Order of Growth

A function f(n) is said to be growing faster than g(n) if

$$\lim_{n\to\infty} \frac{g(n)}{f(n)} = 0$$

OR,

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \infty$$

f(n) and g(n) represent Time taken.

$n \geq 0$

$f(n), g(n) \geq 0$

$f(n) = n + 1$
$g(n) = 1000$

$f(n) = n^2 + n + 6$

$g(n) = 2n + 5$

$$\lim_{n\to\infty} \frac{2n+5}{n^2+n+6}$$

$$= \lim_{n\to\infty} \frac{\frac{2}{n}+\frac{5}{n^2}}{1+\frac{1}{n}+\frac{6}{n^2}}$$

$$= \lim_{n\to\infty} \frac{0+0}{1+0+0}$$

$$= 0$$

Direct way to find and compare Growths

1. Ignore lower order terms
2. Ignore leading term constant

Example: $f(n) = 2n^2 + n + 6$, Order of growth : n^2 (Quadratic)

$g(n) = 100n + 3$, Order of growth : n (Linear)

How do we compare terms?

$C < \log(\log n) < \log n < n^{1/3} < n^{1/2} < n < n^2 < n^3 < n^4 < 2^n < n^n$

1. $f(n) = c_1 \log n + c_2$

   $g(n) = c_3 n + c_4 \log(\log n) + c_5$

   $=$ $f(n) : \log n$

   $g(n) : n$

   ⇨ $g(n)$ is a bad algorithm

2. $f(n) = c_1 n^2 + c_2 n + c_3$

   $g(n) = c_4\, n \log n + c_5 n + c_6$

   $=$ $f(n) : n^2$

   $g(n) : n \log n$

   ⇨ $f(n)$ is a bad algorithm