that $\cos A = \cos B$, show that $\angle A = \angle B$.**

Sol. Draw a right angled ΔABC, right angled at C. With reference to $\angle A$, base $= AC$, hypotenuse $= AB$ and with reference to $\angle B$, base $= BC$ and hypotenuse $= AB$

Given, $\quad \cos A = \cos B$

$\Rightarrow \quad \frac{AC}{AB} = \frac{BC}{AB}$

$\Rightarrow \quad AC = BC$

$\therefore \quad \angle B = \angle A$

[since, in a triangle angles opposi to equal sides are also equal] **Hence prove**

Q7. If $\cot\theta = \frac{7}{8}$, evaluate

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$ (ii) $\cot^2\theta$

Sol. Given, $\cot\theta = \frac{7}{8} \Rightarrow \frac{B}{P} = \frac{7}{8}$

Let $B = 7k$ and $P = 8k$

where, k is any positive integer.

Draw a right angled ΔPQR, right angled at Q.

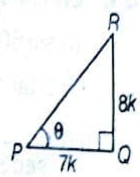
In right angled ΔPQR,

$$H^2 = B^2 + P^2$$

$\Rightarrow$ $H^2 = (7k)^2 + (8k)^2$ [by Pythagoras theorem]

$\Rightarrow$ $H^2 = 49k^2 + 64k^2 = 113k^2$

$\Rightarrow$ $H = k\sqrt{113}$ [taking positive square root since, side cannot be negative]



$\therefore$ $\sin\theta = \frac{P}{H} = \frac{8k}{k\sqrt{113}} = \frac{8}{\sqrt{113}}$

and $\cos\theta = \frac{B}{H} = \frac{7k}{k\sqrt{113}} = \frac{7}{\sqrt{113}}$

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{1^2-\sin^2\theta}{1^2-\cos^2\theta}$

[$\because (a+b)(a-b) = a^2 - b^2$]

$$= \frac{1-\left(\frac{8}{\sqrt{113}}\right)^2}{1-\left(\frac{7}{\sqrt{113}}\right)^2} = \frac{1-\frac{64}{113}}{1-\frac{49}{113}} = \frac{\frac{113-64}{113}}{\frac{113-49}{113}} = \frac{49}{64}$$

(ii) $\cot^2\theta = \left(\frac{7}{8}\right)^2 = \frac{49}{64}$

Q8. If $3\cot A = 4$, check whether

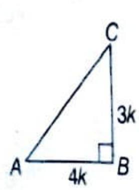
$$\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A \text{ or not.}$$

Sol. Given, $3\cot A = 4$

$\Rightarrow$ $\cot A = \frac{4}{3} \Rightarrow \frac{B}{P} = \frac{4}{3}$

Let $B = 4k$ and $P = 3k$

where, k is any positive integer.



Draw a right angled ΔABC, right angled at B.

In right angled ΔABC,

$$H^2 = P^2 + B^2$$ [by Pythagoras theorem]

$\Rightarrow$ $H^2 = (3k)^2 + (4k)^2$

$= 9k^2 + 16k^2 = 25k^2$

$\Rightarrow$ $H = 5k$ [taking positive square root since, side cannot be negative]

$\therefore$ $\tan A = \frac{1}{\cot A} = \frac{3}{4}$ $\left[\because \tan\theta = \frac{1}{\cot\theta}\right]$

$\cos A = \frac{B}{H} = \frac{4k}{5k} = \frac{4}{5}$ and $\sin A = \frac{P}{H} = \frac{3k}{5k} = \frac{3}{5}$

Now, $\text{LHS} = \frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2}$ $\left[\because \tan\theta = \frac{3}{4}\right]$

$$= \frac{1-\frac{9}{16}}{1+\frac{9}{16}} = \frac{\frac{16-9}{16}}{\frac{16+9}{16}} = \frac{7}{25} \quad \text{...(i)}$$

and $\text{RHS} = \cos^2 A - \sin^2 A$

$$= \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \frac{16-9}{25} = \frac{7}{25} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), LHS = RHS **Hence proved.**

Q9. In ΔABC, right angled at B, if $\tan A = \frac{1}{\sqrt{3}}$, then find the value of

(i) $\sin A\cos C + \cos A\sin C$.

(ii) $\cos A\cos C - \sin A\sin C$.

Sol. Given, $\tan A = \frac{1}{\sqrt{3}}$

$\Rightarrow$ $\frac{P}{B} = \frac{1}{\sqrt{3}}$

Let $P = k$ and $B = \sqrt{3}k$, where k is any positive integer.

Draw a right angled ΔABC, right angled at B.

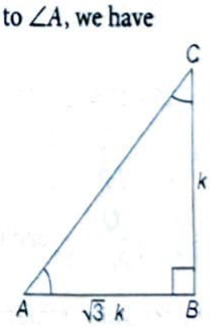
In right angled ΔABC,

$$H^2 = B^2 + P^2$$ [by Pythagoras theorem]

$\Rightarrow$ $H^2 = (\sqrt{3}k)^2 + (k)^2 = 3k^2 + k^2 = 4k^2$

$\therefore$ $H = 2k$ [taking positive square root since, side cannot be negative]

With reference to $\angle A$, we have



Base $= AB = \sqrt{3}k$, perpendicular $= BC = k$

and hypotenuse $= AC = 2k$

With reference to $\angle C$, we have

Base $= BC = k$, perpendicular $= AB = \sqrt{3}k$

and hypotenuse $= AC = 2k$

(i) $\sin A \cos C + \cos A \sin C = \left(\frac{BC}{AC}\right)\left(\frac{BC}{AC}\right) + \left(\frac{AB}{AC}\right)\left(\frac{AB}{AC}\right)$

$\left[\because \sin\theta = \frac{P}{H} \text{ and } \cos\theta = \frac{B}{H}\right]$

$= \left(\frac{k}{2k}\right)\left(\frac{k}{2k}\right) + \left(\frac{\sqrt{3}k}{2k}\right)\left(\frac{\sqrt{3}k}{2k}\right)$

$= \frac{k^2}{4k^2} + \frac{3k^2}{4k^2} = \frac{4k^2}{4k^2} = 1$

(ii) Do same as Part (i). **Ans.** 0

Q 10. In ΔPQR, right angled at Q, $PR + QR = 25$ cm and $PQ = 5$ cm. Determine the values of $\sin P$, $\cos P$ and $\tan P$.

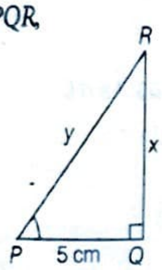
Sol. Let $QR = x$ and $PR = y$

Then, $PR + QR = 25$ [given]

$\Rightarrow y + x = 25$

$\Rightarrow y = 25 - x$ ...(i)

In right angled ΔPQR,



$RQ^2 + PQ^2 = PR^2$

$\Rightarrow x^2 + (5)^2 = y^2$ [by Pythagoras theorem]

$\Rightarrow x^2 + 25 = (25 - x)^2$ [from Eq. (i)]

$\Rightarrow x^2 + 25 = 625 + x^2 - 50x$

$[\because (a-b)^2 = a^2 + b^2 - 2ab]$

$\Rightarrow x^2 - x^2 + 50x = 625 - 25$

$\Rightarrow 50x = 600$

$\Rightarrow x = \frac{600}{50} = 12 = QR$

From Eq. (i),

$y = 25 - x = 25 - 12 = 13 = PR$

Now, $\sin P = \frac{P}{H} = \frac{RQ}{PR} = \frac{12}{13}$

$\cos P = \frac{B}{H} = \frac{PQ}{PR} = \frac{5}{13}$ and $\tan P = \frac{P}{B} = \frac{QR}{PQ} = \frac{12}{5}$

Q 11. State whether the following statements are true or false. Justify your answer.

(i) The value of $\tan A$ is always less than 1.

(ii) $\sec A = \frac{12}{5}$ for some value of $\angle A$.

(iii) $\cos A$ is the abbreviation used for the cosecant of $\angle A$

(iv) $\cot A$ is the product of cot and A.

(v) $\sin\theta = \frac{4}{3}$ for some $\angle\theta$.

Sol. (i) False, because the value of $\tan A$ lies between $-\infty$ a[...] $+\infty$.

(ii) True, because $\sec A$ is always greater than 1.

(iii) False, because $\cos A$ is the abbreviation of cosine of $\angle A$ and for cosecant of $\angle A$, the abbreviation is cosec A.

(iv) False, because $\cot A$ is just a symbol and cannot be separated.

(v) False, because the value of $\sin\theta$ is always less than or equal to 1. Here, $\sin\theta = 4/3$ which is greater than 1, [...] it is not possible for any θ.

EXERCISE 8.2

Q1. Evaluate the following.

(i) $\sin 60° \cos 30° + \sin 30° \cos 60°$

(ii) $2\tan^2 45° + \cos^2 30° - \sin^2 60°$

(iii) $\frac{\cos 45°}{\sec 30° + \text{cosec } 30°}$

(iv) $\frac{\sin 30° + \tan 45° - \text{cosec } 60°}{\sec 30° + \cos 60° + \cot 45°}$

(v) $\frac{5\cos^2 60° + 4\sec^2 30° - \tan^2 45°}{\sin^2 30° + \cos^2 30°}$

Sol. (i) $\sin 60° \cos 30° + \sin 30° \cos 60° = \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2}$

$\left[\because \sin 60° = \cos 30° = \frac{\sqrt{3}}{2} \text{ and } \sin 30° = \cos 60° = \ldots\right]$

$= \frac{3}{4} + \frac{1}{4} = \frac{3+1}{4} = \frac{4}{4} = 1$

(ii) Do same as part (i). **Ans.** 2

(iii) Do same as part (i) **Ans.** $\frac{3\sqrt{2} - \sqrt{6}}{8}$

(iv) $\frac{\sin 30° + \tan 45° - \text{cosec } 60°}{\sec 30° + \cos 60° + \cot 45°}$

$= \frac{\frac{1}{2} + \frac{1}{1} - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + \frac{1}{1}} = \frac{\frac{\sqrt{3} + 2\sqrt{3} - 4}{2\sqrt{3}}}{\frac{4 + \sqrt{3} + 2\sqrt{3}}{2\sqrt{3}}}$

$\left[\because \sin 30° = \cos 60° = \frac{1}{2}, \text{cosec } 60° = \sec 30° = \frac{2}{\sqrt{3}}\right.$

$\left. \cot 45° = \tan 45° = 1\right]$

$= \frac{3\sqrt{3} - 4}{4 + 3\sqrt{3}} = \frac{3\sqrt{3} - 4}{4 + 3\sqrt{3}} \times \frac{4 - 3\sqrt{3}}{4 - 3\sqrt{3}}$

[multiplying numerator and denominator [...] the conjugate of $4 + 3\sqrt{3}$ i.e. $4 - 3\sqrt{\ldots}$

$$= \frac{12\sqrt{3} - 27 - 16 + 12\sqrt{3}}{(4)^2 - (3\sqrt{3})^2}$$

$[\because (a+b)(a-b) = a^2 - b^2]$

$$= \frac{24\sqrt{3} - 43}{16 - 27} = \frac{-(43 - 24\sqrt{3})}{-11} = \frac{43 - 24\sqrt{3}}{11}$$

(v) Do same as part (iv). **Ans.** $\frac{67}{12}$

Q 2. Choose the correct option and justify your choice.

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(i) $\frac{2\tan 30°}{1+\tan^2 30°}$ is equal to

(a) $\sin 60°$ (b) $\cos 60°$ (c) $\tan 60°$ (d) $\sin 30°$

(ii) $\frac{1-\tan^2 45°}{1+\tan^2 45°}$ is equal to

(a) $\tan 90°$ (b) 1 (c) $\sin 45°$ (d) 0

(iii) $\sin 2A = 2\sin A$ is true, when A is equal to

(a) $0°$ (b) $30°$ (c) $45°$ (d) $60°$

(iv) $\frac{2\tan 30°}{1-\tan^2 30°}$ is equal to

(a) $\cos 60°$ (b) $\sin 60°$ (c) $\tan 60°$ (d) $\sin 30°$

Sol. (i) (a) $\frac{2\tan 30°}{1+\tan^2 30°} = \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1+\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{\left(1+\frac{1}{3}\right)}\left[\because \tan 30° = \frac{1}{\sqrt{3}}\right]$

$$= \frac{\frac{2}{\sqrt{3}}}{\left(\frac{3+1}{3}\right)} = \frac{2}{\sqrt{3}} \times \frac{3}{4} = \frac{\sqrt{3}}{2} = \sin 60° \left[\because \sin 60° = \frac{\sqrt{3}}{2}\right]$$

(ii) (d) $\frac{1-\tan^2 45°}{1+\tan^2 45°} = \frac{1-(1)^2}{1+(1)^2}$ $[\because \tan 45° = 1]$

$$= \frac{1-1}{1+1} = \frac{0}{2} = 0$$

(iii) (a) Given, $\sin 2A = 2\sin A$

When $A = 0°$, then $\sin(2 \times 0°) = 2\sin(0°)$

Now, $2\sin(0°) = 2(0) = 0$ $[\because \sin 0° = 0]$

$\therefore$ $0 = 0$

So, for $A = 0°$, the given statement is true.

(iv) (c) Do same as part (i). **Ans.** $\tan 60°$

Q 3. If $\tan(A+B) = \sqrt{3}$ and $\tan(A-B) = 1/\sqrt{3}$, $0° < A+B \le 90°$, $A > B$, find the values of A and B.

CBSE 2020 (Basic), 12, 11, 10

Sol. Do same as Example 6 of Topic 2. **Ans.** $A = 45°$ and $B = 15°$

Q 4. State whether the following statements are true or false. Justify your answer.

(i) $\sin(A+B) = \sin A + \sin B$.

(ii) The value of $\sin\theta$ increases as θ increases.

(iii) The value of $\cos\theta$ increases as θ increases.

(iv) $\sin\theta = \cos\theta$ for all values of θ.

(v) $\cot A$ is not defined for $A = 0°$.

Sol. (i) False, let $A = 60°$ and $B = 30°$

Then, $\sin(A+B) = \sin(60° + 30°) = \sin 90° = 1$

and $\sin A + \sin B = \sin 60° + \sin 30° = \frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{\sqrt{3}+1}{2}$

Hence $\sin(A+B) \neq \sin A + \sin B$.

(ii) True, consider the table given below

θ	$0°$	$30°$	$45°$	$60°$	$90°$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1

From this table, it is clear that the value of $\sin\theta$ increases as θ increases.

(iii) False, consider the table given below

θ	$0°$	$30°$	$45°$	$60°$	$90°$
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0

From this table, it is clear that the value of $\cos\theta$ decreases as θ increases.

(iv) False, because it is only true for $\theta = 45°$

$\because$ $\sin 45° = \frac{1}{\sqrt{2}} = \cos 45°$

(v) True, because if $A = 0°$, then $\cot A = \frac{1}{\tan A}$

$$= \frac{1}{\tan 0°} = \frac{1}{0} = \infty \quad \text{[not defined]}$$

EXERCISE 8.3

Q 1. Express the trigonometric ratios $\sin A$, $\tan A$ and $\sec A$ in terms of $\cot A$.

Sol. (i) $\sin A = \frac{1}{\operatorname{cosec} A} = \frac{1}{\sqrt{1+\cot^2 A}}$

$[\because \operatorname{cosec}^2\theta = 1 + \cot^2\theta \Rightarrow \operatorname{cosec}\theta = \sqrt{1+\cot^2\theta}]$

(ii) $\tan A = \frac{1}{\cot A}$ $[\because \tan\theta \cot\theta = 1]$

(iii) $\sec A = \sqrt{1+\tan^2 A}$

$[\because \sec^2\theta = 1 + \tan^2\theta \Rightarrow \sec\theta = \sqrt{1+\tan^2\theta}]$

$$= \sqrt{1+\left(\frac{1}{\cot A}\right)^2} \quad \left[\because \tan\theta = \frac{1}{\cot\theta}\right]$$

$$= \sqrt{1+\frac{1}{\cot^2 A}} = \sqrt{\frac{\cot^2 A + 1}{\cot^2 A}} = \frac{\sqrt{1+\cot^2 A}}{\cot A}$$

$[\because \sqrt{a^2} = a]$

Q2. Write all the other trigonometric ratios of $\angle A$ in terms of sec A

Sol. (i) We know that

$$\sin^2 A + \cos^2 A = 1$$

$$\Rightarrow \sin^2 A = 1 - \cos^2 A$$

$$\Rightarrow \sin A = \sqrt{1 - \cos^2 A}$$

[taking positive square root]

$$= \sqrt{1 - \frac{1}{\sec^2 A}} \quad \left[\because \cos\theta = \frac{1}{\sec\theta}\right]$$

$$= \sqrt{\frac{\sec^2 A - 1}{\sec^2 A}} = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

$$[\because \sqrt{a^2} = a]$$

(ii) $\cos A = \dfrac{1}{\sec A}$

(iii) We know that $\sec^2 A - \tan^2 A = 1$

$$\Rightarrow \tan^2 A = \sec^2 A - 1$$

$\therefore \tan A = \sqrt{\sec^2 A - 1}$ [taking positive square root]

(iv) $\cot A = \dfrac{1}{\tan A} = \dfrac{1}{\sqrt{\sec^2 A - 1}}$ [using part (iii)]

(v) $\operatorname{cosec} A = \dfrac{1}{\sin A} = \dfrac{\sec A}{\sqrt{\sec^2 A - 1}}$ [using part (i)]

Q3. Choose the correct option. Justify your choice.

(i) $9\sec^2 A - 9\tan^2 A$ is equal to

(a) 1 (b) 9 (c) 8 (d) 0

(ii) $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta)$ is equal to

(a) 0 (b) 1 (c) 2 (d) -1

(iii) $(\sec A + \tan A)(1 - \sin A)$ is equal to

[CBSE 2023 (Basic)]

(a) sec A (b) sin A (c) cosec A (d) cos A

(iv) $\dfrac{1 + \tan^2 A}{1 + \cot^2 A}$ is equal to

(a) $\sec^2 A$ (b) -1 (c) $\cot^2 A$ (d) $\tan^2 A$

Sol. (i) (b) $9\sec^2 A - 9\tan^2 A = 9(\sec^2 A - \tan^2 A)$

$$= 9 \times 1 = 9 \quad [\because \sec^2 A - \tan^2 A = 1]$$

(ii) (c) $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta)$

$$= \left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)\left(1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)$$

$$\left[\because \tan A = \frac{\sin A}{\cos A}, \sec A = \frac{1}{\cos A}, \cot A = \frac{\cos A}{\sin A} \text{ and } \operatorname{cosec} A = \frac{1}{\sin A}\right]$$

$$= \left[\frac{(\cos\theta + \sin\theta) + 1}{\cos\theta}\right] \times \left[\frac{(\sin\theta + \cos\theta) - 1}{\sin\theta}\right]$$

$$= \frac{(\cos\theta + \sin\theta)^2 - 1^2}{\cos\theta \sin\theta}$$

$$[\because (a + b)(a - b) = a^2 - b^2]$$

$$= \frac{\cos^2\theta + \sin^2\theta + 2\cos\theta\sin\theta - 1}{\cos\theta\sin\theta}$$

$$[\because (a + b)^2 = a^2 + b^2 + 2ab]$$

$$= \frac{1 + 2\cos\theta\sin\theta - 1}{\cos\theta\sin\theta} \quad [\because \cos^2 A + \sin^2 A = 1]$$

$$= \frac{2\cos\theta\sin\theta}{\cos\theta\sin\theta} = 2$$

(iii) (d) $(\sec A + \tan A)(1 - \sin A)$

$$= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)(1 - \sin A) \quad \left[\because \sec\theta = \frac{1}{\cos\theta} \text{ and } \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$$

$$= \frac{(1 + \sin A)(1 - \sin A)}{\cos A} = \frac{1 - \sin^2 A}{\cos A}$$

$$[\because (a + b)(a - b) = a^2 - b^2]$$

$$= \frac{\cos^2 A}{\cos A} = \cos A$$

$$[\because \sin^2\theta + \cos^2\theta = 1 \Rightarrow \cos^2\theta = 1 - \sin^2\theta]$$

(iv) (d) $\dfrac{1 + \tan^2 A}{1 + \cot^2 A} = \dfrac{\sec^2 A}{\operatorname{cosec}^2 A}$ $\left[\because 1 + \tan^2\theta = \sec^2\theta \text{ and } 1 + \cot^2\theta = \operatorname{cosec}^2\theta\right]$

$$= \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}} \quad \left[\because \sec\theta = \frac{1}{\cos\theta} \text{ and } \operatorname{cosec}\theta = \frac{1}{\sin\theta}\right]$$

$$= \frac{1}{\cos^2 A} \times \frac{\sin^2 A}{1} = \tan^2 A \quad \left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$$

Q4. Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

(i) $(\operatorname{cosec}\theta - \cot\theta)^2 = \dfrac{1 - \cos\theta}{1 + \cos\theta}$

(ii) $\dfrac{\cos A}{1 + \sin A} + \dfrac{1 + \sin A}{\cos A} = 2\sec A$

(iii) $\dfrac{\tan\theta}{1 - \cot\theta} + \dfrac{\cot\theta}{1 - \tan\theta} = 1 + \sec\theta \operatorname{cosec}\theta$

(iv) $\dfrac{1+\sec A}{\sec A}=\dfrac{\sin^2 A}{1-\cos A}$

(v) $\dfrac{\cos A-\sin A+1}{\cos A+\sin A-1}=\operatorname{cosec} A+\cot A,$ using the identity $\operatorname{cosec}^2 A=1+\cot^2 A.$

(vi) $\sqrt{\dfrac{1+\sin A}{1-\sin A}}=\sec A+\tan A$

(vii) $\dfrac{\sin\theta-2\sin^3\theta}{2\cos^3\theta-\cos\theta}=\tan\theta$ CBSE 2024 (Standard)

(viii) $(\sin A+\operatorname{cosec} A)^2+(\cos A+\sec A)^2=7+\tan^2 A+\cot^2 A$ CBSE 2024 (Basic)

(ix) $(\operatorname{cosec} A-\sin A)(\sec A-\cos A)=\dfrac{1}{\tan A+\cot A}$

(x) $\left(\dfrac{1+\tan^2 A}{1+\cot^2 A}\right)=\left(\dfrac{1-\tan A}{1-\cot A}\right)^2=\tan^2 A$

Sol. (i) LHS $=(\operatorname{cosec}\theta-\cot\theta)^2=\left(\dfrac{1}{\sin\theta}-\dfrac{\cos\theta}{\sin\theta}\right)^2$

$\left[\because \operatorname{cosec} A=\dfrac{1}{\sin A},\ \cot A=\dfrac{\cos A}{\sin A}\right]$

$=\left(\dfrac{1-\cos\theta}{\sin\theta}\right)^2=\dfrac{(1-\cos\theta)^2}{\sin^2\theta}=\dfrac{(1-\cos\theta)^2}{1-\cos^2\theta}$

$[\because \sin^2 A=1-\cos^2 A]$

$=\dfrac{(1-\cos\theta)(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$ $[\because a^2-b^2=(a+b)(a-b)]$

$=\dfrac{1-\cos\theta}{1+\cos\theta}=$ RHS **Hence proved.**

(ii) LHS $=\dfrac{\cos A}{1+\sin A}+\dfrac{1+\sin A}{\cos A}=\dfrac{\cos^2 A+(1+\sin A)^2}{(1+\sin A)\cos A}$

$=\dfrac{\cos^2 A+\sin^2 A+1+2\sin A}{(1+\sin A)\cos A}$

$[\because (a+b)^2=a^2+b^2+2ab]$

$=\dfrac{1+1+2\sin A}{(1+\sin A)\cos A}$ $[\because \sin^2\theta+\cos^2\theta=1]$

$=\dfrac{2+2\sin A}{(1+\sin A)\cos A}$

$=\dfrac{2(1+\sin A)}{(1+\sin A)\cos A}$

$=\dfrac{2}{\cos A}=2\sec A$ $\left[\because \dfrac{1}{\cos\theta}=\sec\theta\right]$

$=$ RHS **Hence proved.**

(iii) LHS $=\dfrac{\tan\theta}{1-\cot\theta}+\dfrac{\cot\theta}{1-\tan\theta}=\dfrac{\dfrac{\sin\theta}{\cos\theta}}{1-\dfrac{\cos\theta}{\sin\theta}}+\dfrac{\dfrac{\cos\theta}{\sin\theta}}{1-\dfrac{\sin\theta}{\cos\theta}}$

$\left[\because \tan A=\dfrac{\sin A}{\cos A},\cot A=\dfrac{\cos A}{\sin A}\right]$

$=\dfrac{\dfrac{\sin\theta}{\cos\theta}}{\dfrac{\sin\theta-\cos\theta}{\sin\theta}}+\dfrac{\dfrac{\cos\theta}{\sin\theta}}{\dfrac{\cos\theta-\sin\theta}{\cos\theta}}$

$=\dfrac{\sin\theta}{\cos\theta}\times\dfrac{\sin\theta}{\sin\theta-\cos\theta}+\dfrac{\cos\theta}{\sin\theta}\times\dfrac{\cos\theta}{\cos\theta-\sin\theta}$

$=\dfrac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}-\dfrac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$

$=\dfrac{\sin^3\theta-\cos^3\theta}{\cos\theta\sin\theta(\sin\theta-\cos\theta)}$

$=\dfrac{(\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta)}{\cos\theta\sin\theta(\sin\theta-\cos\theta)}$

$[\because x^3-y^3=(x-y)(x^2+y^2+xy)]$

$=\dfrac{1+\sin\theta\cos\theta}{\cos\theta\sin\theta}$ $[\because \sin^2 A+\cos^2 A=1]$

$=\dfrac{1}{\cos\theta\sin\theta}+\dfrac{\sin\theta\cos\theta}{\sin\theta\cos\theta}=\dfrac{1}{\cos\theta\sin\theta}+1$

$=1+\sec\theta\operatorname{cosec}\theta$

$\left[\because \dfrac{1}{\cos A}=\sec A \text{ and } \dfrac{1}{\sin A}=\operatorname{cosec} A\right]$

$=$ RHS **Hence proved.**

(iv) LHS $=\dfrac{1+\sec A}{\sec A}=\dfrac{1+\dfrac{1}{\cos A}}{\dfrac{1}{\cos A}}$ $\left[\because \sec\theta=\dfrac{1}{\cos\theta}\right]$

$=\dfrac{\dfrac{\cos A+1}{\cos A}}{1/\cos A}=\dfrac{\cos A+1}{\cos A}\times\dfrac{\cos A}{1}$

$=\cos A+1=1+\cos A$ ...(i)

RHS $=\dfrac{\sin^2 A}{1-\cos A}=\dfrac{1-\cos^2 A}{1-\cos A}$

$[\because \sin^2\theta+\cos^2\theta=1 \Rightarrow \sin^2\theta=1-\cos^2\theta]$

$=\dfrac{(1+\cos A)(1-\cos A)}{(1-\cos A)}$ $[\because a^2-b^2=(a+b)(a-b)]$

$=1+\cos A$ ...(ii)

From Eqs. (i) and (ii), LHS $=$ RHS **Hence proved.**

(v) LHS $= \dfrac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$

On dividing numerator and denominator by $\sin A$, we get

$$= \dfrac{\dfrac{\cos A}{\sin A} - \dfrac{\sin A}{\sin A} + \dfrac{1}{\sin A}}{\dfrac{\cos A}{\sin A} + \dfrac{\sin A}{\sin A} - \dfrac{1}{\sin A}} = \dfrac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A}$$

$$\left[\because \dfrac{\cos\theta}{\sin\theta} = \cot\theta \text{ and } \dfrac{1}{\sin\theta} = \operatorname{cosec}\theta\right]$$

$$= \dfrac{\cot A + \operatorname{cosec} A - 1}{\cot A + 1 - \operatorname{cosec} A}$$

$$= \dfrac{(\cot A + \operatorname{cosec} A) - (\operatorname{cosec}^2 A - \cot^2 A)}{\cot A + 1 - \operatorname{cosec} A}$$

$$[\because 1 = \operatorname{cosec}^2 A - \cot^2 A]$$

$$= \dfrac{(\cot A + \operatorname{cosec} A) - [(\operatorname{cosec} A + \cot A)(\operatorname{cosec} A - \cot A)]}{\cot A + 1 - \operatorname{cosec} A}$$

$$[\because a^2 - b^2 = (a+b)(a-b)]$$

$$= \dfrac{(\cot A + \operatorname{cosec} A)[1 - (\operatorname{cosec} A - \cot A)]}{\cot A + 1 - \operatorname{cosec} A}$$

$$= \dfrac{(\cot A + \operatorname{cosec} A)[1 - \operatorname{cosec} A + \cot A]}{(\cot A + 1 - \operatorname{cosec} A)}$$

$= \cot A + \operatorname{cosec} A$

$=$ RHS **Hence proved.**

(vi) LHS $= \sqrt{\dfrac{1+\sin A}{1-\sin A}} = \sqrt{\dfrac{1+\sin A}{1-\sin A} \times \dfrac{1+\sin A}{1+\sin A}}$

[multiplying numerator and denominator by $\sqrt{(1+\sin A)}$]

$$= \sqrt{\dfrac{(1+\sin A)^2}{1-\sin^2 A}} = \sqrt{\dfrac{(1+\sin A)^2}{\cos^2 A}}$$

$$[\because \sin^2\theta + \cos^2\theta = 1 \Rightarrow 1 - \sin^2\theta = \cos^2\theta]$$

$$= \dfrac{1+\sin A}{\cos A} = \dfrac{1}{\cos A} + \dfrac{\sin A}{\cos A}$$

$$= \sec A + \tan A \left[\because \dfrac{1}{\cos\theta} = \sec\theta \text{ and } \tan\theta = \dfrac{\sin\theta}{\cos\theta}\right]$$

$=$ RHS **Hence proved.**

(vii) LHS $= \dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \dfrac{\sin\theta(1 - 2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)}$

$$= \dfrac{\sin\theta[1 - 2(1-\cos^2\theta)]}{\cos\theta(2\cos^2\theta - 1)}$$

$$[\because \sin^2 A + \cos^2 A = 1 \Rightarrow \sin^2 A = 1 - \cos^2 A]$$

$$= \dfrac{\sin\theta(1 - 2 + 2\cos^2\theta)}{\cos\theta(2\cos^2\theta - 1)}$$

$$= \dfrac{\sin\theta(2\cos^2\theta - 1)}{\cos\theta(2\cos^2\theta - 1)}$$

$$= \dfrac{\sin\theta}{\cos\theta} = \tan\theta = \text{RHS} \qquad \left[\because \tan A = \dfrac{\sin A}{\cos A}\right]$$

Hence proved.

(viii) LHS $= (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$

$$= \sin^2 A + \operatorname{cosec}^2 A + 2\sin A \operatorname{cosec} A + \cos^2 A + \sec^2 A + 2\cos A \sec A$$

$$[\because (a+b)^2 = a^2 + b^2 + 2ab]$$

$$= (\sin^2 A + \cos^2 A) + (1 + \cot^2 A) + 2\sin A \dfrac{1}{\sin A} + (1 + \tan^2 A) + 2\cos A \dfrac{1}{\cos A}$$

$$\left[\because \operatorname{cosec}^2\theta = 1 + \cot^2\theta, \sec^2\theta = 1 + \tan^2\theta, \operatorname{cosec}\theta = \dfrac{1}{\sin\theta} \text{ and } \sec\theta = \dfrac{1}{\cos\theta}\right]$$

$$= 1 + 1 + \cot^2 A + 2 + 1 + \tan^2 A + 2$$

$$[\because \sin^2\theta + \cos^2\theta = 1]$$

$= 7 + \tan^2 A + \cot^2 A =$ RHS **Hence proved.**

(ix) LHS $= (\operatorname{cosec} A - \sin A)(\sec A - \cos A)$

$$= \left(\dfrac{1}{\sin A} - \dfrac{\sin A}{1}\right)\left(\dfrac{1}{\cos A} - \dfrac{\cos A}{1}\right)$$

$$\left[\because \operatorname{cosec}\theta = \dfrac{1}{\sin\theta} \text{ and } \sec\theta = \dfrac{1}{\cos\theta}\right]$$

$$= \left(\dfrac{1-\sin^2 A}{\sin A}\right)\left(\dfrac{1-\cos^2 A}{\cos A}\right) = \dfrac{\cos^2 A}{\sin A} \times \dfrac{\sin^2 A}{\cos A}$$

$$[\because \sin^2\theta + \cos^2\theta = 1]$$

$= \cos A \sin A$ ...(i)

RHS $= \dfrac{1}{\tan A + \cot A} = \dfrac{1}{\left(\dfrac{\sin A}{\cos A} + \dfrac{\cos A}{\sin A}\right)}$

$$\left[\because \tan\theta = \dfrac{\sin\theta}{\cos\theta}, \cot\theta = \dfrac{\cos\theta}{\sin\theta}\right]$$

$$= \dfrac{1}{\left(\dfrac{\sin^2 A + \cos^2 A}{\cos A \sin A}\right)} = \dfrac{1}{\dfrac{1}{\cos A \sin A}}$$

$$[\because \sin^2\theta + \cos^2\theta = 1]$$

$= 1 \times \dfrac{\cos A \sin A}{1} = \cos A \sin A$ ...(ii)

From Eqs. (i) and (ii), LHS $=$ RHS **Hence proved.**

(x) First, put $\tan A$ and $\cot A$ in terms of $\sin A$ and $\cos A$. Then, simplify it.

REVIEW EXERCISE

Including Competency Based Questions

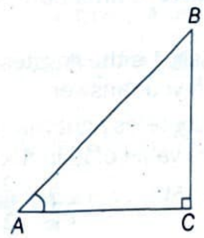
Part I

Multiple Choice Questions

1. If $\tan\theta = \frac{5}{12}$, then the value of $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta}$ is **CBSE 2023 (Standard)**

(a) $\frac{-17}{7}$ (b) $\frac{17}{7}$ (c) $\frac{17}{13}$ (d) $\frac{-7}{13}$

2. In ΔABC, $\angle C$ is right angle. If $\tan A = \frac{8}{7}$, then find the value of $\cot B$. **Competency Based Question**

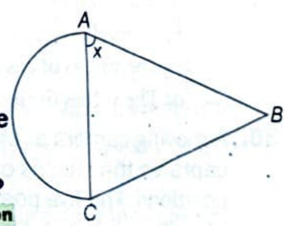


(a) $\frac{7}{8}$ (b) $\frac{8}{7}$ (c) $\frac{7}{\sqrt{113}}$ (d) $\frac{8}{\sqrt{113}}$

3. If $\cos\theta = \frac{1}{\sqrt{2}}$, then $\tan\theta$ is equal to **CBSE 2024 (Basic)**

(a) $\frac{1}{\sqrt{2}}$ (b) 0 (c) 1 (d) $\sqrt{2}+1$

4. ΔABC is an isosceles triangle, with $AB = BC$. A semi-circle of the area equal to that of the triangle is combined with it. What is the value of $\tan x$? **Competency Based Question**



(a) 1 (b) $\frac{1}{4}\pi$ (c) $\frac{1}{2}\pi$ (d) π

5. $\frac{2\tan 30°}{1+\tan^2 30°}$ is equal to **CBSE 2023 (Standard)**

(a) $\sin 60°$ (b) $\cos 60°$ (c) $\tan 60°$ (d) $\sin 30°$

6. If $\sin\theta - \cos\theta = 0$, then the value of θ is **CBSE 2021 Term I (Basic)**

(a) 30° (b) 45° (c) 90° (d) 0°

7. If θ is an acute angle and $\tan\theta + \cot\theta = 2$, then the value of $\sin^3\theta + \cos^3\theta$ is **CBSE 2021 Term I (Standard)**

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{\sqrt{2}}{2}$ (d) $\sqrt{2}$

8. If $\cos\theta = \frac{\sqrt{3}}{2}$ and $\sin\phi = \frac{1}{2}$, then $\tan(\theta+\phi)$ is **CBSE 2024 (Standard)**

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$

(c) 1 (d) not defined

9. The value of $(\tan^2 45° - \cos^2 60°)$ is **CBSE 2022 (Basic)**

(a) 1/2 (b) 1/4 (c) 3/2 (d) 3/4

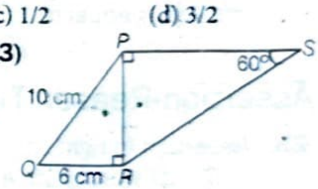
10. If $\cot\theta = \frac{1}{\sqrt{3}}$, the value of $\sec^2\theta + \operatorname{cosec}^2\theta$ is **CBSE 2022 (Standard)**

(a) 1 (b) $\frac{40}{9}$ (c) $\frac{38}{9}$ (d) $5\frac{1}{3}$

11. In a right-angled ΔPQR, $\angle Q = 90°$. If $\angle P = 45°$, then value of $\tan P - \cos^2 R$ is

(a) 0 (b) 1 (c) 1/2 (d) 3/2

Directions (Q. Nos. 12 to 13) In the figure given below, $PQRS$ is a quadrilateral. PR is perpendicular to QR and PS.



Based on the above information, answer the following questions. **Competency Based Question**

12. What is the value of $\tan Q$?

(a) $\frac{3}{5}$ (b) $\frac{1}{2}$ (c) 1 (d) $\frac{4}{3}$

13. What is the length of RS?

(a) 8 units (b) 10 units (c) $8\sqrt{2}$ units (d) $\frac{16}{3}\sqrt{3}$ units

14. $(\cos^4 A - \sin^4 A)$ on simplified form, gives **CBSE 2023 (Standard)**

(a) $2\sin^2 A - 1$ (b) $2\sin^2 A + 1$
(c) $2\cos^2 A + 1$ (d) $2\cos^2 A - 1$

15. If θ is an acute angle of a right angled triangle, then which of the following equation is not true?

(a) $\sin\theta\cot\theta = \cos\theta$ (b) $\cos\theta\tan\theta = \sin\theta$
(c) $\operatorname{cosec}^2\theta - \cot^2\theta = 1$ (d) $\tan^2\theta - \sec^2\theta = 1$

16. If $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ and $x\sin\theta = y\cos\theta$, then $x^2 + y^2$ is equal to **CBSE 2023 (Standard)**

(a) 0 (b) 1/2 (c) 1 (d) 3/2

17. $1 - \cos^2 A$ is equal to **CBSE Sample Paper (Basic)**

(a) $\sin^2 A$ (b) $\tan^2 A$ (c) $1 - \sin^2 A$ (d) $\sec^2 A$

18. $\dfrac{\cos^2\theta}{\sin^2\theta} - \dfrac{1}{\sin^2\theta}$, in simplified form, is

(a) $\tan^2\theta$ (b) $\sec^2\theta$ (c) 1 (d) -1

19. If $\sec\theta - \tan\theta = \dfrac{1}{3}$, then the value of $(\sec\theta + \tan\theta)$ is **CBSE 2023 (Standard)**

(a) $\dfrac{4}{3}$ (b) $\dfrac{2}{3}$ (c) $\dfrac{1}{3}$ (d) 3

20. $(1+\tan^2 A)(1+\sin A)(1-\sin A)$ is equal to

(a) $\dfrac{\cos^2 A}{\sec^2 A}$ (b) 1 (c) 0 (d) 2

21. If $\sin\theta = \dfrac{3}{4}$, then $\dfrac{(\sec^2\theta - 1)\cos^2\theta}{\sin\theta}$ equals **CBSE 2023 (Basic)**

(a) $\dfrac{3}{5}$ (b) $\dfrac{3}{4}$ (c) $\dfrac{4}{3}$ (d) $\dfrac{9}{16}$

22. If $\sec\theta + \tan\theta = p$, then $\tan\theta$ is **CBSE 2021 Term I (Standard)**

(a) $\dfrac{p^2+1}{2p}$ (b) $\dfrac{p^2-1}{2p}$ (c) $\dfrac{p^2-1}{p^2+1}$ (d) $\dfrac{p^2+1}{p^2-1}$

23. Given that $\sin\theta = \dfrac{a}{b}$, find $\cos\theta$

(a) $\dfrac{b}{\sqrt{b^2-a^2}}$ (b) $\dfrac{b}{a}$ (c) $\dfrac{\sqrt{b^2-a^2}}{b}$ (d) $\dfrac{a}{\sqrt{b^2-a^2}}$

24. If $a\cot\theta + b\,\text{cosec}\,\theta = p$ and $b\cot\theta + a\,\text{cosec}\,\theta = q$, then $p^2 - q^2$ is equal to **CBSE 2022 (Standard)**

(a) $a^2 - b^2$ (b) $b^2 - a^2$ (c) $a^2 + b^2$ (d) $b - a$

Assertion-Reason Type Questions

25. **Assertion** In right angled triangles, ΔABC and ΔDEF ($\angle C = \angle F = 90°$), $\angle B$ and $\angle E$ are acute angles, such that $\sin B = \sin E$, then $\angle B = \angle E$.

Reason $\Delta ABC \sim \Delta DEF$.

(a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
(c) Assertion is correct but Reason is incorrect.
(d) Assertion is incorrect but Reason is correct.

26. **Assertion** The equation $\sec^2\theta = \dfrac{4xy}{(x+y)^2}$ is only possible, when $x = y$.

Reason $\sec^2\theta \ge 1$ and therefore $(x-y)^2 \le 0$.

(a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
(c) Assertion is correct but Reason is incorrect.
(d) Assertion is incorrect but Reason is correct.

27. **Assertion** For $0 < \theta \le 90°$, $\text{cosec}\,\theta - \cot\theta$ and $\text{cosec}\,\theta + \cot\theta$ are reciprocal of each other.

Reason $\text{cosec}^2\theta - \cot^2\theta = 1$ **CBSE 2023 (Basic)**

(a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
(c) Assertion is correct but Reason is incorrect.
(d) Assertion is incorrect but Reason is correct.

28. **Assertion** $\cos^2 A - \sin^2 A = 1$ is a trigonometric identity.

Reason An equation involving trigonometric ratios of an angle is called trigonometric identity, if it is true for all values of the angles involved.

(a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
(c) Assertion is correct but Reason is incorrect.
(d) Assertion is incorrect but Reason is correct.

Case Study Based Questions

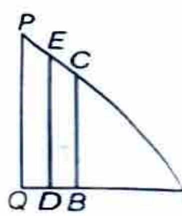
29. In the given figure, ΔABC, ΔADE and ΔAQP are three right angled triangles.

Based on the above information, answer the following questions.

(i) The value of $\sin A$ is the greatest for ΔPQA. Do you agree? Justify your answer.

(ii) ΔABC is an isosceles right triangle, right-angled at B. What is the value of $2\sin A \times \cos A$?

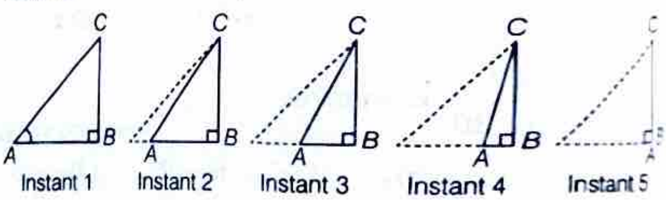
(a) $\dfrac{1}{2}$ (b) 1 (c) $\dfrac{3}{2}$ (d) 2



(iii) Which one of the following statements is true about trigonometric ratios in a right triangle?

(a) The values of cot and tan vary from 0 to 1.
(b) The values of sin and cos vary from 0 to 1.
(c) The values of cos and sec vary from 0 to 1.
(d) The values of sin and cosec vary from 0 to 1.

30. A moving camera at the top of a 4 m high building captures the images of a walking man at five different positions. The five positions are shown in the figure below.



Based on the above information, answer the following questions.

(i) Describe the change in the value of $\sin A$.

(ii) Which of the following is not true for position 5?

(a) $AB = 0$ (b) $BC = AC$ (c) $BC = 1$ (d) $CA = AB$

(iii) In the isosceles ΔABC, BD is the altitude and $\angle ABC = 120°$. What is the value $\cos C$?

Part II

Very Short Answer Type Questions

1. If $\tan A = 3/4$, then find the value of $\frac{1}{\sin A} + \frac{1}{\cos A}$.

CBSE Sample Paper 2020 (Standard)

2. If $7\tan\theta = 4$, then find the value of $\frac{7\sin\theta - 3\cos\theta}{7\sin\theta + 3\cos\theta}$.

3. If $\sqrt{3}\tan\theta = 3\sin\theta$, then find the value of $\sin^2\theta - \cos^2\theta$. **Competency Based Question**

4. Evaluate $\sin^2 60° + 2\tan 45° - \cos^2 30°$.

CBSE Sample Paper 2023 (Basic)

5. Given that, $\sin\alpha = \frac{1}{2}$ and $\cos\beta = \frac{1}{2}$, what is the value of $(\alpha + \beta)$? **NCERT Exemplar**

6. Prove that

$$\sec A(1 - \sin A)(\sec A + \tan A) = 1.$$

7. If $\cos A + \cos^2 A = 1$, find the value of $\sin^2 A + \sin^4 A$

NCERT Exemplar

8. If $x = 2\sin^2\theta$ and $y = 2\cos^2\theta + 1$, then find $x + y$.

9. If $4x = \operatorname{cosec}\theta$ and $\frac{4}{x} = \cot\theta$, find the value of $4\left[x^2 - \frac{1}{x^2}\right]$. **Competency Based Question**

Short Answer Type Questions

10. If $\tan A = \sqrt{2} - 1$, prove that

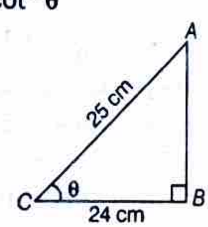
$$\frac{\tan A}{1 + \tan^2 A} = \frac{\sqrt{2}}{4}.$$

11. In right angled ΔACB, $\angle C = 90°$, $AB = 29$ units and $BC = 21$ units. If $\angle ABC = \theta$, find $\cos^2\theta - \sin^2\theta$ and $\sin^2\theta + \cos^2\theta$.

12. With the help of following figure, find the value of

(i) $\sec^2\theta + \tan^2\theta$

(ii) $\operatorname{cosec}^2\theta - \cot^2\theta$



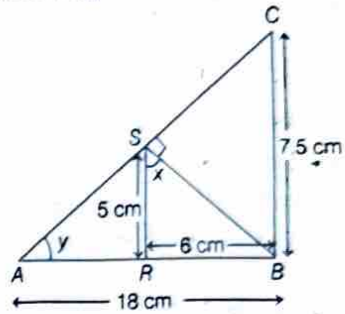
13. If $\cos A = \frac{5}{13}$, then verify that

$$\frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 - \cot A} = \cos A + \sin A$$

CBSE 2024 (Basic)

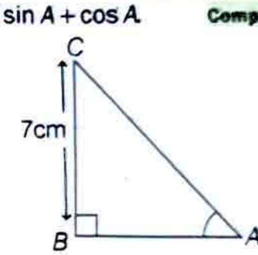
14. In the given figure, ΔABC is right angled triangle, right angle at B. ΔBSC is right angled triangle, right angle at S and ΔBRS is right angled triangle, right angle at R, $AB = 18$ cm, $BC = 7.5$ cm, $RS = 5$ cm, $RB = 6$ cm, $\angle BSR = x$ and $\angle SAB = y$. **Competency Based Question**

Find the following

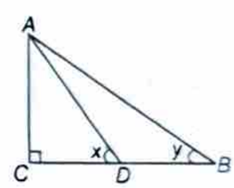


(i) $\tan x$ (ii) $\sin y$ (iii) $\cos y$

15. In the given figure, ΔABC is a right angled triangle, right angle at B. If $BC = 7$ cm and $AC - AB = 1$ cm, find the value of $\sin A + \cos A$. **Competency Based Question**



16. In the given figure, if D is the mid-point of BC, find the value of $\frac{\tan x}{\tan y}$.



17. If $\tan\theta + \frac{1}{\tan\theta} = 2$, find the value of $\cot^2\theta + \frac{1}{\cot^2\theta}$.

18. Prove that $\sqrt{\frac{\sec A + \tan A}{\sec A - \tan A}} \cdot \sqrt{\frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1}} = 1$.

19. If $\sqrt{3}\cot^2\theta - 4\cot\theta + \sqrt{3} = 0$, then find the value of $\tan^2\theta + \cot^2\theta$.

20. If $\tan\theta = 1$ and $\sin\phi = \frac{1}{\sqrt{2}}$, then find the value of $\cos(\theta + \phi)$, where θ and ϕ are both acute angles.

Competency Based Question

21. Show that $2(\cos^4 60° + \sin^4 30°) - (\tan^2 60° + \cot^2 45°) + 3\sec^2 30° = 1$.

22. Two famous bowlers of Indian Cricket team Navdeep Saini and Jaspreet Bumrah throw a ball at an angle of A and B respectively in such a way that $\sin(A-B)=\frac{1}{2}$ and $\cos(A+B)=0$, $0°<A+B\leq 90°$, $A>B$.

(i) Find the angles of both bowlers at which they throw a ball.

(ii) Find the values of the following trigonometric ratios $\tan A$, $\operatorname{cosec}(A-B)$, $\sec B$. **Competency Based Question**

23. Suppose $\sin\theta_1+\sin\theta_2+\sin\theta_3=3$, $0°<\theta_1,\theta_2,\theta_3\leq 90°$,

(i) Find the value of $\cos\theta_1+\cos\theta_2+\cos\theta_3$.

(ii) Calculate the value of $\tan(225°-\theta_1-\theta_2)$.

(iii) Evaluate $\sin(120°-\theta_3)$. **Competency Based Question**

24. Calculate $A+B$ when $\tan A=\frac{2}{3}$ and $\tan B=\frac{3}{2}$, if it is given that $\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$ where, A and B are acute angles. **Competency Based Question**

25. If $\operatorname{cosec}(A-B)=2$, $\cot(A+B)=\frac{1}{\sqrt{3}}$, $0°<(A+B)\leq 90°$, $A>B$, then find A and B.

26. If $\tan\theta=\frac{20}{21}$, then prove that $\frac{1-\sin\theta+\cos\theta}{1+\sin\theta+\cos\theta}=\frac{3}{7}$.

27. Prove that $\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}=2\sec^2\theta$. **CBSE 2020 (Basic)**

28. Prove that $\frac{1-\tan^2\theta}{1+\tan^2\theta}=\cos^2\theta-\sin^2\theta$.

29. Prove that $\frac{\sin^4\theta+\cos^4\theta}{1-2\sin^2\theta\cdot\cos^2\theta}=1$.

30. Simplify $\frac{\sin^3\theta+\cos^3\theta}{\sin\theta+\cos\theta}+\sin\theta\cos\theta$.

31. Prove that $\frac{\cot A-\cos A}{\cot A+\cos A}=\frac{\cos^2 A}{(1+\sin A)^2}$. **CBSE 2023 (Basic)**

32. Prove that $\frac{\cot\theta+\operatorname{cosec}\theta-1}{\cot\theta-\operatorname{cosec}\theta+1}=\frac{1+\cos\theta}{\sin\theta}$. **CBSE 2020, 2016 (Standard)**

33. If $\sin^6 A+\cos^6 A+3\sin^2 A\cdot\cos^2 A+4=k$, then find the value of k.

34. If $\sec\theta=x+\frac{1}{4x}$, then find the value of $\sec\theta+\tan\theta$.

35. Prove that $\sqrt{\sec^2\theta+\operatorname{cosec}^2\theta}=\tan\theta+\cot\theta$. **NCERT Exemplar**

36. Eliminate θ from the following equations.

(i) $x=a\sec\theta$, $y=b\tan\theta$

(ii) $x=k+a\cos\theta$, $y=h+b\sin\theta$

37. If $m\sin\theta+n\cos\theta=p$ and $m\cos\theta-n\sin\theta=q$, then prove that $m^2+n^2=p^2+q^2$.

38. Prove that $\frac{\sin\theta}{\cot\theta+\operatorname{cosec}\theta}=2+\frac{\sin\theta}{\cot\theta-\operatorname{cosec}\theta}$.

39. If $7\sin^2 A+3\cos^2 A=4$, prove that $\tan A=\frac{1}{\sqrt{3}}$.

40. If $\cos\theta+\sin\theta=\sqrt{2}\cos\theta$, prove that $\cos\theta-\sin\theta=\sqrt{2}\sin\theta$. **CBSE 2019, 14, 1[illegible]**

41. Ram was trying hard to prove $a\cos\theta-b\sin\theta=\pm\sqrt{a^2+b^2-c^2}$, when $a\sin\theta+b\cos\theta=c$. Her classmate Swati gave her a hint of squaring both sides of $a\sin\theta+b\cos\theta=c$ and proceed further. With her hint, Ram was able to solve the problem and he thanks Swati for this hint. So, write the solution of the above problem. **Competency Based Question**

42. Prove that $\frac{\sin A-\cos A+1}{\sin A+\cos A-1}=\frac{1}{\sec A-\tan A}$. **CBSE Sample Paper 2020 (Basic)**

43. If $\cos\theta-\sin\theta=x$ and $\sin\theta+\cos\theta=y$, then show that $x^2+y^2=2$.

44. If $\cot\theta=3x-\frac{1}{12x}$, show that $\cot\theta+\operatorname{cosec}\theta=6x$ or $-\frac{1}{6x}$.

45. If $\frac{\cos\alpha}{\cos\beta}=m$ and $\frac{\cos\alpha}{\sin\beta}=n$, show that $(m^2+n^2)\cos^2\beta=n^2$. **Competency Based Question**

46. Prove that $\frac{1}{(\sec x-\tan x)}-\frac{1}{\cos x}=\frac{1}{\cos x}-\frac{1}{\sec x+\tan x}$

47. Prove that $(1+\tan A\tan B)^2+(\tan A-\tan B)^2=\sec^2 A\sec^2 B$.

48. If $a\cos\theta-b\sin\theta=c$, prove that $a\sin\theta+b\cos\theta=\pm\sqrt{a^2+b^2-c^2}$.

49. If $\sin\theta+2\cos\theta=1$, prove that $2\sin\theta-\cos\theta=2$. **NCERT Exemplar**

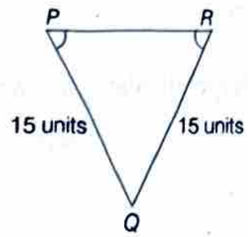
50. If $\sin A=\frac{1}{\sqrt{5}}$ and $\sin B=\frac{1}{\sqrt{10}}$, find the values of $\cos A$ and $\cos B$. Hence, using the formula $\cos(A+B)=\cos A\cos B-\sin A\sin B$, show that $A+B=45°$. **Competency Based Question**

51. Prove that $\sec^2 A-\left(\frac{\sin^2 A-2\sin^4 A}{2\cos^4 A-\cos^2 A}\right)=1$

52. If $1+\sin^2\theta=3\sin\theta\cos\theta$, prove that $\tan\theta=1$ or $\frac{1}{2}$. **NCERT Exemplar**

Long Answer Type Questions

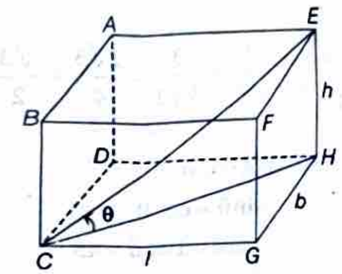
53. In the figure below, $5\sin P = 4$.



(Note The figure is not to scale)

What is the length of PR? Draw a diagram and show your steps. **Competency Based Question**

54. Shown below is a cuboid. Its length is l units, breadth b units and height h units.



(i) Express $\cos\theta$ in terms of l, b and h.

(ii) If the figure was a cube, what would be the value of $\cos\theta$?

Show your work. **Competency Based Question**

55. Prove that

$$\cot^2 A\left[\frac{\sec A-1}{1+\sin A}\right]+\sec^2 A\left[\frac{\sin A-1}{1+\sec A}\right]=0.$$

56. Prove that $\dfrac{\sin A-2\sin^3 A}{2\cos^3 A-\cos A}=\tan A$.

57. $\dfrac{1}{\operatorname{cosec}\theta-\cot\theta}-\dfrac{\cot\theta}{\cos\theta}=\cot\theta$ **Competency Based Question**

58. If $\dfrac{1}{\sin\theta-\cos\theta}=\dfrac{\operatorname{cosec}\theta}{\sqrt{2}}$, prove that

$$\left(\frac{1}{\sin\theta+\cos\theta}\right)^2=\frac{\sec^2\theta}{2}.$$

Competency Based Question

59. Prove that

$$\frac{\operatorname{cosec}^2 x-\sin^2 x\cot^2 x-\cot^2 x}{\sin^2 x}=1.$$

60. Prove that

$$\left[\frac{1}{\sec^2\theta-\cos^2\theta}+\frac{1}{\operatorname{cosec}^2\theta-\sin^2\theta}\right]\sin^2\theta\cos^2\theta$$
$$=\frac{1-\sin^2\theta\cos^2\theta}{2+\sin^2\theta\cos^2\theta}.$$

61. Prove that

$$\frac{(1+\cot\theta+\tan\theta)(\sin\theta-\cos\theta)}{\sec^3\theta-\operatorname{cosec}^3\theta}=\sin^2\theta\cos^2\theta.$$

62. If $\tan A = a\tan B$ and $\sin A = b\sin B$, prove that $\cos^2 A=\dfrac{b^2-1}{a^2-1}$.

63. If $\operatorname{cosec}\theta-\sin\theta=m$ and $\sec\theta-\cos\theta=n$, then prove that $(m^2n)^{2/3}+(mn^2)^{2/3}=1$.

64. If $\tan\theta+\sec\theta=l$, then prove that $\sec\theta=\dfrac{l^2+1}{2l}$. **NCERT Exemplar**

65. If $\sec A=\dfrac{17}{8}$, show that $\dfrac{3-4\sin^2 A}{4\cos^2 A-3}=\dfrac{3-\tan^2 A}{1-3\tan^2 A}$.

66. Show that $(\sin^8 A-\cos^8 A)$
$$=(2\sin^2 A-1)(1-2\sin^2 A\cos^2 A).$$

67. If $\operatorname{cosec}\theta+\cot\theta=p$, then prove that $\cos\theta=\dfrac{p^2-1}{p^2+1}$. **NCERT Exemplar**

68. Prove that $(\operatorname{cosec} A-\sin A)(\sec A-\cos A)$
$$=\frac{1}{\tan A+\cot A}.$$

69. If $\operatorname{cosec} A-\cot A=q$, show that

$$\frac{q^2-1}{q^2+1}+\cos A=0.$$

70. Prove that $2\sec^2\theta-\sec^4\theta-2\operatorname{cosec}^2\theta$
$$+\operatorname{cosec}^4\theta=\cot^4\theta-\tan^4\theta.$$

71. If $x=r\sin A\cos B$, $y=r\sin A\sin B$ and $z=r\cos A$, show that $x^2+y^2+z^2=r^2$.

72. Prove that

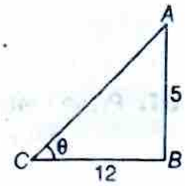
$$\left(\tan\theta+\frac{1}{\cos\theta}\right)^2+\left(\tan\theta-\frac{1}{\cos\theta}\right)^2=2\left(\frac{1+\sin^2\theta}{1-\sin^2\theta}\right).$$

73. Prove that $l^2m^2(l^2+m^2+3)=1$, if $\operatorname{cosec}\theta-\sin\theta=l$ and $\sec\theta-\cos\theta=m$.

HINTS & SOLUTIONS

Part I

1. (a) Given, $\tan\theta = \frac{5}{12}$



We know that $\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$

By Pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

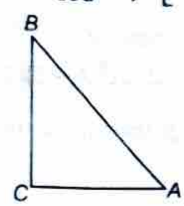
$$\Rightarrow \quad AC^2 = 5^2 + 12^2 = 25 + 144 = 169$$

$$\Rightarrow \quad AC = 13 \qquad \text{[neglecting '– ve' sign]}$$

$$\therefore \quad \sin\theta = \frac{AB}{AC} = \frac{5}{13} \text{ and } \cos\theta = \frac{BC}{AC} = \frac{12}{13}$$

Thus, $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = \frac{\frac{5}{13} + \frac{12}{13}}{\frac{5}{13} - \frac{12}{13}} = \frac{5+12}{5-12} = \frac{-17}{7}$

2. (b) In ΔACB, $\tan A = \frac{BC}{AC} = \frac{8}{7}$ $\left[\because \tan\theta = \frac{\text{Perpendicular}}{\text{Base}}\right]$



$$\cot B = \frac{BC}{AC} \qquad \left[\because \cot\theta = \frac{\text{Base}}{\text{Perpendicular}}\right]$$

$$\Rightarrow \quad \cot B = \frac{8}{7}$$

3. (c) Given, $\cos\theta = \frac{1}{\sqrt{2}}$

Since, $\cos\theta = \frac{1}{\sqrt{2}} = \cos 45° \Rightarrow \theta = 45°$

Now, $\tan\theta = \tan 45° = 1$

Hence, the value of $\tan\theta$ is 1.

4. (c) In the isosceles ΔABC, BP is perpendicular to AC.

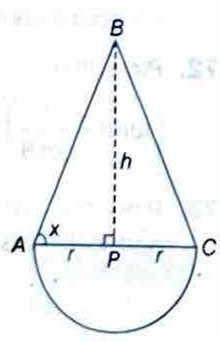
Let $BP = h$

Since, area of semi-circle

= Area of triangle

$$\frac{1}{2}\pi r^2 = \frac{1}{2} \cdot 2r \cdot h$$

$$\frac{1}{2}\pi r = h$$



[r = Radius of semi-circle]

$$h = \frac{\pi r}{2}$$

Now, $\tan x = \frac{\text{Perpendicular}}{\text{Base}} = \frac{h}{r} = \frac{\pi r}{2} \cdot \frac{1}{r}$

$$\tan x = \frac{\pi}{2}$$

5. (a) We know, $\tan 30° = \frac{1}{\sqrt{3}}$

So, $\frac{2\tan 30°}{1 + \tan^2 30°} = \frac{2 \times \frac{1}{\sqrt{3}}}{1 + \left(\frac{1}{\sqrt{3}}\right)^2}$

$$\Rightarrow \quad \frac{\frac{2}{\sqrt{3}}}{1 + \frac{1}{3}} = \frac{2}{\sqrt{3}} \times \frac{3}{3+1} = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2} = \sin 60°$$

6. (b) Given, $\sin\theta - \cos\theta = 0$

$$\Rightarrow \quad \sin\theta = \cos\theta$$

$$\Rightarrow \quad \tan\theta = 1 \Rightarrow \theta = 45°$$

7. (c) If $\theta = 45°$, then $\tan\theta + \cot\theta = 2$

Now, $\sin^3 45° + \cos^3 45°$ $[\because \theta = 45°$

$$= \left(\frac{1}{\sqrt{2}}\right)^3 + \left(\frac{1}{\sqrt{2}}\right)^3$$

$$= \frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

8. (a) Given, $\cos\theta = \frac{\sqrt{3}}{2}$ and $\sin\phi = \frac{1}{2}$

Since, $\cos\theta = \frac{\sqrt{3}}{2} = \cos 30° \Rightarrow \theta = 30°$

and $\quad \sin\phi = \frac{1}{2} = \sin 30° \Rightarrow \phi = 30°$

$\because \tan(\theta + \phi) = \tan(30° + 30°) = \tan(60°) = \sqrt{3}$

$\therefore \tan(\theta + \phi) = \sqrt{3}$

9. (d) Given, $\tan^2 45° - \cos^2 60°$

$$= 1 - \left(\frac{1}{2}\right)^2 \qquad [\because \tan 45° = 1, \cos 60° = 1/2]$$

$$= 1 - \frac{1}{4} = \frac{3}{4}$$

10. (d) We have, $\quad \cot\theta = \frac{1}{\sqrt{3}}$

$$\cot\theta = \cot 60° \qquad \left[\because \cot 60° = \frac{1}{\sqrt{3}}\right]$$

$$\theta = 60°$$

$$\text{Now, } \sec^2\theta + \text{cosec}^2\theta = \sec^2 60^\circ + \text{cosec}^2 60^\circ$$

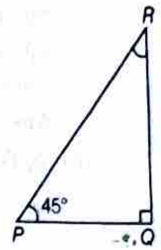
$$= (2)^2 + \left(\frac{2}{\sqrt{3}}\right)^2 \quad \left[\begin{array}{l} \because \sec 60^\circ = 2, \\ \text{cosec } 60^\circ = \dfrac{2}{\sqrt{3}} \end{array}\right]$$

$$= 4 + \frac{4}{3} = \frac{16}{3} = 5\frac{1}{3}$$

11. (c) Given, in ΔPQR,

$\angle Q = 90^\circ$

$\angle P = 45$



So, $\angle R = 180^\circ - (P + Q)$,

$\angle R = 180^\circ - (90^\circ + 45^\circ)$

and $\angle R = 45^\circ$

So, $\tan P - \cos^2 R = \tan 45^\circ - \cos^2 45^\circ$ $\left[\begin{array}{l} \because \tan 45^\circ = 1, \\ \cos 45^\circ = 1/\sqrt{2} \end{array}\right]$

$$= 1 - \left(\frac{1}{\sqrt{2}}\right)^2$$

$\tan P - \cos^2 R = 1/2$

12. (d) Using Pythagoras theorem in ΔPQR, we get

$PR = 8$ cm

Now, in ΔPRQ, $\tan Q = \dfrac{PR}{QR} = \dfrac{8}{6} = \dfrac{4}{3}$

13. (d) **Hint** Similar as Question 12. Ans. $\dfrac{16\sqrt{3}}{3}$

14. (d) We have, $\cos^4 A - \sin^4 A$

$$= (\cos^2 A + \sin^2 A)(\cos^2 A - \sin^2 A)$$

$$= 1(\cos^2 A - \sin^2 A) \quad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$= \cos^2 A - (1 - \cos^2 A) = 2\cos^2 A - 1$$

15. (d) From option (a),

$$\sin\theta\cot\theta = \sin\theta \times \frac{\cos\theta}{\sin\theta} = \cos\theta$$

$\therefore$ Option (a) is true

From option (b), $\cos\theta\tan\theta = \cos\theta \times \dfrac{\sin\theta}{\cos\theta} = \sin\theta$

$\therefore$ Option (b) is true

From option (c), $\text{cosec}^2\theta - \cot^2\theta = \dfrac{1}{\sin^2\theta} - \dfrac{\cos^2\theta}{\sin^2\theta}$

$$= \frac{1 - \cos^2\theta}{\sin^2\theta} = \frac{\sin^2\theta}{\sin^2\theta} = 1$$

$$[\because \sin^2 A + \cos^2 A = 1]$$

$\therefore$ Option (c) is true.

16. (c) We have, $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$

$\Rightarrow (x\sin\theta)\sin^2\theta + (y\cos\theta)\cos^2\theta = \sin\theta\cos\theta$

$\Rightarrow x\sin\theta(\sin^2\theta) + (x\sin\theta)\cos^2\theta = \sin\theta\cos\theta$

$$[\because y\cos\theta = x\sin\theta]$$

$\Rightarrow x\sin\theta(\sin^2\theta + \cos^2\theta) = \sin\theta\cos\theta$

$\Rightarrow x\sin\theta = \sin\theta\cos\theta \quad [\because \sin^2\theta + \cos^2\theta = 1]$

$\Rightarrow x = \cos\theta$

Now, $x\sin\theta = y\cos\theta$

$\Rightarrow \cos\theta\sin\theta = y\cos\theta$

$\Rightarrow y = \sin\theta$

Hence, $x^2 + y^2 = \cos^2\theta + \sin^2\theta = 1$

17. (a) We know trigonometric identity.

$$\sin^2 A + \cos^2 A = 1 \Rightarrow 1 - \cos^2 A = \sin^2 A$$

18. (d) Given, $\dfrac{\cos^2\theta}{\sin^2\theta} - \dfrac{1}{\sin^2\theta} = \dfrac{\cos^2\theta - 1}{\sin^2\theta} = \dfrac{-\sin^2\theta}{\sin^2\theta} = -1$

$$[\because \sin^2 A + \cos^2 A = 1 \Rightarrow \cos^2 A - 1 = -\sin^2 A]$$

19. (d) We know that

$$\sec^2\theta - \tan^2\theta = 1$$

$\Rightarrow (\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$

$$\left[\because \text{given}, \sec\theta - \tan\theta = \frac{1}{3}\right]$$

$\Rightarrow \dfrac{1}{3}(\sec\theta + \tan\theta) = 1$

$\Rightarrow \sec\theta + \tan\theta = 3$

20. (b) We have, $(1 + \tan^2 A)(1 + \sin A)(1 - \sin A)$

$$= \left(1 + \frac{\sin^2 A}{\cos^2 A}\right)(1 - \sin^2 A) \left[\begin{array}{l} \because \tan A = \dfrac{\sin A}{\cos A} \\ (a-b)(a+b) = a^2 - b^2 \end{array}\right]$$

$$= \frac{\cos^2 A + \sin^2 A}{\cos^2 A}(1 - \sin^2 A)$$

$$= \frac{1}{\cos^2 A}\cdot\cos^2 A \quad \left[\begin{array}{l} \because \cos^2 A + \sin^2 A = 1 \\ \cos^2 A = 1 - \sin^2 A \end{array}\right]$$

$$= 1$$

21. (b) Given, $\sin\theta = \dfrac{3}{4}$

We know that $\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \cos^2\theta = 1 - \sin^2\theta$

$\Rightarrow \cos^2\theta = 1 - \left(\dfrac{3}{4}\right)^2 = 1 - \dfrac{9}{16} = \dfrac{7}{16}$

We know, $\sec^2\theta = \dfrac{1}{\cos^2\theta} = \dfrac{16}{7}$

Thus, $\dfrac{(\sec^2\theta - 1)\cos^2\theta}{\sin\theta} = \dfrac{\left(\dfrac{16}{7} - 1\right)\dfrac{7}{16}}{\dfrac{3}{4}} = \dfrac{9}{7} \times \dfrac{7}{16} \times \dfrac{4}{3} = \dfrac{3}{4}$

22. (b) $\sec\theta + \tan\theta = p$ ... (i)

$\sec\theta - \tan\theta = \frac{1}{p}$ $[\because \sec^2 A - \tan^2 A = 1]$ ... (ii)

On subtracting Eq. (ii) from Eq. (i), we get

$$2\tan\theta = p - \frac{1}{p}$$

$$\Rightarrow \tan\theta = \frac{1}{2}\left[\frac{p^2-1}{p}\right]$$

23. (c) Given, $\sin\theta = \frac{a}{b}$

We know that $\sin^2\theta + \cos^2\theta = 1 \Rightarrow \cos^2\theta = 1 - \frac{a^2}{b^2}$

$$\Rightarrow \cos\theta = \frac{\sqrt{b^2 - a^2}}{b}$$

24. (a) We have, $a\cot\theta + b\,\text{cosec}\,\theta = p$

$b\cot\theta + a\,\text{cosec}\,\theta = q$

After squaring both sides, we get

$a^2\cot^2\theta + b^2\text{cosec}^2\theta + 2ab\cot\theta\,\text{cosec}\,\theta = p^2$ ...(i)

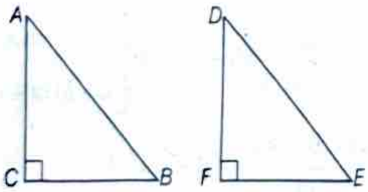
$b^2\cot^2\theta + a^2\text{cosec}^2\theta + 2ab\cot\theta\,\text{cosec}\,\theta = q^2$ ...(ii)

Now, $p^2 - q^2 = (a^2 - b^2)\cot^2\theta - (a^2 - b^2)\,\text{cosec}^2\theta$ [using Eqs. (i) and (ii)]

$= (a^2 - b^2)(\cot^2\theta - \text{cosec}^2\theta)$ $[\because \cot^2\theta - \text{cosec}^2\theta = 1]$

$= a^2 - b^2$

25. (a)



In ΔABC and ΔDEF, $\sin B = \sin E \Rightarrow \angle B = \angle E$

and $\angle C = \angle F$ [each 90°]

$\therefore$ By AA criteria, $\Delta ABC \sim \Delta DEF$

26. (a) Since, $\sec^2\theta = \frac{4xy}{(x+y)^2} \geq 1$

$$\Rightarrow 4xy \geq (x+y)^2$$

$$\Rightarrow x^2 + y^2 + 2xy - 4xy \leq 0$$

$$\Rightarrow (x-y)^2 \leq 0 \Rightarrow x = y$$

27. (a) Given, $\text{cosec}\,\theta - \cot\theta$

Reciprocal of this expression $= \frac{1}{\text{cosec}\,\theta - \cot\theta}$

On multiplying and dividing by $\text{cosec}\,\theta + \cot\theta$,

$$\frac{1}{\text{cosec}\,\theta - \cot\theta} \times \frac{\text{cosec}\,\theta + \cot\theta}{\text{cosec}\,\theta + \cot\theta} = \frac{\text{cosec}\,\theta + \cot\theta}{\text{cosec}^2\theta - \cot^2\theta}$$

$= \text{cosec}\,\theta + \cot\theta$ $[\because \text{cosec}^2\theta - \cot^2\theta = 1]$

28. (d) We have, $\cos^2 A - \sin^2 A = 1$

Put $A = 45°$, we get

$$\cos^2 45° - \sin^2 45° = \left(\frac{1}{\sqrt{2}}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2} - \frac{1}{2} = 0$$

$\therefore$ This is not a trigonometry identity.

29. (i) The value of $\sin A$ does not depend on side lengths but it is a ratio of the side lengths and depends upon measure of $\angle A$. So, all triangles have the same value of $\sin A$.

Ans. No

(ii) (b) For an isosceles right triangle, $\tan A = 1$

$$\Rightarrow \frac{\sin A}{\cos A} = 1$$

$$\Rightarrow (\sin A - \cos A)^2 = 0$$

$$\Rightarrow \sin^2 A + \cos^2 A - 2\sin A\cos A = 0$$

$$\Rightarrow 1 - 2\sin A\cos A = 0$$

$[\because \sin^2 A + \cos^2 A = 1]$

$$\Rightarrow 2\sin A\cos A = 1$$

(iii) (b) The value of sin and cos vary from 0 to 1.

30. (i) The value of $\sin A$ increases as the measure of $\angle A$ increases. It reaches to 1, when the person comes directly under the camera.

(ii) (c) For position 5, $BC \neq 1$

(iii) Find using suitable trigonometric ratios.

Ans. $\cos C = \cos 30° = \frac{\sqrt{3}}{2}$

Part II

1. Given, $\tan A = \frac{3}{4}$

$\tan A = \frac{P}{B}$

$\Rightarrow P = 3k$ and $B = 4k$

As, $H = \sqrt{(3k)^2 + (4k)^2} = 5k$

$\sin A = \frac{P}{H} = \frac{3k}{5k} = \frac{3}{5}$

$\cos A = \frac{B}{H} = \frac{4k}{5k} = \frac{4}{5}$

$$\frac{1}{\sin A} + \frac{1}{\cos A} = \frac{5}{3} + \frac{5}{4} = \frac{20+15}{12} = \frac{35}{12}$$

2. Hint $\tan\theta = \frac{4}{7}$, now divide numerator and denominator by $\cos\theta$, we get $\frac{7\tan\theta - 3}{7\tan\theta + 3}$.

Then, put value of $\tan\theta$ and simplify.

Ans. $\frac{1}{7}$

3. Hint $\sqrt{3}\tan\theta = 3\sin\theta \Rightarrow \dfrac{\sqrt{3}\sin\theta}{\cos\theta} - 3\sin\theta = 0$

$$\Rightarrow \quad \sqrt{3}\sin\theta\left(\frac{1}{\cos\theta} - \sqrt{3}\right) = 0$$

$$\Rightarrow \quad \sin\theta = 0 \text{ or } \cos\theta = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \sin\theta = 0 \text{ or } \sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{\frac{2}{3}}$$

Now, put value of $\sin\theta$ after rewriting the given expression as $2\sin^2\theta - 1$ **Ans.** -1 or $\frac{1}{3}$.

4. $\sin^2 60° + 2\tan 45° - \cos^2 30°$

$$= \left(\frac{\sqrt{3}}{2}\right)^2 + 2(1)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= \frac{3}{4} + 2 - \frac{3}{4} = 2$$

5. Hint Given, $\sin\alpha = \frac{1}{2}$ and $\cos\beta = \frac{1}{2}$

$$\Rightarrow \quad \sin\alpha = \sin 30° \quad \left[\because \sin 30° = \frac{1}{2}\right]$$

and $\quad \cos\beta = \cos 60° \quad \left[\because \cos 60° = \frac{1}{2}\right]$

$$\Rightarrow \quad \alpha = 30° \text{ and } \beta = 60°$$

$$\therefore \quad \alpha + \beta = 30° + 60° = 90° \textbf{ Ans. } 90°$$

6. Hint Express $\sec A$ and $\tan A$ in terms of $\sin A$ and $\cos A$, then use $\cos^2\theta = 1 - \sin^2\theta$

7. Hint $\cos A = 1 - \cos^2 A \Rightarrow \cos A = \sin^2 A$ **Ans.** 1

8. $x + y = 2\sin^2\theta + 2\cos^2\theta + 1 = 2[\sin^2\theta + \cos^2\theta] + 1$

$= 2 + 1 \quad [\because \sin^2 A + \cos^2 A = 1]$

$= 3$

9. Hint $4x = \operatorname{cosec}\theta \Rightarrow x = \dfrac{\operatorname{cosec}\theta}{4}$

and $\quad \dfrac{4}{x} = \cot\theta \Rightarrow \dfrac{1}{x} = \dfrac{\cot\theta}{4}$

Now, $4\left[x^2 - \dfrac{1}{x^2}\right] = 4\left[\left(\dfrac{\operatorname{cosec}\theta}{4}\right)^2 - \left(\dfrac{\cot\theta}{4}\right)^2\right]$

Ans. 1/4

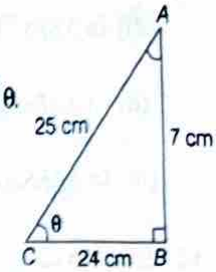
10. Hint Put the value of $\tan A$ in LHS and simplify.

11. Hint First do same as Question 1 of NCERT Folder Exercise 8.1, then put value of $\cos\theta$ and $\sin\theta$ in given expression. **Ans.** $\dfrac{41}{841}, 1$

12. Hint Given, hypotenuse = 25 cm and base B = 24 cm

Apply Pythagoras theorem, we get

$AB = 7$ cm



(i) $\sec\theta = \dfrac{25}{24}, \tan\theta = \dfrac{7}{24}$

Put these value in $\sec^2\theta + \tan^2\theta$.

Ans. (i) $\dfrac{337}{288}$

(ii) $\operatorname{cosec}\theta = \dfrac{25}{7}, \cot\theta = \dfrac{24}{7}$

Put these values in $\operatorname{cosec}^2\theta - \cot^2\theta$

Ans. (ii) 1

13. Given, $\cos A = \dfrac{5}{13} = \dfrac{\text{Base}(B)}{\text{Hypotenuse}(H)}$

By Pythagoras theorem, we get

$$H^2 = P^2 + B^2$$

$$\Rightarrow \quad (13)^2 = P^2 + (5)^2$$

$$\Rightarrow \quad 169 = P^2 + 25$$

$$\Rightarrow \quad 169 - 25 = P^2$$

$$\Rightarrow \quad 144 = P^2$$

$$\Rightarrow \quad P = 12$$

Now, we have to verify that

$$\frac{\cos A}{1-\tan A} + \frac{\sin A}{1-\cot A} = \cos A + \sin A$$

$$\text{LHS} = \frac{\cos A}{1-\tan A} + \frac{\sin A}{1-\cot A}$$

$$= \frac{\frac{5}{13}}{1-\frac{12}{5}} + \frac{\frac{12}{13}}{1-\frac{5}{12}}$$

$$= \frac{\frac{5}{13}}{\frac{5-12}{5}} + \frac{\frac{13}{13}}{\frac{12-5}{12}} = \frac{5}{13} \times \left(-\frac{5}{7}\right) + \frac{12}{13} \times \frac{12}{7}$$

$$= -\frac{25}{91} + \frac{144}{91} = \frac{119}{91} = \frac{17}{13}$$

$$\text{RHS} = \cos A + \sin A$$

$$= \frac{5}{13} + \frac{12}{13} = \frac{17}{13}$$

LHS = RHS **Hence proved.**

14. Hint Given, $\angle CBA = 90°, \angle BRS = 90°$, $\angle BSC = 90°, BC = 7.5$ cm, $RS = 5$ cm, $BR = 6$ cm and $AB = 18$ cm

Then, $\quad AR = AB - RB = 18 - 6 = 12$ cm

In ΔARS, using Pythagoras theorem, we get

$$AS^2 = AR^2 + RS^2 = (12)^2 + (5)^2$$

$$\Rightarrow \quad AS^2 = 144 + 25 = 169$$

$$\Rightarrow \quad AS = \sqrt{169}$$

$\therefore \quad AS = 13$ cm [taking positive square root since, side cannot be negative]

(i) In ΔBRS, $\tan x = \frac{P}{B} = \frac{BR}{RS} = \frac{6}{5}$

(ii) In ΔARS, $\sin y = \frac{P}{H} = \frac{SR}{AS} = \frac{5}{13}$

(iii) In ΔARS, $\cos y = \frac{B}{H} = \frac{AR}{AS} = \frac{12}{13}$

15. **Hint** Do same as Example 5 of Topic 1. **Ans.** $\frac{31}{25}$

16. **Hint** Use $\tan\theta = \frac{P}{B}$ and $BC = 2CD$. **Ans.** 2

17. **Hint** Given, $\tan\theta + \frac{1}{\tan\theta} = 2$

On squaring both sides, we get

$$\left(\tan\theta + \frac{1}{\tan\theta}\right)^2 = (2)^2 \quad \textbf{Ans. } 2$$

18. **Hint** Use $\sec A = \frac{1}{\cos A}$, $\tan A = \frac{\sin A}{\cos A}$ and $\operatorname{cosec} A = \frac{1}{\sin A}$ in LHS of given equation and simplify.

19. **Hint** Let $\cot\theta = x$

So, the given equation can be rewritten as

$$\sqrt{3}x^2 - 4x + \sqrt{3} = 0$$

Using quadratic formula, we get

$$x = \sqrt{3}, \frac{1}{\sqrt{3}} \text{ i.e. } \cot\theta = \sqrt{3}, \frac{1}{\sqrt{3}}$$

So, $\tan\theta = \frac{1}{\sqrt{3}}, \sqrt{3}$. **Ans.** $\frac{10}{3}$

20. **Hint** Since, $\tan\theta = 1$ and $\sin\phi = \frac{1}{\sqrt{2}}$,

$\therefore \theta = 45^\circ$ and $\phi = 45^\circ$

Now, $\cos(\theta + \phi) = \cos(45^\circ + 45^\circ) = \cos(90^\circ) = 0$

21. **Hint** Put $\cos 60^\circ = \sin 30^\circ = \frac{1}{2}$

$\tan 60^\circ = \sqrt{3}, \cot 45^\circ = 1, \sec 30^\circ = \frac{2}{\sqrt{3}}$ in LHS and simplify.

22. (i) Given, $\sin(A - B) = \frac{1}{2}$

$$\Rightarrow \quad \sin(A - B) = \sin 30^\circ$$

$$\Rightarrow \quad A - B = 30^\circ \quad \ldots\text{(i)}$$

and $\cos(A + B) = 0$

$$\Rightarrow \quad \cos(A + B) = \cos 90^\circ$$

$$\Rightarrow \quad A + B = 90^\circ \quad \ldots\text{(ii)}$$

On solving Eqs. (i) and (ii), we get

$$A = 60^\circ \text{ and } B = 30^\circ$$

Navdeep Saini and Jaspreet Bumrah throwing ball at an angle of 60° and 30°, respectively.

(ii) $\tan A = \sqrt{3}$, $\operatorname{cosec}(A - B) = 2$ and $\sec B = \frac{2}{\sqrt{3}}$

23. (i) We know that the maximum value of $\sin\theta$ is 1.

$$\therefore \quad \sin\theta_1 + \sin\theta_2 + \sin\theta_3 = 3$$

$$\Rightarrow \quad \sin\theta_1 = \sin\theta_2 = \sin\theta_3 = 1$$

$$\Rightarrow \quad \theta_1 = \theta_2 = \theta_3 = 90^\circ$$

Now, $\cos\theta_1 + \cos\theta_2 + \cos\theta_3$

$$= \cos 90^\circ + \cos 90^\circ + \cos 90^\circ$$

$$= 0 + 0 + 0 = 0$$

(ii) **Hint** $225^\circ - \theta_1 - \theta_2 = 225^\circ - 90^\circ - 90^\circ = 45^\circ$

Ans. 1

(iii) **Hint** $120^\circ - \theta_3 = 120^\circ - 90^\circ = 30^\circ$ **Ans.** $\frac{1}{2}$

24. **Hint** Given, $\tan A = 2/3$, $\tan B = 3/2$

Since, $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$$\tan(A + B) = \frac{2/3 + 3/2}{1 - 2/3 \cdot 3/2} = \frac{(4 + 9)/6}{0}$$

$$\tan(A + B) = \frac{13}{0} = \infty$$

$$A + B = 90^\circ$$

25. **Hint** Do same as Example 6 of Topic 2

Ans. $A = 45^\circ, B = 15^\circ$

26. **Hint** $\sec\theta = \sqrt{1 + \tan^2\theta} = \sqrt{1 + \left(\frac{20}{21}\right)^2} = \sqrt{1 + \frac{(20)^2}{(21)^2}} = \frac{29}{21}$

On dividing numerator and denominator of LHS by $\cos\theta$, we get $\frac{\sec\theta - \tan\theta + 1}{\sec\theta + \tan\theta + 1}$.

Now, put the value of $\sec\theta$ and $\tan\theta$ then simplify.

27. **Hint** LHS $= \frac{1}{1 + \sin\theta} + \frac{1}{1 - \sin\theta} = \frac{1 - \sin\theta + 1 + \sin\theta}{(1 + \sin\theta)(1 - \sin\theta)}$

$$= \frac{2}{(1 - \sin^2\theta)} = \frac{2}{\cos^2\theta} \quad [\because 1 - \sin^2 A = \cos^2 A]$$

$$= 2\sec^2\theta = \text{RHS}$$

Hence proved.

28. **Hint** LHS $= \frac{1 - \tan^2\theta}{1 + \tan^2\theta} = \frac{1 - \frac{\sin^2\theta}{\cos^2\theta}}{1 + \frac{\sin^2\theta}{\cos^2\theta}} \quad \left[\because \tan A = \frac{\sin A}{\cos A}\right]$

$$= \frac{\frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta}}{\frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta}} = \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta}$$

$$= \cos^2\theta - \sin^2\theta \quad [\because \cos^2 A + \sin^2 A = 1]$$

Hence proved.

29. **Hint** In LHS add and subtract the term $(2\sin^2\theta\cos^2\theta)$ in the numerator and then simplify the expression using $\sin^2\theta + \cos^2\theta = 1$.

30. Hint Given, $\dfrac{\sin^3\theta+\cos^3\theta}{\sin\theta+\cos\theta}+\sin\theta\cos\theta$

$$=\frac{(\sin\theta+\cos\theta)(\sin^2\theta+\cos^2\theta-\sin\theta\cos\theta)}{(\sin\theta+\cos\theta)}+(\sin\theta\cos\theta)$$

$[\because a^3+b^3=(a^2-ab+b^2)(a+b)]$

Ans. 1

31. LHS $=\dfrac{\cot A-\cos A}{\cot A+\cos A}=\dfrac{\dfrac{\cos A}{\sin A}-\cos A}{\dfrac{\cos A}{\sin A}+\cos A}\quad\left[\because \cot A=\dfrac{\cos A}{\sin A}\right]$

$$=\frac{\cos A-\cos A\sin A}{\cos A+\cos A\sin A}=\frac{\cos A(1-\sin A)}{\cos A(1+\sin A)}$$

$$=\frac{(1-\sin A)}{(1+\sin A)}\frac{(1+\sin A)}{(1+\sin A)}=\frac{1-\sin^2 A}{(1+\sin A)^2}$$

$$=\frac{\cos^2 A}{(1+\sin A)^2}=\text{RHS}$$ **Hence proved.**

32. LHS $=\dfrac{\cot\theta+\operatorname{cosec}\theta-1}{\cot\theta-\operatorname{cosec}\theta+1}$

$$=\frac{(\cot\theta+\operatorname{cosec}\theta)-(\operatorname{cosec}^2\theta-\cot^2\theta)}{\cot\theta-\operatorname{cosec}\theta+1}$$

$[\because \operatorname{cosec}^2 A-\cot^2 A=1]$

$$=\frac{(\operatorname{cosec}\theta+\cot\theta)\{1-(\operatorname{cosec}\theta-\cot\theta)\}}{\cot\theta-\operatorname{cosec}\theta+1}$$

$$=\frac{(\operatorname{cosec}\theta+\cot\theta)(1-\operatorname{cosec}\theta+\cot\theta)}{\cot\theta-\operatorname{cosec}\theta+1}$$

$$=\operatorname{cosec}\theta+\cot\theta=\frac{1}{\sin\theta}+\frac{\cos\theta}{\sin\theta}=\frac{1+\cos\theta}{\sin\theta}=\text{RHS}$$

33. Hint Given, $\sin^6 A+\cos^6 A+3\sin^2 A\cos^2 A+4=k$

$[\because a^3+b^3=(a+b)^3-3ab(a+b)]$

$$(\sin^2 A)^3+(\cos^2 A)^3+3\sin^2 A\cos^2 A+4$$
$$=(\sin^2 A+\cos^2 A)^3-3\sin^2 A\cos^2 A(\sin^2 A+\cos^2 A)+3\sin^2 A\cos^2 A+4$$
$$=1-3\sin^2 A\cos^2 A+3\sin^2 A\cos^2 A+4=5$$

$[\because \sin^2 A+\cos^2 A=1]$

Ans. $k=5$

34. Hint $\sec\theta=x+\dfrac{1}{4x}$

$$\Rightarrow\quad \sec^2\theta=x^2+\left(\frac{1}{4x}\right)^2+2\cdot x\cdot\frac{1}{4x}$$

$$\Rightarrow \sec^2\theta-1=x^2+\frac{1}{16x^2}-\frac{1}{2}$$

$$\Rightarrow\quad \tan^2\theta=\left(x-\frac{1}{4x}\right)^2$$

$$\Rightarrow\quad \tan\theta=\pm\left(x-\frac{1}{4x}\right)$$ **Ans.** $2x, 1/2x$

35. Hint Use $\sec\theta=\dfrac{1}{\cos\theta}$ and $\operatorname{cosec}\theta=\dfrac{1}{\sin\theta}$ in LHS and $\tan\theta=\dfrac{\sin\theta}{\cos\theta}, \cot\theta=\dfrac{\cos\theta}{\sin\theta}$ in RHS

36. (i) **Hint** First we have, $\sec\theta=\dfrac{x}{a}$ and $\tan\theta=\dfrac{y}{b}$, then use $\sec^2\theta-\tan^2\theta=1$

Ans. $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$

(ii) **Hint** First we have, $\cos\theta=\dfrac{x-k}{a}, \sin\theta=\dfrac{y-h}{b}$, then use $\sin^2\theta+\cos^2\theta=1$

Ans. $\dfrac{(x-k)^2}{a^2}+\dfrac{(y-h)^2}{b^2}=1$

37. Hint Given, $m\sin\theta+n\cos\theta=p$ ...(i)

$m\cos\theta-n\sin\theta=q$ ...(ii)

On squaring Eqs.(i) and (ii) and adding them, we get

$$(m\sin\theta+n\cos\theta)^2+(m\cos\theta-n\sin)^2=p^2+q^2$$

Now, simplify the above equation to get the required expression.

38. Hint Simplify LHS and RHS separately and use $\sin^2\theta+\cos^2\theta=1$

39. Hint Given, $7\sin^2 A+3\cos^2 A=4$

$$\Rightarrow\quad \cos^2 A(7\tan^2 A+3)=4$$
$$\Rightarrow\quad 7\tan^2 A+3=4\sec^2 A$$

Now use, $\sec^2 A-\tan^2 A=1$

40. Hint Given, $\cos\theta+\sin\theta=\sqrt{2}\cos\theta$

On squaring both sides, we get

$$\cos^2\theta+\sin^2\theta+2\sin\theta\cos\theta=2\cos^2\theta$$
$$\Rightarrow\quad \sin^2\theta=\cos^2\theta-2\sin\theta\cos\theta$$
$$\Rightarrow\quad \sin^2\theta+\sin^2\theta=\cos^2\theta-2\sin\theta\cos\theta+\sin^2\theta$$
$$\Rightarrow\quad 2\sin^2\theta=(\cos\theta-\sin\theta)^2$$

41. Given that $a\sin\theta+b\cos\theta=c$

On squaring both sides of above equation, we get

$$(a\sin\theta+b\cos\theta)^2=c^2$$
$$\Rightarrow\quad a^2\sin^2\theta+b^2\cos^2\theta+2ab\sin\theta\cos\theta=c^2$$
$$\Rightarrow a^2(1-\cos^2\theta)+b^2(1-\sin^2\theta)+2ab\sin\theta\cos\theta=c^2$$
$$\Rightarrow a^2-a^2\cos^2\theta+b^2-b^2\sin^2\theta+2ab\sin\theta\cos\theta=c^2$$
$$\Rightarrow a^2+b^2-c^2=a^2\cos^2\theta+b^2\sin^2\theta-2ab\sin\theta\cos\theta$$
$$\Rightarrow (a\cos\theta-b\sin\theta)^2=a^2+b^2-c^2$$
$$\Rightarrow\quad a\cos\theta-b\sin\theta=\pm\sqrt{a^2+b^2-c^2}$$ **Hence proved.**

42. Hint Divide numerator and denominator by $\cos A$ and use the identity $\sec^2 A - \tan^2 A = 1$ to get the desired result.

43. Hint Given $\cos\theta - \sin\theta = x$ ...(i)

$\sin\theta + \cos\theta = y$ ...(ii)

On squaring Eq. (i) and (ii) both side, we get

$\cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta = x^2$

$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = y^2$

Now, $x^2 + y^2 = 2\cos^2\theta + 2\sin^2\theta - 2\sin\theta\cos\theta + 2\sin\theta\cos\theta$

$= 2(\cos^2\theta + \sin^2\theta)$

$= 2$ $[\because \sin^2\theta + \cos^2\theta = 1]$

44. Hint Do same as Question 34.

45. Hint Put values of m and n in LHS and RHS of given equation separately and show the equality.

46. Hint Convert $\sec x$ and $\tan x$ into $\sin x$ and $\cos x$ using, $\sec x = \frac{1}{\cos x}$ and $\tan x = \frac{\sin x}{\cos x}$ in both LHS and RHS separately and simplify.

47. Hint First, simplify the LHS of given equation using $(a+b)^2 = a^2 + 2ab + b^2$ and $(a-b)^2 = a^2 - 2ab + b^2$, then use $1 + \tan^2\theta = \sec^2\theta$.

48. Hint $(a\cos\theta - b\sin\theta)^2 + (a\sin\theta + b\cos\theta)^2$

$= (a^2\cos^2\theta + b^2\sin^2\theta - 2ab\sin\theta\cos\theta) + (a^2\sin^2\theta + b^2\cos^2\theta + 2ab\sin\theta\cos\theta)$

$\Rightarrow c^2 + (a\sin\theta + b\cos\theta)^2 = a^2 + b^2$

49. Hint Given, $\sin\theta + 2\cos\theta = 1$

On squaring both sides, we get

$\sin^2\theta + 4\cos^2\theta + 4\sin\theta\cos\theta = \sin^2\theta + \cos^2\theta$

$\Rightarrow \sin^2\theta + 4\cos^2\theta = \sin^2\theta + \cos^2\theta - 4\sin\theta\cos\theta$

$\Rightarrow 4\sin^2\theta + 4\cos^2\theta = 4\sin^2\theta + \cos^2\theta - 4\sin\theta\cos\theta$

[$\because$ adding $3\sin^2\theta$ both sides]

$\Rightarrow 4(\sin^2\theta + \cos^2\theta) = (2\sin\theta - \cos\theta)^2$

50. Hint Use, $\cos^2\theta = 1 - \sin^2\theta$ to find $\cos A$ and $\cos B$.

51. Hint Let LHS $= \frac{1}{\cos^2 A} - \frac{\sin^2 A(1 - 2\sin^2 A)}{\cos^2 A(2\cos^2 A - 1)}$

$\left[\because \sec^2\theta = \frac{1}{\cos^2\theta}\right]$

$= \frac{1}{\cos^2 A} - \frac{\sin^2 A[1 - 2(1-\cos^2 A)]}{\cos^2 A(2\cos^2 A - 1)}$

$[\because \sin^2\theta = 1 - \cos^2\theta]$

52. Hint Given, $1 + \sin^2\theta = 3\sin\theta\cos\theta$

[dividing both sides by $\cos$

$\Rightarrow \sec^2\theta + \tan^2\theta = 3\tan\theta$

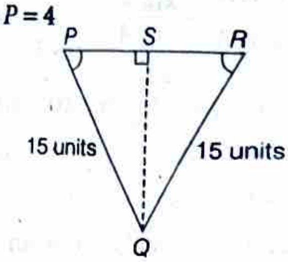
$\Rightarrow 2\tan^2\theta - 3\tan\theta + 1 = 0$ $[\because \sec^2\theta = 1 + \tan^2$

Let, $\tan\theta = x$, so above equation becomes

$2x^2 - 3x + 1$

Solve for $x = \tan\theta$ using quadratic formula.

53. Given, $5\sin P = 4$



From figure, in ΔPSQ

$\sin P = \frac{SQ}{PQ}$

$SQ = PQ \times \sin P$

$= 15 \times 4/5 = 12$ units

Using Pythagoras theorem, we get

$PS^2 = PQ^2 - SQ^2$

$= 15^2 - 12^2$

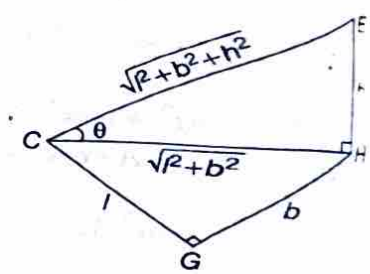
$= 225 - 144 = 81$

So, $PS = 9$

So, length of $PR = 2 \times 9 = 18$ units.

54. In ΔHGC,

Using Pythagoras theorem,



$CH^2 = CG^2 + GH^2$

$CH^2 = l^2 + b^2$

$CH = \sqrt{l^2 + b^2}$

Now, in ΔCHE

$CE^2 = CH^2 + HE^2$

$= (\sqrt{l^2 + b^2})^2 + h^2$

$CE^2 = l^2 + b^2 + h^2$

$CE = \sqrt{l^2 + b^2 + h^2}$

(i) $\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{CH}{CE} = \frac{\sqrt{l^2 + b^2}}{\sqrt{l^2 + b^2 + h^2}}$

(ii) In cube, all sides are equal. So, $l = b = h$

$\cos\theta = \frac{\sqrt{l^2 + l^2}}{\sqrt{l^2 + l^2 + l^2}} = \sqrt{\frac{2}{3}}$

55. Hint Use $\cot A = \dfrac{\cos A}{\sin A}$ and $\sec A = \dfrac{1}{\cos A}$ in LHS of given equation and simplify it.

56. To prove $\dfrac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A$

Let LHS $= \dfrac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \dfrac{\sin A(1 - 2\sin^2 A)}{\cos A(2\cos^2 A - 1)}$

$$= \frac{\sin A(\sin^2 A + \cos^2 A - 2\sin^2 A)}{\cos A(2\cos^2 A - (\sin^2 A + \cos^2 A))} \quad [\because \sin^2 A + \cos^2 A = 1]$$

$$= \frac{\sin A(\cos^2 A - \sin^2 A)}{\cos A(\cos^2 A - \sin^2 A)}$$

$= \tan A =$ RHS **Hence proved**

57. Given, $\dfrac{1}{\operatorname{cosec}\theta - \cot\theta} - \dfrac{\cot\theta}{\cos\theta} = \cot\theta$

Taking LHS $= \dfrac{1}{\operatorname{cosec}\theta - \cot\theta} - \dfrac{\cot\theta}{\cos\theta}$

$$= \frac{1}{\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}} - \frac{\cos\theta}{\sin\theta\cos\theta} \quad \left[\because \operatorname{cosec}\theta = \frac{1}{\sin\theta},\ \cot\theta = \frac{\cos\theta}{\sin\theta}\right]$$

$$= \frac{\sin\theta}{1-\cos\theta} - \frac{1}{\sin\theta} = \frac{\sin^2\theta - 1 + \cos\theta}{\sin\theta(1-\cos\theta)}$$

$$= \frac{1 - \cos^2\theta - 1 + \cos\theta}{\sin\theta(1-\cos\theta)} \quad [\because \sin^2\theta = 1 - \cos^2\theta]$$

$$= \frac{\cos\theta(1-\cos\theta)}{\sin\theta(1-\cos\theta)} = \frac{\cos\theta}{\sin\theta}$$

$= \cot\theta$

$=$ RHS

58. Given, $\dfrac{1}{\sin\theta - \cos\theta} = \dfrac{\operatorname{cosec}\theta}{\sqrt{2}}$

On squaring both sides, we get

$$\frac{1}{\sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta} = \frac{\operatorname{cosec}^2\theta}{2} \quad [\because (a-b)^2 = a^2 + b^2 - 2ab]$$

$$\Rightarrow \frac{1}{1 - 2\sin\theta\cos\theta} = \frac{\operatorname{cosec}^2\theta}{2} \quad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$\Rightarrow \frac{2}{\operatorname{cosec}^2\theta} = 1 - 2\sin\theta\cos\theta$$

$$\Rightarrow 2\cos\theta\sin\theta = 1 - 2\sin^2\theta$$

$$\Rightarrow 2\cos\theta\sin\theta = \sin^2\theta + \cos^2\theta - 2\sin^2\theta$$

$$\Rightarrow 2\cos\theta\sin\theta = \cos^2\theta - \sin^2\theta \quad [\because \sin^2\theta = 1 - \cos^2\theta]$$

$$= 2\cos^2\theta - 1$$

$$\Rightarrow 2\cos^2\theta = 1 + 2\sin\theta\cos\theta$$

$$\Rightarrow \frac{2}{\sec^2\theta} = 1 + 2\sin\theta\cos\theta$$

$$\Rightarrow \frac{\sec^2\theta}{2} = \frac{1}{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta}$$

$$\Rightarrow \frac{\sec^2\theta}{2} = \frac{1}{(\sin\theta + \cos\theta)^2}$$

59. Given, $\dfrac{\operatorname{cosec}^2 x - \sin^2 x\cot^2 x - \cot^2 x}{\sin^2 x} = 1$

$$= \frac{\frac{1}{\sin^2 x} - \sin^2 x \cdot \frac{\cos^2 x}{\sin^2 x} - \frac{\cos^2 x}{\sin^2 x}}{\sin^2 x} \quad \left[\because \operatorname{cosec} x = \frac{1}{\sin x},\ \cot x = \frac{\cos x}{\sin x}\right]$$

$$= \frac{\frac{1-\cos^2 x}{\sin^2 x} - \cos^2 x}{\sin^2 x} = \frac{\frac{\sin^2 x}{\sin^2 x} - \cos^2 x}{\sin^2 x} \quad [\because \sin^2 x = 1 - \cos^2 x]$$

$$= \frac{1 - \cos^2 x}{\sin^2 x} = \frac{\sin^2 x}{\sin^2 x}$$

$= 1$

60. Hint LHS $= \left[\dfrac{1}{\frac{1}{\cos^2\theta} - \cos^2\theta} + \dfrac{1}{\frac{1}{\sin^2\theta} - \sin^2\theta}\right]\sin^2\theta\cos^2\theta$

$$= \left[\frac{\cos^2\theta}{1-\cos^4\theta} + \frac{\sin^2\theta}{1-\sin^4\theta}\right]\sin^2\theta\cos^2\theta$$

$$= \left[\frac{\cos^2\theta}{(1+\cos^2\theta)(1-\cos^2\theta)} + \frac{\sin^2\theta}{(1-\sin^2\theta)(1+\sin^2\theta)}\right]\sin^2\theta\cos^2\theta$$

$$= \left[\frac{\cos^2\theta}{(1+\cos^2\theta)\sin^2\theta} + \frac{\sin^2\theta}{\cos^2\theta(1+\sin^2\theta)}\right]\sin^2\theta\cos^2\theta$$

61. Hint Let LHS $= \dfrac{\left[1 + \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}\right](\sin\theta - \cos\theta)}{\frac{1}{\cos^3\theta} - \frac{1}{\sin^3\theta}}$

62. **Hint** $\cot B = \frac{a}{\tan A}$ and $\operatorname{cosec} B = \frac{b}{\sin A}$

We know that $\operatorname{cosec}^2 B - \cot^2 B = 1$

$$\Rightarrow \quad \frac{b^2}{\sin^2 A} - \frac{a^2}{\tan^2 A} = 1$$

63. **Hint** Given, $\operatorname{cosec} \theta - \sin \theta = m$

and $\quad \sec \theta - \cos \theta = n$

$$\Rightarrow \quad \frac{1}{\sin \theta} - \sin\theta = m$$

and $\quad \frac{1}{\cos \theta} - \cos \theta = n$

$$\Rightarrow \quad \frac{1-\sin^2\theta}{\sin\theta} = m \text{ and } \frac{1-\cos^2\theta}{\cos\theta} = n$$

$$\Rightarrow \quad \frac{\cos^2\theta}{\sin\theta} = m \text{ and } \frac{\sin^2\theta}{\cos\theta} = n$$

Now, put the values of m and n in LHS of given equation.

64. **Hint** Given, $\quad \sec\theta + \tan\theta = l \quad$ ...(i)

$$\Rightarrow \quad \frac{(\sec\theta+\tan\theta)(\sec\theta-\tan\theta)}{(\sec\theta-\tan\theta)} = l$$

$$\Rightarrow \quad \frac{\sec^2\theta-\tan^2\theta}{\sec\theta-\tan\theta} = l$$

$$\Rightarrow \quad \sec\theta - \tan\theta = \frac{1}{l} \quad \text{...(ii)}$$

Hence, find $\sec\theta$ using Eqs. (i) and (ii).

65. **Hint** First $\tan A = \sqrt{\sec^2 A - 1}$

$$= \sqrt{\left(\frac{17}{8}\right)^2 - 1} = \frac{15}{8}$$

Now, LHS $= \frac{3-4\sin^2 A}{4\cos^2 A - 3}$

$$= \frac{3\sec^2 A - 4\tan^2 A}{4 - 3\sec^2 A}$$

[dividing numerator and denominator by $\cos^2 A$]

Now, put values of $\sec^2 A$ and $\tan^2 A$ in LHS and RHS and hence verify the equality.

66. **Hint** Let LHS $= (\sin^4 A)^2 - (\cos^4 A)^2$

$$= (\sin^4 A - \cos^4 A)(\sin^4 A + \cos^4 A)$$

$[\because (a^2 - b^2) = (a+b)(a$

$$= [(\sin^2 A)^2 - (\cos^2 A)^2][(\sin^2 A)^2 + (\cos^2$$

$$= (\sin^2 A + \cos^2 A)(\sin^2 A - \cos^2 A)$$

$[(\sin^2 A + \cos^2 A)^2 - 2\sin^2 A\cos$

$[\because a^2 + b^2 = (a+b)^2 - 2$

$$= 1 \cdot [\sin^2 A - (1-\sin^2 A)][1^2 - 2\sin^2 A\cos^2 A]$$

$[\because \sin^2\theta + \cos^2\theta$

$$= (2\sin^2 A - 1)(1 - 2\sin^2 A\cos^2 A)$$

$= \text{RHS}$

Hence prove

67. **Hint** Put the value of p in RHS of given equality and simplify to obtain it equal to LHS.

68. **Hint** Use $\operatorname{cosec} A = \frac{1}{\sin A}$, $\sec A = \frac{1}{\cos A}$, $\tan A = \frac{\sin A}{\cos A}$ and $\cot A = \frac{\cos A}{\sin A}$ in LHS and RHS separately and henc show the equality.

69. **Hint** Put value of q in LHS of given equation and simplify it.

70. **Hint** Consider, simplify it

$$\text{LHS} = \sec^2\theta(2-\sec^2\theta) - \operatorname{cosec}^2\theta(2-\operatorname{cosec}^2\theta)$$

$$= \sec^2\theta(2-1-\tan^2\theta) - \operatorname{cosec}^2\theta(2-1-\cot^2\theta)$$

$$= (1+\tan^2\theta)(1-\tan^2\theta) - (1+\cot^2\theta)(1-\cot^2\theta)$$

$[\because \sec^2 A = 1 + \tan^2 A$ and $\operatorname{cosec}^2 A = 1 + \cot^2 A$

71. **Hint** $x^2 + y^2 + z^2 = r^2\sin^2 A\cos^2 B$

$$+ r^2 \sin^2 A \sin^2 B + r^2\cos^2$$

$$= r^2\sin^2 A(\cos^2 B + \sin^2 B) + r^2\cos^2$$

72. **Hint** Let LHS $= \left(\frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)^2 + \left(\frac{\sin\theta}{\cos\theta} - \frac{1}{\cos\theta}\right)^2$

$$= \frac{(\sin\theta+1)^2}{\cos^2\theta} + \frac{(\sin\theta-1)^2}{\cos^2\theta}$$

73. **Hint** Simplify l and m in terms of $\sin\theta$ and $\cos\theta$ and then put the value of l and m in LHS of the equation.

Mind Map

INTRODUCTION TO TRIGONOMETRY

Study of relationships between the sides and angles of a right triangle

Trigonometric Identities

(i) $\sin^2 A + \cos^2 A = 1$

(ii) $1 + \tan^2 A = \sec^2 A$

(iii) $\cot^2 A + 1 = \text{cosec}^2 A$

e.g. Express the ratios $\cos A$, $\tan A$ and $\sec A$ in terms of $\sin A$.

$\sin^2 A + \cos^2 A = 1$

$\cos^2 A = 1 - \sin^2 A$

$\cos A = \sqrt{1 - \sin^2 A}$

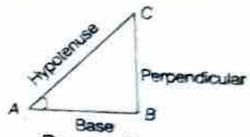
$\tan A = \dfrac{\sin A}{\sqrt{1 - \sin^2 A}}$

$\sec A = \dfrac{1}{\sqrt{1 - \sin^2 A}}$

Trigonometric Ratios of Some Specific Angles

$\angle A$	0°	30°	45°	60°	90°
sin A	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
cos A	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
tan A	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞
cot A	∞	$\sqrt{3}$	1	$1/\sqrt{3}$	0
sec A	1	$2/\sqrt{3}$	$\sqrt{2}$	2	∞
cosec A	∞	2	$\sqrt{2}$	$2/\sqrt{3}$	1

Trigonometric Ratios



$\sin A = \dfrac{\text{Perpendicular}}{\text{Hypotenuse}} = \dfrac{BC}{AC}$

$\cos A = \dfrac{\text{Base}}{\text{Hypotenuse}} = \dfrac{AB}{AC}$

$\tan A = \dfrac{\text{Perpendicular}}{\text{Base}} = \dfrac{BC}{AB}$

$\text{cosec } A = \dfrac{\text{Hypotenuse}}{\text{Perpendicular}} = \dfrac{AC}{BC}$

$\sec A = \dfrac{\text{Hypotenuse}}{\text{Base}} = \dfrac{AC}{AB}$

$\cot A = \dfrac{\text{Base}}{\text{Perpendicular}} = \dfrac{AB}{BC}$