CHAPTER-2

HCF AND LCM

HCF : Highest common factor
HCD : Highest common division
GCD : Greatest common division
LCM : Least common multiply.

∴ HCF = max, greatest, divided by.
∴ LCM = least, smallest, divisible by, together.

1) traffic (together) = LCM
2) Tumbling Ball : LCM - If some ball tumble after every 2, 4, 6, 8, 10, 12, sec. In 30 min how many times they tumble together.

2	2, 4, 6, 8, 10, 12
2	1, 2, 3, 4, 5, 6
2	1, 1, 3, 2, 5, 3
3	1, 1, 3, 1, 5, 3
5	1, 1, 1, 1, 5, 1
	1, 1, 1, 1, 1, 1

LCM = 2 × 2 × 2 × 3 × 5
= 120 sec.

No. of times = $\frac{30}{2}$ = 15 + 1 = 16 times
↓
add + 1 extra

Type-1

Q. Find largest no which can divides x, y, z, by leaving a, b, c remainder respectively.

⇒ $\frac{x}{N} = a$, $\frac{y}{N} = b$, $\frac{z}{N} = c$

N = hcf of [(x-a), (y-b), (z-c)]

Type-2

q. Find the largest no which can divide x, y, z by leaving same remainder in each case.

→ Let N, q ∈ I

$\frac{x}{N} = a$, $\frac{y}{N} = a$, $\frac{z}{N} = a$

N = HCF of [|x-y|, |y-z|, |z-x|]

Type-3

q. Find the smallest no. which is divisible by x, y, z leaving a, b, c as a remainder respectively.

→ Let N,

$\frac{N}{x} = a$, $\frac{N}{y} = b$, $\frac{N}{z} = c$

∴ k = x-a, y-b, z-c ∴ [k value same, if diff. then k = 0].

∴ N = LCM of (x, y, z) - k

Type-4

q. Find smallest no. which is divisible by x, y, z, by leaving same remainder in each case.

→ Let = N

$\frac{N}{x} = r$, $\frac{N}{y} = r$, $\frac{N}{z} = r$

∴ N = LCM of (x, y, z) + r

Type-5

$N_1 \times N_2 = HCF \times LCM$

$N_1 \times N_2 \times N_3 = HCF \times LCM$
(for 3 nos.)

HCF AND LCM

Q1 Find the HCF of 42, 63 and 140.

⇒

42	63	140
2 | 42	3 | 63	2 | 140
3 | 21	3 | 21	2 | 70
7 | 7	7 | 7	5 | 35
1	1	7 | 7
		1

$42 = 2 \times 3 \times 7$

$63 = 3 \times 3 \times 7$

$140 = 2 \times 2 \times 5 \times 7$

Ans: 7

Q2. Find the HCF of $a^2b^4c^6$, $b^3c^8a^4$ and $a^8b^6c^2$.

→

$= a^2b^4c^6, b^3c^8a^4, a^8b^6c^2$

$= a^2b^3c^2$ (common)

Ans) $a^2b^3c^2$

Q3. Find the HCF of 0.63, 1.05 and 2.1.

→ $0.63 = \frac{63}{100}$

63	105	210
3 | 63	5 | 105	2 | 210
3 | 21	3 | 21	5 | 105
7 | 7	7 | 7	3 | 21
1	1	7 | 7
		1

$1.05 = \frac{105}{100}$

$2.10 = \frac{210}{100}$

$63 = 3 \times 3 \times 7$

$100 = 5 \times 5 \times 2 \times 2$

$210 = 2 \times 5 \times 3 \times 7$

$\therefore HCF = 3 \times 7 = \frac{21}{100} = 0.21$

a) 0.21

Q4. Find the H.C.F of $2^2 3^2 5^5$, $2^3 3^2 5^2 7$ and $2^4 3^4 7^2 11$.

Ans. From this taking a common.

So, $H.C.F = 2^2 3^2 5$

a) $2^2 3^2 5$

Q5. Find the HCF of $\frac{2}{3}, \frac{8}{9}$ and $\frac{10}{27}, \frac{64}{81}$

Ans: In this, $HCF = \frac{HCF(2, 8, 64, 10)}{LCM(3, 9, 27, 81)}$

$= \frac{HCF(2)}{81}$

$= \frac{2}{81}$

b) $\frac{2}{81}$

Q6. Find the maximum number of students among who 1001 pens and 910 pencils can be distributed in such a way that each student gets same number of pens and same number of

Find HCF

pencils.

Ans: $1001 \div 910$

$910 \overline{)1001}$ (1
910
91 | 910
91
0

1) 91

Q7. Find the greatest (H.C.F) possible length of a scale that can used to measure exactly the following length of cloth 3m; 5m 10cm; and 12m 90 cm.

Ans: 3m = 300cm,
5m = 500cm
10cm = 10cm
12m = 1200cm
90cm = 90cm. → 1290cm

∴ finding the H.C.F of all this numbers,

10	300	510	1290
3	30	51	129
	10,	17,	63

HCF = 10 × 3 = 30.

1) 30.

Q8. Find the greatest possible length of scale to measure exactly following lengths 20 feet 9 inches; 17 feet 6 inches; and 21 feet 3 inch. and 13 feet 9 inch

Ans 20 feet = 20 × 12 = 240 inch
13 × 12 + 9 = 165
17 × 12 + 6 = 210
21 × 12 + 3 = 255

Find HCF.

5 | 165, 210, 255
3 | 33, 42, 51
11, 14, 17

HCF = 5 × 3 = 15 inch.

b) 1 feet 3 inch (15 inch)

Q9 Three containers have mixture of milk & water 403 liters, 713 liter and 496 liters resp. Find greatest measurement which can measure the mixture.

Ans: 403, 713, 496

403) 713 (1
403
310) 403 (3
310
93) 310 (3
279
31

31) 496 (16
31
186
186
xxx

c) 31 liter.

10. Traffic light at three different points are changing respectively at 24, 48 and 72 second. If all three are changed together at 7:10:24 hours, when will next change at three place together?

Ans. Traffic light : L.C.M. : together

2 | 24, 48, 72
2 | 12, 24, 36
2 | 6, 12, 18
3 | 3, 6, 9
1, 2, 3

LCM = 2 × 2 × 2 × 3 × 1 × 2 × 3 = 144

LCM = 144 ⇒ 2 min 24 sec.

9:10:24
2:24
9:12:48

c) 9:12:48 hours.

Q11. A, B, and C start at same time in same direction to run around a circular stadium. A complete one round is 252 sec, B is 308 sec & C is 198 sec....

→

198 252 308 = LCM

LCM = 2772 sec.

60) 2772 (
x
= 46 m 12 sec.

d) 46 sec 12 min

Q12. A, B, and C start at the same time in same direction to run around a circular stadium of length 12 km & their speeds are 3 km/h 4 km/h and 6 km/h resp. After what time will they meet again at starting point?

Ans

	Time A	B	C
12 Time →	$\frac{12}{3}$ = 4h	$\frac{12}{4}$ = 3h	$\frac{12}{6}$ = 2h

LCM of 4, 3 & 2 hour

LCM = 4 × 3 = 12 h.

b) 12h

Q13. The smallest number is exactly divisible by 2, 4, 3, 5, 6, 8 & 10 when 7 is subtracted from the number what is the number?

Ans: LCM of 2, 4, 3, 5, 6, 8, 10 is.

3	3, 6, 8, 10
2	1, 2, 8, 10
4	1, 1, 4, 5
5	1, 1, 1, 5
	1, 1, 1, 1

LCM = 3 × 2 × 4 × 5 = 120

So, AN is 120 + 7 (subtracted)

= 127

c) 127

Q14. The smallest no 'to which is 8 added, is exactly divisible by, 10, 12, 15 and 20.

Ans Number = LCM of (10, 12, 15, 20) − 8

2	10, 12, 15, 20
5	5, 6, 15, 10
3	1, 6, 3, 2
2	1, 2, 1, 2
	1, 1, 1, 1

∴ LCM = 2 × 5 × 3 × 2

= 60.

Number = 60 − 8 = 52

c) 52

Q15. Which is the smallest no. that can be subtracted from 1936 so that one being divided by 9, 10, 15 the remainder is 7 every time?

Ans:

LCM of 9, 10, 15

9 | 9, 10, 15	2 | 9, 10, 15
10 | 1, 10, 15	3 | 9, 5, 15
15 | 1, 1, 15	3 | 3, 5, 15
1, 1, 1	5 | 1, 5, 5
	1, 1, 1

LCM = 2 × 3 × 3 × 5 = 90

```
90 ) 1936 (
    -180
    ----
     136
     -90
     ---
      46   (R)
     -39
     ---
       7
```

d) 39

Q16. The smallest no. that will be divisible by 4, 6, 8, 12 and is leaving a remainder 2 in each case is,

Ans: N = LCM of (4, 6, 8, 12, 16) + Remainder

48 + 2 = 50.

b) 50

Q17. Find the greatest number that will divide 187, 233, 279 so as to leave same remainder in each case?

Ans:

N = HCF of $[|x-y|, |y-z|, |z-x|]$

$= HCF\ of\ [|87-2331|, |233-2791|, |2791-1871|]$

$= HCF\ [46, 46, 92]$

$\Rightarrow 46$

c) 46.

Q18. The numbers 2272 and 875 divided by a three digit no. N, gives the same remainder. The sum of digit of N is -

Ans:

b) 10.

Q19. The numbers 1305, 4665 and 6905 are divided by four digit no. N & gives same remainder. The sum of digit of N is.

Ans: $N = HCF\ of\ [|x-y|, |y-z|, |z-x|]$

$= [3360, 2240, 5600]$

By division method

$2240 \overline{)3360} (1$
-2240
$1120 \overline{)2240}$
2240
0000

$1120 \overline{)5600} (5$
5600
0

$HCF = 1120 = 1+1+2+0 = 4$

a) 4

Q20. The greatest no. which divides 110 & 128 leaving same remainder 2 in each case, is,

Ans: HCF of $[a-y, y-z]$

HCF of $[|110-128|, |128-110|]$

HCF of $(18, 18)$ is 18.

S) b) 18

Q21. The greatest no. which can divide 122 & 243 & leave remainder 2 & 3 resp. is.

Ans: HCF. of $[(x-a)(y-b)]$

$= (122-2)(243-3)$

$= (120, 240)$

$\therefore$ HCF $= 120$.

d) 120

Q22. The least no, which when divided by 12, 15, & 16 leaves a 7, 10 & 11 as remainder resp. is,

Ans: $K = x-a, y-b,$ & $z-c$

$= 12-7, 15-10, 16-11$

$\therefore K = 5, 5, 5$

$N = LCM$ of $(x, y, z) - K$

$= (12, 15, 16) - K$

$= 240 - 5$

$= 235$

b) 235

Q23. The least number, which when divided by 5, 6, 7 and 8 leaves a remainder 3, but when it is divided by 9 leaves no remainder, is -

Ans: N = LCM of (5, 6, 7, 8) + r

5 | 5, 6, 7, 8	⇒ 840 + 3
2 | 1, 6, 7, 8	= 843
3 | 1, 3, 7, 4	
4 | 1, 1, 7, 4	
7 | 1, 1, 7, 1	
1, 1, 1, 1	LCM = 840

N = LCM of 3 + 3
= 840 * 2 + 3
= 1683 (if this no. not divide by 9 then find LCM of 3, ⇒ 840 * 3 + 3

b) 1683

Q24. The least number, which when divided by 20, 25, 30 & 40 leaves remainder 14, 19, 29 & 34 resp, is

Ans: K = x - a, y - b, z - c, w - d
= 20 - 14, 25 - 19, 30 - 29, 40 - 34
K = 6, 6, 6, 6.

LCM of (20, 25, 30, 40) - K
= 1400 - 6
= 1394

b) 1394

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Q25 Find a largest 5 digit no. exactly divisible by 12, 16, 18, 24 & 32.

Ans: Largest 5 digit no is 99999

Find LCM of 12, 16, 18, 24 & 32

2 | 18, 24, 32
3 | 9, 12, 16
3 | 3, 4, 16
2 | 1, 4, 16
2 | 1, 1, 4
2 | 1, 1, 1

LCM = 288

288) 99999 (
↓
863

N = 99999 − 63
= 99936

Q) 99936

Q26 Find the ~~her~~ smallest 5 digit no exactly divisible by 16, 24, 36 & 54.

Ans: LCM of 16, 24, 36, 54

2 | 16, 24, 36, 54
2 | 8, 12, 18, 27
3 | 4, 6, 9, 27
2 | 4, 2, 3, 9
2 | 2, 1, 3, 9
3 | 1, 1, 3, 9
3 | 1, 1, 1, 3

LCM = 432

$$
\begin{array}{r|l}
402 & 100000 \;(\;23 \\
 & 864 \\
\hline
 & 1360 \\
 & 1296 \\
\hline
 & 64
\end{array}
$$

add; $432 - 64 = 368$

b) 10368

Q27. Find largest four-digit no. which divided by 12, 18, 21 & 24 leaves remainder of 6 in each case.

Ans: LCM of 12, 18, 21, & 24 is 504

$$
\begin{array}{r|l}
504 & 9999 \;(\;19 \\
 & -504 \\
\hline
 & 4959 \\
 & -4536 \\
\hline
 & 423
\end{array}
$$

$\therefore 9999 - 423 = 9576$

$9576 + 6 = 9582$

a) 9582

Q28 The LCM of two no is 1296 & HCF is 96. If one of no is 864 then other is,

Ans: $N_1 \times N_2 = LCM \times HCF$

$864 \times N_2 = 1296 \times 96$

$N_2 = \frac{1296 \times 96}{864}$

$N_2 = 144$

a) 144

Q9 The HCF of two no. is 11 & LCM is 7700 If one of the num is 275, then other is,

Ans: $N_1 \times N_2 = HCF \times LCM$

$275 \times N_2 = 11 \times 7700$

$N_2 = 77 \times 4 =$

c) 308

Q10. The LCM of two no is 495 & their HCF is 5. If sum of no is 100, then their diff is,

Ans $N_1 \times N_2 = LCM \times HCF$

$N_1 \times N_2 = 495 \times 5$ —(I)

$N_1 + N_2 = 100$ —(II).

$N_1 \times N_2 = 495 \times 5$

$N_1 (100 - N_2) \times N_2 = 2475$

$N_1 = 45$

$N_2 = 55$ & diff $= 55 - 45 = 10$.

a) 10.

Q11 The product of LCM & HCF of two number is 24. The diff of two no is 2. Find no.

Ans: $N_1 \times N_2 = LCM \times HCF$

$N_1 \times N_2 = 24$

$N_1 + N_2 = 2$

b) 6 and 4.

Q32. The LCM of two no is 45 times of HCF. If one no is 125 & sum of HCF & LCM of two no is 1150, the other no is,

Ans: $N1 \times N2 = H \times L$

$N1 \times N2 = 25 \times 25 \times 45$

$N2 = \frac{25 \times 25 \times 45}{125}$

$\therefore N2 = 225$

$H + L = 1150$

$H + 45H = 1150$

$46H = 1150$

$H = 25$

c) 225.

Q33. product of two co-prime no is 117. Their LCM should be

Ans. we know that

∴ co-prime always have HCF = 1

$9 \times 4 = 12$

b) 117.

Q34. The L.C.M of three diff. no is 120. which of the following can't be their HCF?

Ans: LCM = H.C.F × coprime factor (C.F)

d) 35 (The no which no divide 120 completely)

Q35. The HCF of two no is 8. which one of following can never be their LCM?

Ans:

d) 60 (60 never completely divid. by 87.

Qs6. HCF of 3240, 3600 & third no, is 36 & their LCM is $2^4 \cdot 3^5 \cdot 5^2 \cdot 7^2$. The third no is-

Ans: $3240 = 2^3 \times 3^4 \times 5$

$3600 = 2^4 \times 3^2 \times 5^2$

$HCF = 2^2 \times 3^2$ $\quad$ $LCM = 2^4 \times 3^5 \times 5^2 \times 7^2$

product of Max power: $2^2 \times 3^5 \times 7^2$

Q) $2^2 \cdot 3^5 \cdot 7^2$

Q37. The ratio of two no is 4:5 & their HCF is 2. The LCM is.

Ans: $N_1 : N_2 = 4:5$

$4x - 5x \rightarrow HCF = 2 = (x)$

$x \quad x$

$8 \quad 10$

LCM → 2 | 8, 10

2 | 4, 5

2 | 2, 5

5 | 1, 5

1, 1

LCM = 40

Q) 40

Q38. The ratio of two no is 3:2 & their LCM is 72. Their HCF is

Ans $L.M = HCF \times C.F_1 \times C.F_2$

$72 = HCF \times 3 \times 2$

$H = 72/6$

$H = 12$

Q) 12

Qs9. The sum of two no.'s is 36 & their HCF is 4. How many no. of pairs may be possible-

AN: $N_1 + N_2 = 36$

$HCF \times CF_1 + HCF \times CF_2 = 36$

$4 \times CF_1 + 4 \times CF_2 = 36$

$CF_1 + CF_2 = 9$

$\therefore 1 + 8 = 9$

$2 + 7 = 9$

$3 + 6 = 9$ * (this is not co prime factor)

$4 + 5 = 9$

$\therefore$ Co-prime factor = Not common of HCF's

(3) 3

Qu0. A no. when divided by 10 leaves remainder 9, when divided by 9 leaves a remainder of 8, when divided by 8 leaves a remainder of 7 & so on. when divided by 2 leaves a remainder of 1. Find no.

AN:

(2) 2519 Ans