PA-31

Pre-Algebra

Distance and Angle

11.1 Angle and Line Relationships
11.2 Congruent Triangles
11.3 Rotations

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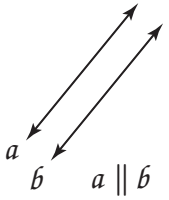
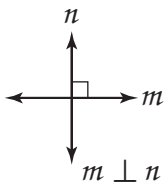
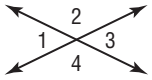
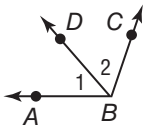
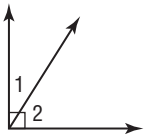
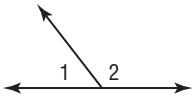
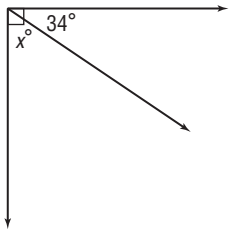
11-1 Study Guide and Intervention

Angle and Line Relationships

Line and Angle Relationships					
Parallel Lines	**Perpendicular Lines**	**Vertical Angles**	**Adjacent Angles**	**Complementary Angles**	**Supplementary Angles**
 $a \parallel b$	 $m \perp n$	 $\angle 1 \cong \angle 3$ $\angle 2 \cong \angle 4$	 $m\angle ABC =$ $m\angle 1 + m\angle 2$	 $m\angle 1 + m\angle 2 = 90°$	 $m\angle 1 + m\angle 2 = 180°$

Example **In the figure at the right, classify the relationship between the pairs of angles shown. Then find the value of x.**



The angles are complementary. The sum of their measures is 90°.

$$m\angle x + 34 = 90$$ Write the equation.

$$m\angle x + 34 - 34 = 90 - 34$$ Subtract 34 from each side.

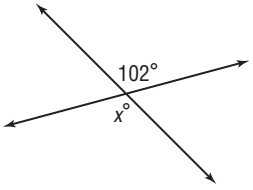
$$m\angle x = 56$$ Simplify.

So, $m\angle x$ is 56°.

Exercises

Classify the pairs of angles shown. Then find the value of x in each figure.

1.



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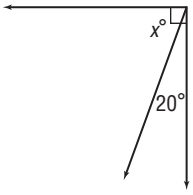

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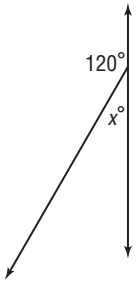

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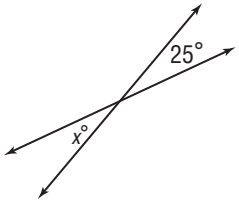
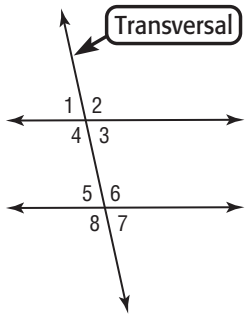


Lesson 11-1

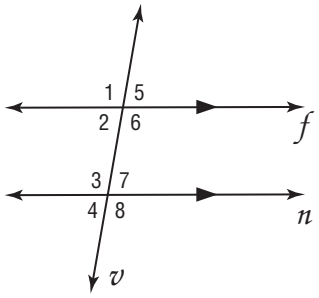
11-1 Study Guide and Intervention *(continued)*

Angle and Line Relationships

Names of Special Angles		
Interior angles lie inside the parallel lines.	∠3, ∠4, ∠5, ∠6	
Exterior angles lie outside the parallel lines.	∠1, ∠2, ∠7, ∠8	
Alternate interior angles are on opposite sides of the transversal and inside the parallel lines.	∠3 and ∠5, ∠4 and ∠6	
Alternate exterior angles are on opposite sides of the transversal and outside the parallel lines.	∠1 and ∠7, ∠2 and ∠8	
Corresponding angles are in the same position on the parallel lines in relation to the transversal.	∠1 and ∠5, ∠2 and ∠6, ∠3 and ∠7, ∠4 and ∠8	

When a transversal intersects two parallel lines, pairs of alternate exterior angles, alternate interior angles, and corresponding angles are congruent.

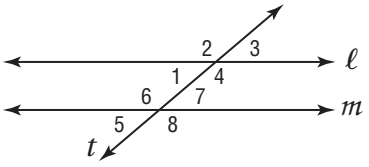
Example **In the figure, $f \parallel n$ and v is a transversal. If $m\angle 3 = 100°$, find $m\angle 1$ and $m\angle 6$.**



Since ∠1 and ∠3 are corresponding angles, they are congruent. So, $m\angle 1 = 100°$. Since ∠3 and ∠6 are alternate interior angles, they are congruent. So, $m\angle 6 = 100°$.

Exercises

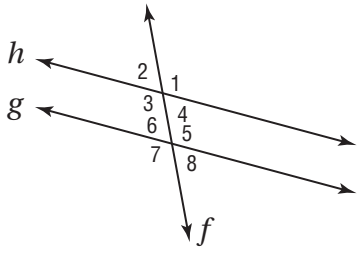
In the figure on the right, $l \parallel m$ and t is a transversal. If $m\angle 1 = 61.2°$ and the $m\angle 6 = 118.8°$, find the measure of each angle.



1. ∠7 **2.** ∠3 **3.** ∠4

4. ∠8 **5.** ∠5 **6.** ∠2

In the figure on the right, $g \parallel h$ and f is a transversal. If $m\angle 1 = 125°$ and the $m\angle 6 = 55°$, find the measure of each angle.



7. ∠2 **8.** ∠4 **9.** ∠5

10. ∠3 **11.** ∠8 **12.** ∠7

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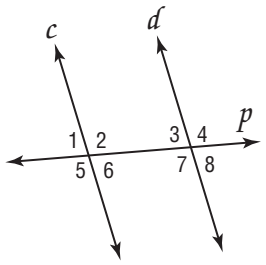
11-1 Skills Practice

Angle and Line Relationships

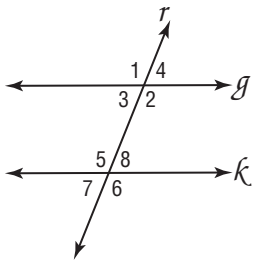
In the figure at the right, $c \parallel d$ and p is a transversal. If $m\angle 5 = 110°$, find the measure of each angle.



1. $\angle 6$ **2.** $\angle 8$

3. $\angle 2$ **4.** $\angle 4$

In the figure at the right, $g \parallel k$ and r is a transversal. If $m\angle 7 = 60°$, find the measure of each angle.

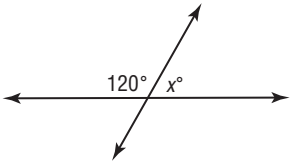


5. $\angle 4$ **6.** $\angle 6$

7. $\angle 5$ **8.** $\angle 3$

Classify the pairs of angles shown. Then find the value of x in each figure.

9.



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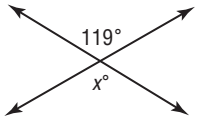

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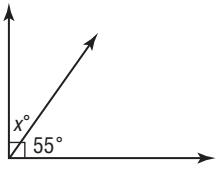

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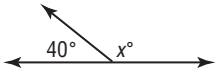

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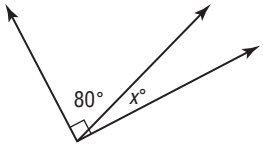

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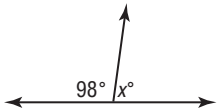

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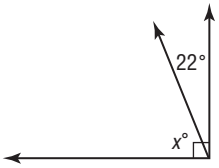

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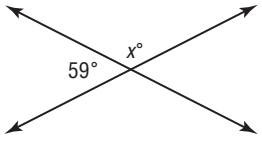

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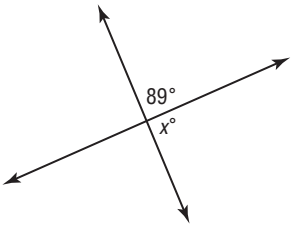

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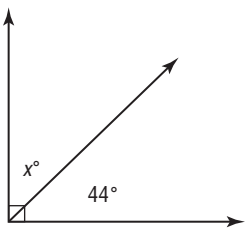

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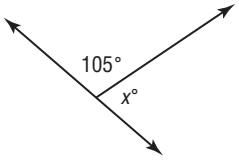

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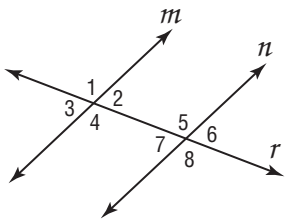
11-1 Practice

Angle and Line Relationships

In the figure at the right, $m \parallel n$ and r is a transversal. If $m\angle 2 = 45°$, find the measure of each angle.



1. $\angle 4$

2. $\angle 5$

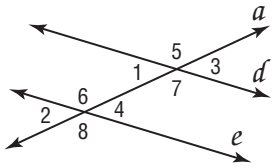
3. $\angle 7$

4. $\angle 8$

5. $\angle 6$

6. $\angle 3$

In the figure at the right, $d \parallel e$ and a is a transversal. If $m\angle 5 = 143°$, find the measure of each angle.



7. $\angle 7$

8. $\angle 6$

9. $\angle 4$

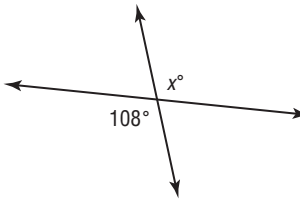
10. $\angle 2$

11. $\angle 1$

12. $\angle 8$

Classify the pairs of angles shown. Then find the value of x in each figure.

13.



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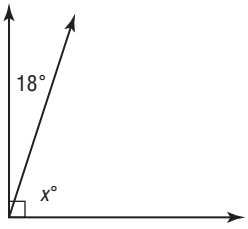

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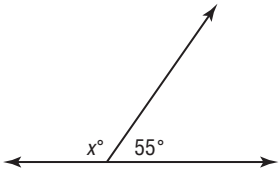

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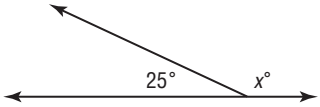

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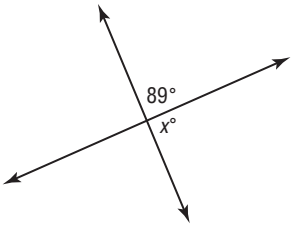

19. Angles Q and R are complementary. Find $m\angle R$ if $m \angle Q = 24°$.

20. Find $m\angle J$ if $m\angle K = 29°$ and $\angle J$ and $\angle K$ are supplementary.

21. The measures of angles A and B are equal and complementary. What is the measure of each angle?

22. ALGEBRA Angles G and H are complementary. If $m\angle G = 3x + 6$ and $m\angle H = 2x - 11$, what is the measure of each angle?

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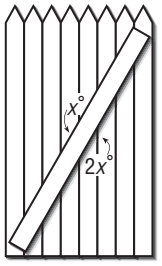
11-1 Word Problem Practice

Angle and Line Relationships

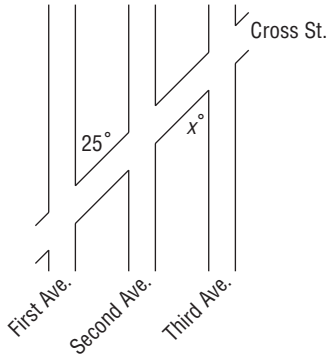
1. PROPERTY LINES The front and back property lines of Michaela's land are parallel lines. If the angle between the west side property line and back property line is 106°, what is the angle between the front property line and west side property line?

2. SCISSORS Archie opened up a pair of scissors so that the angle between the blades is 38°. What is the angle between the handles?

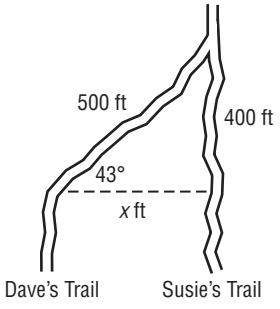
3. FENCING The sections of fence in Sioban's yard have diagonal supports as shown. The top side of the diagonal support makes an angle of $x°$ with the fence slats. The bottom side makes an angle that is twice the measure of the top angle. Find the measures of both angles.



4. MAPS In the following map, First Avenue, Second Avenue, and Third Avenue are parallel. Cross Street intersects all three avenues. First Avenue and Cross Street meet at a 25° angle. What angle does the intersection of Third Avenue and Cross Street make?



5. HIKING Dave and Susie are walking on parallel trails in the woods. Dave's trail turns to the right 43° and meets up with Susie's trail.



a. At what angle does Dave's trail meet Susie's trail?

b. How far apart were Dave and Susie's trails originally?

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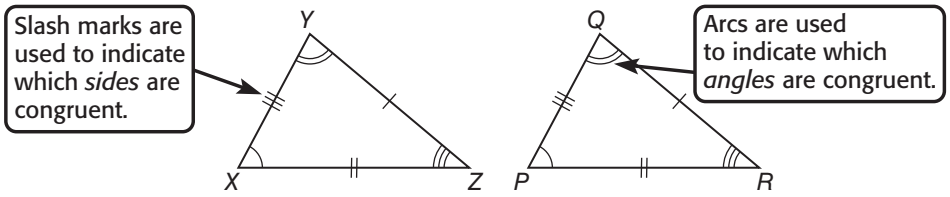
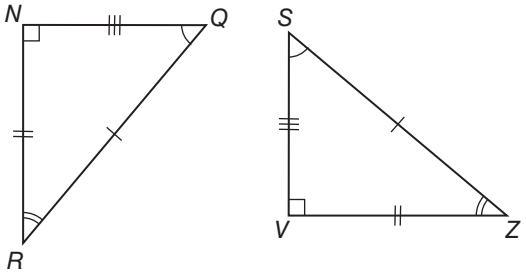
11-2 Study Guide and Intervention

Congruent Triangles

Corresponding Parts of Congruent Triangles	
Words	Two triangles are **congruent** if they have the same size and shape. If two triangles are congruent, their corresponding sides are congruent and their corresponding angles are congruent.
Model	Slash marks are used to indicate which *sides* are congruent. Arcs are used to indicate which *angles* are congruent. 
Symbols	Congruent Angles: $\angle X \cong \angle P$, $\angle Y \cong \angle Q$, $\angle Z \cong \angle R$ Congruent Sides: $\overline{XY} \cong \overline{PQ}$, $\overline{YZ} \cong \overline{QR}$, $\overline{XZ} \cong \overline{PR}$

Example **Name the corresponding parts in the congruent triangles shown. Then write a congruence statement.**



Corresponding angles:

$\angle Q \cong \angle S$, $\angle R \cong \angle Z$, $\angle N \cong \angle V$

Corresponding sides:

$\overline{SZ} \cong \overline{QR}$, $\overline{ZV} \cong \overline{RN}$, $\overline{VS} \cong \overline{NQ}$

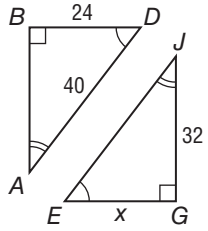
$\triangle NQR \cong \triangle VSZ$

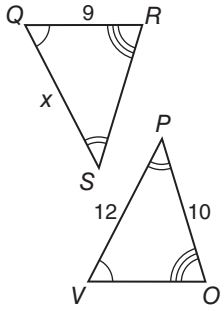
Exercises

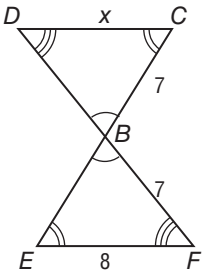
Complete each congruence statement if $\triangle DFH \cong \triangle PWZ$.

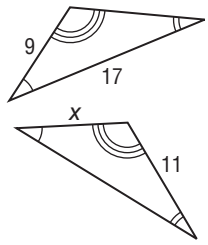
1. $\angle F \cong$ ____ **2.** $\angle P \cong$ ____ **3.** $\overline{DH} \cong$ ____ **4.** $\overline{ZW} \cong$ ____

Find the value of x for each pair of congruent triangles.

5. 

6. 

7. 

8. 

Lesson 11-2

NAME _____ DATE _____ PERIOD _____

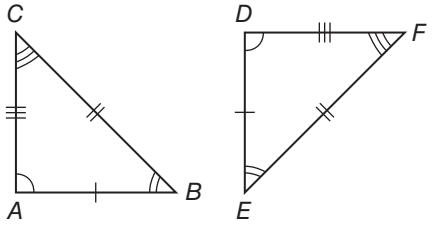
11-2 Study Guide and Intervention *(continued)*

Congruent Triangles

Identify Congruent Triangles Two triangles are congruent if and only if all pairs of corresponding angles are congruent and all pairs of corresponding sides are congruent.

Example **Determine whether the triangles shown are congruent. If so, name the corresponding parts and write a congruence statement.**

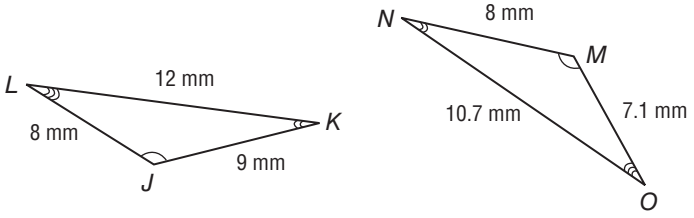
a.



Corresponding angles: The arcs indicate that $\angle A \cong \angle D$, $\angle B \cong \angle E$, and $\angle C \cong \angle F$.
Corresponding sides: The side measures indicate that $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, and $\overline{CA} \cong \overline{FD}$.

Since all pairs of corresponding angles and sides are congruent, the triangles are congruent. One congruence statement is $\Delta ABC \cong \Delta DEF$.

b.

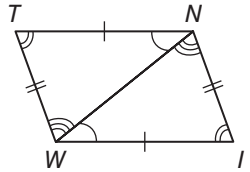


Although the arcs indicate that $\angle J \cong \angle M$, $\angle K \cong \angle N$, and $\angle L \cong \angle O$, the side measures indicate that no sides are congruent with one another. Therefore, the triangles are *not* congruent.

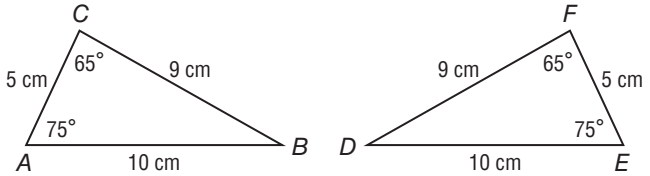
Exercises

Determine whether the triangles shown are congruent. If so, name the corresponding parts and write a congruence statement.

1.



2.



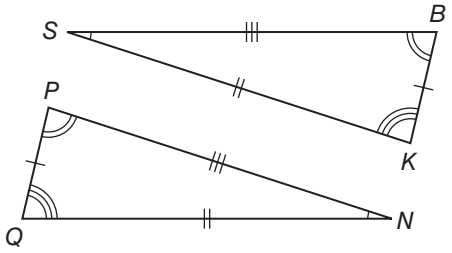
NAME _____ DATE _____ PERIOD _____

11-2 Skills Practice

Congruent Triangles

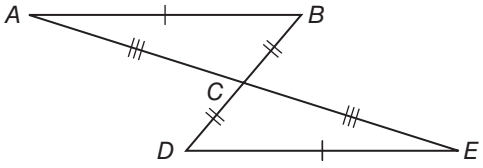
Name the corresponding parts in each pair of congruent triangles. Then complete the congruence statement.

1.



$\triangle KBS \cong$ _____

2.



$\triangle ACB \cong$ _____

Complete each congruence statement if $\triangle MRU \cong \triangle ACF$.

3. $\angle R \cong$?

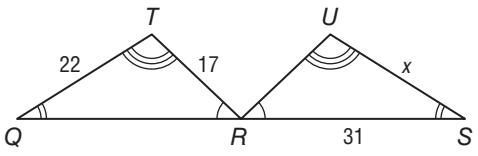
4. $\overline{CA} \cong$?

5. $\overline{MU} \cong$?

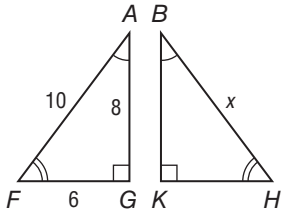
6. $\angle A \cong$?

Find the value of x for each pair of congruent triangles.

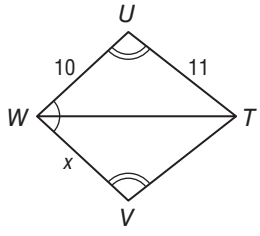
7.



8.

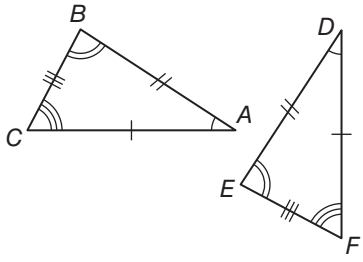


9.

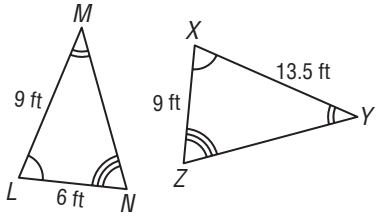


Determine whether the triangles shown are congruent. If so, name the corresponding parts and write a congruence statement.

10.



11.



NAME _____ DATE _____ PERIOD _____

11-2 Practice

Congruent Triangles

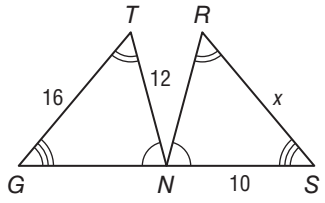
Complete the congruence statement if $\triangle CMH \cong \triangle PLF$ and $\triangle DNO \cong \triangle AET$.

1. $\angle M \cong$? **2.** $\overline{MC} \cong$? **3.** $\overline{DN} \cong$? **4.** $\angle A \cong$?

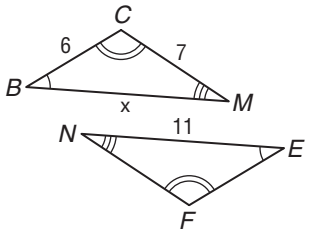
5. $\overline{FL} \cong$? **6.** $\angle C \cong$? **7.** $\overline{TE} \cong$? **8.** $\angle O \cong$?

Find the value of x for each pair of congruent triangles.

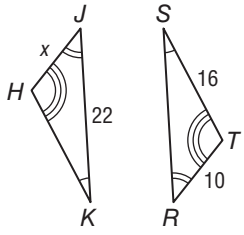
9.



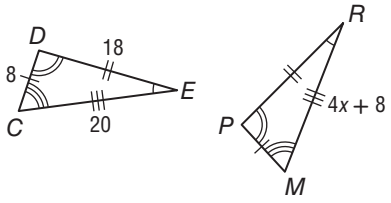
10.



11.

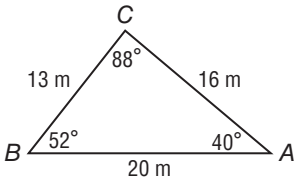


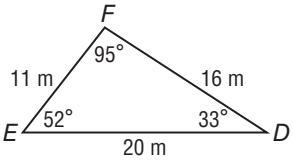
12. ALGEBRA If $\triangle DEC \cong \triangle PRM$, what is the value of x?



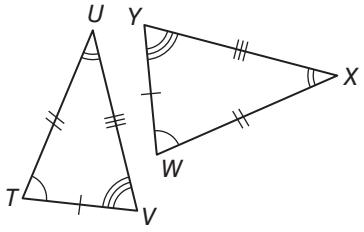
Determine whether the triangles shown are congruent. If so, name the corresponding parts and write a congruence statement.

13.





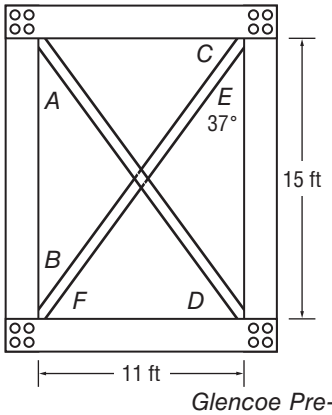
14.



ARCHITECTURE For Exercises 15 and 16, use the diagram of the Eiffel Tower truss at the right and the fact that $\triangle ACB \cong \triangle DFE$.

15. Find the distance between A and B.

16. What is the measure of $\angle B$?

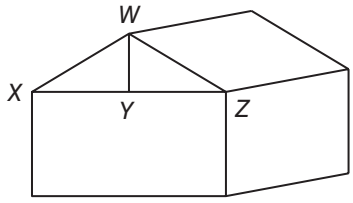


NAME _____ DATE _____ PERIOD _____

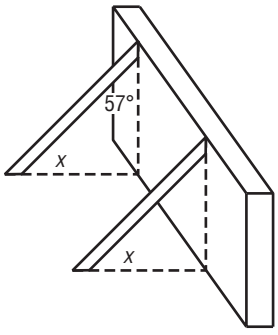
11-2 Word Problem Practice

Congruent Triangles

1. ROOFING The structure of a roof can be broken into congruent triangles. If side $\overline{WX}$ is 12 feet long, what is the length of side $\overline{WZ}$?

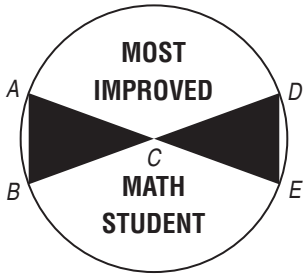


2. CONSTRUCTION Braces are often used to support walls during the construction of a house. If the two braces used in the following house are the same length and perpendicular to the ground, what is the measure of the angle x where the braces meet the ground?

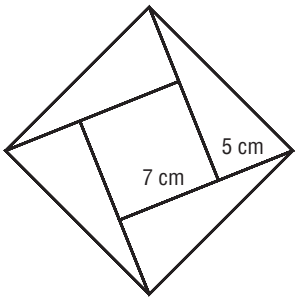


3. FLOWER BEDS Jane has two congruent flower beds in her backyard. The flower beds are triangular in shape. If the longest side of one flower bed is 12 feet, how long is the longest side of the other flower bed?

4. AWARDS The award for Most Improved Math Student in Mrs. Pike's classroom is a circle containing two congruent triangles connected at a vertex. What side in ΔABC corresponds to $\overline{DE}$?



5. KITES Joey's kite is made up of 4 congruent right triangles and 1 square as shown in the diagram below.



a. What are the three side lengths of each triangle?

b. What is the perimeter of Joey's kite?

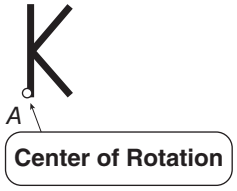
Lesson 11-2

NAME _____ DATE _____ PERIOD _____

11-3 Study Guide and Intervention

Rotations

Rotations A **rotation** is a transformation in which a figure is turned around a fixed point. This point is called the **center of rotation.** A rotated figure has the same size and shape as the original figure.

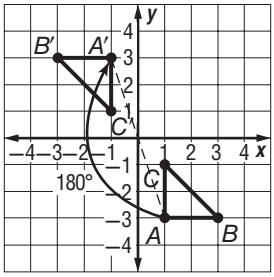
Original Figure	Angle of Clockwise Rotation		
	90°	180°	270°
			

Example **Triangle *ABC* has vertices *A*(1, −3), *B*(3, −3), and *C*(1, −1). Graph the figure and its image after it is rotated 180° clockwise about the origin.**

Step 1 Graph $\triangle ABC$ on a coordinate plane.

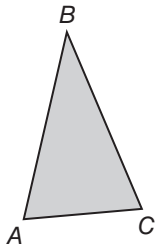
Step 2 Graph point A' after a 180° clockwise rotation about the origin.

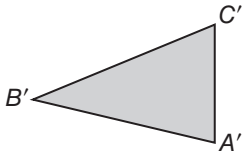
Step 3 Graph the remaining vertices after 180° rotations about the origin. Then connect the vertices to form $\triangle A'B'C'$.



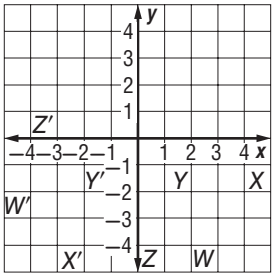
Exercises

1. Draw the figure at the right after a 270° clockwise rotation about point *B*.





2. A figure has vertices *W*(2, −4), *X*(4, −2), *Y*(2, −2), and *Z*(0, −4). Graph the figure and its image after a clockwise rotation of 90° about the origin.



Lesson 11-3

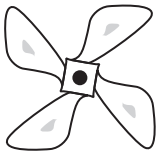
NAME _____ DATE _____ PERIOD _____

11-3 Study Guide and Intervention *(continued)*

Rotations

Rotational Symmetry A complete rotation of a figure is 360° because a circle has 360°. A figure that can be turned about its center less than 360° and match the original figure is said to have **rotational symmetry.** If the figure matches itself *only* after a 360° turn, it does not have rotational symmetry.

Example **TOYS Determine whether the pinwheel at the right has rotational symmetry. If it does, describe the angle of rotation.**



The pinwheel can match itself in four positions.

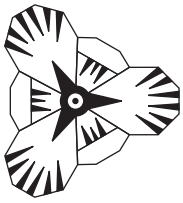
The pattern repeats in 4 even intervals.

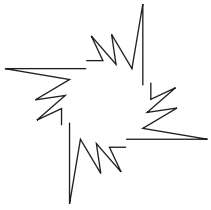
So, the angle of rotation is 360° ÷ 4 or 90°.

Exercises

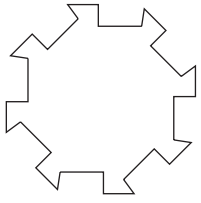
Determine whether each figure has rotational symmetry. If it does, describe the angle of rotation.

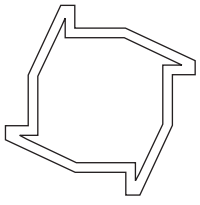
1. 

2. 

3. 

4. 

5. 

6. 

NAME _____ DATE _____ PERIOD _____

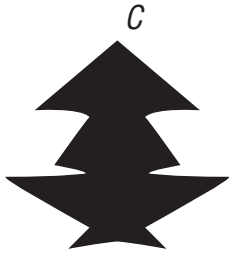
11-3 Skills Practice

Rotations

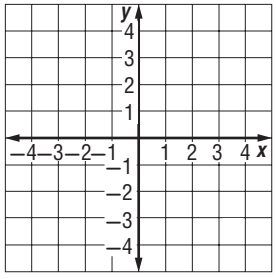
Draw each figure after the rotation described.

1. 90° clockwise rotation about point B

2. 180° clockwise rotation about point C



3. A figure has vertices $A(1, 1)$, $B(1, 3)$, $C(3, 3)$, and $D(4, 1)$. Graph the figure and its image after a rotation of 90° clockwise about the origin.

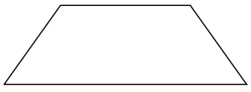


Determine whether each figure has rotational symmetry. If it does, describe the angle of rotation.

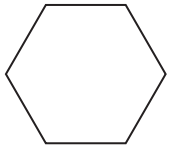
4.

5.

6.



7.



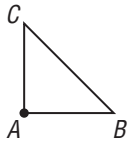
NAME _____ DATE _____ PERIOD _____

11-3 Practice

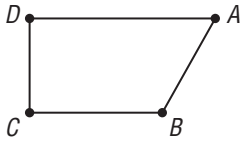
Rotations

Draw each figure after the rotation described.

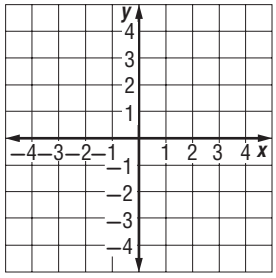
1. 270° clockwise rotation about point A



2. 180° clockwise rotation about point A

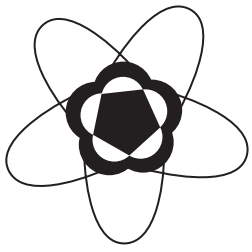


3. A figure has vertices $A(1, 3)$, $B(1, 5)$, and $C(5, 4)$. Graph the figure and its image after a rotation of 90° clockwise about the origin.



Determine whether each figure has rotational symmetry. If it does, describe the angle of rotation.

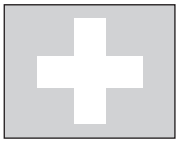
4.



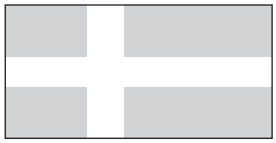
5.

6. FLAGS Many countries have cooperated to build the International Space Station. The flags below represent three of them.

United Kingdom



Switzerland



Sweden

a. Which flags have rotational symmetry?

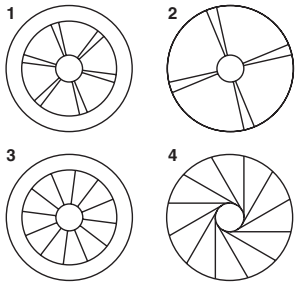
b. Describe the angle of rotation for each flag.

NAME _____ DATE _____ PERIOD _____

11-3 Word Problem Practice

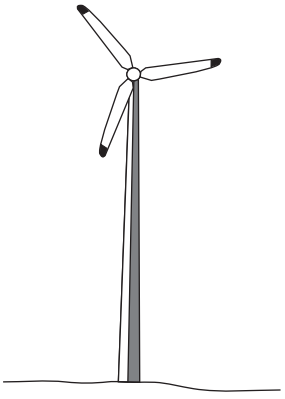
Rotations

1. HUBCAPS Corinne noticed that hubcaps are a good source of objects that have rotational symmetry. One day she observed the four hubcaps shown below. Determine the angle of rotation for each hubcap.



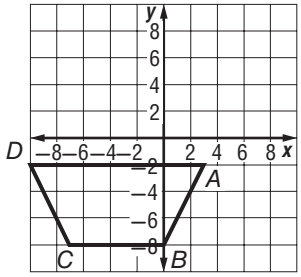
2. GEOMETRY If vertex U of quadrilateral $STUV$ is located in Quadrant IV and the quadrilateral is rotated 180° about the origin, in what quadrant will vertex U' lie?

3. WINDMILLS In Southern California, there are miles of three-bladed windmills that are used to produce electricity. Determine if the blades of the windmills have rotational symmetry. If so, describe the angle of rotation.

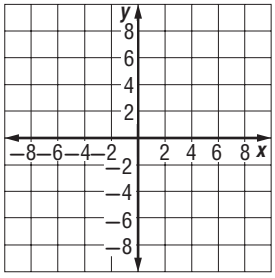


4. ALPHABET Examine each capital letter in the alphabet and determine which letters have rotational symmetry at 180°.

5. GEOMETRY Trapezoid $ABCD$ is shown below.

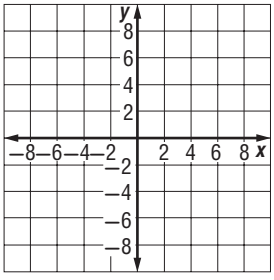


a. Graph the trapezoid after a clockwise rotation of 90° about the origin.



b. What are the coordinates of the new trapezoid?

c. Graph the trapezoid after a *counterclockwise* rotation of 270° about the origin.



d. What do you notice about the two images graphed in parts a and c?

Lesson 11-3

NAME _____ DATE _____ PERIOD _____

11-1 Study Guide and Intervention

Angle and Line Relationships

Line and Angle Relationships

Parallel Lines	Perpendicular Lines	Vertical Angles	Adjacent Angles	Complementary Angles	Supplementary Angles
$a \parallel b$	$m \perp n$	$\angle 1 \cong \angle 3$ $\angle 2 \cong \angle 4$	$m\angle ABC =$ $m\angle 1 + m\angle 2$	$m\angle 1 + m\angle 2 = 90°$	$m\angle 1 + m\angle 2 = 180°$

Example **In the figure at the right, classify the relationship between the pairs of angles shown. Then find the value of x.**

The angles are complementary. The sum of their measures is 90°.

$m\angle x + 34 = 90$ Write the equation.

$m\angle x + 34 - 34 = 90 - 34$ Subtract 34 from each side.

$m\angle x = 56$ Simplify.

So, $m\angle x$ is 56°.

Exercises

Classify the pairs of angles shown. Then find the value of x in each figure.

1.

vertical angles; 102

2.

complementary angles; 70

3.

supplementary angles; 60

4.

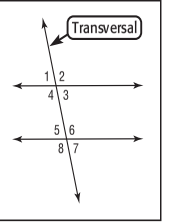
vertical angles; 25

Lesson 11-1

NAME _____ DATE _____ PERIOD _____

11-1 Study Guide and Intervention *(continued)*

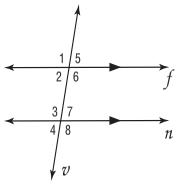
Angle and Line Relationships

Names of Special Angles		
Interior angles lie inside the parallel lines.	∠3, ∠4, ∠5, ∠6	
Exterior angles lie outside the parallel lines.	∠1, ∠2, ∠7, ∠8	
Alternate interior angles are on opposite sides of the transversal and inside the parallel lines.	∠3 and ∠5, ∠4 and ∠6	
Alternate exterior angles are on opposite sides of the transversal and outside the parallel lines.	∠1 and ∠7, ∠2 and ∠8	
Corresponding angles are in the same position on the parallel lines in relation to the transversal.	∠1 and ∠5, ∠2 and ∠6, ∠3 and ∠7, ∠4 and ∠8	

When a transversal intersects two parallel lines, pairs of alternate exterior angles, alternate interior angles, and corresponding angles are congruent.

Example **In the figure, $f \parallel n$ and v is a transversal.**

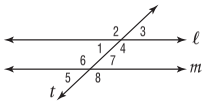
If $m\angle 3 = 100°$, find $m\angle 1$ and $m\angle 6$.



Since ∠1 and ∠3 are corresponding angles, they are congruent. So, $m\angle 1 = 100°$. Since ∠3 and ∠6 are alternate interior angles, they are congruent. So, $m\angle 6 = 100°$.

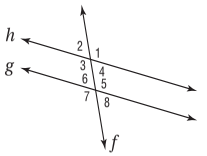
Exercises

In the figure on the right, $l \parallel m$ and t is a transversal. If $m\angle 1 = 61.2°$ and the $m\angle 6 = 118.8°$, find the measure of each angle.



1. ∠7 **61.2°**
2. ∠3 **61.2°**
3. ∠4 **118.8°**
4. ∠8 **118.8°**
5. ∠5 **61.2°**
6. ∠2 **118.8°**

In the figure on the right, $g \parallel h$ and f is a transversal. If $m\angle 1 = 125°$ and the $m\angle 6 = 55°$, find the measure of each angle.



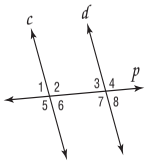
7. ∠2 **55°**
8. ∠4 **55°**
9. ∠5 **125°**
10. ∠3 **125°**
11. ∠8 **55°**
12. ∠7 **125°**

NAME _____ DATE _____ PERIOD _____

11-1 Skills Practice

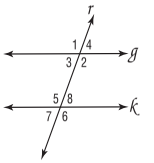
Angle and Line Relationships

In the figure at the right, $c \parallel d$ and p is a transversal. If $m\angle 5 = 110°$, find the measure of each angle.



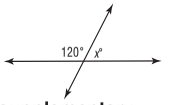
1. ∠6 **70°**
2. ∠8 **70°**
3. ∠2 **110°**
4. ∠4 **110°**

In the figure at the right, $g \parallel k$ and r is a transversal. If $m\angle 7 = 60°$, find the measure of each angle.

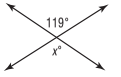


5. ∠4 **60°**
6. ∠6 **120°**
7. ∠5 **120°**
8. ∠3 **60°**

Classify the pairs of angles shown. Then find the value of x in each figure.

9. 

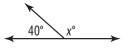
supplementary angles; 60

10. 

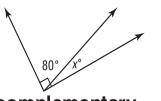
vertical angles; 119

11. 

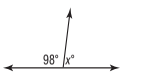
complementary angles; 35

12. 

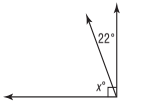
supplementary angles; 140

13. 

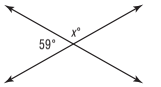
complementary angles; 10

14. 

supplementary angles; 82

15. 

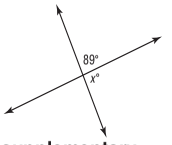
complementary angles; 68

16. 

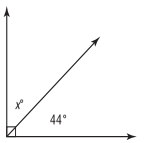
supplementary angles; 121

17. 

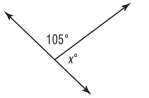
supplementary angles; 174

18. 

supplementary angles; 91

19. 

complementary angles; 46

20. 

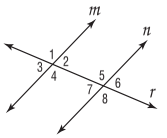
supplementary angles; 75

NAME _____ DATE _____ PERIOD _____

11-1 Practice

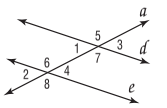
Angle and Line Relationships

In the figure at the right, $m \parallel n$ and r is a transversal. If $m\angle 2 = 45°$, find the measure of each angle.



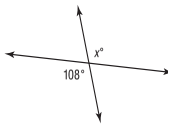
1. $\angle 4$ **135°**
2. $\angle 5$ **135°**
3. $\angle 7$ **45°**
4. $\angle 8$ **135°**
5. $\angle 6$ **45°**
6. $\angle 3$ **45°**

In the figure at the right, $d \parallel e$ and a is a transversal. If $m\angle 5 = 143°$, find the measure of each angle.

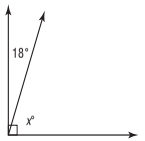


7. $\angle 7$ **143°**
8. $\angle 6$ **143°**
9. $\angle 4$ **37°**
10. $\angle 2$ **37°**
11. $\angle 1$ **37°**
12. $\angle 8$ **143°**

Classify the pairs of angles shown. Then find the value of x in each figure.

13. 

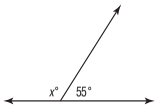
vertical angles; 108

14. 

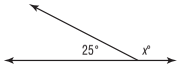
complementary angles; 72

15. 

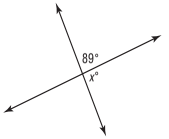
supplementary angles; 9

16. 

supplementary angles; 125

17. 

supplementary angles; 155

18. 

supplementary angles; 91

19. Angles Q and R are complementary. Find $m\angle R$ if $m\angle Q = 24°$. **66°**

20. Find $m\angle J$ if $m\angle K = 29°$ and $\angle J$ and $\angle K$ are supplementary. **151°**

21. The measures of angles A and B are equal and complementary. What is the measure of each angle? **45°**

22. **ALGEBRA** Angles G and H are complementary. If $m\angle G = 3x + 6$ and $m\angle H = 2x - 11$, what is the measure of each angle? **$m\angle G = 63°$; $m\angle H = 27°$**

NAME _____ DATE _____ PERIOD _____

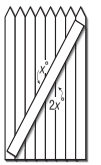
11-1 Word Problem Practice

Angle and Line Relationships

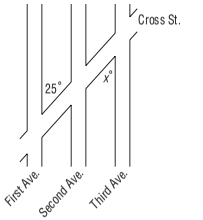
1. **PROPERTY LINES** The front and back property lines of Michaela's land are parallel lines. If the angle between the west side property line and back property line is 106°, what is the angle between the front property line and west side property line? **74°**

2. **SCISSORS** Archie opened up a pair of scissors so that the angle between the blades is 38°. What is the angle between the handles? **38°**

3. **FENCING** The sections of fence in Sioban's yard have diagonal supports as shown. The top side of the diagonal support makes an angle of $x°$ with the fence slats. The bottom side makes an angle that is twice the measure of the top angle. Find the measures of both angles. **60°, 120°**

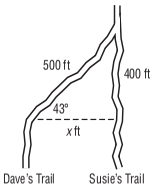


4. **MAPS** In the following map, First Avenue, Second Avenue, and Third Avenue are parallel. Cross Street intersects all three avenues. First Avenue and Cross Street meet at a 25° angle. What angle does the intersection of Third Avenue and Cross Street make?



25°

5. **HIKING** Dave and Susie are walking on parallel trails in the woods. Dave's trail turns to the right 43° and meets up with Susie's trail.



a. At what angle does Dave's trail meet Susie's trail? **47°**

b. How far apart were Dave and Susie's trails originally? **300 ft**

NAME _____ DATE _____ PERIOD _____

11-2 Study Guide and Intervention

Congruent Triangles

Corresponding Parts of Congruent Triangles

Words Two triangles are **congruent** if they have the same size and shape.
If two triangles are congruent, their corresponding sides are congruent and their corresponding angles are congruent.

Model

Symbols Congruent Angles: $\angle X \cong \angle P$, $\angle Y \cong \angle Q$, $\angle Z \cong \angle R$
Congruent Sides: $\overline{XY} \cong \overline{PQ}$, $\overline{YZ} \cong \overline{QR}$, $\overline{XZ} \cong \overline{PR}$

Example **Name the corresponding parts in the congruent triangles shown. Then write a congruence statement.**

Corresponding angles:
$\angle Q \cong \angle S$, $\angle R \cong \angle Z$, $\angle N \cong \angle V$

Corresponding sides:
$\overline{SZ} \cong \overline{QR}$, $\overline{ZV} \cong \overline{RN}$, $\overline{VS} \cong \overline{NQ}$

$\triangle NQR \cong \triangle VSZ$

Exercises

Complete each congruence statement if $\Delta DFH \cong \Delta PWZ$.

1. $\angle F \cong$ ___$\angle W$___
2. $\angle P \cong$ ___$\angle D$___
3. $\overline{DH} \cong$ ___$\overline{PZ}$___
4. $\overline{ZW} \cong$ ___$\overline{HF}$___

Find the value of x for each pair of congruent triangles.

5.

24

6.

12

7.

8

8.

9

11

Lesson 11-2

NAME _____ DATE _____ PERIOD _____

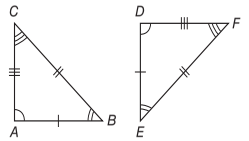
11-2 Study Guide and Intervention *(continued)*

Congruent Triangles

Identify Congruent Triangles Two triangles are congruent if and only if all pairs of corresponding angles are congruent and all pairs of corresponding sides are congruent.

Example **Determine whether the triangles shown are congruent. If so, name the corresponding parts and write a congruence statement.**

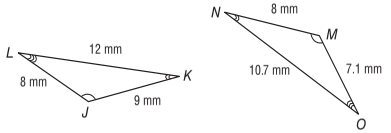
a.



Corresponding angles: The arcs indicate that $\angle A \cong \angle D$, $\angle B \cong \angle E$, and $\angle C \cong \angle F$.
Corresponding sides: The side measures indicate that $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, and $\overline{CA} \cong \overline{FD}$.

Since all pairs of corresponding angles and sides are congruent, the triangles are congruent. One congruence statement is $\triangle ABC \cong \triangle DEF$.

b.

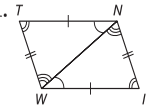


Although the arcs indicate that $\angle J \cong \angle M$, $\angle K \cong \angle N$, and $\angle L \cong \angle O$, the side measures indicate that no sides are congruent with one another. Therefore, the triangles are *not* congruent.

Exercises

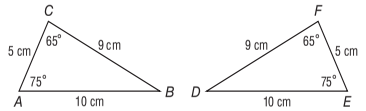
Determine whether the triangles shown are congruent. If so, name the corresponding parts and write a congruence statement.

1.



congruent;
$\angle N \cong \angle W$, $\angle T \cong \angle I$, $\angle W \cong \angle N$;
$\overline{NT} \cong \overline{WI}$, $\overline{TW} \cong \overline{IN}$, $\overline{WN} \cong \overline{NW}$;
$\triangle NTW \cong \triangle WIN$

2.



congruent;
$\angle C \cong \angle F$, $\angle A \cong \angle E$, $\angle B \cong \angle D$;
$\overline{CA} \cong \overline{FE}$, $\overline{AB} \cong \overline{ED}$, $\overline{BC} \cong \overline{DF}$;
$\triangle CAB \cong \triangle FED$

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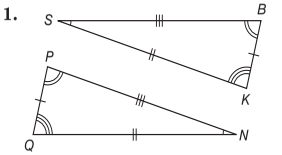
11-2 Skills Practice

Congruent Triangles

Name the corresponding parts in each pair of congruent triangles. Then complete the congruence statement.

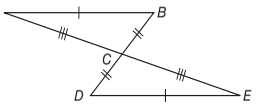
1.



$\triangle KBS \cong$ $\triangle QPN$

$\overline{BK} \cong \overline{PQ}$, $\overline{SB} \cong \overline{NP}$, $\overline{KS} \cong \overline{QN}$,
$\angle S \cong \angle N$, $\angle B \cong \angle P$, $\angle K \cong \angle Q$

2.



$\triangle ACB \cong$ $\triangle ECD$

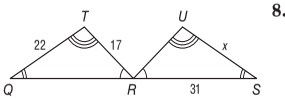
$\overline{BC} \cong \overline{DC}$, $\overline{AC} \cong \overline{EC}$, $\overline{AB} \cong \overline{ED}$,
$\angle B \cong \angle D$, $\angle A \cong \angle E$, $\angle ACB \cong \angle ECD$

Complete each congruence statement if $\triangle MRU \cong \triangle ACF$.

3. $\angle R \cong$? $\angle C$ **4.** $\overline{CA} \cong$? $\overline{RM}$ **5.** $\overline{MU} \cong$? $\overline{AF}$ **6.** $\angle A \cong$? $\angle M$

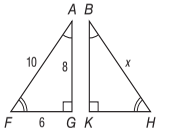
Find the value of x for each pair of congruent triangles.

7.



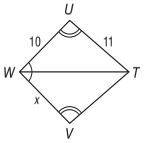
22

8.



10

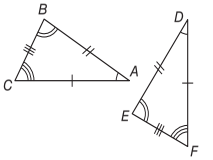
9.



10

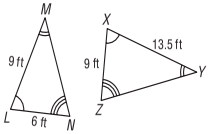
Determine whether the triangles shown are congruent. If so, name the corresponding parts and write a congruence statement.

10.



congruent; $\angle A \cong \angle D$, $\angle B \cong \angle E$,
$\angle C \cong \angle F$; $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$,
$\overline{CA} \cong \overline{FD}$; $\triangle ABC \cong \triangle DEF$

11.



not congruent

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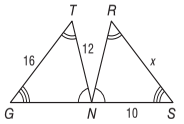
11-2 Practice

Congruent Triangles

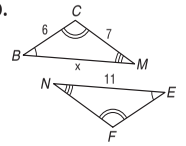
Complete the congruence statement if $\triangle CMH \cong \triangle PLF$ and $\triangle DNO \cong \triangle AET$.

1. $\angle M \cong$? $\angle L$
2. $\overline{MC} \cong$? $\overline{LP}$
3. $\overline{DN} \cong$? $\overline{AE}$
4. $\angle A \cong$? $\angle D$
5. $\overline{FL} \cong$? $\overline{HM}$
6. $\angle C \cong$? $\angle P$
7. $\overline{TE} \cong$? $\overline{ON}$
8. $\angle O \cong$? $\angle T$

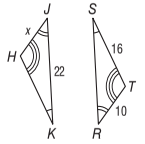
Find the value of x for each pair of congruent triangles.

9. 

16

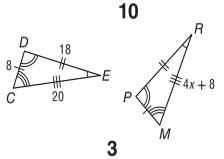
10. 

11

11. 

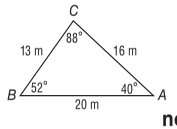
10

12. **ALGEBRA** If $\triangle DEC \cong \triangle PRM$, what is the value of x?

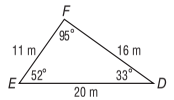


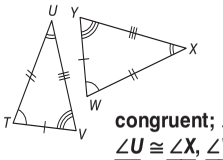
3

Determine whether the triangles shown are congruent. If so, name the corresponding parts and write a congruence statement.

13. 

not congruent



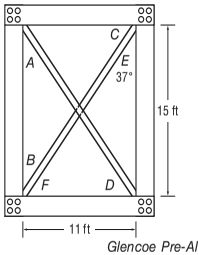
14. 

congruent; $\angle T \cong \angle W$, $\angle U \cong \angle X$, $\angle V \cong \angle Y$; $\overline{TU} \cong \overline{WX}$, $\overline{UV} \cong \overline{XY}$, $\overline{VT} \cong \overline{YW}$; $\triangle TUV \cong \triangle WXY$

ARCHITECTURE For Exercises 15 and 16, use the diagram of the Eiffel Tower truss at the right and the fact that $\triangle ACB \cong \triangle DFE$.

15. Find the distance between A and B. **15 ft**

16. What is the measure of $\angle B$? **37°**



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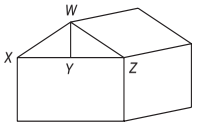
11-2 Word Problem Practice

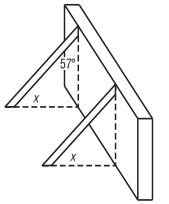
Congruent Triangles

1. **ROOFING** The structure of a roof can be broken into congruent triangles. If side $\overline{WX}$ is 12 feet long, what is the length of side $\overline{WZ}$?



12 feet

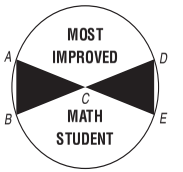
2. **CONSTRUCTION** Braces are often used to support walls during the construction of a house. If the two braces used in the following house are the same length and perpendicular to the ground, what is the measure of the angle x where the braces meet the ground?



33°

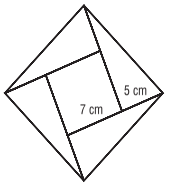
3. **FLOWER BEDS** Jane has two congruent flower beds in her backyard. The flower beds are triangular in shape. If the longest side of one flower bed is 12 feet, how long is the longest side of the other flower bed? **12 feet**

4. **AWARDS** The award for Most Improved Math Student in Mrs. Pike's classroom is a circle containing two congruent triangles connected at a vertex. What side in $\triangle ABC$ corresponds to $\overline{DE}$?



$\overline{AB}$

5. **KITES** Joey's kite is made up of 4 congruent right triangles and 1 square as shown in the diagram below.



   a. What are the three side lengths of each triangle?
   5 cm, 12 cm, 13 cm

   b. What is the perimeter of Joey's kite?
   52 cm

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11-3 Study Guide and Intervention

Rotations

Rotations A **rotation** is a transformation in which a figure is turned around a fixed point. This point is called the **center of rotation.** A rotated figure has the same size and shape as the original figure.

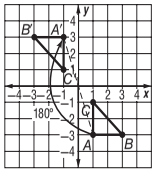
Original Figure	Angle of Clockwise Rotation		
	90°	180°	270°
			

Example **Triangle ABC has vertices $A(1, -3)$, $B(3, -3)$, and $C(1, -1)$. Graph the figure and its image after it is rotated 180° clockwise about the origin.**

Step 1 Graph $\triangle ABC$ on a coordinate plane.

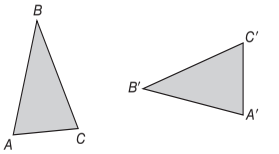
Step 2 Graph point A' after a 180° clockwise rotation about the origin.

Step 3 Graph the remaining vertices after 180° rotations about the origin. Then connect the vertices to form $\triangle A'B'C'$.

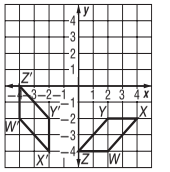


Exercises

1. Draw the figure at the right after a 270° clockwise rotation about point B.



2. A figure has vertices $W(2, -4)$, $X(4, -2)$, $Y(2, -2)$, and $Z(0, -4)$. Graph the figure and its image after a clockwise rotation of 90° about the origin.



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11-3 Study Guide and Intervention *(continued)*

Rotations

Rotational Symmetry A complete rotation of a figure is 360° because a circle has 360°. A figure that can be turned about its center less than 360° and match the original figure is said to have **rotational symmetry.** If the figure matches itself *only* after a 360° turn, it does not have rotational symmetry.

Example **TOYS Determine whether the pinwheel at the right has rotational symmetry. If it does, describe the angle of rotation.**

The pinwheel can match itself in four positions.

The pattern repeats in 4 even intervals.

So, the angle of rotation is 360° ÷ 4 or 90°.

Exercises

Determine whether each figure has rotational symmetry. If it does, describe the angle of rotation.

1. 

yes; 60°

2. 

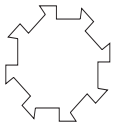
yes; 120°

3. 

yes; 90°

4. 

no

5. 

yes; 45°

6. 

yes; 90°

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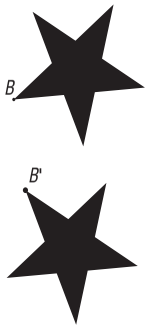
11-3 Skills Practice

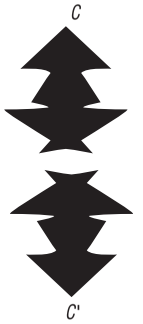
Rotations

Draw each figure after the rotation described.

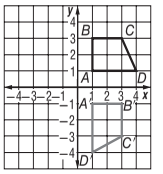
1. 90° clockwise rotation about point B



2. 180° clockwise rotation about point C



3. A figure has vertices $A(1, 1)$, $B(1, 3)$, $C(3, 3)$, and $D(4, 1)$. Graph the figure and its image after a rotation of 90° clockwise about the origin.



Determine whether each figure has rotational symmetry. If it does, describe the angle of rotation.

4. 

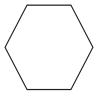
yes; 180°

5. 

no

6. 

no

7. 

yes; 60°

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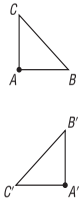
11-3 Practice

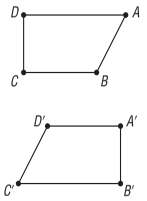
Rotations

Draw each figure after the rotation described.

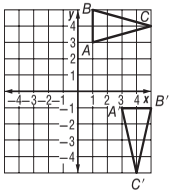
1. 270° clockwise rotation about point A



2. 180° clockwise rotation about point A

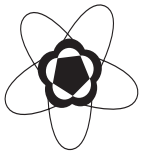


3. A figure has vertices $A(1, 3)$, $B(1, 5)$, and $C(5, 4)$. Graph the figure and its image after a rotation of 90° clockwise about the origin.



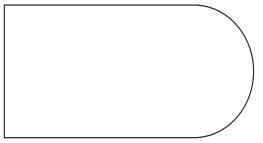
Determine whether each figure has rotational symmetry. If it does, describe the angle of rotation.

4.



yes; 72°

5.



no

6. FLAGS Many countries have cooperated to build the International Space Station. The flags below represent three of them.

United Kingdom

Switzerland

Sweden

a. Which flags have rotational symmetry? **United Kingdom; Switzerland**

b. Describe the angle of rotation for each flag.
United Kingdom: 180°; Switzerland: 90°

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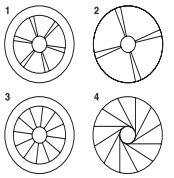
11-3 Word Problem Practice

Rotations

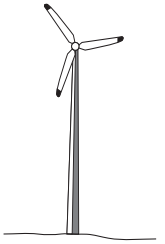
1. HUBCAPS Corinne noticed that hubcaps are a good source of objects that have rotational symmetry. One day she observed the four hubcaps shown below. Determine the angle of rotation for each hubcap.



1. 60°; 2. 90°; 3. 30°; 4. 30°

2. GEOMETRY If vertex U of quadrilateral $STUV$ is located in Quadrant IV and the quadrilateral is rotated 180° about the origin, in what quadrant will vertex U' lie? **Quadrant III**

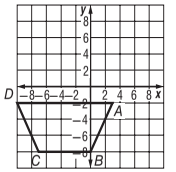
3. WINDMILLS In Southern California, there are miles of three-bladed windmills that are used to produce electricity. Determine if the blades of the windmills have rotational symmetry. If so, describe the angle of rotation.



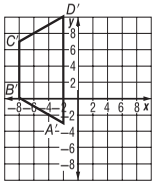
yes; 120°

4. ALPHABET Examine each capital letter in the alphabet and determine which letters have rotational symmetry at 180°.
H, I, N, O, S, X, Z

5. GEOMETRY Trapezoid $ABCD$ is shown below.

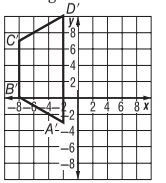


a. Graph the trapezoid after a clockwise rotation of 90° about the origin.



b. What are the coordinates of the new trapezoid?
$A'(2, -3)$, $B'(-8, 0)$, $C'(-8, 7)$, $D'(-2, 9)$

c. Graph the trapezoid after a *counterclockwise* rotation of 270° about the origin.



d. What do you notice about the two images graphed in parts a and c?
They are the same image.

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11-3 Enrichment

Rotations

Imagine a game of concentration. Each box represents a card in the game. Choose two cards with figures that would have rotational symmetry if they were put together. Cross them out. Rearrange the bold letters of the unmatched cards to solve the riddle.

O	**I**	**O**	**R**
R	**E**	**D**	**T**
A	**C**	**F**	**X**
V	**T**	**D**	**I**

Who earns a living by driving his customers away?

A ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___

taxi driver