Analog and Digital Electronics

UNIT II
Frequency Response

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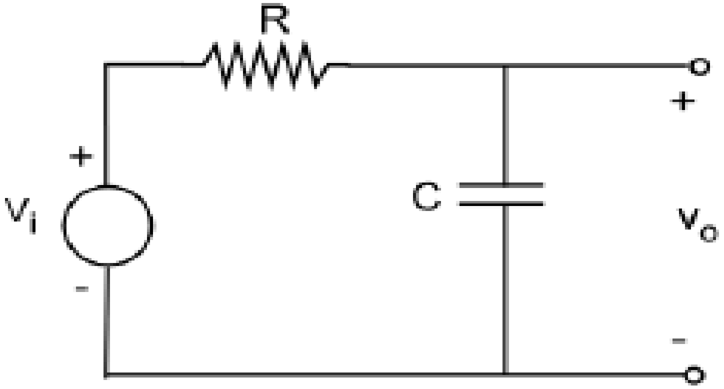
Low-Pass Circuit



In frequency domain:

$$V_o = \frac{V_i}{R + \frac{1}{j\omega C}} \cdot \frac{1}{j\omega C}$$

$$V_o = \frac{V_i}{1 + j\omega RC} \Rightarrow A_v = \frac{V_o}{V_i} = \frac{1}{1 + j\omega RC}$$

$$A_v = \frac{1}{1 + j\omega RC} = \frac{1}{1 + jf / f_o}$$

Low-Pass Circuit

$$f_o = \frac{1}{2\pi RC} = \frac{1}{2\pi\tau}$$

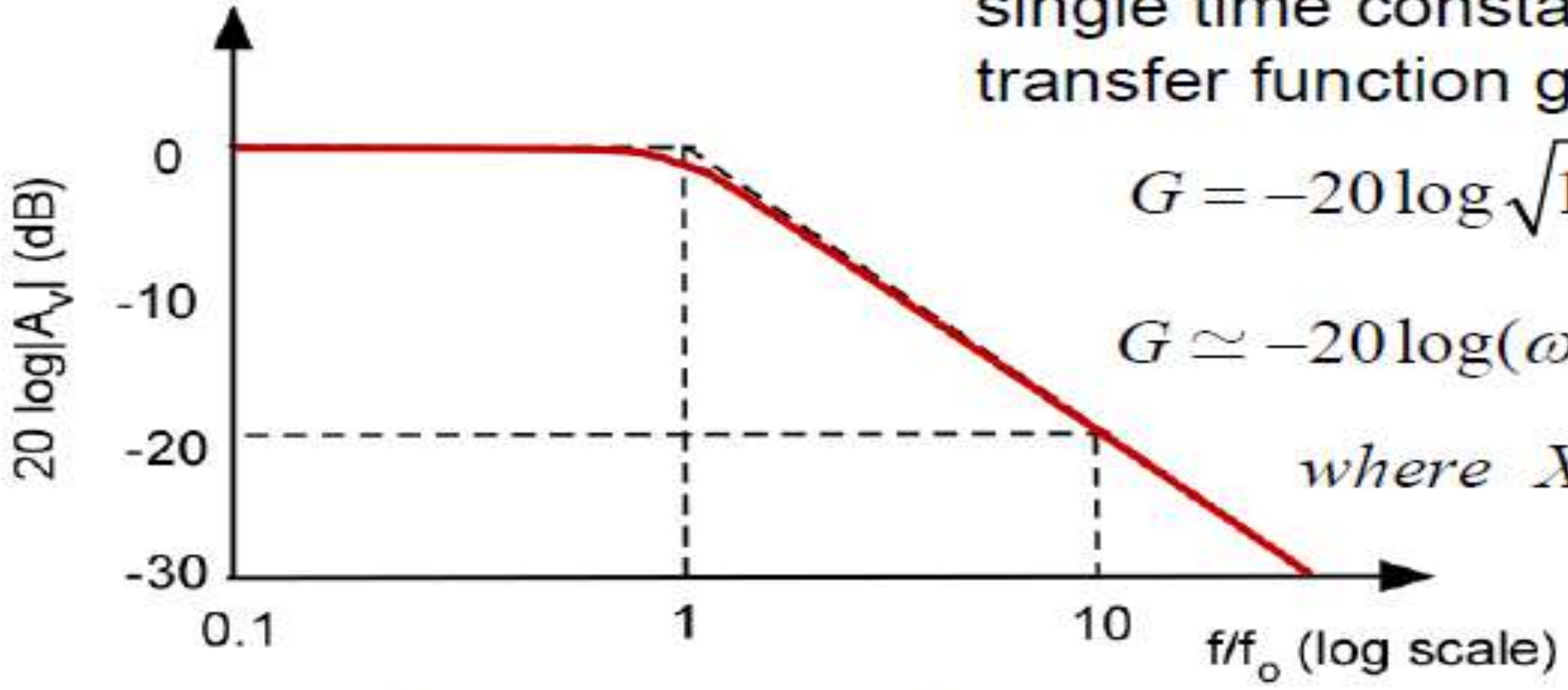
$\tau = RC = \text{time constant}$

At very high frequencies, the single time constant (STC) transfer function goes as

$$G = -20\log\sqrt{1+(\omega/\omega_o)^2}$$

$$G \simeq -20\log(\omega/\omega_o) = -20X$$

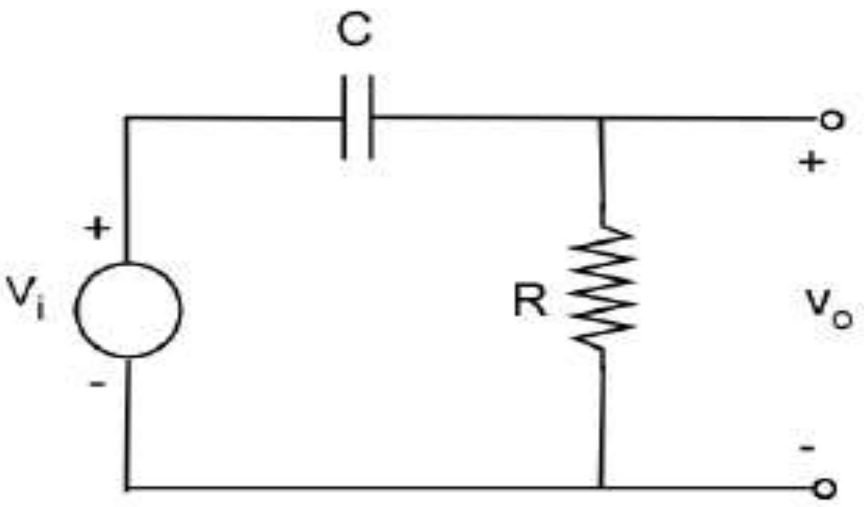
$$where \;\; X = \log(\omega/\omega_o)$$

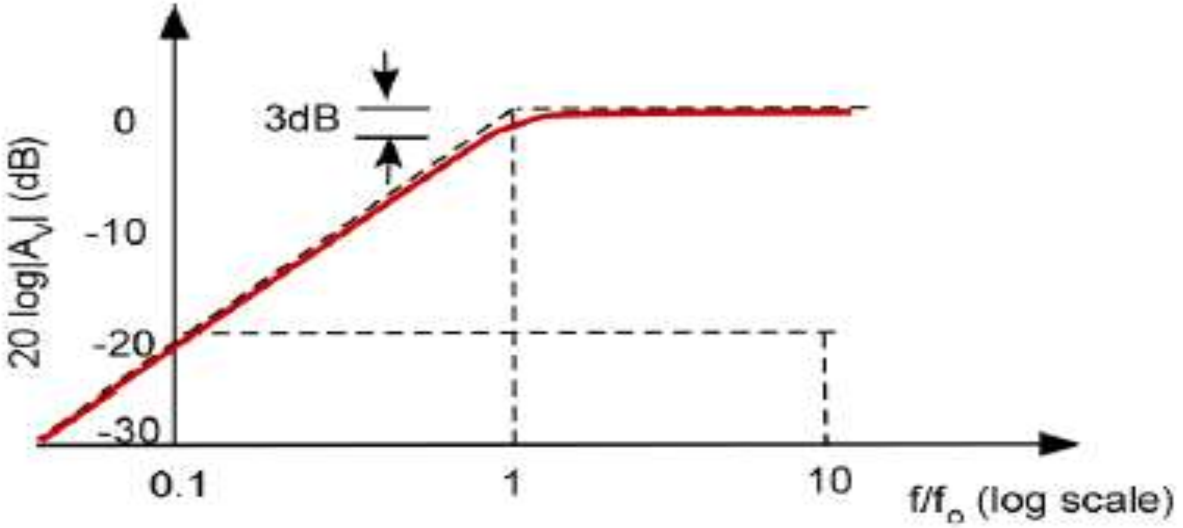


At high frequencies, slope of curve is –20 dB

$$if \;\; X = 1\,(\omega = 10\omega_o),\, decrease\, is - 20\; dB \Rightarrow -20dB/decade$$

High-Pass Circuit





$$V_o = \frac{V_i R}{R + \dfrac{1}{j\omega C}} = \frac{V_i}{1 + \dfrac{1}{j\omega RC}}$$

$$A_v = \frac{V_o}{V_i} = \frac{1}{1 - j\dfrac{1}{2\pi fRC}} = \frac{1}{1 - jf_o / f}$$

Octave & Decade

If $f_2 = 2f_1$, then f_2 is one octave above f_1

If $f_2 = 10f_1$, then f_2 is one decade above f_1

$$\#\ of\ octaves = \log_2 \frac{f_2}{f_1} = 3.32 \log_{10} \frac{f_2}{f_1}$$

$$\#\ of\ decades = \log_{10} \frac{f_2}{f_1}$$

2 GHz is one octave above 1 GHz

10 GHz is one decade above 1 GHz

Frequency Response

3-dB points are points where the magnitude is divided by $2^{1/2}$ (power is halved) $|1+j| = 2^{1/2}$

A_{dB}= 20log1.414 = 3 dB

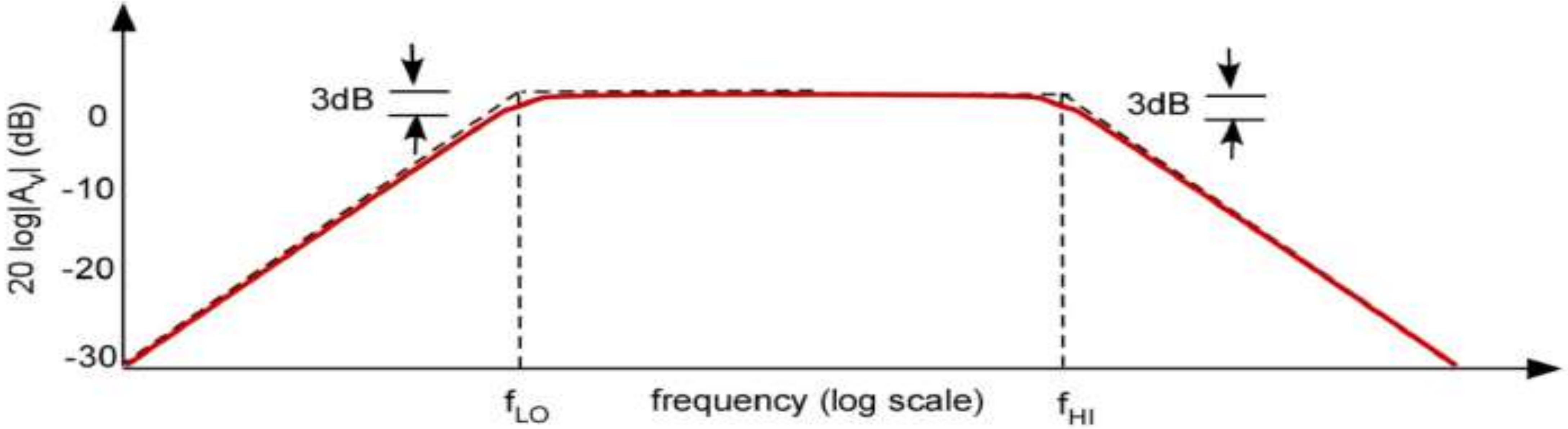
Amplifier has intrinsic gain A_o

Low-pass characteristics is: $$\frac{1}{1 + jf / f_{hi}}$$

High-pass characteristics is: $$\frac{jf / f_{lo}}{1 + jf / f_{lo}}$$

Overall gain $A(f)$ is $$A_o \cdot \frac{jf / f_{lo}}{1 + jf / f_{lo}} \cdot \frac{1}{1 + jf / f_{hi}}$$

Octave & Decade



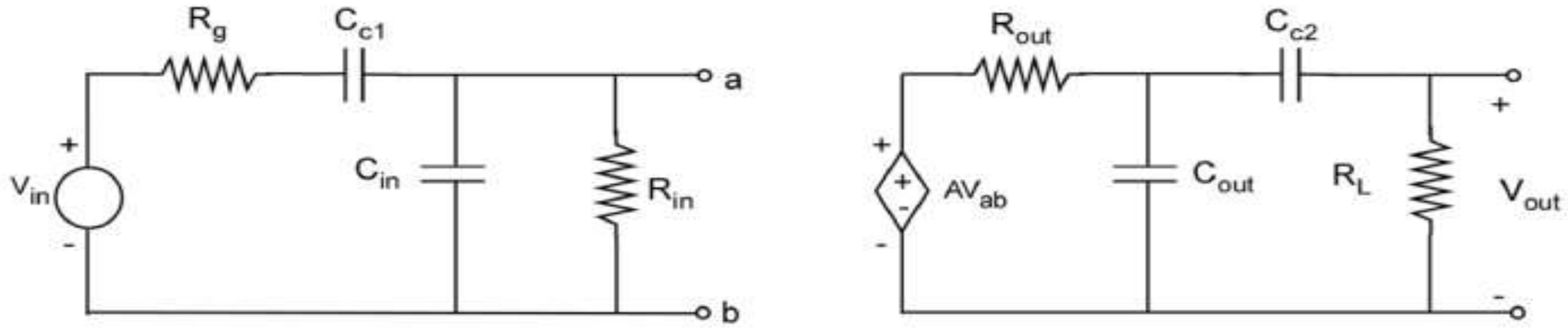
Overall gain $A(f)$ is

$$A(f) = A_o \cdot \frac{jf / f_{lo}}{1 + jf / f_{lo}} \cdot \frac{1}{1 + jf / f_{hi}}$$

Model for general Amplifying Element

C_{c1} and C_{c2} are coupling capacitors (large) ➔ μF

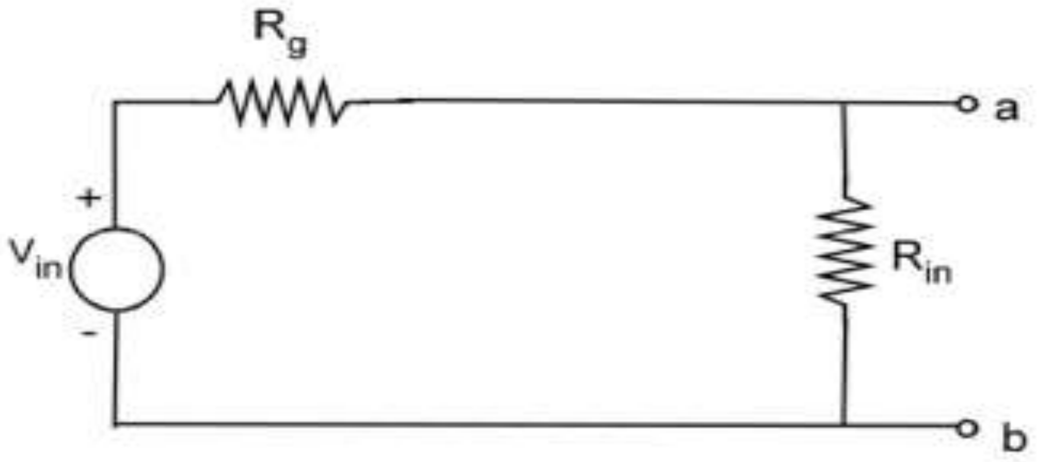
C_{in} and C_{out} are parasitic capacitors (small) ➔ pF

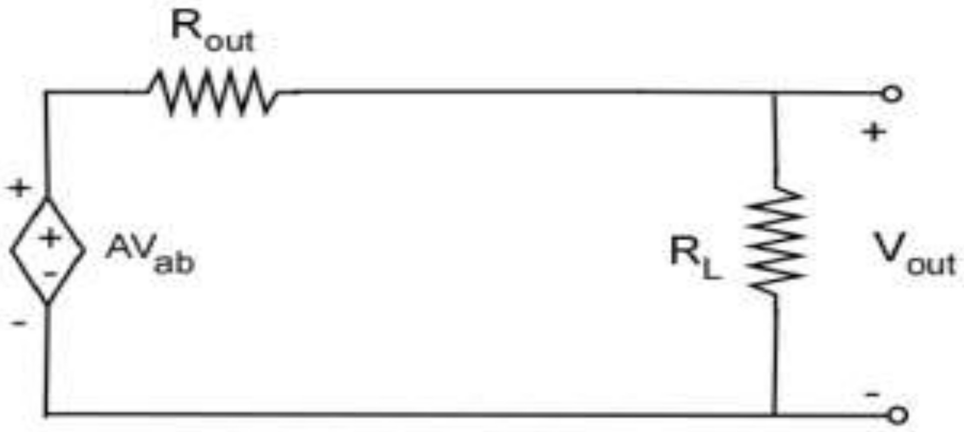


Midband Frequencies

- Coupling capacitors are short circuits

- Parasitic capacitors are open circuits

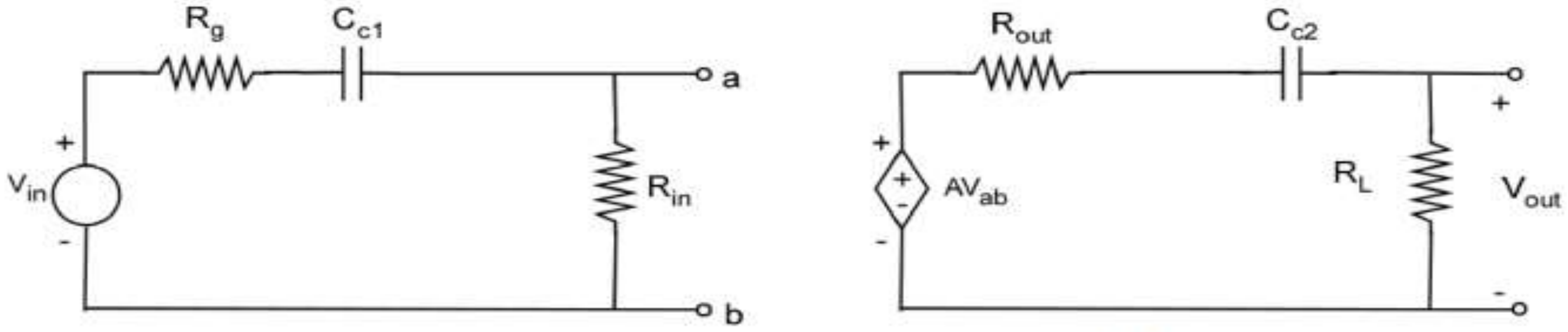




$$A_{MB} = \frac{v_{out}}{v_{in}} = \frac{R_{in}}{R_g + R_{in}} A \frac{R_L}{R_{out} + R_L}$$

Low Frequency Model

- Coupling capacitors are present
- Parasitic capacitors are open circuits



$$v_{ab} = \frac{v_{in} R_{in}}{R_g + R_{in} + \frac{1}{j\omega C_{c1}}} = \frac{v_{in} j\omega C_{c1} R_{in}}{1 + j\omega C_{c1}(R_g + R_{in})}$$

$$v_{ab} = v_{in} \frac{R_{in}}{R_g + R_{in}} \cdot \frac{j\omega C_{c1}(R_g + R_{in})}{\left[1 + j\omega C_{c1}(R_g + R_{in})\right]}$$

Low Frequency Model

$$define\ \ f_{l1} = \frac{1}{2\pi\left(R_g + R_{in}\right)C_{c1}}\ \ and\ \ f_{l2} = \frac{1}{2\pi\left(R_L + R_{out}\right)C_{c2}}$$

$$v_{ab} = v_{in}\frac{R_{in}}{R_g + R_{in}} \cdot \frac{jf / f_{l1}}{1 + jf / f_{l1}}$$

$$Similarly,\ \ v_{out} = Av_{ab}\frac{R_L}{R_L + R_{out}} \cdot \frac{jf / f_{l2}}{1 + jf / f_{l2}}$$

Low Frequency Model

$$Overall\ gain = \frac{v_{out}}{v_{in}} = \frac{R_{in}}{R_g + R_{in}} \cdot A \cdot \frac{R_L}{R_L + R_{out}} \cdot \frac{jf / f_{l1}}{1 + jf / f_{l1}} \cdot \frac{jf / f_{l2}}{1 + jf / f_{l2}}$$

$$\frac{v_{out}}{v_{in}} = A_{MB} \cdot \frac{jf / f_{l1}}{1 + jf / f_{l1}} \cdot \frac{jf / f_{l2}}{1 + jf / f_{l2}}$$

Example

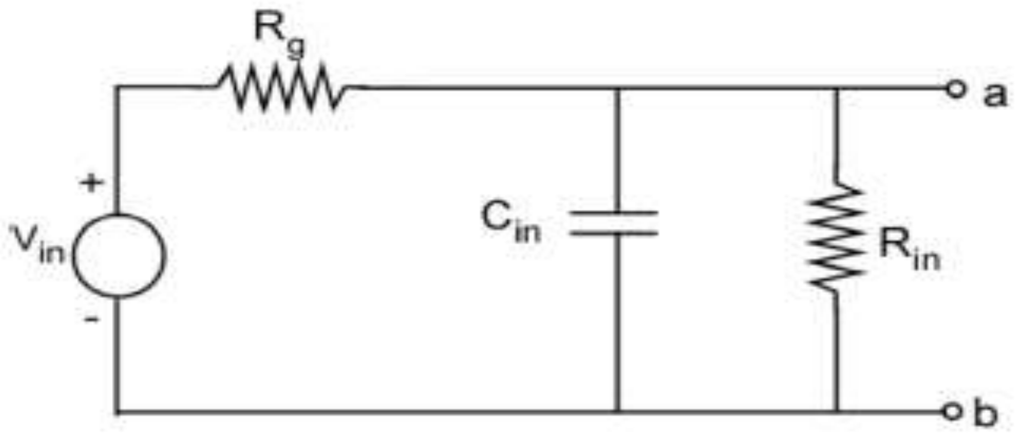
R_{out} = 3 kΩ, R_g=200 Ω, R_{in}=12 kΩ, R_L=10 kΩ
C_{c1}=5 μF and C_{c2}=1 μF

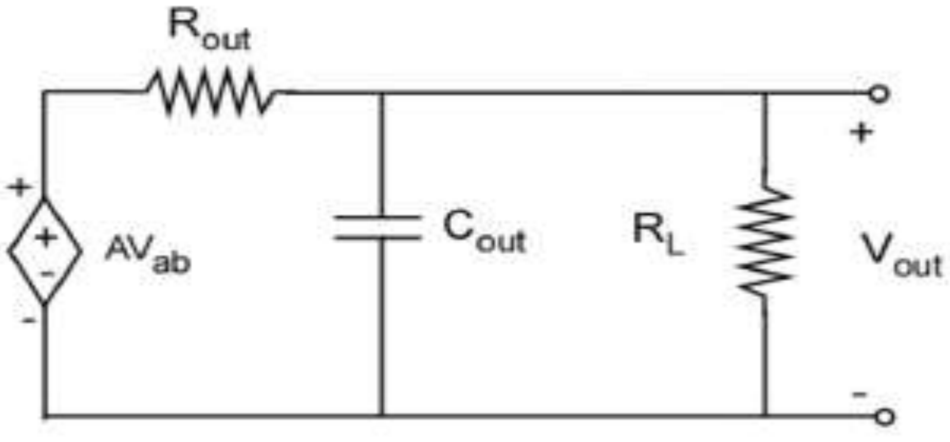
$$f_{l1} = \frac{1}{2\pi(12,200 \times 5 \times 10^{-6})} = 2.61\,Hz$$

$$f_{l2} = \frac{1}{2\pi(13,000 \times 10^{-6})} = 12.2\,Hz$$

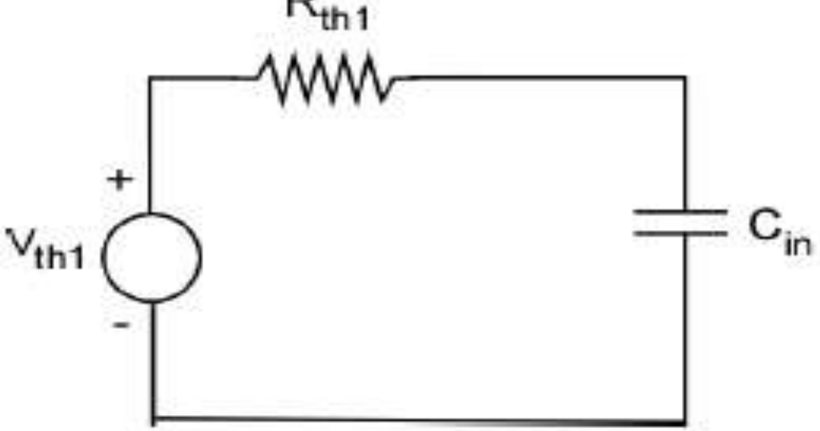
High Frequency Model

- Assume coupling capacitors are short
- Account for parasitic capacitors





Potential Thevenin equivalent for input as seen by C_{in}



$$V_{th1} = \frac{v_{in} R_{in}}{R_g + R_{in}}$$

$$R_{th1} = R_g \parallel R_{in}$$

High Frequency Model

$$v_{ab} = \frac{v_{in}R_{in}}{R_g + R_{in}} \cdot \frac{1}{1 + j\omega C_{in}R_{th1}}$$

$$v_{ab} = \frac{v_{in}R_{in}}{R_g + R_{in}} \cdot \frac{1}{1 + jf / f_{h1}} \quad where \quad f_{h1} = \frac{1}{2\pi R_{th1}C_{in}}$$

$$Likewise \; v_{out} = \frac{Av_{ab}R_L}{R_{out} + R_L} \cdot \frac{1}{1 + j\omega C_{out}R_{th2}}$$

$$with \quad R_{th2} = R_{out} \,||\, R_L$$

$$v_{out} = \frac{Av_{ab}R_L}{R_L + R_{out}} \cdot \frac{1}{1 + jf / f_{h2}} \quad where \quad f_{h2} = \frac{1}{2\pi R_{th2}C_{out}}$$

High Frequency

Overall gain is:

$$\frac{v_o}{v_i} = A \cdot \frac{R_{in}}{R_{in} + R_g} \cdot \frac{R_L}{R_L + R_{out}} \cdot \frac{1}{1 + jf / f_{h1}} \cdot \frac{1}{1 + jf / f_{h2}}$$

or

$$\frac{v_o}{v_i} = A_{MB} \cdot \frac{1}{1 + jf / f_{h1}} \cdot \frac{1}{1 + jf / f_{h2}}$$

Important Remarks

– An arbitrary network's transfer function can be described in terms of its s-domain representation

– s is a complex number $s = \sigma + j\omega$

– The impedance (or admittance) or networks can be described in the s domain

Transfer Function Representation

$$T(s) = \frac{a_m s^m + a_{m-1} s^{m-1} + \ldots + a_1 s + a_0}{s^n + b_{n-1} s^{n-1} + \ldots + b_1 s + b_0}$$

The coefficients *a* and *b* are real and the order *m* of the numerator is smaller than or equal to the order *n* of the denominator

A stable system is one that does not generate signal on its own.

For a stable network, the roots of the denominator should have negative real parts

Transfer Function Representation

In general, the transfer function of an amplifier can be expressed as

$$F_H(s) = a_m \frac{(s - Z_1)(s - Z_2)\ldots(s - Z_m)}{(s - P_1)(s - P_2)\ldots(s - P_m)}$$

$Z_1, Z_2, \ldots Z_m$ are the **zeros** of the transfer function

$P_1, P_2, \ldots P_m$ are the **poles** of the transfer function

s is a complex number $s = \sigma + j\omega$

3dB Frequency Determination

$$A(s) \equiv A_M F_H(s)$$

- Designer is interested in midband operation
- However needs to know upper 3-dB frequency
- In many cases some conditions are met:
  - Zeros are infinity or at very high frequencies
  - One of the poles (ω_{P1}) is at much lower frequency than other poles (➔dominant pole)
- If the conditions are met then $F_H(s)$ can be approximated by:

$$F_H(s) \equiv \frac{1}{1 + s / \omega_{P1}} \quad \text{and we have} \quad \omega_H \cong \omega_{P1}$$

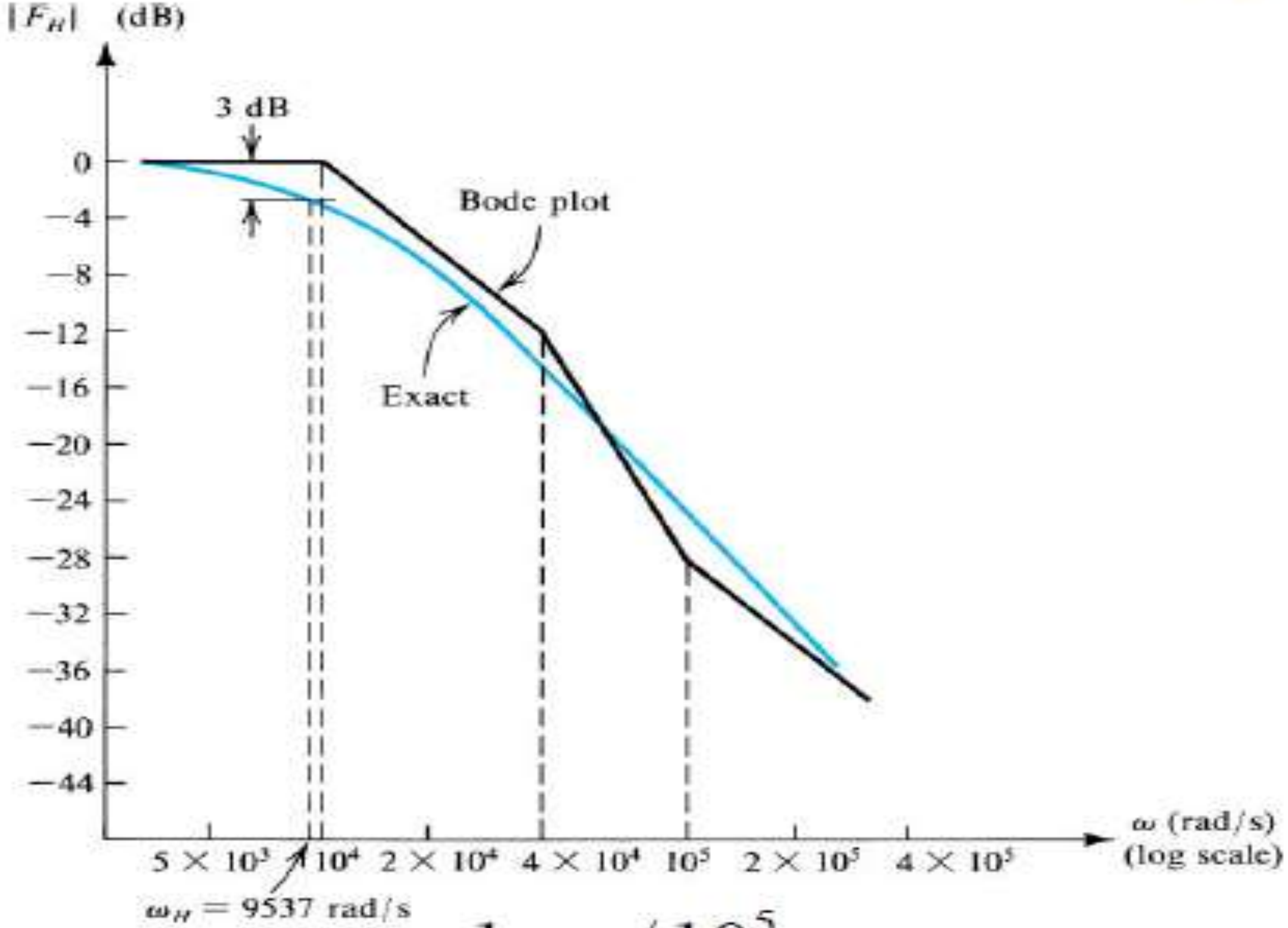
3dB Frequency Determination

If the lowest frequency pole is at least 4 times away from the nearest pole or zero, it is a **dominant pole**

If there is no dominant pole, the 3-dB frequency ω_H can be approximated by:

$$\omega_H \cong 1/\sqrt{\left(\frac{1}{\omega_{P1}^2}+\frac{1}{\omega_{P2}^2}+\ldots\right)-2\left(\frac{1}{\omega_{Z1}^2}+\frac{1}{\omega_{Z2}^2}+\ldots\right)}$$

High-Frequency Behavior - Example



$$F_H(s) \equiv \frac{1 - s/10^5}{\left(1 + s/10^4\right)\left(1 + s/4\times10^4\right)}$$

Open-Circuit Time Constants

$$F_H(s) = \frac{1 + a_1 s + a_2 s^2 + \ldots + a_n s^n}{1 + b_1 s + b_2 s^2 + \ldots b_n s^n}$$

The coefficients *a* and *b* are related to the frequencies of the zeros and poles respectively.

$$b_1 = \frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}} + \ldots + \frac{1}{\omega_{pn}}$$

b_1 can be obtained by summing the individual time constants of the circuit using the *open-circuit time constant method*

Open-Circuit Time Constant Method

- The time constant of each capacitor in the circuit is evaluated. It is the product of the capacitance and the resistance seen across its terminals with:
  - All other internal capacitors open circuited
  - All independent voltage sources short circuited
  - All independent current sources opened

- The value of b_1 is computed by summing the individual time constants

$$b_1 = \sum_{i=1}^{n} C_i R_{io}$$

Open-Circuit Time Constant Method

- **An approximation can be made by using the value of b_1 to determine the 3dB upper frequency point ω_H**
- **If the zeros are not dominant and if one of the poles P1 is dominant, then**

$$b_1 \simeq \frac{1}{\omega_{P1}}$$

Assuming that the 3-dB frequency will be approximately equal to ω_{P1}

$$\omega_H \approx \frac{1}{b_1} = \frac{1}{\sum_i C_i R_{io}}$$