

Big O Notation (Upper Bound on Order of Growth)

We say $f(n) = O(g(n))$ if there exist constants c and n_0 such that $f(n) \leq cg(n)$ for all $n \geq n_0$

Example: $f(n) = 2n + 3$ can be written as $O(n)$.

$$f(n) \leq cg(n) \text{ for all } n \geq n_0$$

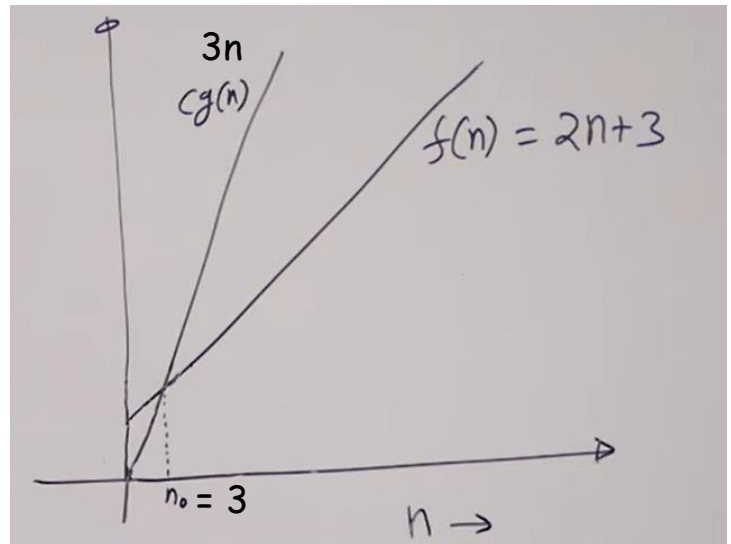
$$(2n + 3) \leq cn \text{ for all } n \geq n_0$$

Placed, $c = 3$

$$\Rightarrow (2n + 3) \leq 3n$$

$$\Rightarrow 3 \leq n$$

$$\Rightarrow n_0 = 3$$



$$f(n) = 3n^2 + 2n + 100$$

$$= O(n^2)$$

Another example:

$$f(n) = 4n + \log n + 30$$

$$= O(n)$$

$$\{n/4, 2n + 3, n/100 + \log n, n + 10000, n/10000, 100, \dots\} \in O(n)$$

$$\{n^2 + n, 2n^2, n^2 + 1000n, n^2 + 2\log n, n^2/1000, \dots\} \in O(n^2)$$

$$\{1000, 2, 1, 3, 10000, 100000, 1000000, \dots\} \in O(1)$$