Modeling and Simulation, CS302

Lab-2

Due Date -Friday 27th, January.

- You can submit the last problem in Lab-1 with this lab.

1. **Models of innovation diffusion:** Fig. 1 shows the diffusion of innovations. Initially, the market share of a new product is zero. However, due to certain factors there are a group of people called early innovators who initially adopt the product (technology). As time progresses the product increases its market share due to many factors such as advertising, distribution of prices and contact between users and finally saturates to a maximum value. Behavior observed in Fig. 1 or some variant of it is commonly observed in a multitude of problems (e.g number of twitter users, people using smart phones, market share of apple phones etc.). In this problem we take a look at some of the models specifically for such problems.

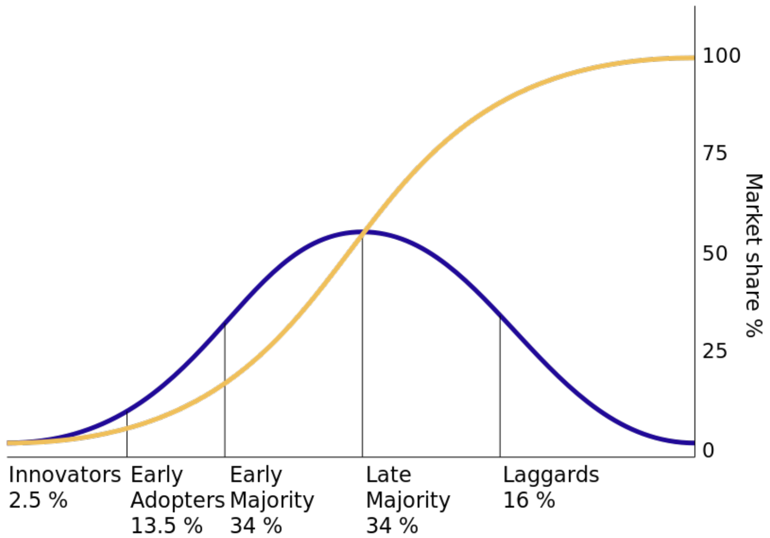

FIG. 1: The diffusion of innovations according to Rogers. With successive groups of consumers adopting the new technology (shown in blue), its market share (yellow) will eventually reach the saturation level. The yellow curve is known as the logistic function. (figure and caption taken from wikipedia)

Typically the mathematical model for such problems have the form:

$$\dot{N} = \alpha(t)(C - N(t)) \tag{1}$$

where, $\alpha(t)$ is the coefficient of diffusion, C is the maximum number of potential users of the product and $N(t)$ is the total number who have adopted the product till time t. Three models are common:

- external influence model in which $g(t) = p$, where p is a constant and captures the innovators or people who adopt the product on their own without being influenced by others.
- internal influence model $g(t) = qN(t)/C$, in which the rate $g(t)$ now captures the adoption due to the effect of the other users.
- mixed influence model (Bass): $g(t) = p + qN(t)/C$, which captures both the effects.

Assuming that the market share can be approximated by the percentage of users and the population size to be large ($C = 10^5$)

(a) First consider the external influence model and study the effect of variation of p. Plot similar to Fig. 1 both the market share and groups of consumers adopting the product.

(b) Similar to (a) but now consider the internal influence model. Study the effect of varying q. Can you comment on the aspect of the problem which the logistic equation can not capture.

(c) Similar to (a) but consider the mixed influence model. Can you identify how the missing aspect in (b) is already accounted for in the mixed model. What processes according to you would be captured by the constants p and q. Comment on the behavior observed with a rational choice of parameters.

(d) (**Extension idea: Not to be submitted**) Quite often we observe that one product loses its share to another one. Take the example of social network platform like orkut which was very popular and then sort of lost its popularity to facebook or IBM machines losing to Microsoft. Can you think of a realistic model for such phenomenon?

2. We have been modeling population using Logistic equation. The logistic equation has the property that population eventually reaches a steady state value given by the carrying capacity. In many situations of interest the carrying capacity is not a constant but rather is itself dynamic. One reason for this is technological advancement. The results of these technological advancement are not visible immediately but do so over a period of time. For

example, it might be adopted at a changing rate. In such situations the rate of change of population can be written as:

$$\dot{P} = \alpha P \left(1 - \frac{P}{K(t)}\right) \tag{2}$$

where in Eq. (2) the carrying capacity $K(t)$ is a function of time. Let us assume that the carrying capacity varies in a fashion shown in Fig. 2. This captures the above mentioned features of carrying capacity. It is initially at a lower value K_1 then as new technologies are adopted it slowly increases and finally settles to a new value $K_1 + K_2$. In Fig. 2 $K_1 = 20$ and $K_2 = 50$.

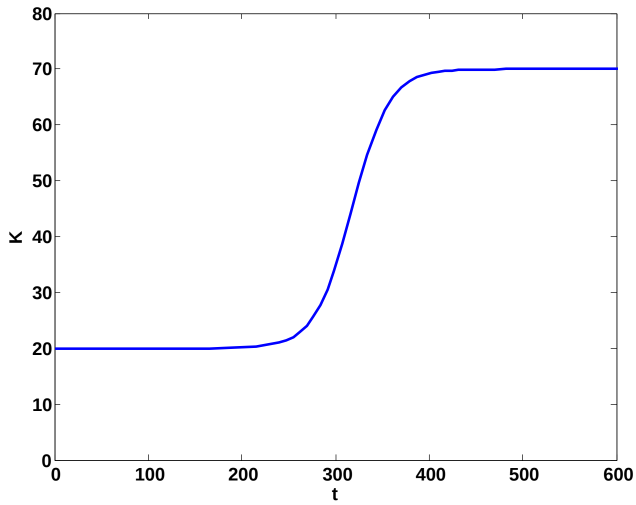

FIG. 2: Dynamically varying carrying capacity

(a) Model the dynamics of the carrying capacity as shown in Fig. 2 through a differential equation. Explain the parameters used.

(b) Which of the parameter(s) you have used in your expression when changed would change the time the carrying capacity spends at the lower value K_1. Now attempt to produce a figure similar to Fig. 2. What is the initial condition and why?

(c) Now solve Eq. (2). Using reasonable values from part (b) above, what according to you should be a reasonable value of α. If you think that α can take any positive value then taking a range of values comment on the different behaviors observed.

(d) Compare and contrast the current case with the situation in which the carrying capacity is a constant.