Analysis of common loops

```
# First Example loop
i = 0
while i < n:
        i = i + c
        # some other constant
        # time work
```

```
# Equivalent for loop
for i in range(0, n, c):
        # some constant work
```

Order of Growth of these two loops are : $\ominus(n)$

```
# Second Example loop
i = n
while i > 0:
        i = i - c
        # some other constant
        # time work
```

```
# Equivalent for loop
for i in range(n, 0, -c):
        # some constant work
```

Order of Growth of these two loops are : $\ominus(n)$

```
# Third Example loop
i = 1
while i < n:
        i = i * c
        # some other constant
        # time work
```

$1, c, c^2, \ldots\ldots, c^{k-1}$

$\Rightarrow c^{k-1} < n$

$\Rightarrow k < \log_c n + 1$

Then, Order of Growth: $\ominus(\log n)$

```
# Fourth Example loop
i = n
while i > 1:
        i = i / c
        # some other constant
        # time work
```

Order of Growth: $\Theta(\log n)$

```
# Fifth Example loop
i = 2
while i < n:
        i = i ** c
        # some other constant
        # time work
```

$2, 2^c, (2^c)^c, \ldots\ldots, ((2^c)^c)^{c\ldots\ldots}$

- ⇨ $2, 2^c, 2^{c\hat{}2}, \ldots\ldots, 2^{c\hat{}(k-1)}$
- ⇨ $2^{c\hat{}(k-1)} < n$
- ⇨ $c^{k-1} < \log_2 n$
- ⇨ $k - 1 < \log_c \log_2 n$
- ⇨ $k < \log_c \log_2 n + 1$

Then, Order of Growth: $\Theta(\log (\log n))$

```
# Sixth Example subsequent loops
i = 0
while i < n:
        i = i + 2
```

$\Theta(n)$

```
i = 1
while i < n:
        i = i * 3
```

$\Theta(\log n)$

```
i = 1
while i < 100:
        i = i + 1
```

$\Theta(1)$

$\Theta(n) + \Theta(\log n) + \Theta(1)$

Order of Growth: $\Theta(n)$

```
# Seventh Example nested loops
i = 0
while i < n:
        j = 1
        while j < n:
                j = j * 2
        i = i + 1
        # some other constant
        # time work
```

$\Theta(n \log n)$

Order of Growth: $\Theta(n \log n)$

```
# Eighth Example mixed loops
i = 0
while i < n:
        j = 1
        while j < n:
                j = j * 2
        i = i + 1
i = 0
while i < n:
        j = 1
        while j < n:
                j = j + 1
        i = i + 1
```

$\Theta(n \log n)$

$\Theta(n^2)$

$\Theta(n \log n) + \Theta(n^2) = \Theta(n^2)$

Order of Growth: $\Theta(n^2)$

```
# Ninth Example multiple input
i = 0
while i < n:
        j = 1
        while j < n:
                j = j * 2
        i = i + 1
i = 0
while i < m:
        j = 1
        while j < m:
                j = j + 1
        i = i + 1
```

$\ominus(n \log n)$

$\ominus(m^2)$

$\ominus(n \log n) + \ominus(m^2) = \ominus(n \log n + m^2)$

Order of Growth: $\ominus(n \log n + m^2)$