CHAPTER 5

Arithmetic Progressions (AP)

In this chapter, we will study the sequence (list of numbers) in which succeeding terms are obtained by adding a fixed number to the preceding terms. We will find the formulae to find the value of any term and the sum of terms for this sequence.

Sequence (List of Numbers)

A list of numbers arranged in a definite order according to some definite rule, is called a sequence.

e.g.

(i) 1, 2, 3, 4, 5, 6, ...

In this sequence each number is 1 more than the number preceding it, except first number.

(ii) 2, 4, 8, 16, ...

In this sequence, each number is obtained on multiplying the preceding number by 2, except first number.

The various numbers occurring in a sequence are called its **terms**.

Consider, a sequence $a_1, a_2, a_3, a_4 \ldots$

Here, a_1 is called first term, a_2 is called second term and so on. Its nth term is denoted by a_n, which is also known as the general term of sequence.

A sequence is said to be finite or infinite accordingly it has a finite or infinite number of terms.

Progression

Those sequences whose terms always follow certain patterns are called progressions.

e.g.

(i) The sequence 3, 5, 7, 9 ... is a progression, as each term can be found by adding 2 to the term preceding it.

(ii) The sequence 3, 10, 18, 21, 35 .... . is not a progression as its terms are not following any certain pattern.

Note It All progressions are sequences but all sequences need not be progressions.

*In this chapter, we will study about one of the progressions, **arithmetic progression**.*

TOPIC 01 Introduction to Arithmetic Progressions

Arithmetic Progressions

An Arithmetic Progression (AP) is a list of numbers in which each term is obtained by adding a fixed number to the preceding term except the first term.

This fixed number is called the common difference (d) of the AP. It can be positive, negative or zero.

In other words, a list of numbers $a_1, a_2, a_3, \ldots, a_n$ is called an arithmetic progression (AP), if there exists a constant number d (called common difference) such that

$$\begin{aligned} a_2 - a_1 &= d \\ a_3 - a_2 &= d \\ a_4 - a_3 &= d \\ &\vdots \\ a_n - a_{n-1} &= d \text{ and so on.} \end{aligned}$$

Each of the numbers in this list is called a **term**.

In general, $a, a+d, a+2d, a+3d, \ldots$ represent an arithmetic progression, where a is the **first term** and d is **the common difference**. This is called general form of an AP.

If number of terms in an AP is finite, then it is called a **finite AP**, otherwise it is called an **infinite AP**. Such APs do not have a last term.

Note It If the common difference of an AP is zero i.e. $d = 0$, then each term of the AP will be same as the first term of the AP.

Example 1. Find the common difference of the following AP's.

(i) $3, -2, -7, -12, \ldots$

(ii) $11, 11, 11, 11, \ldots$

(iii) $5\frac{1}{2}, 9\frac{1}{2}, 13\frac{1}{2}, 17\frac{1}{2}, \ldots$

(iv) $\sqrt{3}, \sqrt{12}, \sqrt{27}, \sqrt{48}, \ldots$ CBSE 2019

Sol. (i) Given, AP is $3, -2, -7, -12, \ldots$

Here, $a_1 = 3,\ a_2 = -2,\ a_3 = -7,\ a_4 = -12$ and so on.

$\therefore$ Common difference $(d) = a_2 - a_1 = -2 - 3 = -5$

(ii) Given, AP is $11, 11, 11, 11, \ldots$

Here, $a_1 = 11, a_2 = 11, a_3 = 11, a_4 = 11$ and so on.

$\therefore$ Common difference $(d) = a_2 - a_1 = 11 - 11 = 0$

(iii) Given, AP is $5\frac{1}{2}, 9\frac{1}{2}, 13\frac{1}{2}, 17\frac{1}{2}, \ldots\ldots$

Here, $a_1 = 5\frac{1}{2},\ a_2 = 9\frac{1}{2},\ a_3 = 13\frac{1}{2},\ a_4 = 17\frac{1}{2}$ and so on.

$\therefore$ Common difference $(d) = a_2 - a_1 = 9\frac{1}{2} - 5\frac{1}{2}$

$$= \frac{19}{2} - \frac{11}{2} = \frac{8}{2} = 4$$

(iv) Given, AP is $\sqrt{3}, \sqrt{12}, \sqrt{27}, \sqrt{48} \ldots$

Here, $a_1 = \sqrt{3}, a_2 = \sqrt{12} = 2\sqrt{3}, a_3 = \sqrt{27} = 3\sqrt{3}$ and $a_4 = \sqrt{48} = 4\sqrt{3}$ and so on.

$\therefore$ Common difference $(d) = a_2 - a_1 = 2\sqrt{3} - \sqrt{3} = \sqrt{3}$

Method to Check an AP When a List of Numbers is Given

Sometimes, a list of numbers or sequence is given and we have to check that this sequence is an AP or not. For this, we find the differences of consecutive terms. If these differences are same, then given list of numbers or sequence is an AP, otherwise not.

Example 2. Examine that the list of numbers 13, 10, 7, 4,... form an AP. If it forms an AP, then write the next two terms.

Sol. Here, we have

$$\begin{aligned} a_2 - a_1 &= 10 - 13 = -3, \\ a_3 - a_2 &= 7 - 10 = -3, \\ a_4 - a_3 &= 4 - 7 = -3 \text{ and so on} \end{aligned}$$

Since, difference of any two consecutive terms is same, so the given list of numbers form an AP.

Now, the next two terms are $4 + (-3) = 1$ and $1 + (-3) = -2$.

Example 3. Examine that the list of numbers obtained from following situation, will be in the form of an AP. "Amount left with Sandeep (in ₹) out of the total amount of ₹ 12000 which he had in the beginning, when he spends ₹ 500 in the beginning of every month."

Sol. Given, in the beginning Sandeep had = ₹ 12000

Also, in the beginning of every month, he spend = ₹ 500

So, in the beginning of 1st month, he had amount, $t_1 =$ ₹ 12000

In the beginning of 2nd month, he had amount,

$t_2 = 12000 - 500 =$ ₹ 11500

In the beginning of 3rd month, he had amount,

$t_3 = 11500 - 500 =$ ₹ 11000

In the beginning of 4th month, he had amount,

$t_4 = 11000 - 500 =$ ₹ 10500 and so on.

Now, the list of amounts is $12000, 11500, 11000, 10500, \ldots$

Here, $t_2 - t_1 = t_3 - t_2 = t_4 - t_3 = -500$

i.e. $t_{k+1} - t_k$ is same everytime.

So, the above list of numbers forms an AP.

Method to Write an AP When First Term and Common Difference are Given

To write an AP, the minimum information required is to know the first term a and the common difference d of the arithmetic progression. Then, we put the values of a and d in $a, a+d, a+2d, a+3d, \dots$ to get the required AP.

Example 4. **Write an AP having 4 as the first term and −3 as the common difference.**

Sol. Given, first term $(a) = 4$ and common difference $(d) = -3$

On putting the values of a and d in general form $a, a+d, a+2d, a+3d, \dots$, we get

$4, 4-3, 4+2(-3), 4+3(-3), \dots$

$4, 1, 4-6, 4-9$, or $4, 1, -2, -5, \dots$

which is the required AP.

Try These 5.1

1. **Which of the following form an AP? Justify your answer.**
   (i) 1, 1, 2, 2, 3, 3, ... NCERT Exemplar
   (ii) $5, 2, -1, -4, -7, \dots$
   (iii) 0.3, 0.33, 0.333, ...
   (iv) $4, 4+\sqrt{2}, 4+2\sqrt{2}, 4+3\sqrt{2}, \dots$
2. **Write the common difference of the given AP** $\sqrt{3}, 2\sqrt{3}, 3\sqrt{3}, \dots$
3. **Write the first term and common difference of the following APs:**
   (i) 5, 8, 11, 14, ... (ii) $\frac{1}{4}, \frac{5}{4}, \frac{9}{4}, \frac{13}{4}, \dots$
4. **Find the common difference and the next two terms of the following APs :**
   (i) 6, 12, 18,... . (ii) 5, 13, 21, 29, ...
5. **Find the first four terms of an AP, whose first term a and the common difference d are given as follows :**
   (i) $a = -2, d = -2$ (ii) $a = -1, d = 4$
6. **Examine that the list of numbers 7, 13, 19, 25, ... forms an AP. If they form an AP, then write the next two terms.**
7. **Which of the following are APs? If they form an AP, find the common difference d and write three more terms.**
   (i) 4, 8, 12, 16, ... (ii) $1^2, 2^2, 3^2, 4^2, \dots$
8. **Write next three terms of the given AP**
   $(a+b), (a+1)+b, (a+1)+(b+1), \dots$ NCERT Exemplar
9. **Examine that the list of numbers obtained from the following situation, will be in the form of an AP or not.**
   (i) 'The amount of money in the account every year, when ₹ 100 is deposited at compound interest at 4% per annum'.
   (ii) The cost of digging a well after every metre of digging, when it cost ₹ 300 for the first metre and rises by ₹ 65 for each subsequent metre.

Answers

1. (i) No (ii) Yes (iii) No (iv) Yes
2. $\sqrt{3}$
3. (i) 5, 3 (ii) $\frac{1}{4}, 1$
4. (i) $d = 6$, The next two terms are 24 and 30.
   (ii) $d = 8$, The next two terms are 37 and 45.
5. (i) $-2, -4, -6$ and -8 (ii) $-1, 3, 7, 11$
6. Yes; 31, 37
7. (i) $a_1 = a = 4, a_2 = 8, a_3 = 12$
   So, $a_2 - a_1 = 8 - 4 = 4$
   and $a_3 - a_2 = 12 - 8 = 4$
   Since, $a_2 - a_1 = a_3 - a_2$
   So, the given list of numbers forms an AP.
   Here, $a = 4$ and $d = 4$
   So, next three terms are a_5, a_6, a_7, respectively.
   So, $a_5 = a + 4d = 4 + 4 \times 4 = 20$
   $a_6 = a + 5d = 4 + 5 \times 4 = 24$
   $a_7 = a + 6d = 4 + 6 \times 4 = 28$
   (ii) $a_1 = 1^2 = 1, a_2 = 2^2 = 4, a_3 = 3^2 = 9$
   $\therefore a_2 - a_1 = 4 - 1 = 3, a_3 - a_2 = 9 - 4 = 5$
   Since, $a_2 - a_1 \neq a_3 - a_2$
   $\therefore$ The given list of numbers does not form an AP.
8. $(a+2)+(b+1), (a+2)+(b+2), (a+3)+(b+2)$
9. (i) No (ii) Yes, consecutive terms differ by same numbers.

TOPIC 02 nth Term of an AP

If the first term of an AP is a and its common difference is d, then its nth term is given by the formula

$$a_n = a + (n-1)d$$

The nth term of an AP is also called its **general term.**

If there are n terms in an AP, then nth term is known as last term of an AP and it is denoted by l, which is given by the formula

$$l = a + (n-1)d$$

where, a is first term and d is common difference.

Example 1. **Find the 20th term of the AP : 7, 3, −1, −5 ...**

Sol. Given, AP is $7, 3, -1, -5, \dots$.

Here, $a = 7$ and $d = 3 - 7 = -4$

Since, nth term, $a_n = a + (n-1)d$

On putting $n = 20$, we get

$a_{20} = a + (20-1)d = 7 + 19(-4)$

$= 7 - 19 \times 4 = 7 - 76 = -69$

Hence, 20th term of given sequence is -69.

Types of Problems Based on nth Term of an AP

Type I Problems Based on Finding n, when nth Term or Last Term of an AP are Given

In this type of problems, an AP with last term is given or AP containing n terms is given (or nth term of an AP is given) and we have to find the value of n.

Example 2. How many terms are there in the AP 3, 6, 9, 12, ..., 111?

Sol. Given, AP is 3, 6, 9, 12, ..., 111.

Here, $a = 3$ and $d = 6 - 3 = 3$

Let there be n terms in the given AP.

Then, nth term $= 111$

$\Rightarrow \quad a+(n-1)d = 111 \quad [\because a_n = a+(n-1)d]$

$\Rightarrow \quad 3+(n-1)\times 3 = 111$

$\Rightarrow \quad 3(1+n-1) = 111$

$\Rightarrow \quad n = \dfrac{111}{3} \Rightarrow n = 37$

Hence, the given AP contains 37 terms.

Example 3. Which term of the AP 21, 18, 15, ... is −81? Also, is any term 0? Give reason. CBSE 2024 (Basic)

Sol. Given, AP is 21, 18, 15, ...

Here, $a = 21$ and $d = 18 - 21 = -3$

Let nth term of given AP be − 81.

Then, $a_n = -81$

$\Rightarrow \quad a+(n-1)d = -81 \quad [\because a_n = a+(n-1)d)]$

On putting the values of a and d, we get

$21+(n-1)(-3) = -81$

$\Rightarrow \quad 21-3n+3 = -81$

$\Rightarrow \quad 24-3n = -81$

$\Rightarrow \quad -3n = -81-24 = -105$

$\Rightarrow \quad n = \dfrac{-105}{-3} = 35$

Hence, 35th term of given AP is − 81.

Now, we want to know that if there is any n for which $a_n = 0$.

If such any n is there, then we have

$21+(n-1)(-3) = 0 \Rightarrow 3(n-1) = 21$

$\Rightarrow \quad n-1 = 7 \Rightarrow n = 8$

$\Rightarrow$ So, eighth term is 0.

Example 4. Check whether 200 is a term of the list of numbers 7, 11, 15, 19, ...

Sol. Given, list of numbers 7, 11, 15, 19, ...

Here, $11-7 = 15-11 = 19-15 ... = 4$

So, it an AP with first term, $a = 7$

and common difference, $d = 4$

Let 200 be a term, say the nth term of this AP.

We know that $a_n = a+(n-1)d$

$\therefore \quad 200 = 7+(n-1)(4)$

$\Rightarrow \quad 193 = (n-1)(4)$

$\Rightarrow \quad n-1 = \dfrac{193}{4} \Rightarrow n = \dfrac{197}{4} = 49\dfrac{1}{4}$

But the number of terms cannot be a fraction.

$\therefore$ 200 is not a term of the given AP.

Example 5. Which term of the AP 3, 15, 27, 39,... will be 120 more than its 21st term? CBSE 2019

Sol. Given, AP is 3, 15, 27, 39, ...

Here, $a = 3$ and $d = (15 - 3) = 12$

$\therefore$ 21st term is given by

$T_{21} = a+(21-1)d = a+20d = 3+20\times 12 = 243$

Required term $= (243+120) = 363$

Let it be nth term.

Then, $T_n = 363 \quad [\because T_n$ means nth term$]$

$\Rightarrow \quad a+(n-1)d = 363$

$\Rightarrow \quad 3+(n-1)\times 12 = 363$

$\Rightarrow \quad 12n = 372 \Rightarrow n = 31$

Hence, 31st term is the required term.

Type II Problems Based on Finding the AP or nth Term or Both, when its Two Terms are Given

In this type of problems, two terms (or relation between two terms of an AP) is given and we have to find the AP and nth term of AP.

Example 6. Determine the AP whose 3rd term is 5 and the 7th term is 9. CBSE 2024 (Standard)

Sol. We have,

$a_3 = a+(3-1)d = a+2d = 5 \quad$ ... (i)

$a_7 = a+(7-1)d = a+6d = 9 \quad$ ... (ii)

From Eqs. (i) and (ii), we get

$a = 3$ and $d = 1$

Hence, the required AP is 3, 4, 5, 6, ...

Example 7. If pth term of an AP is q and qth term is p then prove that its nth term is $(p+q-n)$. CBSE 2023 (Standard)

Sol. Given, pth term of AP $= q$

qth term of AP $= p$

To prove, nth term of AP $= p+q-n$

Let a be the first term and d be the common difference of the AP. Since, pth term is q.

$\Rightarrow \quad a_p = q \Rightarrow a+(p-1)d = q \quad$ ...(i)

Similarly, qth term is p.

$\Rightarrow \quad a_q = p \Rightarrow a+(q-1)d = p \quad$ ...(ii)

On subtracting Eq. (ii) from Eq. (i), we get

$$(p-1-q+1)d = q-p$$

$$\Rightarrow \quad (p-q)d = q-p \Rightarrow d = -1$$

From Eq. (i), $a+(p-1)(-1) = q$

$$\Rightarrow \quad a-p+1 = q \Rightarrow a = p+q-1$$

$\therefore$ nth term, $a_n = a+(n-1)d$

$$= p+q-1+(n-1)(-1)$$

$$= p+q-1-n+1$$

$$= p+q-n$$

Hence proved.

Type III Word Problems

In this type of problems, first we write the AP or list of numbers (sequence) with the help of given information and then find the required value.

Example 8. How many two-digit numbers are divisible by 7? CBSE 2019

Sol. Two-digit numbers are $10, 11, 12, 13, 14, 15, \ldots, 97, 98, 99$ in which only $14, 21, 28, \ldots, 98$ are divisible by 7.

Here, $21-14 = 28-21\ldots = 7$.

So, this list of numbers forms an AP, whose first term $(a) = 14$ and common difference $(d) = 7$.

Let there be n terms in the above sequence, such that $a_n = 98$

$$\Rightarrow \quad a+(n-1)d = 98 \quad [\because a_n = a+(n-1)d]$$

$$\Rightarrow \quad 14+(n-1)7 = 98$$

$$\Rightarrow \quad 14+7n-7 = 98$$

$$\Rightarrow \quad 7n = 91$$

$$\Rightarrow \quad n = \frac{91}{7} = 13$$

Hence, there are 13 two-digit numbers divisible by 7.

Example 9. A sum of ₹ 2000 is invested at 7% simple interest per year. Calculate the interest at the end of each year. Do these interest form an AP? If so, then find the interest at the end of 20th year making use of this fact.

Sol. Given, initial money $P =$ ₹ 2000

Rate of interest, $R = 7\%$ per year; Time, $T = 1, 2, 3, 4, \ldots$

We know that simple interest is given by the following formula

$$\text{SI} = \frac{PRT}{100}$$

$\therefore$ SI at the end of 1st year $= \dfrac{2000 \times 7 \times 1}{100} =$ ₹ 140

SI at the end of 2nd year $= \dfrac{2000 \times 7 \times 2}{100} =$ ₹ 280

SI at the end of 3rd year $= \dfrac{2000 \times 7 \times 3}{100} =$ ₹ 420

Thus, the required list of numbers is 140, 280, 420,... .

Here, $280-140 = 420-280\ldots = 140$

So, above list of numbers forms an AP, whose first term $(a) = 140$ and common difference $(d) = 140$.

Now, SI at the end of 20th year will be equal to 20th term of the above AP.

$$\therefore a_{20} = a+(20-1)d$$

$$= 140+19 \times 140 = 140+2660 = 2800$$

Hence, the interest at the end of 20th year will be ₹ 2800.

nth Term from the End of an AP

Let a be the first term, d be the common difference and l be the last term of an AP, then nth term from the end can be found by the formula

$$n\text{th term from the end} = l-(n-1)d$$

Example 10. Determine the 10th term from the end of the AP 4, 9, 14, ..., 254.

Sol. Given, AP is $4, 9, 14, \ldots, 254$.

Here, $l =$ last term $= 254$

$d =$ common difference $= 9-4 = 5$

$\therefore$ 10th term from the end $= l-(10-1)d = l-9d$

$$= 254-9 \times 5 = 254-45 = 209$$

Alternate Method

On reversing the given AP, new AP is $254, \ldots, 14, 9, 4$.

Here, first term $(a) = 254$

and common difference $(d) = -5$

Now, 10th term of new AP $= a_{10}$

$$= 254+(10-1)(-5)$$

$$= 254-9 \times 5 = 209$$

Hence, 10th term from the end of given AP is 209.

Try These 5.2

1. **Find 11th term of AP $-5, -5/2, 0, 5/2, \ldots$** NCERT Exemplar

2. **Fill in the blanks in the following table, given that a is the first term, d is the common difference and a_n is the nth term of the AP.**

	a	d	n	a_n
(i)	5	4	5	...
(ii)	3	...	15	31
(iii)	...	-3	11	10
(iv)	10	5	...	25

3. **For what value of n, are the nth terms of two APs $30, 27, 24, \ldots$ and $6, 11, 16, \ldots$ equal?**

4. If in an AP, $a = 15, d = -3$ and $a_n = 0$, then find the value of n. CBSE 2019

5. If $d = -4$, $n = 7$ and $a_n = 4$, then find the value of the first term. NCERT Exemplar

6. The first term of an AP is 5, common difference is 3 and the last term is 80. Find the number of terms.

7. Check if 0 is a term of the AP 31, 28, 25, ...

8. Find p, if the given value of x is the pth term of the following AP 25, 50, 75, 100, ... ; $x = 1000$

9. The 11th term of an AP is 80 and the 16th term is 110. Find the 31st term.

10. What will be the value of $a_8 - a_4$ for the following AP 4, 9, 14, ... 254?

11. Which term of AP 3, 8, 13, 18, ... will be 130 more than its 31st term?

12. Two APs have the same common difference. The difference between their 50th terms is 200, what is the difference between their 500th terms?

13. In a flower bed, there are 43 rose plants in the first row, 41 in the second, 39 in the third and so on. There are 11 rose plants in the last row. How many rows are there in the flower bed?

14. A man starts repaying a loan as first instalment of ₹ 100. If he increases the instalment by ₹ 5 every month, then what amount he will pay in the 30th instalment?

15. How many numbers lie between 10 and 300, which divided by 4 leave a remainder 3 ?

16. A sum of ₹ 5000 is invested at 8% simple interest per annum. Calculate the interest at the end of each year. Do these interests form an AP? Find the interest at the end of 30th year.

17. The taxi fare after each kilometre, when the fare is ₹ 15 for the first kilometre and ₹ 8 for each additional kilometre, does not form an AP as the total fare (in ₹) after each kilometre is 15, 8, 8 ,8, ... Is the statement true? Give reason. NCERT Exemplar

18. Find the 6th term from the end of the AP 17, 14, 11, ... (– 40).

19. In an AP, if $a = 10$, $d = 5$ and $n = 100$, then find the value of a_{100} and also find the 50th term from the end.

Answers

1. Do same as Example 1. **Ans.** 20

2. **Hint** Use the general term formula, $a_n = a + (n-1)d$

 (i) $a = 5, d = 4, n = 5, a_n = ?$
 So, $a_n = 5 + (5-1) \times 4 = 21$

 (ii) $a = 3, n = 15, a_n = 31, d = ?$
 So, $31 = 3 + (15-1)d \Rightarrow d = 2$

 (iii) $a = ?, d = -3, n = 11, a_n = 10$
 So, $10 = a + (11-1)(-3) \Rightarrow a = 40$

 (iv) $a = 10, d = 5, n = ?, a_n = 25$
 So, $25 = 10 + (n-1)5 \Rightarrow n = 4$

3. **Hint** For the AP 30, 27, 24, ...

$$a_1 = 30,\ d_1 = 27 - 30 = -3$$

So, $a_n = a_1 + (n-1)d_1 = 30 + (n-1)(-3)$ ... (i)

Now, for the AP, 6, 11, 16, ...

$$b_1 = 6,\ d_2 = 11 - 6 = 5$$

So, $b_n = b_1 + (n-1)d_2 = 6 + (n-1)5$ ... (ii)

Now, to find the value of n for which the nth terms of two AP's are equal, we have

$$a_n = b_n$$

Ans. 4

4. **Hint** $a_n = a + (n-1)d$ **Ans.** 6

5. **Hint** $a_n = a + (n-1)d$ **Ans.** 28

6. **Hint** Solve as Example 2. **Ans.** 26

7. Solve as Example 4. **Ans.** No

8. **Hint** Here, $a_p = 1000$ **Ans.** 40

9. Solve as Example 6 and find a_{31}. **Ans.** 200

10. **Hint** First find a_8, a_4 and then find $a_8 - a_4$. **Ans.** 20

11. Solve as Example 5. **Ans.** 57th

12. Let the common difference of both APs be d.
 The first AP be $a_1, a_2, \ldots, a_n$
 and the second AP be $b_1, b_2, \ldots, b_n$.
 Now, $a_{50} - b_{50} = 200$
 $\Rightarrow [a_1 + (50-1)d] - [b_1 + (50-1)d] = 200$
 $\Rightarrow \quad a_1 + 49d - b_1 - 49d = 200$
 $\Rightarrow \quad a_1 - b_1 = 200$ ... (i)
 Now, consider $a_{500} - b_{500} = [a_1 + (500-1)d] - [b_1 + (500-1)d]$
 $= a_1 + 499d - b_1 - 499d$
 $= a_1 - b_1$
 $= 200$ [From Eq. (i)]
 Thus, difference between the 500th terms of two APs is 200.

13. **Hint** First term $a = 43$, common difference $d = -2, a_n = 11$
 Ans. 17

14. **Hint** $a = 100, d = 5$ **Ans.** ₹245

15. **Hint** The required numbers are 11,15,19, ..., 299.
 Ans. 73

16. Solve as Example 9. **Ans.** 400, 800, 1200, ...; Yes, ₹ 12000

17. **Hint** The total fare (in ₹) after each kilometre is 15, 23, 31, 39, ... **Ans.** Yes

18. **Hint** 6th term from the end $= l - (6-1)d$
 Ans. –25

19. **Hint** $a_n = a + (n-1)d$
 Then, $a_{100} = a + 99d$
 and then same as Example 10.
 Ans. 505, 260

TOPIC 03 Sum of First n Terms of an AP

If first term of an AP is a and its common difference is d, then the sum of its first n terms S_n, is given by the formula

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

This formula can also be written as

$$S_n = \frac{n}{2}[a + a_n]$$

where, a_n = nth term of the AP

If l is the last term of an AP of n terms, then $a_n = l$ and the sum of all the terms is given by this formula

$$S_n = \frac{n}{2}[a + l]$$

Example 1. **Find the sum of the first 20 terms of the AP** $-\frac{29}{3}, -9, -\frac{25}{3}, -\frac{23}{3}, \ldots\ldots$. CBSE 2023 (Basic)

Sol. Given, AP is $-\frac{29}{3}, -9, -\frac{25}{3}, -\frac{23}{2}, \ldots$ 20 terms

$$a = -\frac{29}{3}, d = -9 + \frac{29}{3} = \frac{-27+29}{3} = \frac{2}{3}$$

$$n = 20$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow S_{20} = \frac{20}{2}\left[2\left(-\frac{29}{3}\right) + (19)\frac{2}{3}\right] = 20\left[-\frac{29}{3} + \frac{19}{3}\right]$$

$$= 20 \times \left(-\frac{10}{3}\right) = -\frac{200}{3}$$

Example 2. **The ratio of the 11th term to the 18th term of an AP is 2 : 3. Find the ratio of the 5th term to the 21st term. Also, find the ratio of the sum of first 5 terms to the sum of first 21 terms.** CBSE 2023 (Standard)

Sol. Given, $\frac{a_{11}}{a_{18}} = \frac{2}{3}$

$$\Rightarrow \frac{a+10d}{a+17d} = \frac{2}{3} \quad [\because a_n = a + (n-1)d]$$

$$\Rightarrow 3a + 30d = 2a + 34d$$

$$a = 4d \quad \ldots(\text{i})$$

$$\therefore \frac{a_5}{a_{21}} = \frac{a+4d}{a+20d} = \frac{2a}{a+5a} \quad [\text{using Eq. (i)}]$$

$$= \frac{2a}{6a} = \frac{1}{3}$$

$$\therefore a_5 : a_{21} = 1 : 3$$

Now, $\frac{S_5}{S_{21}} = \frac{\frac{5}{2}(2a+4d)}{\frac{21}{2}(2a+20d)}$ $\left[\because S_n = \frac{n}{2}[2a + (n-\right.$

$$= \frac{5(2a+a)}{21(2a+5a)} = \frac{5}{21} \times \frac{3}{7} = \frac{15}{147} = \frac{5}{49} \text{ [using E}$$

$$\therefore S_5 : S_{21} = 5 : 49$$

Example 3. **Find the sum of the first 17 terms of an AP whose 4th and 9th terms are -15 and -30 respectiv**

Sol. Here, $a_4 = -15$ and $a_9 = -30$

As, $a_n = a + (n-1)d$

$\therefore$ $a_4 = a + (4-1)d = -15 \Rightarrow a + 3d = -15$

and $a_9 = a + (9-1)d = -30 \Rightarrow a + 8d = -30$

From Eqs. (i) and (ii), we get

$$a = -6, d = -3$$

So, $S_{17} = \frac{17}{2}[2 \times (-6) + (17-1)(-3)]$

$\left[\because S_n = \frac{n}{2}[2a + (n-\right.$

$$= \frac{17}{2}[-12 - 48] = -510$$

$$\therefore S_{17} = -510$$

Example 4. **Find the sum of first 24 terms of an AP, whose nth term is given by $a_n = 3 + 2n$**

Sol. We have, $a_n = 3 + 2n$

$\therefore$ $a_1 = 3 + 2 = 5$

$a_2 = 3 + 2 \times 2 = 7$

$a_3 = 3 + 2 \times 3 = 9$

$a_4 = 3 + 2 \times 4 = 11$

$\vdots \quad \vdots \quad \vdots$

So, the list of number becomes 5, 7, 9, 11, ...

Here, $7 - 5 = 9 - 7 = 11 - 9 = 2$ and so on.

So, it forms an AP with common difference, $d = 2$

To find S_{24}, we have, $n = 24$, $a = 5$ and $d = 2$

$\therefore S_{24} = \frac{24}{2}[2 \times 5 + (24-1) \times 2]$ $\left[\because S_n = \frac{n}{2}[2a + (n-\right.$

$$= 12(10 + 46) = 672$$

So, the sum of first 24 terms of the list of numbers is

Alternate Method

Given, nth term of an AP, $a_n = 3 + 2n$

Clearly, sum of first 24 terms,

$$(S_{24}) = \frac{24}{2}(a + a_{24}) = 12(5 + 51)$$

$[\because a_1 = 3 + 2 = 5$ and $a_{24} = 3 + 2 \times 24 = 3 + 48$

$$= 12 \times 56 = 672$$

Different Types of Problems Based on the Sum of n Terms of an AP

Type I Problems Based on Finding the Sum of First m Terms, When Sum of First p Terms and q Terms are Given

In this type of problems, we form two equations in a and d with the help of given information and solve them to get a and d. Then, find the sum of required number of terms.

Example 5. **If the sum of first 6 terms of an AP is 36 and that of the first 16 terms is 256, then find the sum of first 10 terms.** CBSE 2023 (Standard)

Sol. Given, $S_6 = 36$

$$\Rightarrow \frac{6}{2}[2a+(6-1)d] = 36 \quad \left[\because S_n = \frac{n}{2}[2a+(n-1)d]\right]$$

$$\Rightarrow 3(2a+5d) = 36$$

$$\Rightarrow 2a+5d = 12 \quad \text{...(i)}$$

Also, $S_{16} = 256$

$$\Rightarrow \frac{16}{2}[2a+(16-1)d] = 256$$

$$\Rightarrow 8(2a+15d) = 256$$

$$\Rightarrow 2a+15d = 32 \quad \text{...(ii)}$$

On subtracting Eq. (i) from Eq. (ii), we get

$$10d = 20$$

$$\Rightarrow d = 2$$

On putting $d = 2$ in Eq. (i), we get

$$2a = 12 - 5(2) = 2$$

$$\Rightarrow a = 1$$

$$\therefore S_{10} = \frac{10}{2}[2(1)+(10-1)2]$$

$$= 5(2+18) = 100$$

Hence, the sum of first 10 terms is 100.

Type II Problems Based on Finding the Number of Terms, When Sum of Terms and AP are Given

In this type of problems, we first find a and d and then assume that number of terms is n. After that, we use the formula of sum of first n terms to calculate n.

Example 6. **How many terms of the AP $20, 19\frac{1}{3}, 18\frac{2}{3}, \ldots$ must be taken, so that their sum is 300?**

Sol. Given, AP is $20, 19\frac{1}{3}, 18\frac{2}{3}, \ldots$

Here, $a = 20$ and $d = 19\frac{1}{3} - 20 = \frac{58}{3} - 20 = \frac{58-60}{3} = \frac{-2}{3}$

Let n terms of given AP be required to get sum 300.

We know that

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$\Rightarrow 300 = \frac{n}{2}\left[2(20)+(n-1)\left(\frac{-2}{3}\right)\right]$$

$$[\because a = 20 \text{ and } d = -2/3]$$

$$\Rightarrow 600 = n\left[40 - \frac{2}{3}n + \frac{2}{3}\right]$$

$$\Rightarrow 600 = \frac{1}{3}[120n - 2n^2 + 2n]$$

$$\Rightarrow 600 \times 3 = 122n - 2n^2$$

$$\Rightarrow 1800 + 2n^2 - 122n = 0$$

$$\Rightarrow 2[n^2 - 61n + 900] = 0$$

$$\Rightarrow n^2 - 61n + 900 = 0$$

$$\Rightarrow n^2 - 36n - 25n + 900 = 0$$

$$\Rightarrow n(n-36) - 25(n-36) = 0$$

$$\Rightarrow (n-36)(n-25) = 0$$

$$\Rightarrow n = 36 \text{ or } 25$$

Since, a is positive and d is negative, so both values of n are possible.

Hence, the sum of 25 terms of given AP

= Sum of 36 terms of given AP = 300

Note It If a is positive and d is negative, then we get two values of n, because some terms will be positive and some others negative, and will cancel out each other.

Type III Problem Based on Finding the nth Term, When the Sum of First n Terms is Given

If S_n and S_{n-1} is the sum of first n and $(n-1)$ terms of an AP, then its nth term a_n is given by

$$a_n = S_n - S_{n-1}$$

Example 7. **If in an AP, $a = 2$ and $S_{10} = 335$, then its 10th term is** CBSE 2024 (Basic)

(a) 55 **(b)** 65 **(c)** 68 **(d)** 58

Sol. (b) In a given AP, first term, $a = 2$

and sum of first 10 terms, $S_{10} = 335$

Since, $S_n = \frac{n}{2}[2a+(n-1)d]$

$$\therefore S_{10} = \frac{10}{2}[2\times 2+(10-1)d]$$

$$\Rightarrow 335 = 5[4+9d] \Rightarrow 4+9d = 67$$

$$\Rightarrow 9d = 63$$

$$\Rightarrow d = \frac{63}{9} = 7$$

$\therefore$ Common difference, $d = 7$

Also, nth term $a_n = a+(n-1)d$

$$\therefore a_{10} = a+(10-1)d$$

$$= a+9d$$

$$= 2+9\times 7 = 2+63 = 65$$

Hence, the 10th term is 65.

Example 8. **If S_n is the sum of first n terms of an AP given by $S_n = 3n^2 - 4n$, then find the nth term.** CBSE 2019

Sol. Given, $$S_n = 3n^2 - 4n \quad \text{...(i)}$$

On replacing n by $(n-1)$ in Eq. (i), we get

$$S_{n-1} = 3(n-1)^2 - 4(n-1)$$

nth term of the AP, $a_n = S_n - S_{n-1}$

$\therefore \quad a_n = (3n^2 - 4n) - [3(n-1)^2 - 4(n-1)]$

$\Rightarrow \quad a_n = 3[n^2 - (n-1)^2] - 4[n - (n-1)]$

$\Rightarrow \quad a_n = 3[n^2 - n^2 + 2n - 1] - 4[n - n + 1]$

$\Rightarrow \quad a_n = 3(2n-1) - 4$

$\Rightarrow \quad a_n = 6n - 3 - 4 \Rightarrow a_n = 6n - 7$

Thus, the nth term of the AP $= 6n - 7$.

Example 9. **If the sum of first n terms of an AP is n^2, then find its 10th term.** CBSE 2019

Sol. Given, $S_n = n^2$

We know that

$$T_n = S_n - S_{n-1} = n^2 - (n-1)^2 = n^2 - n^2 + 2n - 1$$

$\Rightarrow \quad T_n = 2n - 1$

Now, 10th term $= T_{10} = 2 \times 10 - 1 = 20 - 1 = 19$

Example 10. **Find the sum of the first 100 positive integers.**

Sol. Let $S = 1 + 2 + 3 + \ldots + 100$

Here, $a = 1$ and the last term l is 100.

$\therefore \quad S_n = \frac{n}{2}(a + l)$

$$\Rightarrow S_{100} = \frac{100(1+100)}{2} = \frac{100 \times 101}{2} = 50 \times 101 = 5050$$

Note The sum of first n positive integers is given by $S_n = \frac{n(n+1)}{2}$

Type IV Word Problems

Example 11. **Find the sum of integers between 100 and 200 which are (i) divisible by 9 (ii) not divisible by 9.** CBSE 2023 (Standard)

Sol. (i) The integers between 100 and 200 divisible by 9 are 108, 117, 126, ..., 198.

$\therefore$ Required sum $= 108 + 117 + 126 + \ldots + 198$

Here, first term, $a = 108$

Common difference, $d = 9$

Last term, $l = 198$

$\therefore l = a_n = a + (n-1)d = 198$

$\Rightarrow \quad 108 + (n-1)9 = 198 \Rightarrow (n-1)9 = 90$

$\Rightarrow \quad n - 1 = 10 \Rightarrow n = 11$

Sum of n terms, $\quad S_n = \frac{n}{2}[a + l]$

$\Rightarrow \quad S_{11} = \frac{11}{2}[108 + 198] = \frac{11}{2} \times 306 = 1683$

(ii) Numbers between 100 and 200 are 101, 102, 103 ... [

$\therefore$ Total number of terms, $n = 99$

First term, $a = 101$

Common difference, $d = 1$

$\therefore \quad S_{99} = \frac{99}{2}[101 + 199] = \frac{99}{2} \times 300 = 14850$

So, the sum of integers between 100 and 200 which are not divisible by $9 = 14850 - 1683 = 13167$

Example 12. **A man repays a loan of ₹ 3250 by ₹ 20 i[n] the first month and then increases the payment by ₹ 15 every month. How long will it take him to clear the loan**

Sol. Given, total amount of loan = ₹ 3250

Amount paid in first month = ₹ 20

and amount increase every month = ₹ 15

Clearly, the amounts of repayment form an AP with fir[st] term, $(a) = 20$ and common difference, $(d) = 15$

Let the loan be cleared in n months.

Then, $\quad S_n = 3250$

$\Rightarrow \quad \frac{n}{2}[2a + (n-1)d] = 3250$

$\Rightarrow \quad \frac{n}{2}[2(20) + (n-1)15] = 3250$

$\Rightarrow \quad n[40 + 15n - 15] = 3250 \times 2$

$\Rightarrow \quad 25n + 15n^2 = 6500$

$\Rightarrow \quad 3n^2 + 5n - 1300 = 0$ [dividing both sides b

$\Rightarrow 3n^2 + 65n - 60n - 1300 = 0$

$\Rightarrow n(3n + 65) - 20(3n + 65) = 0$

$\Rightarrow \quad (n - 20)(3n + 65) = 0$

$\Rightarrow \quad n = 20$ or $n = \frac{-65}{3}$

Since, n should be a positive integer, so neglect $n = \frac{-65}{3}$

$\therefore n = 20$

Hence, the loan is cleared in 20 months.

Example 13. **Kanika got her pocket money on Jan 1s 2008. She puts ₹ 1 on day 1, ₹ 2 on day 2, ₹ 3 on day and continued doing so till the end of the month, from this money into her piggy bank. She also spent ₹ 204 o her pocket money and found that at the end of the mo she still had ₹ 100 with her. How much was her pocke money for the month?** NCERT Exem

Sol. Let her pocket money be ₹x.

Now, she takes ₹1 on day 1, ₹2 on day 2, ₹3 on day 3 a[nd] so on till the end of the month, from this money.

Clearly, the amounts that she takes every day of the month, form an AP, in which number of terms is 31, f[irst] term $(a) = 1$ and common difference $(d) = 2 - 1 = 1$.

Now, sum of first 31 terms, $(S_{31}) = \frac{31}{2}[2 \times 1 + (31 - 1$

$\left[\because \text{sum of } n \text{ terms, } (S_n) = \frac{n}{2}[2a + (n -\right.$

$$= \frac{31}{2}(2+30) = \frac{31 \times 32}{2} = 31 \times 16 = 496$$

So, Kanika takes ₹496 till the end of the month from her pocket money.

Also, she spent ₹204 of her pocket money and found that at the end of the month, she still has ₹100 with her.

According to the above condition, we have

$$(x - 496) - 204 = 100$$

$$\Rightarrow \quad x - 700 = 100 \Rightarrow x = ₹\,800$$

Hence, ₹ 800 was her pocket money for the month.

Try These 5.3

1. **Find the sum of the following APs**
   (i) 50, 46, 42, ... to 10 terms
   (ii) 10, 15, 20, ... to 20 terms
2. **Find the sum of the first 20 terms of the AP 8, 12.5, 17, 21.5, ...**
3. **Find the sum of the following APs**
   (i) $7 + 10 + 13 + \ldots + 94$ (ii) $35 + 32 + 29 + \ldots + 17$
4. **Find the sum of first 15 multiples of 4.**
5. **Find the sum of even positive integers between 1 and 200.**
6. **Find the sum of all even numbers between 101 and 999.**
7. **Find the sum of first 25 terms of an AP whose nth term is given by $a_n = 7 + 3n$.**
8. **If sum of first 6 terms of an AP is 30 and that of the first 21 terms is 420, then find the sum of first 10 terms.** CBSE 2024 (Standard)
9. **Find the sum of first 21 terms of an AP whose 2nd term is 8 and 4th term is 4.**
10. **Find the common difference of an AP whose first term is 8, last term is 65 and the sum of all its terms is 730.** CBSE 2023 (Standard)
11. **If the sum of first p terms of an AP is q and the sum of first q terms is p, then find the sum of first $(p+q)$ terms.**
12. **Find the number of terms of the AP 64,60,56,...., so that their sum is 544.**
13. **The first term of an AP is 10 and common difference is 4, find the number of terms if its sum is 1344.**
14. **If S_n is the sum of the first n terms of an AP given by $S_n = 2n^2 + 5n$, then find its nth term.** CBSE 2019
15. **If the sum of first n terms of an AP is given by $S_n = 3n^2 + n$, then find the nth term of the AP. Also, find the AP.** NCERT Exemplar
16. **In an AP,**
    (i) Given $a = 7$, $d = -5$, $a_n = -28$, find n, a_{30} and S_n.
    (ii) Given $a = 4$, $d = 6$, $S_n = 310$, find n and a_n.
17. **228 logs are to be stacked in a store in the following manner: 30 logs in the bottom, 28 in the next row, then 26 and so on, in how many rows can these 228 logs be stacked? How many logs are there in the last row?**
18. **Yasmeen saves ₹ 32 during the first month, ₹ 36 in the second month and ₹ 40 in the third month. If she continues to save in this manner, then in how many months will she save ₹ 2000?** NCERT Exemplar
19. **The ages of the students in a class are in AP, whose common difference is 4 months. If the youngest student is 8 yr old and the sum of the ages of all the students is 168 yr, then find the number of students in the class.**
20. **Prove that the sum of later half of $2n$ terms of an AP is equal to one-third of the sum of the first $3n$ terms.**
21. **The sum of first six terms of an arithmetic progression is 42. The ratio of its 10th term to its 30th term is 1 : 3. Calculate the first and the 13th term of the AP.**
22. **A man arranges to pay off a debt of ₹ 3600 by 40 annual instalments which are in AP. When 30 of the instalments are paid, he dies leaving one-third of the debt unpaid. Find the value of the 8th instalment.**
23. **The sum of the first five terms and the sum of the first seven terms of an AP is 167. If the sum of the first ten terms of this AP is 235, then find the sum of its first twenty terms.** NCERT Exemplar
24. **Priya is preparing for the Bicycle marathon. Her racing bicycle has a device to calculate the number of kilometres she cycled. She decides to increase the distance she cycles everyday by a fixed number of kilometres.**
    (i) On the first day Priya cycled 8 km. In 10 days she cycled a total of 170 km. How many kilometres did she cycle on the 3rd day?
    (ii) Priya plans to go on a cycle tour from Bangalore to Mangalore covering 425 km. She travels 20 km on day 1 and increases the distance covered each day by 5 km. In how many days will she reach her destination? Competency Based Question

Answers

1. **Hint** Solve as Example 1.
   (i) 320 (ii) 1150
2. **Hint** Solve as Example 1. **Ans.** 1015
3. (i) Here, $a = 7$, $d = 10 - 7 = 3$, $a_n = 94$
   Since, $a_n = a + (n-1)d$
   $$94 = 7 + (n-1)\,3$$
   $$\Rightarrow (n-1) = \frac{94-7}{3} = \frac{87}{3} = 29 \Rightarrow n = 30$$
   Now, $S_n = \frac{n}{2}[2a + (n-1)d]$
   $$S_{30} = \frac{30}{2}[2(7) + (30-1)\,3]$$
   $$= 15[14 + 29 \times 3] = 1515$$

(ii) $a=35, d=32-35=-3, a_n=17$

$\therefore \quad a_n=a+(n-1)d$

$17=35+(n-1)(-3)$

$\therefore \quad n=7$

Now, $S_7=\frac{7}{2}[2(35)+6(-3)]=182$

4. Hint Multiples of 4 are 4, 8, 12, 16, 20, ... which forms an AP with $a=4$ and $d=4$. Now, find S_{15}. **Ans.** 480

5. Hint Even integer between 1 and 200 are 2, 4, 6, 8, ... 198, which forms an AP with $a=2$ and $d=2$.

Now, let $a_n=198 \Rightarrow n=99$. After this, find S_{99}. **Ans.** 9900

6. Hint Given, $a=102, d=2, l=998$

$\because l=a+(n-1)d \Rightarrow 998=102+(n-1)(2) \Rightarrow n=449$

Now, $S_n=\frac{n}{2}(a+l)=\frac{449}{2}(102+998)$ **Ans.** 246950

7. Hint Solve as Example 4. **Ans.** 1150

8. Hint Solve as Example 5. **Ans.** 90

9. Hint Solve as Example 3. **Ans.** −210

10. Let a be the first term and d be the common difference of AP.

Given, $a=8, a_n=l=65, S_n=730$

$$S_n=\frac{n}{2}(a+l)$$

$$\Rightarrow \quad 730=\frac{n}{2}(8+65)$$

$$\Rightarrow \quad 730\times 2=73n \Rightarrow n=\frac{730\times 2}{73}=20$$

General term, $a_n=a+(n-1)d$

$$\Rightarrow \quad 65=8+(20-1)d \Rightarrow 19d=57 \Rightarrow d=3$$

Hence, the common difference is 3.

11. Hint We have, $S_p=q$ and $S_q=p$.

On subtracting, we get

$$2a+(p+q-1)d=-2.$$

Now, $S_{p+q}=\frac{p+q}{2}[2a+(p+q-1)d]$

$$=\left(\frac{p+q}{2}\right)(-2)=-(p+q)$$ **Ans.** $-(p+q)$

12. Hint Solve as Example 6. **Ans.** 16, 17

13. Here $a=10, d=4, S_n=1344$

As, $\quad S_n=\frac{n}{2}[2a+(n-1)d]$

$\Rightarrow 1344=\frac{n}{2}[20+(n-1)4]$

$\Rightarrow 2688=n[20+4n-4]$

$\Rightarrow 4n^2+16n-2688=0$

$\Rightarrow \quad n^2+4n-672=0$

$\Rightarrow \quad n=24, -28$

Since, $n=-28$ cannot be possible

$\therefore \quad n=24$

14. Hint Solve as Example 8.

Ans. $3+4n$

15. Hint Solve as Example 8.

Ans. $6n-2$; 4, 10, 16, ...

16. (i) **Hint** $a_n=a+(n-1)d$

$-28=7+(n-1)(-5)$

$\therefore \quad n=8$

$S_8=\frac{8}{2}[2(7)+7\times(-5)]=-84$

$a_{30}=a+29d=7+29(-5)=-138$

(ii) Here $a=4, d=6, S_n=310$

$S_n=\frac{n}{2}[8+(n-1)6]=310$

$\therefore \quad n=10$

$a_{10}=a+9d=4+9(6)=58$

17. Hint Solve as Example 12. **Ans.** $n=12$ and $l=8$

18. Hint Solve as Example 12. **Ans.** 25 months

19. Number of student in the class $=16$

20. Hint To prove $S_{2n}-S_n=\frac{1}{3}S_{3n}$

21. Hint Given, $S_6=42$ and $\frac{a_{10}}{a_{30}}=\frac{1}{3}$. This implies $a=d$.

Ans. $a_1=2$ and $a_{13}=26$

22. Hint Let the amount of instalments be $a, a+d, a+2d$, ...

Then, according to given condition, we have,

$$S_{40}=3600 \text{ and } S_{30}=3600-\frac{1}{3}\times 3600=2400$$

$\Rightarrow 2a+39d=180$ and $2a+29d=160$

Now, find a and d, and then find a_8.

Ans. $a_8=$ ₹65

23. Hint Solve as Example 5. **Ans.** $S_{20}=970$

24. (i) Let a be the first term and d be the common difference

So, $a=8$ km, $S_n=170$ where $n=10$

Since, $\quad S_n=\frac{n}{2}[2a+(n-1)d]$

$\Rightarrow \quad S_{10}=\frac{10}{2}[2\times 8+(10-1)d]=170$

$\Rightarrow 5[16+9d]=170$

$\Rightarrow \quad 9d=18 \Rightarrow d=2$

$\therefore \quad a_3=a+2d=8+2\times 2=12$

Thus, she cycled 12 km distance on 3rd day.

(ii) $a=20, d=5, S_n=425$

$\therefore \quad S_n=\frac{n}{2}[2a+(n-1)d]$

$\Rightarrow \quad 425=\frac{n}{2}[2(20)+(n-1)5]$

$\Rightarrow \quad 850=n[40+5n-5]$

$\Rightarrow n^2+7n-170=0$

$\therefore n=-17$ or $n=10$

Since, number of terms cannot be negative.

$\therefore n=10$

NCERT FOLDER

Complete Solutions of All Exercises of NCERT Textbook

EXERCISE 5.1

Q1. In which of the following situations, does the list of numbers involved make an arithmetic progression and why?

(i) The taxi fare after each kilometre when the fare is ₹ 15 for the first kilometre and ₹ 8 for each additional kilometre.

(ii) The amount of air present in a cylinder when a vacuum pump removes 1/4 of the air remaining in the cylinder at a time.

(iii) The cost of digging a well after every metre of digging, when it costs ₹ 150 for the first metre and rises by ₹ 50 for each subsequent metre.

(iv) The amount of money in the account every year, when ₹ 10000 is deposited at compound interest at 8% per annum.

Sol. (i) According to the question, the fare for journey of 1 km, 2 km, 3 km, 4 km, ... are ₹ 15, ₹ (15 + 8), ₹ (15 + 2 × 8), ₹ (15 + 3 × 8) and so on i.e. 15, 23, 31, 39, ...

Here, each term is obtained by adding 8 to the preceding term except first term. So, it forms an AP.

(ii) Let the amount of air present in the cylinder be y units.

According to the question,

Remaining air in the cylinder after using vacuum pump first time $= y - \frac{y}{4} = \frac{3y}{4}$

Remaining air in the cylinder after using vacuum pump second time

$$= \frac{3y}{4} - \frac{1}{4} \times \frac{3y}{4} = \frac{12y - 3y}{16} = \frac{9y}{16} \text{ and so on.}$$

Thus, the list of numbers is $y, \frac{3y}{4}, \frac{9y}{16}, \ldots$

Here, $\frac{3y}{4} - y = -\frac{y}{4}$ $[\because a_2 - a_1]$

and $\frac{9y}{16} - \frac{3y}{4} = \frac{9y - 12y}{16} = -\frac{3y}{16}$ $[\because a_3 - a_2]$

Since, $\frac{3y}{4} - y \neq \frac{9y}{16} - \frac{3y}{4}$

i.e. difference between any two consecutive terms is not same. Hence, it does not form an AP.

(iii) According to the question, the cost of digging for the first metre, second metre, third metre,... are respectively ₹ 150, ₹ (150 + 50), ₹ (150 + 2 × 50), ... and so on.

Thus, the list of numbers is 150, 200, 250, ...

Here, each term is obtained by adding ₹ 50 to the preceding term except first term. So, it forms an AP.

(iv) We know that amount, $A = P\left(1 + \frac{R}{100}\right)^T$.

Here, $P =$ ₹ 10000, $R = 8\%$ per annum and $T = 1, 2, 3, \ldots$

Now, amount in 1st year $= 10000\left(1 + \frac{8}{100}\right)^1$

$$= 10000 \times \frac{108}{100}$$

$=$ ₹ 10800

Amount in 2nd year $= 10000\left(1 + \frac{8}{100}\right)^2$

$$= \frac{10000 \times 108 \times 108}{100 \times 100}$$

$=$ ₹ 11664 and so on.

Thus, the list of numbers is 10000, 10800, 11664, ...

Here, $10800 - 10000 = 800$

and $11664 - 10800 = 864$

i.e. the common difference is not same.

Hence, it does not form an AP.

Q2. Write first four terms of the AP, when the first term a and the common difference d are given as follows

(i) $a = 10, d = 10$ (ii) $a = -2, d = 0$

(iii) $a = 4, d = -3$ (iv) $a = -1, d = \frac{1}{2}$

(v) $a = -1.25, d = -0.25$

Sol. (i) Given, $a = 10$ and $d = 10$

We know that $a, a + d, a + 2d, a + 3d, \ldots$ are in AP.

Here, $a = 10$

$a + d = 10 + 10 = 20$

$a + 2d = 10 + 2 \times 10 = 30$

and $a + 3d = 10 + 3 \times 10 = 40$

Thus, the first four terms of the AP are 10, 20, 30 and 40.

(ii) Do same as Part (i). **Ans.** $-2, -2, -2$ and -2.

(iii) Do same as Part (i). **Ans.** 4, 1, -2 and -5.

(iv) Do same as Part (i). **Ans.** $-1, -\frac{1}{2}, 0$ and $\frac{1}{2}$.

(v) Given, $a = -1.25$ and $d = -0.25$

We know that $a, a+d, a+2d, a+3d, \dots$ are in AP.

Here, $a = -1.25$

$a+d = -1.25 - 0.25 = -1.50$

$a+2d = -1.25 + 2(-0.25) = -1.75$

and $a+3d = -1.25 + 3(-0.25) = -2.00$

Thus, the first four terms of the AP are

$-1.25, -1.50, -1.75$ and -2.00.

Q3. For the following AP's, write the first term and the common difference

(i) 3, 1, –1, –3, ... (ii) – 5, – 1, 3, 7, ...

(iii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}, \dots$ (iv) 0.6, 1.7, 2.8, 3.9, ...

Sol. (i) First term $(a) = 3$

and common difference $(d) =$ 2nd term $-$ 1st term

$= 1 - 3 = -2$

(ii) Do same as Part (i). **Ans.** $a = -5,\ d = 4$

(iii) Do same as Part (i). **Ans.** $a = \frac{1}{3},\ d = \frac{4}{3}$

(iv) Do same as Part (i). **Ans.** $a = 0.6,\ d = 1.1$

Q4. Which of the following are AP's? If they form an AP, then find the common difference *d* and write three more terms.

(i) 2, 4, 8, 16, ...

(ii) $2, \frac{5}{2}, 3, \frac{7}{2}, \dots$

(iii) – 1. 2, – 3.2, – 5.2, – 7.2, ...

(iv) $-10, -6, -2, 2, \dots$

(v) $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}, \dots$

(vi) 0.2, 0.22, 0.222, 0.2222, ...

(vii) $0, -4, -8, -12, \dots$

(viii) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \dots$

(ix) $1, 3, 9, 27, \dots$

(x) $a, 2a, 3a, 4a, \dots$

(xi) $a, a^2, a^3, a^4, \dots$

(xii) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \dots$

(xiii) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, \dots$

(xiv) $1^2, 3^2, 5^2, 7^2, \dots$

(xv) $1^2, 5^2, 7^2, 73, \dots$

Sol. (i) Here, we have

$a_2 - a_1 = 4 - 2 = 2$, and $a_3 - a_2 = 8 - 4 = 4$

Since, $a_2 - a_1 \neq a_3 - a_2$

Therefore, the given list of numbers does not form an AP.

(ii) Here, we have

$a_2 - a_1 = \frac{5}{2} - 2 = \frac{5-4}{2} = \frac{1}{2}$,

$a_3 - a_2 = 3 - \frac{5}{2} = \frac{6-5}{2} = \frac{1}{2}$,

$a_4 - a_3 = \frac{7}{2} - 3 = \frac{7-6}{2} = \frac{1}{2}$ and so on.

Since, the difference of any two consecutive terms is same. Therefore, the given list of numbers forms an AP and its common difference (d) is $\frac{1}{2}$.

Now, next three terms of this AP are,

$a_5 = a_4 + d = \frac{7}{2} + \frac{1}{2}$

$[\because a_5 = a + 4d = a + 3d + d = a_4 + d]$

$= \frac{7+1}{2} = \frac{8}{2} = 4$

$a_6 = a_5 + d = 4 + \frac{1}{2} = \frac{9}{2}$

and $a_7 = a_6 + d = \frac{9}{2} + \frac{1}{2} = \frac{9+1}{2} = \frac{10}{2} = 5$

(iii) Here, we have

$a_2 - a_1 = -3.2 - (-1.2) = -3.2 + 1.2 = -2$

$a_3 - a_2 = -5.2 - (-3.2) = -5.2 + 3.2 = -2$,

$a_4 - a_3 = -7.2 - (-5.2) = -7.2 + 5.2 = -2$

and so on.

Since, the difference of any two consecutive terms is same. Therefore, the given list of numbers forms an AP and its common difference (d) is -2.

Now, next three terms of this AP are,

$a_5 = a_4 + d = -7.2 + (-2) = -9.2$

$a_6 = a_5 + d = -9.2 + (-2) = -11.2$

and $a_7 = a_6 + d = -11.2 + (-2) = -13.2$

(iv) Do same as Part (iii).

Ans. Yes, $d = 4$ and next three terms are 6, 10, 14

(v) Here, we have

$a_2 - a_1 = 3 + \sqrt{2} - 3 = \sqrt{2}$,

$a_3 - a_2 = 3 + 2\sqrt{2} - (3 + \sqrt{2}) = \sqrt{2}$,

$a_4 - a_3 = 3 + 3\sqrt{2} - (3 + 2\sqrt{2}) = \sqrt{2}$ and so on.

Since, the difference of any two consecutive terms is same. Therefore, the given list of numbers forms an AP and its common difference (d) is $\sqrt{2}$.

Now, next three terms of this AP are,

$a_5 = a_4 + d = 3 + 3\sqrt{2} + \sqrt{2} = 3 + 4\sqrt{2}$

$a_6 = a_5 + d = 3 + 4\sqrt{2} + \sqrt{2} = 3 + 5\sqrt{2}$

and $a_7 = a_6 + d = 3 + 5\sqrt{2} + \sqrt{2} = 3 + 6\sqrt{2}$

(vi) Here, we have $a_2 - a_1 = 0.22 - 0.2 = 0.02$

and $a_3 - a_2 = 0.222 - 0.22 = 0.002$

Since, $a_2 - a_1 \neq a_3 - a_2$, therefore the given list of numbers does not form an AP.

(vii) Do same as Part (i).

Ans. Yes, $d = -4$ and next three terms are $-16, -20, -24$.

(viii) Here, we have

$$a_2 - a_1 = -\frac{1}{2} - \left(-\frac{1}{2}\right) = -\frac{1}{2} + \frac{1}{2} = 0,$$
$$a_3 - a_2 = -\frac{1}{2} - \left(-\frac{1}{2}\right) = -\frac{1}{2} + \frac{1}{2} = 0,$$
$$a_4 - a_3 = -\frac{1}{2} - \left(-\frac{1}{2}\right) = -\frac{1}{2} + \frac{1}{2} = 0$$

and so on.

Since, the difference of any two consecutive terms is same. Therefore, the given list of numbers forms an AP and its common difference (d) is 0.

Now, next three terms of this AP are

$$a_5 = a_4 + d = -\frac{1}{2} + 0 = -\frac{1}{2}$$
$$a_6 = a_5 + d = -\frac{1}{2} + 0 = -\frac{1}{2}$$
and $a_7 = a_6 + d = -\frac{1}{2} + 0 = -\frac{1}{2}$

(ix) Do same as Part (i). **Ans.** Not an AP.

(x) Here, we have

$$a_2 - a_1 = 2a - a = a,$$
$$a_3 - a_2 = 3a - 2a = a,$$
$a_4 - a_3 = 4a - 3a = a$ and so on.

Since, the difference of any two consecutive terms is same. Therefore, the given list of numbers forms an AP and its common difference (d) is a.

Now, next three terms of this AP are,

$$a_5 = a_4 + d = 4a + a = 5a$$
$$a_6 = a_5 + d = 5a + a = 6a$$
and $a_7 = a_6 + d = 6a + a = 7a$

(xi) Here, we have

$$a_2 - a_1 = a^2 - a = a(a-1),$$
and $a_3 - a_2 = a^3 - a^2 = a^2(a-1)$

Since, $a_2 - a_1 \neq a_3 - a_2$

Therefore, the given list of numbers does not form an AP.

(xii) Here, we have

$$a_2 - a_1 = \sqrt{8} - \sqrt{2} = 2\sqrt{2} - \sqrt{2} = \sqrt{2},$$
$$a_3 - a_2 = \sqrt{18} - \sqrt{8} = 3\sqrt{2} - 2\sqrt{2} = \sqrt{2},$$
$a_4 - a_3 = \sqrt{32} - \sqrt{18} = 4\sqrt{2} - 3\sqrt{2} = \sqrt{2}$ and so on.

Since, the difference of any two consecutive terms is same. Therefore, the given list of numbers forms an AP and its common difference (d) is $\sqrt{2}$.

Now, next three terms of this AP are,

$$a_5 = a_4 + d = 4\sqrt{2} + \sqrt{2} = 5\sqrt{2} = \sqrt{50}$$
$$a_6 = a_5 + d = 5\sqrt{2} + \sqrt{2} = 6\sqrt{2} = \sqrt{72}$$
and $a_7 = a_6 + d = 6\sqrt{2} + \sqrt{2} = 7\sqrt{2} = \sqrt{98}$

(xiii) Here, we have

$$a_2 - a_1 = \sqrt{6} - \sqrt{3} = \sqrt{3 \times 2} - \sqrt{3} = \sqrt{3}(\sqrt{2} - 1)$$
and $a_3 - a_2 = \sqrt{9} - \sqrt{6}$
$$= 3 - \sqrt{3 \times 2} = \sqrt{3}(\sqrt{3} - \sqrt{2})$$

Since, $a_2 - a_1 \neq a_3 - a_2$

Therefore, the given list of numbers does not form an AP.

(xiv) Here, we have $a_2 - a_1 = 3^2 - 1^2 = 9 - 1 = 8$

and $a_3 - a_2 = 5^2 - 3^2 = 25 - 9 = 16$

Since, $a_2 - a_1 \neq a_3 - a_2$

Therefore, the given list of numbers does not form an AP.

(xv) Here, we have $a_2 - a_1 = 5^2 - 1^2 = 25 - 1 = 24$,

$$a_3 - a_2 = 7^2 - 5^2 = 49 - 25 = 24,$$
and $a_4 - a_3 = 73 - 7^2 = 73 - 49 = 24$ and so on.

Since, the difference of any two consecutive terms is same. Therefore, the given list of numbers forms an AP and its common difference (d) is 24.

Now, next three terms of this AP are,

$$a_5 = a_4 + d = 73 + 24 = 97$$
$$a_6 = a_5 + d = 97 + 24 = 121$$
and $a_7 = a_6 + d = 121 + 24 = 145$

EXERCISE 5.2

Q1. Fill in the blanks in the following table, given that a is the first term, d the common difference and a_n the nth term of the AP.

	a	d	n	a_n
(i)	7	3	8	...
(ii)	−18	...	10	0
(iii)	...	−3	18	−5
(iv)	−18.9	2.5	...	3.6
(v)	3.5	0	105	...

Sol. (i) Here, $a = 7, d = 3$ and $n = 8$

The nth term of an AP is $a_n = a + (n-1)d$

$\therefore \quad a_8 = 7 + (8-1)3 = 7 + 7 \times 3 = 28$

(ii) Here, $a = -18$, $n = 10$ and $a_n = 0$

We know that the nth term of an AP is

$$a_n = a + (n-1)d$$

On putting the given values, we get

$$0 = -18 + (10-1)d$$
$$\Rightarrow \quad 18 = 9d \Rightarrow d = \frac{18}{9} = 2$$

(iii) Here, $n = 18$, $d = -3$ and $a_n = -5$

We know that the nth term of an AP is

$$a_n = a + (n-1)d$$

On putting the given values, we get

$$-5 = a + (18-1)(-3)$$

$$\Rightarrow \quad a = -5 + 51 = 46$$

(iv) Here, $a = -18.9$, $d = 2.5$ and $a_n = 3.6$

We know that the nth term of an AP is

$$a_n = a + (n-1)d$$

On putting the given values, we get

$$3.6 = -18.9 + (n-1)2.5$$

$$\Rightarrow 3.6 + 18.9 = (n-1)2.5$$

$$\Rightarrow \quad n - 1 = 9 \Rightarrow n = 9 + 1 = 10$$

(v) Do same as part (i) **Ans.** $a_n = 3.5$

Q2. Choose the correct choice in the following and justify.

(i) 30th term of the AP 10, 7, 4, ... is

(a) 97 (b) 77 (c) -77 (d) -87

(ii) 11th term of the AP : $-3, -\frac{1}{2}, 2, \ldots$ is

(a) 28 (b) 22 (c) -38 (d) $-48\frac{1}{2}$

Sol. (i) (c) Here, first term $(a) = 10$,

and common difference $(d) = 7 - 10 = -3$

We know that the nth term of an AP is

$$a_n = a + (n-1)d$$

$$\therefore \quad a_{30} = 10 + (30-1)(-3) = -77$$

(ii) (b) Do same as part (i) **Ans.** $a_{11} = 22$

Q3. In the following APs, find the missing terms of the boxes.

(i) $2, \square, 26$ (ii) $\square, 13, \square, 3$

(iii) $5, \square, \square, 9\frac{1}{2}$ (iv) $-4, \square, \square, \square, \square, 6$

(v) $\square, 38, \square, \square, \square, -22$

Sol. (i) We know that $a, a+d, a+2d, \ldots$ is the general form of an AP. We have, $a = 2$ and $a + 2d = 26$

$$\Rightarrow 2 + 2d = 26 \qquad [\because a = 2]$$

$$\Rightarrow \quad 2d = 26 - 2 = 24$$

$$\Rightarrow \quad d = \frac{24}{2} = 12$$

Hence, the missing term $= a + d = 2 + 12 = 14$

(ii) We know that $a, a+d, a+2d, a+3d, \ldots$ is the general form of an AP.

$\therefore$ We have, $a + d = 13$ ...(i)

and $a + 3d = 3$ ...(ii)

On subtracting Eq. (i) from Eq. (ii), we get

$$2d = -10$$

$$\Rightarrow \quad d = -5$$

On putting $d = -5$ in Eq. (ii), we get

$$a + 3(-5) = 3$$

$$\Rightarrow \quad a = 18$$

Hence, the missing terms are $a = 18$ and

$$a + 2d = 18 + 2(-5) = 18 - 10 = 8$$

(iii) We know that $a, a+d, a+2d, a+3d, \ldots$ is the general form of an AP.

$\therefore$ We have, $a = 5$

and $a + 3d = 9\frac{1}{2} = \frac{19}{2}$

On subtracting Eq. (i) from Eq. (ii), we get

$$3d = \frac{19}{2} - 5 = \frac{19-10}{2} = \frac{9}{2}$$

$$\Rightarrow \quad 3d = \frac{9}{2}$$

$$\Rightarrow \quad d = \frac{1}{3} \times \frac{9}{2} = \frac{3}{2}$$

Hence, the missing terms are

$$a + d = 5 + \frac{3}{2} = \frac{10+3}{2} = \frac{13}{2} = 6\frac{1}{2}$$

and $a + 2d = 5 + 2 \times \frac{3}{2} = 5 + 3 = 8$

(iv) Do same as part (iii) **Ans.** $-2, 0, 2, 4$

(v) Do same as part (ii) **Ans.** $53, 23, 8, -7$

Q4. Which term of the AP : 3, 8, 13, 18, ... is 78?

Sol. Do same as Example 3 of Topic 2.

Ans. 16th term is 78.

Q5. Find the number of terms in each of the following APs :

(i) 7, 13, 19, ..., 205 (ii) $18, 15\frac{1}{2}, 13, \ldots, -47$

Sol. Do same as Example 2 of Topic 2.

Ans. (i) 34 terms (ii) 27 terms

Q6. Check whether -150 is a term of the AP : 11, 8, 5, 2, ...

Sol. Here, $a = 11$ and $d = 8 - 11 = -3$

Assume that, -150 be the nth term of the given AP.

We know that the nth term of an AP is

$$a_n = a + (n-1)d$$

$$\Rightarrow \quad -150 = 11 + (n-1)(-3)$$

$$\Rightarrow \quad n - 1 = \frac{161}{3}$$

$$\Rightarrow \quad n = \frac{161}{3} + 1 = \frac{164}{3}$$

But n should be a positive integer.

So, -150 is not a term of the given AP.

Q7. Find the 31st term of an AP, whose 11th term is 38 and the 16th term is 73.

Sol. Let a be the first term and d be the common difference of given AP.

We know that the nth term of an AP is $a_n = a + (n-1)d$

$\therefore$ 11th term, $a_{11} = a + 10d = 38$ ...(i)

$[\because a_{11} = 38, \text{given}]$

and 16th term, $a_{16} = a + 15d = 73$ ...(ii)

$[\because a_{16} = 73, \text{given}]$

On subtracting Eq. (i) from Eq. (ii), we get

$$5d = 35 \Rightarrow d = \frac{35}{5} = 7$$

On putting the value of d in Eq. (i), we get

$$a + 10 \times 7 = 38 \Rightarrow a = 38 - 70 = -32$$

Now, 31st term, $a_{31} = a + 30d$

$$= -32 + 30 \times 7 = -32 + 210 = 178$$

Q8. An AP consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term.

Sol. Let a be the first term and d be the common difference of given AP.

Now, as the nth term of an AP is $a_n = a + (n-1)d$

$\therefore$ $a_3 = a + 2d = 12$ $[\because a_3 = 12, \text{given}]$ ...(i)

and $a_{50} = a + 49d = 106$ ...(ii)

$[\because a_{50} = 106, \text{given}]$

On subtracting Eq. (i) from Eq. (ii), we get

$$47d = 94$$

$$\Rightarrow d = \frac{94}{47} = 2$$

On putting the value of d in Eq. (i), we get

$$a + 2 \times 2 = 12 \Rightarrow a = 12 - 4 = 8$$

Now, 29th term, $a_{29} = a + (29-1)d$

$$= 8 + 28 \times 2 = 8 + 56 = 64$$

Q9. If the 3rd and 9th terms of an AP are 4 and −8 respectively, then which term of this AP is zero?

Sol. Let a be the first term and d be the common difference of given AP.

$\because$ The nth term of an AP is $a_n = a + (n-1)d$

$\therefore$ $a_3 = a + 2d = 4$ $[\because a_3 = 4, \text{given}]$ ...(i)

and $a_9 = a + 8d = -8$ $[\because a_9 = -8, \text{given}]$ ...(ii)

On subtracting Eq. (i) from Eq. (ii), we get

$$6d = -12$$

$$\Rightarrow d = \frac{-12}{6} = -2$$

On putting the value of d in Eq. (i), we get

$$a + 2 \times (-2) = 4 \Rightarrow a - 4 = 4 \Rightarrow a = 4 + 4 = 8$$

Let the nth term of this AP be zero, i.e.

$$a_n = 0 \Rightarrow a + (n-1)d = 0$$

$\Rightarrow$ $8 + (n-1)(-2) = 0$ $[\because a = 8, d = -2]$

$$\Rightarrow n - 1 = \frac{-8}{-2} = 4$$

$$\Rightarrow n = 4 + 1 = 5$$

Hence, 5th term of this AP is zero.

Q10. The 17th term of an AP exceeds its 10th term by 7. Find the common difference. CBSE 2019

Sol. Let a be the first term and d be the common difference of given AP.

Now, according to the question, $a_{17} = a_{10} + 7$

$$\Rightarrow a_{17} - a_{10} = 7$$

$\Rightarrow$ $[a + (17-1)d] - [a + (10-1)d] = 7$

$[\because a_n = a + (n-1)d]$

$$\Rightarrow (a + 16d) - (a + 9d) = 7$$

$$\Rightarrow 7d = 7$$

$$\Rightarrow d = 1$$

Hence, the common difference of this AP is 1.

Q11. Which term of the AP : 3, 15, 27, 39, ... will be 132 more than its 54th term?

Sol. Do same as Example 5 of Topic 2.

Ans. 65th term is 132 more than the 54th term of this AP.

Q12. Two AP's have the same common difference. The difference between their 100th terms is 100. What is the difference between their 1000th terms?

Sol. Let the two APs be $a_1, a_2, a_3, \dots, a_n \dots$ and $b_1, b_2, b_3, \dots, b_n \dots$

Also, let d be the same common difference of two APs.

Then, the nth term of first AP is

$$a_n = a_1 + (n-1)d$$

and the nth term of second AP is

$$b_n = b_1 + (n-1)d$$

Now, $a_n - b_n = [a_1 + (n-1)d] - [b_1 + (n-1)d]$

$$\Rightarrow a_n - b_n = a_1 - b_1, \forall n \in N$$

$\therefore$ $a_{100} - b_{100} = a_1 - b_1 = 100$ [given]

So, $a_{1000} - b_{1000} = a_1 - b_1$

$\Rightarrow$ $a_{1000} - b_{1000} = 100$ $[\because a_1 - b_1 = 100]$

Hence, the difference between their 1000th terms is also 100 for all $n \in N$.

Q13. How many three-digit numbers are divisible by 7?

Sol. We know that 105 is the first and 994 is the last three-digit numbers divisible by 7. Thus, we have to determine the number of terms in the list 105, 112, 119, ..., 994.

Clearly, the successive difference of the terms is same.

So, above list of numbers forms an AP, with first term $(a) = 105$

and common difference $(d) = 112 - 105 = 7$

Let there be n terms in the AP.

Then, nth term $= 994$

$\Rightarrow \quad 105 + (n-1)7 = 994 \quad [\because a_n = a + (n-1)d]$

$\Rightarrow \quad 7(n-1) = 994 - 105$

$\Rightarrow \quad 7(n-1) = 889$

$\Rightarrow \quad n = 127 + 1 = 128$

Hence, there are 128 numbers of three-digit which are divisible by 7.

Q 14. How many multiples of 4 lie between 10 and 250?

Sol. We know that 12 is the first integer between 10 and 250, which is a multiple of 4. Also, when we divide 250 by 4, the remainder is 2. Therefore, $250 - 2 = 248$ is the greatest integer divisible by 4 and lying between 10 and 250.

Thus, we have to find the number of terms in an AP, whose first term $(a) = 12$, last term $(l) = 248$ and common difference $(d) = 4$.

Let the nth term of this AP be $a_n = 248$

Then, $\quad 12 + (n-1)4 = 248 \quad [\because a_n = a + (n-1)d]$

$\Rightarrow \quad 4(n-1) = 248 - 12$

$\Rightarrow \quad 4(n-1) = 236 \Rightarrow n = 59 + 1 = 60$

Hence, there are 60 multiples of 4 lying between 10 and 250.

Q 15. For what value of n, the nth terms of two AP's : 63, 65, 67, ... and 3, 10, 17,... are equal?

Sol. Given, APs are 63, 65, 67, ... and 3, 10, 17, ...

Here, first term of first AP $(a_1) = 63$

common difference of first AP $(d_1) = 65 - 63 = 2$

first term of second AP $(a_2) = 3$

and common difference of second AP $(d_2) = 10 - 3 = 7$

According to the question, nth term of both AP's are equal.

$\therefore \quad 63 + (n-1)2 = 3 + (n-1)7 \; [\because a_n = a + (n-1)d]$

$\Rightarrow \quad 7(n-1) - 2(n-1) = 63 - 3$

$\Rightarrow \quad (n-1)(7-2) = 60$

$\Rightarrow \quad (n-1) = \frac{60}{5} = 12$

$\Rightarrow \quad n = 12 + 1 = 13$

Hence, the 13th term of the two given AP's are equal.

Q 16. Determine the AP whose 3rd term is 16 and the 7th term exceeds the 5th term by 12.

Sol. Let a be the first term and d be the common difference of given AP.

Given that the third term of the AP is

$a_3 = 16$

$\Rightarrow \quad a + 2d = 16 \quad [\because a_n = a + (n-1)d]$ ...(i)

Also, it is given that

7th term of an AP $= 12 +$ 5th term of an AP i.e.

$a_7 = 12 + a_5$

$\Rightarrow \quad a_7 - a_5 = 12$

$\Rightarrow \quad (a + 6d) - (a + 4d) = 12$

$\Rightarrow \quad 2d = 12 \Rightarrow d = 6$

On putting $d = 6$ in Eq. (i), we get

$a + 2 \times 6 = 16$

$\Rightarrow \quad a = 16 - 12 = 4$

We know that general form of an AP is

$a, a+d, a+2d, a+3d, \dots$.

Then, the required AP is

$4, 4+6, 4+2\times 6, 4+3\times 6, \dots$, i.e. $4, 10, 16, 22, \dots$.

Q 17. Find the 20th term from the last term of the AP : 3, 8, 13, ... ,253.

Sol. Do same as Example 10 of Topic 2. **Ans. 158**

Q 18. The sum of the 4th and 8th terms of an AP is 24 the sum of the 6th and 10th terms is 44. Find th first three terms of the AP.

Sol. Let a be the first term and d be the common differenc given AP.

Given, $\quad a_4 + a_8 = 24$

$\therefore \quad (a + 3d) + (a + 7d) = 24 \quad [\because a_n = a + (n-$

$\Rightarrow \quad a + 5d = 12$

[dividing both sides

and $\quad a_6 + a_{10} = 44$

$\Rightarrow \quad (a + 5d) + (a + 9d) = 44$

$\Rightarrow \quad 2a + 14d = 44$

$\Rightarrow \quad a + 7d = 22$

[dividing both sides

On subtracting Eq. (i) from Eq. (ii), we get

$2d = 10 \Rightarrow d = 5$

On putting $d = 5$ in Eq. (i), we get

$a + 25 = 12 \Rightarrow a = -13$

Hence, the first three terms are

$a, (a+d)$ and $(a+2d)$

i.e. $-13, (-13+5)$ and $(-13 + 2\times 5)$

i.e. $-13, -8$ and -3.

Q 19. Subba Rao started work in 1995 at an annual sa of ₹ 5000 and received an increment of ₹ 200 eac year. In which year, did his income reach ₹ 7000?

Sol. The annual salary received by Subba Rao in the years 1995, 1996, 1997, ... are ₹ 5000, ₹ 5200, ₹ 5400, ..., respectively.

Here, the list of numbers 5000, 5200, 5400, ..., forms an AP with first term $(a) = 5000$

and common difference $(d) = 200$.

Let nth term of this AP be $a_n = 7000$

$\Rightarrow \quad 7000 = a + (n-1)d$ $\quad [\because a_n = a + (n-1)d]$

$\Rightarrow \quad 7000 = 5000 + (n-1)(200)$

$\Rightarrow \quad 200(n-1) = 7000 - 5000 = 2000$

$\Rightarrow \quad n - 1 = \frac{2000}{200} = 10 \Rightarrow n = 10 + 1 = 11$

Thus, in 11th year of his service, Subba Rao received an annual salary of ₹ 7000.

Q 20. Ramkali saved ₹ 5 in the first week of a year and then increased her weekly savings by ₹ 1.75. If in the nth week, her weekly savings becomes ₹ 20.75, find n.

Sol. Ramkali's savings in the subsequent weeks are respectively ₹ 5, ₹ 5 + ₹ 1.75, ₹ 5 + 2 × ₹ 1.75, ₹ 5 + 3 × ₹ 1.75,...

i.e. ₹ 5, ₹ 6.75, ₹ 8.50, ₹ 10.25,...

Clearly, Ramkali's savings forms an AP with first term $(a) = 5$ and common difference $(d) = 1.75$

Thus, in nth week, her savings will be ₹ $5 + (n-1) \times 1.75$

$\Rightarrow \quad 5 + (n-1) \times 1.75 = 20.75$

[$\because$ in the nth week saving amount = ₹ 20.75]

$\Rightarrow \quad (n-1) \times 1.75 = 20.75 - 5 = 15.75$

$\Rightarrow \quad n - 1 = \frac{15.75}{1.75} = 9$

$\Rightarrow \quad n = 9 + 1 = 10$

EXERCISE 5.3

Q 1. Find the sum of the following APs :

(i) 2, 7, 12, ..., to 10 terms

(ii) $-37, -33, -29, \ldots$, to 12 terms

(iii) 0.6, 1.7, 2.8, ..., to 100 terms

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \ldots$, to 11 terms

Sol. (i) Given, AP is 2, 7, 12, ..., to 10 terms.

Here, $a = 2, d = 7 - 2 = 5$ and $n = 10$

$\because$ Sum of first n terms of an AP,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$\therefore$ On putting $a = 2, d = 5$ and $n = 10$, we get

$$S_{10} = \frac{10}{2}[2 \times 2 + (10-1)5]$$

$$= 5(4 + 9 \times 5) = 5(4 + 45) = 5 \times 49 = 245$$

(ii) Given, AP is $-37, -33, -29, \ldots$ to 12 terms.

Here, $a = -37$,

$d = -33 - (-37) = -33 + 37 = 4$ and $n = 12$

Do same as Part (i). **Ans.** -180

(iii) Given, AP is 0.6, 1.7, 2.8, ... to 100 terms.

Here, $a = 0.6$, $d = 1.7 - 0.6 = 1.1$ and $n = 100$

Do same as Part (i). **Ans.** 5505

(iv) Given, AP is $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \ldots$ to 11 terms.

Here, $a = \frac{1}{15}$, $d = \frac{1}{12} - \frac{1}{15} = \frac{5-4}{60} = \frac{1}{60}$ and $n = 11$

Do same as Part (i). **Ans.** $\frac{33}{20}$

Q 2. Find the sums given below :

(i) $7 + 10\frac{1}{2} + 14 + \ldots + 84$

(ii) $34 + 32 + 30 + \ldots + 10$

(iii) $-5 + (-8) + (-11) + \ldots + (-230)$

Sol. (i) The given numbers are $7, 10\frac{1}{2}, 14, \ldots 84$

$\because 10\frac{1}{2} - 7 = 14 - 10\frac{1}{2} = \ldots = \frac{7}{2}$

$\therefore$ The given numbers forms an AP.

Here, first term, $a = 7$,

common difference, $d = 10\frac{1}{2} - 7 = 3\frac{1}{2} = \frac{7}{2}$

and last term, $l = a_n = 84$

$\because \quad a_n = a + (n-1)d$

$\therefore \quad 84 = 7 + (n-1)\frac{7}{2}$ $\quad \left[\because a = 7 \text{ and } d = \frac{7}{2}\right]$

$\Rightarrow \quad n - 1 = 22 \Rightarrow n = 23$

$\because$ Sum of n terms of an AP, $S_n = \frac{n}{2}(a + l)$

$\therefore$ Sum of 23 terms $(S_{23}) = \frac{23}{2}(7 + 84) = \frac{23}{2} \times 91$

$$= \frac{2093}{2} = 1046\frac{1}{2}$$

(ii) Do same as Part (i). **Ans.** 286

(iii) Do same as Part (i). **Ans.** -8930

Q 3. In an AP :

(i) given $a = 5, d = 3, a_n = 50$, find n and S_n.

(ii) given $a = 7, a_{13} = 35$, find d and S_{13}.

(iii) given $a_{12} = 37, d = 3$, find a and S_{12}.

(iv) given $a_3 = 15, S_{10} = 125$, find d and a_{10}.

(v) given $d = 5, S_9 = 75$, find a and a_9.

(vi) given $a = 2, d = 8, S_n = 90$, find n and a_n.

(vii) given $a = 8, a_n = 62, S_n = 210$, find n and d.

(viii) given $a_n = 4, d = 2, S_n = -14$, find n and a.

(ix) given $a = 3, n = 8, S = 192$, find d.

(x) given $l = 28, S = 144$, and there are total 9 terms. Find a.

Sol. (i) Here, $a=5,\ d=3$ and $a_n=50$

$\because$ $a_n=a+(n-1)d$

$\therefore$ $5+(n-1)3=50$

$\Rightarrow$ $(n-1)=\frac{45}{3}=15 \Rightarrow n=15+1=16$

On putting $n=16$, $a=5$ and $l=a_n=50$ in $S_n=\frac{n}{2}(a+l)$, we get

$$S_{16}=\frac{16}{2}(5+50)=8\times 55=440$$

Hence, $n=16$ and $S_n=440$.

(ii) Here, $a=7$ and $a_{13}=35$

$\because$ $a_{13}=35$

$\therefore$ $a+12d=35$ $[\because a_n=a+(n-1)d]$

$\Rightarrow$ $7+12d=35$ $[\because a=7]$

$\Rightarrow$ $d=\frac{28}{12}=\frac{7}{3}$

On putting $n=13, a=7$ and $l=a_{13}=35$ in $S_n=\frac{n}{2}(a+l)$, we get

$$S_{13}=\frac{13}{2}(7+35)=\frac{13}{2}\times 42=13\times 21=273$$

Hence, $d=\frac{7}{3}$ and $S_{13}=273$

(iii) Here, $a_{12}=37$ and $d=3$

Then, $a_{12}=37$

$\Rightarrow$ $a+11d=37$ $[\because a_n=a+(n-1)d]$

$\Rightarrow$ $a+11(3)=37$ $[\because d=3]$

$\Rightarrow$ $a=37-33=4$

On putting $n=12, a=4$ and $l=a_{12}=37$ in $S_n=\frac{n}{2}(a+l)$, we get

$$S_{12}=\frac{12}{2}(4+37)=6\times 41=246$$

Hence, $a=4$ and $S_{12}=246$.

(iv) Here, $a_3=15$ and $S_{10}=125$

$\because$ $a_3=15$

$\therefore a+2d=15$ ...(i)

$[\because a_n=a+(n-1)d]$

Also, $S_{10}=125$

$\therefore$ $\frac{10}{2}[2a+(10-1)d]=125 \left[\because S_n=\frac{n}{2}\{2a+(n-1)d\}\right]$

$\Rightarrow$ $2a+9d=25$ [dividing by 5] ...(ii)

On multiplying Eq. (i) by 2 and then subtracting Eq. (ii) from it, we get

$2(a+2d)-(2a+9d)=2\times 15-25$

$\Rightarrow$ $4d-9d=30-25 \Rightarrow -5d=5$

$\Rightarrow$ $d=-\frac{5}{5}=-1$

Now, $a_{10}=a+9d$

$=(a+2d)+7d$

$=15+7(-1)$ [from Eq.(

$=15-7=8$

Hence, $d=-1$ and $a_{10}=8$.

(v) Here, $d=5$ and $S_9=75$

$\because$ $S_9=75$

$\therefore$ $\frac{9}{2}[2a+(9-1)5]=75$

$\left[\because S_n=\frac{n}{2}\{2a+(n-1)\right.$

$\Rightarrow$ $\frac{9}{2}(2a+40)=75$

$\Rightarrow$ $9a+180=75$

$\Rightarrow$ $a=\frac{-105}{9}=\frac{-35}{3}$

Now, $a_9=a+8d=\frac{-35}{3}+8\times 5$

$[\because a_n=a+(n-$

$=\frac{-35+120}{3}=\frac{85}{3}$

Hence, $a=\frac{-35}{3}$ and $a_9=\frac{85}{3}$.

(vi) Here, $a=2, d=8$ and $S_n=90$

$\because$ $S_n=90$

$\therefore$ $\frac{n}{2}[2\times 2+(n-1)8]=90 \left[\because S_n=\frac{n}{2}\{2a+(n-1\right.$

$\Rightarrow$ $\frac{n}{2}(4+8n-8)=90$

$\Rightarrow$ $\frac{n}{2}(8n-4)=90$

$\Rightarrow$ $n(4n-2)=90$

$\Rightarrow$ $4n^2-2n-90=0$

$\Rightarrow$ $2n^2-n-45=0$

$\Rightarrow$ $(n-5)(4n+18)=0$

$\Rightarrow$ $n=5$ or $\frac{-9}{2}$

Since, n cannot be negative.

$\therefore$ $n=5$

Now, $a_5=2+(5-1)8$

$[\because a_n=a+(n-1)d, a=2$ and $d=$

$=2+32=34$

Hence, $n=5$ and $a_n=34$.

(vii) Here, $a=8, a_n=62$ and $S_n=210$

$\because$ $S_n=210$

$\therefore$ $\frac{n}{2}(a+a_n)=210$ $\left[\because S_n=\frac{n}{2}(a+a_n\right.$

$\Rightarrow \quad \frac{n}{2}(8+62)=210 \qquad [\because a=8 \text{ and } a_n=62]$

$\Rightarrow \quad n=210\times\frac{2}{70}=3\times2=6$

Thus, $\quad a_n=62$

$\Rightarrow \quad a_6=62$

$\Rightarrow \quad a+5d=62 \qquad [\because a_n=a+(n-1)d]$

$\Rightarrow \quad 8+5d=62 \qquad [\because a=8]$

$\Rightarrow \quad 5d=62-8=54$

$\Rightarrow \quad d=\frac{54}{5}$

Hence, $d=\frac{54}{5}$ and $n=6$.

(viii) Here, $a_n=4, d=2$ and $S_n=-14$

$\because \quad a_n=4$

$\therefore a+(n-1)\,2=4 \qquad [\because a_n=a+(n-1)d]$

$\Rightarrow \quad a=4-2(n-1) \qquad \ldots(i)$

Also, $\quad S_n=-14$

$\therefore \quad \frac{n}{2}(a+a_n)=-14 \qquad \left[\because S_n=\frac{n}{2}(a+a_n)\right]$

$\Rightarrow \quad n(a+4)=-28 \qquad [\because a_n=4]$

$\Rightarrow n[4-2(n-1)+4]=-28 \qquad$ [from Eq. (i)]

$\Rightarrow \quad n(4-2n+2+4)=-28$

$\Rightarrow \quad n(-n+5)=-14$

[dividing both sides by 2]

$\Rightarrow \quad -n^2+5n=-14$

$\Rightarrow n^2-5n-14=0$

$\Rightarrow \quad (n-7)(n+2)=0 \quad$ [by factorisation method]

$\Rightarrow n=7$ or $n=-2$

Since, n cannot be negative.

$\therefore \quad n=7$

On putting $n=7$ in Eq. (i), we get

$a=4-2(7-1)=4-2\times6=4-12=-8$

Hence, $n=7$ and $a=-8$.

(ix) Here, $a=3, n=8$ and $S=192$

$\because \quad S=\frac{n}{2}[2a+(n-1)d]$

$\therefore \quad 192=\frac{8}{2}[2\times3+(8-1)d]$

$\Rightarrow 192=4(6+7d)$

$\Rightarrow \quad 48=6+7d \qquad$ [dividing both sides by 4]

$\Rightarrow \quad d=\frac{42}{7}=6$

Hence, $\quad d=6$

(x) Here, $l=28, S=144$

and $\quad n=9$

$\because \quad S=144$

$\therefore \quad \frac{n}{2}(a+l)=144 \quad \left[\because S=\frac{n}{2}(\text{first term}+\text{last term})\right]$

$\Rightarrow \quad \frac{9}{2}(a+28)=144 \Rightarrow a+28=32$

Hence, $\quad a=4$

Q4. How many terms of the AP : 9, 17, 25, ... must be taken to give a sum of 636?

Sol. Do same as Example 6 of Topic 3.

Ans. The sum of 12 terms is 636.

Q5. The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

Sol. Let n be the number of terms in the given AP and d be the common difference.

Given, first term $(a)=5$, last term $(l)=45$ and sum of n terms $(S_n)=400$.

$\because \quad S_n=\frac{n}{2}(a+l)$

$\Rightarrow \quad 400=\frac{n}{2}(5+45)$

$\Rightarrow \quad n=\frac{400\times2}{50}=8\times2=16$

Now, $\quad l=45$

$\Rightarrow \quad a+(n-1)d=45 \qquad [\because l=a_n=a+(n-1)d]$

$\Rightarrow \quad 5+(16-1)d=45 \qquad [\because a=5, n=16]$

$\Rightarrow \quad d=\frac{40}{15}=\frac{8}{3}$

$\therefore$ The number of terms is 16 and the common difference is $\frac{8}{3}$.

Q6. The first and the last terms of an AP are 17 and 350 respectively. If the common difference is 9, then how many terms are there and what is their sum?

Sol. Given, first term, $a=17$, last term, $l=350$ and common difference, $d=9$.

Let there are n terms in the given AP. Then, $l=a_n=350$

$\Rightarrow \quad a+(n-1)d=350 \qquad [\because a_n=a+(n-1)d]$

$\Rightarrow \quad 17+(n-1)9=350 \qquad [\because a=17 \text{ and } d=9]$

$\Rightarrow \quad n-1=\frac{333}{9}=37$

$\Rightarrow \quad n=37+1=38$

On putting $a=17$, $l=350$ and $n=38$ in $S_n=\frac{n}{2}(a+l)$, we get

$$S_{38}=\frac{38}{2}(17+350)=19(367)=6973$$

Hence, there are 38 terms in the AP having their sum 6973.

Q7. Find the sum of first 22 terms of an AP, in which $d = 7$ and 22nd term is 149.

Sol. Given, $d = 7$ and $a_{22} = 149$

$\because \quad a_n = a + (n-1)d$

$\therefore \quad a + (22-1)d = 149$

$\Rightarrow \quad a + 21 \times 7 = 149$

$\Rightarrow \quad a = 149 - 147 = 2$

On putting $a = 2, n = 22$ and $d = 7$ in

$S_n = \frac{n}{2}[2a + (n-1)d]$, we get

$S_{22} = \frac{22}{2}[2 \times 2 + (22-1)7]$

$= 11(4 + 21 \times 7) = 11(4 + 147) = 11 \times 151 = 1661$

Hence, the sum of first 22 terms is 1661.

Q8. Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

Sol. Given, second term, $a_2 = 14$ and third term, $a_3 = 18$

$\because \quad a_n = a + (n-1)d$

We have, $\quad a + d = 14 \quad \ldots(i)$

and $\quad a + 2d = 18 \quad \ldots(ii)$

On subtracting Eq. (i) from Eq. (ii), we get

$d = 4$

On putting $d = 4$ in Eq. (i), we get

$a + 4 = 14$

$\Rightarrow \quad a = 14 - 4 = 10$

$\because$ Sum of first n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore$ Sum of first 51 terms, $S_{51} = \frac{51}{2}[2 \times 10 + (51-1) \times 4]$

$[\because a = 10$ and $d = 4]$

$= \frac{51}{2}[20 + 50 \times 4] = \frac{51}{2}(20 + 200) = \frac{51}{2} \times 220$

$= 51 \times 110 = 5610$

Q9. If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, then find the sum of first n terms.

Sol. Given, $S_7 = 49$ and $S_{17} = 289$.

Let a be the first term and d be the common difference of given AP. Then,

$S_7 = 49$

$\Rightarrow \quad \frac{7}{2}[2a + 6d] = 49 \Rightarrow a + 3d = 7 \quad \ldots(i)$

and $\quad S_{17} = 289$

$\Rightarrow \frac{17}{2}[2a + 16d] = 289$

$\Rightarrow \quad a + 8d = 17 \quad \ldots(ii)$

On subtracting Eq. (i) from Eq. (ii), we get

$5d = 10 \Rightarrow d = 2$

On substituting $d = 2$ in Eq. (i), we get

$a + 6 = 7 \Rightarrow a = 1$

Now, $S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}[2 + (n-1)2]$

$= n[1 + n - 1] = n^2$

Q10. Show that $a_1, a_2, \ldots, a_n, \ldots$ form an AP, where a_n is defined as below :

(i) $a_n = 3 + 4n$ (ii) $a_n = 9 - 5n$

Also, find the sum of the first 15 terms in each case.

Sol. (i) Given, $a_n = 3 + 4n$

Clearly, $a_{n+1} - a_n = 3 + 4(n+1) - (3 + 4n)$

$= 3 + 4n + 4 - 3 - 4n = 4$, which is independent of n.

So, given sequences forms an AP with

first term $(a) = a_1 = 3 + 4 = 7$ and

common difference $(d) = a_2 - a_1 = 11 - 7 = 4$.

$\because$ Sum of n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore \quad S_{15} = \frac{15}{2}[2 \times 7 + (15-1)4]$

$= \frac{15}{2} \times 70 = 15 \times 35 = 525$

(ii) Do same as Part (i). **Ans.** -465

Q11. If the sum of the first n terms of an AP is $4n - n^2$, then what is the first term (i.e. S_1)? What is the sum of first two terms? What is the second term? Similarly, find the 3rd, the 10th and the nth terms.

Sol. Given, the sum of first n terms,

$S_n = 4n - n^2$

On putting $n = 1$ in Eq. (i), we get

$S_1 = 4 \times 1 - 1^2 = 4 - 1 = 3$

Thus, sum of first term = 3

On putting $n = 2$ in Eq. (i), we get

$S_2 = 4 \times 2 - 2^2 = 8 - 4 = 4$

Thus, sum of first two terms = 4

$\because$ The nth term of an AP, $a_n = S_n - S_{n-1}$

$\therefore$ Second term $= S_2 - S_1 = 4 - 3 = 1$

On putting $n = 3$ in Eq. (i), we get

$S_3 = 4 \times 3 - 3^2 = 12 - 9 = 3$

$\therefore$ Third term $= S_3 - S_2 = 3 - 4 = -1$

On putting $n = 9$ in Eq. (i), we get

$S_9 = 4 \times 9 - 9^2 = 36 - 81 = -45$

Again, putting $n = 10$ in Eq. (i), we get

$S_{10} = 4 \times 10 - 10^2 = 40 - 100 = -60$

$\therefore$ 10th term $= S_{10} - S_9$

$$= -60-(-45) = -60+45 = -15$$

Now, on replacing n by $n-1$ in Eq. (i), we get

$$S_{n-1} = 4(n-1)-(n-1)^2$$
$$= 4n-4-n^2+2n-1 = -n^2+6n-5$$

$\therefore$ nth term $= S_n - S_{n-1}$

$$= 4n-n^2-(-n^2+6n-5)$$
$$= 4n-n^2+n^2-6n+5 = 5-2n$$

Q 12. Find the sum of the first 40 positive integers divisible by 6.

Sol. The first 40 positive integers divisible by 6 are 6, 12, 18, ..., 240 . Clearly, these numbers forms an AP with first term, $a = 6$, common difference, $d = 6$ and last term, $l = 240$.

$\because$ Sum of n terms, $S_n = \frac{n}{2}[a+l]$

$\therefore$ Sum of first 40 terms,

$$S_{40} = \frac{40}{2}[6+240] = 20 \times 246 = 4920$$

Hence, the sum of first 40 positive integers divisible by 6 is 4920.

Q 13. Find the sum of the first 15 multiples of 8.

Sol. The first 15 multiples of 8 are

$$8 \times 1, 8 \times 2, 8 \times 3, ..., 8 \times 15$$

i.e. $8, 16, 24, ..., 120$ which are in AP.

Here, first term $(a) = 8$, last term $(l) = 120$ and number of terms $(n) = 15$.

$\because$ Sum of n terms, $S_n = \frac{n}{2}(a+l)$

$\therefore$ Sum of 15 terms, $S_{15} = \frac{15}{2}(8+120) = \frac{15}{2} \times 128$

$$= 15 \times 64 = 960$$

Hence, the sum of first 15 multiples of 8 is 960.

Q 14. Find the sum of the odd numbers between 0 and 50.

Sol. The odd numbers between 0 and 50 are 1, 3, 5,..., 49 which form an AP.

Here, first term $(a) = 1$, last term $(l) = 49$ and common difference $(d) = 3-1 = 2$.

Let there be n numbers in the AP.

Then, nth term $(a_n) = a+(n-1)d = l$

$\Rightarrow$ $1+(n-1)(2) = 49$

$\Rightarrow$ $n-1 = 24 \Rightarrow n = 25$

Now, sum of n terms, $S_n = \frac{n}{2}(a+l)$

$\therefore$ Sum of 25 terms, $S_{25} = \frac{25}{2}(1+49) = \frac{25}{2} \times 50$

$$= 25 \times 25 = 625$$

Q 15. A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: ₹ 200 for the first day, ₹ 250 for the second day, ₹ 300 for the third day etc. The penalty for each succeeding day being ₹ 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days?

Sol. Given, the penalty for each succeeding day is ₹ 50 more than the preceding day, therefore the penalties for the first day, the second day, the third day etc., will form an AP.

Here, $a = 200, d = 250-200 = 50$ and $n = 30$.

Clearly, the money required by the contractor to pay as penalty, if he delayed the work by 30 days, will be S_{30}.

We know that

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$\therefore \quad S_{30} = \frac{30}{2}[2 \times 200 + (30-1)50]$$

$$= 15(400+1450) = 15 \times 1850 = 27750$$

Hence, the contractor has to pay ₹ 27750, if he delayed the work.

Q 16. A sum of ₹ 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is ₹ 20 less than its preceding prize, then find the value of each of the prizes.

Sol. Here, ₹ 700 is to be used to give 7 cash prizes.

Let the value of first prize be ₹ a. Then, value of second prize ₹ $(a-20)$, value of third prize be ₹ $[(a-20)-20]$ and so on.

Thus, we get an AP whose first term is a and common difference is -20.

Since, $\quad S_7 = 700$

$$\therefore \quad \frac{7}{2}[2a+(7-1)(-20)] = 700 \quad \left[\because S_n = \frac{n}{2}\{2a+(n-1)d\}\right]$$

$\Rightarrow$ $2a - 120 = 200$

$\Rightarrow$ $2a = 320$

$\Rightarrow$ $a = \frac{320}{2} = 160$

Hence, value of 1st prize = ₹ 160

Value of 2nd prize = ₹160 − ₹20 = ₹ 140

Value of 3rd prize = ₹140 − ₹20 = ₹ 120

Value of 4th prize = ₹120 − ₹20 = ₹ 100

Value of 5th prize = ₹100 − ₹20 = ₹ 80

Value of 6th prize = ₹80 − ₹20 = ₹ 60

and value of 7th prize = ₹60 − ₹20 = ₹ 40

Q 17. **In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g. section of class I will plant 1 tree, a section of class II will plant 2 trees and so on till class XII. There are three sections of each class. How many trees will be planted by the students?**

Sol. According to the question, there are three sections of each class, so the number of trees that are planted by students of class I, class II, class III,... and class XII, are respectively, $1\times 3, 2\times 3, 3\times 3, \ldots$ and 12×3.

Thus, we get the following list of numbers, 3, 6, 9, ..., 36

Clearly, it forms an AP.

Here, the first term, $a = 3$

common difference, $d = 6-3 = 3$ and the last term, $l = 36$

Let the last term of this AP be its nth term.

Then, $a_n = a+(n-1)d = l$

$\Rightarrow \quad 3+(n-1)(3) = 36 \Rightarrow (n-1)3 = 33$

$\Rightarrow \quad n-1 = 11 \Rightarrow n = 12$

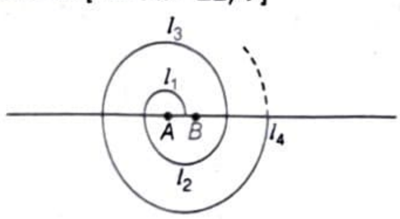
Hence, the number of the trees planted by the students

$= $ Sum of 12 terms of above AP

$= \frac{12}{2}(3+36) \qquad \left[\because S_n = \frac{n}{2}(a+l)\right]$

$= 6\times 39 = 234$

Q 18. **A spiral is made up of successive semi-circles, with centres alternately at A and B, starting with centre at A, of radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm, ... as shown in below figure. What is the total length of such a spiral made up of thirteen consecutive semi-circles? [Take $\pi = 22/7$]**



[**Hint** Length of successive semi-circles are $l_1, l_2, l_3, l_4, \ldots$ with centres at $A, B, A, B, \ldots$, respectively.]

Sol. Length of spiral made up of thirteen consecutive semi circles

$= (\pi\times 0.5 + \pi\times 1.0 + \pi\times 1.5 + \pi\times 2.0 + \ldots + \pi\times 6.5)$

[$\because$ circumference of semi-circle $= \pi r$, where, r is radius of circle]

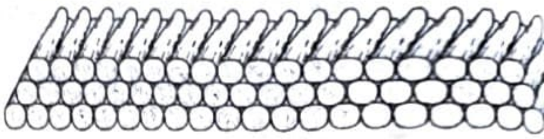
$= 0.5\pi\,[1+2+3+4+\ldots+13]$

$= \pi\times 0.5\times\frac{13}{2}[2\times 1+(13-1)\times 1]$

[$\because$ the numbers 1, 2, 3, ..., 13, forms an AP with $a=1$, $d = 2-1 = 1$. Also, $S_n = \frac{n}{2}\{2a+(n-1)d\}$]

$= \frac{22}{7}\times\frac{5}{10}\times\frac{13}{2}\times 14 = 143\,\text{cm}$

Q 19. **200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 1 the row next to it and so on (see below figure). In how many rows are the 200 logs placed and many logs are in the top row?**



Sol. Number of logs stacked in each row form a sequenc 20, 19, 18, 17, ..., which is an AP with first term, $a =$ common difference, $d = 19 - 20 = -1$.

Suppose number of rows is n, then $S_n = 200$

$\Rightarrow \frac{n}{2}[2\times 20+(n-1)(-1)] = 200$

$\left[\because S_n = \frac{n}{2}\{2a+(n\right.$

$\Rightarrow \quad 400 = 40n - n^2 + n$

$\Rightarrow \quad n^2 - 41n + 400 = 0$

$\Rightarrow \quad n^2 - 25n - 16n + 400 = 0$ [by factorisa

$\Rightarrow \quad n(n-25) - 16(n-25) = 0$

$\Rightarrow \quad (n-25)(n-16) = 0$

$\Rightarrow \quad n = 16$ or $n = 25$

Hence, the number of rows is either 25 or 16.

When $n = 16$,

$a_n = a+(n-1)d = 20+(16-1)(-1) = 20-15$

When $n = 25$,

$a_n = a+(n-1)d = 20+(25-1)(-1)$

$= 20 - 24 = -4$

[$\because$ number of logs cannot be neg

Hence, the number of rows is 16 and number of logs the top row is 5.

Q 20. **In a potato race, a bucket is placed at the startin point, which is 5 m from the first potato and the o potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line (see below figu**

A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs the bucket to drop it in and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to r

[**Hint** To pick up the first potato and the second potato, the distance (in metres) run by a competitor is $2\times 5 + 2\times(5+3$

Sol. The distance run by the competitor to picks up the first potato, second potato, third potato, fourth potato, ... are respectively $2 \times 5, 2 \times (5+3), 2 \times (5+3+3)$, $2 \times (5+3+3+3)$ ... i.e. 10, 16, 22, 28, ...

Clearly, it is an AP with first term, $a = 10$ and common difference, $d = 16 - 10 = 6$.

$\because$ Sum of first n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore$ Sum of first 10 terms, $S_{10} = \frac{10}{2}[2 \times 10 + (10-1) \times 6]$ $[\because n = 10, \text{ given}]$

$= 5(20 + 54) = 5 \times 74 = 370$

Hence, the total distance the competitor has to run is 370 m.

EXERCISE 5.4

Q 1. Which term of the AP : 121, 117, 113, ... is its first negative term?

[**Hint** Find n for which $a_n < 0$.]

Sol. Given, AP is 121, 117, 113, ...

Here, first term, $a = 121$

and common difference, $d = 117 - 121 = -4$

Let the nth term of this AP be the first negative term.

Then, $a_n < 0$

$\Rightarrow a + (n-1)d < 0 \Rightarrow 121 + (n-1)(-4) < 0$

$\Rightarrow 125 - 4n < 0 \Rightarrow 125 < 4n \Rightarrow 4n > 125$

$\Rightarrow n > \frac{125}{4} \Rightarrow n > 31\frac{1}{4}$

So, least integral value of n is 32.

Hence, 32nd term of the given AP is the first negative term.

Q 2. The sum of the third and the seventh terms of an AP is 6 and their product is 8. Find the sum of first sixteen terms of the AP.

Sol. Let the first term and the common difference of the given AP be a and d, respectively.

According to the question,

Third term + Seventh term = 6

$\Rightarrow [a + (3-1)d] + [a + (7-1)d] = 6$ $[\because a_n = a + (n-1)d]$

$\Rightarrow (a + 2d) + (a + 6d) = 6$

$\Rightarrow a + 4d = 3$ ...(i)

and Third term $\times$ Seventh term = 8

$\Rightarrow (a + 2d)(a + 6d) = 8$

$\Rightarrow \{(a + 4d) - 2d\}\{(a + 4d) + 2d\} = 8$

$\Rightarrow (3 - 2d)(3 + 2d) = 8$ [using Eq. (i)]

$\Rightarrow 9 - 4d^2 = 8$ $[\because (a-b)(a+b) = a^2 - b^2]$

$\Rightarrow 4d^2 = 9 - 8$

$\Rightarrow d^2 = \frac{1}{4} \Rightarrow d = \pm\frac{1}{2}$

When $d = \frac{1}{2}$, then from Eq. (i), we get

$a + 4\left(\frac{1}{2}\right) = 3$

$\Rightarrow a + 2 = 3$

$\Rightarrow a = 3 - 2 \Rightarrow a = 1$

Now, sum of first sixteen terms of the AP,

$S_{16} = \frac{16}{2}[2a + (16-1)d]$ $\left[\because S_n = \frac{n}{2}\{2a + (n-1)d\}\right]$

$= 8[2a + 15d] = 8\left[2(1) + 15\left(\frac{1}{2}\right)\right]$

$= 8\left(2 + \frac{15}{2}\right) = 8\left(\frac{19}{2}\right) = 76$

When $d = -\frac{1}{2}$, then from Eq. (i), we get

$a + 4\left(-\frac{1}{2}\right) = 3 \Rightarrow a - 2 = 3 \Rightarrow a = 5$

Now, sum of first sixteen terms of this AP,

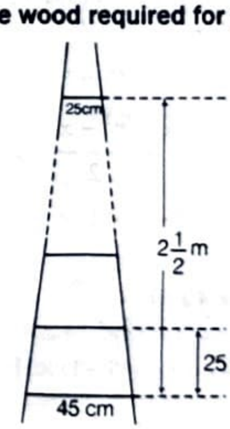
$S_{16} = \frac{16}{2}[2a + (16-1)d]$ $\left[\because S_n = \frac{n}{2}\{2a + (n-1)d\}\right]$

$= 8(2a + 15d) = 8\left[2(5) + 15\left(-\frac{1}{2}\right)\right]$

$= 8\left(10 - \frac{15}{2}\right) = 8\left(\frac{5}{2}\right) = 20$

Hence, the sum of first sixteen terms, $S_{16} = 20$ or 76.

Q 3. A ladder has rungs 25 cm apart (see figure). The rungs decrease uniformly in length from 45 cm at the bottom to 25 cm at the top. If the top and the bottom rungs are $2\frac{1}{2}$ m apart, then what is the length of the wood required for the rungs?



[**Hint** Number of rungs $= \frac{\text{Distance between top and bottom rungs}}{\text{Distance between two rungs}} + 1$]

Sol. According to the question,

Number of rungs

$$= \frac{\text{Distance between top and bottom rungs}}{\text{Distance between two rungs}} + 1$$

$$= \frac{2\frac{1}{2}\text{ m}}{25 \text{ cm}} + 1 = \frac{\frac{5}{2} \times 100 \text{ cm}}{25 \text{ cm}} + 1 \quad [\because 1 \text{ m} = 100 \text{ cm}]$$

$$= \frac{250 \text{ cm}}{25 \text{ cm}} + 1 = 10 + 1 = 11$$

Hence, there are 11 rungs.

The length of the wood required for the rungs

$$= \text{Sum of length of 11 rungs} = \frac{11}{2}(25 + 45)$$

$\left[\begin{array}{l}\because \text{ length of rungs forms an AP with} \\ \text{first term } (a) = 25 \text{ and last term } (l) = 45 \text{ and } S_n = \frac{n}{2}(a + l)\end{array}\right]$

$$= \frac{11}{2} \times 70 = 11 \times 35 = 385 \text{ cm}$$

Hence, the length of the wood required for the rungs is 385 cm.

Q 4. The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it. Find the value of x. [Hint $S_{x-1} = S_{49} - S_x$]

Sol. The numbers on the houses of a row are 1, 2, 3, ..., 49.

Clearly, this list of numbers forms an AP with

$$a = 1 \text{ and } d = 2 - 1 = 1$$

According to the question, we have

$$S_{x-1} = S_{49} - S_x \quad \text{...(i)}$$

$$\because \quad S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\therefore \quad S_{x-1} = \frac{x-1}{2}[2 \times 1 + (x - 1 - 1) \times 1]$$

$$= \frac{x-1}{2}(2 + x - 2)$$

$$= \frac{(x-1)x}{2} = \frac{x^2 - x}{2},$$

$$S_x = \frac{x}{2}[2 \times 1 + (x-1) \times 1]$$

$$= \frac{x}{2}(x+1) = \frac{x^2 + x}{2}$$

and $$S_{49} = \frac{49}{2}[2 \times 1 + (49 - 1) \times 1]$$

$$= \frac{49}{2}[2 + 48]$$

$$= \frac{49}{2} \times 50 = 49 \times 25 = 1225$$

Now, on substituting above values in Eq(i), we get

$$\frac{x^2 - x}{2} = 1225 - \frac{x^2 + x}{2}$$

$$\Rightarrow \quad \frac{x^2 - x}{2} + \frac{x^2 + x}{2} = 1225$$

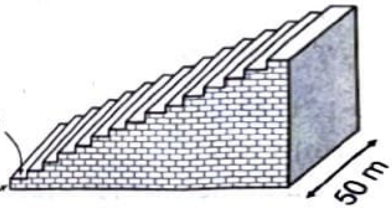
$$\Rightarrow \quad \frac{x^2 - x + x^2 + x}{2} = 1225$$

$$\Rightarrow \quad x^2 = 1225 \Rightarrow x = \pm 35$$

Since, x is a counting number.

$\therefore$ Taking positive sign, we get $x = 35$.

Q 5. A small terrace at a football ground comprises of 15 steps each of which is 50 m long and built of solid concrete. Each step has a rise of $\frac{1}{4}$ m and a tread of $\frac{1}{2}$ m (see figure). Calculate the total volume of concrete required to build the terrace.



[Hint Volume of concrete required to build the first step $= \frac{1}{4} \times \frac{1}{2} \times 50 \text{ m}^3$]

Sol. Clearly, volume of concrete required to build the I step, II step, III step,.. are respectively,

$$\frac{1}{4} \times \frac{1}{2} \times 50, \left(2 \times \frac{1}{4}\right) \times \frac{1}{2} \times 50, \left(3 \times \frac{1}{4}\right) \times \frac{1}{2} \times 50$$

i.e. $\frac{50}{8}, 2 \times \frac{50}{8}, 3 \times \frac{50}{8}, \ldots$

Now, total volume of concrete,

$$V = \frac{50}{8} + 2 \times \frac{50}{8} + 3 \times \frac{50}{8} + \ldots = \frac{50}{8}[1 + 2 + 3 + \ldots]$$

Note that the numbers in the bracket forms an AP with first term $(a) = 1$, common difference $(d) = 2 - 1 = 1$ and number of terms $(n) = 15$

$$\therefore \quad V = \frac{50}{8} \times \frac{15}{2}[2 \times 1 + (15 - 1) \times 1]$$

$$\left[\because S_n = \frac{n}{2}\{2a + (n-1)d\}\right]$$

$$= \frac{50}{8} \times \frac{15}{2} \times (2 + 14)$$

$$= \frac{25 \times 15}{8} \times 16 = 750 \text{ m}^3$$

Hence, the total volume of concrete required to build the terrace is 750 m^3.

REVIEW EXERCISE

Including Competency Based Questions

Part I

Multiple Choice Questions

1. If $-5, x, 3$ are three consecutive terms of an AP then the value of x is CBSE 2023 (Basic)
(a) -2 (b) 2 (c) 1 (d) -1

2. What cannot be the difference between four consecutive terms of an arithmetic progression? Competency Based Question
(a) 0, 0, 0 (b) $-2, -2, -2$ (c) 2, 3, 4 (d) $\frac{2}{7}, \frac{2}{7}, \frac{2}{7}$

3. The common difference of the AP $\frac{1}{2x}, \frac{1-4x}{2x}, \frac{1-8x}{2x}, \ldots$ is CBSE 2024 (Standard)
(a) $-2x$ (b) -2 (c) 2 (d) $2x$

4. The 8th term of an AP is 17 and its 14th term is 29. The common difference of this AP is CBSE 2023 (Basic)
(a) 3 (b) 2 (c) 5 (d) -2

5. Is an sequence defined by $a_n = 2n^2 + 1$ forms an AP? NCERT Exemplar
(a) Yes (b) Not
(c) Cannot be determined (d) None of these

6. The 21st term of an AP whose first two terms are -3 and 4, is
(a) 17 (b) 137 (c) 143 (d) -143

7. The common difference of the AP whose nth term is given by $a_n = 3n + 7$, is CBSE 2023 (Standard)
(a) 7 (b) 3 (c) $3n$ (d) 1

8. Let a be a sequence defined by $a_1 = 1, a_2 = 1$ and $a_n = a_{n-1} + a_{n-2}$ for all $n > 2$, then the value of $\frac{a_4}{a_3}$ is Competency Based Question
(a) $\frac{2}{3}$ (b) $\frac{5}{4}$ (c) $\frac{4}{5}$ (d) $\frac{3}{2}$

9. Which term of the AP 5, 15, 25, ... will be 130 more than its 31st term? NCERT Exemplar
(a) 42 (b) 44 (c) 46 (d) 48

10. Two APs have the same common difference. The first term of one of these is -1 and that of the other is -8. Then, the difference between their 4th terms is CBSE Sample Paper 2023 (Standard)
(a) 1 (b) 8 (c) 7 (d) 9

11. The 11th term from the end of the AP 10, 7, 4, ..., -62 is CBSE 2023 (Standard)
(a) 25 (b) 16 (c) -32 (d) 0

12. If the sum of the first n terms of an AP be $3n^2 + n$ and its common difference is 6, then its first term is CBSE 2024, 23 (Standard)
(a) 2 (b) 3 (c) 1 (d) 4

13. The sum of the first 50 odd natural numbers is CBSE 2023 (Basic)
(a) 5000 (b) 2500 (c) 2550 (d) 5050

Assertion-Reason Type Questions

14. **Assertion (A)** $-5, \frac{-5}{2}, 0, \frac{5}{2}, \ldots$ is an arithmetic progression.
Reason (R) The terms of an arithmetic progression cannot have both positive and negative rational numbers. CBSE Sample Paper 2023 (Standard)
(a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
(c) Assertion is correct but Reason is incorrect.
(d) Assertion is incorrect but Reason is correct.

15. **Assertion (A)** The first term of an AP is m and its common difference is p, then the 13th term is $a + 10p$.
Reason (R) In an AP, $S_n - S_{n-1} = a_n$.
(a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
(c) Assertion is correct but Reason is incorrect.
(d) Assertion is incorrect but Reason is correct.

16. **Assertion (A)** Sum of first 10 even natural numbers is 120.
Reason (R) If a is the first term, l is the last term and d is the common difference of an AP, then nth term from the end is given by $l - (n-1)d$.
(a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
(c) Assertion is correct but Reason is incorrect.
(d) Assertion is incorrect but Reason is correct.

17. Assertion (A) If the nth term of an AP be $(2n^2 - 1)$, then the sum of its first n terms is n^3.

Reason (R) If a, l and n are first term, last term and number of terms of an AP, respectively, then $S_n = \frac{n}{2}(a + l)$.

(a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
(c) Assertion is correct but Reason is incorrect.
(d) Assertion is incorrect but Reason is correct.

18. Assertion (A) In an AP, $S_n = n^2 + n$, then $T_{20} = 40$.

Reason (R) In an AP, $a_n - a_{n-1} = d$.

(a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
(c) Assertion is correct but Reason is incorrect.
(d) Assertion is incorrect but Reason is correct.

Case Based Type Questions

19. Your friend Veer wants to participate in a 200 m race. He can currently run that distance in 51 s and with each day of practice it takes him 2 s less. He wants to do in 31 s. **CBSE Question Bank**

Based on the above information, answer the following questions.

(i) Which of the following terms are in AP for the given situation?
(a) 51, 53, 55.... (b) 51, 49, 47.... (c) – 51, – 53, – 55.... (d) 51, 55, 59...

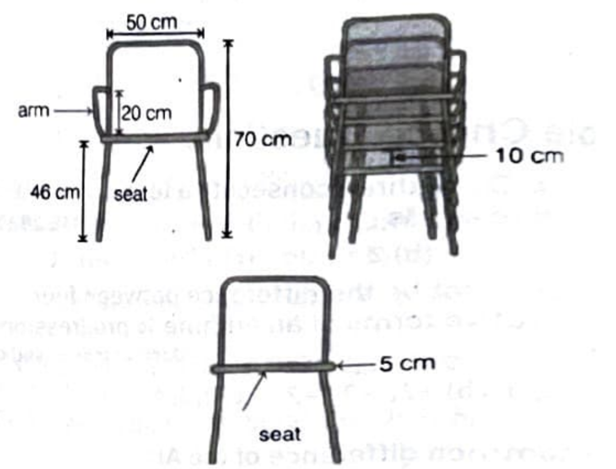
(ii) What is the minimum number of days he needs to practice till his goal is achieved?
(a) 10 (b) 12 (c) 11 (d) 9

(iii) Which of the following term is not in the AP of the above given situation?
(a) 41 (b) 30 (c) 37 (d) 39

(iv) If nth term of an AP is given by $a_n = 2n + 3$, then common difference of an AP is
(a) 2 (b) 3 (c) 5 (d) 1

(v) The value of x, for which $2x$, $x + 10$, $3x + 2$ are three consecutive terms of an AP
(a) 6 (b) – 6 (c) 18 (d) – 18

20. A chair is available in two models with arms and without arms. A person bought 20 chairs of each model. After use, he stacked the chairs in a storero The height of the storeroom is 1.55 m. The dimensions of the chair and how they are stacked s shown in the figure below.



The dimensions of the seat (in cm) is $50 \times 40 \times 5$.
Based on the above information, answer the follow questions.

(i) What is the height of the stack shown in the figu
(a) 30 cm (b) 76 cm (c) 100 cm (d) 110 c

(ii) What is the maximum number of chairs with ar that can be stacked in the storeroom?
(a) 4 (b) 8 (c) 9 (d) 10

(iii) What is the maximum number of chairs without arms that can be stacked in the storeroom?

(iv) What is the minimum number of columns (stack in which all chairs can be stacked in the storeroom?

(v) What can be the dimensions of the storeroom (in m)?
(a) $0.3 \times 1.5 \times 1.55$ (b) $0.4 \times 1.75 \times 1.55$
(c) $0.5 \times 0.4 \times 1.55$ (d) $1 \times 2.75 \times 1.55$

21. India is competitive manufacturing location due to t low cost of manpower and strong technical and engineering capabilities contributing to higher qua production runs. The production of TV sets in a fact increases uniformly by a fixed number every year. It produced 16000 sets in 6th year and 22600 in 9th year. **CBSE Questio**

Based on the above information, answer the following questions:

(i) Find the production during first year.

(ii) Find the production during 8th year.

(iii) Find the production during first 3 yr.

(iv) In which year, the production is ₹ 29200.

(v) Find the difference of the production during 7th year and 4th year.

22. Your elder brother wants to buy a car and plans to take loan from a bank for his car. He repays his total loan of ₹ 118000 by paying every month starting with the first instalment of ₹ 1000. If he increases the instalment by ₹ 100 every month, answer the following : **CBSE Question Bank**

(i) The amount paid by him in 30th instalment is
(a) 3900 (b) 3500 (c) 3700 (d) 3600

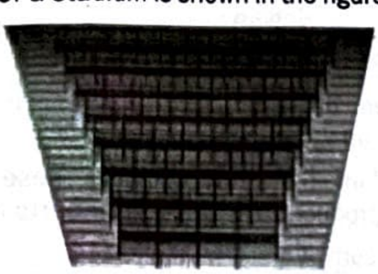
(ii) The amount paid by him in the 30 instalments is
(a) 37000 (b) 73500 (c) 75300 (d) 75000

(iii) What amount does he still have to pay after 30th instalment?
(a) 45500 (b) 49000 (c) 44500 (d) 54000

(iv) If total instalments are 40, then amount paid in the last instalment?
(a) 4900 (b) 3900 (c) 5900 (d) 9400

(v) The ratio of the 1st instalment to the last instalment is
(a) 1 : 49 (b) 10 : 49 (c) 10 : 39 (d) 39 : 10

23. Stadium seating surrounds the centre pitch. Each row in the seating is positioned at a slightly higher level than the one in front of it. A safe seating-standing section of a stadium is shown in the figure below.



There are 20 rows in the section. Each row in the section is 700 mm in height, excluding the seat and has one more seat than the previous row, starting from the second row. The first row has 4 seats.

Based on the above information, answer the following questions.

(i) Sidharth is seating in the centre seat of Row 12 in the section. How many seats are on his left?
(a) 5 (b) 7 (c) 8 (d) 24

(ii) What is the seating capacity of the section?
(a) 80 (b) 210 (c) 270 (d) 840

24. Sam is standing in Row 15 and Ronit is standing in Row 1.

Based on the above information, answer the following questions.

(i) How much higher is Sam's row than Ronit's?

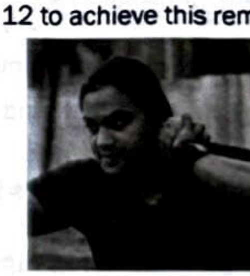
(ii) The height of the section is measured from the foot of the first row to the last row. What is the height of the section?

25. Manpreet Kaur is the national record holder for women in the shot-put discipline. Her throw of 18.86 m at the Asian Grand Prix in 2017 is the biggest distance for an Indian female athlete. **CBSE 2023 (Standard)**

Keeping her as a role model, Sanjitha is determined to earn gold in Olympics one day.

Initially her throw reached 7.56 m only. Being an athlete in school, she regularly practiced both in the mornings and in the evenings and was able to improve the distance by 9 cm every week.

During the special camp for 15 days, she started with 40 throws and every day kept increasing the number of throws by 12 to achieve this remarkable progress.



Based on the above information, answer the following questions.

(i) How many throws Sanjitha practiced on 11th day of the camp?

(ii) What would be Sanjitha's throw distance at the end of 6 months?

Or When will she be able to achieve a throw of 11.16 m?

(iii) How many throws did she do during the entire camp of 15 days ?

Part II

Very Short Answer Type Questions

1. $\sqrt{2}, \sqrt{18}, \sqrt{50}, \sqrt{98} \ldots$
Is the above pattern in AP? Justify your answer.
Competency Based Question

2. What is the difference between consecutive terms of an arithmetic progression $-13, -8, -3, 2$?

3. Consider the list of numbers given below.
$\frac{1}{2} + k, \frac{2}{3} + k, \frac{3}{4} + k, \frac{4}{5} + k, \ldots$ where k is an integer.
Is the above list of numbers an arithmetic progression? Justify your answer.
Competency Based Question

4. For what value of k will $k + 9, 2k - 1$ and $2k + 7$ are the consecutive terms of an AP.

5. Find the common difference of the AP $\frac{1}{p}, \frac{1-p}{p}, \frac{1-2p}{p}, \ldots$. **CBSE 2020 (Standard), 2013**

6. In an AP, if the common difference (d) is -4 and the seventh term (a_7) is 4, then find the first term. **CBSE 2018**

7. Find the common difference of the AP 4, 9, 14, ...
If the first term changes to 6 and the common difference remains the same, then write the new AP.
Competency Based Question

8. Find the 19th term of the following sequence.
$$t_n = \begin{cases} n^2, & \text{where } n \text{ is even} \\ n^2 - 1, & \text{where } n \text{ is odd} \end{cases}$$

9. Find the 7th term of the sequence whose nth term is given by $a_n = (-1)^{n-1} \cdot n^3$.

10. Find the 25th term of the AP $-5, -\frac{5}{2}, 0, \frac{5}{2}, \ldots$

11. If $a_n = 5 - 11n$, then find the common difference.

12. For AP, if $a_{18} - a_{14} = 32$, then find the common difference d.

13. If the common difference of an AP is 5, then what is $a_{18} - a_{13}$? **NCERT Exemplar**

14. Find the value of $a_{25} - a_{15}$ for the AP 6, 9, 12, 15, ...

15. If 7 times the seventh term of the AP is equal to 5 times the fifth term, then find the value of its 12th term.

16. Which term of the AP 21, 42, 63, 84, ... is 210? **NCERT Exemplar**

17. Is 68 a term of the AP 7,10, 13,...?

18. Find the 7th term from the end of the AP 7, 10, 13, ... ,184.

19. If the nth term of an AP is $(2n + 1)$, then find the sum of its first three terms.

Short Answer Type Questions

20. In which of the following situations, do the list of numbers involved form an AP? Give reason for your answer.
(i) The number of bacteria in certain food item after each second, when they double in every second.
(ii) Number of students left in the school auditorium from the total strength of 800 students, when they leave the auditorium in batches of 20 students.
Competency Based Question

21. If the 2nd term of an AP is 13 and 5th term is 25, then what is its 7th term? **NCERT Exemplar**

22. The 16th term of an AP is 1 more than twice its 8th term. If the 12th term of an AP is 47, then find its nth term.

23. The eighth term of an AP is half its second term and the eleventh term exceeds one-third of its fourth term by 1. Find the 15th term. **Competency Based Question**

24. The fourth term of an AP is 11. The sum of the fifth and seventh terms of the AP is 24. Find its common difference.

25. If the nth terms of the two AP's 9, 7, 5,... and 24, 21, 18,... are the same, then find the value of n. Also, find that term. **NCERT Exemplar**

26. The angles of a triangle are in AP. The greatest angle is twice the least. Find all the angles of the triangle.
NCERT Exemplar

27. Find the number of natural numbers between 101 and 999 which are divisible by both 2 and 5.

28. How many multiples of 9 lie between 10 and 300?

29. If four numbers are in AP such that their sum is 50 and the greatest number is 4 times the least, then find the numbers. **Competency Based Question**

30. Find how many two-digit numbers are divisible by 6.

31. The 4th term of an AP is zero. Prove that the 25th term of the AP is three times its 11th term.

32. Which term of the AP 120, 116, 112, ... is first negative term?

33. Split 207 into three parts such that these are in AP and the product of the two smaller parts is 4623.

34. Find the sum of first 8 multiples of 3. **CBSE 20**

35. Find the sum of the first 25 terms of an AP, whose nth term is given by $a_n = 7 - 3n$.

36. An AP 5, 8, 11, ... has 40 terms. Find the last term. Also, find the sum of the last 10 terms.

37. If $\dfrac{1+3+5+\ldots \text{ upto } n \text{ terms}}{2+5+8+\ldots \text{ upto 8 terms}} = 9$, then find the value of n.

38. Sum of the first n terms of an AP is $5n^2 - 3n$. Find the AP and also find its 16th term.

39. Find the sum of first 24 terms of the AP $a_1, a_2, a_3, \ldots$, if it is known that $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$

40. The sum of first, third and seventeenth terms of an AP is 216. Find the sum of the first 13 terms of the AP.

41. The sum of 5th and 9th terms of an AP is 72 and sum of 7th and 12th terms is 97. Find the AP. **NCERT Exemplar**

42. If an AP has $a = 1$, $a_n = 20$ and $S_n = 399$, then find the value of n.

43. If 12th term of an AP is 213 and the sum of its four terms is 24, then what is the sum of its first 10 terms?

44. Find the common difference of an AP whose first term is 5 and the sum of its first 4 terms is half the sum of the next 4 terms.

45. The sum of the first three terms of an AP is 33. If the sum of the twice of first and the third term exceeds the third term by 28, then find the AP. **NCERT Exemplar**

46. The sum of the first term and the fifth term of an ascending AP is 26 and the product of the second term by the fourth term is 160. Find the sum of the first seven terms of this AP. **Competency Based Question**

47. The sum of the first n terms of an AP whose first term is 8 and the common difference is 20, is equal to the sum of first $2n$ terms of another AP whose first term is -30 and the common difference is 8. Find the value of n. **NCERT Exemplar**

48. If S_n denotes the sum of first n terms of an AP, then show that the common difference d of the AP is given by $d = S_n - 2S_{n-1} + S_{n-2}$.

49. The sum of the first two terms of an arithmetic progression is the same as the sum of the first seven terms of the same arithmetic progression.

    Can such an arithmetic progression exist? Justify your answer. **Competency Based Question**

50. Find the sum of all three-digit natural numbers, which are multiples of 11.

51. Find the sum of all two-digit numbers greater than 50 which when divided by 7 leaves remainder 4.

52. Find the sum of those integers between 1 and 500, which are multiples of 2 as well as of 5. **NCERT Exemplar**
    [**Hint** Take the LCM of 2 and 5.]

53. In an AP, if $S_n = n^2p$ and $S_m = m^2p$, $m \neq n$, then prove that $S_p = p^3$.

54. The sum of first n, $2n$ and $3n$ terms of an AP are S_1, S_2 and S_3, respectively.
    Prove that $S_3 = 3(S_2 - S_1)$.

55. The sum of first n terms of three AP's are S_1, S_2 and S_3. The first term of each AP is unity and their common differences are 1, 2 and 3, respectively.
    Prove that $S_1 + S_3 = 2S_2$. **Competency Based Question**

56. Show that the sum of first n even natural numbers is equal to $\left(1+\dfrac{1}{n}\right)$ times the sum of first n odd natural numbers.

57. If there are $(2n+1)$ terms in an AP, then prove that the ratio of the sum of odd terms and the sum of even terms is $(n+1) : n$.

58. If a clock strikes one at one O'clock, two at two O'clock and so on, but does not strike at half hours, then total number of times the bell will be stuck in 24 h, are 156. Pradeep at once said, "It is true". Do you agree with Pradeep? **Competency Based Question**

Long Answer Type Questions

59. The ratio of the 11th term to the 18th term of an AP is 2 : 3. Find the ratio of 5th term to 21st term and also the ratio of the sum of the first 5 terms to sum of the first 21 terms. **CBSE 2023 (Standard)**

60. If the mth term of an AP is $\dfrac{1}{n}$ and nth term is $\dfrac{1}{m}$, then show that its mnth term is 1. **CBSE 2023 (Standard)**

61. Each year, a tree grows 5 cm less than it did the preceding year. If it grew by 1m in the first year, then in how many years will it have ceased growing?

62. 14 21, 28, 35, ... and 26, 39, 52, 65, ... are two arithmetic progressions such that the pth term of the first arithmetic progression is the same as the qth term of the second arithmetic progression. Derive a relationship between p and q. Show your work. **Competency Based Question**

63. Two arithmetic progressions have the same first term. The common difference of one progression is 4 more than the other progression. 124th term of the first arithmetic progression is the same as 42nd term of the second.

    Find one set of possible values of the common differences. Show your work. **Competency Based Question**

64. The pth, qth and rth terms of an AP are a, b and c, respectively. Show that
    $a(q-r) + b(r-p) + c(p-q) = 0$
    CBSE Sample Paper 2021 Term I (Basic)

65. If $a_1, a_2, \ldots, a_{n-2}, a_{n-1}, a_n$ are in AP, then prove that

$$\frac{1}{a_1 \cdot a_n} + \frac{1}{a_2 \cdot a_{n-1}} + \frac{1}{a_3 \cdot a_{n-2}} + \ldots + \frac{1}{a_n \cdot a_1} = \frac{2}{a_1 + a_n}\left(\frac{1}{a_1} + \frac{1}{a_2} + \ldots + \frac{1}{a_n}\right)$$

66. Solve the equation

$$-4 + (-1) + 2 + \ldots + x = 437.$$

CBSE 2023 (Standard), NCERT Exemplar

67. The sum of first four terms of an AP is 32 and the ratio of third and fourth term is 7:8. Find the terms of an AP. **CBSE 2020 (Standard)**

68. The first term of an AP is 22, the last term is -6 and the sum of all the terms is 64. Find the number of terms of the AP. Also, find the common difference. **CBSE 2023 (Basic)**

69. The sum of the first 7 terms of an AP is 63 and that of its next 7 terms is 161. Find the AP.

70. An AP consists of 37 terms. The sum of the three middle most terms is 225 and the sum of the last three terms is 429. Find the AP. **NCERT Exemplar**

71. Find the sum of the two middle most terms of the AP $-\frac{4}{3}, -1, \frac{-2}{3}, \ldots 4\frac{1}{3}$.

[**Hint** Sum of the two middle most terms $= a_9 + a_{10}$]

NCERT Exemplar ; CBSE 2010

72. If the sum of first p terms of an AP is same as the sum of its first q terms ($p \neq q$), then show that the sum of its first $(p+q)$ terms is zero.

73. Show that the sum of an AP whose first term is a, second term is b and the last term is c, is equal to

$$\frac{(b + c - 2a)(a + c)}{2(b - a)}$$

CBSE 2020 (Standard), NCERT Exemplar

74. A person buys cash certificates of ₹ 125. Thereafter, every year, he buys these certificates of the value exceeding previous year purchase by ₹ 25. Find the total value of certificates purchased by him in 20th year.

75. If pth term of an AP is a and qth term is b. Prove that the sum of its $(p+q)$th term is

$$\frac{p+q}{2}\left[a + b + \frac{(a-b)}{p-q}\right].$$

76. A student of class X gets pocket money from his mo everyday. Out of the pocket money, she saves mone poor people in her locality. On Ist day she saves ₹ 36 On each succeeding day she increases her saving b ₹ 4.5.

(i) Which mathematical concept and formula is be to used here for solving this question?

(ii) Find the amount saved by student 22th day.

77. Jaspal Singh repays his total loan of ₹ 118000 by paying every month starting with the first instalme ₹ 1000. If he increases the instalment by ₹ 100 ev month.

(i) What amount will be paid by him in the 30th instalment?

(ii) What amount of loan does he still have to pay a 30th instalment? **NCERT Exemplar; CBSE 20**

78. 360 bricks are stacked in the following manner, 30 bricks in the bottom row, 29 bricks in the next 28 bricks in the row next to it, and so on. In how m rows, 360 bricks are placed and how many bricks there in the top row?

79. The students of a school decided to beautify the school on the annual day by fixing colourful flags the straight passage of the school. They have 27 f to be fixed at intervals of every 2 m. The flags are stored at the position of the middle most flag. Ruc was given the responsibility of placing the flags. R kept her books where the flags were stored. She c carry only one flag at a time.

(i) How much distance did she cover in completin this job and returning back to collect her books

(ii) What is the maximum distance she travelled carrying a flag? **Competency Based Q**

HINTS & SOLUTIONS

Part I

1. (d) Since, $-5, x, 3$ is in AP.

$$\therefore \quad x - (-5) = 3 - x$$
$$\Rightarrow \quad x + 5 = 3 - x$$
$$\Rightarrow \quad 2x = -2$$
$$\Rightarrow \quad x = -1$$

2. (c) We know that in an AP the difference between consecutive terms is fixed and is called the commo difference of AP.

So, 2, 3, 4 is not the possible difference between consecutive terms of an AP.

3. (b) Given AP is $\frac{1}{2x}, \frac{1-4x}{2x}$,

So, $a_1 = \frac{1}{2x}, a_2 = \frac{1-4x}{2x}, a_3 = \frac{1-8x}{2x}, \ldots$

$\therefore \quad a_2 - a_1 = \frac{1-4x}{2x} - \frac{1}{2x} = \frac{1-4x-1}{2x} = \frac{-4x}{2x} = -2$

$a_3 - a_2 = \frac{1-8x}{2x} - \frac{1-4x}{2x}$

$= \frac{1-8x-1+4x}{2x} = \frac{-4x}{2x} = -2$

Since, $a_2 - a_1 = a_3 - a_2 = -2$

and common difference, $d = a_2 - a_1 = a_3 - a_2 =$ so on

$\therefore \quad d = -2$

4. (b) Given, $a_8 = 17, a_{14} = 29$

Let a be the first term and d be the common difference.

Since, $a_8 = 17$

$\Rightarrow \quad a + 7d = 17 \quad \ldots(i)$

Also, $\quad a_{14} = 29$

$\Rightarrow \quad a + 13d = 29 \quad \ldots(ii)$

On subtracting Eq. (i) from Eq. (ii), we get

$6d = 12 \Rightarrow d = 2$

5. (b) We have, $a_n = 2n^2 + 1$

Replacing n by $(n+1)$ in a_n, we obtain

$a_{n+1} = 2(n+1)^2 + 1 = 2n^2 + 4n + 3$

and $a_{n+1} - a_n = (2n^2 + 4n + 3) - (2n^2 + 1) = 4n + 2$

Clearly, $a_{n+1} - a_n$ is not independent of n.

So, the given sequence is not an AP.

6. (b) Given, first two terms of an AP are $a = -3$

and $\quad a + d = 4 \Rightarrow -3 + d = 4$

Common difference, $d = 7$

$\therefore a_{21} = a + (21-1)d \quad [\because a_n = a + (n-1)d]$

$= -3 + (20)\,7 = -3 + 140 = 137$

7. (b) Given, $a_n = 3n + 7$

If $n = 1, a_1 = 3(1) + 7 = 3 + 7 = 10$

So, first term $(a) = 10$

If $n = 2, a_2 = 3(2) + 7 = 6 + 7 = 13$

We know that $d = a_2 - a_1$

$\therefore \quad d = 13 - 10 = 3$

8. (d) We have, $a_1 = 1, a_2 = 1$

and $a_n = a_{n-1} + a_{n-2}$ for all $n > 2$

On putting $n = 3$ and 4, we get

$a_3 = a_2 + a_1 = 1 + 1 = 2$

$a_4 = a_3 + a_2 = 2 + 1 = 3$

Now, $\quad \frac{a_4}{a_3} = \frac{3}{2}$

9. (b) Solve as Example 5 of Topic 2.

10. (c) Let the common difference of two APs be d.

Given, first term of one AP $(a_1) = -1$

and first term of second AP $(b_1) = -8$

$\therefore$ Fourth term of 1st AP $(a_4) = a_1 + (4-1)d = -1 + 3d$

and Fourth term of 2nd AP $(b_4) = b_1 + (4-1)d$

$= -8 + 3d$

$\therefore a_4 - b_4 = (-1 + 3d) - (-8 + 3d)$

$= -1 + 3d + 8 - 3d = 7$

11. (c) Solve as Example 10 of Topic 2.

12. (d) Let a be the first term and common difference be d.

Given, $d = 6$

Sum of n terms $= 3n^2 + n$

Sum of 1st term = First term, $a = 3(1)^2 + 1 = 3 + 1 = 4$

13. (b) First 50 odd natural numbers 1, 3, 5, 7, 4, 11, ... 50 terms

It forms an AP with

First term $(a) = 1$, common difference $(d) = 3 - 1 = 2$

and number of terms $(n) = 50$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow \quad S_{50} = \frac{50}{2}[2(1) + (50-1)(2)]$

$= 25[2 + (49)(2)] = 25 \times 2 \times 50 = 2500$

14. (c) Solve as Question 4 of NCERT Folder Exercise 5.1.

Ans. Yes

15. (d) **Hint** $a = m, d = p, a_{13} = a + 12d$

16. (d) **Hint** Solve as Example 10 of Topic 3. **Ans.** No

17. (a) We have, $a_n = 2n^2 - 1$

$\therefore \quad a_1 = 1$

Now, $S_n = \frac{n}{2}(a + l) = \frac{n}{2}(1 + 2n^2 - 1) = n^3$

18. (a) **Hint** Solve as Example 9 of Topic 3.

19. (i) (b) In first day, Veer takes 51 s to complete the 200 m race. But in each day he takes 2 s lesser than the previous days.

Thus, AP series will formed

51, 49, 47, ...

(ii) (c) Since, Veer wants to achieve the race in 31 s.

Let Veer takes n days to achieve the target.

$\therefore \quad T_n = a + (n-1)d$

Here, $a = 51, d = 49 - 51 = -2$

$\therefore \quad 31 = 51 + (n-1)(-2)$

$\Rightarrow \quad (n-1)2 = 20 \Rightarrow (n-1) = 10 \Rightarrow n = 11$

Hence, he needs minimum 11 days to achieve the goal.

(iii) (b) In an AP series, we get the series of odd terms.

Hence, term 30 is not an AP.

(iv) (a) Given, $a_n = 2n + 3$

$\therefore$ Common difference $= a_{n+1} - a_n$

$= 2(n+1) + 3 - (2n+3)$
$= 2n + 2 + 3 - 2n - 3 = 2$

(v) (a) Given, terms $2x$, $x + 10$, $3x + 2$ are in AP.

$$\therefore \quad x + 10 = \frac{2x + (3x+2)}{2}$$

$$\Rightarrow \quad 2x + 20 = 5x + 2$$

$$\Rightarrow \quad 3x = 18 \Rightarrow x = 6$$

20. (i) (c) AP formed by the heights of the chairs,
$70, 70+10, 70+10+10, 70+10+10+10$
i.e. $70, 80, 90, 100$
Here, first term, $a = 70$
Common difference, $d = 10$
Height of the stack = nth term of AP = 100 cm

(ii) (c) Given, height of the storeroom = 1.55 m = 155 cm
$\therefore$ Maximum height of the stack ≤ 155

$$\therefore \quad a_n \le 155$$
$$\Rightarrow \quad a + (n-1)d \le 155$$
$$\Rightarrow \quad 70 + (n-1)10 \le 155$$
$$\Rightarrow \quad (n-1)10 \le 85$$
$$\Rightarrow \quad n - 1 \le 8.5$$
$$\Rightarrow \quad n \le 9.5$$

Since, number of chairs cannot be in decimal.
$\therefore \; n = 9$

(iii) AP formed by height of chairs without arms is
$70, 70+5, 70+5+5, 70+5+5+5, \ldots$
i.e. $70, 75, 80, 85, \ldots$
Here, first term, $a = 70$
common difference, $d = 5$
Maximum height of the stack ≤ 155

$$\therefore \quad a_n \le 155$$
$$\Rightarrow \quad a + (n-1)d \le 155$$
$$\Rightarrow \quad 70 + (n-1)5 \le 155$$
$$\Rightarrow \quad (n-1)5 \le 85$$
$$\Rightarrow \quad n - 1 \le 17$$
$$\Rightarrow \quad n \le 18$$

$\therefore$ Maximum number of chairs that can be stacked in the storeroom = 18

(iv) Minimum number of columns (stacks) in which all chairs can be stacked in the storeroom = 4

(v) (d) Given, height of the storeroom = 1.55 m
Length of one chair = 0.5 m
and breadth of one chair = 0.4 m
Also, minimum number of columns (stacks) in which all chairs can be stacked in the storeroom = 4
$\therefore$ Only possible dimensions of the storeroom
$= 1 \times 2.75 \times 1.55$

21. (i) Let the production of TV sets in first year be a.
Then, production in the next consecutive years $a + d, a + 2d, \ldots$
Thus, we get the sequence, $a, a + d, a + 2d, \ldots$
This is an AP sequence, whose first term = a and common difference = d.
Given, $T_6 = 16000$ and $T_9 = 22600$

$$\therefore \quad a + (6-1)d = 16000$$
and $a + (9-1)d = 22600 \quad [\because T_n = a + (n-1)d]$
$$\Rightarrow \quad a + 5d = 16000$$
and $a + 8d = 22600$

On subtracting Eq. (i) from Eq. (ii), we get
$$3d = 22600 - 16000$$
$$\Rightarrow \quad 3d = 6600$$
$$\Rightarrow \quad d = 2200$$

On putting $d = 2200$ in Eq. (i), we get
$$a + 5 \times 2200 = 16000$$
$$\Rightarrow \quad a = 16000 - 11000 = 5000$$

Hence, the production during first year is 5000.

(ii) The production during 8th yr is
$$T_8 = a + (8-1)d$$
$$= 5000 + 7 \times 2200$$
$$= 5000 + 15400$$
$$= 20400$$

Hence, the production during 8th yr is 20400.

(iii) The production during first 3 yrs,
$$S_3 = \frac{3}{2}[2a + (3-1)d]$$
$$= \frac{3}{2}[2 \times 5000 + 2 \times 2200]$$
$$= 3[5000 + 2200]$$
$$= 3 \times 7200$$
$$= 21600$$

(iv) Let in nth year, the production is 29200.
$$\because \quad T_n = a + (n-1)d$$
$$\therefore \quad 29200 = 5000 + (n-1)2200$$
$$\Rightarrow (n-1)2200 = 24200$$
$$\Rightarrow \quad (n-1) = \frac{24200}{2200}$$
$$\Rightarrow \quad n - 1 = 11$$
$$\Rightarrow \quad n = 12$$

(v) The difference of the production during 7th yr and 4th yr
$$= T_7 - T_4$$
$$= a + (7-1)d - [a + (4-1)d]$$
$$= 6d - 3d = 3d$$
$$= 3 \times 2200 = 6600$$

22. (i) (a) Since, he pays first instalment of ₹ 1000 and next consecutive months he pay the instalment are 1100, 1200, ...

Thus, we get the AP sequence, 1000, 1100, 1200, ...

Here, $a = 1000$, $d = 1100 - 1000 = 100$

Now, $T_{30} = a + (30 - 1)\,d$

$= 1000 + 29 \times 100 = 1000 + 2900 = 3900$

Hence, the amount paid by him in 30th instalment is ₹ 3900.

(ii) (b) Now, $S_{30} = \frac{30}{2}[2a + (30 - 1)d]$

$= 15(2 \times 1000 + 29 \times 100)$

$= 15(2000 + 2900) = 15 \times 4900 =$ ₹ 73500

(iii) (c) After 30th instalment, he still have to pay

$= 118000 - 73500 = 44500$

(iv) (a) The amount in last 40th instalment is

$T_{40} = a + (40 - 1)d$

$= 1000 + 39 \times 100$

$= 1000 + 3900 =$ ₹ 4900

(v) (b) The ratio of 1st instalment to the last instalment is $\frac{1000}{4900}$ i.e. $\frac{10}{49}$.

23. (i) (b) Given, total number of rows = 20

Number of seats in first row = 4

And given, every consecutive row has one more seat than the other.

∴ Number of seats from an AP, 4, 5, 6, 7, 8 ...

First term, $a = 4$

Common difference, $d = 1$

Number of seats in 12th row,

$a_{12} = a + (12 - 1)d$

$a_{12} = 4 + 11 \times 1 = 15$

Given, Siddharth is seating in the center seat of row 12.

Number of seats on his left $= \frac{15 - 1}{2} = 7$

(ii) (c) Seating capacity of the section

$= 4 + 5 + 6 + 7 + 8$ upto 20th term.

∴ $S_{20} = \frac{20}{2}[2 \times 4 + (20 - 1)1]$

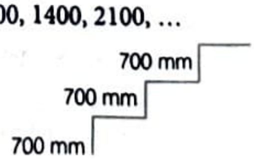
$= 10[8 + 19] = 270$

24. (i) Given, each row in the section is 700 mm in height.

∴ Heights of rows form an AP,

700, (700 + 700), (700 + 700 + 700), ...

i.e. 700, 1400, 2100, ...



First term, $a = 700$

Common difference, $d = 700$

Height of Sam's row $= a_{15}$

and height of Ronit's row $= a_1$

∴ Required height $= a_{15} - a_1$

$= a + 14d - (a + 0d)$

$= 14d = 14 \times 700$

$= 9800$ mm

(ii) Since, the height of the section is measured from the foot of the first row to the last row, therefore the height of the section $= 20 \times 700 = 14000$ mm

25. (i) Number of throws on first day $a = 40$

Number of throws on second day $= 40 + 12 = 52$

AP formed by number of throws is

40, 52, (52 + 12) ...

(I day) (II day) (III day)

Here, common difference, $d = 12$

Number of throws she practiced on 11th day

$a_{11} = a + (11 - 1)\,d$

$= 40 + 10 \times 12 = 160$

(ii) Distance of Sanjitha's throw on first day, $a = 7.56$ m

Distance of Sanjitha throw after a week

$= (7.56 + 0.09)$ m $= 7.65$ m

∴ AP formed by the distances is

7.56, 7.65, 7.65 + (0.09), ...

(I week) (II week) (III week)

Here, common difference, $d = 0.09$ m

Sanjitha's throw distance at the end of 6 months (i.e. 26 weeks)

$a_{26} = a + (26 - 1)d$

$= 7.56 + 25 \times 0.09$

$= 7.56 + 2.25 = 9.81$ m

Or

Here, $a_n = 11.16 = a + (n - 1)d$

$\Rightarrow 7.56 + (n - 1)\,0.09 = 11.16$

$\Rightarrow \quad n - 1 = \frac{3.6}{0.09} \Rightarrow n = 41$

∴ On 41st week, she will be able to achieve a throw of 11.16 m.

(iii) Total throws done by her during the entire camp of 15 days $= S_n = \frac{n}{2}[2a + (n - 1)\,d]$

where, n is total number of terms

$\Rightarrow \quad S_{15} = \frac{15}{2}[2 \times 40 + 14 \times 12]$

$= 15[40 + 84]$

$= 15 \times 124 = 1860$

Part II

1. $a_1 = \sqrt{2}, a_2 = \sqrt{18} = 3\sqrt{2},$
$a_3 = \sqrt{50} = 5\sqrt{2}, a_4 = \sqrt{98} = 7\sqrt{2}$
So, $a_2 - a_1 = 3\sqrt{2} - \sqrt{2} = 2\sqrt{2}$
$a_3 - a_2 = 5\sqrt{2} - 3\sqrt{2} = 2\sqrt{2}$
$a_4 - a_3 = 7\sqrt{2} - 5\sqrt{2} = 2\sqrt{2}$
Since, $a_2 - a_1 = a_3 - a_2 = a_4 - a_3$
$\therefore$ The given series is in AP because the common difference is the same which is $2\sqrt{2}$.

2. $a_1 = -13, a_2 = -8, a_3 = -3, a_4 = 2$
$a_2 - a_1 = -8 - (-13) = 5$
$a_3 - a_2 = -3 - (-8) = 5$
$a_4 - a_3 = 2 - (-3) = 5$
$\therefore\ a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = 5$

3. Hint Solve as Question 4 of NCERT Folder Exercise 5.1. **Ans.** No

4. Hint $a_2 - a_1 = a_3 - a_2$
$\Rightarrow (2k-1)-(k+9) = (2k+7)-(2k-1)$ **Ans.** $k = 18$

5. Hint Common difference = Second term − First term
$$d = \frac{1-p}{p} - \frac{1}{p} \quad \textbf{Ans. } -1$$

6. Given, $d = -4$ and $a_7 = 4$
Let first term of an AP be a.
$\because\quad a_n = a + (n-1)d$
$\therefore\quad a_7 = a + (7-1)(-4)$
$\Rightarrow\quad 4 = a + (6)(-4)$
$\Rightarrow\quad 4 = a - 24$
$\Rightarrow\quad a = 4 + 24$
$\Rightarrow\quad a = 28$

7. Given AP, 4, 9, 14 ...
First term = 4 and common difference = 5
Now, we need to make new AP with first term 6 and common difference 5
$\therefore$ First term = 6, second term = 6 + 5 = 11
$\therefore$ New AP is 6, 11, 16, 21, ...

8. Hint For 19th term i.e. for $n = 19$ which is odd, we take
$t_n = n^2 - 1 = (19)^2 - 1 = 360$

9. Hint 7th term $(a_7) = (-1)^{7-1} \cdot (7)^3$. **Ans.** 343

10. Hint $a = -5,\ d = -\frac{5}{2} + 5 = \frac{5}{2}$
$\therefore$ 25th term of given AP, $a_n = a + (n-1)d$
$$= -5 + (25-1) \times \frac{5}{2} = 55$$

11. Hint Common difference, $d = a_{n+1} - a_n$
$= 5 - 11(n+1) - (5 - 11n)$
$= 5 - 11n - 11 - 5 + 11n = -11$

12. Hint $[a + (18-1)d] - [a + (14-1)d] = 32$ **Ans.** d

13. Hint $a_{18} - a_{13} = (a + 17d) - (a + 12d) = 5d$ **Ans.**

14. Given AP, 6, 9, 12, 15 ......
So, $a = 6$ and $d = 9 - 6 = 3$
$a_{25} - a_{15} = a + 24d - (a + 14d)$
$= a + 24d - a - 14d$
$= 10d = 10 \times 3 = 30$

15. Let the nth term of an AP be a_n.
According to question,
7 times the 7th term = 5 times its 5th term
$7a_7 = 5a_5$
$\Rightarrow\quad 7(a + 6d) = 5(a + 4d)$
$\Rightarrow\quad 7a + 42d = 5a + 20d$
$\Rightarrow\quad 2a = -22d$
$\Rightarrow\quad a = -11d$
Then, $a_{12} = a + 11d = -11d + 11d$
$a_{12} = 0$

16. Hint Do same as Example 3 of Topic 2. **Ans.** 10th term

17. Hint Do same as Question 6 of NCERT Folder Exercise 5.2.
Ans. No

18. Hint Do same as Example 10 of Topic 2. **Ans.** 16

19. Hint Given, $a_n = 2n + 1$
On putting, $n = 1, 2, 3$, we get
$a_1 = 2(1) + 1 = 3 = \text{First term} = (a)$
$a_2 = 2(2) + 1 = 5$
$a_3 = 2(3) + 1 = 7$
Now, $S_n = \frac{n}{2}(a + l)$
$\Rightarrow\quad S_3 = \frac{3}{2}[3 + 7]$ **Ans.** 15

20. Hint Solve as Example 3 of Topic 1.
Ans. (i) It is not forming an AP.
(ii) It is forming an AP.

21. Hint Do same as Question 7 of NCERT Folder Exercise 5.2. **Ans.** 33

22. Hint Given, $a_{12} = 47$
$\Rightarrow\quad a + 11d = 47$
and $\quad a_{16} = 1 + 2a_8$ [by given cond
$\Rightarrow [a + (16-1)d] = 1 + 2[a + (8-1)d]$
$\Rightarrow\quad a - d = -1$
On solving Eqs. (i) and (ii), we get
$d = 4$ and $a = 3$
$\therefore\quad a_n = 3 + (n-1)4 = 4n - 1$

23. **Hint** We have, $a_8 = \frac{1}{2}a_2$ and $a_{11} = \frac{1}{3}a_4 + 1$

$\Rightarrow \quad a + 13d = 0$ and $2a + 27d = 3$. **Ans.** 3

24. **Hint** Given, $a_4 = 11 \Rightarrow a + 3d = 11$ ...(i)

and $a_5 + a_7 = 24 \Rightarrow a + 5d = 12$ ...(ii)

On solving Eqs. (i) and (ii), we get

Ans. $d = \frac{1}{2}$

25. **Hint** Do same as Question 15 of NCERT Folder Exercise 5.2.

Hint To find 13th term, put $n = 13$ in nth term of any of the two AP

Ans. $n = 13$ and nth term is -15.

26. **Hint** Let the angles be $a°, (a + d)°, (a + 2d)°$

Then, we get $a + a + d + a + 2d = 180$ and $a + 2d = 2a$

Ans. $40°, 60°, 80$

27. **Hint** Natural numbers between 101 and 999 divisible by both 2 and 5 i.e. divisible by 10, are 110, 120, 130, ..., 990

Here, $120 - 110 = 130 - 120 = \ldots = 10$.

So, it is an AP with first term $(a) = 110$, common difference $(d) = 120 - 110 = 10$ and last term $(l) = 990$.

Let $l = a_n = a + (n - 1)d$

Then, $990 = 110 + (n - 1) \times 10$

Ans. $n = 89$

28. **Hint** Do same as Question 14 of NCERT Folder Exercise 5.2. **Ans.** 32

29. **Hint** Let $a + (a + d) + (a + 2d) + (a + 3d) = 50$

$\Rightarrow \quad 2a + 3d = 25$ ...(i)

and $\quad a + 3d = 4a$

$\Rightarrow \quad a = d$ ...(ii)

On solving Eqs. (i) and Eqs. (ii), we get

$\therefore \quad a = d = 5$

Ans. 5, 10, 15 and 20

30. **Hint** Do same as Example 8 of Topic 2. **Ans.** 15

31. **Hint** Let a and d be the first term and common difference of the given AP, respectively.

Given $\quad a_4 = 0$

$\Rightarrow \quad a + 3d = 0$

$\Rightarrow \quad a = -3d$ ...(i)

To prove $\quad a_{25} = 3a_{11}$

Proof Clearly, $\quad a_{25} = a + 24d$

$= -3d + 24d$ [from Eq. (i)]

$\Rightarrow \quad a_{25} = 21d$ ...(ii)

Also, $\quad a_{11} = a + 10d$

$= -3d + 10d$ [from Eq. (i)]

$\Rightarrow \quad a_{11} = 7d$

$\Rightarrow \quad 3a_{11} = 21d$ ...(iii)

From Eqs. (ii) and (iii), we get

$a_{25} = 3a_{11}$ **Hence proved.**

32. **Hint** Do same as Question 1 of NCERT Folder Exercise 5.4 **Ans.** 32nd term

33. **Hint** Let the three parts be $a, a + d$ and $a + 2d$ and let $d > 0$.

Then, $\quad a + a + d + a + 2d = 207$

$\Rightarrow \quad a + d = 69$ ...(i)

and product of smaller parts = 4623

$\Rightarrow \quad a(a + d) = 4623 \Rightarrow a = 67$

On putting $a = 67$ in Eq. (i), we get

$\Rightarrow \quad d = 2$

Ans. First part, $a = 67$

Second part, $a + d = 69$

Third part, $a + 2d = 71$

Hence, three parts of 207 are 67, 69 and 71.

34. **Hint** Do same as Q. 13 of NCERT Folder Exercise. 5.3. **Ans.** 108

35. **Hint** Do same as Example 4 of Topic 3. **Ans.** -800

36. Given AP, 5, 8, 11... upto 40 terms

Here, $a = 5, d = 3$ and $n = 40$

$a_{40} = a + (40 - 1)d$

$= 5 + 39 \times 3 = 5 + 117 = 122$

Hence, the last term is 122.

Now, we need to find sum of last 10 terms

Last 10 terms will be from the 31st to the 40th term.

Now, 31st term $= a_{31}$

$= a + (n - 1)d$

$= 5 + (31 - 1)3$

$= 5 + 30 \times 3 = 95$

So, our last 10th term will be 95, 98, ..., 122

$$S_n = \frac{n}{2}[a + l] = \frac{10}{2}[95 + 122] = 5 \times 217 = 1085$$

$\therefore$ Required sum is 1085.

37. **Hint** Given, $\dfrac{1 + 3 + 5 + \ldots \text{ upto } n \text{ terms}}{2 + 5 + 8 + \ldots \text{ upto } 8 \text{ term}} = 9$

$$\Rightarrow \quad \frac{\frac{n}{2}[2(1) + (n - 1)2]}{\frac{8}{2}[2(2) + (8 - 1)3]} = 9$$ **Ans.** $n = 30$

38. **Hint** Do same as Q. 11 of NCERT Folder Exercise. 5.3. **Ans.** 2, 12, 22, ..., 152

39. Hint $a + (a + 4d) + (a + 9d) + (a + 14d) + (a + 19d) + (a + 23d) = 225$

$\Rightarrow \quad 2a + 23d = 75 \quad \ldots(i)$

Now, $S_{24} = \frac{24}{2}[2a + (24 - 1)d]$

$= 12(2a + 23d)$

Ans. 900

40. Hint Third term $a_3 = a + 2d$

and seventeenth term $a_{17} = (a + 16d)$

Also, $a_1 + a_3 + a_{17} = 216$

$\Rightarrow a + a + 2d + a + 16d = 216$

$\Rightarrow \quad a + 6d = 72 \quad \ldots(i)$

$S_{13} = \frac{13}{2}[2a + (13 - 1)d]$

$= \frac{13}{2}[2(a + 6d)]$ **Ans.** 936

41. Hint Do same as Question 18 of NCERT Folder Exercise 5.2. **Ans.** 6, 11, 16, 21, 26, ...

42. Hint Do same as Question 3 (vii) of NCERT Folder Exercise 5.3. **Ans.** $n = 38$

43. Hint $a_{12} = 213 \Rightarrow a + 11d = 213 \quad \ldots(i)$

and $S_4 = 24$

$\Rightarrow \frac{4}{2}[2a + (4 - 1)d] = 24$

$\Rightarrow \quad 2a + 3d = 12 \quad \ldots(ii)$

On solving Eqs. (i) and (ii), we get

$a = \frac{-507}{19}$ and $d = \frac{414}{19}$

Now, $S_{10} = \frac{10}{2}\left[2 \times \left(\frac{-507}{19}\right) + (10 - 1) \times \frac{414}{19}\right]$

Ans. 713.68

44. Hint $S_4 = \frac{1}{2}(\text{Sum of } a_5, a_6, a_7, a_8) = \frac{1}{2}(S_8 - S_4)$

$\Rightarrow \quad 2S_4 = S_8 - S_4 \Rightarrow 3S_4 = S_8 \quad \ldots(i)$

Clearly, $S_4 = \frac{4}{2}[2a + (4 - 1)d] \left[\because S_n = \frac{n}{2}\{2a + (n - 1)d\}\right]$

$= 2[2(5) + 3 \times d] \quad [\because a = 5]$

$\Rightarrow \quad S_4 = 20 + 6d$

and $S_8 = \frac{8}{2}[2 \times 5 + (8 - 1)d]$

$= \frac{8}{2}(10 + 7d) = 40 + 28d$

On putting the values of S_4 and S_8 in Eq. (i), we get

$3(20 + 6d) = 40 + 28d$

$\Rightarrow \quad 60 + 18d = 40 + 28d$

$\Rightarrow \quad 20 = 10d \Rightarrow d = 2$

Hence, common difference is 2.

45. Hint Let the three terms in AP be $a, a + d, a + 2d$

$\Rightarrow \quad a + a + d + a + 2d = 33 \Rightarrow a + d = 11$

By second condtion,

$2a + a + 2d = 28 + a + 2d \Rightarrow a = 14$

Hence, $d = 11 - 14 = -3$ **Ans.** 14, 11, 8, ...

46. Hint $a_1 + a_5 = 26 \Rightarrow a + 2d = 13$

Also, we have $a_2 \times a_4 = 160$

$\Rightarrow \quad (a + d) \times (a + 3d) = 160$

$\Rightarrow (13 - 2d + d)(13 - 2d + 3d) = 160$ [from E

$\Rightarrow \quad (13)^2 - (d)^2 = 160$

$\therefore \quad d = \pm 3$

On putting $d = 3$ in Eq. (i), we get $a = 7$

Now, the sum of first seven terms,

$S_7 = \frac{7}{2}[2 \times a + (7 - 1)d] = 112$

On putting $d = -3$ in Eq. (i), we get $a = 19$

Now, $S_7 = \frac{7}{2}[2(19) + (7 - 1)(-3)] = 70$ **Ans.** 112, 70

47. Hint Given, $S_n = S_{2n}$

$\frac{n}{2}[2 \times 8 + (n - 1) \times 20] = \frac{2n}{2}[2 \times (-30) + (2n - 1) \times 8]$

After solving, we get value of n. **Ans.** $n = 11$

48. Hint RHS $= S_n - 2S_{n-1} + S_{n-2}$

$= (S_n - S_{n-1}) - (S_{n-1} - S_{n-2})$

$= a_n - a_{n-1} \quad [\because a_n = S_n - S$

$= [a + (n - 1)d] - [a + (n - 1 - 1)d]$

$[\because a_n = a + (n$

$= a + (n - 1)d - a - (n - 2)d$

$= (n - 1 - n + 2)d = d =$ LHS

$\therefore \quad d = S_n - 2S_{n-1} + S_{n-2}$

49. Given, $S_2 = S_7$

$\Rightarrow \frac{2}{2}[2a + (2 - 1)d] = \frac{7}{2}[2a + (7 - 1)d]$

[where, a is the first term and the common differe

$\Rightarrow \quad 2a + d = 7(a + 3d)$

$\Rightarrow \quad 5a = -20d$

$\Rightarrow \quad a + 4d = 0 \Rightarrow a_5 = 0$

Thus, 5th term is 0.

For example, if the series of AP is as follows :

$-4, -3, -2, -1, 0, 1, 2, \ldots$ where $a_5 = 0$ and sum of two terms is -7 which is equal to sum of first seven ter

$\therefore$ Such arithmetic progression can exist.

50. Hint All three-digit natural numbers, multiples of 1 are 110, 121, 132, ..., 990.

Here, common difference,

$121 - 110 = 132 - 121 = \ldots = 11.$

So, it is an AP with first term, $a = 110$, common difference, $d = 11$ and last term, $l = 990$.

Let $\quad l = a_n = a + (n-1)d$

$\therefore \quad 990 = 110 + (n-1) \times 11$

$\Rightarrow \quad n = 81$

$\because \quad S_n = \frac{n}{2}[a + l]$

$\therefore \quad S_{81} = \frac{81}{2}[110 + 990]$ **Ans.** $S_{81} = 44550$

51. **Hint** All two-digit numbers greater than 50 which when divided by 7 leaves remainder 4, are

$$53, 60, 67, \ldots, 95.$$

which forms an AP with first term $a = 53$, common difference, $d = 60 - 53 = 7$ and last term $l = 95$.

$a_n = l = a + (n-1)d$

$\therefore \quad 95 = 53 + (n-1)7 \Rightarrow n = 7$

Now, required sum $= 53 + 60 + 67 + \ldots + 95$ **Ans.** 518

52. **Hint** Consider 10, 20, 30, 40, ... 490.

According to question,

$490 = 10 + (n-1) \times 10 \quad [\because a = 10, a_n = 490]$

On solving, we get $n = 49$

and $\quad S_n = \frac{n}{2}[2 \times 10 + (n-1) \times 10]$ **Ans.** 12,250

53. **Hint** $S_n = n^2 p$

$\Rightarrow \quad 2a + (n-1)d = 2np \quad \ldots$(i)

Also, $S_m = m^2 p$

$\Rightarrow \quad 2a + (m-1)d = 2mp \quad \ldots$(ii)

On subtracting Eqs. (ii) from Eq. (i), we get

$2a + (n-1)d - 2a - (m-1)d = 2np - 2mp$

$\Rightarrow \quad (n - 1 - m + 1)d = 2p(n-m)$

$\Rightarrow \quad (n-m)d = 2p(n-m)$

$\Rightarrow \quad d = 2p \quad \ldots$(iii)

Put this in Eq. (i), we get

$2a + (n-1)(2p) = 2np \Rightarrow a = p$

Now, $S_p = \frac{p}{2}[2a + (p-1)d] = \frac{p}{2}[2p + (p-1)2p] = p^3$

$\therefore \quad S_p = p^3$

54. According to the question,

$S_1 = S_n = \frac{n}{2}[2a + (n-1)d] \quad \ldots$(i)

$S_2 = S_{2n} = \frac{2n}{2}[2a + (2n-1)d] \quad \ldots$(ii)

and $S_3 = S_{3n} = \frac{3n}{2}[2a + (3n-1)d] \quad \ldots$(iii)

Now, $S_2 - S_1 = \frac{2n}{2}[2a + (2n-1)d] - \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}[2\{2a + (2n-1)d\} - \{2a + (n-1)d\}]$

$= \frac{n}{2}[2a + (3n-1)d]$

$\therefore \quad 3(S_2 - S_1) = \frac{3n}{2}[2a + (3n-1)d]$

$\Rightarrow 3(S_2 - S_1) = S_3 \quad$ [using Eq. (iii)]

or $\quad S_3 = 3(S_2 - S_1) \quad$ **Hence proved.**

55. **Hint** Do same as Question 54.

56. **Hint** $S_1 = 2 + 4 + 6 + \ldots + 2n$

$= \frac{n}{2}[2 \times 2 + (n-1)2] = n(n+1)$

$S_2 = 1 + 3 + 5 + \ldots + 2n - 1$

$= \frac{n}{2}[2 \times 1 + (n-1) \times 2] = n^2$

$\therefore \quad \frac{S_1}{S_2} = \frac{n(n+1)}{n^2} = \frac{n+1}{n} = 1 + \frac{1}{n}$

$\therefore \quad S_1 = \left(1 + \frac{1}{n}\right)S_2$

57. **Hint** Let a_k denotes the kth term of the given AP. Then,

$a_k = a + (k-1)d$

Now, let S_1 = Sum of odd terms

$\Rightarrow \quad S_1 = a_1 + a_3 + a_5 + \ldots + a_{2n+1}$

$\Rightarrow \quad S_1 = \frac{n+1}{2}(a_1 + a_{2n+1})$

$\Rightarrow \quad S_1 = \frac{n+1}{2}[a + a + (2n + 1 - 1)d]$

$\Rightarrow \quad S_1 = (n+1)(a + nd)$

and $\quad S_2$ = Sum of even terms

$\Rightarrow \quad S_2 = a_2 + a_4 + a_6 + \ldots + a_{2n}$

$\Rightarrow \quad S_2 = \frac{n}{2}(a_2 + a_{2n})$

$\Rightarrow \quad S_2 = \frac{n}{2}[(a + d) + \{a + (2n-1)\}d]$

$\Rightarrow \quad S_2 = n(a + nd)$

Clearly, required ratio $= S_1 : S_2$

$= (n+1)(a + nd) : n(a + nd)$

$= (n+1) : n \quad$ **Hence proved.**

58. Yes, total number of times the bell will be struck in first

$12 \text{ h} = \frac{12(12+1)}{2} = 78$

$\therefore$ Total number of times the bell will be struck in

$24 \text{ h} = 2 \times 78 = 156$

59. **Hint** Given, $\frac{a + 10d}{a + 17d} = \frac{2}{3}$

Then, find $\frac{a + 4d}{a + 20d}$ and $\frac{\frac{5}{2}(2a + 4d)}{\frac{21}{2}(2a + 20d)}$ **Ans.** 1 : 3; 5 : 49.

60. **Hint** Let a be the first term and d be the common difference of an AP.

$$m\text{th term} = \frac{1}{n}$$

$$\text{So}, a_m = a + (m-1)d = \frac{1}{n} \quad \ldots\text{(i)}$$

$$n\text{th term} = \frac{1}{m}$$

$$\text{So}, a_n = a + (n-1)d = \frac{1}{m} \quad \ldots\text{(ii)}$$

On subtracting Eq. (ii) from Eq. (i), we get

$$a + (m-1)d - a - (n-1)d = \frac{1}{n} - \frac{1}{m}$$

$$(m-1-n+1)d = \frac{m-n}{mn}$$

$$\Rightarrow \quad (m-n)d = \frac{(m-n)}{mn}$$

$$\Rightarrow \quad d = \frac{1}{mn}$$

From Eq. (i), we get

$$a + (m-1)\cdot\frac{1}{mn} = \frac{1}{n} \Rightarrow a = \frac{1}{mn}$$

Now, $a_{mn} = a + (mn-1)d$

$$= \frac{1}{mn} + (mn-1)\cdot\frac{1}{mn}$$

$$= 1$$

61. **Hint** Given that, tree grow 5 cm or 0.05 m less than preceding year.

$\therefore$ The following sequence can be formed.

$$1, (1-0.05), (1-2\times 0.05), \ldots, 0$$

i.e. $1, 0.95, 0.90, \ldots, 0$ which is an AP.

Here, $a = 1, d = 0.95 - 1 = -0.05$ and $l = 0$

Let $\quad l = a_n = a + (n-1)d$

Then, $0 = 1 + (n-1)(-0.05)$

Ans. 21

Hence, in 21 yrs, tree will have ceased growing.

62. Equate the p th term of the first arithmetic progression, a_p, to the q th term of the second arithmetic progression, b_q, as $a_p = b_q$.

Since, $a_p = b_q$

$$\Rightarrow \quad a + (p-1)\times d_1 = b + (q-1)\times d_2$$

where a, b are the first terms and d_1, d_2 are the common differences of the given arithmetic progressions respectively.

$$\Rightarrow \quad 14 + (p-1)\times 7 = 26 + (q-1)\times 13$$

$$\Rightarrow \quad (p-1)7 - 13(q-1) = 12$$

$$\Rightarrow 7p - 13q = 12 + 7 - 13 = 6$$

$$\Rightarrow \quad 7p - 13q = 6$$

63. Considers $d_1 = d_2 + 4$ or $d_2 = d_1 + 4$, where d_1 and d_2 are the common differences of the two arithmetic progressions.

Also, $a_{124} = b_{42} \Rightarrow a_1 + 123d_1 = b_1 + 41d_2$

where a_1, a_{124}, b_1 and b_{42} are the 1st, 124th, 1st and 42nd terms of the two arithmetic progressions, respectively.

$$\Rightarrow \quad a_1 + 123d_1 = a_1 + 41d_2 \quad [\because a_1 = b_1]$$

Solve the above two equations and the value of $d_1 = \ldots$ and $d_2 = -6$ or $d_1 = 2$ and $d_2 = 6$.

One set of possible value of common difference is as follows

$$a_{124} = b_{42}$$

$$\Rightarrow a_1 + 123(4 + d_2) = b_1 + 41d_2$$

$$\Rightarrow \quad 492 + 123d_2 = 41d_2 \Rightarrow d_2 = -6$$

64. Let A be the first term and D be the common difference of AP.

$$T_p = a = A + (p-1)D = (A-D) + pD \quad \ldots\text{(i)}$$

$$T_q = b = A + (q-1)D = (A-D) + qD \quad \ldots\text{(ii)}$$

$$T_r = c = A + (r-1)D = (A-D) + rD \quad \ldots\text{(iii)}$$

Here, we have got two unknowns A and D which are to be eliminated.

We multiplying Eqs. (i), (ii) and (iii) by $q-r, r-p$ and $p-q$ respectively,

$$a(q-r) = (A-D)(q-r) + Dp(q-r)$$

$$b(r-p) = (A-D)(r-p) + Dq(r-p)$$

$$c(p-q) = (A-D)(p-q) + Dr(p-q)$$

On adding above three equations., we get

$$a(q-r) + b(r-p) + c(p-q)$$

$$= (A-D)[q-r+r-p+p-q]$$

$$+D[pq - pr + qr - pq + rp - rq]$$

$$= 0$$

65. **Hint** Consider, $\dfrac{1}{a_1} + \dfrac{1}{a_n} = \dfrac{a_1 + a_n}{a_1 \cdot a_n}$

$$\frac{1}{a_2} + \frac{1}{a_{n-1}} = \frac{a_2 + a_{n-1}}{a_2 \cdot a_{n-1}}$$

$$= \frac{a_1 + d + a_n - d}{a_2 \cdot a_{n-1}}$$

$$[\because a_n - a_{n-1} = d \Rightarrow a_{n-1} = a_n - d]$$

$$= \frac{a_1 + a_n}{a_2 \cdot a_{n-1}}$$

Similarly, $\dfrac{1}{a_3} + \dfrac{1}{a_{n-2}} = \dfrac{a_1 + a_n}{a_3 \cdot a_{n-2}}$

$$\vdots \qquad \vdots \qquad \vdots$$

$$\frac{1}{a_n} + \frac{1}{a_1} = \frac{a_1 + a_n}{a_1 \cdot a_n}$$

On adding all the above, we get

$$2\left[\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}+\ldots+\frac{1}{a_n}\right]$$

$$=(a_1+a_n)\left[\frac{1}{a_1\cdot a_n}+\frac{1}{a_2\cdot a_{n-1}}+\frac{1}{a_3\cdot a_{n-2}}+\ldots+\frac{1}{a_n\cdot a_1}\right]$$

$$\Rightarrow \frac{1}{a_1\cdot a_n}+\frac{1}{a_2\cdot a_{n-1}}+\frac{1}{a_3\cdot a_{n-2}}+\ldots+\frac{1}{a_n\cdot a_1}$$

$$=\frac{2}{a_1+a_n}\left(\frac{1}{a_1}+\frac{1}{a_2}+\ldots+\frac{1}{a_n}\right)$$ **Hence proved.**

66. Hint $-4, -1, 2, \ldots, x$ forms an AP with first term, $a=-4$ and common difference $d=-1-(-4)=3$

Also, last term, say $a_n = x$

$\because$ nth term of an AP, $a_n = a+(n-1)\,d$

$\Rightarrow \quad x=-4+(n-1)\,3$

$\Rightarrow \quad n=\dfrac{x+7}{3}$ ...(ii)

$\because$ Sum of first n terms, $S_n=\dfrac{n}{2}\,[2a+(n-1)\,d]$

$$S_{\frac{x+7}{3}}=\frac{x+7}{2\times 3}\left[2(-4)+\left(\frac{x+4}{3}\right)\cdot 3\right]$$

$\Rightarrow \quad x^2+3x-2650=0$

Hence, $\quad x=50, -53$ $\quad$ [$\because$ sum $=437$]

$\therefore \quad x=50$

67. Hint Let a_1, a_2, a_3, a_4 be the first four terms of AP.

$\therefore \quad a_1+a_2+a_3+a_4=32$

$a+a+d+a+2d+a+3d=32$

[$\because a_1=a_1, a_n=a+(n-1)\,d$]

$\Rightarrow \quad 4a+6d=32$

$\Rightarrow \quad 2a+3d=16$ ... (i)

Now, $\quad \dfrac{a_3}{a_4}=\dfrac{7}{8}$

$\Rightarrow \quad \dfrac{a+2d}{a+3d}=\dfrac{7}{8}$

$\Rightarrow \quad 8a+16d=7a+21d$

$\Rightarrow \quad a=5d$ ... (ii)

From Eq. (i),

$2a+3d=16$

$2(5d)+3d=16$

$10d+3d=16$

$13\,d=16$

$\therefore \quad d=\dfrac{16}{13}$

Then, $\quad a=5\times\dfrac{16}{13}=\dfrac{80}{13}$

$\therefore \quad a=\dfrac{80}{13}, a+d=\dfrac{80}{13}+\dfrac{16}{13}=\dfrac{96}{13},$

$a+2d=\dfrac{80}{13}+\dfrac{32}{13}=\dfrac{112}{13}$

$a+3d=\dfrac{80}{13}+\dfrac{48}{13}=\dfrac{128}{13}$

68. Given, first term, $a=22$

last term, $l=-6$

and sum of n terms, $S_n=64$

$\Rightarrow \quad \dfrac{n}{2}[a+l]=64$

$\Rightarrow \quad \dfrac{n}{2}[22-6]=64$

$\Rightarrow \quad n=8$

Now, $l=a+(n-1)d$,

where d is the common difference

$\Rightarrow \quad -6=22+7d$

$\Rightarrow \quad 7d=-28$

$\Rightarrow \quad d=-4$

69. Given, $S_7=63$

$\therefore \quad \dfrac{7}{2}[2a+6d]=63$ $\quad \left[\because S_n=\dfrac{n}{2}[2a+(n-1)d]\right]$

$\Rightarrow \quad 2a+6d=18$ ...(i)

Also, sum of next 7 terms is given 161.

$\therefore S_{14}=S_{\text{first } 7}+S_{\text{next } 7}=63+161=224$

$\Rightarrow \quad \dfrac{14}{2}[2a+13d]=224$

$\Rightarrow \quad 2a+13d=32$ ...(ii)

On subtracting Eq (i) from Eq. (ii), we get

$(2a+13d)-(2a+6d)=32-18$

$\Rightarrow \quad 7d=14$

$\Rightarrow \quad d=2$

Putting $d=2$ in Eq (i), we get

$2a+6\times 2=18$

$\Rightarrow \quad 2a=18-12$

$\Rightarrow \quad 2a=6$

$\Rightarrow \quad a=3$

$\therefore$ Required AP is $a, a+d, a+2d, a+3d, \ldots$

i.e. $\quad 3, 3+2, 3+2\times 2, 3+3\times 2, \ldots$

or $\quad 3, 5, 7, 9, \ldots$

70. Hint $a_{18}+a_{19}+a_{20}=225$

$a_{35}+a_{36}+a_{37}=429$

Ans. 3, 7, 11, 15...

71. **Hint** $a = \frac{-4}{3}, d = -1 - \left(-\frac{4}{3}\right) = \frac{1}{3}$

Find $a_9 + a_{10}$ **Ans.** 3

72. **Hint**
$$S_p = S_q$$
$$\frac{p}{2}[2a + (p-1)d] = \frac{q}{2}[2a + (q-1)d]$$
$$\Rightarrow \quad p[2a + (p-1)d] = q[2a + (q-1)d]$$
$$\Rightarrow (p-q)[2a + (p+q)d - d] = 0$$
$$\Rightarrow \quad 2a + (p+q-1)d = 0$$
$$\Rightarrow \quad S_{p+q} = 0$$

73. **Hint** From AP, we have
$$d = b - a$$
$$c = a + (n-1)(b-a)$$
$$\Rightarrow \quad n = \frac{c + b - 2a}{b - a} \quad \ldots(i)$$

and now we have to prove that $\left[S_n = \frac{(b + c - 2a)(a + c)}{2(b-a)}\right]$

Then, LHS $= S_n = \frac{n}{2}[2a + (n-1)d]$
$$= \frac{n}{2}[2a + (n-1)(b-a)]$$
$$= \frac{c + b - 2a}{2(b-a)}[2a + c - a] \quad \text{[using (i)]}$$
$$= \frac{(c + b - 2a)}{2(b-a)}(a + c) = \text{RHS}$$ **Hence proved.**

74. **Hint** Do same as Question15 of NCERT Folder Exercise 5.3. **Ans.** ₹ 7250

75. Let the first term be A and common difference D.

Now, $T_p = a \Rightarrow A + (p-1)D = a$
$$\Rightarrow \quad (A - D) + pD = a \quad \ldots(i)$$
and $T_q = b \Rightarrow A + (q-1)D = b$
$$\Rightarrow \quad (A - D) + qD = b \quad \ldots(ii)$$
On subtracting Eq. (ii) from Eq. (i), we get
$$(p-q)D = a - b$$
$$\Rightarrow \quad D = \frac{a-b}{p-q} \quad \ldots(iii)$$
On adding Eqs. (i) and (ii), we get
$$2A + (p + q - 2)D = a + b$$
$$\Rightarrow 2A + (p + q - 1)D = a + b + D$$
$$= a + b + \frac{a-b}{p-q}$$
[from Eq. (iii)] ...(iv)

Now, $S_{p+q} = \frac{p+q}{2}[2A + (p + q - 1)D]$
$$= \frac{p+q}{2}\left[a + b + \frac{a-b}{p-q}\right]$$ [from E

76. (i) **Hint** Sum of AP $= \frac{n}{2}[2a + (n-1)d]$, where a is term, d is common difference and n is number terms of an AP.

(ii) ₹ 1831.5

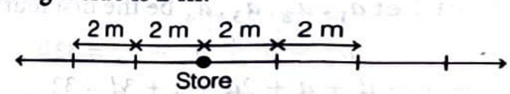
77. (i) **Hint** Clearly, the amount (in ₹) of instalment in first month, second month, third month...are, respectively, 1000, 1100, 1200, 1300, ... which fo an AP, with first term, $a = 1000$ and common difference, $d = 100$.

Now, amount paid in 30th instalment,
$$a_{30} = 1000 + (30 - 1)100$$
$$= ₹\,3900$$

(ii) Amount paid in 30 instalments,
$$S_{30} = \frac{30}{2}[2 \times 1000 + (30 - 1)100]$$
$$= ₹\,73500$$
Remaining amount of loan that he has to pay
$$= ₹\,118000 - ₹\,73500$$
$$= ₹\,44500$$

78. **Hint** Do same as Question 19 of NCERT Folder Exercise 5.3. **Ans.** 16 Rows 15 Bricks.

79. (i) Since, distance between the store and first flag towards right side is 2 m.



∴ Distance covered by Ruchi to place the flag fir towards right side of the store = 4 m

Distance covered by Ruchi to place the second fla towards right side of the store = 8 m

Distance covered by Ruchi to place the third flag towards right side of the store = 12 m

Similarly, distance covered by Ruchi to place the thirteenth flag towards right side of the store
$$= a + 12d = 4 + 12 \times 4 = 52 \text{ m}$$
Now, total distance covered in completing the job
$$= 2S_{13} = 2\left\{\frac{13}{2}(2 \times 4 + 12 \times 4)\right\}$$
$$= 13(8 + 48)$$
$$= 13 \times 56 = 728 \text{ m}$$

(ii) Maximum distance covered by Ruchi for carrying a flag is 52 m.

Mind Map

ARITHMETIC PROGRESSION (AP)

Sequence A list of numbers arranged in a definite order according to some rule is called sequence.
.e.g. 2, 4, 6, 8, ... each number is obtained by adding 2 to the preceding number, except first number.

Arithmetic Progression A list of numbers in which each term is obtained by adding a fixed number to the preceding term except the first term is called an arithmetic progression.
e.g. 3, 8, 13, 18, 23,...... is an AP, as each term is obtained by adding 5 to the preceding term.
Generally, $a_1, a_2, a_3, \ldots\ldots$ is an AP, where a_1 is the first term, a_2 is the second term and so on. Its nth term is denoted by a_n.
The fixed number is called the common difference (d)
i.e. $d = a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \ldots$
The terms of an AP are as follows
$a_1 = a$
$a_2 = a + d$
$a_3 = a + 2d$
| |
and so on.

nth Term of an AP If a is the first term and d is the common difference, then its nth term is
$a_n = a + (n-1)d$
The nth term of an AP is called its general term or last term l.
$\therefore\ l = a_n = a + (n-1)d$

nth Term from the End of an AP
Let a be the first term and d be the common difference and l be the last term of an AP, then nth term from the end can be found by the formula
nth term from the end $= l - (n-1)d$

Sum of First n Terms of an AP
If first term of an AP is a and its common difference is d, then the sum of its first n terms is given by
$S_n = \frac{n}{2}[2a + (n-1)d]$
$= \frac{n}{2}[a + a_n]$
where, a_n is nth term of an AP.
$= \frac{n}{2}[a + l]$ $[\because l = a_n]$