Lab 1 report

Aditya Joglekar 201401086 Rajdeep Pinge 201401103

January 2017

1

A)

The rate is high initially while it decreases to zero as time passes. This suggests that the equation to model would be as follows:

$$T' = -K(T(t) - T_0) \tag{1}$$

B)

We can solve the equation analytically to option the following equation

T = $T_r - e^{-kt+C}$

where C = $\ln(To - Tr)$

C)

We can obtain C from the initial conditions. However, we also need to determine the constant which governs the rate at which the liquid cools. For this we use the condition after 1 hour and substitute it in the analytical equation. K equals 1.333 . K must have units of 1 over time for the equation to make sense dimensionally.

D)

Substituting the required values in the equation gives the answer to be around 0.285 hours or around 17 minutes.

E)

Refer to Figure 22:. Clearly the numerical solution for time steps 1 and 0.5 deviate from the analytical solution. Though, time step equal to 0.05 is the most accurate, even time step equal ot 0.1 does a good job of approximating the solution. Hence, 0.1 will be a more sensible value for the time step.

2

A)

Here, we assume that the drug directly reaches the blood stream. Then the only thing we need to model is how does the drug dissipate from the blood. Assume that the outflow rate of drug is proportional to the amount of the drug present in the blood at an instant.

The rate equation will be:

$$y' = k_1 y \tag{2}$$

y is the amount of drug in blood stream. We seperate the variables and integrate.

$\int \frac{1}{y} dy = -k_2 \int dt$

$\ln y = -k_2 t + C$

C = $\ln(yo)$

y= yo $e^{-(}k_2 t)$

Thus, the drug level in blood is expected to decrease exponentially/

B)

Here, we assume that drug does not reach the blood instantaneously. **Refer to equations (3) and (4)**.

x can solved by variable seperation as in A) while we can solve for y using Laplace transform technique

Taking Laplace transform on both sides,

$$sL(y) - yo = \frac{k_1 xo}{s + k_1} - k_2 L(y) \tag{3}$$

Simplifying and collecting like terms gives us the equation

$$L(y) = \frac{yo}{s + k2} + K[\frac{1}{s + k1} - \frac{1}{s + k2}] \tag{4}$$

Taking inverse laplace transform gives

$$y = K[e^{-}k1t - e^{-}k2t] \tag{5}$$

This is basically the subtraction of an exponential term from another and the plot of this function agrees with the **Figure 2** obtained numerically.

C)

We need to model pills taken at regular intervals. We know that once a pill is taken it dissipates exponentially. When we take a new pill at a certain time, it simply causes a sudden spike in the value of the pill present at that time instant. We can model this by a sequence of the amount of drug after the $n^{th} dosage$.

For instance, consider the following equations

$Q_1 = Q_0$
$Q_2 = Q_0(1 + e^{-kt})$
$Q_3 = (Q_0(1 + e^{-kt}))e^{-kt} + Q_0$

Thus Q_n will contain a geometric series and thus simplifies to,

$$Q_n = \frac{Q_0}{1 - e^{-kt}} \tag{6}$$

Thus in the limiting case, the amount of drug in the blood will saturate to Q_0 which basically is the total dosage which is present in a single pill.

3

A)

This is the implementation for Q1 b). Here we assume that a single pill has been consumed and that it reaches the gastro intestinal track instantaneously. Then it starts getting absorbed into the blood stream. Using the standard compartment model, the rate of dissipation of the drug from the gt tract is assumed to be equal to the rate of absorbtion into the blood stream. This assumes, that all the drug is absorbed into the blood stream with no wastage.

The rate equations are as follows:

$$x' = -k_1 x \tag{7}$$

$$y' = k_1 x - k_2 y \tag{8}$$

x and y represent the concentration of the drug in the gastro-intestinal tract and blood respectively. We consider this system to be a two compartment model and assume that the rates of inflow and outflow are proportional to the amount of the drug present in the tract and the blood.

We run the simulation to produce the following plots for x and y with respect to time.

Refer to Figure 1 and 2, we see the expected behaviour of the level of drug according to the rate equations. In the gastrointestinal tract there is no inflow of the drug only outflow. Hence, the amount of drug in the gt tract

continues to fall until it reaches 0. In the beginning the rate of outflow is high as x is present in a larger quantity but as x decreases the rate also goes down. Hence, we get a graph going to zero with a slope which is levelling out in time

The equation for y is more interesting as there is both an inflow as well as an outflow. Initially, there is no drug present in the blood. Thus the rate of change of y is high, initially. However as time passes, the amount of the drug in the blood starts to increase and begins to counter the inflow. Also after a certain amount of time, the inflow of drug stops as the gt tract is completely exhausted of the drug, hence from this point onwards we only have an outflow in y. Thus this plot shows a behaviour similiar to Figure 1 from this point onwards. Thus, we have an increase to a maximum and then an exponential dissipation afterwards.

The maximum value of the drug in the blood is about 60% of the the amount present in the drug. This is a result of the delay in the drug from the gt tract to the blood stream.

B)

We now change the value of k2 keeping k1 constant. This essentially means that we keep the outflow of the drug from the gt tract unchanged and change the rate of dissipation of the drug in the blood. For a faster rate of outflow, we expect the graph to reach a maximum quicker and go down faster. The maximum reached should also be lesser. The overall nature of the graph should however remain similar.

Refer to Figure 3: Clearly for larger values of k2, a lesser maximum is reached faster followed by an exponential dissipation. The slope of descent too is steeper for a larger value of k2. For a larger k2, the amount of time the drug remains in the blood too is lesser. Note that k1 remains fixed and hence the behaviour in the gt tract remains unchanged.

C)

Now we keep k2 constant and instead vary k1. As k1 is present in both rate equations both the figures x and y will vary with respect to time.

Refer to Figure 4 and 5. For larger k1, the outflow rate from gt is higher. Hence we see a faster dissipation of the drug into the blood. Hence the the drug reaches the blood faster. When k1 dominates k2, the level of the drug in blood can reach upto 90% of the pill's content after which dissipation kicks in. For lower values of k1, the slope of y is much lesser as k1 does not dominate much. Hence it takes longer to reach the maximum and the value reached is lower.

D)

Here, we assume that there is a constant supply of the drug to the gt tract instead of a single discrete dose from a pill. The rate equations are thus as follows,

$$x' = C - k_1 x \tag{9}$$

$$y' = k_1 x - k_2 y \tag{10}$$

Refer to Figure 6 The rate of change in x higher initially. As x starts to rise, the rate starts decreasing till we reach the fixed point at x= C/k1

Refer to Figure 7 As y increases the rate goes on decreasing. Eventually as since x saturates, the inflow in the blood becomes constant. Once x saturates, y behaves exactly like x as the rate equations becomes similar. The increase in y continues to decrease the rate till we reach a fixed point. There is a point of inflection early on in y. This is because intially x incraeses faster and hence the inflow in the blood dominates. But soon, the increase in y starts decreasing the rate of y.

E)

Now we assume that the medicine is taken for half hour every 6 hours. Thus, in a period of 6 hours. The rate equation of x is of the form of for the first half hour eq (1) and of the form eq (3) for the rest of the period. Hence the behaviour in these periods is a combination of the behaviours seen above.

Figure 8:Clearly for the first half hour of every period of the dosage. The constant inflow to the gt causes X to increase. The increase in X also causes an increase in outflow from the gt to the blood. This leads to a steady decrease in the rate of change. After the dosage is stopped for the rest of the period, the amount of x continually decreases in a familiar exponential manner. Thus we get a figure similar to the saw tooth.

Figure 9: In every period, initially the rate of y with time is high. As y increases the rate starts decreasing. After the half hour dosage time, x starts decreasing and hence the inflow to the blood starts decreasing hence, the slope of y starts decreasing. However, before it can start its downward ascent, the next dosage starts and hence the periodic behaviour is repeated.

It can be mathematically shown that a series of such periods leads to y saturating to a certain value. This is because the value of y at any time, can be written as an exponential term with an offset. Thus we get a geometric sequence of exponential terms which adds up to a finite value after a long time has passed. **Figure 10:** Clearly shows for y saturates around a finite value.

F)

The given dosage will prove fatal when it hits the theauraptic limit after 126.5 hours.

G)

Figure 11: If the dosage is done with a period of 6 hrs, then the dosage does not become fatal. It saturates around 16 units.

For the given conditions

4

A)

This question is at least cosmetically similar to the previous problem. It is a little more complex considering that the rate equations depend on parameters which are influenced by gender, weight, age and the amount of food ingested

Refer to Figure 12 and 13

We have taken the weight of an average male to be around 80 kg and that of a female to be around 65 kg. For a male of 80 kg, on an empty stomach, with 3 drinks we get the referred plots. The amount of alcohol in the gt tract obviously goes down exponentially as expected. It is the rate equation for the g tract which is interesting.

$$x' = I - k_1 x \tag{11}$$

$$y' = k_2 x - \frac{k_3 y}{(y + M)}$$

(12)

For A) I is 0 as it is taken care of by an initial condition. Hence, first y increases quickly as y is less. As y starts to increase the rate starts to decrease. When x is depleted, y starts to decrease. If y is greater than M considerably, the slope becomes constant. That why the plot is a straight line for some time. When y becomes comparable to M the rate is less than 1 and decreases very slowly. Drinking alcohol here on an empty stomach will lead to a dizzy feeling

Refer to Figure 14 and 15. Keeping everything else same, for a full stomach, the concentration of alcohol in the blood is much lower. It is lower than 0.05 and hence will not lead to dizziness. Otherwise the behaviour is pretty similar.

Refer to Figure 16 and 17 I took the weight of the female to be around 65 kgs. The graphs obtained were identical. This was coincidental as the value of C turned out to be very close for both the 80 kg man and 65kg woman. So next I compared the results with that of an 80kg woman.

Refer to Figure 18 and 19 As the C for a woman is more, the concentration of alcohol ingested is lesser. But the outflow from the blood is also lesser. Hence, a woman of weight 80 on an empty stomach will be less intoxicated than a man but her intoxication will last a little longer as the outflow rate is lower.

We also noticed that the more weight you have the lesser intoxicated you will be. This is simply because your C will be higher and hence the concentration of alcohol will be much lower.

B)

Now for the continuous we replace I with a constant value indicating constant drinking.

Refer to Figure 20,21 Now if we supply a constant inflow of alcohol into the gi tract. Thus, the x continues to rise till it reaches a fixed point.

When, x saturates, the inflow for y becomes constant. Till then, y ¿¿ M, hence the overall rate of change of y becomes a positive constant. Hence, y will rise indefinitely.

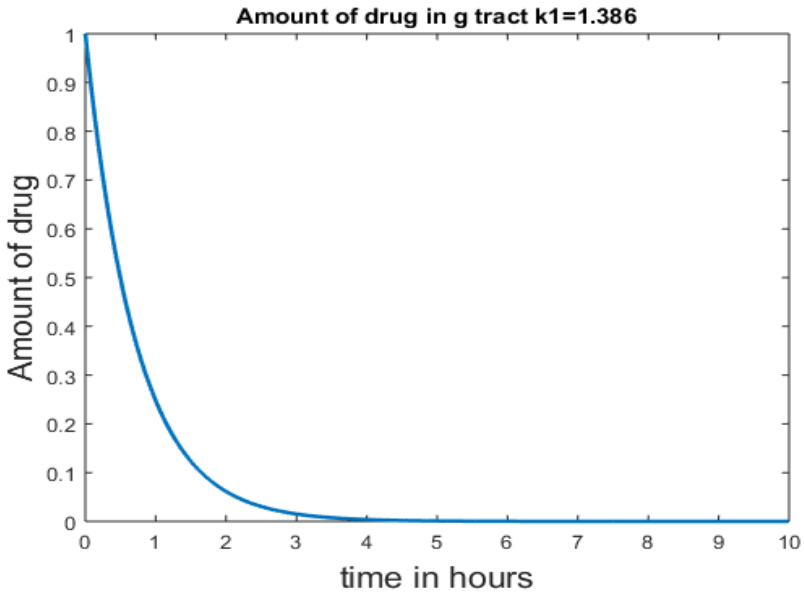

Figure 1:

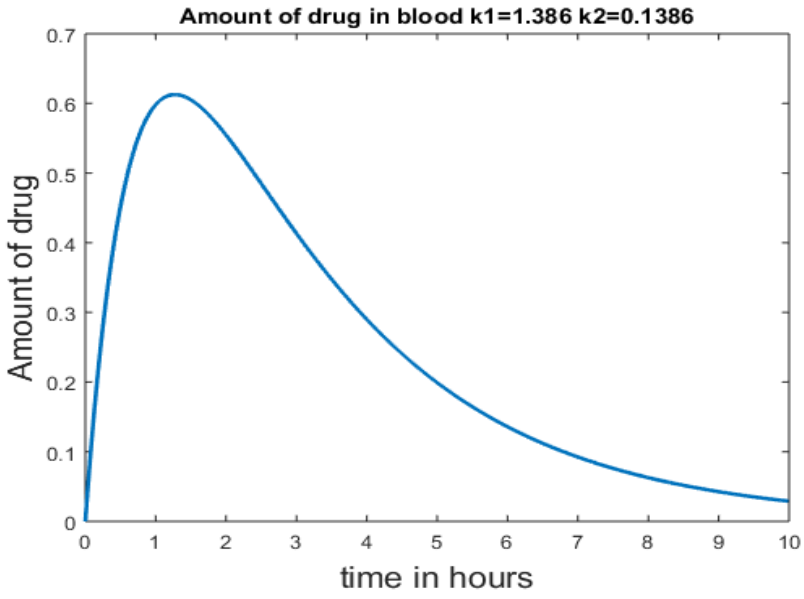

Figure 2:

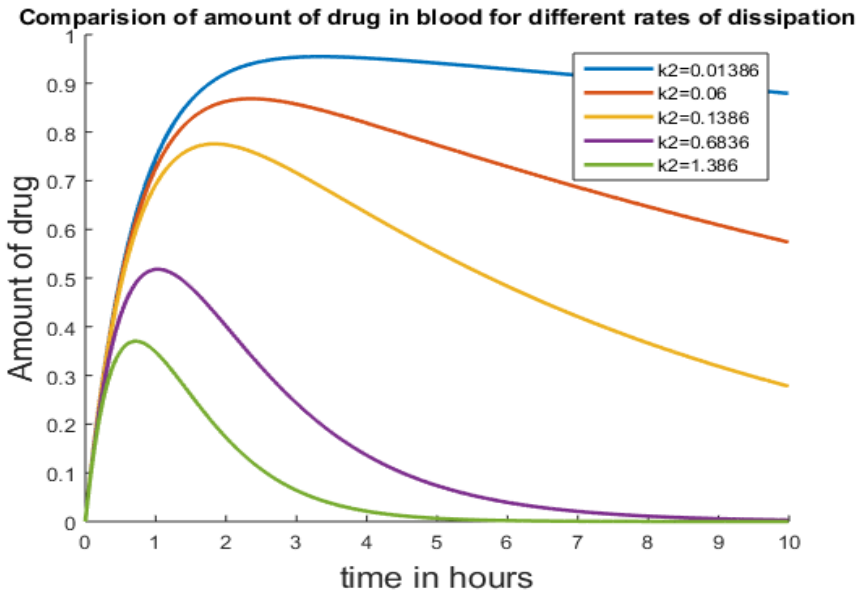
Comparision of amount of drug in blood for different rates of dissipation
k2=0.01386
k2=0.06
k2=0.1386
k2=0.6836
k2=1.386
Amount of drug
time in hours
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
1
2
3
4
5
6
7
8
9
10

Figure 3:

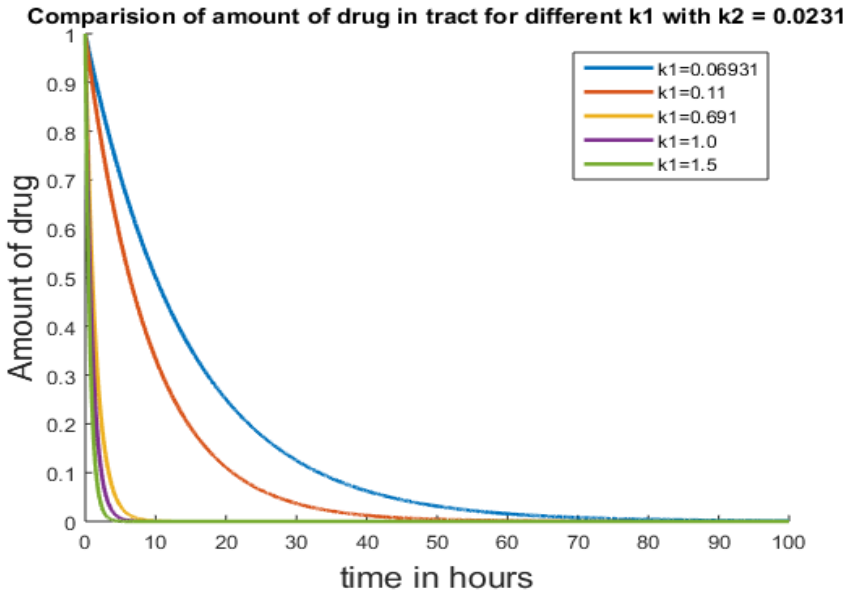

Figure 4:

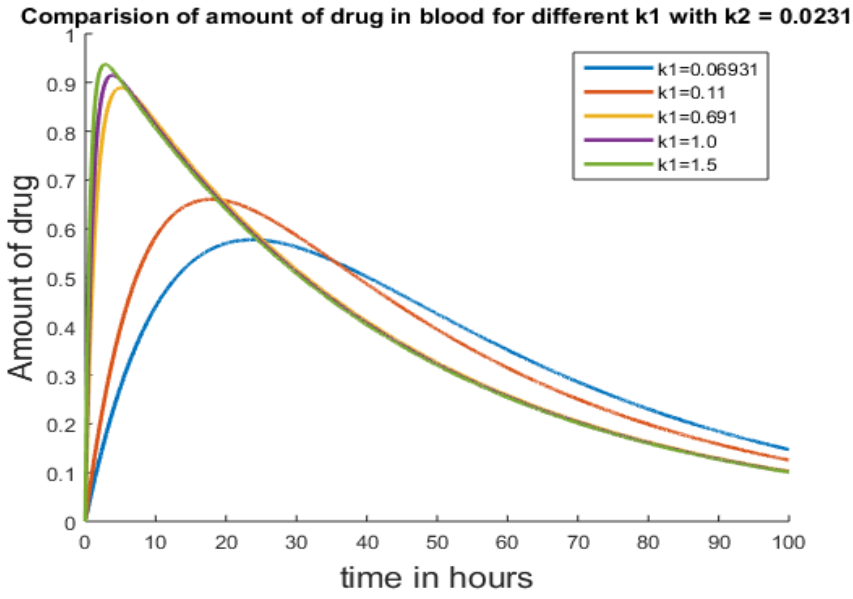

Figure 5:

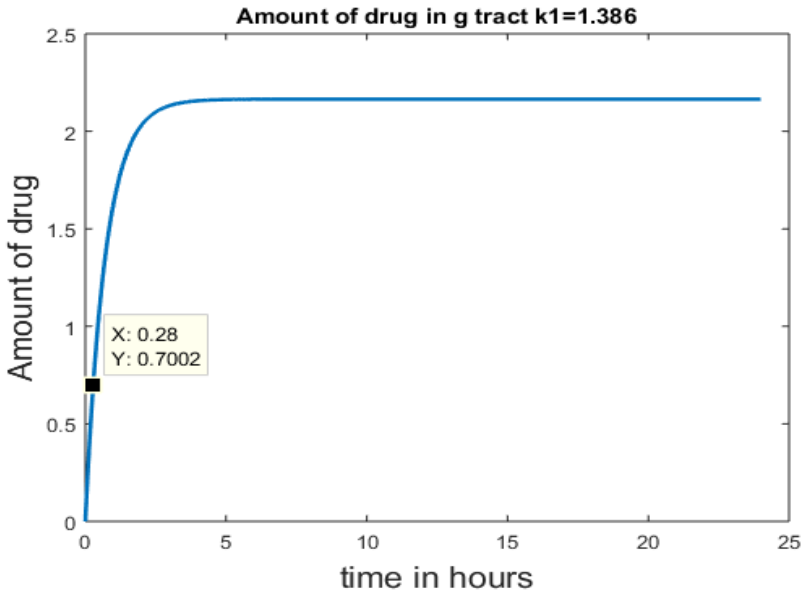

Figure 6:

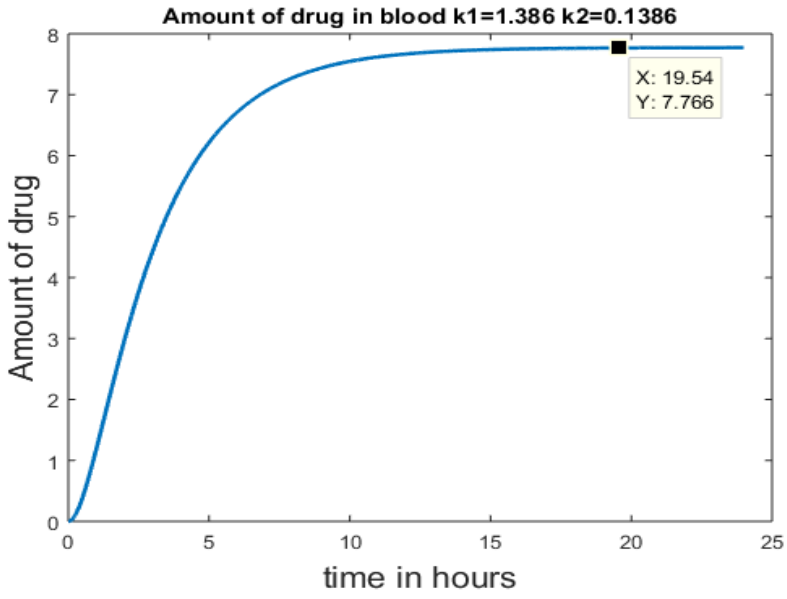

Figure 7:

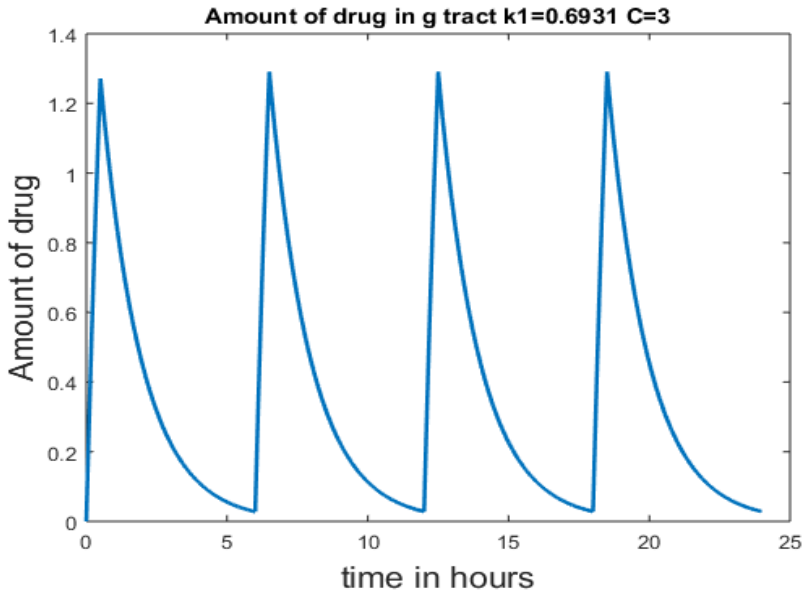

Figure 8:

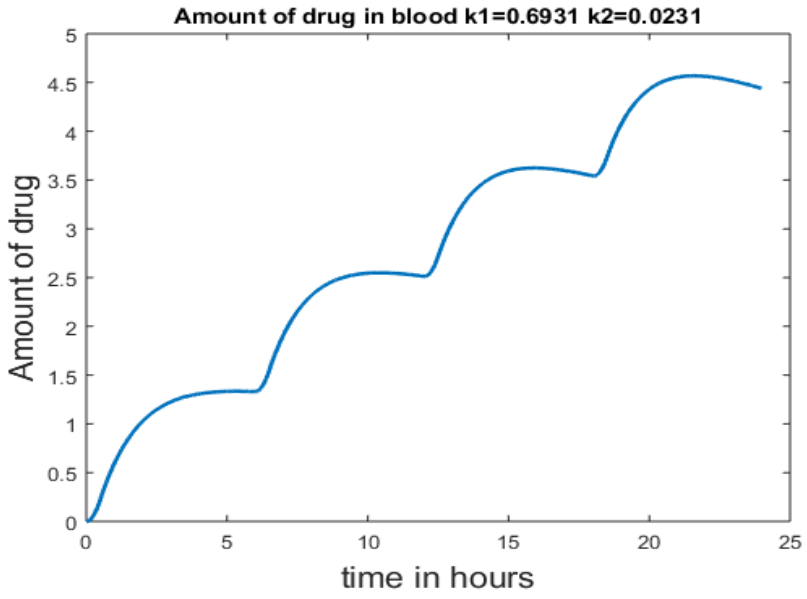

Figure 9:

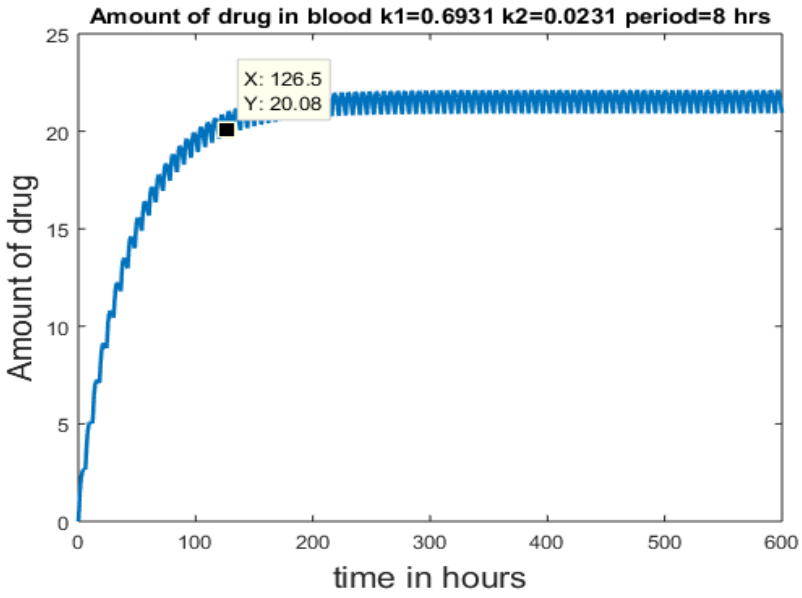

Figure 10:

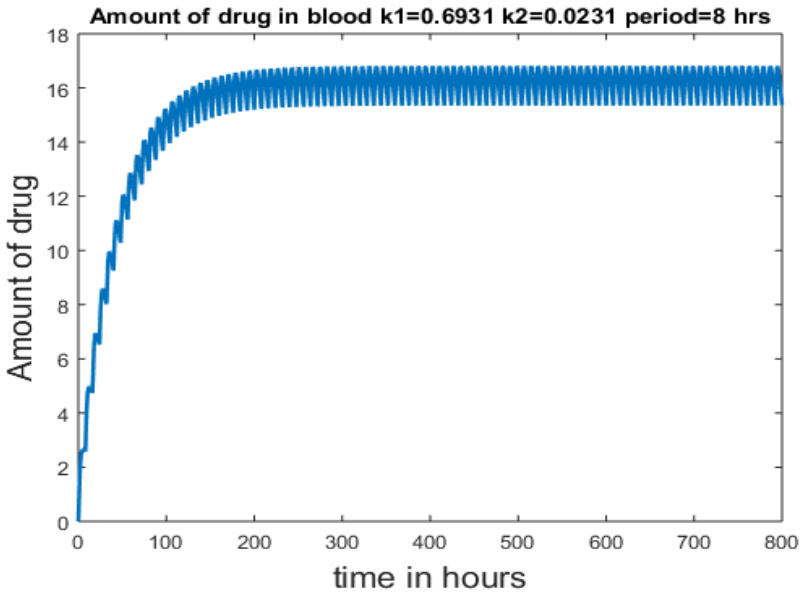

Figure 11:

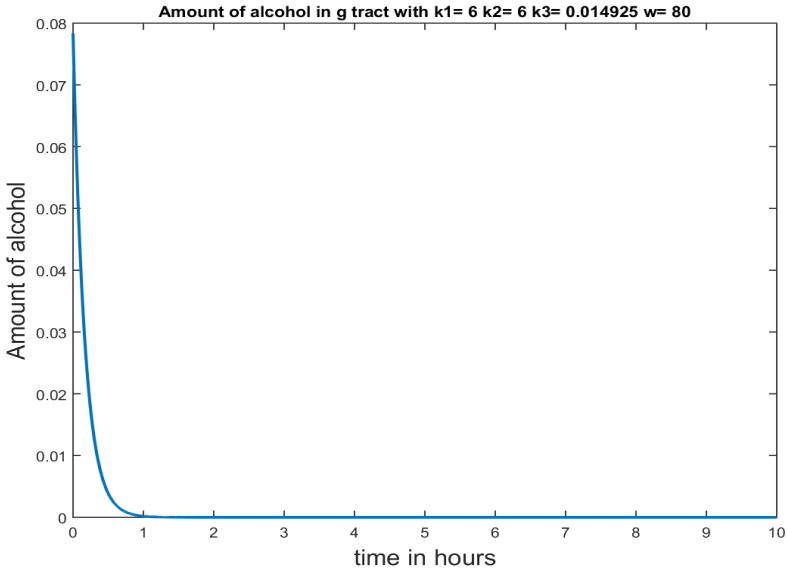

Figure 12:

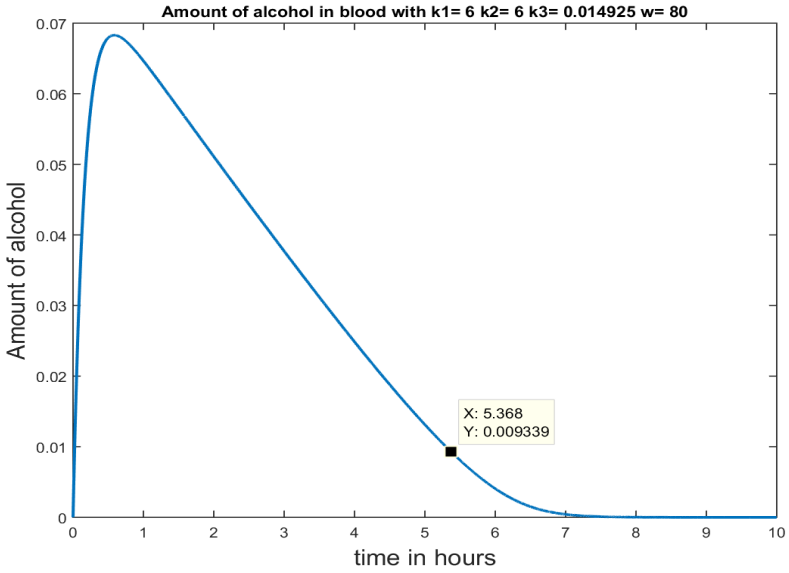

Figure 13:

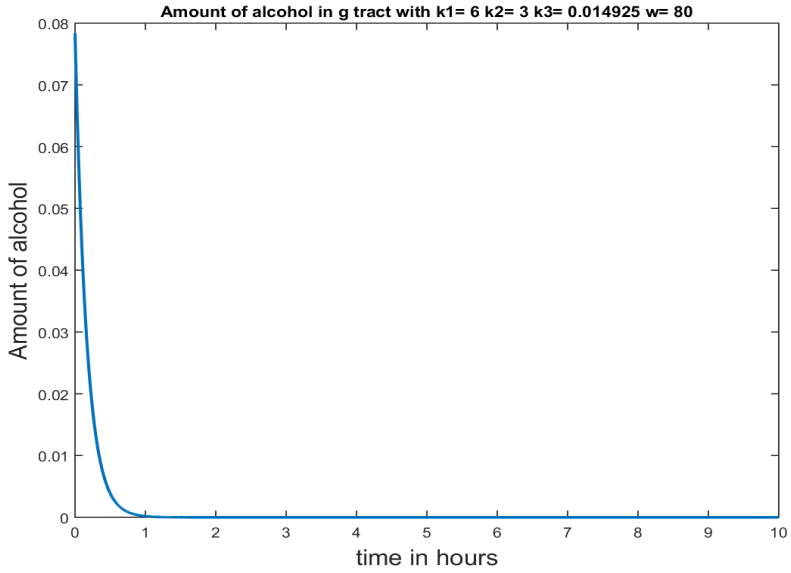

Figure 14:

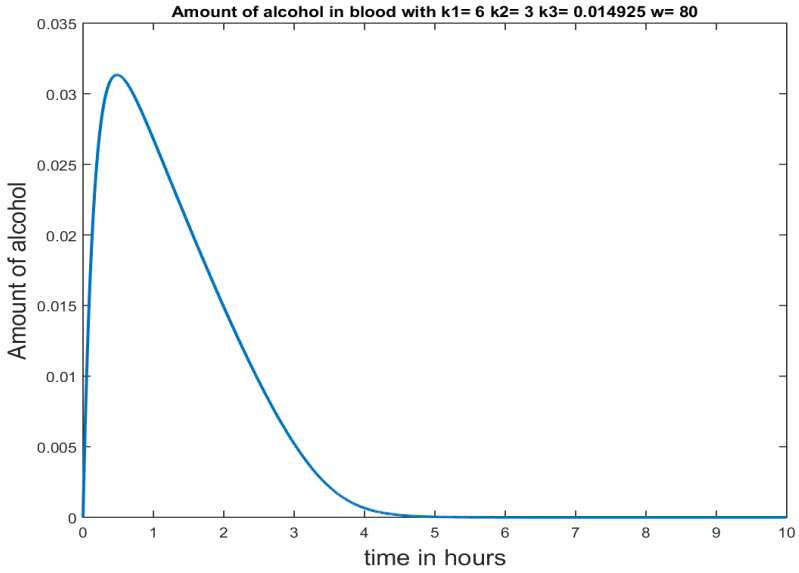

Figure 15:

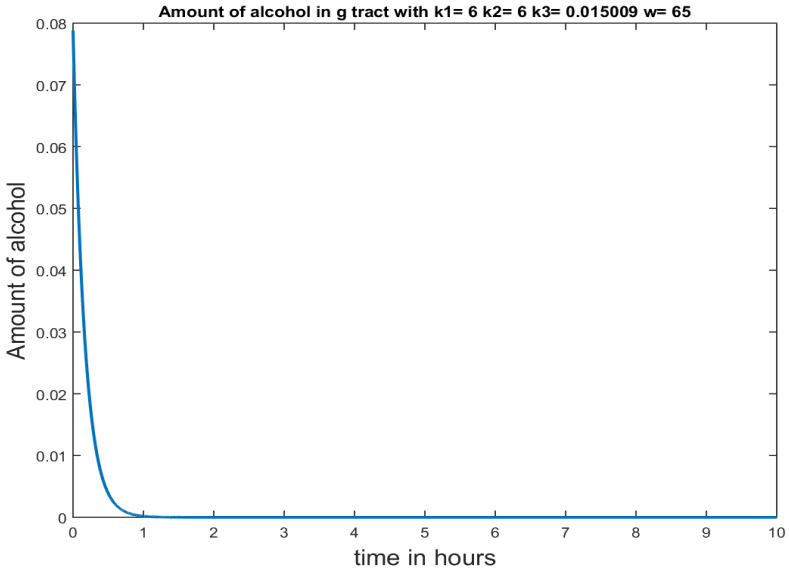

Figure 16:

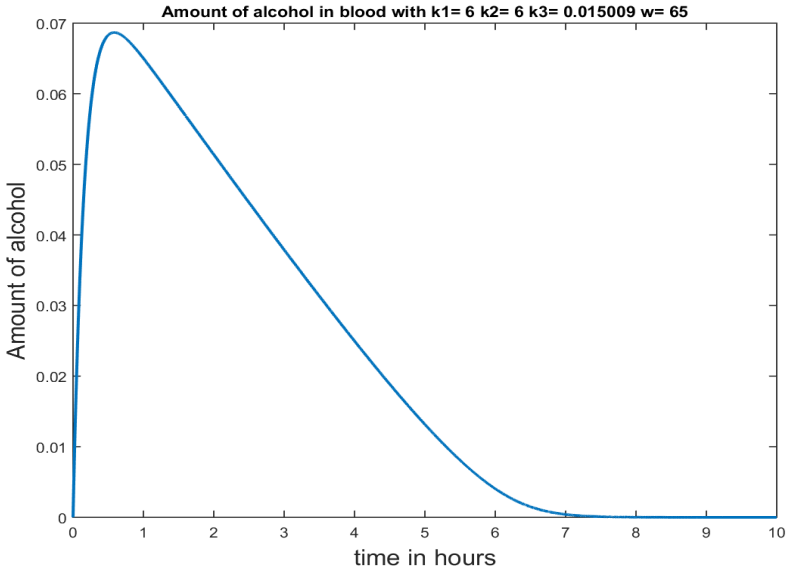

Figure 17:

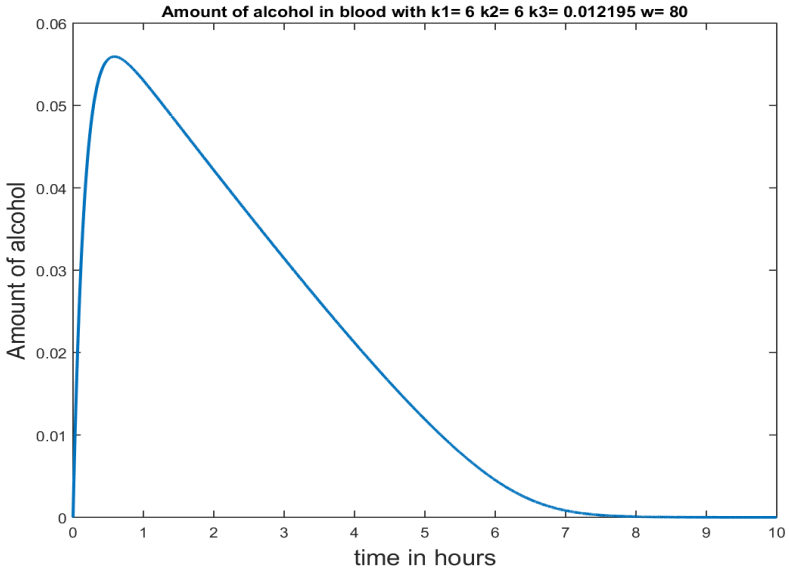

Figure 18:

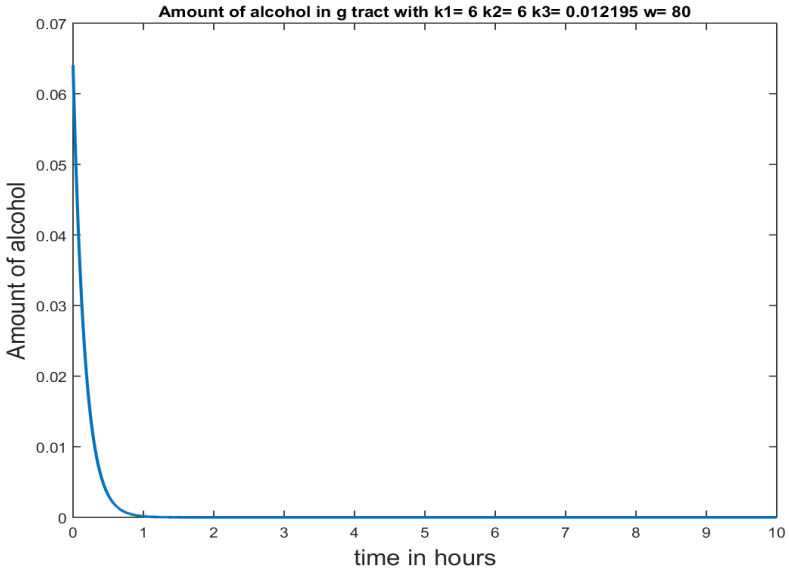

Figure 19:

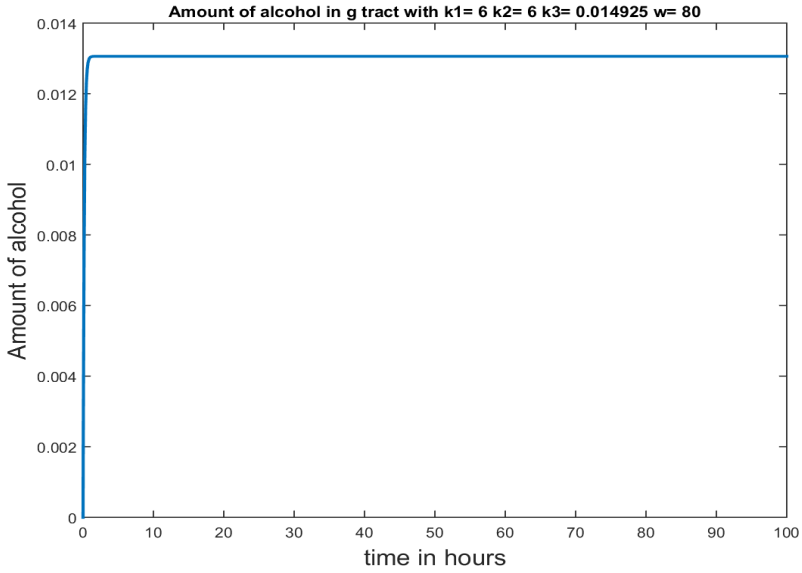

Figure 20:

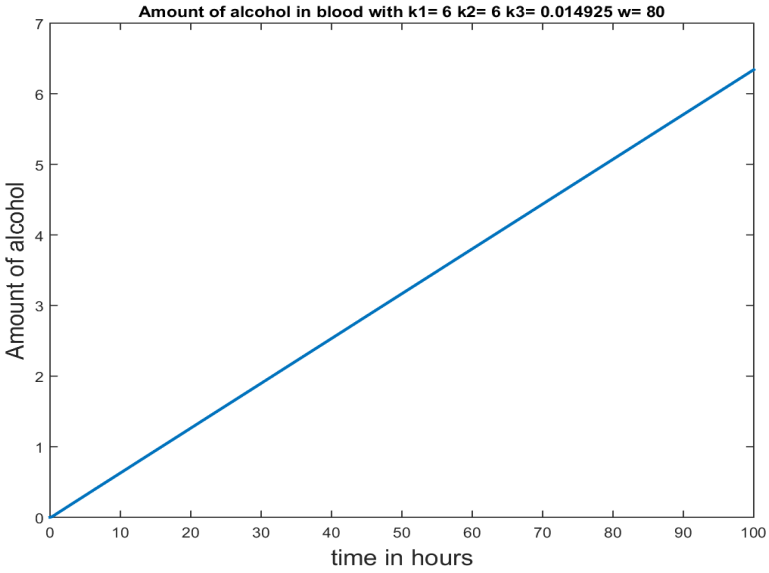

Figure 21:

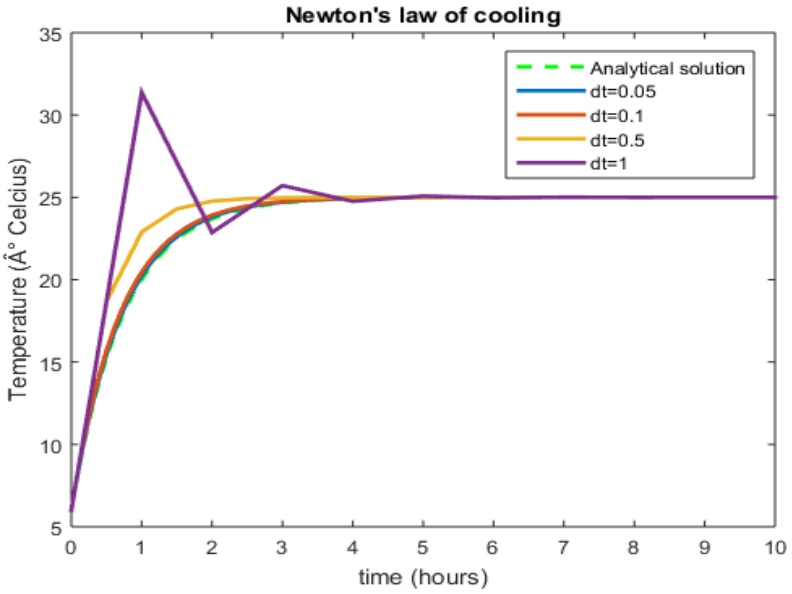
Newton's law of cooling
Analytical solution
dt=0.05
dt=0.1
dt=0.5
dt=1
Temperature (Â° Celcius)
time (hours)
35
30
25
20
15
10
5
0
1
2
3
4
5
6
7
8
9
10

Figure 22: