CS302: Modeling And Simulation Lab3 Report

Rajdeep Pinge 201401103, Aditya Joglekar 201401086

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1 Q1: SIR model

1.1 (Modeling Inuenza) Implement the SIR model of Inuenza and obtain Fig. 6.2 (the basic plots of S, I and R vs time).

Here we simply implement the basic SIR model which we have studied in class. There are 3 classes S,I and R with S flowing into I eventually flowing into R. There is a rate of for the transition of I into R. The rate for S moving into I is given by

$$\beta = kb/N \tag{1}$$

Where k denotes the number of contacts an infectious person has per unit time.**(Assumption)**It is assumed to be constant and independent of the population density. This is a rather simplifying assumption. One would actually expect this to be higher for regions where population is denser. b is the probability of infection transmission that given a meeting happens between an infected and a susceptible. We are more concerned with the ratio of I (infected), so we divide this by N.

β is further multiplied with the product of S and I. The product captures the inherent interactive nature of infection transmission. It can happen only when the members of the two groups interact. Thus, the S' is given by

$$S' = -\beta SI \tag{2}$$

Note that the process of I turning into R is a simple flow process. So change is simply proportional to the I. It does not have any interaction elements like the above. Infected people recover on their own after some time.**(Assumption)** It is assumed that a recovered person becomes immune to the disease. This gives us the R' equation.

$$R' = \gamma I \tag{3}$$

Thus, I' is

$$I' = -\beta IS - \gamma I \tag{4}$$

These are the set of SIR equations. We simulate the model by giving appropriate values to the parameters. Initially assume there is only 1 infected individual.

We get the following plot of evolution of S, I and R over time.

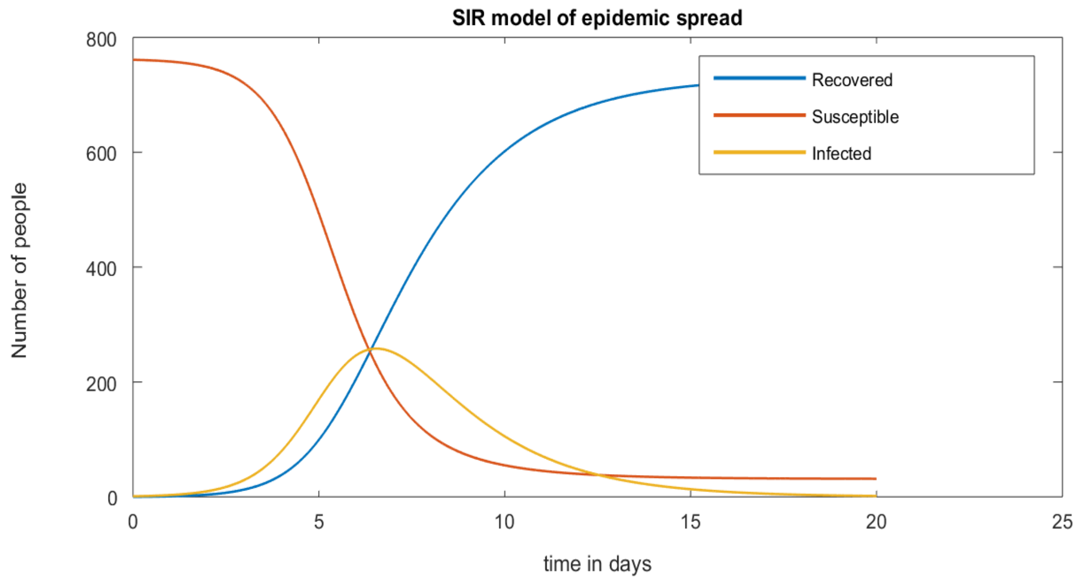

Figure 1: Q1: Basic SIR plot

Observations:

- As expected, the graph of S is an inverted logistic function due to the interaction product.
- As I infects more and more people from S, S steadily decreases.
- Here, I increases to a maximum and then decreases. Thus, we say that an epidemic has occurred.
- Eventually, all the infected are collected into R.
- Interestingly, it can be shown that the maximum of I occurs when R' and S' are maximum. This is because at this moment there are more I than at any other point. Hence the number of Susceptibles infected will be maximum here. This also means that the change in recovered is maximum here.

1.2 Project 1- Effect of vaccination. From the projects section implement problem 1.

Here, we consider the effect of vaccination on the susceptible population and the effect it has on the spread, intensity and duration of the epidemic.

We are told 15% are vaccinated each day. Question is 15% of what. There are several interpretations possible. We have explored two of them.

In one interpretation, we **assume it to be 15% of population at start of the day**. So we simply deduct 15% of the population after each day has passed. This assumption could be valid in a situation where the susceptibles are limited to a small area. It actually is ideal as it quickly removes a lot of susceptibles from danger. It could easily be the case in say a boarding school as is described in the given problem.

We thus, get the following plot for this assumption

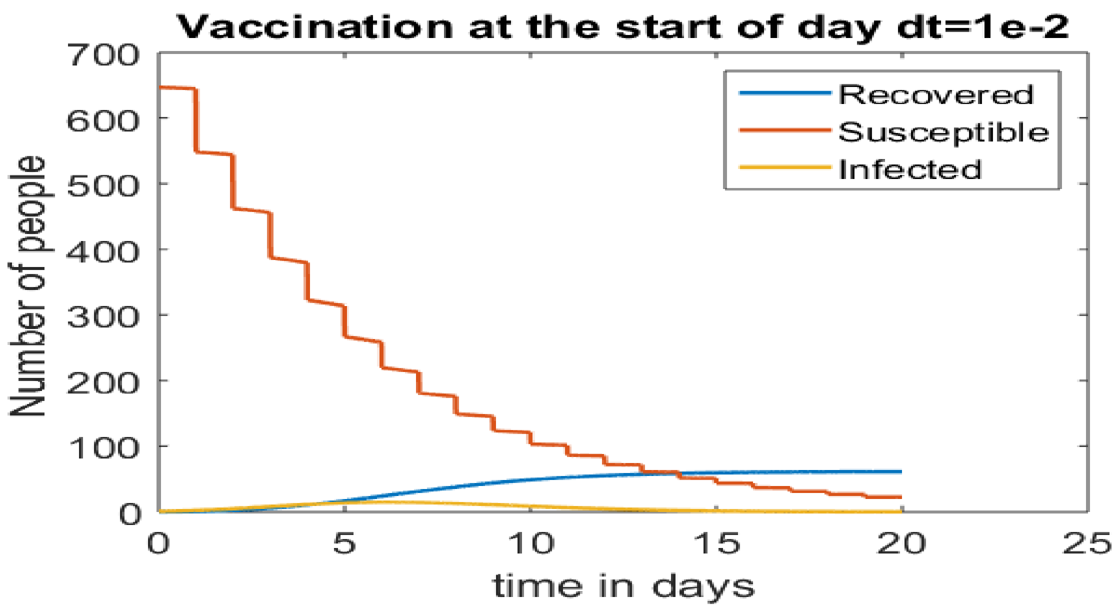

Figure 2: Proj 1 - Vaccination at the start of the day

Observations:

- Firstly,because of our assumption, we reduce the susceptibles by 15% each day. These people immediately are out of danger of an infection. Thus, in this model, suddenly, the number of susceptibles decreases. The suceptible graph thus has a lot of jerks.

- As a result of this, there are lesser people for I to infect. Hence I fails to grow to the extent of the original figure. This implies that vaccination is effective in curbing the amount of infections and hence the severity of the epidemic in general is far lesser. Here, the yellow curve is barely able to rise up, such is the effectiveness of the infection.

- Correspondingly, the recovered curve also does not rise much. Lesser the infected, lesser are the people who have to recuperate.

- The peak of the epidemic too reaches faster.

- Since, I is quite less compared to susceptibles here, the interaction term is not very influential in S' equation. Hence, the bhaviour of S is almost one of exponential decay.

- The duration for which I stays above negligible levels is also lesser for obvious reasons.

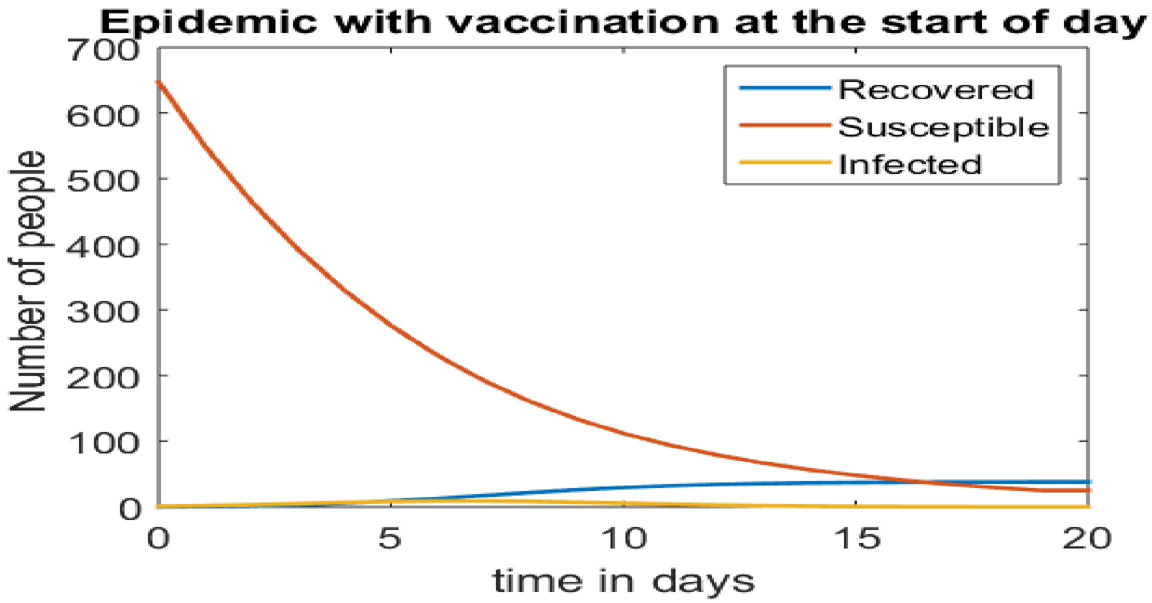

Figure 3: Discrete version dt=1 day - SIR simulation with 15% vaccination at start of day

Observations:

- Here, we have implemented a discrete version of the same problem. We assume that the unit of time 1 day. We do not compute for the time instants in between. The two plots (Fig.2 and Fig.3) are very similiar as the difference in unit of time should not matter much.
- Even here we note that the vaccination seems to be very effective. The infected population is not allowed to grow and hence shows a very flat curve.

We now vary the percentage of people vaccinated and check the impact on the duration and severity of the epidemic.

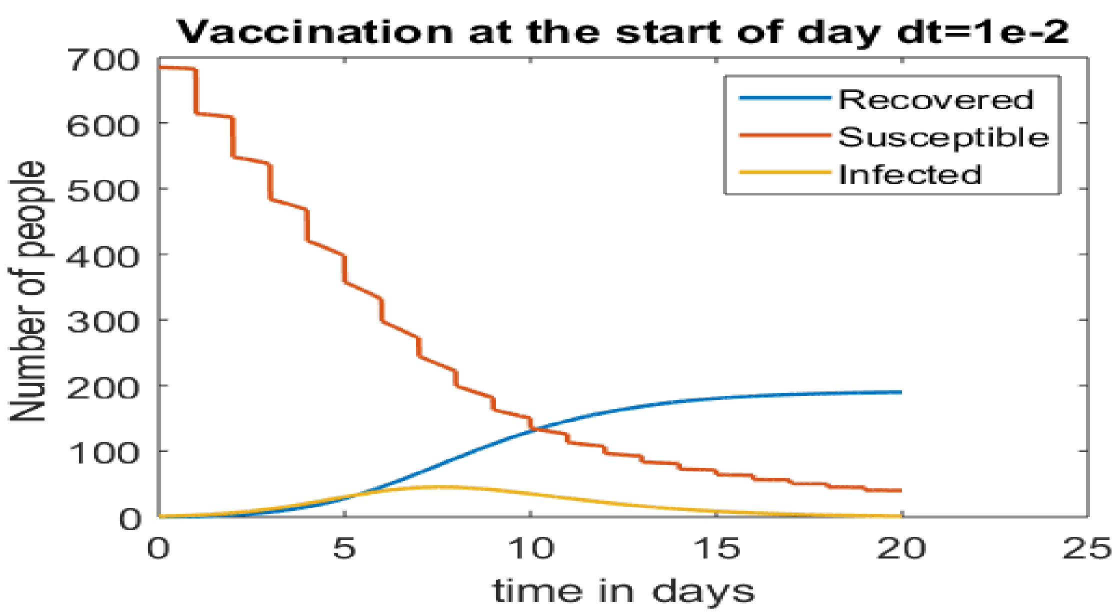

Figure 4: SIR with 10% vaccination at start of each day

Observations:

- As the boys vaccinated here are lesser, the drop in the susceptible graph is lesser.
- The I is able to grow more similarly to the one without vaccination.
- The I reaches a higher peak and remain significant for a longer duration.
- Consequently, the number of people recovered also are larger in number.

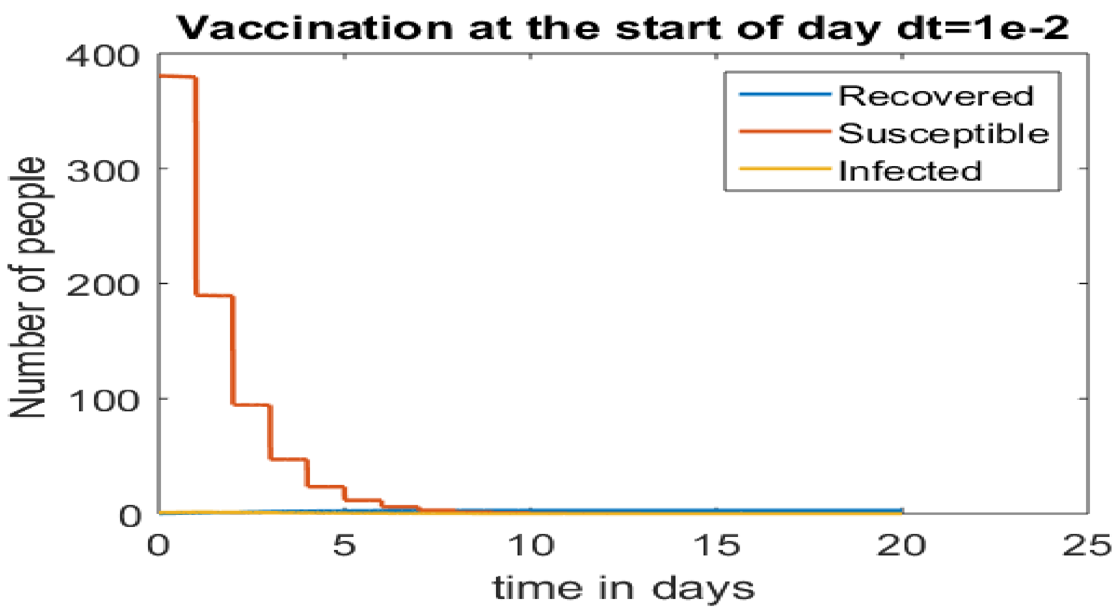

Figure 5: SIR with 50% vaccination at start of each day

Observations:

- This is a situation where aggressive efforts are made to curb the epidemic. More than half of the population is vaccinated every day.
- Notice, that there are barely an infections seen. The epidemic hardly occurs.
- This means that the rate of vaccination is far larger than the rate at which the infection can spread.

1.3 Project II - Introducing a delay in starting the vaccination process

Here we investigate what is the effect of delaying the start of immunization.

We must consider a delay of 3 days. So, the only difference from the previous question is that people will be removed from the susceptible start due to vaccination only after 3 days. This is significant as we know that the spread of infection is often exponential in nature at the start of the epidemic. So, a small difference in terms of time can have vastly different results as far as the spread of the epidemic is concerned.

Such information can be useful for Government and Medical agencies to determine how fast would they have to move in to contain the spread of infection.

The only change which we make in the equations is that the outflow term due to vaccination must be considered only after 3 days. After 3 days, the equations of the two scenarios are exactly the same.

1.3.1 Delay of 3 days

A delay of three days with the rate of vaccination and other parameters same as that of the previous example.

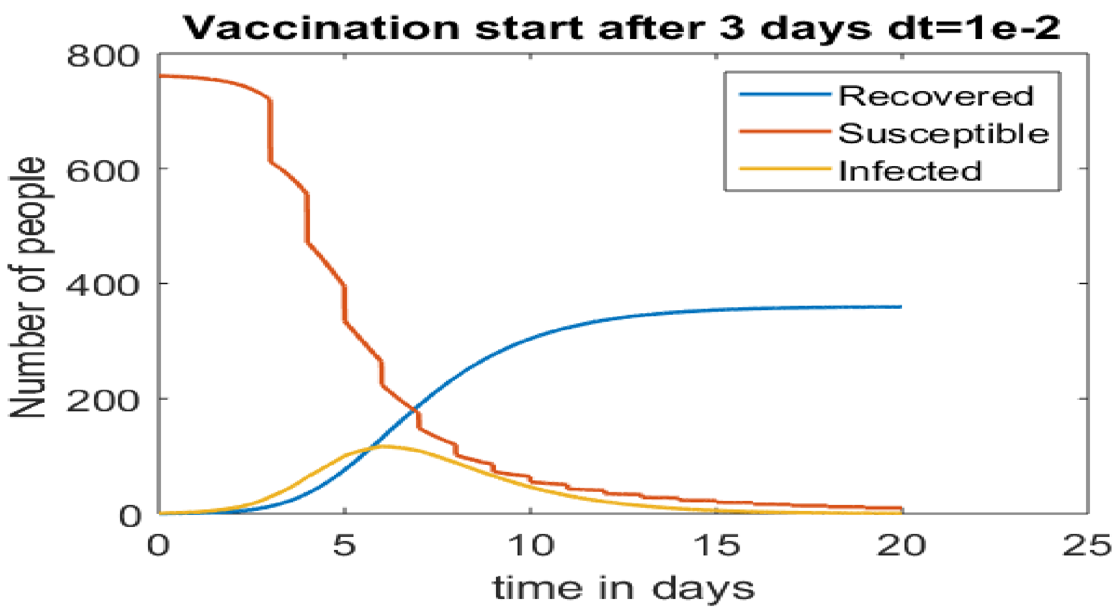

Figure 6: Delay of 3 days

Observations:

- There is no vaccination performed for the first 3 days. Hence, the graph shows the jumps only from the forth day. Thus, initially the behaviour is similar to the basic SIR model.
- However, this causes a rather serious issue of the epidemic breaking out. Notice that here we can see a proper increase in the number of I following a behaviour similar to the basic model.
- Contrast this with the Fig.2. Inspite of having the same rate of vaccination(15% at start of each day), the one with the delay shows a far serious outbreak of the epidemic. The maximum I i s around 25% of the population as compared to just around 1% in the earlier example.
- This means that in case of an epidemic it is important for the authorities to move in as soon as possible. Even a relatively small delay can lead to a far worse case of epidemic.

1.3.2 Varying the interval of vaccination

Here we examine the effect of changing the interval when the vaccination is administered.
Consider an interval of 5 days.

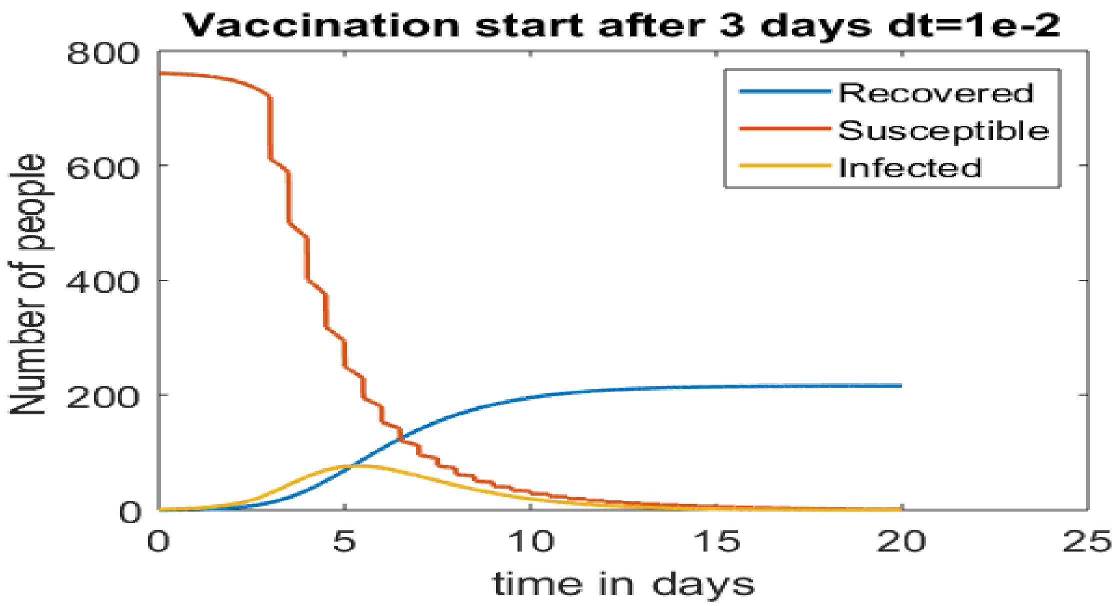

Figure 7: Vaccination after every half a day

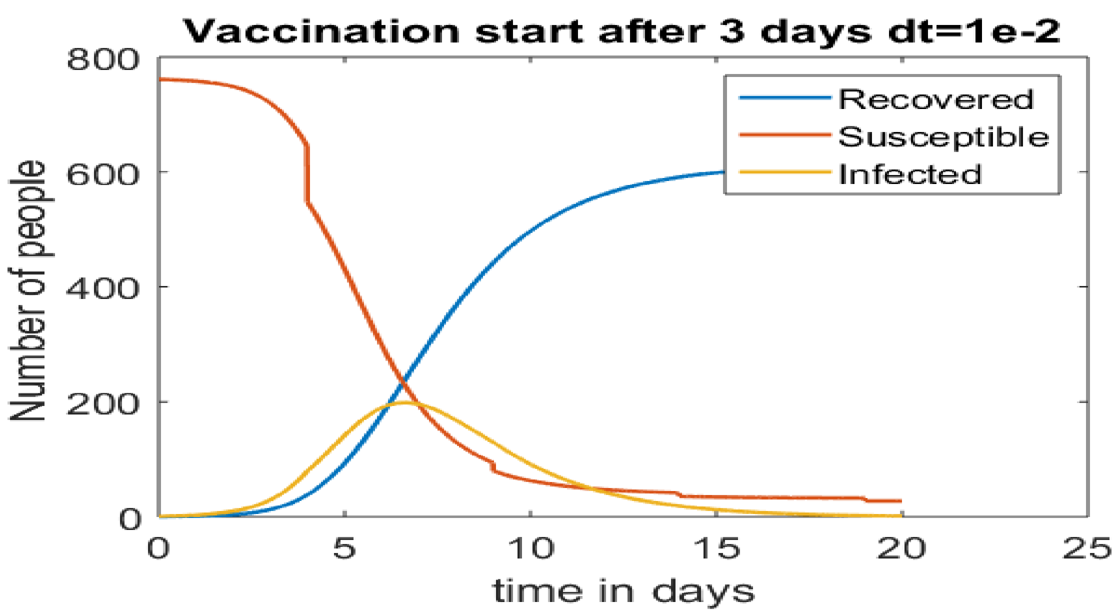

Figure 8: Vaccination after every five days

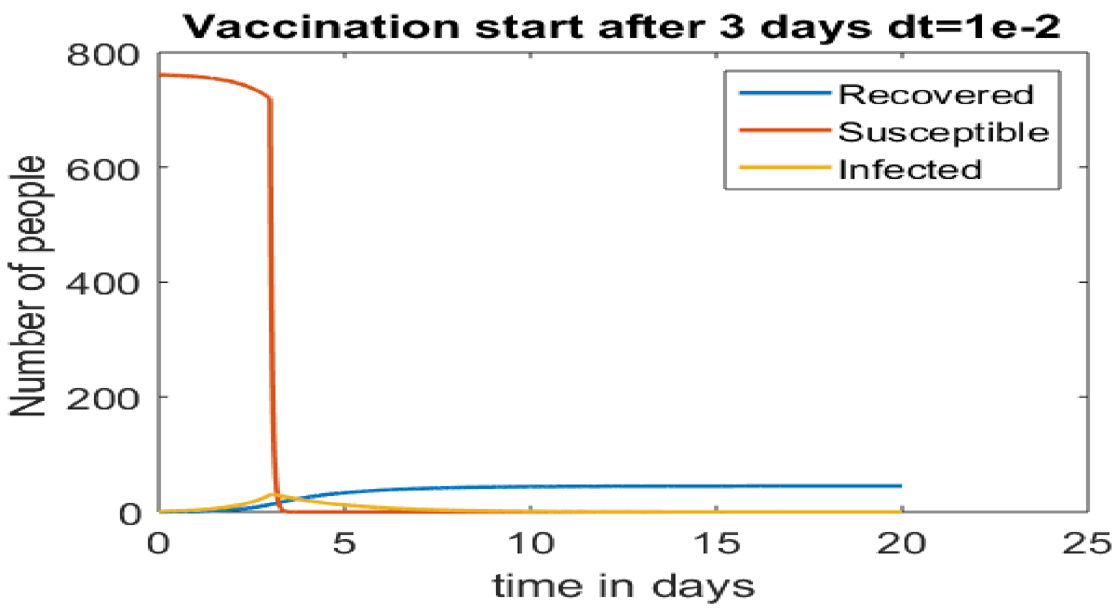

Figure 9: Continuous Vaccination

Observations:

- Naturally, if we have larger interval between vaccinations, it is more conducive for the epidemic.
- Fig 9 on the other hand depicts a constant onslaught of vaccination against the infection. Here we vaccinate people 100 times a day.
- The epidemic is curbed more effectively as a result.
- An interesting point to note. Compare Fig. 9 with Fig. 2. The epidemic evolution is almost similar. This shows the importance of starting the immunization process early. Starting it from day 1 is equivalent to vaccinating people 100 times a day if there is a delay of 3 days!

2 Q2: SARS Model

This is a 9 compartment model which builds on the basic SIR and SEIR models. An important factor in this model is the quarantine factor - q. A certain number of people are quarantined so as to try and prevent them from either getting the diseases or spreading the disease in case they have it.

One main assumption here is that the natural birth and natural death is not considered and the total population remains constant throughout.

2.1 Basic SARS model

As per the given conditions, we have built a model and the results of the simulation are shown below.

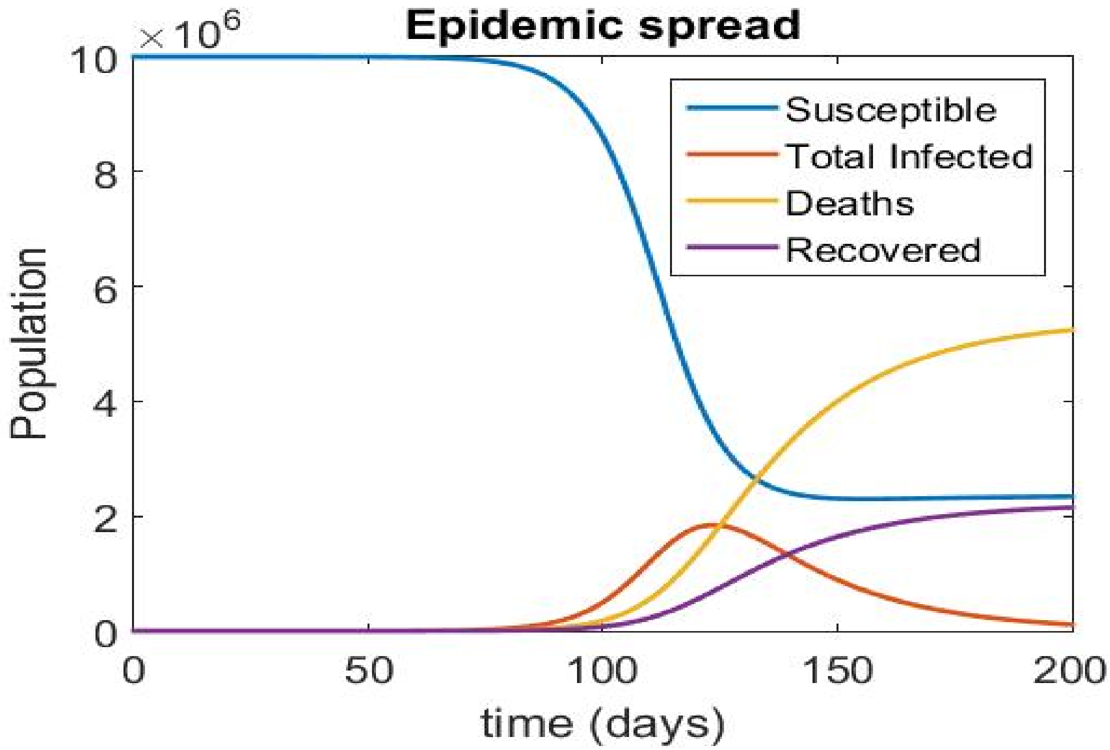

Figure 10: Basic SARS model

Observations:

- The above graph shows the variation in various categories of population for given conditions (in book Q5.)
- As there is no input to the system, the number of susceptible decrease and finally achieve a steady state.
- As susceptible decrease, the number of infected (and/or exposed). dead and recovered start increasing.
- The number of infected initially increase and reach a peak which is the extreme condition of epidemic and then starts decreasing. The decrease is because number of infected depends on number of susceptible. As the number of susceptible decrease, the new patients added to infected category is smaller as compared to the number of patients moving out from the infected category to dead or recovered category. Hence the number of infected decrease.

- The graphs of Deaths and Recovered show similar nature. Both initially very low, start increasing and finally reach a steady state. This is because both these graphs are dependent on the number of infected and as the number of infected increases, both these graphs start increasing and show concave up nature in the first part. Later on, as infected start decreasing, the rate of increase of these two graphs decreases and they show concave down behaviour.

2.1.1 Varying q - fraction of people quarantined

In this part, we vary q - the fraction of people quarantined initially. Intuitively, as this fraction increases, more people are quarantined beforehand and hence lesser amount of people can spread the disease. Hence the number of infected and number of deaths would decrease and the severity of epidemic would decrease.

we calculate basic reproduction number to find out if there will be an epidemic. The equation is:

$$R = k * b * (1 - q)/(v + m + w) \tag{5}$$

Where v: recovery rate, m: death rate and w: isolation rate, b: contacts that result in transmission and k: number of contacts that each infected has per day.

For given values of k, b, v, m, w, we get

$$k * b = 0.6, v + m + w = 0.2 => k * b/(v + m + w) = 2 \tag{6}$$

$$R = 3 * (1 - q) \tag{7}$$

For it to not become an epidemic and to remain in control, R should be leas that 1.

$$R = 3 - 3q < 1 \tag{8}$$

$$2 < 3q \tag{9}$$

$$q > 2/3 \tag{10}$$

$$q > 0.6666 \tag{11}$$

We must study the model for all the values of q both below and above 0.6666.

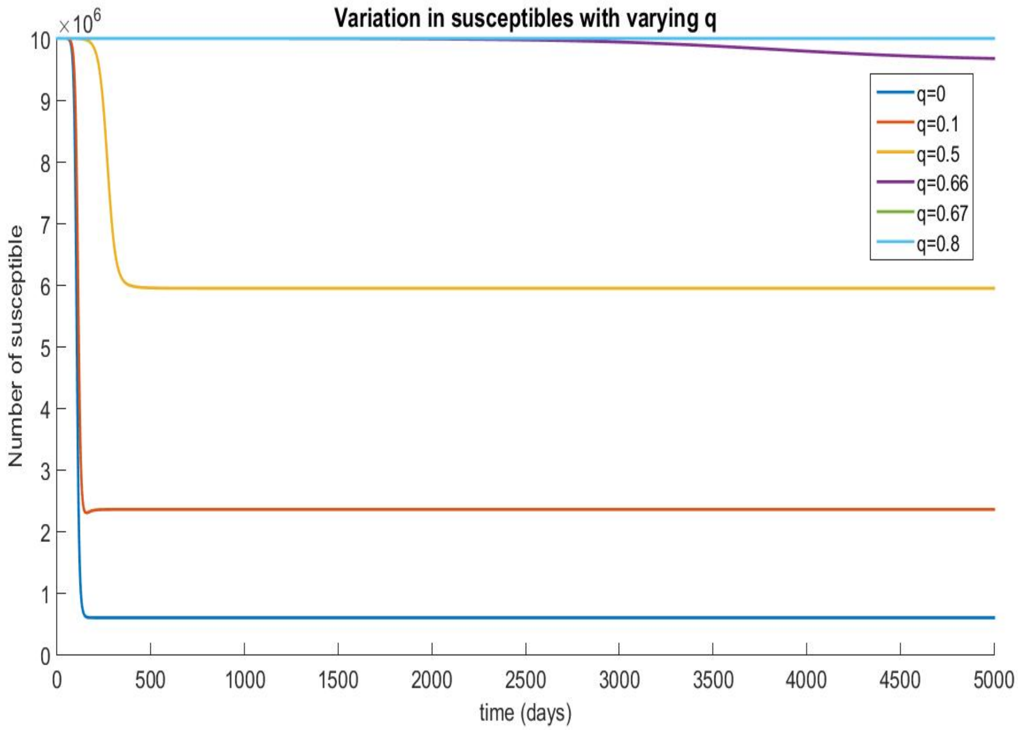

Figure 11: Variation in number of susceptible with varying q

Observations:

- In the above graph, we look at how the number of susceptible vary w.r.t. variation in q. W
- When q=0, it is simple SIR model with the lowermost dark blue line indicating that number of susceptible decreases most in this case.
- It might seem that this is good because susceptible are decreasing but in reality this is worst case because total population is constant and hence reduction in susceptible means increase in infectious and other compartments which increases severity of epidemic.
- As q increases, the number of susceptible stabilizes to a larger value. This is because more people have been quarantined beforehand and have been prevented from spreading disease. The stability is because, the infected undetected people are so less that they cannot spread the disease among susceptible.
- For $q \geq 0.67$, the condition to prevent an epidemic, the number of susceptible remains same that is none of them is infected. This is essentially how we should prevent the epidemic.
- Also in epidemic cases, as q increases, the spread of the epidemic i.e. the sharp decrease in the graphs happen after a few days and happens over a larger period of time. This is because due to quarantine effect. lesser people are able to spread the disease.

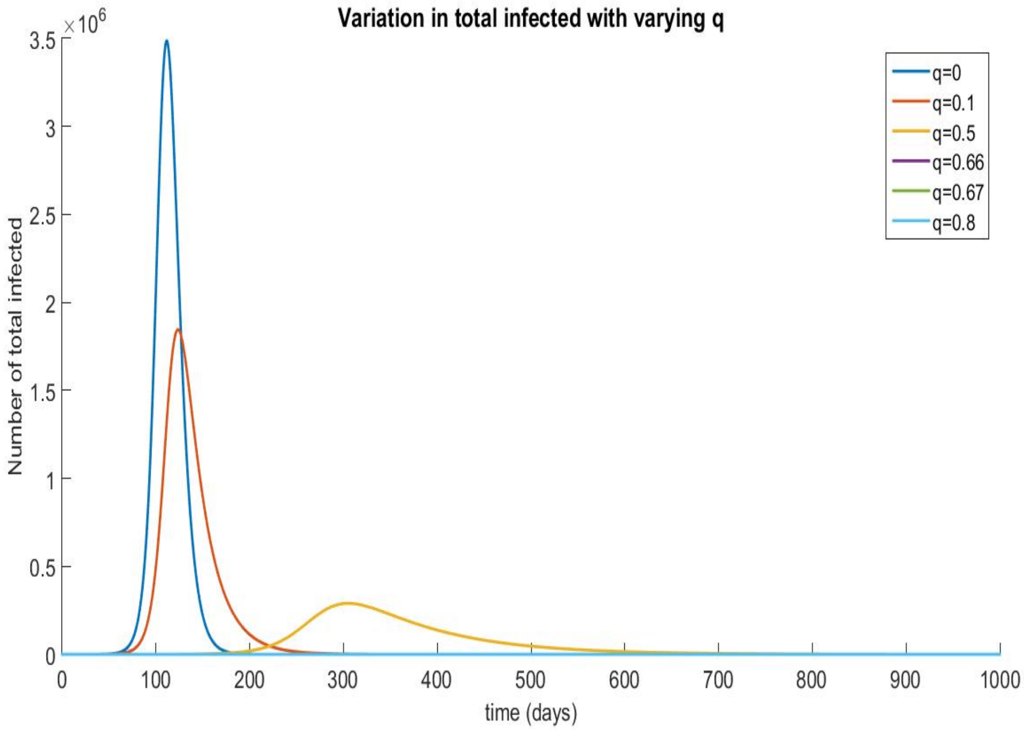

Figure 12: Variation in total number of infected with varying q

Observations:

- This graphs shows the variation in total infected as the value of q is changed.
- We observe that as q increases, the peak achieved by the graphs decreases, i.e. in worst case, less number of people get infected. This must be attributed to the fact that more people are quarantined and hence there are less people to spread to disease.
- Another observation is that as q increases, the spread of the graphs(width) increases, this shows that although the severity of epidemic decreases, the epidemic lasts for longer duration. This may be because as number of infected decrease, it is difficult to detect them and hence the duration over which they act increases.
- For $q \geq 0.67$ the number of infected are almost negligible indicating the avoidance of epidemic when sufficient number of people are quarantined.
- An important point to ponder over is the location of graphs when infected rise significantly. This shows that if lesser people are quarantined, the epidemic spreads very quickly and if more are quarantined, it takes time for the epidemic to take place.

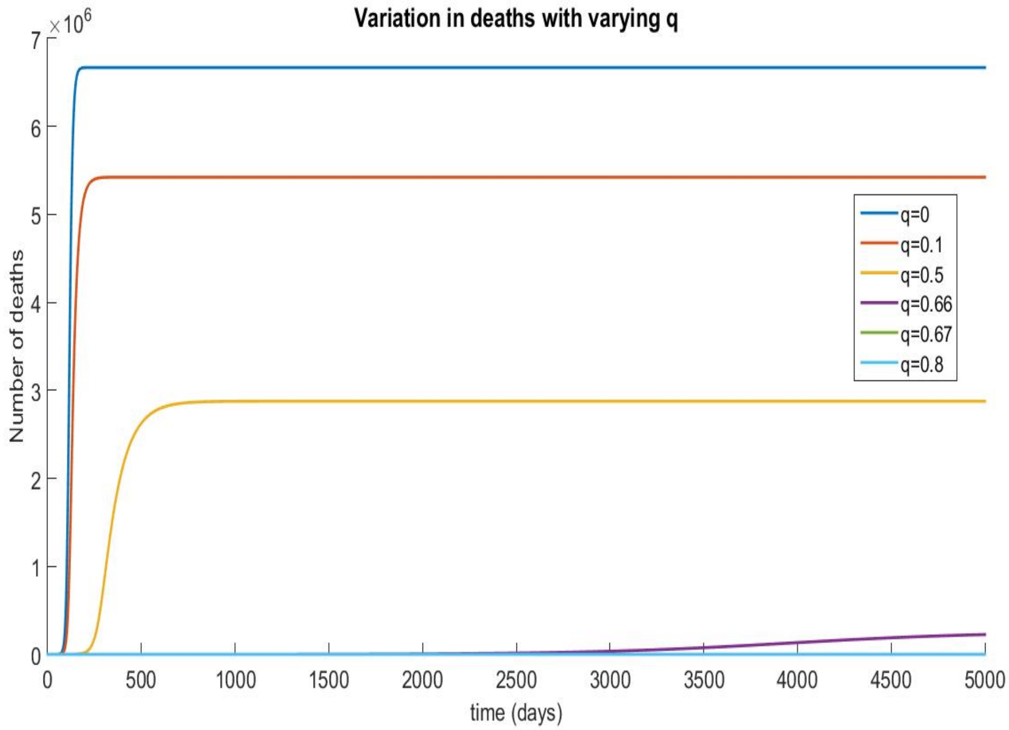

Figure 13: Variation in number of deaths with varying q

Observations:

- The above graph looks at the number of deaths with variation in q.
- for q=0, almost $6.5 * 10^6$ people die i.e. almost 65% of the total population of 10^7 dies. This implies severe epidemic situation.
- Even for q = 0.1, 50% of the people eventually die.
- But for $q \geq 0.67$, we see no deaths because there is no epidemic and hence the disease is not allowed to spread.
- Also for small q, most of the death occur over a small period of time while as q increases (look at q = 0.66), the number of deaths decreases due to more number of people quarantined and hence lesser spread of diseases and deaths occur over a larger period of time.

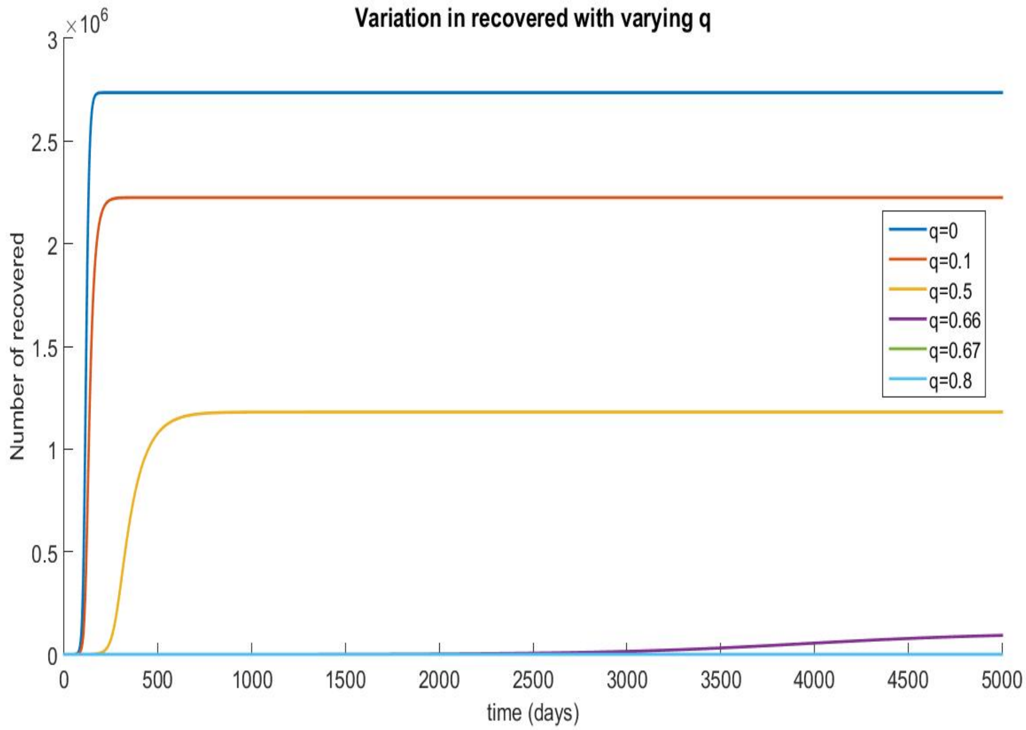

Figure 14: Variation in number of recovered with varying q

Observations:

- Intuitively the above graph seems contradictory because it shows that as the q increases, the number of people recovered decreases.
- But this is the case because number of recovered depends on number of infected and if more people are quarantined, lesser are infected and hence lesser are recovered.
- So, the decrease in recovered is actually because less number of people were infected in the first place.
- This graph shows similar behaviour to the graph of deaths.
- for $q \geq 0.67$ we see that the recovered are almost 0 because there is no epidemic and hence no person is infected.

2.2 Test the above model for other ranges of k from 5 to 20 per day.

k : Number of contacts that an infected person has with another person per day (per unit time). Intuitively, if the number of contacts increase, then the probability of transmission of disease goes up. Hence severity of the epidemic increases.

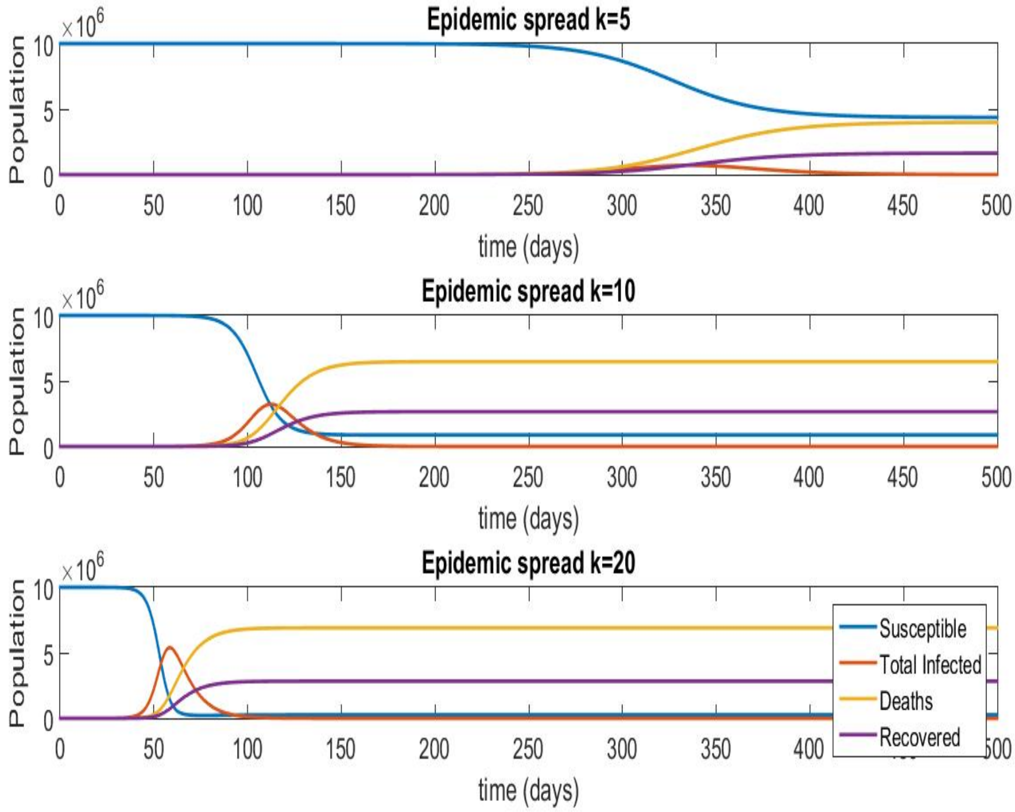

Figure 15: Comparing all the terms for varying k values

Observations:

- The above 3 graphs show behaviour of different quantities as number of contacts made by an infected person increase.
- If there are less contacts, then the spread of the disease is less and hence epidemic occurs after a long time. also the epidemic is spread over a long duration as shows by the red graphs of total infected
- The peaks of the total infected graphs increase with k and the number of deaths also increase with k indicating that the severity of the epidemic increases if the infected person comes in contact with more number of people.
- Also if the contacts increase, more number of people get infected at an earlier time. Hence the epidemic spreads at earlier time than when contacts are more.

2.3 Test the SARS model for ranges of 1/(v + m + w) from 1 to 5 days.

1/(v + m + w) is called average duration of infectiousness i.e. for how much time a person infected with SARS with have the disease before the person gets into a different compartment which is one of infectious isolated or Recovered or Dead.

We know that such a situation will occur when the person is in infectious undetected compartment and is spreading the disease. This value shows on an average, for how much time a person would remain in infectious undetected compartment.

we work with 3 cases:

- CASE I: Death rate is varying with other two rates remaining constant as given.
- CASE II: Recovery rate is varying with other two rates constant.
- CASE III: Isolation rate is varying i.e. the number of infectious quarantined and infectious undetected people who are isolated is varying with other two rates constant.

The rates are varied such that they change the average duration of infectiousness from 1 to 5 days. With the lower value of variable in the denominator indicating higher duration of infectiousness.

2.3.1 CASE I: Death rate varying

Although this situation is highly unlikely, because the death rate cannot change, we must look at how the situation will pan out if it were to change.

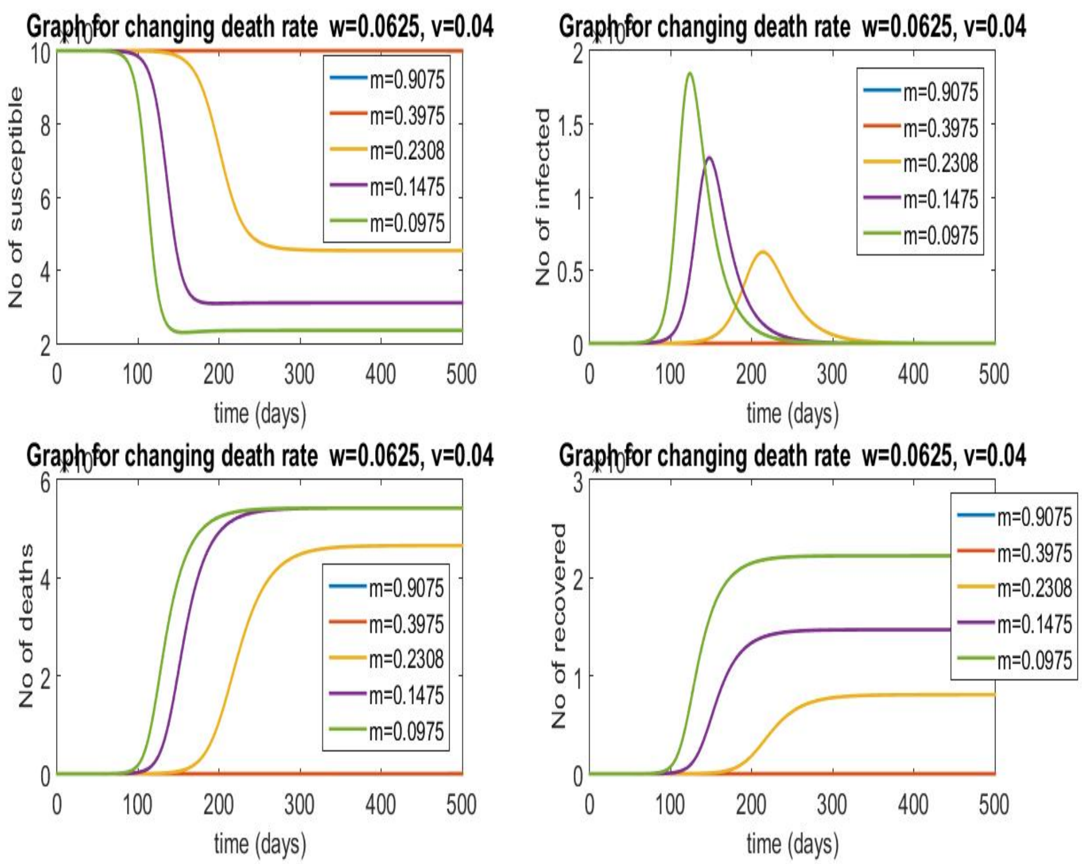

Figure 16: Graphs of all the terms for varying death rate m

Observations:

- As the death rate decreases, more people remain in the infected region. Hence more people can spread the disease and hence in the graph of number of infected we see higher peaks. Similarly we see the number of susceptible dropping more for lower death rate.
- Also because the death rate is less, the increased infected people must recover i.e. more number of people will be recovered as shown in the corresponding graph.
- We see and interesting trend here wherein the number of death is same for two of the smallest death rates. This may be because the situation is such that this is the maximum number of deaths that can occur.
- Another important observation is that if the death rate is too high, the infected people will immediately die and won't spread the disease. This means that there will be no epidemic as indicated by all the graphs.

2.3.2 CASE II: Recovery rate varying

Recovery rate will change in the cases where medicine is improved and is distributed to more number of people.

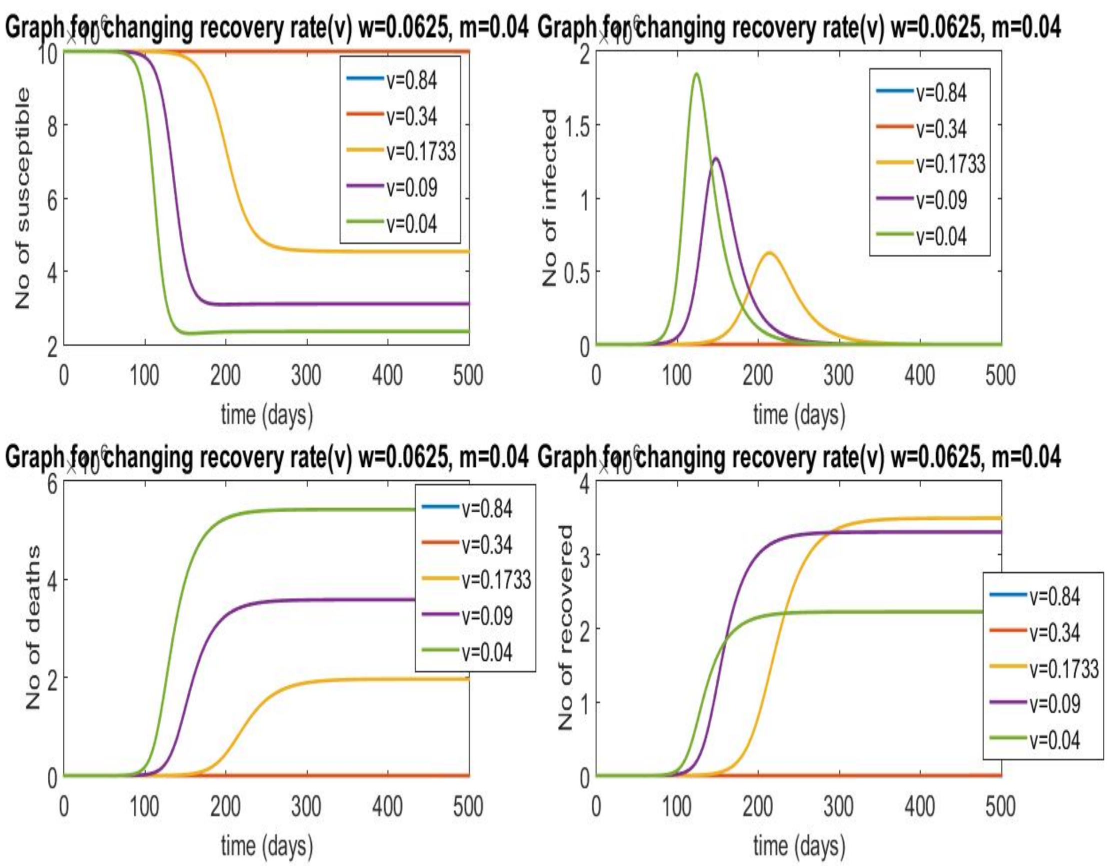

Figure 17: Graphs of all the terms for varying recovery rate v

Observations:

- As visible in the bottom right graph, as the recovery rate rate increases, the number of recovered increases and the time period at which recovery happens is shifted rightwards.
- Looking at infected, as the recovery rate decreases while the total population remaining constant, the rate of people going into recovery compartment decreases. Hence more people remain infected. Thereby infecting more and more people. Hence the graph of infected has higher peak for low recovery rate.
- Since no of deaths are dependent on no of infected, as recovery rate decreases, more people are in the infected region. Hence more number of people die. Therefore the trend seen in the bootom left graph is justified.
- Also since more people become infected, susceptible population decreases as rate of recovery decreases.
- If rate of recovery increases too much, v=0.34 or v=0.84, the duration of infectiousness is 1 or 2 days and the infected people recover immediately without spreading the infection. Hence epidemic is avoided. Therefore number of infected, dead and recovered are almost zero and all remain in the susceptible region.

2.3.3 CASE III: Isolation rate varying

This rate can most easily be changed by increasing the number of infected people that are being isolated.

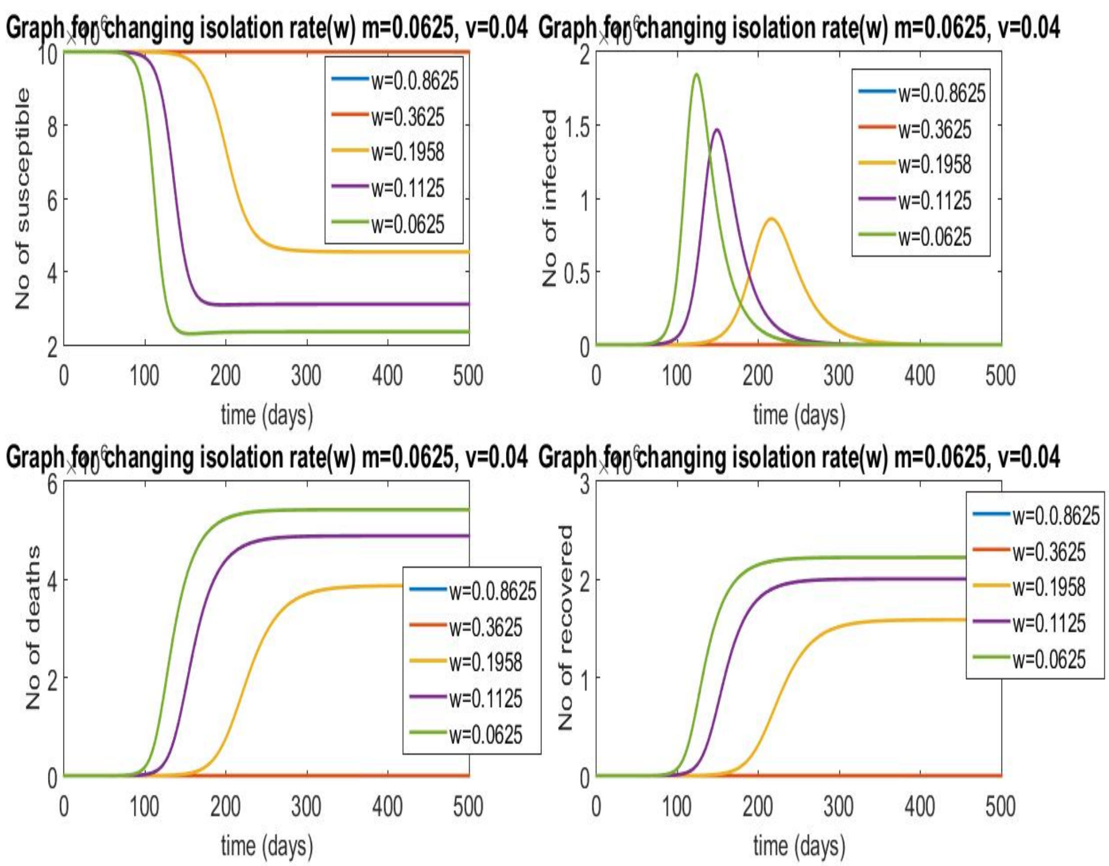

Figure 18: Graphs of all the terms for varying rate of isolation w

Observations:

- This is straightforward. As the isolation rate increases, the number of infectious people who are undetected and can spread the disease decreases. This means that the total infected population decreases.
- As total infected decrease, The number of deaths and recoveries also decrease.
- similarly no. of susceptible increase with increasing isolation rate. This must be because as isolation increases beyond a certain limit, all the infected people are immediately isolated and hence there is no more spread of the disease.

2.4 Adjust the model of Project 5 so that the simulation is allowed to run for a while before quarantine and isolation measures that reduce R to below 1 are instituted. Discuss the implications on the number of people quarantined and on the health care system of not taking aggressive measures initially.

This means that initially it is simple SEIR model without quarantine or isolation measures. These measures are started some time later which is quite logical because the response is not immediate.

When the measures are started, it is to reduce the R - reproduction number to below 1 which will imply that there is no epidemic and the disease is under control.

2.4.1 q = 0.2 when quarantine measures are started

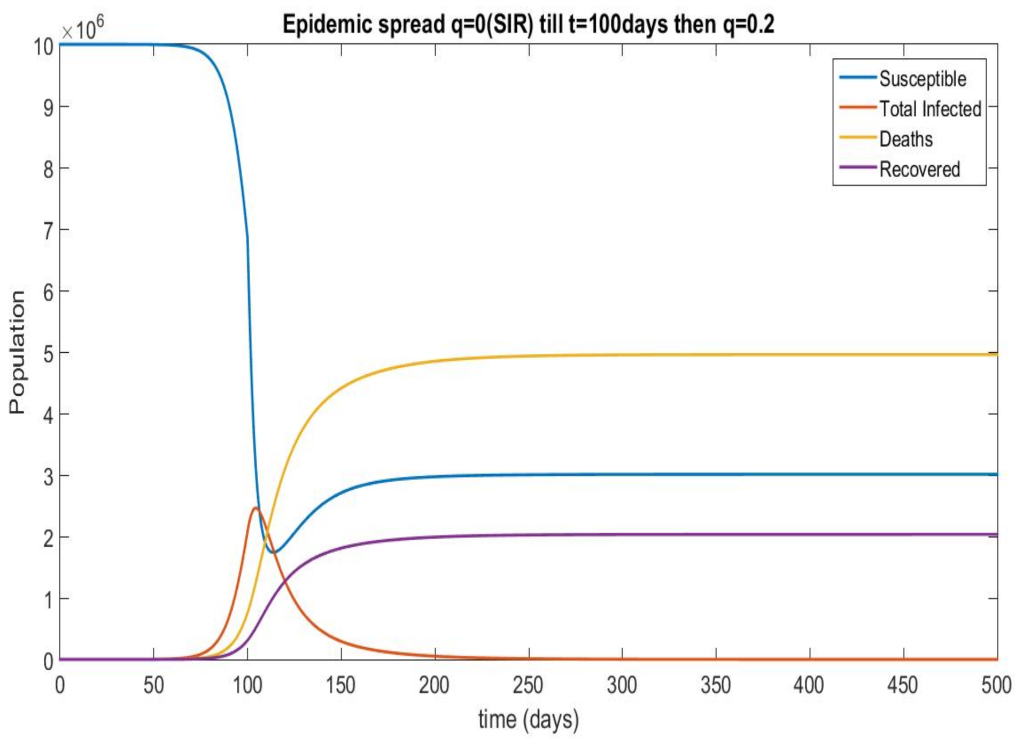

Figure 19: Quarantined measures are taken 100 days after the start of the epidemic q=0.2.

Observations:

- Here we see an interesting obeservation. Initially, the number of susceptible starts decreasing and the infected start increasing due to normal epidemic with no quarantine measures. But once the quarantine measures are started at t = 100 days, the infected start decreasing immediately.

- Also since the number of quarantined increases, the number of susceptible quarantined also increases. This means that the rate of outflow decreases which the rate of inflow increases for the susceptible compartment. Hence the number of susceptible start increasing till it balances out.

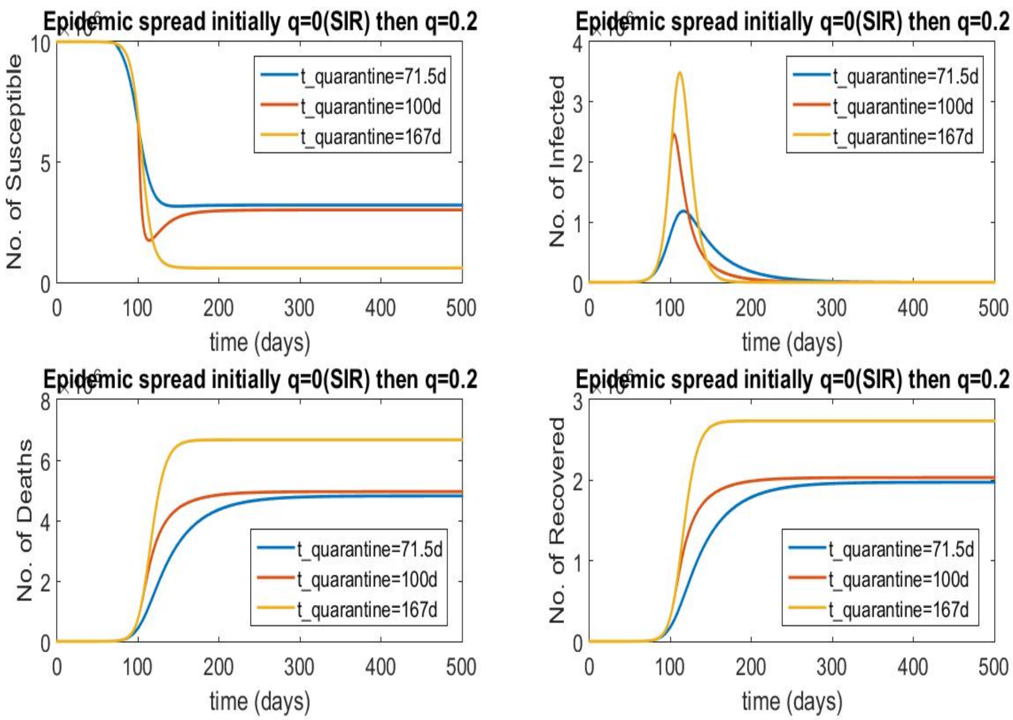

Figure 20: Quarantined measures are taken at different time after the start of the epidemic q=0.2.

Observations:

- The above graphs compare the numbers when quarantine measures are started at different times.
- In the top left graph, the yellow plot reaches steady state even before the quarantine measures are started. So there is no further variation in that plot. The red plot shows variation because quarantine measures are started just at the time of epidemic spread . Therefore there is a kink. The Blue plot shows that if the quarantine measures are started much before the spread, the same steady state can be achieved
- In the graph of infected, comparing red and yellow plots, if the quarantine measures are started during the disease spread, the number of infected decreases sharply. Also if the measures are started much before, the peak number of infected decreases much more but the epidemic may last longer.
- The graphs of deaths and recoveries show similar trends. The yellow graphs show that the epidemic had caused devastation because the measures were taken too late and had no effect. The red and blue graph show that if the measures are taken before the epidemic, it would be most useful but even if they are taken just when epidemic starts, the result would be same.

2.4.2 q = 0.8 when quarantine measures are started

Note here that to reduce R below 1, q should be greater than 0.66. Hence q = 0.8

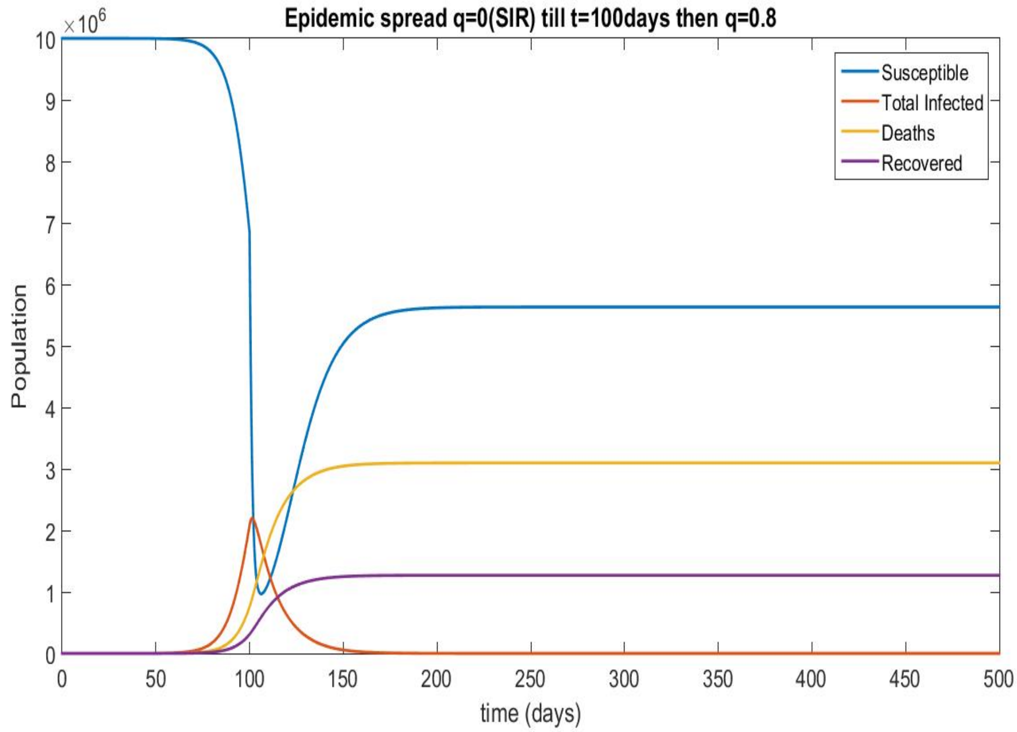

Figure 21: Quarantined measures are taken 100 days after the start of the epidemic q=0.8.

Observations:

- Here we have increased the quarantine rate to 0.8 which should prevent the epidemic.
- But if epidemic has already started, then this would not prevent but rather control the epidemic.
- Here the number of infected decreases even more sharply due to serious quarantine measures.
- The number of susceptible increases due to high number of susceptible quarantine which contribute to increases inflow.

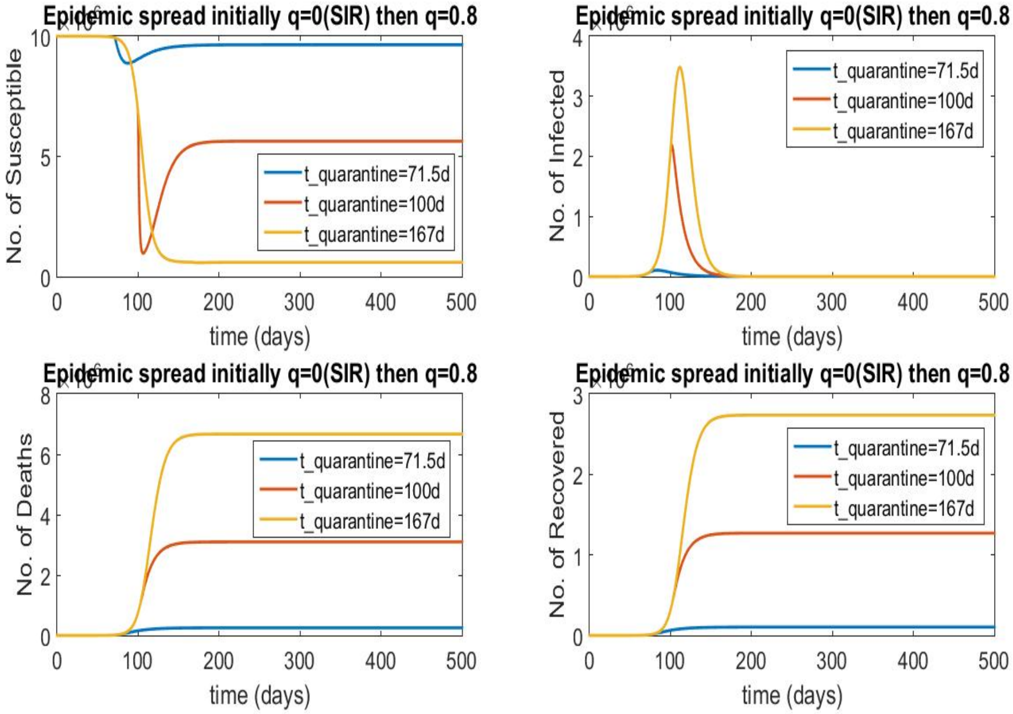

Figure 22: Quarantined measures are taken at different time after the start of the epidemic q=0.8.

Observations:

- Here we can clearly observe from the blue graphs that if the measure of quarantine and isolation are taken before the spread of the epidemic, them the epidemic would be controlled to a large extent.
- The rest two graphs show similar nature to the previous ones. The yellow plots show that if the measures are taken after the epidemic, there would be no effect of the measures. The red graph shows the effect of measures if taken suring the spread.

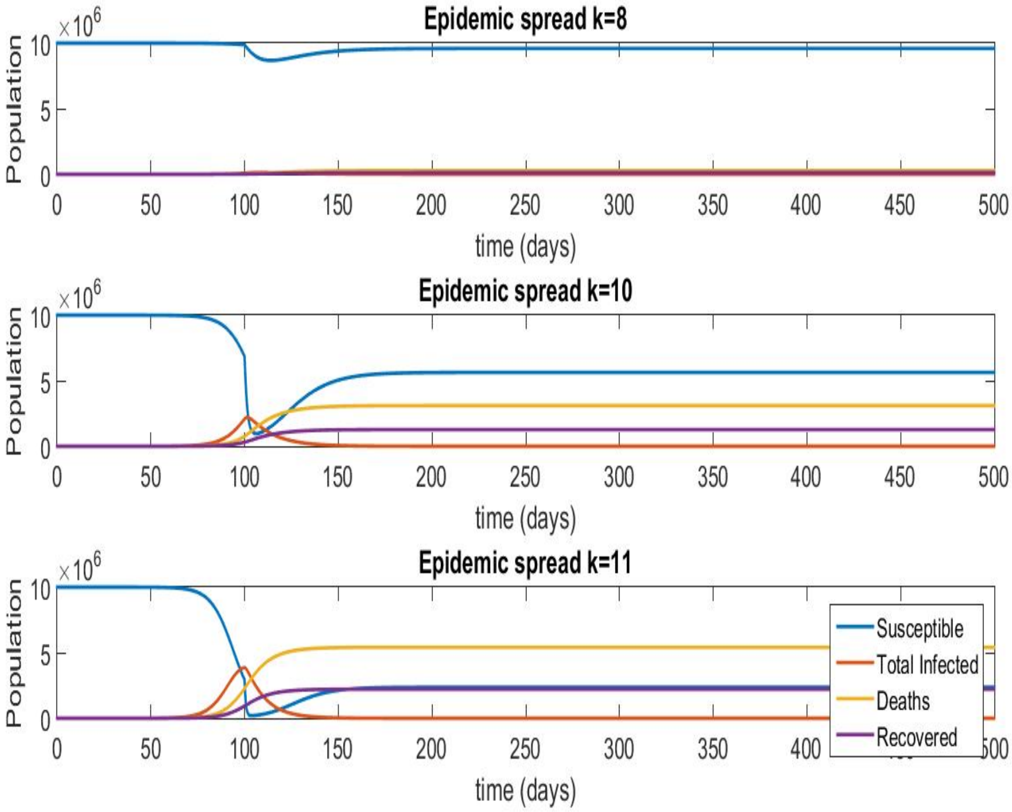

Figure 23: Graph of above problem for varying k Quarantine start time = 100days q=0.8

Observations:

- Here we are applying quarantine measures just when t = 100 days and the epidemic is spreading.
- The graph shows nature of various quantities for varying k: The number of contacts that an infected person has per day.
- Note that the nature of the graphs is same as in Figure 21:
- As the number of contacts increase, the number of infected increases much more and hence the number of deaths also increases.
- For lesser number of contacts, it is as good as having no epidemic. This means that there is certain threshold number of contacts which an infected person must have in order for there to be an epidemic.