SDSE Homework 1

Due date: February 18th, 11:59pm

Problem 1 `[ 12 = 4 + 4 + 4 points ]`

Find whether each of the following functions $p(x)$ are valid pdfs over the sample space Ω.

- You may use a plot of the function to prove positivity. If you do so, make sure to label the axes of your plot. You can use the software of your choice to make the plot.
- You may do the proofs by hand, or with a symbolic solver such as Python's *sympy* or Matlab's symbolic math toolbox. If you use a symbolic solver, please include a screenshot or a printout of your code.

(a) $p(x) = \frac{1}{3}\, x^3 \quad , \quad \Omega = [0, 1]$

(b) $p(x) = 1/x \quad , \quad \Omega = \mathbb{N}^+$ (the positive natural numbers $\{1, 2, 3, \ldots\}$)

(c) $p(x) = 2^{-x} \quad , \quad \Omega = \mathbb{N}$ (the natural numbers, including 0)

Problem 2 `[ 17 = 2 + 2 + 2 + 3 + 2 + 3 + 3 points ]`

The joint pdf of a pair of discrete random variables X (with $\Omega_X = \{1, 2\}$) and Y (with $\Omega_Y = \{3, 4, 5\}$) is given in the table below.

$Y =$ \ $X =$	1	2
3	$1/8$	$1/4$
4	$1/16$	$1/2$
5	$1/16$	0

Compute the following,

(a) $p_X(x)$ (You may report these as labeled plots or as a mathematical equation with conditions (e.g. Eq. 2.87 in the reader).

(b) $P(Y \geq 4 \mid X = 1)$

(c) $E[X]$

(d) $Var[X]$

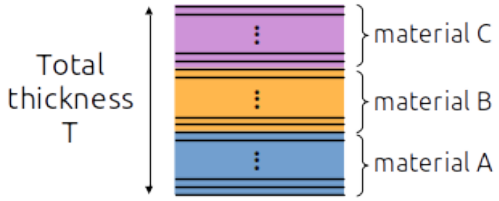
(e) $E[Y]$

(f) $Var[Y]$

(g) $Cov(X, Y)$

Problem 3 [8 = 4 + 4 points]

3D printers create objects by printing layers of material one on top of the other. Different materials have different mean thicknesses and standard deviations. A particular part is made by stacking 10 layers of material A, followed by 15 layers of material B, and 30 layers of material C. The properties of these materials are shown in the table below (note the units). Assume that the layer thicknesses are all independent. Compute the mean (in mm) and standard deviation (in μm) of the total thickness T of the part.



Material	Average thickness [mm]	Standard deviation [μm]
A	0.2	4
B	0.1	3
C	0.05	0.5

Problem 4 [19 = 2 + 2 + 3 + 4 + 4 + 4 points]

A study of washing machines found their lifetime T to follow an exponential distribution:

$$p_T(t) = 0.1e^{-0.1t}$$

with t measured in years.

(a) What is the mean lifetime of washing machines?

(b) What is the standard deviation?

(c) What percentage of washing machine are expected to fail within 10 years?

(d) What is the median life of washing machines? (The "median" is defined as the value of t for which $P(T < t) = 0.5$).

(e) A small hotel owns five washing machines. Their lifetimes are independent. What is the probability that none of the machines will fail within the next 3 years?

(f) What is the probability that all five machines will fail within the next 15 years?

Problem 5 [3 points]

Iron pipes used in a cooling system are required to have a roughness coefficient of between 0.2 mm and 0.3 mm. A manufacturing process is known to produce pipes with normally distributed roughnesses with mean 0.25 mm and a standard deviation of 0.03 mm. What percentage of these pipes is expected to meet the specification?

Note: You may either use the probability table provided on the bCourses page for this course, or use SciPy's /stats.norm class as demonstrated in lecture. If you use tables, please round the lookup value to the nearest one in the table (instead of interpolating between values). If you use SciPy, please screenshot or paste your code into your submission.