Goal:

To exemplify the bullwhip effect in a supply chain and how its behaviour changes with different lead times and the weight forecasts place on the most recent data.

Idea:

There is a supply chain consisting of a manufacturer who supplies to a distributor who supplies to a wholesaler who supplies to a retailer who supplies to consumers. Each member is denoted as a firm in general.

There is a lead time of L + 1 at each step of the supply chain, consisting of 1 period to transmit the order from the customer to the supplier, and L periods to send goods from the supplier to the customer. Hence, order placed by a firm in time period t-1 becomes the demand for its supplier in time period t.

Each firm uses an exponential smoothing method to forecast its future demand based on the previous order placed by its immediate downstream member of the supply chain. All the firms are calculating their forecast, order data and inventory independently. This lack of communication between various members of the supply chain will lead to the Bullwhip effect which will be demonstrated by this activity.

Let us define some parameters useful for carrying out the calculations and keep track of the records and activities.

DX(t) = demand for a firm in time period t

OX(t) = order placed by a firm in time period t

IX(t) = the net inventory at the end of period t, after the replenishment in time period t has been received from the supplier, and after the customer order received in time period t has been deducted from inventory

IPX(t) = the inventory position in period t after the customer order placed at the end of time period t-1 has been received in period t and after items have been sent to the customer in time period t, but before the order with the supplier in time

FX(t) = forecast for a period t calculated by each firm using exponential smoothing method

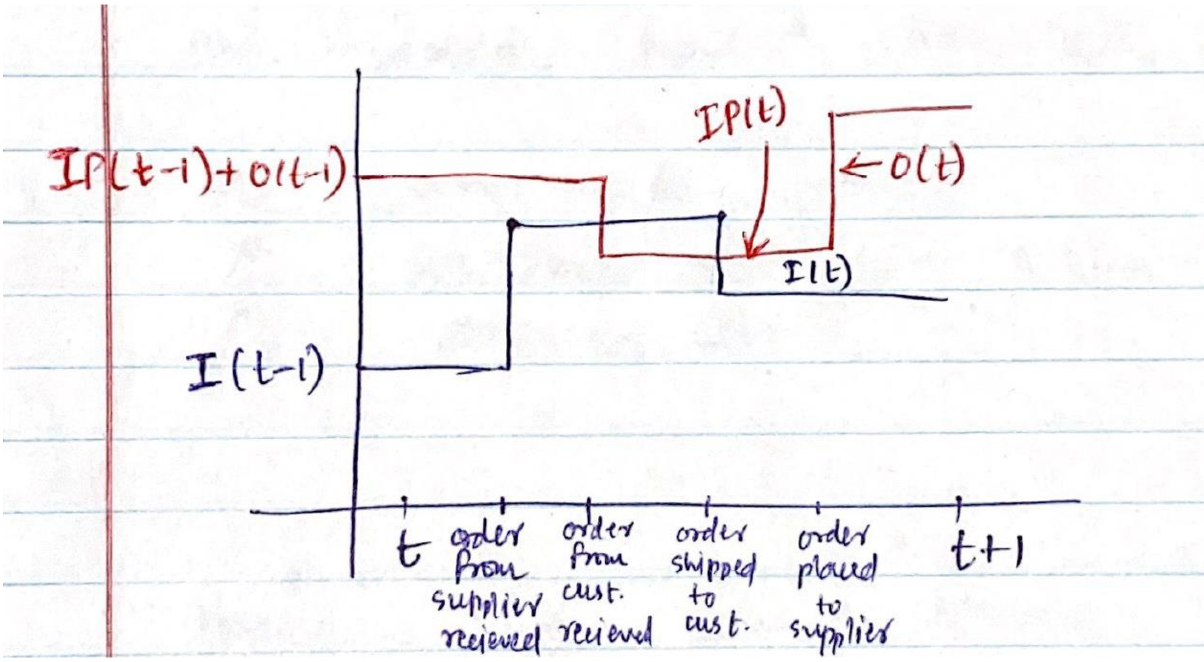
BX(t) = Base stock level in period t (it will be useful in the third case ES-3

X = R, W, D, M respectively for Retailer, Wholesaler, Distributor and Manufacturer

L = Lead time to send goods from supplier to customer

α = Exponential smoothing factor

Following figure represents the change in inventory and inventory positions as the various activities take place during the supply chain.



Following formula are used to calculate inventory, inventory position and forecast.

$$IX(t) = IX(t-1) + OX(t-L-1) – DX(t)$$

$$IPX(t) = IP(t-1) + OX(t-1) – DX(t)$$

$$FX(t+1) = (1-\alpha)\ FX(t) + \alpha\ DX(t)$$

Except for retailer, DX(t) becomes OX(t-1) where orders placed by a downstream member in t-1 period becomes a demand for t period for its immediate upstream member.

There is a common initial condition for each three methods. Demand for retailer is 10 for t=1 and then it is the cycle of period 4 (9, 12, 8, 11) repeating for all t >= 2. IX(0) = 10 for each firm and IPX(0) = 10 + 10L for each firm since 10 units of products are on its way to each firm for a lead time period.

We will consider L = 0, 2 and 4 and α will vary from (0,1) with an increment of 0.01.

Order quantity OX(t) will be chosen using one of the following methods:

1.) Method ES-1

Each firm collects data of its own customer orders and tries to forecast demand for t+1 period using simple exponential forecasting formula

$$FX(t+1) = (1-\alpha)\ FX(t) + \alpha\ DX(t)$$

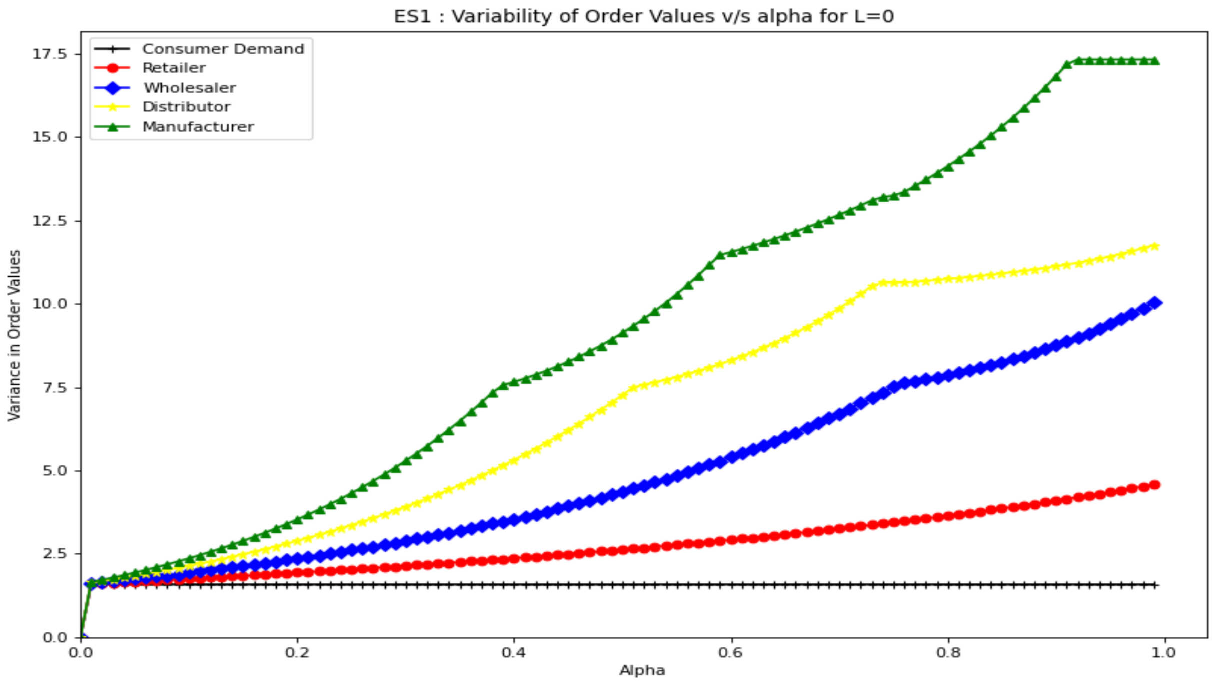
Each firm plans for demand equal to F(t+1) in each time period in the future and since the order placed in time period t will arrive at t+L+1, the order should be sufficient to make inventory level non-negative at the end of period t+1+L. Here, we consider that demand is F(t+1) for period t hence, total demand will be F(t+1)*(L+1) for the period of lead time. Now we don’t want inventory to fall below zero hence order to be placed should be equal to

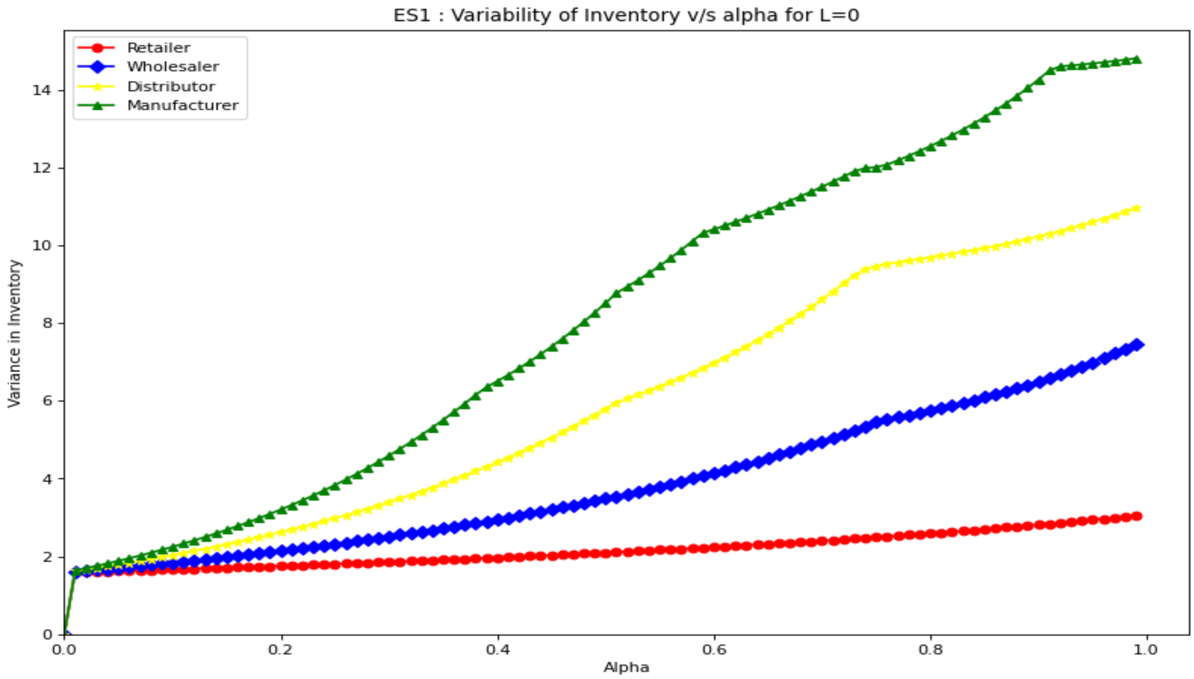
FX(t+1)*(L+1) – IPX(t) and it should be non-negative too. Hence, order quantity will be decided by the following formula for ES-1 model.

$$OX(t) = \max\{ F(t+1)*(L+1) - IPX(t), 0\}$$

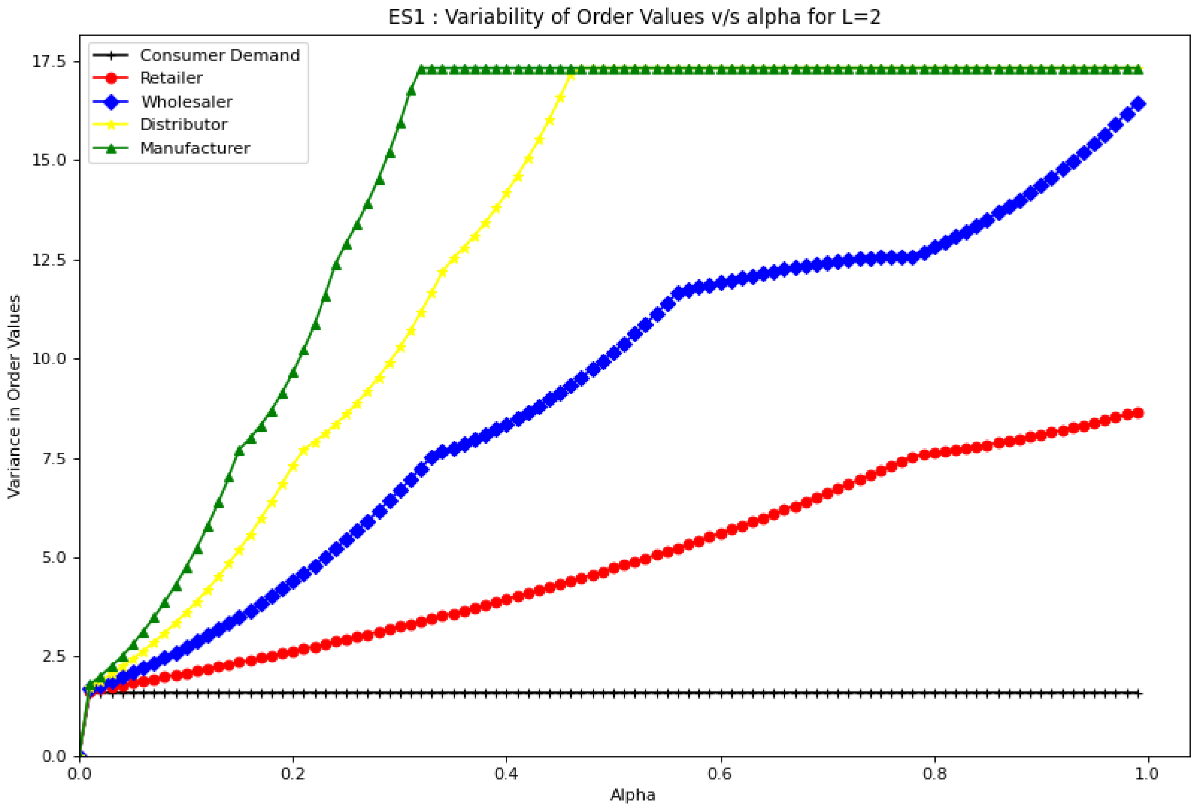
Using the above philosophy, a simulation was run for all four firms of the supply chain with α varying from 0 to 1 with an increment of 0.01 and three different cases taken with L=0, 2, 4. Variances in the order values and inventory of all members plotted against α.

(i) L=0

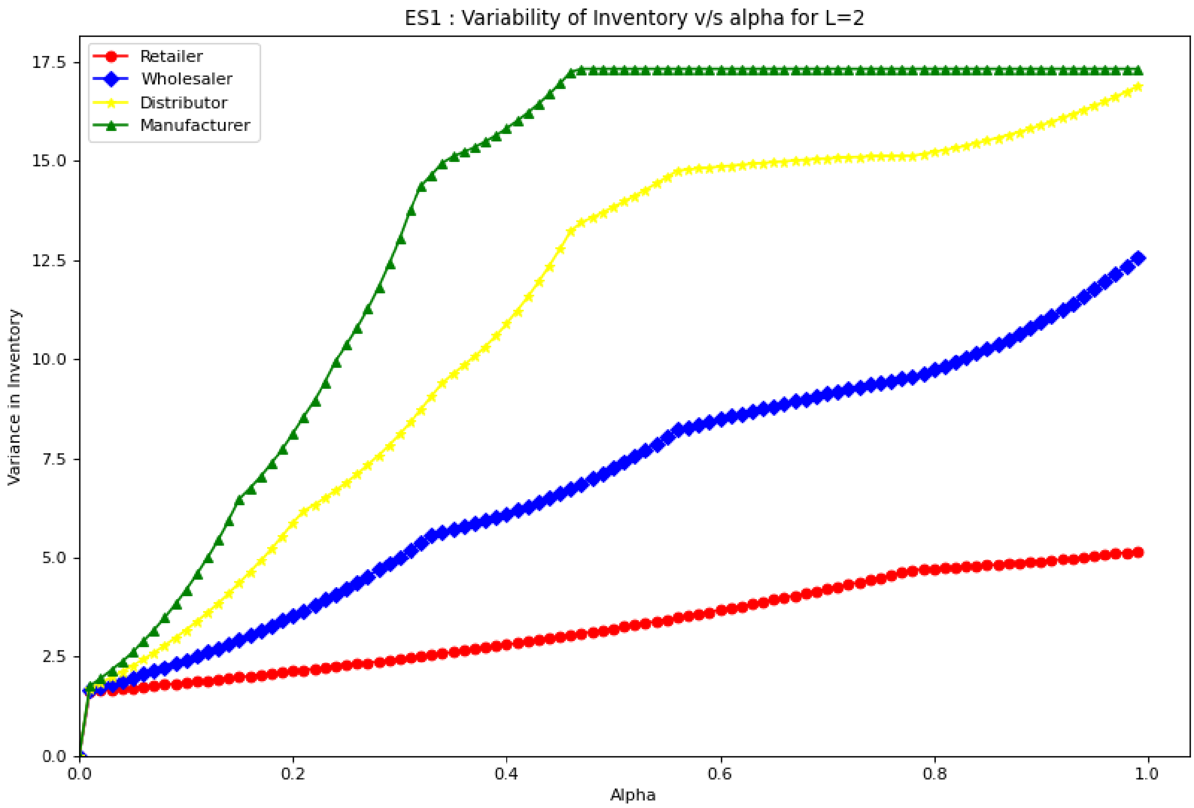




(ii) L=2

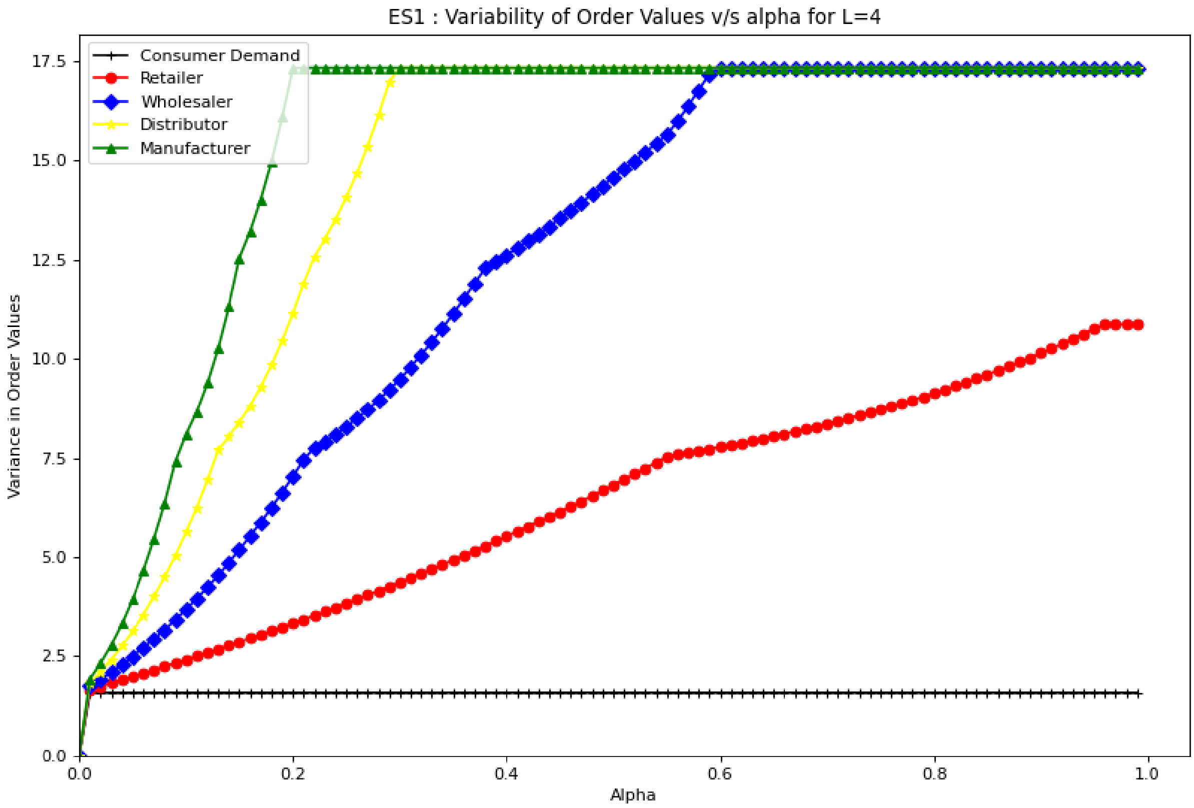


ES1 : Variability of Order Values v/s alpha for L=2
Consumer Demand
Retailer
Wholesaler
Distributor
Manufacturer
Variance in Order Values
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
0.0
0.2
0.4
0.6
0.8
1.0
Alpha

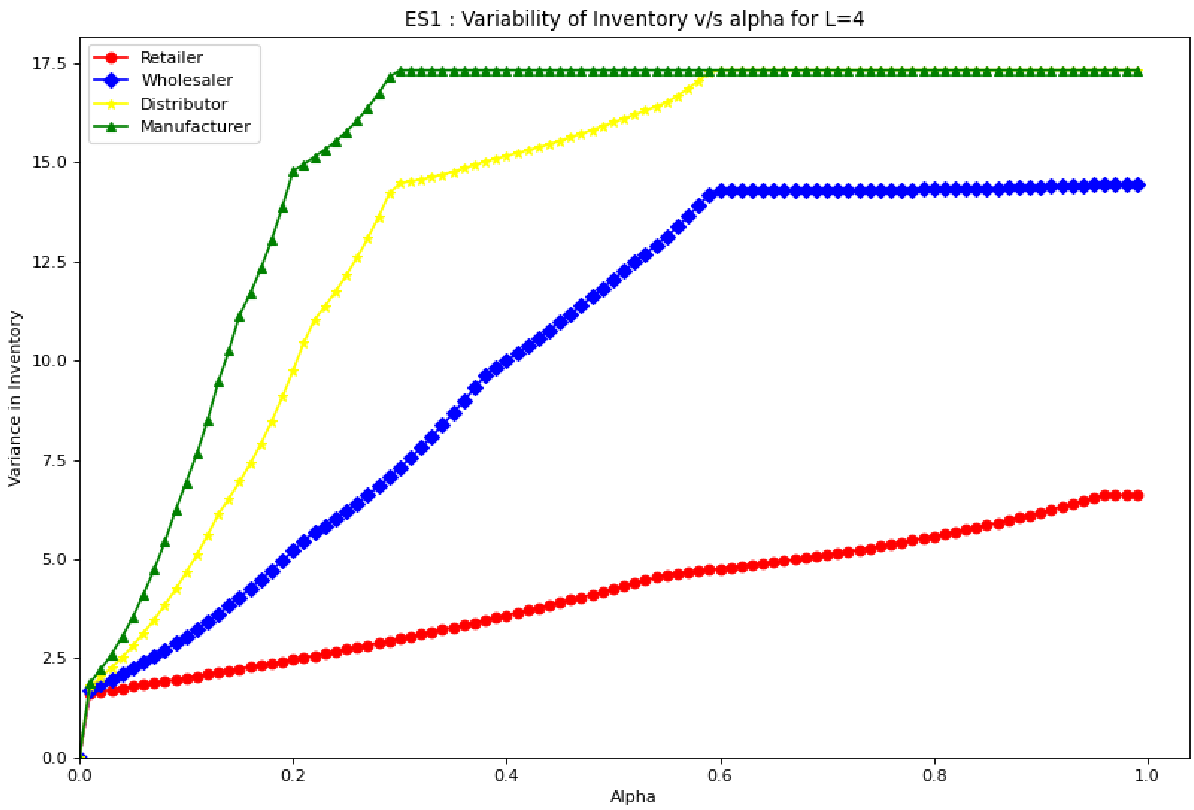


ES1 : Variability of Inventory v/s alpha for L=2
Retailer
Wholesaler
Distributor
Manufacturer
Variance in Inventory
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
0.0
0.2
0.4
0.6
0.8
1.0
Alpha

(iii) L=4



ES1 : Variability of Order Values v/s alpha for L=4
Consumer Demand
Retailer
Wholesaler
Distributor
Manufacturer
17.5
15.0
12.5
10.0
7.5
5.0
2.5
0.0
Variance in Order Values
0.0
0.2
0.4
0.6
0.8
1.0
Alpha



ES1 : Variability of Inventory v/s alpha for L=4
Retailer
Wholesaler
Distributor
Manufacturer
17.5
15.0
12.5
10.0
7.5
5.0
2.5
0.0
Variance in Inventory
0.0
0.2
0.4
0.6
0.8
1.0
Alpha

2.) Method ES-2

Here each firm calculates forecast using the same exponential smoothing but additionally, each firm also calculates variance of its orders by $\sigma^2(t) = \frac{1}{t-1}\sum_{T=1}^{t}[D(T) - \overline{D}(t)]^2$ where $\overline{D}(t) = \frac{1}{t}\sum_{T=1}^{t} D(T)$

The items that the firm orders in period t should arrive in period t + 1 + L, and should be sufficient to make the inventory level at the end of period t + 1 + L nonnegative with probability approximately 0.99, that is, P[I(t + 1 + L) >= 0] ≈ 0.99.

We can calculate the order values using the following calculation.

The item that firm orders in period t should arrive in period t+L+1 and should be sufficient to make the inventory level at the end of period t+L+1 non-negative with probabity approximately equal to 0.99.

$$\therefore P[I(t+L+1) \geq 0] \approx 0.99$$

$$\therefore P\left[O(t) + IP(t) - \sum_{i=t}^{t+L+1} D(i) \geq 0\right] = 0.99$$

$$\therefore P\left[O(t) + IP(t) \geq \sum_{i=t}^{t+L+1} D(i)\right] = 0.99$$

To standardize it, we will subtract mean demand which is $(L+1) \cdot F(t+1)$ for the period of lead time and divide it by standard deviation $\sqrt{(L+1)\sigma^2(t)}$.

$$\therefore P\left[\frac{O(t) + IP(t) - (L+1) \cdot F(t+1)}{\sqrt{(L+1) \cdot \sigma^2(t)}} \geq \frac{\sum_{i=t}^{t+L+1} D(i) - (L+1) F(t+1)}{\sqrt{(L+1) \cdot \sigma^2(t)}}\right] = 0.99$$

$$= Z_{0.99}$$

$$= 2.33$$

$$\therefore \frac{O(t) + IP(t) - (L+1) \cdot F(t+1)}{\sqrt{(L+1) \cdot \sigma^2(t)}} = 2.33$$

$$\therefore O(t) + IP(t) - (L+1) \cdot F(t+1) = 2.33 \cdot \sqrt{(L+1) \cdot \sigma^2(t)}$$

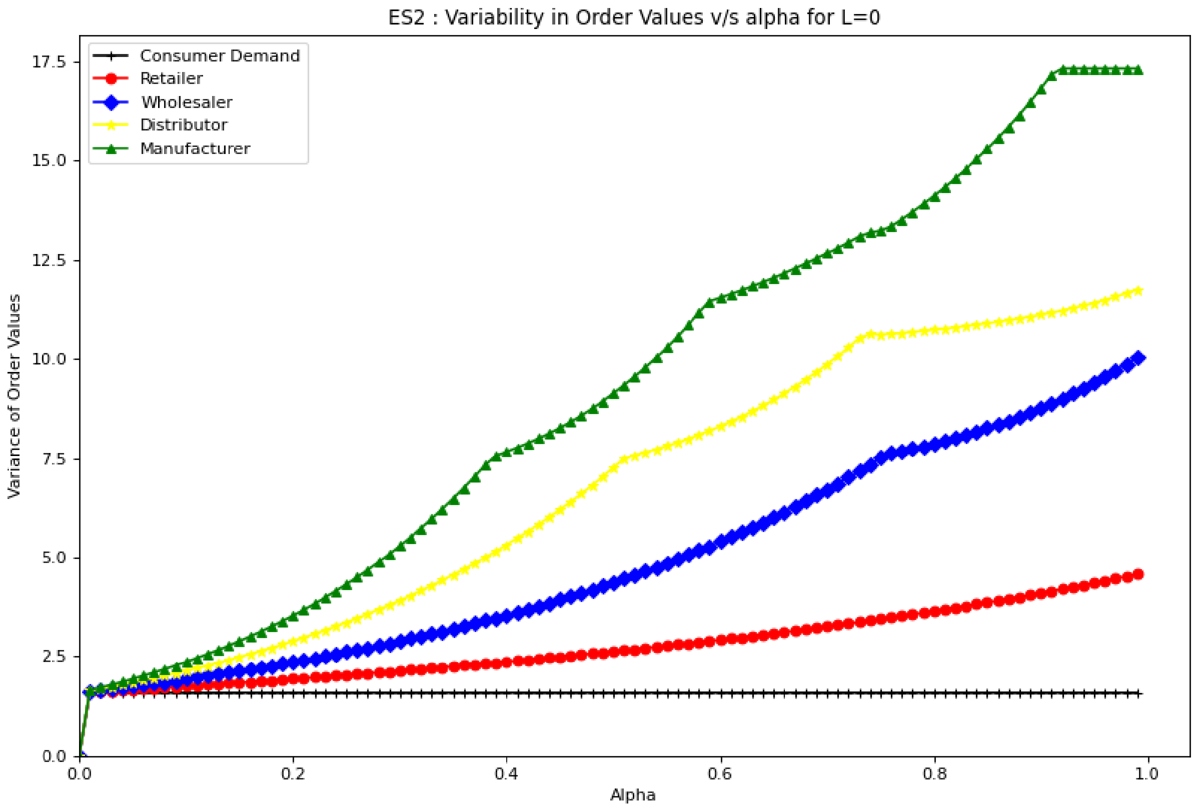
$$\therefore O(t) = (L+1) F(t+1) + 2.33 \sqrt{(L+1) \cdot \sigma^2(t)} - IP(t)$$

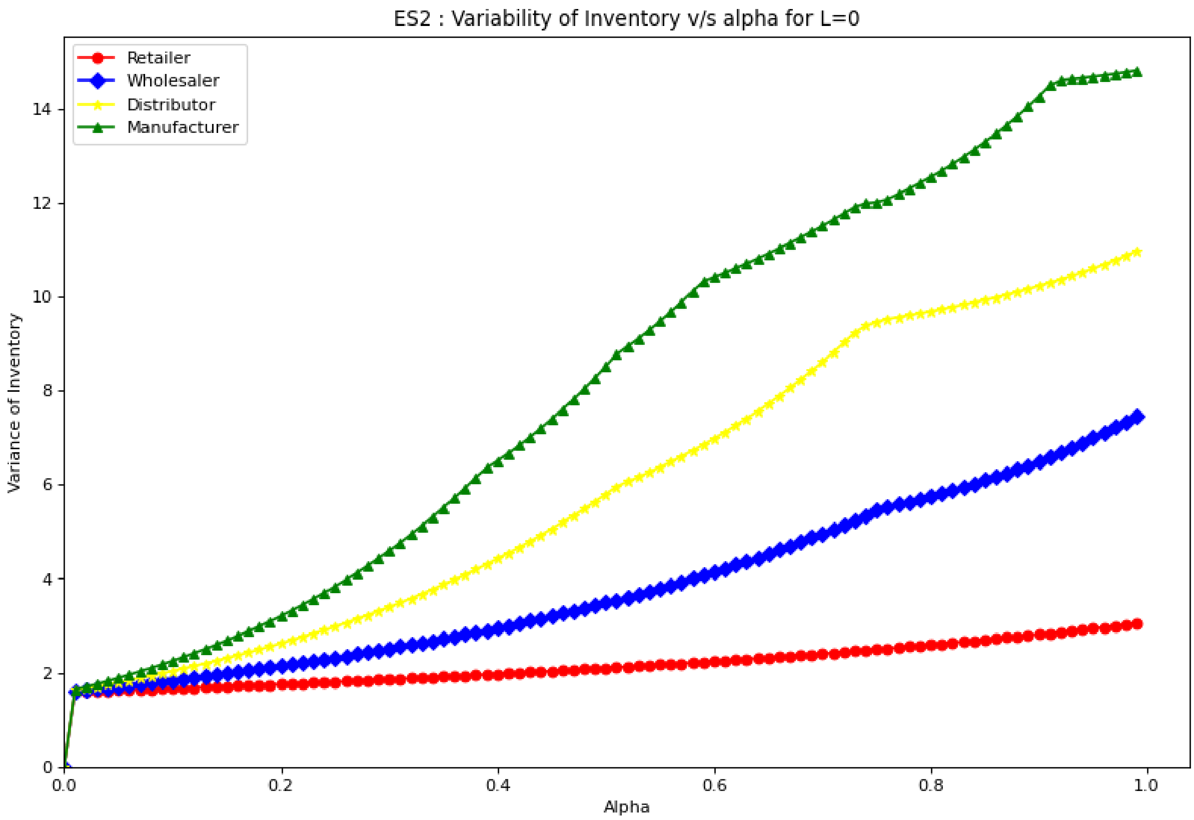
Also, O(t) cannot be negative

$$\therefore O(t) = \max\left\{(L+1) \cdot F(t+1) + 2.33 \sqrt{(L+1) \cdot \sigma^2(t)} - IP(t),\ 0\right\}$$

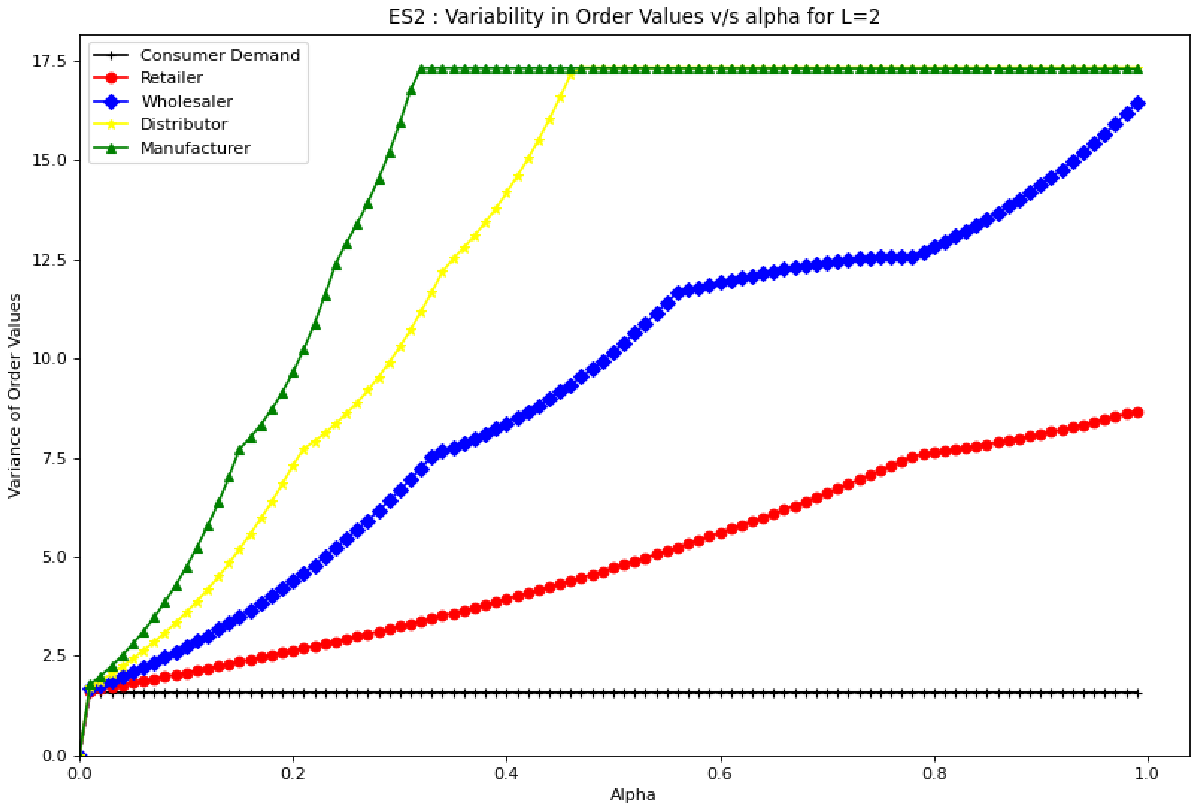
Using the above philosophy, a simulation was run for all four firms of the supply chain with α varying from 0 to 1 with an increment of 0.01 and three different cases taken with L=0, 2, 4. Variances in the order values and inventory of all members plotted against α.

(i) L=0

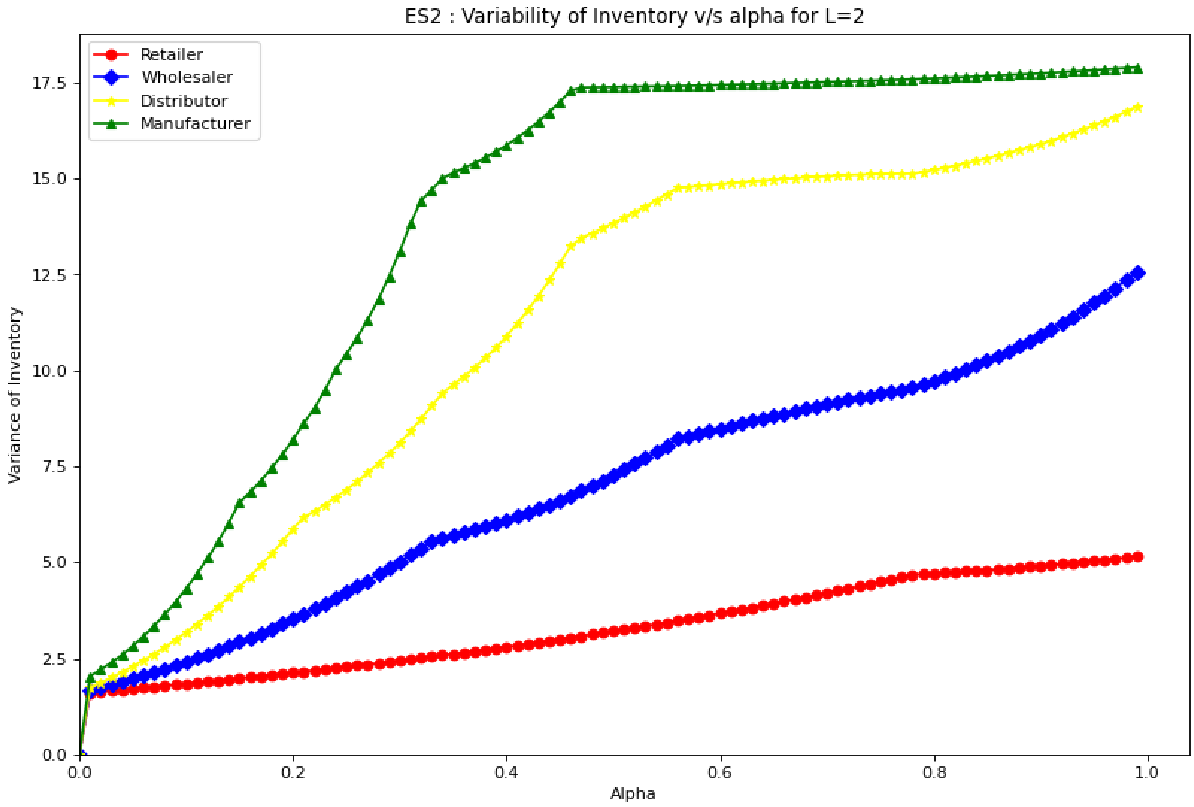




(ii) L=2

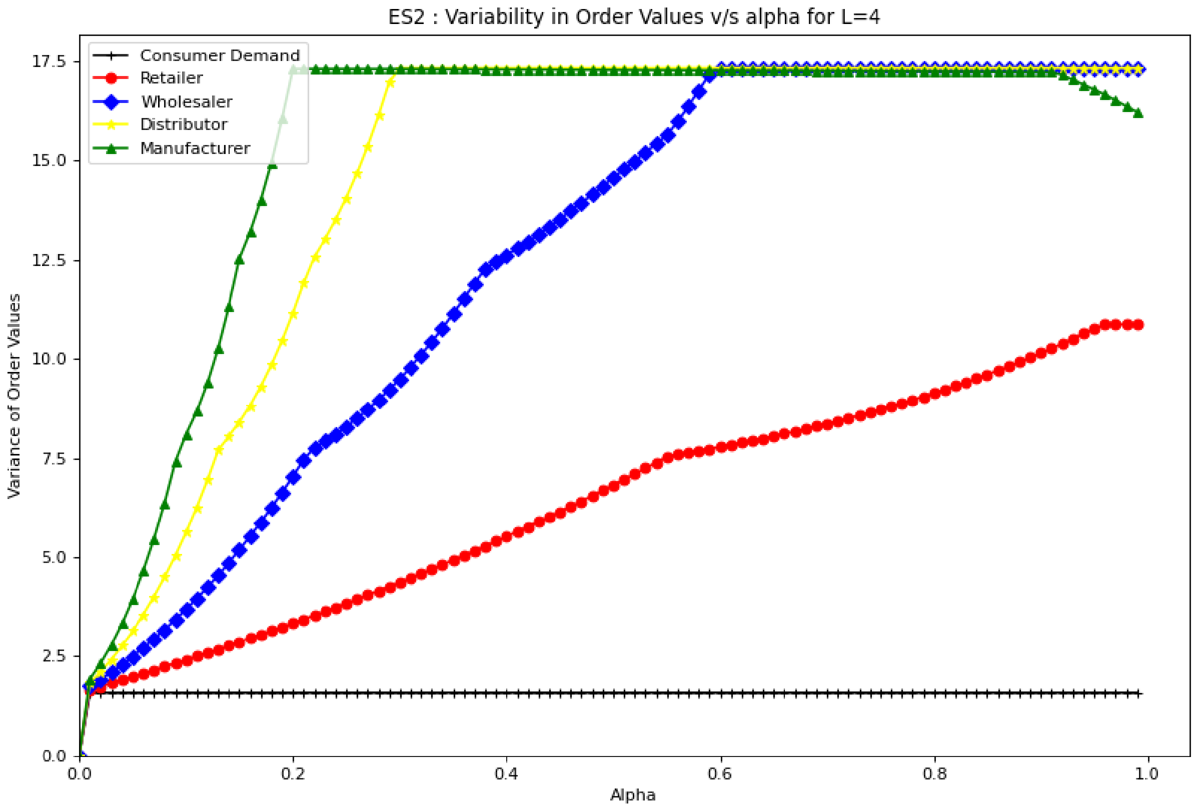


ES2 : Variability in Order Values v/s alpha for L=2
Consumer Demand
Retailer
Wholesaler
Distributor
Manufacturer
Variance of Order Values
Alpha
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
0.0
0.2
0.4
0.6
0.8
1.0

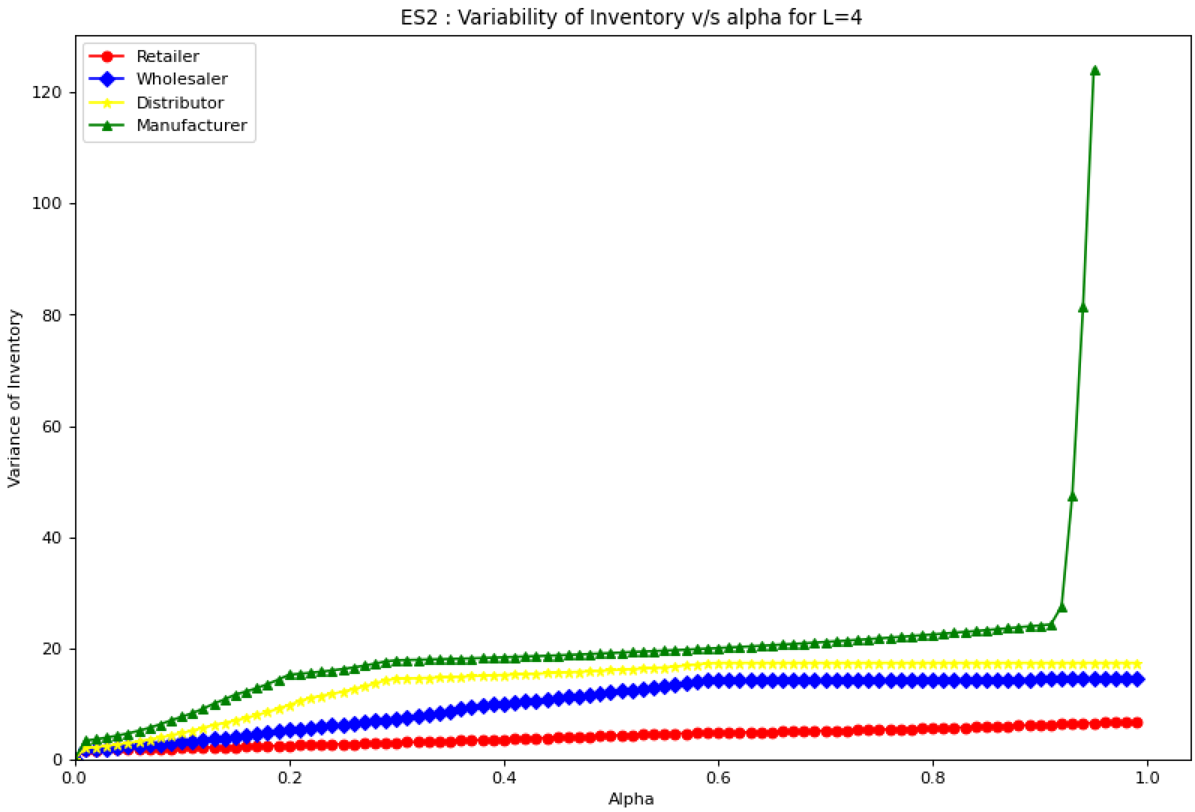


ES2 : Variability of Inventory v/s alpha for L=2
Retailer
Wholesaler
Distributor
Manufacturer
Variance of Inventory
Alpha
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
0.0
0.2
0.4
0.6
0.8
1.0

(iii) L=4



ES2 : Variability in Order Values v/s alpha for L=4
Consumer Demand
Retailer
Wholesaler
Distributor
Manufacturer
Variance of Order Values
Alpha
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
0.0
0.2
0.4
0.6
0.8
1.0



ES2 : Variability of Inventory v/s alpha for L=4
Retailer
Wholesaler
Distributor
Manufacturer
Variance of Inventory
Alpha
0
20
40
60
80
100
120
0.0
0.2
0.4
0.6
0.8
1.0

3.) Method ES-3

This method takes into account the cost of the inventory vs the cost of the backlog. Here also, each firm calculates forecast using the same exponential smoothing but additionally, each firm also calculates variance of its orders by

$$\sigma^2(t) = \frac{1}{t-1}\sum_{T=1}^{t}[D(T) - \overline{D}(t)]^2 \text{ where } \overline{D}(t) = \frac{1}{t}\sum_{T=1}^{t} D(T).$$

Each firm chooses a base stock level B(t) in time period t by minimizing the long-run average cost per unit time. We used the results of the base stock policy. (reference from the book Wallace Hopp, Mark Spearman - Factory Physics (2008))

Base stock BX(t) = θ + Z * σ

Where θ is mean demand which is (L+1) * F(t+1) for our case during the period of lead time and σ is standard deviation calculated by taking square root of $\sigma^2(t)$ * (L+1).

Here Z is decided by using the principle of overage and underage.

Z = $\emptyset^{-1}(b/b+h)$ = $\emptyset^{-1}(4/4+1)$ = 0.8416

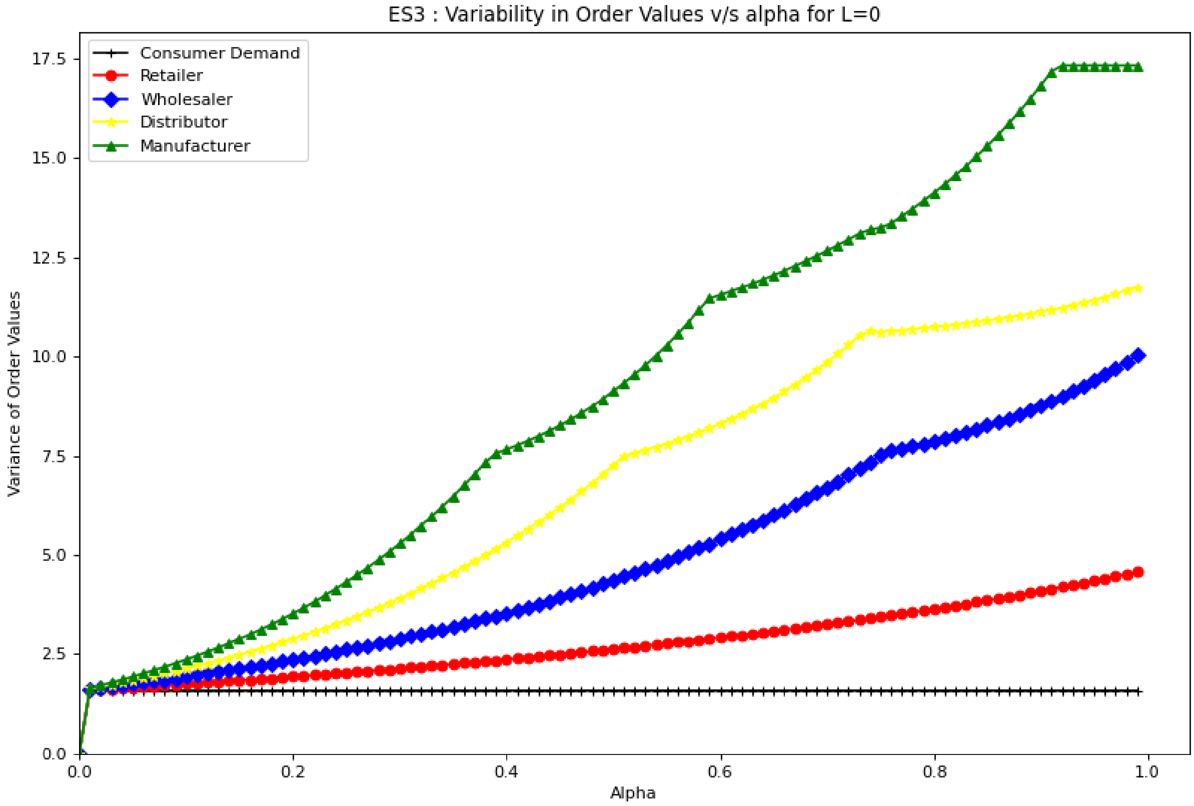
Where, b (underage cost) = cost of backlog = 4$ and h (overage cost) = cost of holding an inventory = 1$

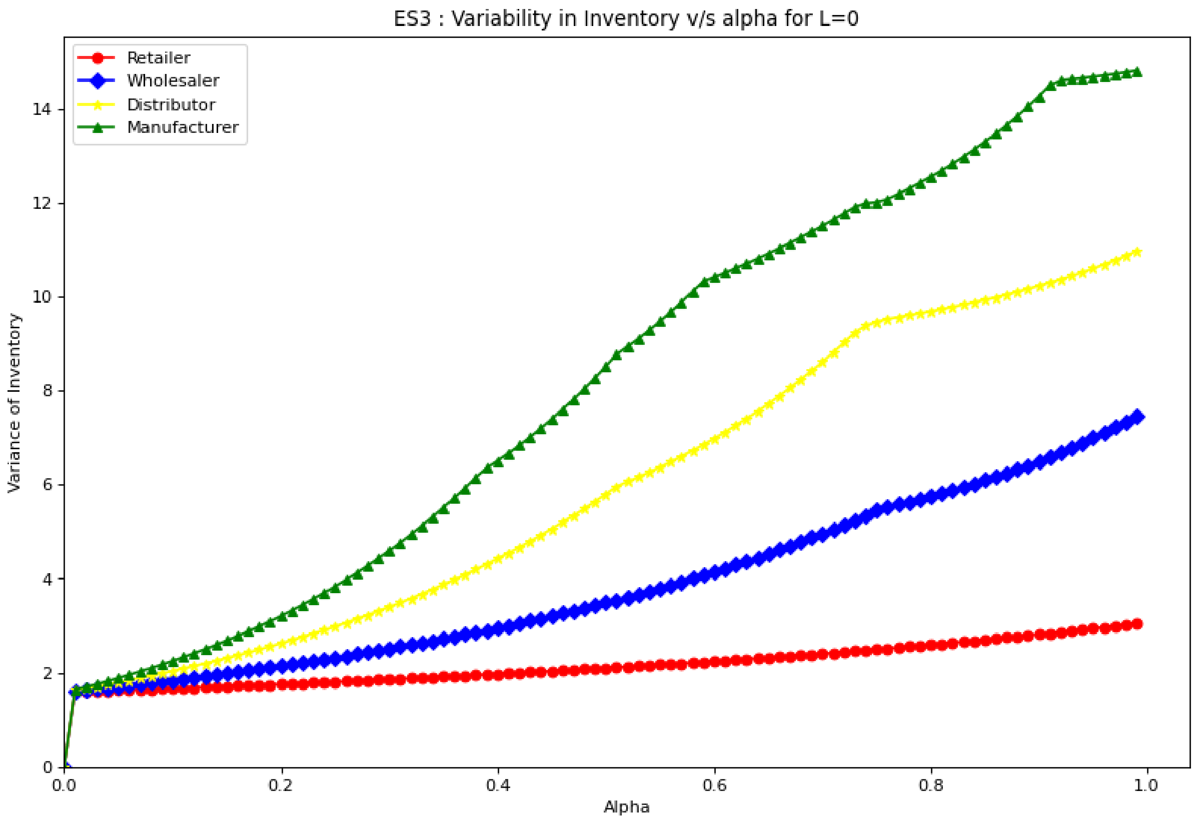
After calculating base stock level, each firm chooses their order value using the following formula

OX(t) = max { BX(t) – IPX(t), 0 }

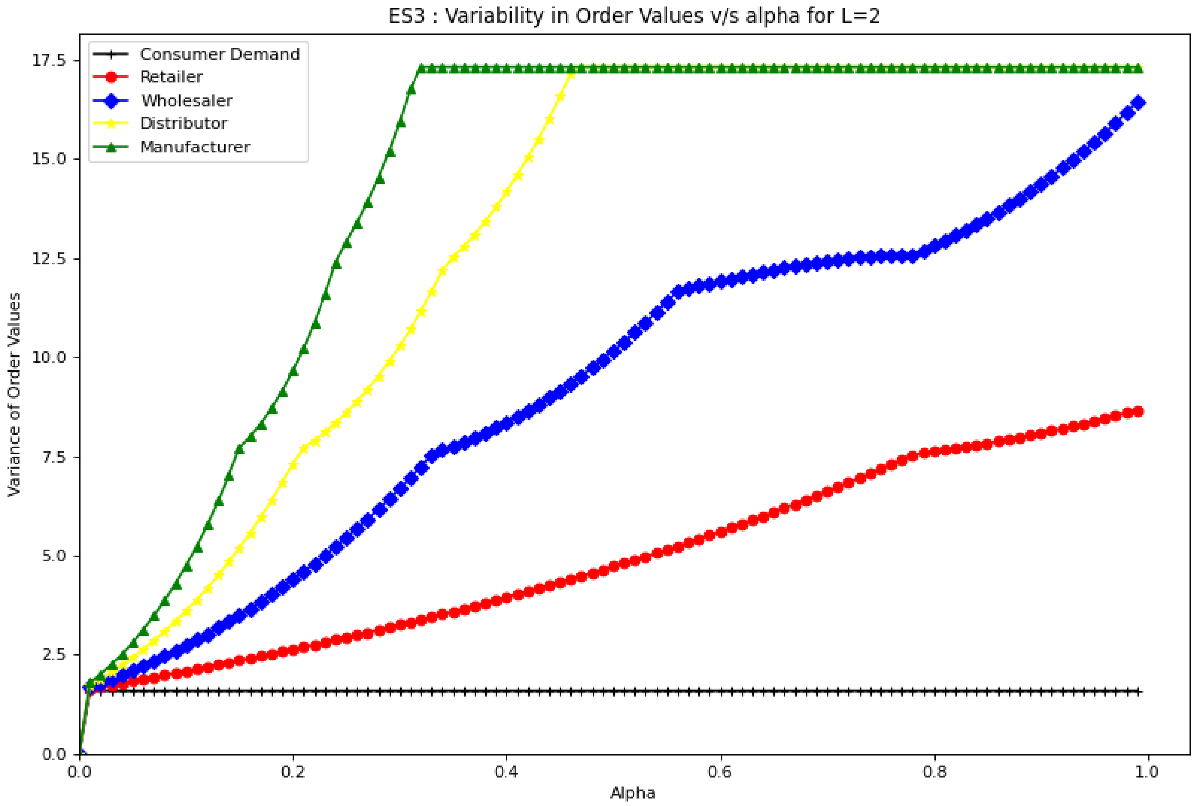
Using the above philosophy, a simulation was run for all four firms of the supply chain with α varying from 0 to 1 with an increment of 0.01 and three different cases taken with L=0, 2, 4. Variances in the order values and inventory of all members plotted against α.

(i) L=0

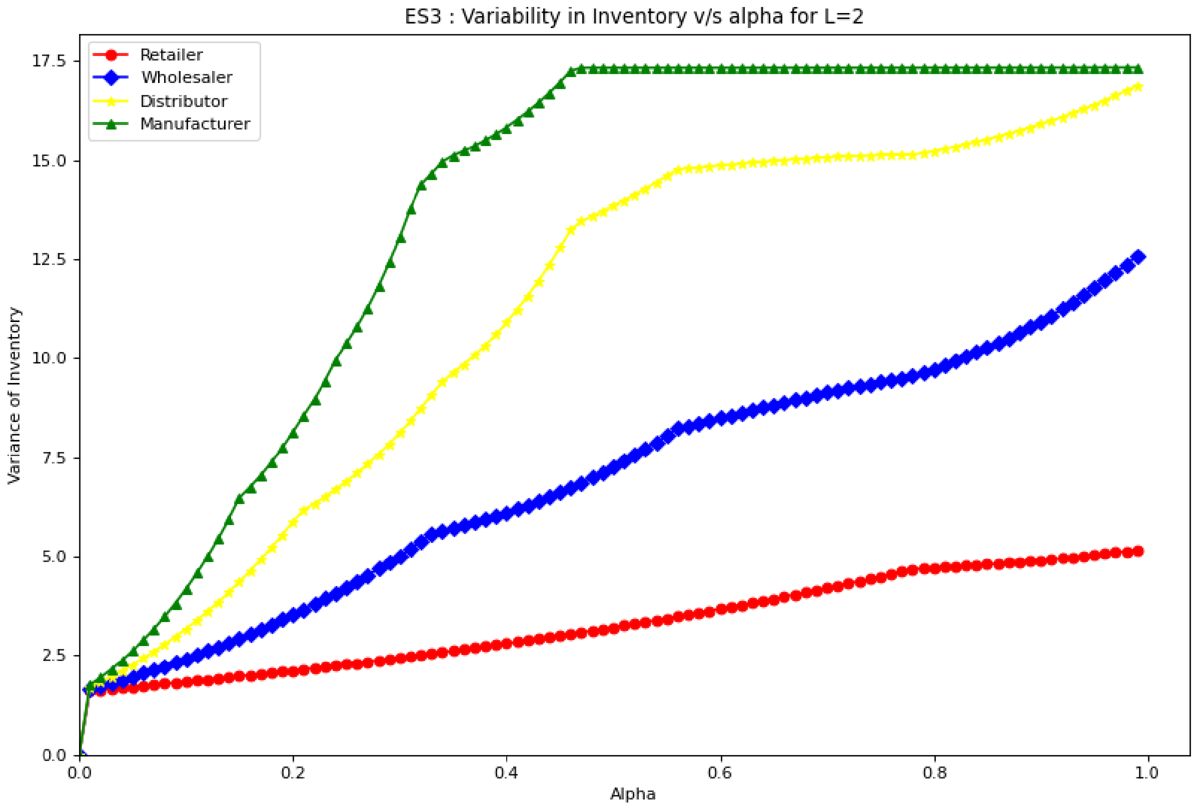

ES3 : Variability in Order Values v/s alpha for L=0
Consumer Demand
Retailer
Wholesaler
Distributor
Manufacturer
Variance of Order Values
Alpha
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
0.0
0.2
0.4
0.6
0.8
1.0


ES3 : Variability in Inventory v/s alpha for L=0
Retailer
Wholesaler
Distributor
Manufacturer
Variance of Inventory
Alpha
0
2
4
6
8
10
12
14
0.0
0.2
0.4
0.6
0.8
1.0

(ii) L=2

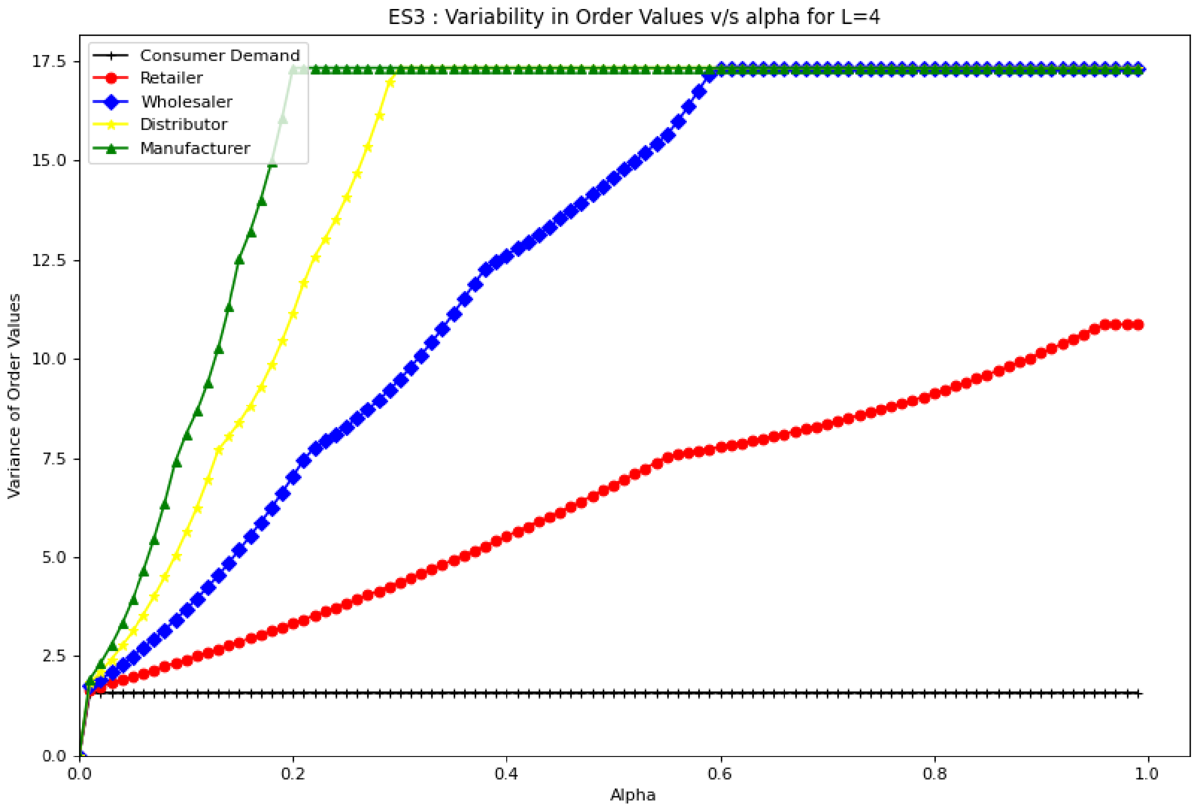


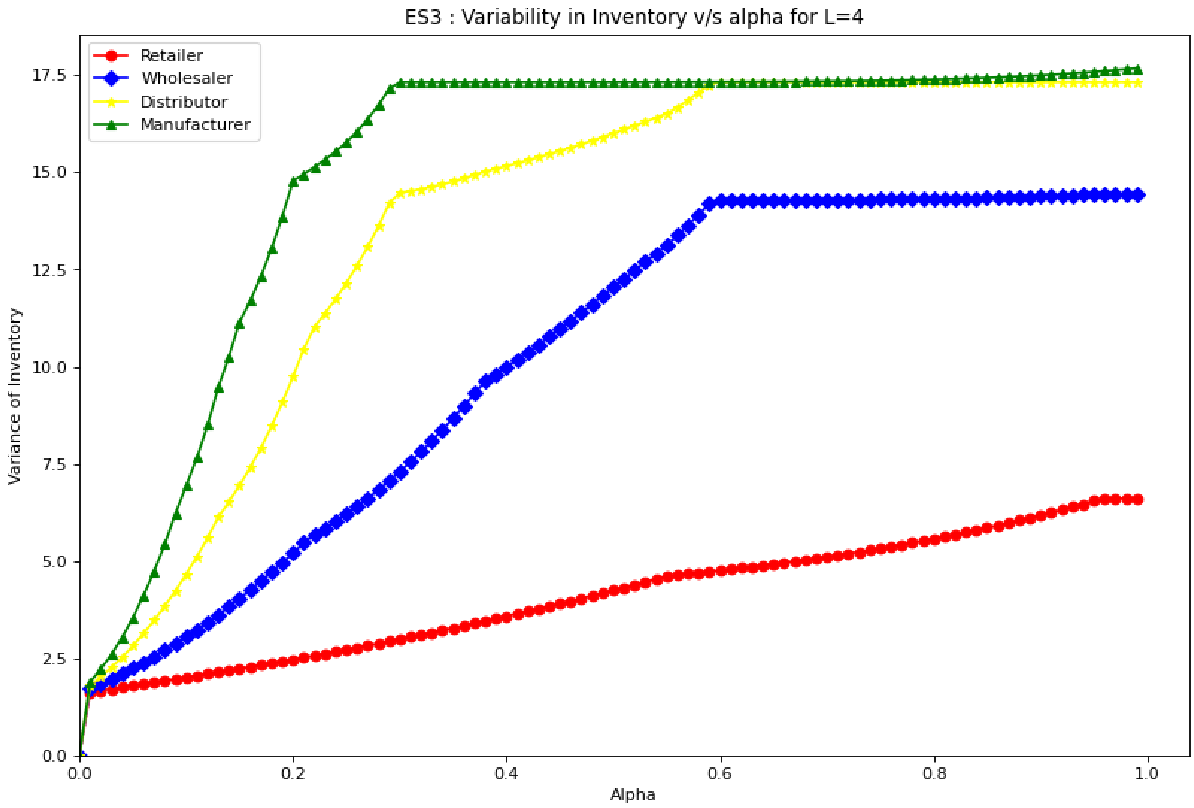
ES3 : Variability in Order Values v/s alpha for L=2
Consumer Demand
Retailer
Wholesaler
Distributor
Manufacturer
Variance of Order Values
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
0.0
0.2
0.4
0.6
0.8
1.0
Alpha



ES3 : Variability in Inventory v/s alpha for L=2
Retailer
Wholesaler
Distributor
Manufacturer
Variance of Inventory
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
0.0
0.2
0.4
0.6
0.8
1.0
Alpha

(iii) L=4


ES3 : Variability in Order Values v/s alpha for L=4
Consumer Demand
Retailer
Wholesaler
Distributor
Manufacturer
Variance of Order Values
Alpha
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
0.0
0.2
0.4
0.6
0.8
1.0


ES3 : Variability in Inventory v/s alpha for L=4
Retailer
Wholesaler
Distributor
Manufacturer
Variance of Inventory
Alpha
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
0.0
0.2
0.4
0.6
0.8
1.0

Conclusion:

It can be seen that variance in end customer order DR(t) is very low since the order values are fixed and repetitive in a cycle of 4. Still, it can be seen very clearly that there is a variability in the order placed by each firm of the supply chain and this variance keep on increasing as we move up the supply chain. Variability in the retailer order is more than that of end consumer demand, variability in the wholesaler order is more than that of the retailer, variability in the distributor order is more than that of the wholesaler and variability in the manufacturer order is more than that of the distributor for all three cases i.e. ES-1, ES-2, ES-3. Here, the **Demand Signal Processing** cause of the Bullwhip effect is visible.

- **Increase in variability with the increase in α:**

It is observed that with the increase in α value, variability in order values as well as inventory increases. As α increases, more weight is put on the demand value of each firm, and less wight on the forecast value. Since demand of downstream member already has some variability and we are putting more weight to it by increased value of α, the forecast values obtained have greater variability which in turn results in the variability of the order value by a firm. This variance flattens out at the value of 17.5.

- **Increase in variability with the increase in L (lead time):**

Also, it is observed that with the increase in the lead time value L, variability in order values and inventory increases. The maximum variance value is of 17.5. It is observed that with increased value of lead time, this peak is achieved even at lower values of α. The longer lead times affect the required safety stock (governed by (L+1) * F(t+1) factor) and it results in even bigger swings in order values of each firm. It demonstrates that the effect of the longer lead time worsening the bullwhip effect.

- **Comparison between three models ES-1/2/3:**

ES-1 is a very basic model which deals with the most recent data. It does not consider the previous history of the orders and inventory level. It tries to forecast demand for the next period using the current period's demand and determines order values considering the recent inventory position. It is a very crude method to plan the orders for the upcoming period.

ES-2 considers the historical data from starting point to the most recent period for determining the order value for the next period. It tries to maintain the service level of 99%. Hence, it is very sensitive to the variance in the order values. Owing to high service level, the safety stock requirement is higher and it makes the variability more sensitive to the variability in orders coming from the downstream nodes. That is why variance value sharply reaches the peak value with the increase in α and L (lead time).

ES-3 is the most practical model of all three as it considers the most important factor which is cost. Just like ES-2, it also considers the historical data from starting point to the most recent period and additionally, it tries to balance the inventory holding cost and back-order cost and determines the base stock level which will result in the lowest total cost. This optimum base stock level is then used to determine the order level.