CS302: Modeling And Simulation
Lab 7 Report

Rajdeep Pinge 201401103, Aditya Joglekar 201401086

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1 Cellular Automata Simulation

In this lab we will use cellular automaton approach to study heat conduction in rod. The relevant chapter (Diusion: Overcoming dierences) from the book is provided as reference. You are supposed to understand the discussion in the chapter and attempt at least two of project questions at the end of the chapter. While you are encouraged to write your own code, you can use the codes shared at the book's webpage as reference. If you are using the shared codes then in your report please state it clearly and provide relevant reference.

NOTE: The Code for the basic heat diffusion problem has been taken from the book's webpage for reference. The link to the code is as follows: https://ics.wofford-ecs.org/toolbox/MATLAB

2 Introduction to the basic model.

The model adopts a cellular automaton approach to model the diffusion of heat through a bar.

The cells in the cellular automaton represent the average temperature over a small local area of the rod.

The update rule is inspired from Newton's law of cooling in which the change in temperature is proportional to the difference in the temperatures.

Also, it is assumed that only the neighbouring cells influence the change in the temperature of a cell.

The update rule for Δ T (change in temperature) is as follows:

$$\Delta T = \sum_{n}(T_n - Tcell)r \tag{1}$$

Here, T_n is the temperature of a neighbouring cell and the summation is done over the Moore neighbourhood [N,S,E,W,NE,NW,SE,SW]. $Tcell$ is temperature at the current cell. r is the coefficient of diffusion which controls how fast the diffusion basically happens. So, the local change in temperature is dependent on the overall difference in Temperature of the current cell and its neighbours.

The basic model uses reflecting boundary conditions wherein the edge rows and columns are extended to ensure that even borderline cells have the entire neighbourhood defined. For an n x n matrix, we first reflect the row 1 upwards and the last row downwards. Then using the the n+2 dimensional last column to the right and n+2 dimensional first column to the left. This gives us a n+2*(m+2) matrix in which each cell of the bar has a neighbour.

At every iteration, each cell updates its temperature according to the above mentioned rule. Initially, we expect a faster change in temperature. But as the temperature diffuses, it leads to a decrease in the temperature differences throughout the rod and one would expect it to go to an equilibrium position. However, we have some special regions known as the heat sources and sinks which are always at the same temperature no matter what. These constant regions maintain differences in temperature and delay the establishment of an equilibrium. Comparing it with an opinion dynamics model, these heat sinks and sources are analogous to the fanatics who never change their opinion. There is an interesting analogy between heat diffusion and how opinions spread. We will try to gain insights using our intuitions about the latter!

The following sections and their respective subsections include the analysis of the projects mentioned in the book at the end of section 10.2: Diffusion: Overcoming Differences

3 Project 2. Develop simulations and animations for the bar modelled in this module using several boundary conditions: three simulations of absorbing boundary conditions with constant values 0, 25, and 50 and periodic boundary conditions. Along with the reflecting boundary conditions, describe the results.

3.1 Simulation using Reflecting boundary Condition

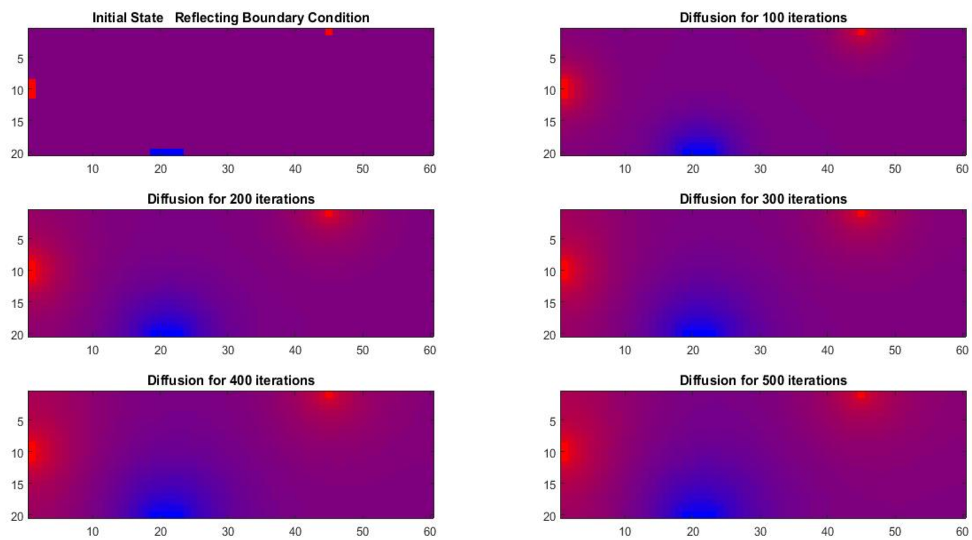

Figure 1: Simulation of heat diffusion for the suggested initial condition with heat sources and sinks(reflecting boundary conditions).

Observations:

- Figure 1 shows the heat diffusion for the suggested initial conditions under reflecting boundary conditions.
- We immediately notice that as time passes, the heat sources and sinks start influencing the temperatures in their neighbourhood. The cells around them begin to update their temperatures accordingly because the sources and sinks are the only ones which have a temperature difference with the cells at least in the beginning.
- The sources and sinks ensure that their is always a heat gradient along which the diffusion can continue to happen until equilibrium is reached. One could imagine the sources/sinks to be fanatical clerics who are hellbent on converting the masses to their faith!

- The hot and cold regions thus spread outward from the sources/sinks.
- An important thing to notice is the effect of diffusion decreases as one goes away from the heat sources and sinks. This is because the cells nearby them first need to change their temperature and then only will they be able to propagate it further. These cells may not be as hot/cold[extreme!] as the sources/sinks. Hence the subsequent change they cause on their neighbours is thus lesser in intensity. Again this has analog to how beliefs spread. A fanatic might convert a person but that person may not be as motivated to spread the belief! Also as we move away from the epicentres, the extremity goes on decreasing.
- Also notice what happens at the interface of a hot and a cold region. These regions tend to stay close to the equilibrium conditions. This is because the hot and the cold regions effect on these cells cancels out. It is if regions which are caught between two opposing opinions tend to become apathetic to them!
- Here we have used reflecting boundary conditions. This means that the extended extra rows/columns are very similar to the actual row/columns on the edges.
- Thus, they will have minimal effect on the simulation because the more similar the cells are the less do they contribute to the diffusion. This is in line with what we want. We would not like boundary conditions drastically influencing the outcomes of the experiment.
- Later, we will examine the effect of other boundary conditions on the simulation.

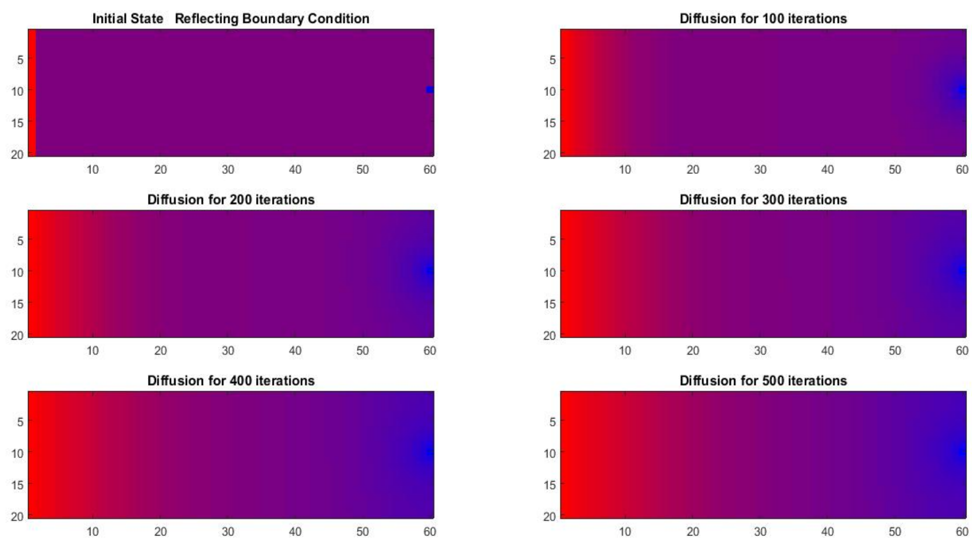

Figure 2: Simulation of heat diffusion for the suggested initial condition with heat sources and sinks(reflecting boundary conditions).

Observations: Heat source at left edge and sink point source on the right

- Here we compare the behaviour of a line source vs a point sink.
- Notice that the line source diffuses heat in horizontal sweeps while the point source leads to semi-circular diffusion.
- Apart from that there is not much to comment on this situation.

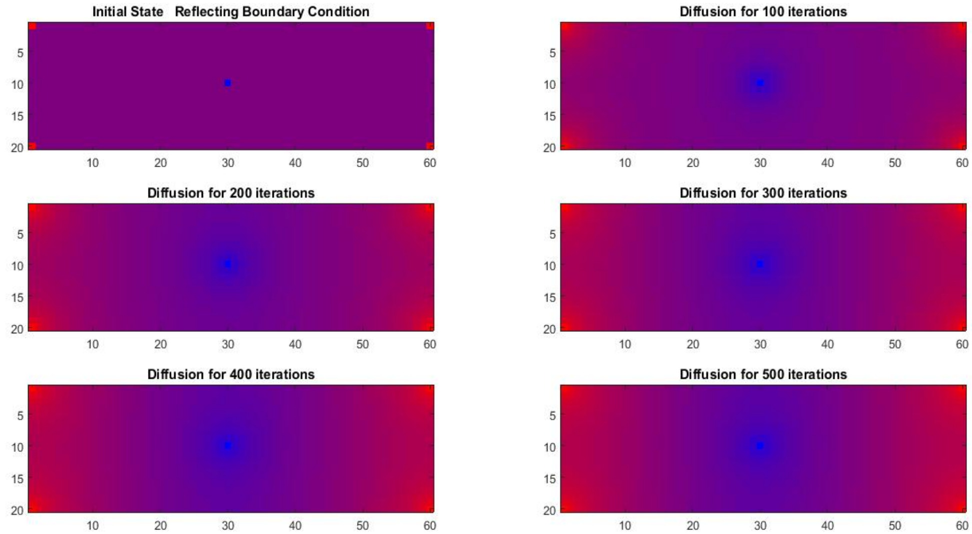

Figure 3:

Observations:

- Here the heat sources try to invade the cooler region in the middle.
- The equilibrium condition is that the sink leads to the development of a cool region in between while the rest is heated up by the heat sources.
- If we run it for a longer duration we would expect the cold region to decrease.

3.2 Simulation using Periodic boundary Condition

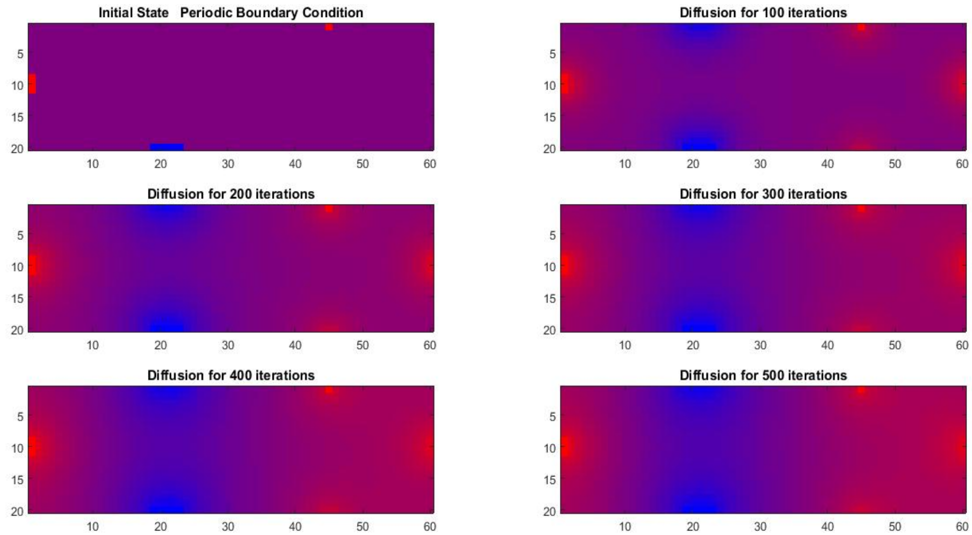

Figure 4: Simulation of 1st case for Periodic Boundary Condition

Observations:

- This is not the correct representation of the behaviour of the system.
- Here due to the periodic condition, the effect of the heat source spill over to the opposite edge which is not correct.
- Periodic conditions give rise to spatial relations which are not physically present. For a small bar this might be problematic.

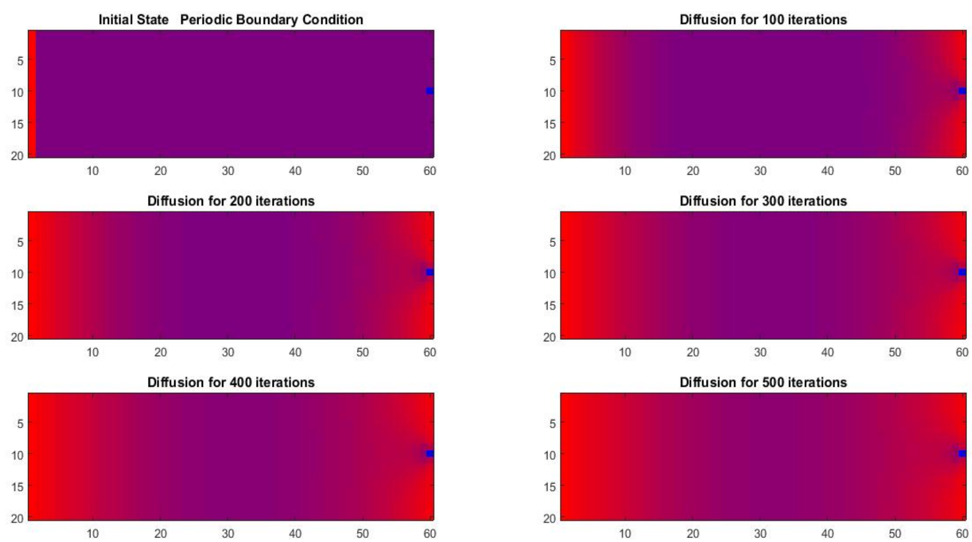

Figure 5: Simulation of 2nd case for Periodic Boundary Condition

Observations:

- Again this representations is not correct because the heat source on one side affects the area around the heat sink on the other side of the the bar.

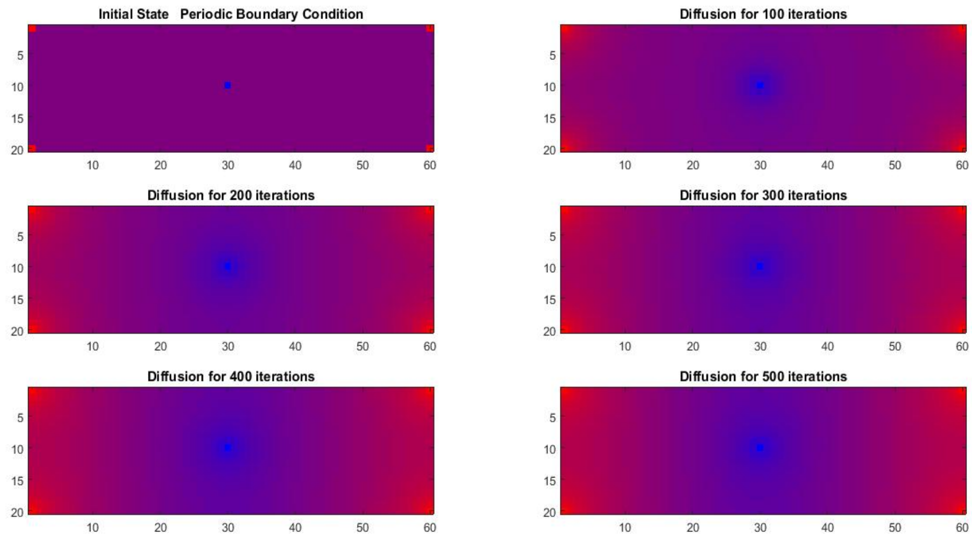

Figure 6: Simulation of 3rd case for Periodic Boundary Condition

Observations:

- In this case, the simulation is same as the reflecting boundary condition due to the symmetry in the situation.
- Hence the behaviour of the system is correctly captured.
- This shows that if the system is periodic or symmetric, then the periodic boundary condition can be used to get correct representations of the system

3.3 Simulation using Absorbing boundary Condition

3.3.1 For boundary at Temp = 0 °C

This condition characterizes the system residing in a cold surroundings.

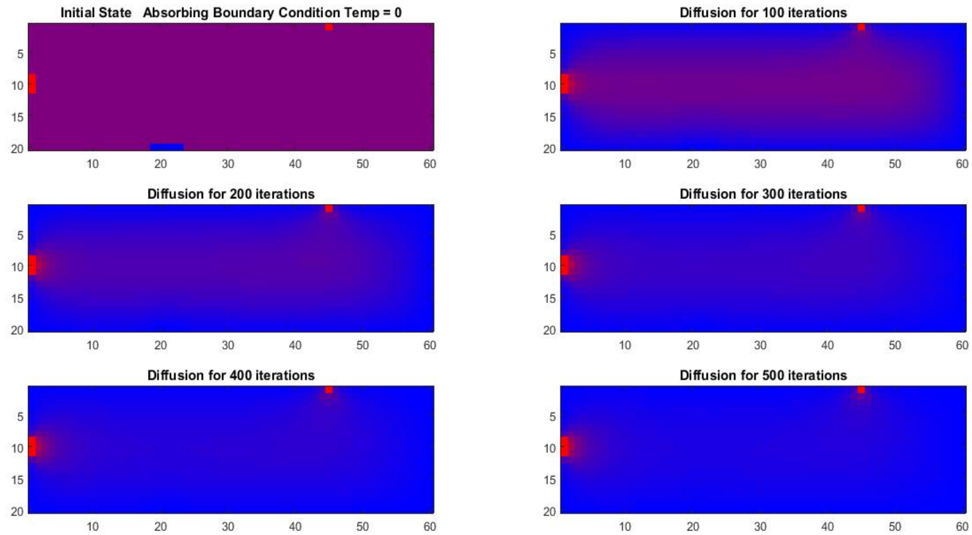

Figure 7: Simulation of 3rd case for Absorbing Boundary Condition with surrounding temp =0 °C

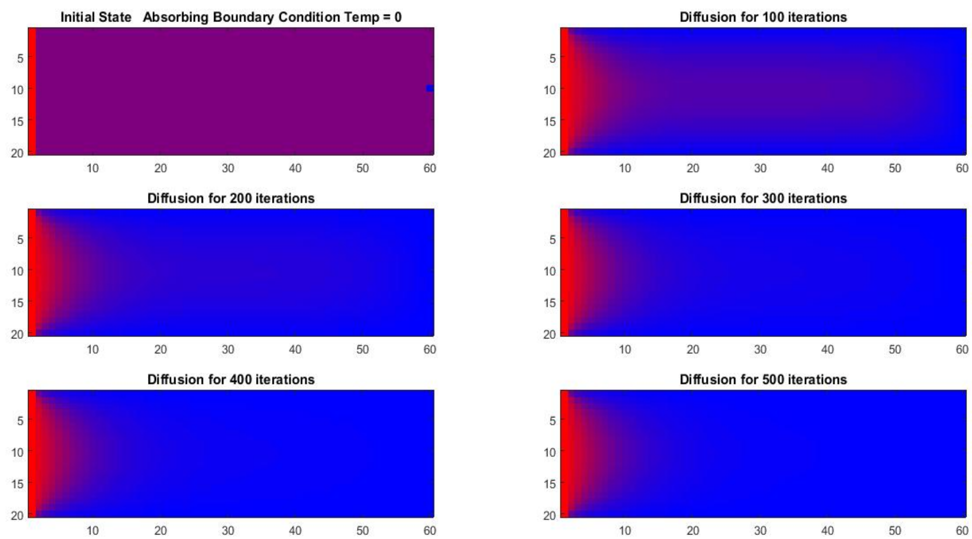

Figure 8: Simulation of 3rd case for Absorbing Boundary Condition with surrounding temp =0 °C

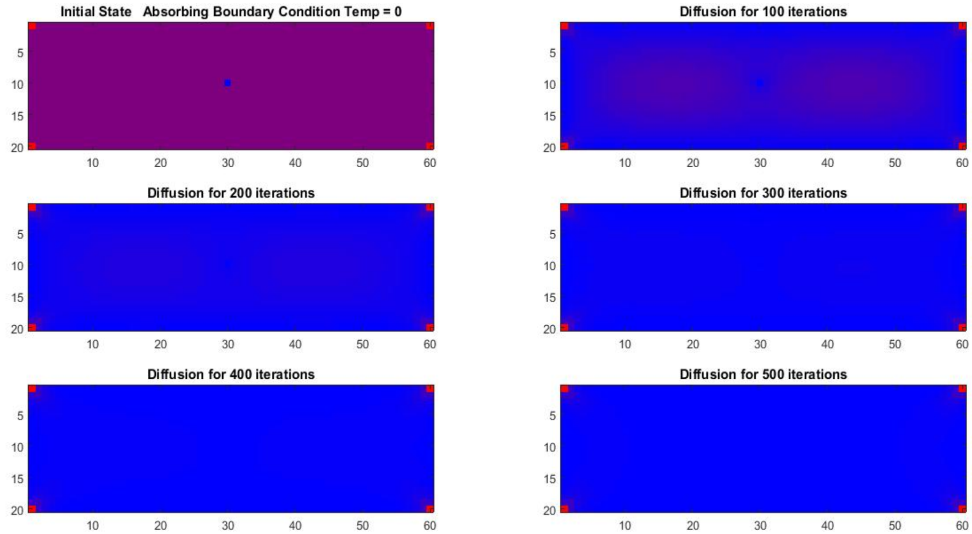

Figure 9: Simulation of 3rd case for Absorbing Boundary Condition with surrounding temp =0 °C

Observations:

- The absorbing boundary conditions lead to maximum influence on the simulation. We will see that changing this boundary condition has a drastic effect on the simulation.
- Here the boundary temperature is a cold sink.
- The situation is akin to keeping the bar wedged between two cold surfaces. Thus the bar quickly cools down to the absorbing conditions temperature
- The heat sources are simply not able to compete with the absorbing conditions.

3.3.2 For boundary at Temp = 50 °C

Here we can consider the system to be enclosed by a body at a higher temperature.

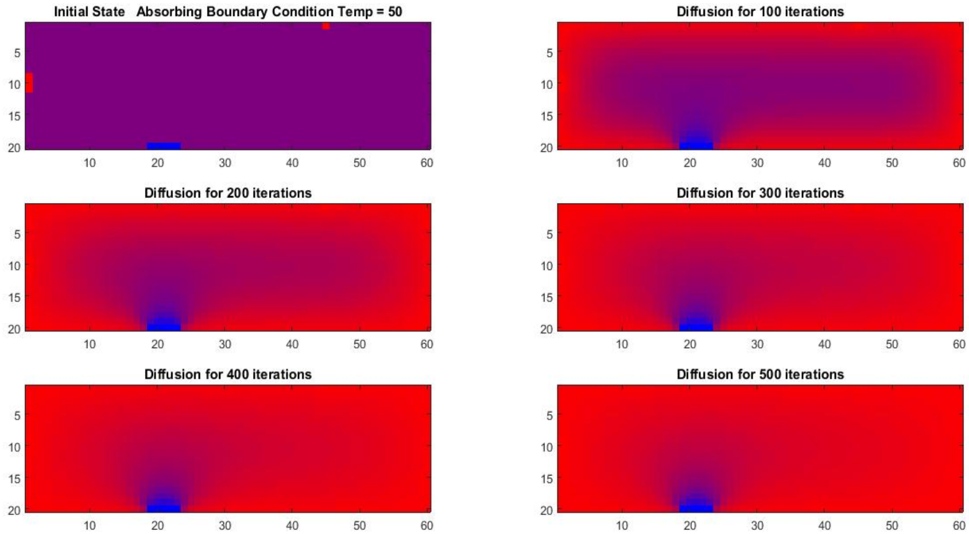

Figure 10: Simulation of 1st case for Absorbing Boundary Condition with surrounding temp = 50 °C

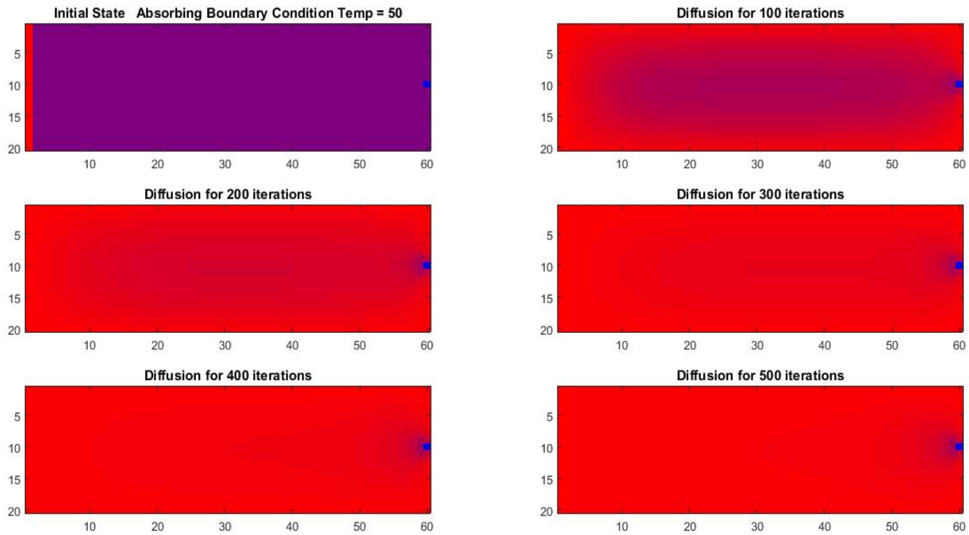

Figure 11: Simulation of 2nd case for Absorbing Boundary Condition with surrounding temp = 50 °C

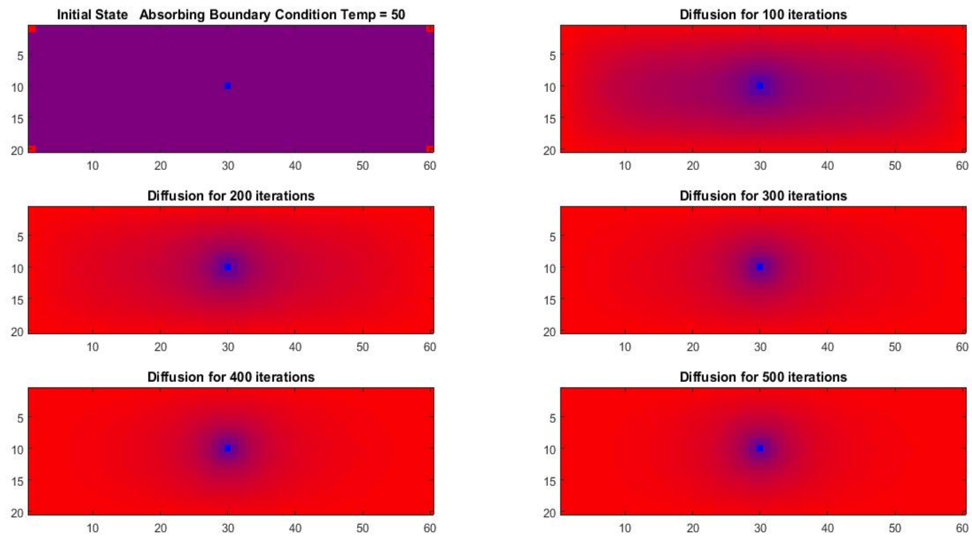

Figure 12: Simulation of 3rd case for Absorbing Boundary Condition with surrounding temp = 50 °C

Observations:

- This is the exact opposite of Fig.7. Here the absorbing conditions favour the bar heating up, this is like keeping a bar in an oven.
- Thus, the sink is restricted only to a small region in the center.

3.3.3 For boundary at Temp = 25 °C

This is the situations in which the surrounding temperature is the room temperature.

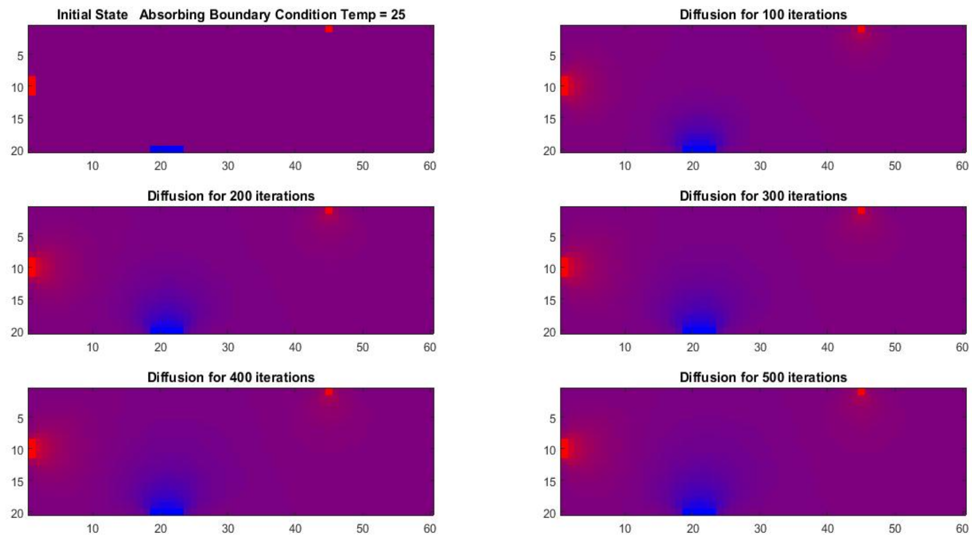

Figure 13: Simulation of 1st case for Absorbing Boundary Condition with surrounding temp = 25 °C

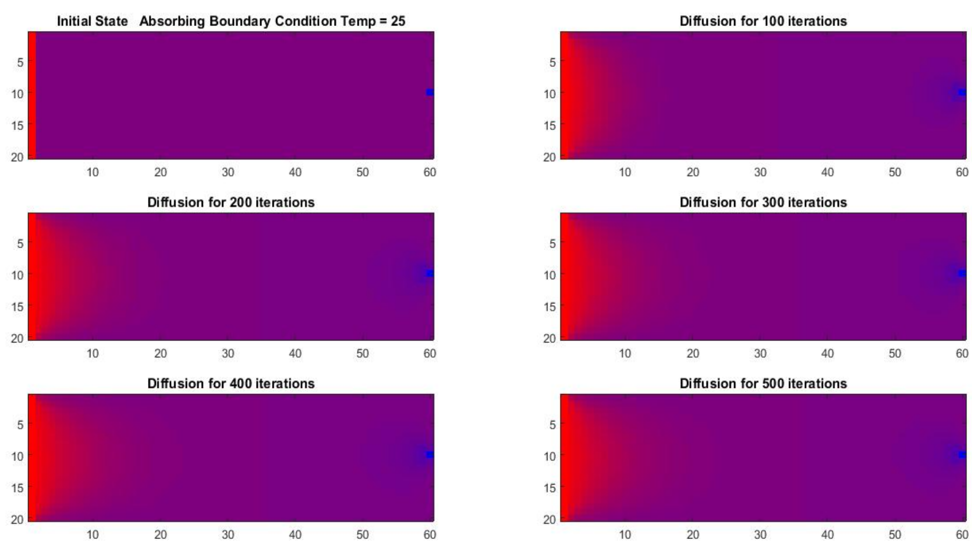

Figure 14: Simulation of 2nd case for Absorbing Boundary Condition with surrounding temp = 25 °C

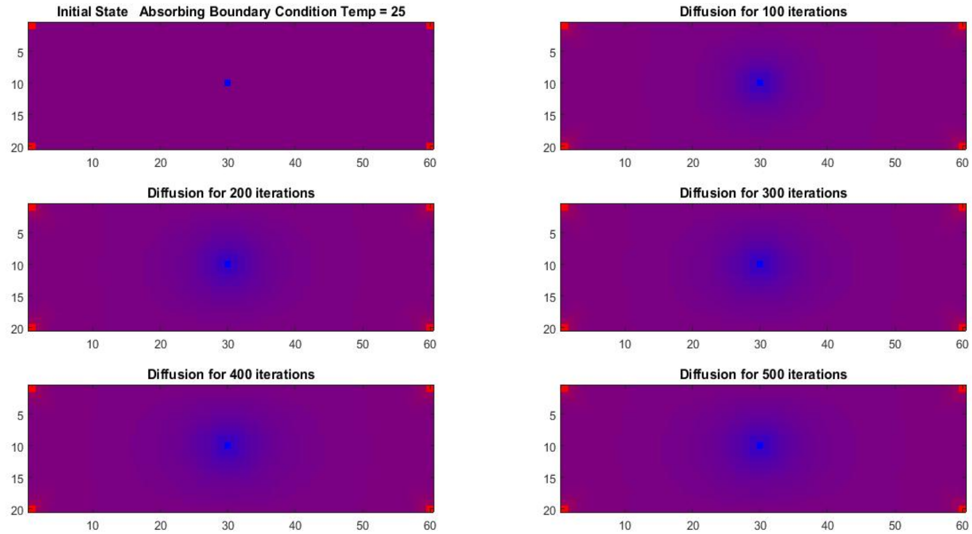

Figure 15: Simulation of 3rd case for Absorbing Boundary Condition with surrounding temp = 25 °C

Observations:

- Here, the absorbing conditions is 25 ie right in between the source and sink temperatures.
- The interesting thing here is that this boundary condition leads to an inhibition of both the heat source and sinks temperature diffusion. This is because it keeps the boundary temperature from rising too much and thus the heat source and sink are not able to influence the temperature much.
- To conclude, for the given experiment, for a limited simulation area, reflection conditions are what should be used as they allow the heat source and sink to influence the heat diffusion and not bias the simulation by the boundary conditions. We must remember that the boundary conditions are just to keep things consistent at the edge of the simulation grid, there certainly should not determine the outcome of the experiment.

3.4 Discuss the advantages and disadvantages of each approach and the situations, such as heat or pollution diffusion, for which each is most appropriate.

3.4.1 Reflecting Boundary Condition:

Advantages:

- Here the surroundings is considered to be at the same temperature as the edge of the system.
- This is useful in situations where the effect on the boundary of the object is quickly transferred to the environment.

Disadvantages:

- Not useful for systems where there is no contact between the system and the surroundings.

Situations:

- This can be used to model situations like air conditions where the air just outside A/C is almost at the same temperature as the air inside A/C.
- Another example is the ionization effect. When electricity or lightning passes through air, it ionizes the atoms nearby. This phenomenon can be modelled by this condition.

3.4.2 Absorbing Boundary Condition:

Advantaged:

- This boundary condition is most beneficial when surroundings are at constant temperature.
- It shows the effect of constant surrounding temperatures on the object.

Disadvantages:

- The effect of environment is much dominant and it can even constrain the effect of sources and sinks of the system.

Situations:

- It can model situations like a refrigerators, large coolers, closed ovens where the system is enclosed by constant temperature.

3.4.3 Periodic Boundary Condition:

Advantages:

- With help of this condition we can model periodic effects happening in the system.
- We can also use this condition to model the continuous behaviour of systems in 2D.

Disadvantages:

- Here, the effect of heat flow can't be modelled properly.
- One of the fundamental requirements of the periodic boundary condition is that it required the system to be very large. So that there is a continuum effect from one edge to the other when the condition is applied.
- unfortunately in problems like heat diffusion when the effect is local and on very small scale, this condition is not accurate and may give wrong interpretations as observed above.

Situations:

- This can be used in situations like heating effect in a circular rod as observed in 2D.
- It can also be used in the situations like conservation of a quantity such as energy so that the effect of conservation or continuity of the system can be modelled.

4 Project 3. Instead of using the formula for diffusion in the section "Heat Diffusion," we employ a filter. Thus, to obtain a value at a site for time t + 1, we add 25% of the site's temperature at time t, 12.5% of the north, east, south, and west cells at time t, and 6.25% of the corner cells to the northeast, southeast, southwest, and northwest. This sum is called a weighted sum with each nutrition value carrying a particular weight as indicated by the table. Revise the model using this configuration and compare the results with that of the module.

Here the diffusion condition has been changed and a filter is applied whereby we take weighted sum of the temperatures of the cells in the neighbourhood along with the cell itself to find out the temperature of the cell at the next instant.

As seen in the previous part, the reflecting boundary condition gives the best representation of the actual situation. Hence we will compare the two models by using reflecting boundary condition for different initial states.

We have 3 starting cases on which we compare the results.

4.1 Results on given starting configuration

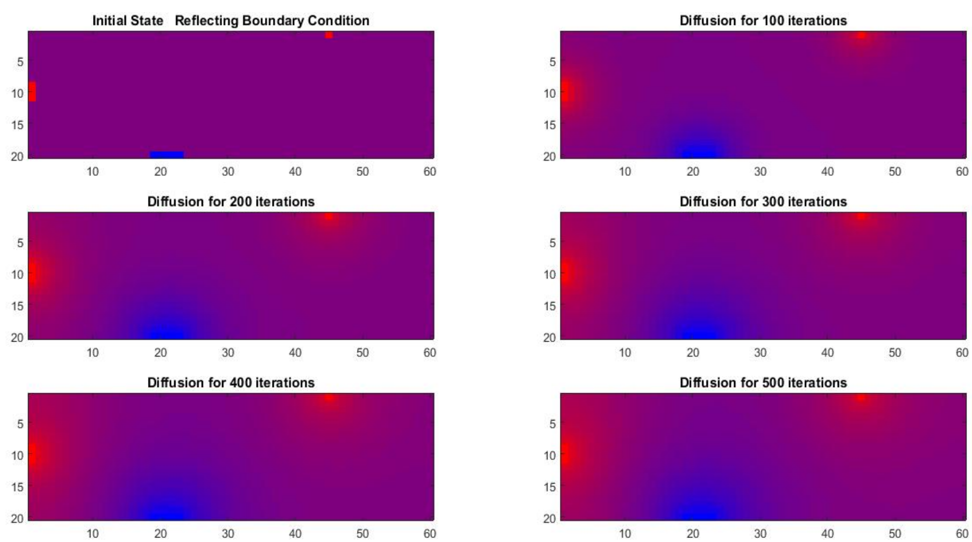

Figure 16: Simulation of bar when normal diffusion condition is applied.

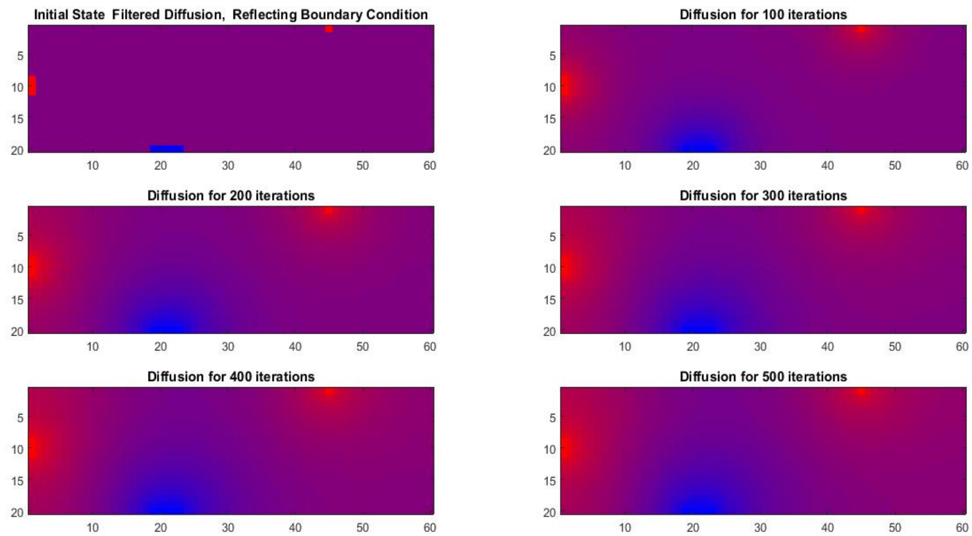

Figure 17: Simulation of bar for filtered diffusion.

Observations:

- Comparing the above two cases, we see very little difference in the rate of diffusion
- The minute difference is not recognizable to human eye.

4.2 Results on starting configuration where heat is applied on left boundary while the cold part is fixed at midpoint on the opposite boundary

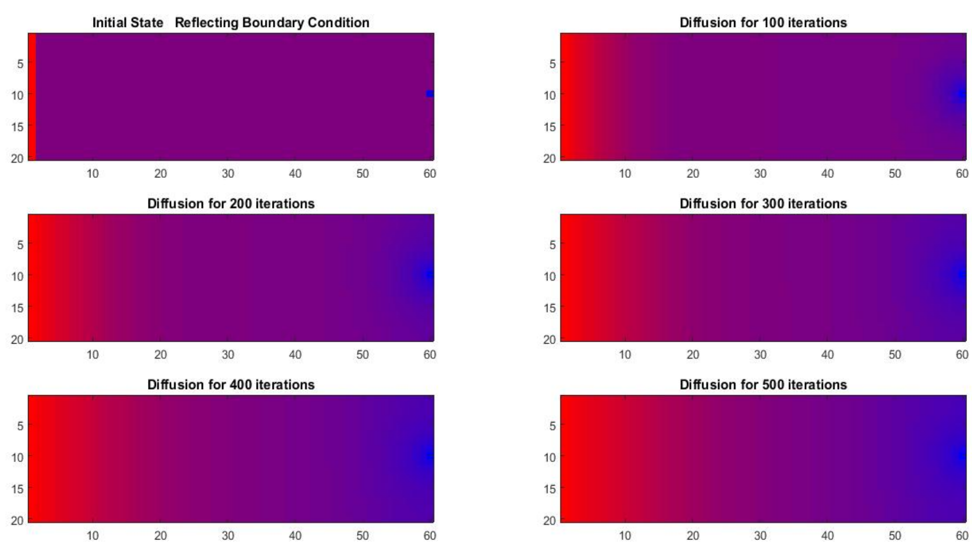

Figure 18: Simulation of bar when normal diffusion condition is applied.

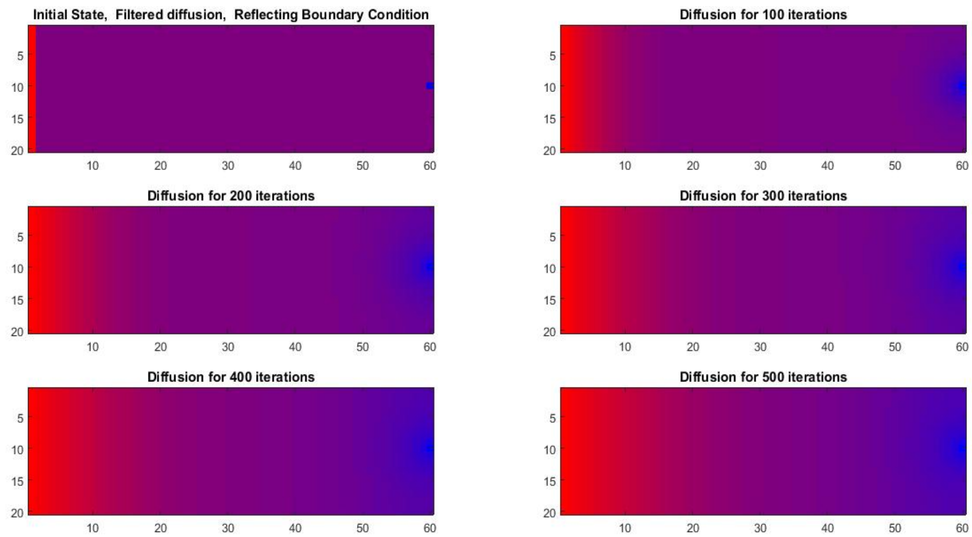

Figure 19: Simulation of bar for filtered diffusion.

Observations:

- Both these situations also match to a large extent.
- But when observed closely, we can see some difference in the diffusion in the cold part at the 500th time step. Here in the first case, i.e the normal diffusion case, the diffusion is slightly more than the weighted average case.

4.3 Starting configuration is such the the hot region is at the 4 corners of the bar while the cold region is at the midpoint of the bar.

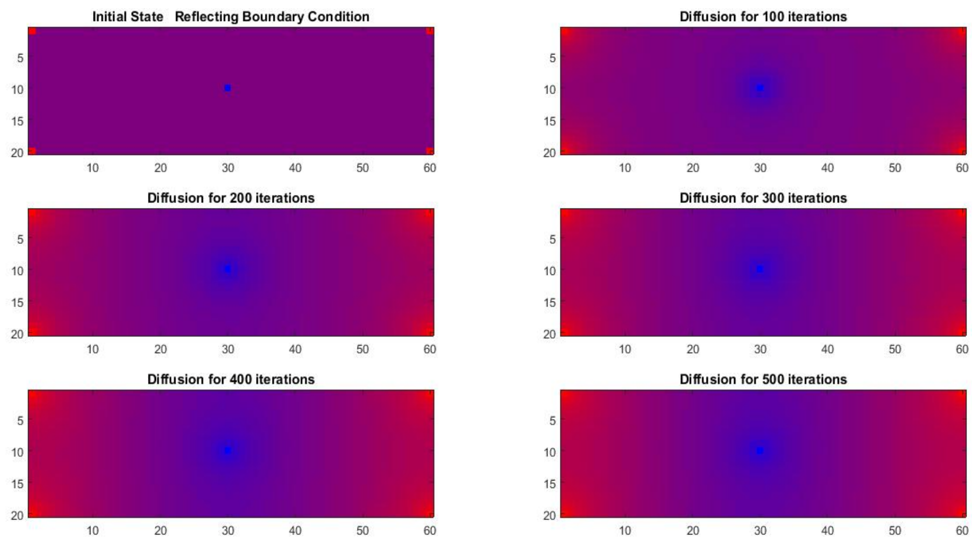

Figure 20: Simulation of bar when normal diffusion condition is applied.

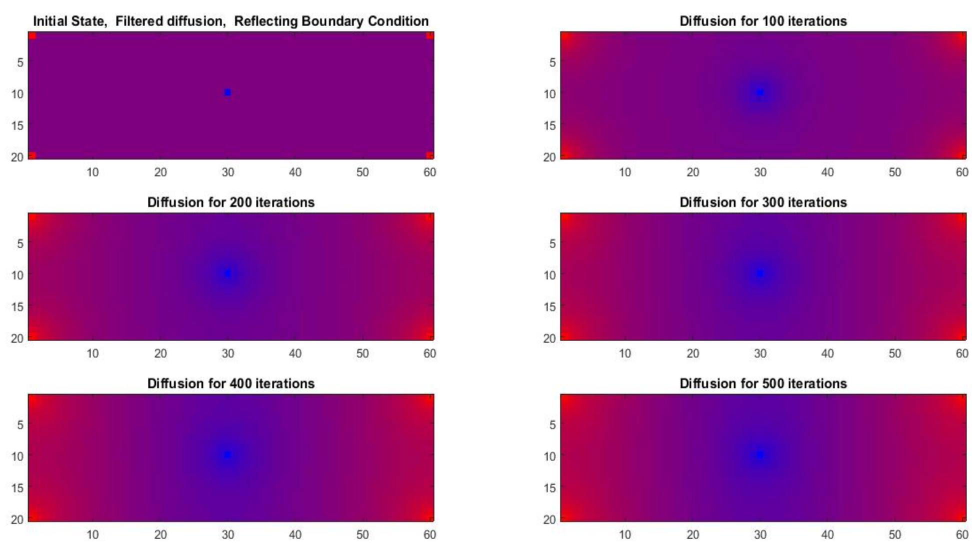

Figure 21: Simulation of bar for filtered diffusion.

Observations:

- In this part also there is no appreciable difference between the two diffusion methods.

4.4 Inference:

All the 3 cases show almost no difference between the two methods of diffusion used. This must be because the coefficients used in the normal diffusion method and the weighted average are almost the same. Hence the difference balances out over large time.

We tweak the weights in the weighted average case to check if we observe any change. Here we take 80% of the site's own temperature, 4% of the temperature of its neighbours on North, East, South and West and 1% contribution of the remaining 4 diagonal neighbours.

Since we got slight difference in the 2nd case, we will make further observations on that case.

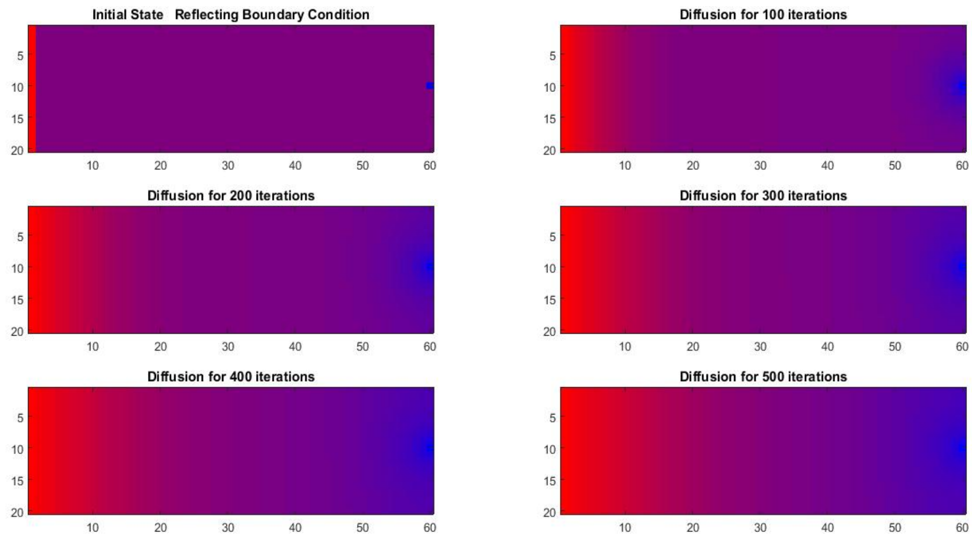

Figure 22: Simulation of bar when normal diffusion condition is applied.

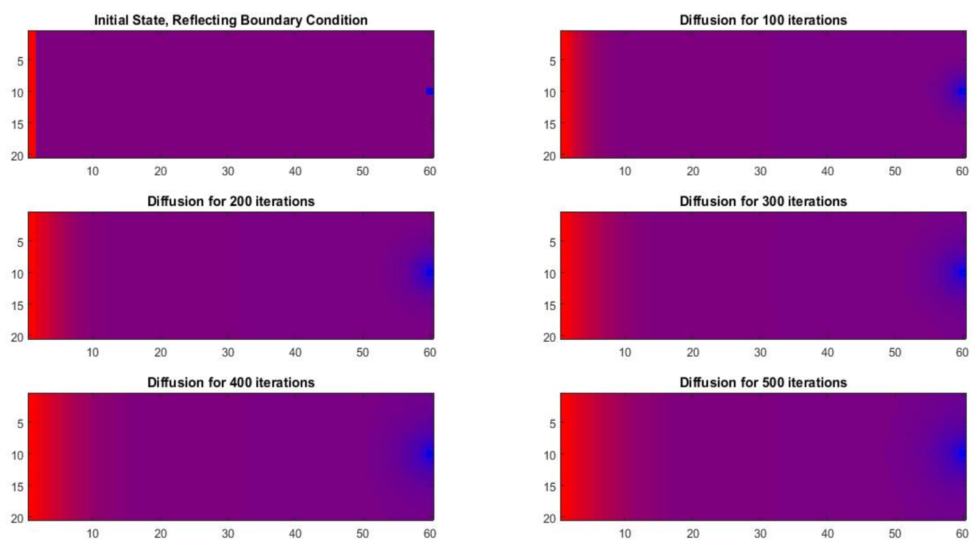

Figure 23: Simulation of bar for filtered diffusion where diffusion condition has been modified.

Observations:

- In the above two graphs we see significant visible difference. This shows that when the weighted average is taken, the system behaviour changes according to the weights applied.

  In reality, application of weights is just the significance level of the neighbours and hence based on cell itself and its neighbours, the changes in the cell are determined.

  Taking analogy of opinion dynamics, the normal diffusion conditions based on Newton's law of Cooling ensures that the next state of the cell that is the opinion of the person at the next time instant is based on the difference between the opinion of the person and the people in the neighbourhood while the weighted sum condition implies that the opinion at the next time instant includes all the opinions in the neighbourhood with certain importance.

  Hence if the person is stubborn and is less likely to change his/her opinion, then the diffusion will be less as can be seen in the weighted sum case while in the difference of opinion case, the shift largely depends on the neighbourhood opinion.

5 Project 4. Effect of External Source on temperature of bar which is initially at a constant temperature

In this part the bar is at a constant uniform temperature initially. An external source of constant temperature is applied on the boundary of the bar. To model this, we take the absorbing boundary condition and consider the surrounding to be at the temperature of the external source and then simulate the situation.

5.1 Model a bar at 100 °C that has a constant application of a 25 °C external source on its boundary. Generate plots of the temperatures at a corner and in the middle of the bar versus time. Describe the shapes of the graphs.

Here the bar is at 100 °C while the temperature of the extended boundary of ghost cells is taken to be the temperature of the external source which is same for all the cells at 25 °C. Hence we use absorbing boundary condition.

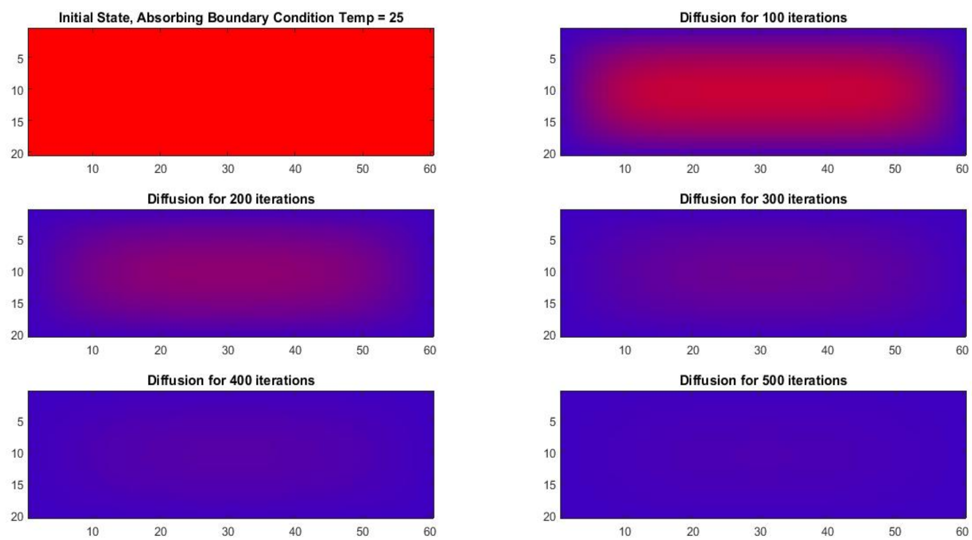

Figure 24: Simulation of bar when an external constant source is applied.

Observations:

- This simulation shows how the state of the bar changes over the time.
- Since the outside temperature is much less that the temperature of the bar, The bar eventually cools down.
- The edges cool down first and then the lack of heat propagates to the middle.

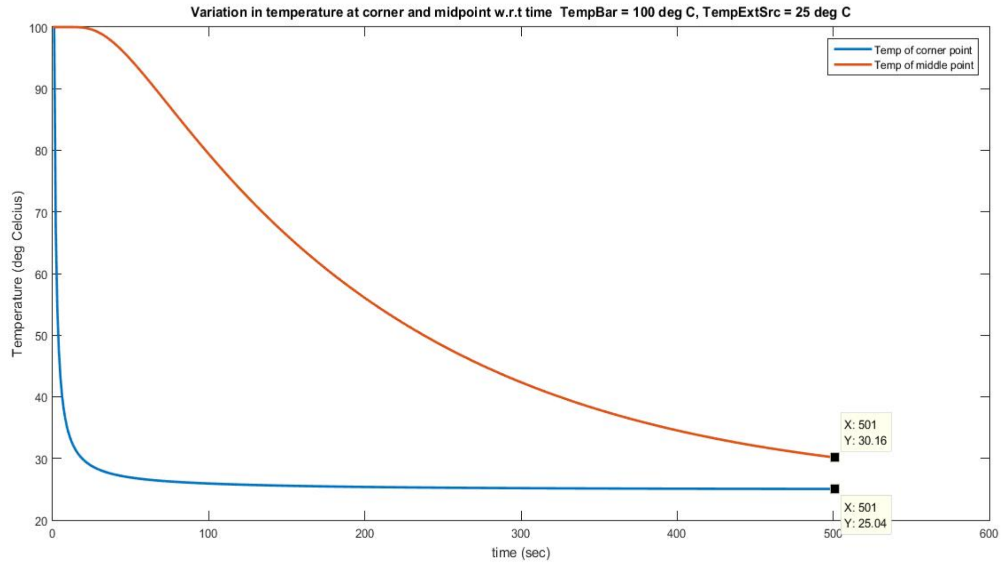

Figure 25: Plots of the temperatures at a corner and in the middle of the bar versus time.

Observations:

- The plots in the above graph show the temperature of two different points on the bar
- Both the points are initially at the same temperature which is the temperature of whole bar at that time instant.
- Over the course of time, the point at the edge of the bar cools down much faster. This is because it is in direct contact with the external source and hence diffusion is faster.
- The point in the middle cools down much slower because it takes a lot of time for the temperature difference between the bar and the surroundings to propagate to the centre. But Eventually, the midpoint also achieves the temperature of the surroundings.

5.2 Repeat Part a with the bar being at −50 °C.

Here the situation has been reversed with the external source being at a higher temperature than the temperature of the bar.

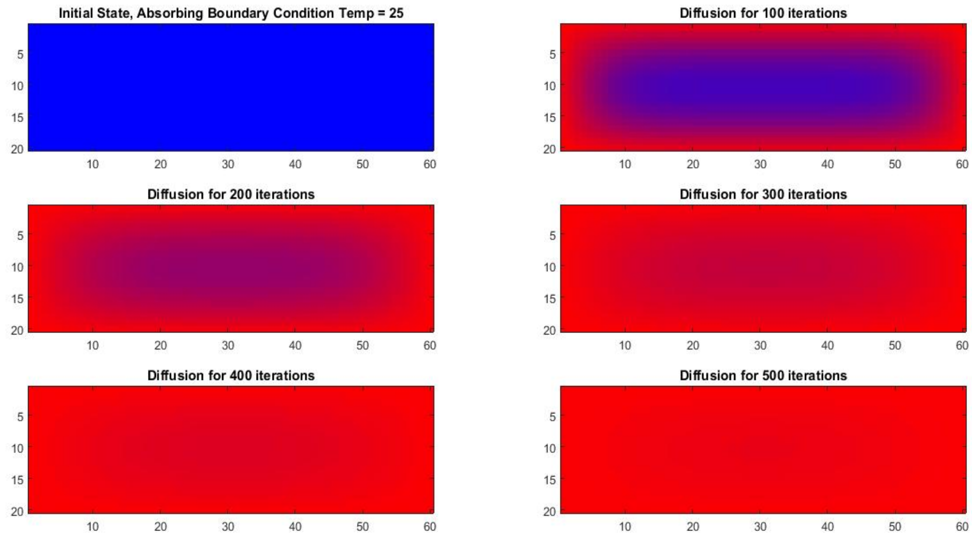

Figure 26: Simulation of bar when an external constant source is applied.

Observations:

- This simulation shows how the state of the bar changes over the time.
- Here the outside temperature is more and hence the bar heat up.
- The temperature of the edges increases first and then eventually due to diffusion of temperature, the centre part also heats up.

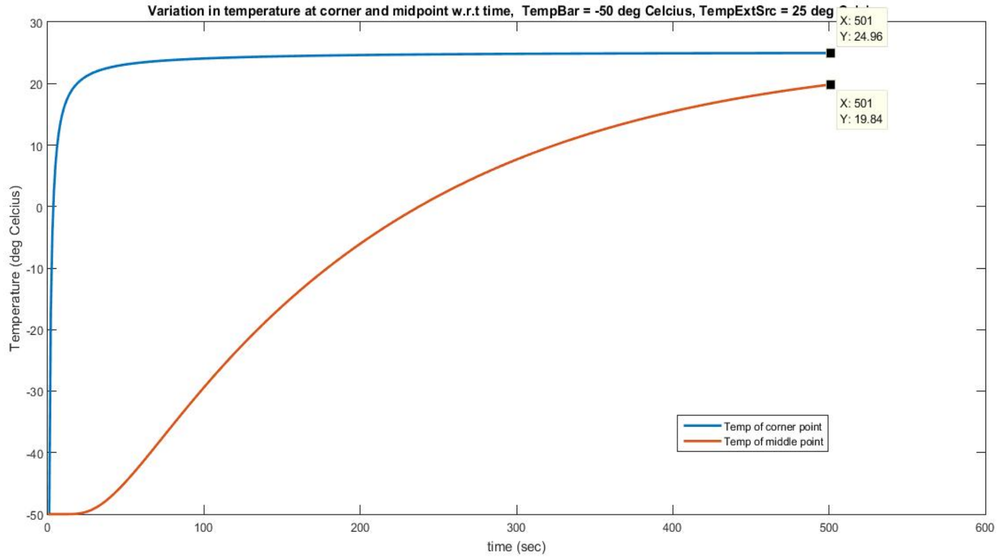

Figure 27: Plots of the temperatures at a corner and in the middle of the bar versus time.

Observations:

- Here we see the opposite behaviour. Here the temperature of the point on the bar increases as time progresses.
- Though the condition is reversed. The nature of the graph remains same in that the point on the edge heats up much faster than the the point in the middle.
- The point in the middle takes a lot of time to heat up because of slow propagation of heat towards centre from the surroundings. But eventually it reaches the temperature of the surroundings.

5.3 Discuss the results.

The results seen are quite intuitive. If the bar is at constant temperature initially and if a constant external source is applied to the bar, then the bar eventually reaches the temperature of the external source or the surroundings which is what is given by the Newton's law of Cooling.

If the temperature of the bar is less than external source then the heat propagates into the bar and the temperature of the bar increases. If the bar is at higher temperature, then heat flows out of the bar into the surroundings reducing the temperature of the bar.

In any case, the propagation of heat starts at the edge of the bar and then it dffuses inwards over time.