CHAPTER - 3

Number system

BODMAS - order of simplification of an expression of numbers.

B = Bracket
O = of
D = Division
M = Multiplication
A = Addition
S = Subtraction.

Q1. What is the unit digit of the product of 207 · 781 · 39 · 94?

Ans:

$207 \times 781 \times 39 \times 94$

7 $7 \times 9 = 63$ $3 \times 4 = 12 = 2$ Ans.

(c) 2.

Q2. What will come in the place of unit digit in the value of $(7^{35}) \cdot (3^{71}) \cdot (11^{55})$?

Ans:

$7^{35} \times 3^{71} \times 11^{55}$ cycle = 4

Divid power by cycle 4 always.

$7^{3} \times 3^{3} \times 11^{3}$ (remainder $\rightarrow 3$)

$4\overline{)35}$
32
3

$= 343 \times 27 \times 1331$

$= 21$ $1 \times 1 = 1$

(c) 1

Q3. Find the numbers of zeros at the end of the product of $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times \cdots \times 99 \times 100$

Ans:

$1 \times 2 \times 3 - - - 100 = 100!$

$$\frac{100!}{5} = \frac{20!}{5} = \frac{4!}{5} \Rightarrow \cancel{5 \times 4} = \cancel{20}$$

$$\begin{array}{r} 20 \\ +\ 4 \\ \hline 24 \end{array}$$

b) 24

Q4. Find the number of zeros at the end of the product of $2 \times 4 \times 6 \times 8 \times 10 \times - - \times 98 \times 100$.

Ans: $2 \times 4 \times 6 \times 8 \times 10 - - \times 100$

$2^{50}[1 \times 2 \times 3 - - \times 50]$

$\rightarrow \cancel{5 \times 2} = \cancel{10}$

$$\frac{50!}{5} = \frac{10!}{5} = \frac{2!}{5}$$

$$\begin{array}{r} 10 \\ +2 \\ \hline 12 \end{array}$$

c) 12

Q5. Find the number of zeros at the end of the product of $10 \times 20 \times 30 \times - - \times 2000$:

Ans $10 \times 20 \times 30 \times - - \times 2000$

$10^{200}[1 \times 2 \times 3 \times - - \times 200]$

$$\frac{200!}{5} = \frac{40!}{5} = \frac{8!}{5} = \text{quotient} = 1$$

$$\begin{array}{r} 200 \\ 40 \\ 8 \\ +\ 1 \\ \hline 249 \end{array}$$

b) 249

Q6. Find the no. of factor of 100.

Ans:

$$\begin{array}{r|l} & 100 \\ 2 & 50 \\ 2 & 25 \\ 5 & 5 \end{array}$$

$100 = 2^2 \times 5^2$

$N = A^p \times B^q \times C^r \ldots$

No. of factor $= (p+1)(q+1)(r+1)$

$= (2+1)(2+1)$

$= 3 \times 3 = 9$

b) 9

Q7. Find the no. of factors of 80!

Ans.

$$\begin{array}{r|l} 2 & 80 \\ 2 & 40 \\ 2 & 20 \\ 2 & 10 \\ 5 & 5 \\ & 1 \end{array}$$

$80 = 2^4 \times 5^1$

$p = 4$ & $q = 1$

$N = A^p \times B^q \times C^r$

So, No. of factors $= (p+1)(q+1)(r+1)$

$= (4+1)(1+1)$

$= 5 \times 2 = 10.$

Q8. Find the sum of factor of 100.

Ans: $100 = 2^2 \times 5^2 \ldots$

$N = A^p \times B^q \times C^r \ldots$

$$\text{SUM} = \frac{A^{p+1}-1}{A-1} \times \frac{B^{q+1}-1}{B-1} \times \frac{C^{r+1}-1}{C-1}$$

$$= \frac{2^{2+1}-1}{2-1} \times \frac{5^{2+1}-1}{5-1}$$

$$= \frac{8-1}{2-1} \times \frac{125-1}{5-1}$$

$$= \frac{7}{1} \times \frac{124}{4}$$

$= 7 \times 31 = 217$

b) 217

Q9. Find the sum of the factors of 50:

Ans

$$2\,|\,50$$
$$5\,|\,25$$
$$5$$

$50 = 2^1 \times 5^2$

$N = A^p \times B^q \times C^r \ldots$

$$\text{sum} = \frac{A^{p+1}-1}{A-1} \times \frac{B^{q+1}-1}{B-1} \times \frac{C^{r+1}-1}{C-1}$$

$$= \frac{2^{1+1}-1}{2-1} \times \frac{5^{2+1}-1}{5-1}$$

$$= 3 \times \frac{124}{4}\ 31$$

$$= 93$$

b) 93

Q10 Find the average of factors of 60:

Ans:

$$2\,|\,60$$
$$2\,|\,30$$
$$3\,|\,15$$
$$5\,|\,5$$
$$1$$

$60 = 2^2 \times 5^1 \times 3^1$

$$\text{No. of factor} = (2+1)(1+1)(1+1)$$
$$= 3 \times 2 \times 2$$
$$= 12$$

$$\text{SUM} = \frac{2^{2+1}-1}{2-1} \times \frac{5^{1+1}-1}{5-1} \times \frac{3^{1+1}-1}{3-1}$$

$$= \frac{7}{1} \times \frac{24}{4} \times 8/2$$

$$= 168$$

$$\text{Avg} = \frac{\text{Total sum}}{\text{total num factor}} = \frac{168}{12} = 14$$

c) 14

Q11: Find the product of factors of 100:

AN: $100 = 2^2 \times 5^2$

product $= (N)^{\frac{\text{No. of factor}}{2}}$

$\therefore$ No. of factors $= (2+1)(2+1)$

$= 3 \times 3 = 9$

so, product $= (100)^{9/2}$

$= (10)^{2 \times 9/2}$

$= (10)^9$.

a) 10^9

Q12. How many 3 digit numbers are completely divisible by 6?

AN:

1   99, 100 ------- 999

$\underbrace{\quad\quad}_{16}$

$6 \overline{)999}(166$ (quotient)

6
39
36
39
36
3

$\therefore$ total $= 166$ (two digit & three digit)

$\therefore$

$6)\overline{99}(16$ $\therefore$ two digit number

6
39
36
3

$\therefore$ 3 digit $= 166 - 16 = 150$

b) 150

Q13 How many 3 digit no. are completely divisible by 3 & 4.

Ans: take LCM of 3 & 4 = 12

1 ——— 99 ——————— 999

12) 999 (83
96
39
36
3

12) 99 (8
96
3

total digit number = 83
total 2 digit = 8
∴ total only 3 digit = 83 − 8
= 75

b) 75.

Q14. what will be the remainder when 17^{200} is divided by 18 ?

Ans: $\frac{17^{200}}{18}$ $\frac{x^n - a^n}{x+a} = R = 0$ if n = even

c) 1

Q14A. Find the remainder when 3^{85} divided by 6.

Q14.B Find the remainder when 2^{70} is divided by 96.

Ans:

Q15. What will be the remainder when $(67^{67}+67)$ is divided by 68?

Ans: $\frac{67^{67}+67}{68}$

$$\frac{x^n+a^n}{x+a} = R = 0 \text{ if } n = \text{odd}$$

$= \frac{67^{67}+1-1+67}{67+1 \quad 67+1}$

$= 0 + \frac{66}{68}$

$= 0+66$

$= 66$

$\Rightarrow 67^{67}+67/68$

$\Rightarrow 67^{67}+1^{67}+66/67+1$

$\Rightarrow$ 66 remainder

b) 66

Q16. Which of the following is no. will completely divide $(49^{15}-1)$?

Ans: $(7^2)^{15}-1$

$= 7^{30}-1$

$= 7^{30}-1^{30}$

$= 7+1 = 8$

$$\frac{x^n-a^n}{x+a} = R = 0 \text{ if } n = \text{even}$$

a) = 8

Q17. A number when divided by 6 leaves a remainder of 3. When the square of the no. is divided by 6, the remainder is,

Ans: Let a number = N

$N \rightarrow 6 \rightarrow (3)^2 \rightarrow 9$

$6)9(1$
$\underline{-6}$
$3 = R$

(d) 3

Q18. A no when divided successively by 4 & 5 leaves remainders 1 & 4 resp. when it is successively divided by 5 & 4, then the respective remainders will be,

Ans:

$N = 37$

4	N		
5	9	→ 1 (R)	= 37
	1	→ 4 (R)	= 1

$= 5 \times 1 + 4 = 9$

Number = quotient × divisor + Remainder.

5	37	
4	7	→ 2
	1	→ 3

b) 2, 3

Q19 A number was divided successively in order by 4, 5 and 6. Then remainders were resp. 2, 3, & 4. The number is.

AN:

4	N		= 214
5	A = 53	→ 2	= 53×4 + 2 = 214
6	B = 10	→ 3	
	1	→	

The number is 214.

a) 214.

Q20. Which one of the following no. will completely divide $(4^{61} + 4^{62} + 4^{63} + 4^{64})$?

AN:
$4^{61} + 4^{62} + 4^{63} + 4^{64}$
$= 4^{61}[1 + 4 + 4^2 + 4^3]$
$= 4^{61}[\times 85)$
$= 4^{61} \times 17 \times 5$
$= 17$ (divisible)

d) 17.

Q21 Which one of the following no. will completely divide $5^{51} + 5^{52} + 5^{53}$?

Ans:
$5^{51} + 5^{52} + 5^{53}$
$= 5^{51}(1 + 5^1 + 5^2)$
$= 5^{51}(31)$

c) 31

Q22 which of the following is the common factor of $(47^{43} + 43^{43})$ and $(47^{47} + 43^{47})$?

Ans: convert this into this form →

$\frac{x^n + a^n}{(x+a)} = R = 0$ when, $n = $ odd

$47^{47} + 43^{47}$

$= \frac{47^{47} + 43^{47}}{47 + 43}$

& same for another

b) $47 + 43$ Ans

Q23 which one of the following num is completely divisible by 99?

Ans: go through option.

$11 \times 9 = 99$

Check on divided by 11.

a) 3573
$10 - 7 = 3$

b) 13595
$11 - 12 = -1$
(0/11) ×

c) 913464
$18 - 9 = 9$
(0/11) ×

d) 114345

$9 - 9 = 0$.

$\cancel{0/11}$ NV

$(11/11) = 0$ - R.

d) 114345 Ans

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Q24. Which one of the following no. is completely divisible by 45?

Ans $45 = 5 \times 9 = 45$

[$\because 9 \to$ digital sum $\frac{}{9}$]

(a) 181560
sum = $\frac{21}{9}$ ✗

(c) 202860 Ans

Q25. The sum of digits of two-digit no. is 7. If digits of no. are interchanged, then no. so formed is greater than original no. by 27. Find original number.

Ans: $x+y=7$ — ①   original $= 10x+y$
interchange $10y+x$

$\therefore 10y+x-(10x+y)=27$
$\therefore 9y-9x=27$
$y-x=3$ — ②.

$y+\cancel{x}=7$
$y-\cancel{x}=3$
$2y=10$
$y=5$ & $x=2$

Number $= xy = 25$

(b) 25

Q26. What is the digit in the blank space of no. 34*7 so that the no. is divisible by 7?

Ans: 34 $\times$ 7 = 7+4 = 11 & 3+8 = 11.
So, blank space 11.

d) 8.

Q27. If the sum of the digits of two-digits no. & the no. formed by reversing its digit is 99, what is the sum of digits of original number?

Ans: Two digit no. $10x + y$ original
$10y + x$ reversing

then,

$10x + y + (10y + x) = 99$

~~$2x = 99 - 11y$~~

~~$2x + 11y = 99$~~

$11x + 11y = 99$

$x + y = 9$

a) 9

Q28 If the sum of the digits of a two-digits no. & the no. formed by reversing its digits is N, which one of the following no. will completely divide by N?

Ans: Let $10x + y$ two digits no. &
$10y + x$ reversing

$= 10x + y + 10y + x =$
$= 11x + 11y$
$= 11(x + y)$

c) 11.

Q29 If the diff. betn. a two-digits no. & a no. formed by reversing its digit is N, which one of the d) following no. will completely divide N.

or, $10x + y - (10y + x) =$

or, $10x + y - 10y - x$

$= 9x - 9y$

$= 9(x - y)$

d) 9

Q30. if the diff beth two digit no. and no. formed by reversing its digits is 45, what is diff beth digits of original no?

Ans:

or, $10x + y - 10y - x = 45$

or, $9x - 9y = 45$

$x - y = 5$

Q31. A 4 digit no. is formed by repeating a 2-digit no. such as 2525, 3232, etc. Any no. of this form is always divisible by

Ans: $x\ y\ x\ y \longrightarrow 1000x + 100y + 10x + y$

y, $10x$, $100y$, $1000x$

$= 1010x + 101y$

$= 101(10x + y)$

if, Note,

$xyxy \rightarrow 101$ (it is Multiple of 101)

$xyzxyz \rightarrow 1001$

$xyzwxyzw = 10001$

d) smallest three-digit prime number.

$\frac{x^n - a^n}{x+a}$ ++ → odd even — + → odd/even | Natural Number

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Q32. $7^{12} - 4^{12}$ is exactly divisible by which of the following?

Ans: divisible means Remainder should be zero.

power is even :- $\frac{x^n - a^n}{x-a}$ = Natural no. , n is odd/even

$\frac{x^n - a^n}{x+a}$ = when n = even $\boxed{R=0}$

$= 7^{12} - 4^{12}$

$= (7^2)^6 - (4^2)^6$

$= 49^6 - 16^6$

$= \frac{49^6 - 16^6}{49-16}$

$= 49 - 16 = 33$

d) 33

Q33. Find the sum of first fifty natural numbers.

Ans: $1 + 2 + \cdots 50 = \frac{n(n+1)}{2}$

where $n = 50$.

$= \frac{50(50+1)}{2} = \frac{\cancel{50}^{25} \times 51}{\cancel{2}} = 1275$

b) 1275

Q34 Find the value of 51, 52, 53, 54 + ... + 100:

Ans: $\underbrace{1 + \cdots 50 + 51 + 52 + \cdots + 100}_{n=100} - \underbrace{[1 + 2 + \cdots 50]}_{n=50}$

so, sum of N natural no = $\frac{n(n+1)}{2}$

$\Rightarrow \frac{n(n+1)}{2} - \frac{n(n+1)}{2}$

$\Rightarrow \frac{100(100+1)}{2} - \frac{50(50+1)}{2}$

$\Rightarrow 3775$

d) 3775

Q25: Find the sum of squares of first 30 natural number.

Ans: $1^2 + 2^2 + \dots + 30^2 = \frac{n(n+1)(2n+1)}{6}$

$n = 30$

$\Rightarrow \frac{30(30+1)(2\times30+1)}{6}$

$\Rightarrow 9455$

a) 9455.

Q26. Find the value of $2^2 + 4^2 + 6^2 + 8^2 + \dots + 20^2$.

AN: $2^2 [1^2 + 2^2 + 3^2 \dots + 10^2]$

$4 [\dots]$ $n = 10$

sum of square $= \frac{n(n+1)(2n+1)}{6}$

$\Rightarrow \frac{10(10+1)(2\times10+1)}{6} \Rightarrow 1540.$

c) 1540.

Q37. Find the value of $1^2+3^2+5^2+\cdots+19^2$.

Ans: $1^2+3^2+\cdots 19^2 = \frac{n(2n+1)(2n-1)}{3}$

so $n=10$. = 1, 3, 5, 7, 9, 11, 13, 15, 17, 19

$1^2+3^2+\cdots 19^2 \Rightarrow \frac{20}{2} = n = 10$ #

$\Rightarrow \frac{10(2\times10+1)(2\times10-1)}{3} = 1330$.

b) 1330.

Q38 If $1^2+2^2+3^2+4^2+\cdots+10^2 = 385$, find value of $2^2+4^2+6^2+\cdots+20^2$.

Ans: $2^2+4^2+6^2+\cdots+20^2$

$= 2^2[1^2+2^2+3^2+\cdots 10^2]$

$= 4\times(385)$

$= 1540$.

b) 1540.

Q39 Find the value of $11^2+12^2+13^2+14^2+\cdots+20^2$

Ans: $11^2+12^2\cdots+20^2$

$\underbrace{[1^2+2^2\cdots 10^2]+11^2+12^2+\cdots+20^2}_{n=20} - \underbrace{[1^2+2^2\cdots 10^2]}_{n=10}$

$\Rightarrow \frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)(2n+1)}{6}$

$\Rightarrow 2485$

Q40. Find the value of $1^3+3^3+5^3+\cdots+29^3$

AN: $1^3+3^3+5^3+\cdots+29^3$ $\frac{30}{2}=15=n$

Formula $=n^2(2n^2-1)$

$=15^2(2\times15^2-1)$

$=101025$

b) 101025

Q41. If $1^3+2^3+3^3+4^3+\cdots+10^3=3025$, Find value of $2^3+4^3+6^3+\cdots+20^3$.

AN: $2^3[1^3+2^3+3^3+\cdots10^3]$

$=2^3[3025]$

$=24200.$

b) 24200.

Q42. Find the sum of all even no. up to 100:

AN: $2+4+6+\cdots100=n(n+1)$

$n=50.$

$=50(50+1)$

$=50\times51$

$=2550.$

a) 2550.

Q43. Find the sum of all odd no. up to 100.

AN: $1+3+\cdots100=n^2=50^2=2500.$

$n=$ no. of odd Number $=100/2=50$

b) 2500.

Q44. Find the no. of prime factor of $6^{20} \cdot 11^{11}$,

Ans: $6^{20} \times 11^{11} \times 21^{21}$

$= (2\times3)^{20} \times 11^{11} \times (7\times3)^{21}$

$= 2^{20}\times3^{20} \times 11^{11} \times 7^{21}\times3^{21}$

= Add all power,

$\Rightarrow$ 93

b) 93.

Q45. Find the no. of prime factor of $14^{14} \cdot 15^{15}$

$\Rightarrow$ $14^{14} \cdot 15^{15}$

$\Rightarrow 2^{14}\times 7^{14} \times 3^{15} \times 5^{15}$

$= 14+14+15+15$

$= 58.$

b) 58.

Q46. What will be remainder when $(27^{27} + 17^{27})$ is divided by 11?

Ans: $\dfrac{27^{27}+17^{27}}{27+17} = R = 0.$ $\left(\dfrac{x^n + a^n}{x+a} = R = 0 \text{ when } n = \text{odd} \right)$

$\Rightarrow 27+17 = 44$

$\Rightarrow 4\times11$ (divided by 4 & 11) so, R = 0.

c) 0.

Q47. If n is a natural no. (n^3-n) will always be divisible by

Ans: $= n^3 - n$

$= n(n^2-1)$ let $n=1$

$= 1(1^2-1)$

$= 1\times0 = 0.$

Let $n = 2$.

$\Rightarrow 2(4+1) \Rightarrow 2\times5 \Rightarrow 6$. If any no. divide by 6. & ~~also~~ ~~divide by 12~~

a) ~~6 only & 12 both~~

a) 6 only

Q48. $(x^n - a^n)$ is completely divisible by $(x-a)$, when.

Ans: $\dfrac{\cancel{x^n - a^n}}{x+a}$ $\boxed{\cancel{R=0}}$ ~~when, n = Even no.~~

~~b) n is any even natural no.~~

or, $\dfrac{x^n - a^n}{x-a} = \boxed{R=0}$ if, $n =$ natural no.

a) n is any natural number.

Q49. $(x^n - a^n)$ is completely divisible by $(x+a)$, when.

Ans: $\dfrac{x^n - a^n}{x+a} = \boxed{R=0}$, when, $n =$ Even no.

b) n is an Even natural Number

Q50. $(x^n + a^n)$ is completely divisible by $(x+a)$, when.

Ans: $\dfrac{x^n + a^n}{x+a} \rightarrow R = 0$, when, $n =$ odd.

c) n is odd natural number

Q51. Which of the following is prime no.?

Ans: a) 1601 b) 221 → 225

take square of nearest of 1601.

⇒ 169

13^2

11 → 7, 5, 3, 2 → check 1601 % 7, 5, 3, 2 ...

X

X

15^2

√ 13 → 11, 7, 5, 13, ...

c) 373 ✓

Q52. Which one of following is a prime no?

Ans:

d) 71

Q53. Find largest 4-digit no. which is divisible by 88?

Ans: 88 = 11 × 8

check divisibility of 8 → last three digit should divisible by 8.

a) 9944.

Q54. If no is divided by 111, the remainder is 31, what will be remainder if it is divide by 37?

Ans: N → 111 → 31, N → 37 31 ÷ 37, R = 31

a) 31.

Q55. On multiplying a no. by 7, the product is a no. made of only the digit 3. The smallest such no is :-

Ans: a) 47619

$$\begin{array}{r} 47619 \\ \times 7 \\ \hline 333333 \end{array}$$

a) 47619