CS302: Modeling And Simulation
Lab 4 Report

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1 Q1: Modeling of Malaria

The aim of this lab is to model the progress of malaria using the methods developed in the lectures. Malaria involves relationship between humans and mosquitoes. Not all mosquitoes spread malaria. Out of the total mosquito population only a fraction have the potential to spread malaria. Amongst these we can think of ones which are already infected and the ones which are not infected (susceptible). Amongst humans all have equal chance of being infected with malaria (all are susceptible). A bite from an infected mosquito can lead to malaria in humans. Alternatively, if a susceptible mosquito bites an infected human they can become infected.

Assumptions:

- Since life expectancy of humans is much larger than that of mosquito we assume that the human population is closed with no births, immigration and deaths except from malaria.
- As soon as an infected mosquito bites a susceptible human, it becomes infected.
- As soon as a susceptible mosquito bites an infected human it becomes infected.
- Because of their relatively short life expectancy mosquito's births and deaths are considered.

Contributing species and Processes

1. **Humans:**
   - susceptible humans get infected due to mosquito bite from an infected mosquito.
   - Infected humans can recover and become susceptible again.
   - Infected humans can die due to malaria.
   - Infected humans can become immune.
2. **Mosquitoes:**
   - Susceptible mosquitoes can become infected when they bite an infected human.
   - Mosquitoes can be born. All mosquitoes are born susceptible.
   - All mosquitoes can die natural deaths.

1.1 Draw the compartment model for malaria spread.

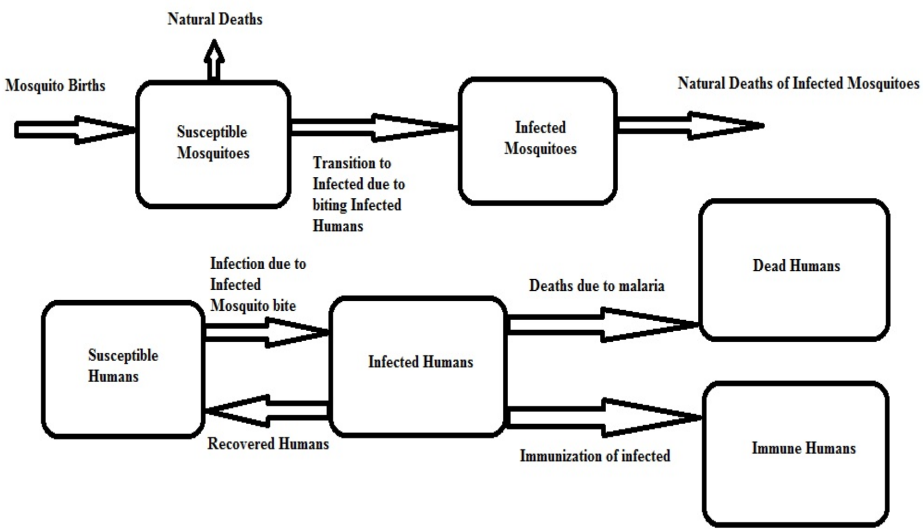

Figure 1: Compartment Model for Malaria

1.2 Identify all the variables and write differential equations for malaria spread assuming that except where interactions takes place all other changes are unconstrained growth and decay.

Variables:

- S_m: Susceptible Mosquitoes
- I_m: Infected Mosquitoes
- S_h: Susceptible Humans
- I_h: Infected Humans
- D_h: Dead Humans
- M_h: Immune Humans
- b: Mosquito Birth Rate
- d_m: Mosquito Death Rate
- N_m: Total Mosquito Population = $S_m + I_m$. This quantity varies with time.
- k: Bites per Mosquito per Unit Time
- N_h: Total Human Population = $S_h + I_h + M_h$
- d_h: Human Death Rate

- m: Human Immunization Rate
- r: Human Recovery Rate i.e. Infected get recovered and directly move back to being susceptible.
- l: Bites by an Infected Mosquito per Unit Time

Equations:

$$S'_m = b * N_m - (k * I_h * S_m)/N_h - d_m * S_m \tag{1}$$

$$I'_m = (k * I_h * S_m)/N_h - d_m * I_m \tag{2}$$

$$S'_h = -(l * I_m * S_h)/N_m + r * I_h \tag{3}$$

$$I'_h = (l * I_m * S_h)/N_m - (d_h + m + r) * I_h \tag{4}$$

$$D'_h = d_h * I_h \tag{5}$$

$$M'_h = m * I_h \tag{6}$$

1.3 Using the following information in your model implement on the computer and comment on the observations (Make a single figure with all the susceptibles and infected):

1. (Initial values (t=0)): Susceptible humans = 300, Infected human = 1, immune humans = 0, susceptible mosquitoes = 300, infected mosquito= 0.
2. Constants: recovery rate of humans = 0.3, immunity rate = 0.01, malaria induced death rate = 0.005, mosquito birth rate = 0.01, probability of mosquito biting a susceptible human = probability of mosquito biting an infected human = 0.3.

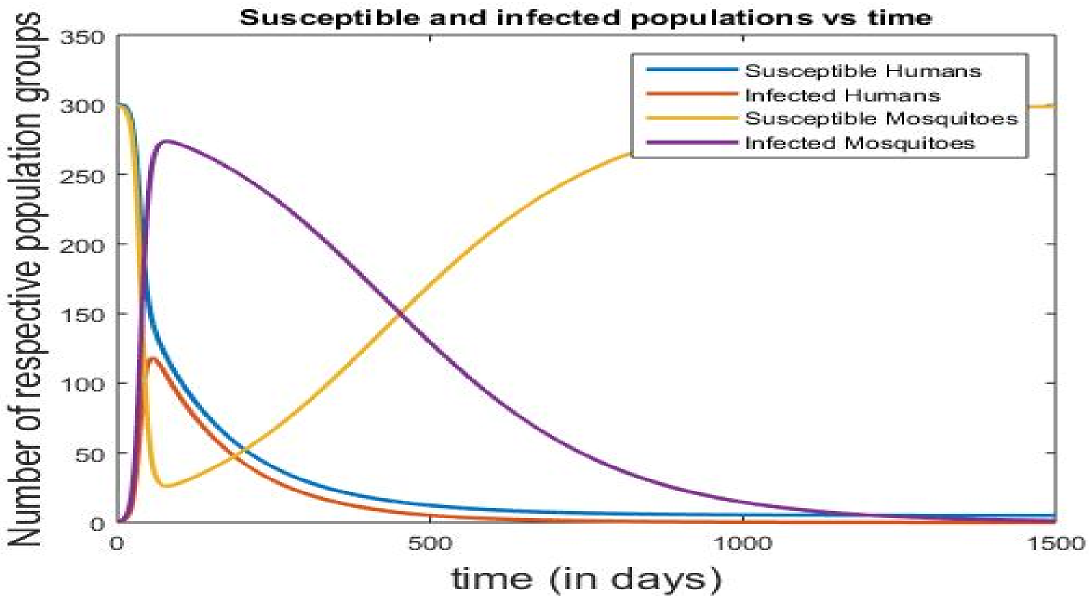

Figure 2: Normal Malaria Model: All susceptible and infected populations

Note, we have set the death rate of mosquitoes equal to the birth rate. Thus, the total number of mosquitoes due to their natural cycle stays the same.

Analyzing this process is a bit more challenging than the SARS model because here the rates of change are an interaction between two different species.

Observations:

- Let us first examine the evolution of Infected Mosquitoes. For the given parameters, we immediately see that there is an epidemic of malaria amongst the mosquitoes. There is a rapid increase to a maximum followed by a much slower decrease.
- Recall that, I_m has an inflow from S_m and outflow due to natural death. When, I_m is in the decreasing phase, I_h is also decreasing, due to this and the fact that new uninfected mosquitoes are being born, S_m goes on increasing. Since, this is an inflow to I_m, I_m itself decreases gently as this influx mitigates the decrease due to mosquito death.
- Infection in humans and mosquitoes is interlocked in the sense that I_m -¿ I_h -¿ I_m. It is a sort of positive feedback loop. The question is when does this stop? That is when does the epidemic start to decline.
- This happens when the I_h population reaches a maximum faster than the I_m and then starts declining. When this happens, soon after, the I_m starts declining, but as mentioned earlier the decline is less as the S_m keeps it up. So, the epidemic really starts declining when the S_h population starts declining. From this point onwards only the S_m increases, but that is because of the inflow due to natural births.
- The reason why I_m reaches a lower and faster maxima is that their are a lot of outflows from the I_h compartment and there is no new replenishment in the form of new susceptible humans being born.
- Finally, notice that once the I_m starts declining, S_m is able to quickly replenish itself, to a constant.

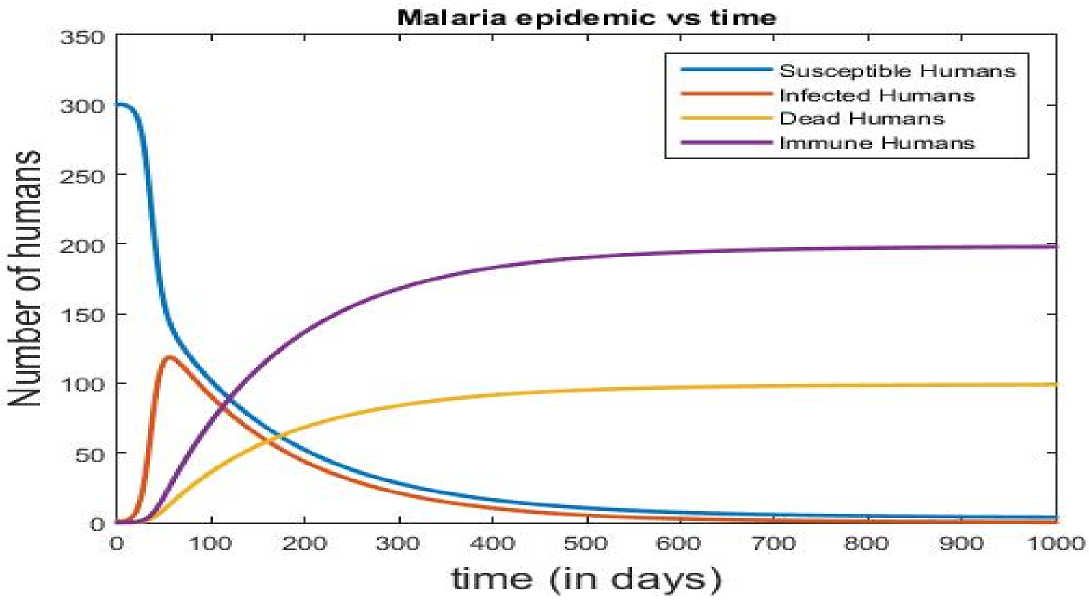

Figure 3: Normal Malaria Model: Effect on All Humans

Observations:

- The above graph shows the effect of malaria spread on all humans.
- Because the immunization rate is double that of the death rate, which is generally the case in real world scenarios, the total immune are almost double that of total deaths after a long time.
- Looking at the graphs of infected and susceptible, initially there are a huge number of susceptible humans hence the probability of infected mosquitoes biting them is high. Hence, initially, a large number of humans get infected and the infected curve rises sharply.
- But after a certain point, the recoveries from infected increase due to higher population in that compartment. Hence the three outflows from the infected compartment become significantly more than the rate of inflow and hence the number of people in the infected compartment start decreasing.
- Also, because of the recoveries, the rate of inflow in the susceptible compartment starts increasing. This explains the reduction in decrease in the blue graph above.
- As the time increases, more number of people become either immune or are dead which a very small amount of people remain susceptible or infected. Hence the susceptible and infected people almost become zero.

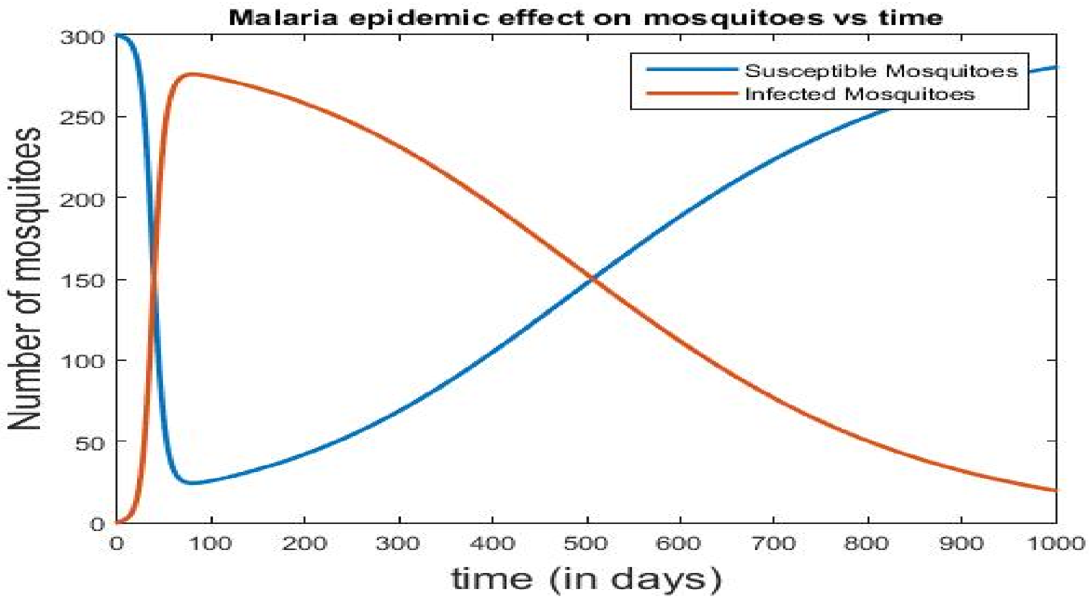

Figure 4: Normal Malaria Model: Effect on mosquitoes

Observations:

- The above graph shows the effect of malaria spread on mosquitoes.
- As can be seen by the red graph, initially, as the number of susceptible mosquitoes is high, the probability of them biting infected humans is also high and hence there is sharp increase in the number of susceptible mosquitoes. We see that this leads to an epidemic of malaria, as the mosquito population rises to a maximum. Note that increase in infected mosquitoes will invariably lead to an increase in number of infected humans leading to an further increase infected mosquitoes. It is a positive feedback loop more commonly known as a 'vicious cycle'.
- But as is typical of constrained growth problems, as the number of infected humans reach a peak and then start decreasing as seen in the previous graph, there are not enough humans nor are there enough susceptible mosquitoes to maintain a certain infected mosquitoes. Hence the rate of inflow is less which is why the red plot comes down.
- Since the rate of inflow in infected compartment is less, the movement of mosquitoes from susceptible to infected compartment is less and hence number of susceptible mosquitoes increase. As the blue graph shows, This value goes on increasing till a certain threshold at which point, the births, deaths and the infections balance out.
- Since, we have considered natural births and deaths for mosquitoes. As the epidemic starts to decline, less and less of the new 'stock' of mosquitoes coming in will be infected, hence the number of susceptible mosquitoes increases steadily.

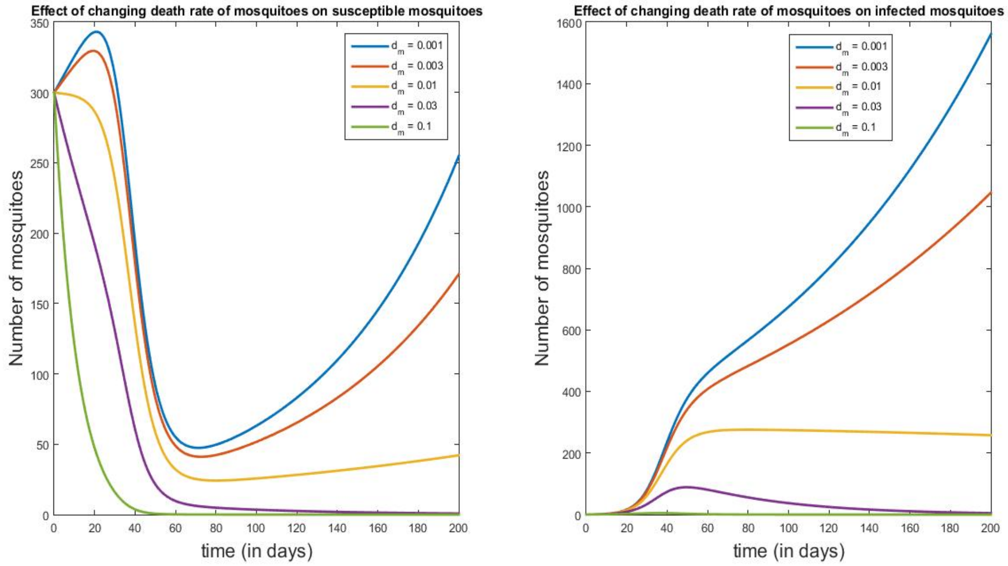

Figure 5: Normal Malaria Model: Effect of changing mosquito death rate

Observations:

- The above two graphs show the effect of changing death rate on population of mosquitoes
- The left graph shows the effect on susceptible mosquitoes. When the death rate is less than the birth rate, initially, the number of mosquitoes start increasing first as can be seen in blue and red plots.
- The opposite result is seen in the green and purple plots where the death rate is more than the birth rate.
- At some point in all the above graphs on left side, there is epidemic of malaria in mosquitoes i.e. susceptible mosquitoes decrease. At this same point, in the right side graph, we see a sharp rise in the infected mosquitoes.
- But after a long time, the nature of the graphs is controlled by the relation between death and birth rate of mosquitoes. If death rate is less than birth rate, then the number of susceptible as well as the number of infected go on increasing. While If the death rate is high, the whole mosquito population dies.
- If death rate is equal to birth rate, both the populations reach a certain constant value because births, deaths and transitions from susceptible to infected balance out.

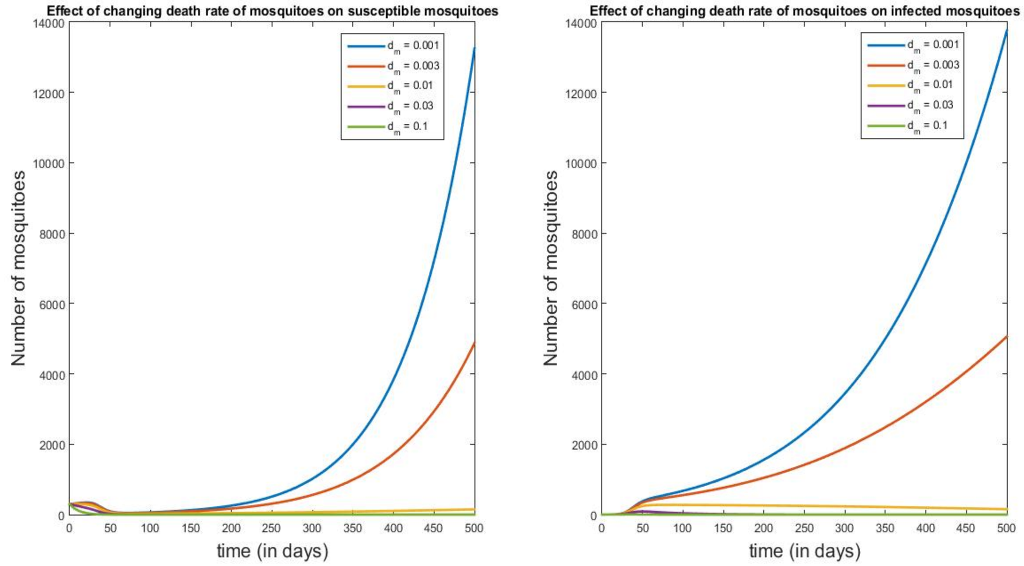

Figure 6: Normal Malaria Model: Effect of changing mosquito death rate

Observations:

- The above graphs shows the long term behaviour when the mosquito death rate is varied.
- As can be observed, the long term behaviour of the graphs is dependent on the relation between birth and death rate of mosquitoes.
- When birth rate is higher, there is an exponential increase in both the populations.
- When death rate is higher, almost all the mosquitoes die.
- When both rates are equal, the population largely remains constant with mild variations due to spread of disease.

1.4 If you we take into account spraying or fumigation which quantity changes in your equations. How does this change reflect in the figure that you made in part 1.3

Spraying increases the death rate of mosquitoes.

Here we assume that continuous spraying is done and accordingly modify the Equations.

Let $dnew_m$ be the new death rate such that $dnew_m = d_m + f$. Where f is deaths due to fumigation per unit time and per mosquito. Therefore we modify the following equations.

$$S'_m = b * N_m - (k * I_h * S_m)/N_h - dnew_m * S_m \tag{7}$$

$$I'_m = (k * I_h * S_m)/N_h - dnew_m * I_m \tag{8}$$

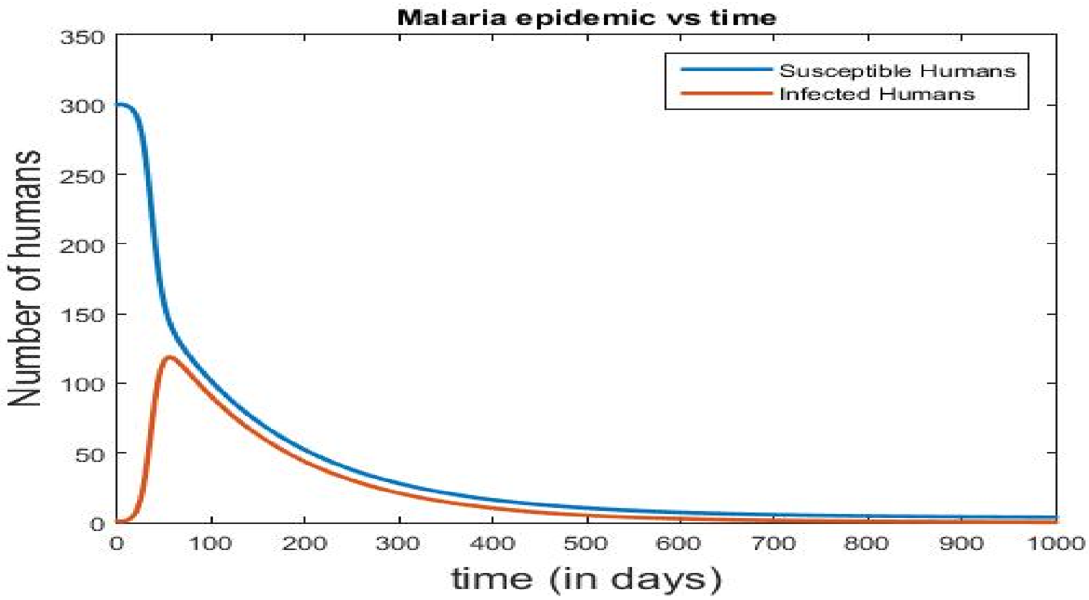

Figure 7: Malaria Model including fumigation: Human susceptible and infected

Observations:

- Surprisingly, even after fumigation, the above graph shows behaviour similar to the normal malaria model as far as Humans are concerned.
- It was expected that the number of infections would go down due to fumigation but this has not been the case.
- This is probably a limitation of the way the model has been built. The I_h equation has a ratio of I_m to total mosquitoes. Now, spraying leads to decrease of all kinds of mosquitoes and hence, the ratio more or less stays the same.

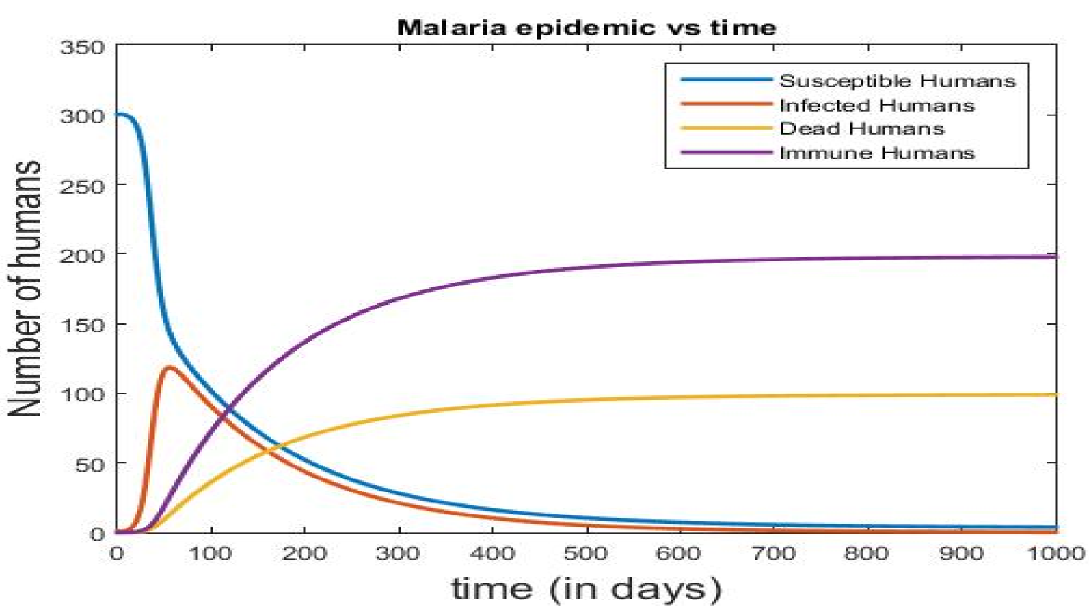

Figure 8: Malaria Model including fumigation: Effect on All Humans

Observations:

- Observing all the humans in various compartments, we see no change in the number of infections nor in any other compartment.
- One possible reason for this may be a problem in the model itself. Because, for susceptible humans, we take probability of the infected mosquitoes $(l * I_m * S_h)/N_m$. But since due to fumigation, both susceptible as well as infected mosquitoes die, the probability of infected mosquitoes in the total population remains same. I_m/N_m is constant. Hence it shows no change in the human population.

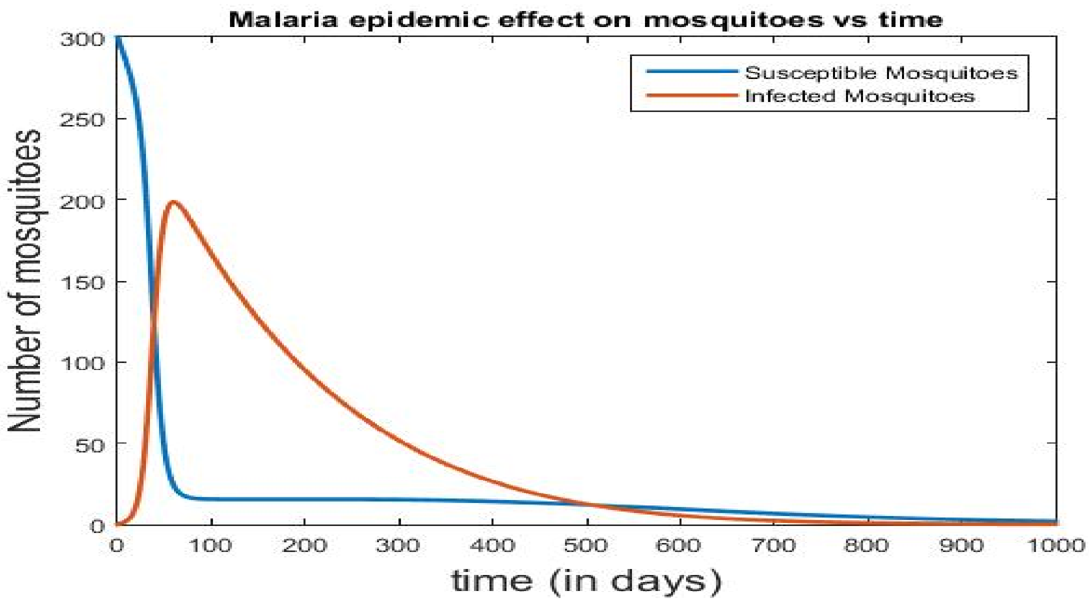

Figure 9: Malaria Model including fumigation: Effect on mosquitoes

Observations:

- This graph shows the effect of fumigation on mosquito population.
- Since fumigation essentially changes the death rate of the mosquito population and because natural birth and death rates are same, the mosquitoes eventually die because of fumigation causing higher number of deaths.
- If we see graph of total population of mosquito, we would see an exponential decrease.
- Note here that the peak of the infected graph does not reach the same level as the normal case because mosquitoes die. Hence in theory this should reduce the severity of epidemic because of lesser transmitting agents.

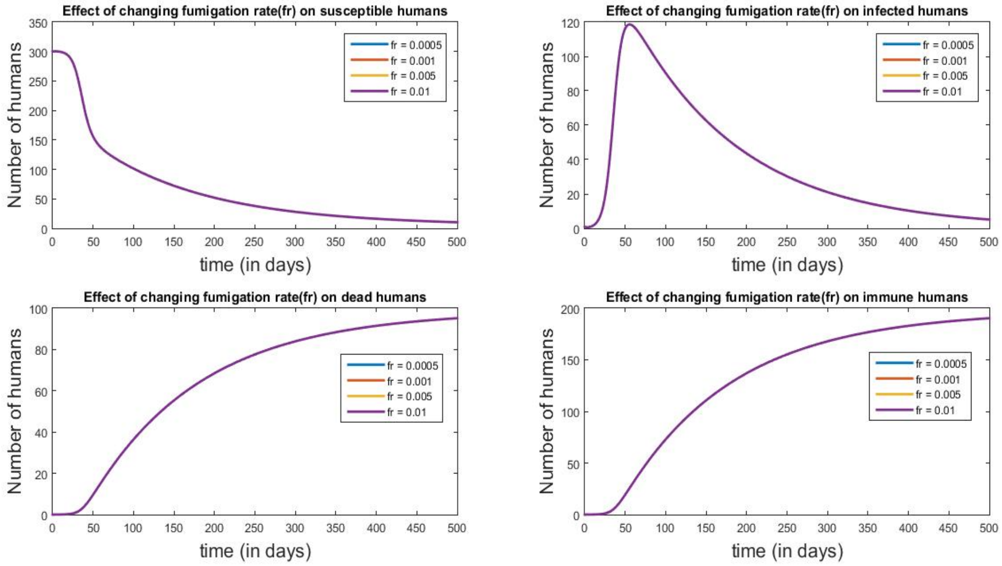

Figure 10: Malaria Model including fumigation: Effect of changing mosquito death rate on humans

Observations:

- As has been explained above for Figure 8, in this case there is no effect of fumigation on the human population. It is difficult to explain this anomaly.
- It is possibly because the spread of epidemic depends on a ratio which remains constant.

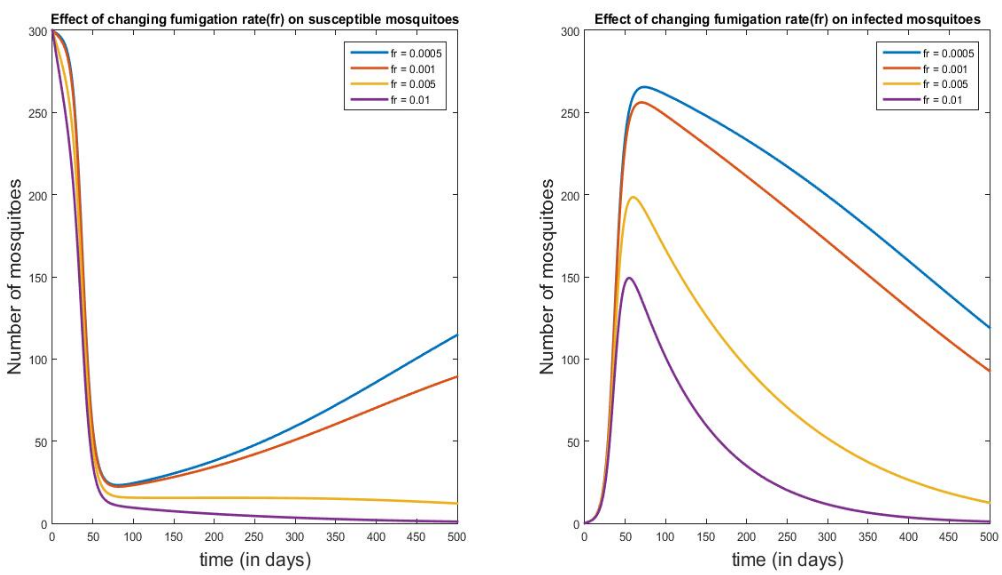

Figure 11: Malaria Model including fumigation: Effect of changing mosquito death rate on mosquitoes

Observations:

- Contrary to the normal case we see an interesting trend here in the mosquito population due to fumigation.

- Note here that natural birth rate = natural death rate.
- As the fumigation rate increases, the peak reached in the infected graph decreases, i.e. many mosquitoes die before becoming infected. Also the susceptible go to zero indicating that almost all mosquitoes die due to fumigation.
- In any case, the number of infected mosquitoes start decreasing after reaching the peak irrespective of the fumigation rate.
- In the graph of susceptible, we see that for low fumigation rates, blue and red graph, even if the total death rate of the mosquito population is high, the susceptible increases once the epidemic outbreak has passed. This seemingly contradictory observation can be explained by the fact that the infected mosquitoes are decreasing at a much faster rate which reduces the transmission of disease into humans and hence reduces the probability of susceptible mosquitoes getting the disease. Hence the total births outweigh the outflow from susceptible compartment.

1.5 If instead of spraying we went for vaccination what would you change and again how does it impact malaria spread.

Due to vaccination, Human susceptible decrease. The **vaccinated susceptible become immune and go into immune compartment.** We assume that immunization happens immediately and once immunized they cannot be infected that is, the immunization is permanent.

We assume that vaccination happens continuously with rate per unit time per unit person.

Here we assume that susceptible humans are vaccinated continuously. Hence we must consider that factor also while writing equations.

Note: This may be an unrealistic assumption. A better way to model this is to assume that vaccination happens at discrete instants of time as done in the SIR model in previous Lab.

Let v be the rate of vaccination. Then we need to modify following equations:

$$S_h' = -(l * I_m * S_h)/N_h + r * I_h - v * S_h \tag{9}$$

$$M_h' = m * I_h + v * S_h \tag{10}$$

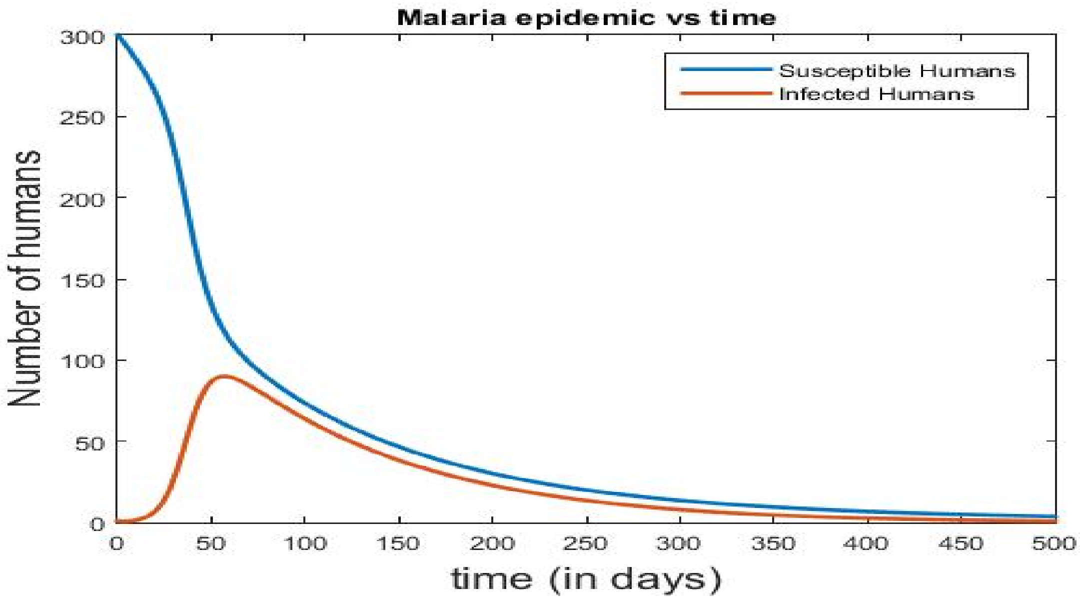

Figure 12: Malaria Model including vaccination: Human susceptible and infected

Observations:

- This graph shows the behaviour of system when continuous vaccination is done.
- Here, the maximum number of susceptible as visible in the red graph is much less than the normal case. Also, the time at which the epidemic occurs has been spread over a larger time and delayed. Which means the epidemic occurs at a later time but lasts for a longer time with reduced severity.
- The susceptible graph is more flatter than the normal case indicating gradual spread of the disease.

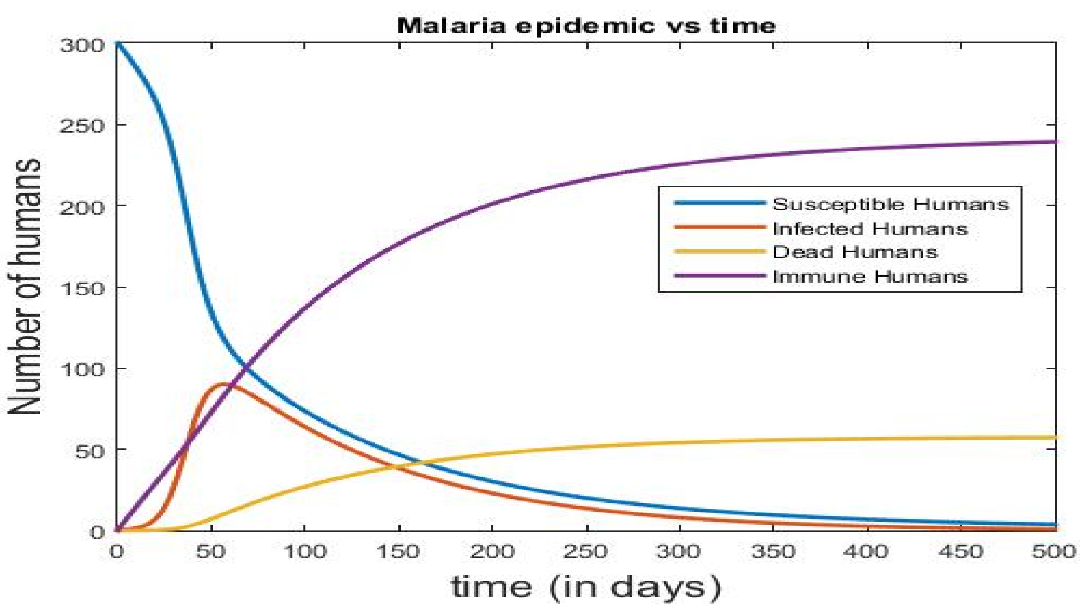

Figure 13: Malaria Model including vaccination: Effect on All Humans

Observations:

- In this graph, we can see the effect of vaccination.
- The immune graph immediately starts increasing while the graph of deaths is close to zero for much larger time before the epidemic outbreak.
- Also the number of people who are immune is much larger than the normal case while the number of deaths are greatly reduced.

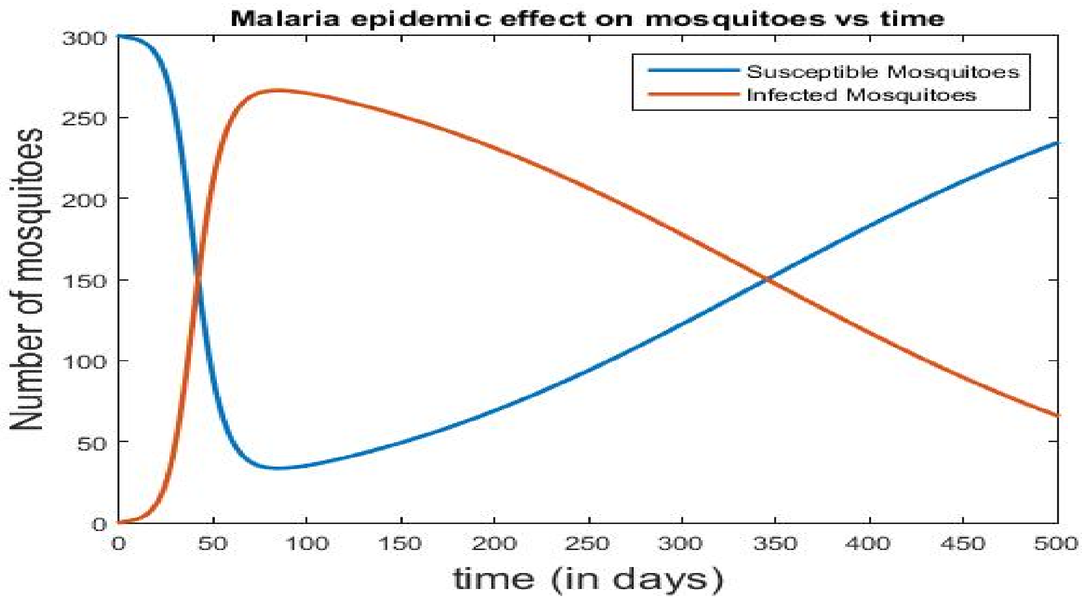

Figure 14: Malaria Model including vaccination: Effect on mosquitoes

Observations:

- The above graph emphasizes that although general trend of the mosquito population remains same but there are many differences.
- The infected population reaches a peak upon the epidemic outbreak but comes down much faster due to the vigorous vaccination which reduces the probability of malaria spreading from humans into mosquitoes.
- The susceptible population starts increasing indicating higher birth rate that the combined infection and death rate in mosquitoes.

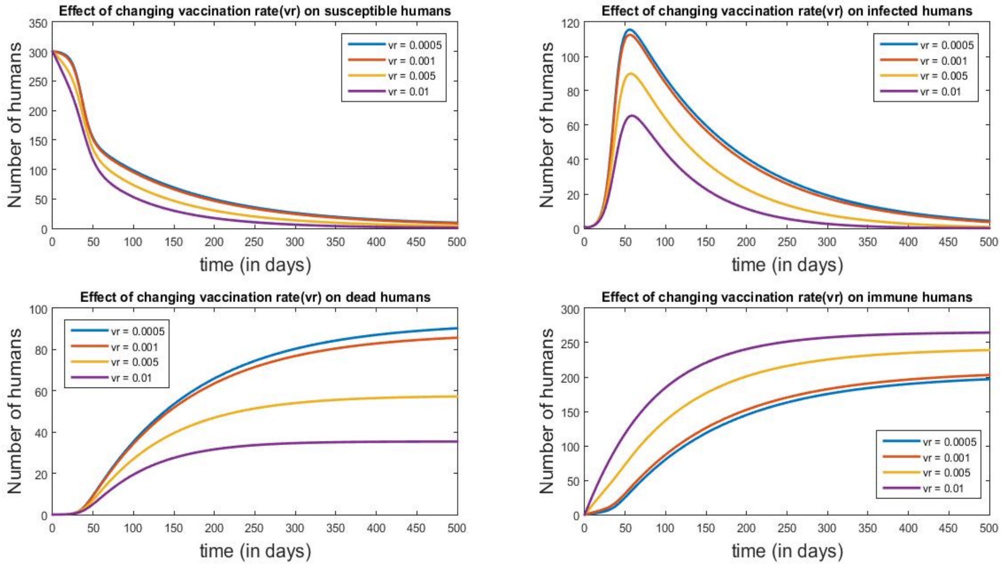

Figure 15: Malaria Model including vaccination

Observations:

- Observing the graph of dead humans, as the vaccination rate increases, the number of deaths reduce drastically.
- Opposite behaviour is seen in the immune graph. Here as vaccination increases, the number of immune humans increase.
- The graph of infected humans at the right top corner shows how the vaccination is instrumental in controlling the spread of epidemic. As the vaccination rate increases, the peak of the infected graph decreases. The spread of the graph also becomes narrow showing that the severity as well as the duration of the epidemic decreases remarkably.
- The graph of susceptible shows least difference among above four. Here as vaccination rate increases, the number of susceptible start decreasing immediately at the start. This is because vaccinated humans become immune. This is better because lesser the number of susceptible, lesser the probability of them getting infected lesser the severity of malariia spread.

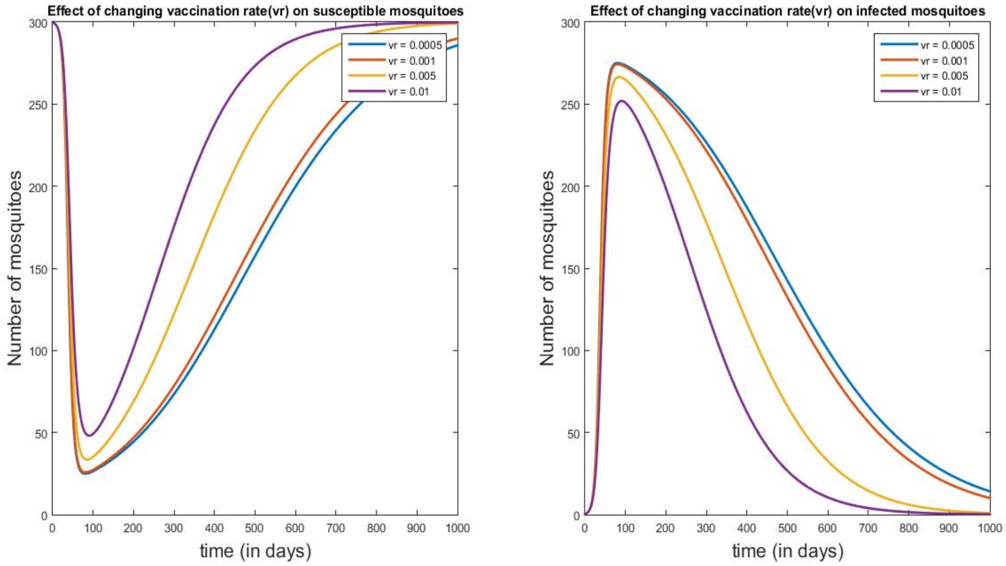

Figure 16: Malaria Model including vaccination

Observations:

- The effect of vaccination on mosquitoes is very interesting to observe.
- We must first observe the effect on infected mosquitoes. As the vaccination rate increases, the highest number of infected mosquitoes decrease. Also the spread of time over which there are infected mosquitoes also decreases. This means that severity of malaria spread decreases. This is a direct consquence of the number of susceptible humans decreasing because of vaccination.
- Since the infected mosquitoes decrease, the birth rate becomes more than death and infection and hence susceptible mosquitoes increase.
- Observe that in no case all the mosquitoes die. But the infected eventually become zero for any vaccination rate. This is because vaccination is a defensive measure not offensive.

1.6 Case of combined fumigation and vaccination

This situation is more closer to reality. Both fumigation and vaccination work simultaneously as a preventive measure for malaria.

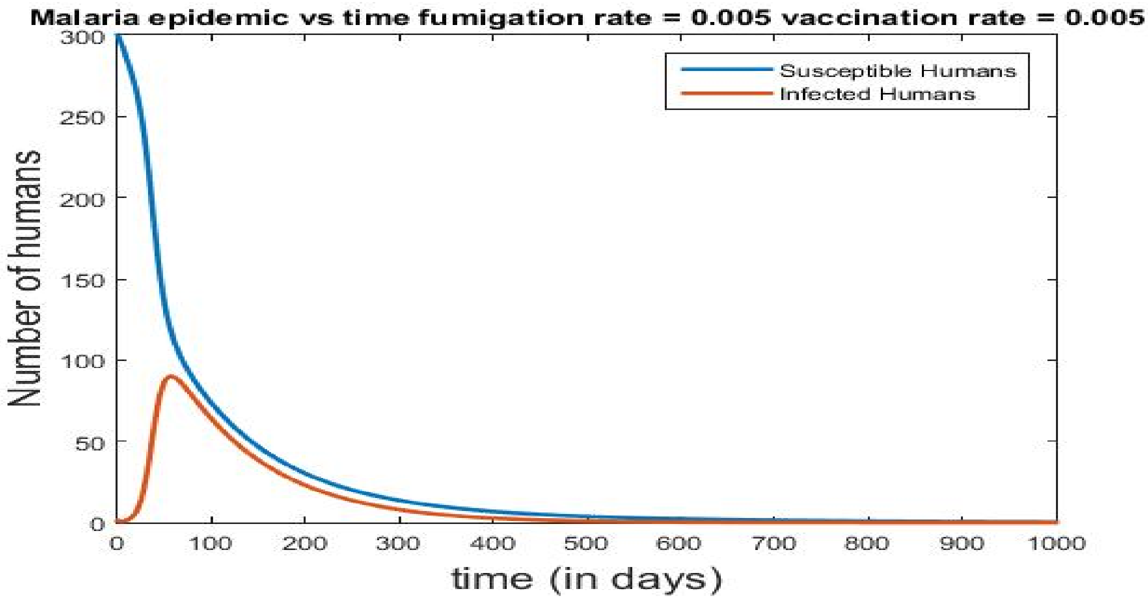

Figure 17: Malaria Model including both fumigation and vaccination: Human susceptible and infected

Observations:

- This graph is similar to the graph of vaccination mainly because as we have shown earlier, there is no indirect effect of fumigation on infected humans.

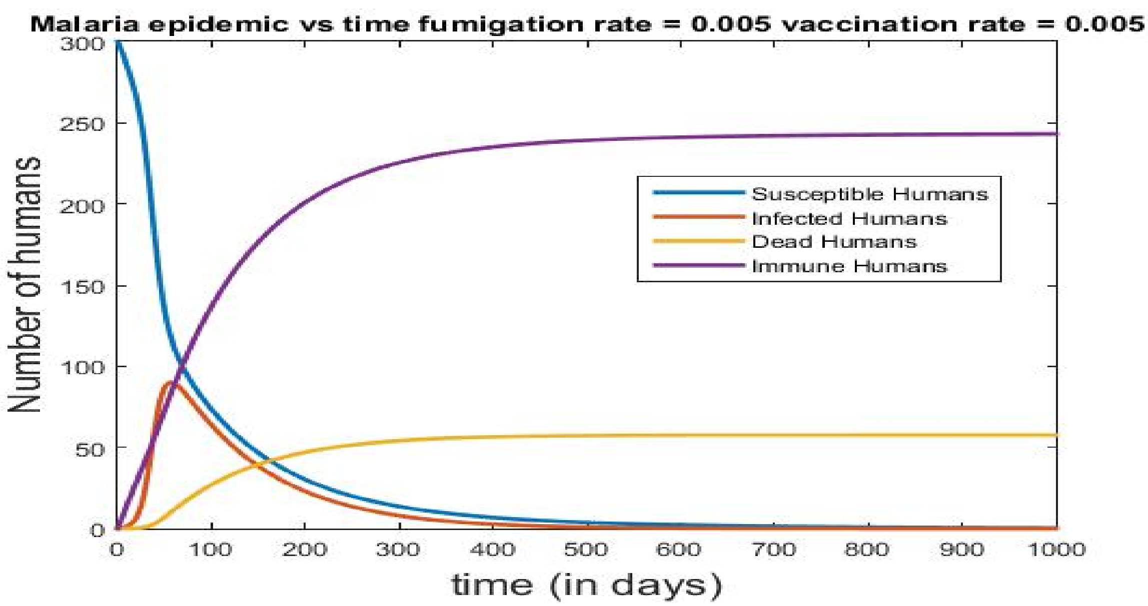

Figure 18: Malaria Model including both fumigation and vaccination: Effect on All Humans

Observations:

- Here also due to vaccination, the number of immune humans increase while the number of deaths decrease.

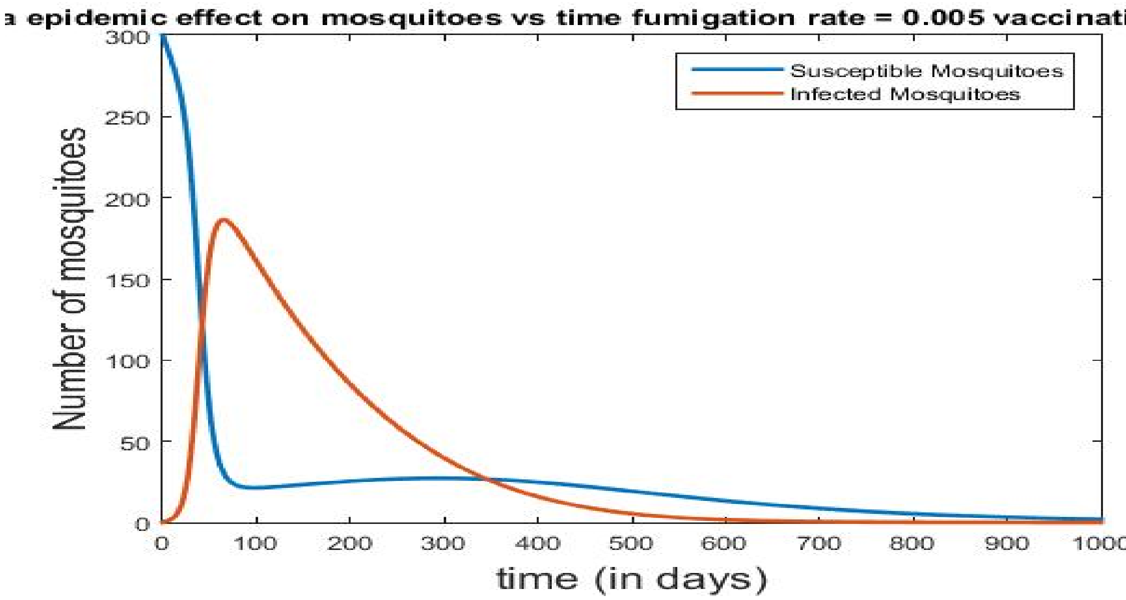

Figure 19: Malaria Model including both fumigation and vaccination: Effect on mosquitoes

Observations:

- This is again an interesting graph. Here, due to fumigation, mosquitoes die and hence number of infected mosquitoes are less that normal case.
- Similarly, the graph of susceptible remains constant or increases slightly because the infected mosquitoes are significant and hence the birth of new susceptible is more.
- But as soon as the infected mosquitoes go down, the susceptible also start going down eventually reaching zero because of the combined effect fumigation as direct consequence and vaccination as indirect consequence.

1.7 Conclusion:

- It was a slightly more complicated model compared to SARS.
- The main reason for added complexity was the indirect infection across two different species. Hence, we had two different infectious interactions, one between $S_h \;\; I_m$ while the other between $S_m \; I_h$.
- The rate equations had a ratio term to capture the probability that a mosquito or a human is infected. This was used to condition the probability of a mosquito biting a susceptible human or a susceptible mosquito biting an infected human.
- The evolution of epidemic was more or less similar to SARS.
- We considered natural births and deaths only for mosquitoes. This lead to interesting evolution of S_h and S_m graphs.
- We finally factored in fumigation and vaccination in our model.
- Our model possibly has a flaw, possibly because we have put ratios to capture the probability that a mosquito or a human is infected. The ratio remains the same inspite of a decrease in mosquitoes by fumigation.
- Fumigation is an active method of protection. It actually kills the infectious vectors. Whereas vaccination is a morr defensive approach. Hence, a combination of both is expected to give best results.