

Analysis of Recursion

```
def fun(n):
    if n == 1:
        return
    for i in range(n):
        print("GFG")
    fun(n/2)
    fun(n/2)
```

$$T(n) = 2T(n/2) + \Theta(n)$$

$$T(1) = \Theta(1)$$

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def fun(n):
    if n == 1:
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    fun(n/2)
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$$T(n) = 2T(n/2) + \Theta(1)$$

$$T(1) = \Theta(1)$$

```
def fun(n):
    if n == 1:
        return
    print(n)
    fun(n - 1)
```

$$T(n) = T(n - 1) + \Theta(1), n > 1$$

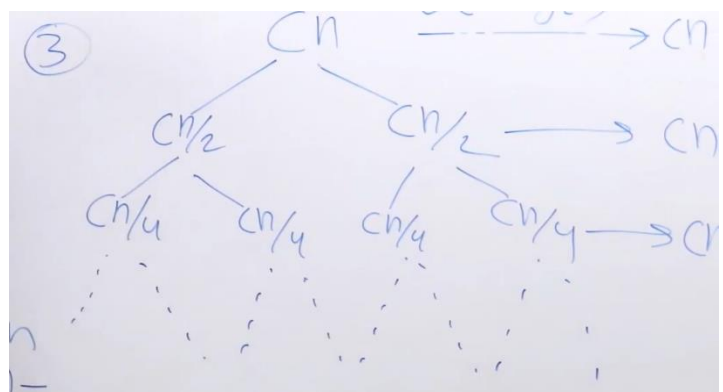
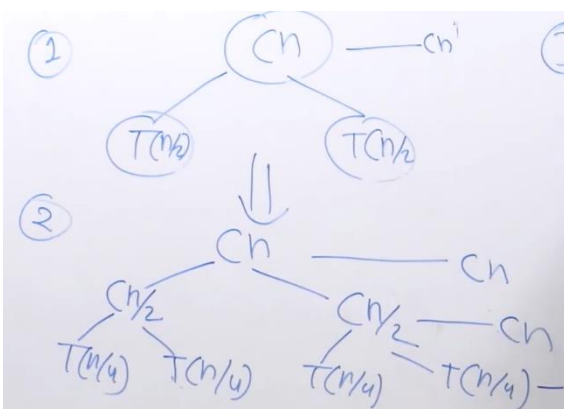
$$T(1) = \Theta(1)$$

$T(n) = 2T(n/2) + Cn$ $T(1) = C$	$Cn + Cn + Cn + \dots + Cn$ $\Rightarrow \log_2 n$ $\Rightarrow Cn \times \log_2 n$ $\Rightarrow \Theta(n \log n)$
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Recursion Tree method

→ We write non-recursive part as root of tree and recursive parts as children.

→ We keep expanding children until we see a pattern.



$$T(n) = T(n - 1) + C$$


$$T(1) = C$$


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$$T(1) = C$$

$$T(n) = 2T(n-1) + C$$


$$T(1) = C$$


①
 

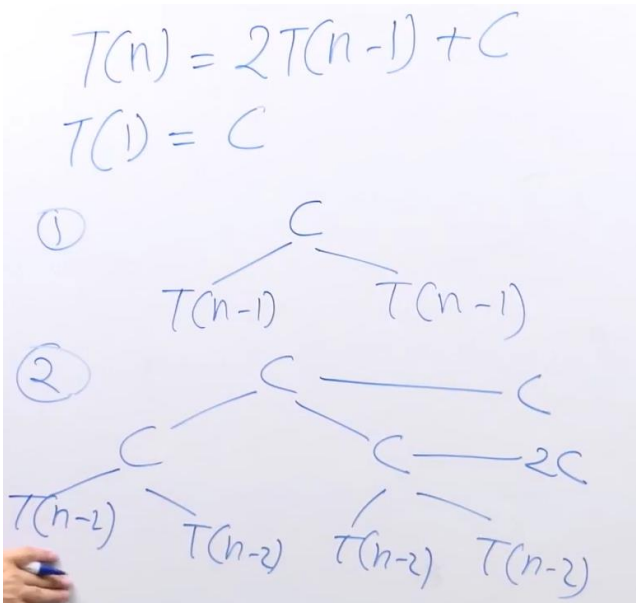
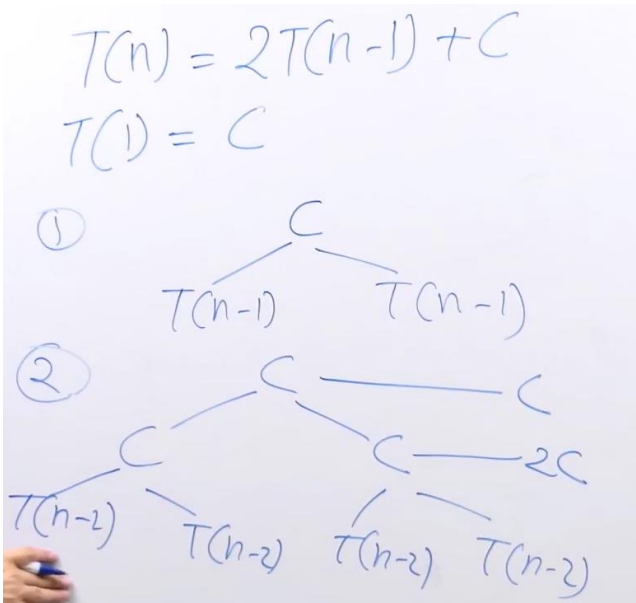
②
 

$$T(n) = 2T(n-1) + C$$

$$T(1) = C$$

①
 

②
 



$$C + 2C + 4C + \dots + C \cdot 2^{n-1} = \Theta(2^n)$$

(3)

2C

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①

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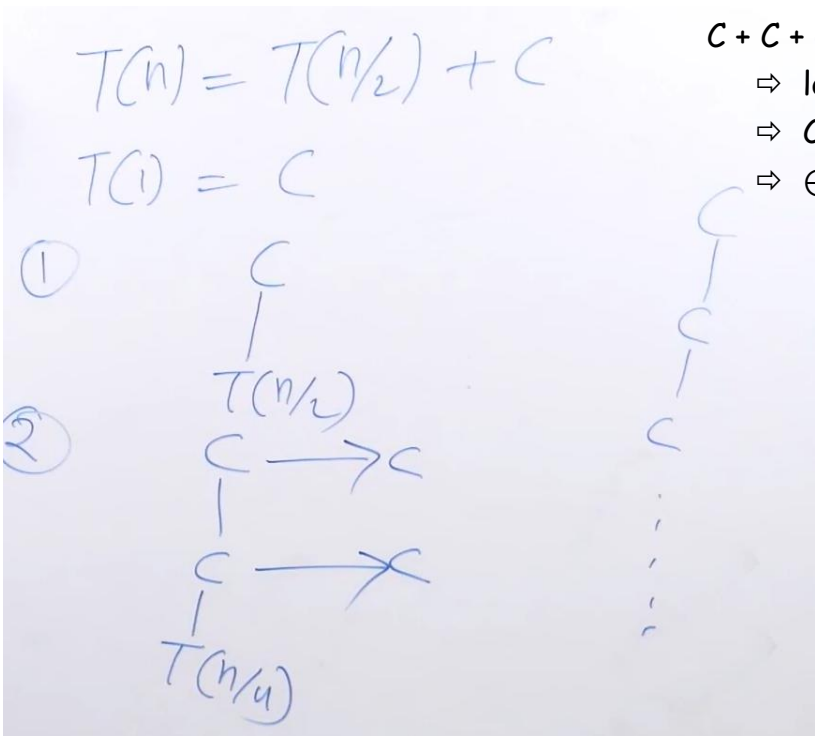
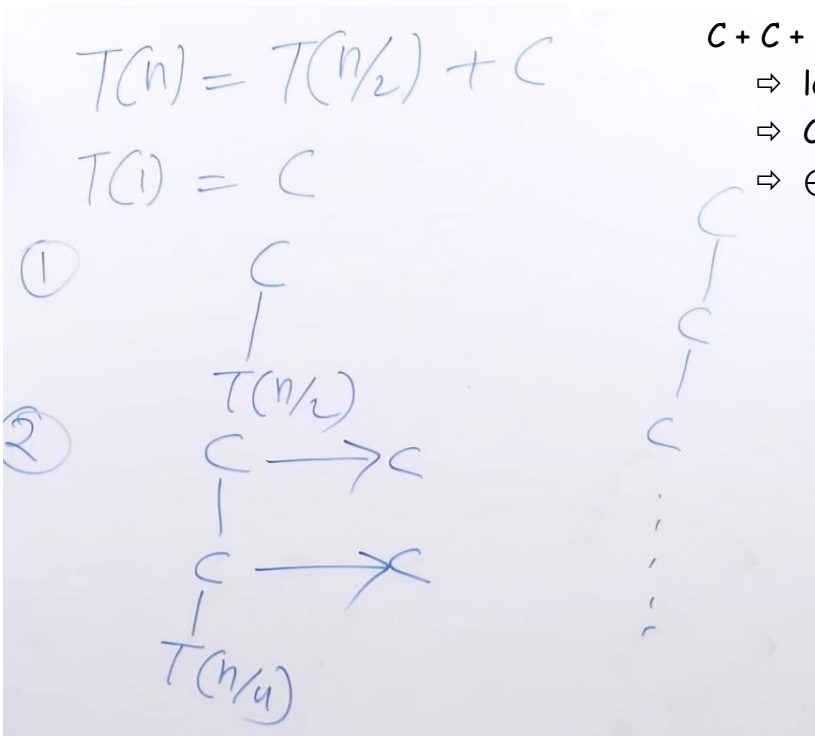
$C + C + \dots + C$
 $\Rightarrow 1$
 $\Rightarrow C$
 $\Rightarrow C$

$T(n) = T(n/2) + C$
 $T(1) = C$

①

②

$C + C + \dots + C$
 $\Rightarrow 1$
 $\Rightarrow C$
 $\Rightarrow C$



$T(n) = T(n/2) + C$
 $T(1) = C$

①

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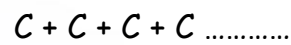
$C + C + \dots + C$
 $\Rightarrow 1$
 $\Rightarrow C$
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$$C + C + C + C \dots\dots\dots$$

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$$C + C + C + C \dots\dots\dots$$



$$T(n) = 2T(n/2) + C$$

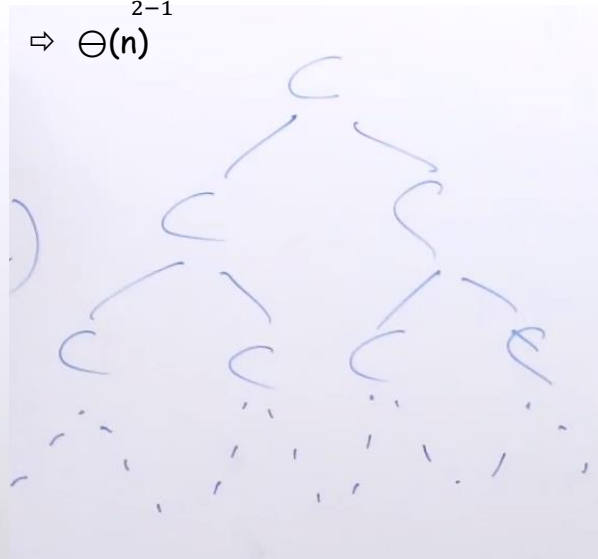
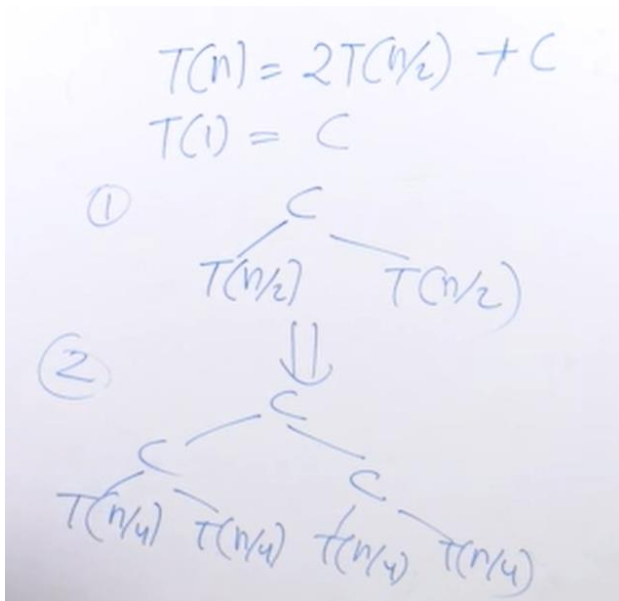
$$T(1) = C$$

$$C + 2C + 4C + \dots$$

$$\Rightarrow \log_2 n$$

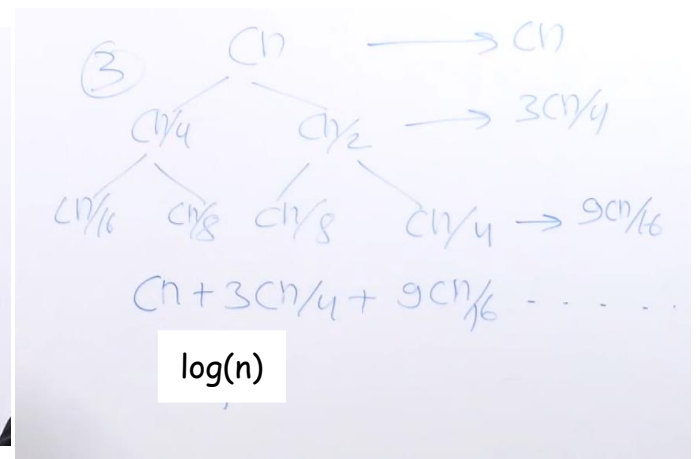
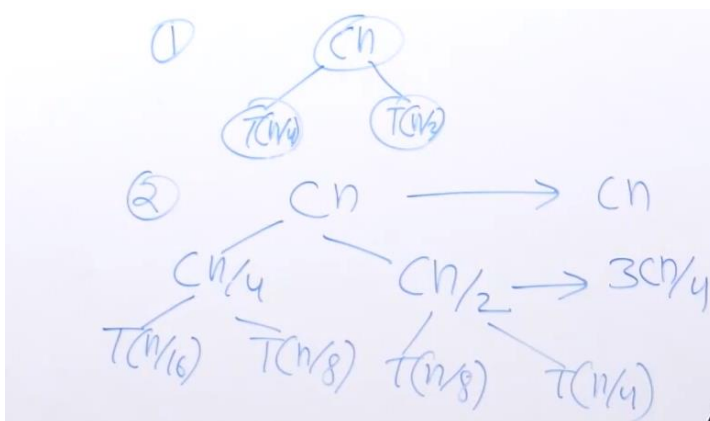
$$\Rightarrow \frac{C \times (2^{\log_2 n} - 1)}{2 - 1}$$

$$\Rightarrow \Theta(n)$$



$$T(n) = T(n/4) + T(n/2) + Cn$$

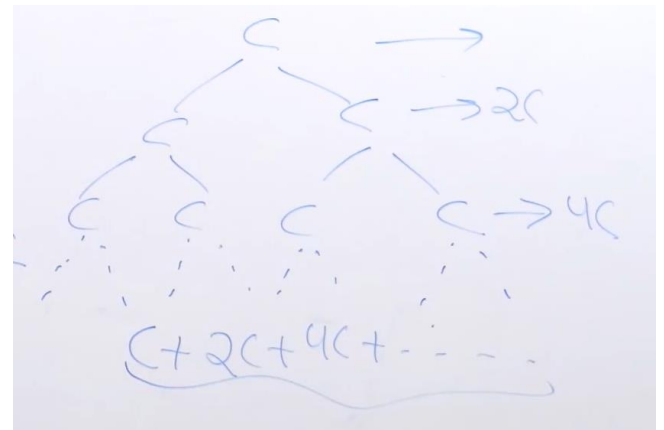
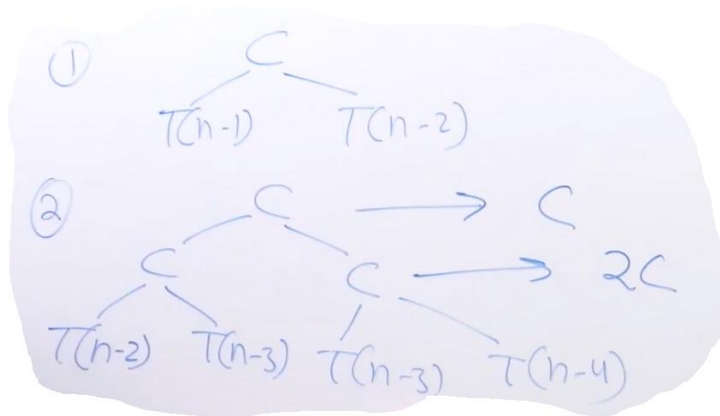
$$T(1) = C$$



$$O(Cn \times 1/(1-3/4)) \Rightarrow O(n)$$

$$T(n) = T(n-1) + T(n-2) + C$$

$$T(1) = C$$



$$C(1 + 2 + 4 + \dots + n \text{ terms}) \Rightarrow O(2^n)$$